**Dynamic modeling of spike count data with Conway-Maxwell Poisson variability**

Ganchao Wei1\* and Ian H. Stevenson2-4

*1 Department of Statistics, University of Connecticut*

*2 Department of Psychological Sciences, University of Connecticut*

*3 Department of Biomedical Engineering, University of Connecticut*

*4 Connecticut Institute for Brain and Cognitive Science, University of Connecticut*

*\* Corresponding Author:* ganchao.wei@uconn.edu

# Abstract

# Introduction

Drift…

(Rokni et al. 2007)

(Chestek et al. 2007)

(Stevenson et al. 2011)

(Tomko and Crapper 1974)

Stability…

(Dickey et al. 2009)

(Steinmetz et al. 2021) [shows that correcting for electrode drift reduces some instability]

[some amount of drift is functional] adaptation/plasticity…

(Lesica et al. 2007)

Variability…

(Ghanbari et al. 2019)

(Stevenson 2016)

(Fenton and Muller 1998)

(Barbieri et al. 2001)

(Maimon and Assad 2009)

(Churchland et al. 2010)

(Churchland et al. 2011)

(Eden et al. 2004)

(Brown et al. 2001)

(DeWeese and Zador 1998)

# Methods

Here we consider a dynamic GLM with Conway-Maxwell Poisson (CMP) observations to describe time-varying spike counts. We first introduce the model. Because of the CMP observations, there’s no closed form estimation. Instead, we infer the model by global Gaussian approximation. Several issues, including challenges coming from CMP distribution and dynamic GLM, are discussed in details later.

## Dynamic Conway-Maxwell Poisson Model

The spike count for a neuron is assumed to follow the CMP distribution, with parameters and . The probability mass function (pmf) of CMP is

, where is the normalizing constant. The parameter controls different dispersion patterns, i.e. equi- (), over- () or under-dispersion (). Three common distributions occur as special cases: (1) the Poisson (), (2) geometric () and (3) Bernoulli ().

For the recordings of neurons up to steps, denote the spikes of these neurons at time bin as , for The corresponding CMP parameters at are and . Previous work has examined the CMP-GLM (Chatla and Shmueli 2018; Sellers and Shmueli 2010), and here we focus on the dynamic version of this GLM. The CMP parameters at are modeled by two log-linear models, and , with and .The state vector is assumed to progress linearly with a Gaussian noise, .

In summary, the neurons follow CMP distributions independently, conditioning on state vector .

The state vector evolves linearly:

## Inference by Gaussian approximation

To fit the model to data we need to estimate the time-varying state vector In this section, we first assume **F** and **Q** are known. Since the observations are CMP distributed, we cannot estimate in closed form. Instead, here we approximate it by a multivariate Gaussian distribution, , with . The parameters of this Gaussian are found by a global Laplace approximation, i.e., and . The log-posterior is given by:

, where is the log-likelihood. The log-posterior is concave (Gupta et al. 2014), and the Markovian structure of the state vector dynamics makes it possible to optimize by Newton-Raphson (NR) in time (Paninski et al. 2010). After the Newton update, we can further quantify the uncertainty for the CMP parameters and the underlying rates (see Appendix).

There are several technical challenges involved with performing the Newton update with CMP observations. Firstly, in order to find the gradient and Hessian we need to calculate moments of and , which have no closed forms (Shmueli et al. 2005). These moments are generally calculated by truncated summation, and when and , truncated summation is computationally costly, since we need many steps for accurate approximation. However, these moments can be efficiently approximated using previous (Chatla and Shmueli 2018; Gaunt et al. 2019) asymptotic results (see Appendix). A second challenge is that the Hessian is not robust to outliers. Since observations are accounted for by both and , outliers often result in the Hessian being close to singular or even positive-definite (see details in Appendix). To ensure the robustness, we use the Fisher scoring where the observed information is replaced by the expected information. Finally, a third challenge is that the Newton updates take a long time to converge if the initial state estimate is far from the maximum of the posterior, especially when is large. To resolve this issue, we use a smoothing estimate with local Gaussian approximation as a “warm start”. The forward filtering stage for Poisson case is derived in the appendix of (Eden et al. 2004), and we can implement CMP filtering following the same rationale. Let and be the mean and variance for the one-step prediction density and and be mean and variance for the posterior density, then, the filtering update for step :

