RMIT UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

EEET2147 Electronic Devices & Physical Devices Simulation

Assignment No.2:

Electrodes Charge Density Distribution Simulation with Method of Moments

Student: ***Wei Han*** ***S3204911***

Prepared For: Prof. A.R. Baghai-Wadji

Abstract:

In this paper, we simulated the charge density distribution for the electrodes which place at the interface of two domains. The upper domain is a semi-infinite free space, the lower domain is a substrate domain has a limited depth. The simulation is based on the Green's function and the Method of Moments.

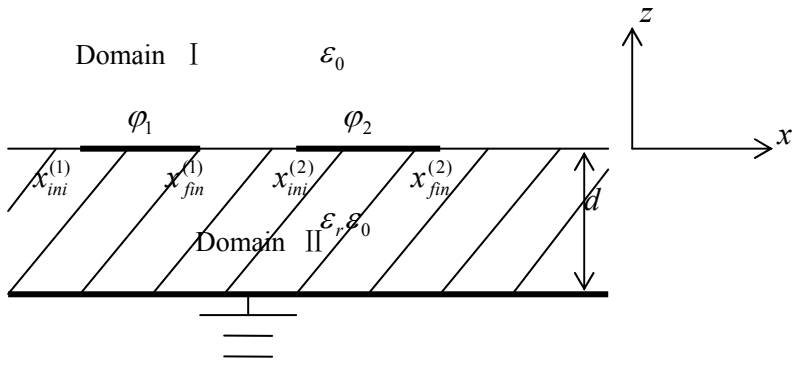
Because the difficult to get Green's function's closed-form in the real space domain, we made a sectioned process for the deduced frequency domain Green's function. We show it is possible and the physical means for it is that the limited depth substrate could be regards as the infinite depth substrate case adding a modification item. In this way, we could reduce computing load significantly.

Based on above results, we simulated a two-electrode system at the same structure with Matlab program. The simulation results meet our expectation. Although the simulation is for the two-electrode system, the deduced results could apply to any number electrode system at the same structure.

Key Words:

Green's Function, Method of Moments, Charge Density Distribution

1. Statement of problem:



In above figure, there are two domains, Domain Ⅰ and Domain Ⅱ. Domain I is semi-infinite free-space which is characterized by ε_0. Domain II is a homogeneous, isotropic dielectric substrate characterized by permittivity constant $\varepsilon = \varepsilon_0\varepsilon_r$ and the thickness as d.

At the boundary of Domain Ⅰ and Domain Ⅱ, there are two electrodes positioned parallel to the $y-$direction and positioned on the substrate-surface $(z=0)$. The electrodes are assumed to have the following $x-$coordinates:

Electrode 1: $x_{ini}^{(1)}$ and $x_{fin}^{(1)}$ (ini: initial and fin: final)

Electrode 2: $x_{ini}^{(2)}$ and $x_{fin}^{(2)}$ (ini: initial and fin: final)

Assume the electrodes 1 and 2 to have the electric potential $\varphi^{(1)}$ and $\varphi^{(2)}$, respectively (measured with respect to an arbitrary reference point).

Assume the backside of the substrate is grounded (zero potential with respect to an arbitrary reference point)

Problem: Find the charge density distribution on the electrodes, the function $\rho(x)$, using the method of moments!

2. Introduction and analysis:

Because the electrodes parallel each other and infinite long along y axis, this means the physical parameters in this problem don't change along y axis. So the problem reduces to a two-dimensional problem in x-z plane.

The basic equations in electromagnetic field are Maxwell Equations:

$$\begin{cases} \vec{\nabla} \times \vec{H} = -\dfrac{\partial \vec{D}}{\partial t} + \vec{J} \\ \vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \cdot \vec{D} = \rho \end{cases}$$

In static electric field, $\dfrac{\partial \vec{B}}{\partial t}$ = 0 and we just need consider electric items in Maxwell equations, so Maxwell equations reduce to:

$$\begin{cases} \vec{\nabla} \times \vec{E} = 0 \\ \vec{\nabla} \cdot \vec{D} = \rho \end{cases}$$

This shows static electric field is a non-curl field, which means we could define a potential in the field as: $\vec{E} = -\nabla \cdot \varphi$

Consider the material equation: $\vec{D} = \varepsilon \cdot \vec{E}$ and $\vec{\nabla} \cdot \vec{D} = \rho$, we could get following static electric field's Possion's equation: $\nabla^2 \cdot \varphi = -\dfrac{\rho}{\varepsilon}$.

