1 $x_t^{(j)}$ and $\mu_t^{(j)}$

Let the t^{th} column observation and firing rate of the j^{th} cluster be $\widetilde{\boldsymbol{y}}_t = vec\left(\{y_{it}^{(j)}|z_i = j\}\right)$ and $\widetilde{\boldsymbol{\lambda}}_t^{(j)} = vec\left(\{\lambda_{it}|z_i = j\}\right)$. The number of neurons in cluster j is $n_j = |\{i : z_i = j\}|$. The corresponding loading for these n_j neurons is $\boldsymbol{C}^{(j)} \in \mathbb{R}^{n_j \times p}$, such that $\log \widetilde{\boldsymbol{\lambda}}_t^{(j)} = \mu_t^{(j)} \mathbf{1}_{n_j} + \boldsymbol{C}^{(j)} \boldsymbol{x}_t^{(j)} = \left(\mathbf{1}_{n_j}, \boldsymbol{C}^{(j)}\right) \cdot \left(\mu_t^{(j)}, \boldsymbol{x}_t^{'(j)}\right)'$. Denote $\widetilde{\boldsymbol{C}}^{(j)} = \left(\mathbf{1}_{n_j}, \boldsymbol{C}^{(j)}\right)$, $\widetilde{\boldsymbol{x}}_t^{(j)} = \left(\mu_t^{(j)}, \boldsymbol{x}_t^{'(j)}\right)'$, $\widetilde{\boldsymbol{x}}^{(j)} = \left(\widetilde{\boldsymbol{x}}_1^{'(j)}, \ldots, \widetilde{\boldsymbol{x}}_T^{'(j)}\right)'$, $\widetilde{\boldsymbol{A}}^{(j)} = diag(f^{(j)}, \boldsymbol{A}^{(j)})$, $\widetilde{\boldsymbol{b}}^{(j)} = \left(g^{(j)}, \boldsymbol{b}^{'(j)}\right)'$ and $\widetilde{\boldsymbol{Q}}^{(j)} = diag(\Sigma^{(j)}, \boldsymbol{Q}^{(j)})$. The full conditional distribution $P(\widetilde{\boldsymbol{x}}^{(j)}|\ldots) = P(\widetilde{\boldsymbol{x}}^{(j)}|\{\widetilde{\boldsymbol{y}}_t\}_{t=1}^T, \widetilde{\boldsymbol{C}}^{(j)}, \widetilde{\boldsymbol{A}}^{(j)}, \widetilde{\boldsymbol{b}}^{(j)}, \widetilde{\boldsymbol{Q}}^{(j)})$ is approximated by a global Laplace approximation, i.e. $P(\widetilde{\boldsymbol{x}}^{(j)}|\ldots) \approx N_{(p+1)T}(\widetilde{\boldsymbol{x}}^{(j)}|\boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}^{(j)}}, \boldsymbol{\Sigma}_{\widetilde{\boldsymbol{x}}^{(j)}})$, with $\boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}^{(j)}} = \arg\max_{\widetilde{\boldsymbol{x}}^{(j)}} P(\widetilde{\boldsymbol{x}}^{(j)}|\ldots)$ and $\boldsymbol{\Sigma}_{\widetilde{\boldsymbol{x}}^{(j)}}) = -\left(\nabla\nabla \log P(\widetilde{\boldsymbol{x}}^{(j)}|\ldots)\big|_{\widetilde{\boldsymbol{x}}^{(j)}=\boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}^{(j)}}}\right)^{-1}$.
The log full conditional distribution $h(\widetilde{\boldsymbol{x}}^{(j)}) = \log P(\widetilde{\boldsymbol{x}}^{(j)}|\ldots)$ is given by:

$$h = \text{const} + \sum_{t=1}^{T} \widetilde{\boldsymbol{C}}^{'(j)}\left(\widetilde{\boldsymbol{y}}_t - \widetilde{\boldsymbol{\lambda}}_t\right) - \frac{1}{2}(\widetilde{\boldsymbol{x}}_1^{(j)} - \widetilde{\boldsymbol{x}}_0^{(j)})'\widetilde{\boldsymbol{Q}}_0^{(j)}(\widetilde{\boldsymbol{x}}_1^{(j)} - \widetilde{\boldsymbol{x}}_0^{(j)}) - \sum_{t=2}^{T} \frac{1}{2}(\widetilde{\boldsymbol{x}}_t^{(j)} - \widetilde{\boldsymbol{A}}^{(j)}\widetilde{\boldsymbol{x}}_{t-1}^{(j)})'\widetilde{\boldsymbol{Q}}^{(j)}(\widetilde{\boldsymbol{x}}_t^{(j)} - \widetilde{\boldsymbol{A}}^{(j)}\widetilde{\boldsymbol{x}}_{t-1}^{(j)})$$

It should be $\boldsymbol{b}_t^{(j)}$, but not $\boldsymbol{b}^{(j)}$! Fix that and then edit the writing!