Then we cant get the smoothing estimate by backward smoother. Although doing smoothing is fast, the estimations will be biased too much for large dynamics. In the forward filtering stage, the Gaussian approximation at each step is conducted locally at the recursive prior . This will be statistically inefficient when recursive prior is too far away from the posterior mode, or when there is a huge “jump” in the state vector. Moreover, using Fisher scoring for robustness makes the algorithm even less efficient. In this case, it is essential to do global Laplace approximation by NR.T

## Robust and quick estimation

For the applications to neural data examined here, we assume that . However, we still need to estimate the process noise . When is small, especially when , different values will have a substantial influence on estimation. One possible way to estimate is to use Expectation Maximization (cite). However, using the Laplace approximation for at E-step breaks the usual guarantee of non-decreasing likelihoods in EM, and, hence, may lead to divergence. This problem is even exacerbated for CMP distribution. To avoid that, we can sample posteriors directly by MCMC. However, the lack of closed-form moments for the CMP distribution makes sampling computationally intensive. Here, to estimate the robustly and quickly, we assume is diagonal and estimate it by maximizing the prediction likelihood in the filtering stage, as in (Wei and Stevenson 2021).

## Neural Data

[need to add section here with description and citations for V1 and HC data, similar to Neural Comp paper or can also look at the J Comp Neuro CMP paper. This should also include details on and ]

# Results

## Figure 1

Code: [figure1\_singleNu\_dirShift.m](https://github.com/weigcdsb/COM_POISSON/blob/main/demo/figure1/figure1_singleNu_dirShift.m)



Simulation settings: 100 directions from 0 to 1 rad. 100 trials. is modeled by with 10 knots, , i.e., over- to under- dispersion.



## Simulation

TBD…

Q optimization (Do we need this? It has been illustrated in SMOOTH-GBLM, and it’s shouldn’t be the focus of this paper)

|  |  |  |  |
| --- | --- | --- | --- |
| Heatmap | Q\_lam  Y = estimated log10(Q\_lam)  X = true log10(Q\_lam)  Color: yellow = small Q\_nu; black = large Q\_nu | Q\_nu  Y = estimated log10(Q\_nu)  X = true log10(Q\_nu)  Color: purple = small Q\_lam; black = large Q\_lam | fitting |
| Chart  Description automatically generated | Chart, scatter chart  Description automatically generated | Chart, scatter chart  Description automatically generated | Graphical user interface  Description automatically generated with low confidence |

## Application

V1 data

Code: [v1\_comparison\_nan.m](https://github.com/weigcdsb/COM_POISSON/blob/main/demo/v1/v1_comparison_nan.m); [compare\_all\_na.m](https://github.com/weigcdsb/COM_POISSON/blob/main/demo/v1/compare_all_na.m); [demo\_v1\_pd.m](https://github.com/weigcdsb/COM_POISSON/blob/main/demo/v1/demo_v1_pd.m)



Hippocampus data

Code: [hc\_comparison\_v2.m](https://github.com/weigcdsb/COM_POISSON/blob/main/demo/hc/hc_comparison_v2.m);



Should we separate the direction (-200 ~ 200) or ignore the direction (0 ~ 200)? I prefer to use -200 ~ 200 (left plot) rather than the right one. Since the direction is different, the (empirical) mean and FF should be separated.

# Discussion

Omitted variables can increase the apparent variability of observations via the law of total variance. For example, in the hippocampus, place cell firing is highly variable on different passes through the field (cite Fenton). This may be partially due to joint selectivity to position, speed, and head direction, as well as the influence of theta phase. Here, rather than model these distinct covariates assuming Poisson observations, we allow the variability to be non-Poisson.