When $\rho = 0$, Possion's equation become to Laplace's equation: $\nabla^2 \cdot \varphi = 0$.

In this problem, there isn't electron charge inside of Domain Ⅰ and Domain Ⅱ, so we just need consider Laplace's equation in these domains.

At the interface between Domain Ⅰ and Domain Ⅱ, consider the continuity of the electric potential and $\vec{\nabla} \cdot \vec{D} = \rho$. We have the following boundary conditions:

$$\begin{cases} \varphi^{I}_{(x,z)}\big|_{z=0^+} = \varphi^{I}_{(x,z)}\big|_{z=0^-} \\ D_z(x,z)\big|_{z=0^+} - D_z(x,z)\big|_{z=0^-} = \rho(x) \end{cases},$$

where $\rho(x)$ is the electric charge density distribution at interface along x axis, which is also the target function in this problem.

At the point z = -d, according the description of problem, the backside of the substrate is grounded, so the potential is zero. Because the Domain I is a semi-infinite free-space, at the $z=\infty$, the potential is zero.

From above analysis, we could restate the conditions of this problem as following:

Electric field holds potential's Laplace equation in x-z plane:

$$(\partial^2 / x^2 + \partial^2 / z^2)\varphi(x,z) = 0 \tag{2.1}$$

Conditions:

Dielectric's material equation:
$$\begin{cases} \vec{D} = \varepsilon_0 \cdot \vec{E} & ,Domain \quad I \\ \vec{D} = \varepsilon_r \varepsilon_0 \cdot \vec{E} & ,Domain \quad II \end{cases} \tag{2.2}$$

Boundary condition at interface:
$$\begin{cases} \varphi^I_{(x,z)}\Big|_{z=0^+} = \varphi^{II}_{(x,z)}\Big|_{z=0^-} \\ D_z(x,z)\Big|_{z=0^+} - D_z(x,z)\Big|_{z=0^-} = \rho(x) \end{cases} \tag{2.3}$$

Boundary condition at the cover: $\varphi^I_{(x,z)}\Big|_{z=\infty} = \varphi^{II}_{(x,z)}\Big|_{z=-d} = 0$ (2.4)

3. Green's Function:

Given the homogeneity of the Laplace's equations, we could solve it with the Method of separate variable.

Considering the try function: $\varphi(x,z) = h(x)\phi(z) = A(k)e^{jkx}e^{\lambda z}$ (3.1)

Substitute equation (3.1) into Laplace equation (2.1), we could get,

$$\Rightarrow \quad \frac{\partial^2}{\partial x^2}\left[A(k)e^{jkx}e^{\lambda z}\right] + \frac{\partial^2}{\partial z^2}\left[A(k)e^{jkx}e^{\lambda z}\right] = 0$$

$$\Rightarrow \quad -k^2 A(k)e^{jkx}e^{\lambda z} + \lambda^2 A(k)e^{jkx}e^{\lambda z} = 0$$

$$\Rightarrow \quad (-k^2 + \lambda^2)A(k)e^{jkx}e^{\lambda z} = 0$$

$$\Rightarrow \quad \lambda^2 = k^2 \qquad \Rightarrow \quad \lambda = \pm|k| \tag{3.2}$$

$$\Rightarrow \quad \varphi(x,z)\Big|_k = A(k)e^{jkx}e^{-|k|z} + B(k)e^{jkx}e^{|k|z} \tag{3.3}$$

Now we need consider boundary condition.

In domain I, when $z=\infty$, have $\varphi^I(x,z)\Big|_{z=\infty} = 0$, substituting it into the equation 3.3, we could get

$B(k) = 0$, so we get the potential equation in Domain I as:

$$\varphi^I(x,z)\Big|_k = A(k)e^{jkx}e^{-|k|z} \tag{3.4}$$

And: $D_z{}^I(x,z)\Big|_k = -\varepsilon_0 \cdot \partial\varphi/\partial z = \varepsilon_0|k|A(k)e^{jkx}e^{-|k|z}$ (3.5)

At the interface between domain I and domain II, where z=0, have,

$$\begin{cases} \varphi^I(x,0^+\big|_k) = A(k)e^{jkx} \\ D_z^I(x,0^+\big|_k) = \varepsilon_0|k|A(k)e^{jkx} \end{cases} \tag{3.6}$$

Now we turn to Domain II.

Consider the given condition: $\varphi^{II}(x,z)\Big|_{z=-d2} = 0$. At the point z=-d, equation (3.3) change to:

$$\varphi^{II}_{(x,z)}\Big|_{z=-d} = C(k)e^{jkx}e^{|k|d} + D(k)e^{jkx}e^{-|k|d} = 0,$$

$$\Rightarrow \quad D(k) = -C(k)e^{2|k|d} \tag{3.7}$$

So we could get the potential equation in Domain II as:

$$\varphi^{II}(x,z)\Big|_k = C(k)e^{jkx}e^{-|k|z} - C(k)e^{2|k|d}e^{jkx}e^{|k|z}$$

$$= C(k)e^{jkx}\left(e^{-|k|z} - e^{2|k|d}e^{|k|z}\right) \tag{3.8}$$

And: $D_z{}^{II}(x,z)\Big|_k = -\varepsilon_0\varepsilon_r \cdot \partial\varphi/\partial z = -\varepsilon_0\varepsilon_r e^{jkx}C(k)(-|k|e^{-|k|z} - |k|e^{2|k|d}e^{|k|z})$

$$= \varepsilon_0\varepsilon_r e^{jkx}C(k)|k|(e^{-|k|z} + e^{2|k|d}e^{|k|z}) \tag{3.9}$$

At the interface between domain I and domain II, where z=0, have,

$$\begin{cases} \varphi^{II}(x,0^-\big|_k) = C(k)e^{jkx}(1-e^{2|k|d}) \\ D_z^{II}(x,0^-\big|_k) = \varepsilon_0\varepsilon_r|k|C(k)e^{jkx}(1+e^{2|k|d}) \end{cases} \tag{3.10}$$

From the boundary conditions $\varphi^I(x,0^+\big|_k) = \varphi^{II}(x,0^-\big|_k)$, have,

$$A(k)e^{jkx} = C(k)e^{jkx}(1-e^{2|k|d})$$

could get: $C(k) = \dfrac{A(k)}{1-e^{2|k|d}}$ (3.11)

so (3.8) become to: $\varphi^{II}(x,z)\Big|_k = \dfrac{A(k)}{1-e^{2|k|d}}e^{jkx}\left(e^{-|k|z} - e^{2|k|d}e^{|k|z}\right)$ (3.12)

(3.9) change to: $D_z{}^{II}(x,z)\Big|_k = \varepsilon_0\varepsilon_r\dfrac{A(k)|k|}{1-e^{2|k|d}}e^{jkx}(e^{-|k|z} + e^{2|k|d}e^{|k|z})$ (3.13)

Integration the equation (3.5) and (3.13) in the wave number domain, we could get:

$$D_z{}^{I}(x,z) = \int_{-\infty}^{+\infty} \varepsilon_0 |k| A(k) e^{jkx} e^{-|k|z} dk \,,$$

And $D_z{}^{II}(x,z) = \int_{-\infty}^{+\infty} \varepsilon_0 \varepsilon_r \frac{A(k)|k|}{1-e^{2|k|d}} e^{jkx} (e^{-|k|z} + e^{2|k|d} e^{|k|z}) dk$

At z=0, have:

$$\begin{cases} D_z^I(x,0^+) = \int_{-\infty}^{+\infty} \varepsilon_0 |k| A(k) e^{jkx} dk \\ D_z^{II}(x,o^-) = \int_{-\infty}^{+\infty} \varepsilon_0 \varepsilon_r \frac{A(k)|k|}{1-e^{2|k|d}} e^{jkx} (1+e^{2|k|d}) dk \end{cases}$$