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# References

**Barbieri R**, **Quirk MC**, **Frank LM**, **Wilson MA**, **Brown EN**. Construction and analysis of non-Poisson stimulus-response models of neural spiking activity. *J Neurosci Methods* 105: 25–37, 2001.

**Brown EN**, **Nguyen DP**, **Frank LM**, **Wilson MA**, **Solo V**. An analysis of neural receptive field plasticity by point process adaptive filtering. *Proc Natl Acad Sci* 98: 12261–12266, 2001.

**Chatla SB**, **Shmueli G**. Efficient estimation of COM–Poisson regression and a generalized additive model. *Comput Stat Data Anal* 121: 71–88, 2018.

**Chestek CA**, **Batista AP**, **Santhanam G**, **Yu BM**, **Afshar A**, **Cunningham JP**, **Gilja V**, **Ryu SI**, **Churchland MM**, **Shenoy K V**. Single-neuron stability during repeated reaching in macaque premotor cortex. *J Neurosci* 27: 10742–10750, 2007.

**Churchland AK**, **Kiani R**, **Chaudhuri R**, **Wang XJ**, **Pouget A**, **Shadlen MN**. Variance as a Signature of Neural Computations during Decision Making. *Neuron* 69: 818–831, 2011.

**Churchland MM**, **Yu BM**, **Cunningham JP**, **Sugrue LP**, **Cohen MR**, **Corrado GS**, **Newsome WT**, **Clark AM**, **Hosseini P**, **Scott BB**, **Bradley DC**, **Smith M a**, **Kohn A**, **Movshon JA**, **Armstrong KM**, **Moore T**, **Chang SW**, **Snyder LH**, **Lisberger SG**, **Priebe NJ**, **Finn IM**, **Ferster D**, **Ryu SI**, **Santhanam G**, **Sahani M**, **Shenoy K V**. Stimulus onset quenches neural variability: A widespread cortical phenomenon. *Nat Neurosci* 13: 369–378, 2010.

**DeWeese M**, **Zador A**. Asymmetric Dynamics in Optimal Variance Adaptation. *Neural Comput* 10: 1179–1202, 1998.

**Dickey AS**, **Suminski A**, **Amit Y**, **Hatsopoulos NG**. Single-unit stability using chronically implanted multielectrode arrays. *J Neurophysiol* 102: 1331–1339, 2009.

**Eden UT**, **Frank LM**, **Barbieri R**, **Solo V**, **Brown EN**. Dynamic Analysis of Neural Encoding by Point Process Adaptive Filtering. *Neural Comput* 16: 971–998, 2004.

**Fenton AA**, **Muller RU**. Place cell discharge is extremely variable during individual passes of the rat through the firing field. *Proc Natl Acad Sci U S A* 95: 3182–3187, 1998.

**Gaunt RE**, **Iyengar S**, **Olde Daalhuis AB**, **Simsek B**, **Robert Gaunt BE**. An asymptotic expansion for the normalizing constant of the Conway-Maxwell-Poisson distribution. *Ann Inst Stat Math* 71: 163–180, 2019.

**Ghanbari A**, **Lee CM**, **Read HL**, **Stevenson IH**. Modeling stimulus-dependent variability improves decoding of population neural responses. *J Neural Eng* 16, 2019.

**Gupta RC**, **Sim SZ**, **Ong SH**. Analysis of discrete data by Conway–Maxwell Poisson distribution. *AStA Adv Stat Anal* 98: 327–343, 2014.

**Lesica NA**, **Jin J**, **Weng C**, **Yeh CI**, **Butts DA**, **Stanley GB**, **Alonso JM**. Adaptation to Stimulus Contrast and Correlations during Natural Visual Stimulation. *Neuron* 55: 479–491, 2007.

**Maimon G**, **Assad J a.** Beyond Poisson: Increased Spike-Time Regularity across Primate Parietal Cortex. *Neuron* 62: 426–440, 2009.