Considering the interface conditions (2.3), $D_z(x,z)\big|_{z=0^+} - D_z(x,z)\big|_{z=0^-} = \rho(x)$, have,

$$\int_{-\infty}^{+\infty} A(k)|k|\varepsilon_0 (1-\varepsilon_r \frac{(1+e^{2|k|d})}{1-e^{2|k|d}}) e^{jkx} dk = \rho(x) = \int_{-\infty}^{+\infty} \rho(k) e^{jkx} dk \qquad (3.14)$$

Where $\rho(k)$ is the electron density in spectrum domain.

$$\Rightarrow \quad A(k)|k|\varepsilon_0 (1-\varepsilon_r \frac{1+e^{2|k|d}}{1-e^{2|k|d}}) = \rho(k)$$

$$\Rightarrow \quad A(k) = \frac{\rho(k)}{|k|\varepsilon_0 (1-\varepsilon_r \frac{1+e^{2|k|d}}{1-e^{2|k|d}})} = \frac{\rho(k)}{|k|\varepsilon_0 (1+\varepsilon_r \frac{e^{-|k|d}+e^{|k|d}}{e^{|k|d}-e^{-|k|d}})}$$

$$\Rightarrow \quad A(k) = \frac{\rho(k)}{|k|\varepsilon_0 (1+\varepsilon_r \coth(|k|d))} \qquad (3.15)$$

From (3.15) could get the potential distribution at z=0 as:

$$\varphi(x,0|_k) = \frac{\rho(k)}{|k|\varepsilon_0 (1+\varepsilon_r \coth(|k|d))} e^{jkx} \qquad (3.16)$$

Integration (3.16) with frequency k, have,

$$\varphi(x,0) = \int_{-\infty}^{+\infty} \frac{1}{2\pi} \frac{\rho(k)}{|k|\varepsilon_0 (1+\varepsilon_r \coth(|k|d))} e^{jkx} dk \qquad (3.17)$$

As we mentioned above, $\rho(k)$ is the electron density in spectrum domain and have: $\rho(x) = \int_{-\infty}^{+\infty} \rho(k) e^{jkx} dk$, so the real domain form of equation (3.17) is:

$$\varphi(x,0) = \int_{-\infty}^{+\infty} G(x-x') \rho(x') dx' \qquad (3.18)$$

Where $G(x-x')$ is the Green's function for this problem.

Comparing equation (3.17) and (3.18), we could see that the Green's in spectrum domain as:

$$G(k) = \frac{1}{|k|\varepsilon_0(1+\varepsilon_r \coth(|k|d))} \quad (3.19)$$

Observing equation (3.19), could find that when $\|k|d| >> 1$, $\coth(|k|d) = 1$, so equation (3.19) could change to: $G(k) = \frac{1}{|k|\varepsilon_0(1+\varepsilon_r)}$. If we choose this value as *kdmax*, we could change equation (3.19) to the following form:

$$G(k) = G_1(k) + G_2(k) \quad (3.20)$$

Where: $G_1(k) = \begin{cases} 0 & when \quad otherwise \\ \frac{1}{|k|\varepsilon_0(1+\varepsilon_r)} & when \quad \|k|d| > kd\max \end{cases}$, and (3.21)

$$G_2(k) = \begin{cases} \frac{1}{|k|\varepsilon_0(1+\varepsilon_r \coth(|k|d))} & when \quad \|k|d| \le kd\max \\ 0 & when \quad otherwise \end{cases} \quad (3.22)$$

Change (3.20) into real space, and considering G(k) is an even function, could get:

$$G(x-x') = \int_{-\infty}^{+\infty} \frac{1}{2\pi} G(k) e^{ik(x-x')} dk = \frac{1}{\pi} \int_0^{+\infty} \left(G_1(k) + G_2(k)\right) e^{ik(x-x')} dk$$

$$= \frac{1}{\pi}\left[\int_0^{+\infty} G_1(k) e^{ik(x-x')} dk + \int_0^{+\infty} G_2(k) e^{ik(x-x')} dk\right] = G_1(x-x') + G_2(x-x') \quad (3.23)$$

Then (3.18) change to:

$$\varphi(x,0) = \int_{-\infty}^{+\infty} \left(G_1(x-x') + G_2(x-x')\right) \rho(x') dx' = \varphi_1(x,0) + \varphi_2(x,0) \quad (3.24)$$

Equation (3.23) and (3.24) mean we could process $G_1(k)$ and $G_2(k)$ separately.

For k=0, and considering the even function property of $G_2(k)$, we have:

$$\lim_{k\to 0} G_2(k) = \lim_{k\to 0} \frac{1}{|k|\varepsilon_0(1+\varepsilon_r \coth(|k|d))} = \lim_{k\to 0^+} \frac{1}{\frac{k}{\tanh(kd)}\varepsilon_0\left(\tanh(kd) + \varepsilon_r\right)}$$

$$= \left. \frac{1}{\frac{1}{d\coth(kd)}\varepsilon_0\left(\tanh(kd)+\varepsilon_r\right)} \right|_{k=0} = \frac{d}{\varepsilon_0\varepsilon_r} \qquad (3.25)$$

Equation (3.25) means the $G_2(k)$ is bounded at k=0.

4. Charge Approximation:

According to the theory of MOM, an orthogonal functions series should be chose as the basis functions. The commonly used could be impulse, pulse, or triangle functions. In this problem, we will choose the pulse function as the basis functions, i.e.,

$$\rho(x) = \sum_{n=1}^{n=N} \rho_n \cdot P_n(x), \qquad (4.1)$$

Where: $P_n(x) = \begin{cases} 1 & if \quad x_n^b \le x \le x_n^e \\ 0 & otherwise \end{cases}$ (4.2)

And: ρ_n is the charge density in nth section, which could be regarded as a constant.

From above, we could get the charge quantity in the Nth section as:

$$q_n = \int_{-\infty}^{+\infty} \rho_n \cdot P_n(x)dx = \rho_n \cdot \int_{-\infty}^{+\infty} P_n(x)dx = \rho_n \cdot (x_n^e - x_n^b) \qquad (4.3)$$

So the equation (4.1) change to: $\rho(x) = \sum_{n=1}^{n=N} q_n \cdot \frac{1}{(x_n^e - x_n^b)} \cdot P_n(x)$ (4.4)

The Fourier transform of equation (4.4) could get as[]:

$$\rho(k) = \sum_{n=1}^{n=N} q_n \cdot \frac{1}{(x_n^e - x_n^b)} \cdot \frac{e^{-jkx_n^e} - e^{-jkx_n^b}}{jk} \qquad (4.5)$$

5. Approximation of $\varphi_1(x)$

We rewrite spectrum domain Green's function $G_1(k)$ (3.21) into following:

$$G_1(k) = \begin{cases} 0 & when \quad otherwise \\ \dfrac{1}{|k|\varepsilon_0(1+\varepsilon_r)} & when \quad \|k|d| > kd\max \end{cases},$$

The real domain form of it is:

$$G_1(x-x') = \int_{-\infty}^{+\infty} \frac{1}{2\pi} G_1(k) e^{ik(x-x')} dk = \int_{-\infty}^{+\infty} \frac{dk}{2\pi} G_1(k)(\cos(k(x-x')) + i\sin(k(x-x'))),$$

Considering $G_1(k)$ is an even function, above relation could change to:

$$G_1(x-x') = 2\int_{-0}^{+\infty}\frac{dk}{2\pi}G_1(k)\cos(k(x-x') \;=\; \frac{1}{\pi}\int_{\frac{kd\max}{d}}^{+\infty}\frac{1}{k\varepsilon_0(1+\varepsilon_r)}\cos(k(x-x')dk$$

$$=\frac{1}{\pi\varepsilon_0(1+\varepsilon_r)}\int_{\frac{kd\max}{d}}^{+\infty}\frac{\cos(k(x-x')}{k}dk=\frac{1}{\pi\varepsilon_0(1+\varepsilon_r)}\left[-\gamma-\ln(\frac{kd\max}{d})-\ln|x-x'|\right] \qquad (5.1)$$

Where γ is Euler's constant.