**Paninski L**, **Ahmadian Y**, **Ferreira DG**, **Koyama S**, **Rahnama Rad K**, **Vidne M**, **Vogelstein J**, **Wu W**. A new look at state-space models for neural data. *J. Comput. Neurosci.* 29Springer: 107–126, 2010.

**Rokni U**, **Richardson AG**, **Bizzi E**, **Seung HS**. Motor learning with unstable neural representations. *Neuron* 54: 653–666, 2007.

**Sellers KF**, **Shmueli G**. A flexible regression model for count data. *https://doi.org/101214/09-AOAS306* 4: 943–961, 2010.

**Shmueli G**, **Minka TP**, **Kadane JB**, **Borle S**, **Boatwright P**. A useful distribution for fitting discrete data: revival of the Conway–Maxwell–Poisson distribution. *J R Stat Soc Ser C (Applied Stat* 54: 127–142, 2005.

**Steinmetz NA**, **Aydin C**, **Lebedeva A**, **Okun M**, **Pachitariu M**, **Bauza M**, **Beau M**, **Bhagat J**, **Böhm C**, **Broux M**, **Chen S**, **Colonell J**, **Gardner RJ**, **Karsh B**, **Kloosterman F**, **Kostadinov D**, **Mora-Lopez C**, **O’Callaghan J**, **Park J**, **Putzeys J**, **Sauerbrei B**, **van Daal RJJ**, **Vollan AZ**, **Wang S**, **Welkenhuysen M**, **Ye Z**, **Dudman JT**, **Dutta B**, **Hantman AW**, **Harris KD**, **Lee AK**, **Moser EI**, **O’Keefe J**, **Renart A**, **Svoboda K**, **Häusser M**, **Haesler S**, **Carandini M**, **Harris TD**. Neuropixels 2.0: A miniaturized high-density probe for stable, long-term brain recordings. *Science (80- )* 372, 2021.

**Stevenson IH**. Flexible models for spike count data with both over- and under- dispersion. *J Comput Neurosci* 41: 29–43, 2016.

**Stevenson IH**, **Cherian A**, **London BM**, **Sachs NA**, **Lindberg E**, **Reimer J**, **Slutzky MW**, **Hatsopoulos NG**, **Miller LE**, **Kording KP**. Statistical assessment of the stability of neural movement representations. *J Neurophysiol* 106: 764–774, 2011.

**Tomko GJ**, **Crapper DR**. Neuronal Variability - Nonstationary Responses to Identical Visual-Stimuli. *Brain Res* 79: 405–418, 1974.

**Wei G**, **Stevenson IH**. Tracking Fast and Slow Changes in Synaptic Weights From Simultaneously Observed Pre- and Postsynaptic Spiking. *Neural Comput* 33: 2682–2709, 2021.

# Appendix

## Approximation of moments for Conway-Maxwell Poisson distribution

In order to efficiently estimate the state-vector for the dynamical CMP model, we make use of a previous for the pmf . Assume and , let , and

Then the moments are (The highlights can be found in the reference, the covariance is calculated by myself):

Here we use this approximation when … and use a truncated summation otherwise.

## Gradient and Hessian of the log-posterior

We estimate the state vector by maximizing the log-posterior with Newton-Raphson updates. Denote .

[add basic equation for NR updates?]

The gradient:

, where , and .

The Hessian:

, where

When , the Hessian may be ill-conditioned or even positive-definite. Because of the factorial, the “outliers” are common. To ensure the robustness, do Fisher scoring, i.e. replace the observed information by the expected information , so that .

## Quantifying Uncertainties

After convergence, we have an approximation of the log-posterior , and we can use this approximation to quantify the state uncertainty as well as uncertainty about the mean rate at each time.

To find the state uncertainty, let , then .

Then to find the uncertainty in the conditional mean rate , we need the covariance matrix of the CMP parameters at time can be calculated by property of multivariate log-normal distribution. The conditional mean firing rate, is , whose variance can be calculated by the Delta method:

We can calculate the moments as in the first section of the appendix, or we can use a simpler approximation when or . Then and .