From equation(3.24), we have:

$$\varphi_1(x,0)=\int_{-\infty}^{+\infty}G_1(x-x')\cdot\rho(x')dx' \qquad (5.2)$$

Where $\rho(x') = \sum_{n=1}^{n=N} q_n \cdot \frac{1}{(x_n^e - x_n^b)} \cdot P_n(x')$ from Equ. (4.4)

From above two equations, could get:

$$\varphi_1(x,0)=\int_{-\infty}^{+\infty}\frac{1}{\pi\varepsilon_0(1+\varepsilon_r)}\left[-\gamma-\ln(\frac{kd\max}{d})-\ln|x-x'|\right]\cdot\rho(x')dx'$$

$$= \frac{1}{\pi\varepsilon_0(1+\varepsilon_r)}\left\{\int_{-\infty}^{+\infty}\left[-\gamma-\ln(\frac{kd\max}{d})\right]\rho(x')dx'-\int_{-\infty}^{+\infty}\left(\ln|x-x'|\right)\rho(x')dx'\right\}$$

$$= \frac{\left[-\gamma-\ln(\frac{kd\max}{d})\right]}{\pi\varepsilon_0(1+\varepsilon_r)}\int_{-\infty}^{+\infty}\rho(x')dx'-\frac{1}{\pi\varepsilon_0(1+\varepsilon_r)}\int_{-\infty}^{+\infty}\left(\ln|x-x'|\right)\rho(x')dx'$$

Given the charge neutral condition, have $\int_{-\infty}^{+\infty}\rho(x')dx' = 0$, so above equation change to:

$$\varphi_1(x,0)= -\frac{1}{\pi\varepsilon_0(1+\varepsilon_r)}\int_{-\infty}^{+\infty}\left(\ln|x-x'|\right)\rho(x')dx' \;=\; \beta\int_{-\infty}^{+\infty}\left(\ln|x-x'|\right)\rho(x')dx' \qquad (5.3)$$

Where: $\beta = -\frac{1}{\pi\varepsilon_0(1+\varepsilon_r)}$ (5.4)

Substitute (4.4) into (5.3) have:

$$\varphi_1(x,0)=\beta\int_{-\infty}^{+\infty}\left(\ln|x-x'|\right)\sum_{n=1}^{n=N}q_n\cdot\frac{1}{(x_n^e-x_n^b)}\cdot P_n(x')dx'$$

$$=\beta\sum_{n=1}^{n=N}q_n\cdot\frac{1}{(x_n^e-x_n^b)}\int_{x_n^b}^{x_n^e}\left(\ln|x-x'|\right)dx' \qquad (5.5)$$

If we have $u = x - x'$, then $dx' = -u$. The integration part in (5.5) could change to:

$$\int_{x-x_n^b}^{x-x_n^e}\left(\ln|u|\right)du = u\ln|u|\Big|_{x-x_n^b}^{x-x_n^e} - \int_{x-x_n^b}^{x-x_n^e}\frac{u}{|u|}d\left(|u|\right)$$

For second part in above equation, if u>0, have: $\frac{u}{|u|}=\frac{u}{u}=1$, and $d\left(|u|\right)$=du; if u<0, have $\frac{u}{|u|}=\frac{u}{-u}$=-1, and $d\left(|u|\right)=d\left(-u\right)$ =-du. So it always equal to:

$$\int_{x-x_n^b}^{x-x_n^e} du \;=\; (x-x_n^e)-(x-x_n^b) = \; -x_n^e - x_n^b$$

So: $$\int_{x-x_n^b}^{x-x_n^e}\left(\ln|u|\right)du = \; (x-x_n^e)\ln|x-x_n^e| - (x-x_n^b)\ln|x-x_n^b| + x_n^e + x_n^b \qquad (5.6)$$

Substitute (5.6) into (5.5), we have,

$$\varphi_1(x,0)=\sum_{n=1}^{n=N}\frac{\beta}{(x_n^e-x_n^b)}\left[(x-x_n^e)\ln|x-x_n^e| - (x-x_n^b)\ln|x-x_n^b| + x_n^e + x_n^b\right]\cdot q_n \quad (5.7)$$

Above is the potential at the substrate interface (z=0) caused by $G_1(k)$ part. If we choose φ_1 as the potential value in the lth section, where $x=\xi^l$, (5.7) could change to:

$$\varphi_1^{\,l}=\varphi_1(\xi^l)=\sum_{n=1}^{n=N}\frac{\beta}{(x_n^e-x_n^b)}\left[(x-x_n^e)\ln|\xi^l-x_n^e| - (\xi^l-x_n^b)\ln|\xi^l-x_n^b| + x_n^e + x_n^b\right]\cdot q_n$$

$$= \sum_{n=1}^{N} AI_{\ln}\cdot q_n, \qquad \text{and } l=1,\cdots N \qquad (5.8)$$

Where: $$AI_{\ln} \;=\; \sum_{n=1}^{n=N}\frac{\beta}{(x_n^e-x_n^b)}\left[(x-x_n^e)\ln|\xi^l-x_n^e| - (\xi^l-x_n^b)\ln|\xi^l-x_n^b| + x_n^e + x_n^b\right] \quad (5.9)$$

6. Approximation of $\varphi_2(x)$ and $\varphi(x)$

In this part, we will consider the effects caused by $G_2(k)$ part, which is:

$$G_2(k) \;=\; \begin{cases} \dfrac{1}{|k|\varepsilon_0(1+\varepsilon_r\coth(|k|d))} & when \quad |k|d \le kd\max \\ 0 & when \quad otherwise \end{cases}$$

Above relation is hard to get a closed form equivalent in real domain. So we will use the charge density's spectrum form, which show in equation (4.5) as:

$$\rho(k)=\sum_{n=1}^{n=N} q_n\cdot\frac{1}{(x_n^e-x_n^b)}\cdot\frac{e^{-jkx_n^e}-e^{-jkx_n^b}}{jk}$$

Substitute above equation into (3.17), we have:

$$\varphi_2(x,0)=\int_{-\infty}^{+\infty}\frac{1}{2\pi}G_2(k)\rho(k)e^{jkx}dk=\sum_{n=1}^{n=N}q_n\cdot\frac{1}{2\pi(x_n^e-x_n^b)}\cdot\int_{-\infty}^{+\infty}G_2(k)\frac{e^{-jk(x_n^e-x)}-e^{-jk(x_n^b-x)}}{jk}dk$$

Same as in $\varphi_1(x)$, if we choose x at the middle point of the l section, and have $x=\xi^l$. We could change above equation to:

$$\varphi_2{}^l=\varphi_2(\xi^l,0)=\sum_{n=1}^{n=N}q_n\cdot\frac{1}{2\pi(x_n^e-x_n^b)}\cdot\int_{-\infty}^{+\infty}G_2(k)\frac{e^{-jk(x_n^e-\xi^l)}-e^{-jk(x_n^b-\xi^l)}}{jk}dk$$

$$=\sum_{n=1}^{n=N}AII_{\ln}\cdot q_n \qquad (6.1)$$

From the reference[1], the coefficients in above relation could get as:

$$AII_{\ln}=\frac{1}{\pi}\int_0^{\infty}G_2(k)\cdot\sin c(\delta_l k)\cdot\sin c(\delta_n k)\cdot\cos(k\left|\xi^l-\xi^n\right|)dk \qquad (6.2)$$

Where δ_l and δ_n are one half of the width of lth and nth sections. ξ^l and ξ^n are the middle coordinate of the lth and nth sections.

Unlike $\varphi_1(x)$, in $\varphi_2(x)$, we need determine the coefficient matrix with numerical integration. So we need discuss the integral limit of the (6.2).

As we discussed in section 3, the upper limit in (6.2) is the constant ***kdmax***=$\left|k\right|d$, which is determined by the nature of function: $\coth(\left|k\right|d)$. Given the value of $\coth(10)$= 1.000000004, which is very close to 1, so if we choose *kdmax* above 10, the accuracy is enough.

As show in relation(3.25), $\lim_{k\to 0}G_2(k)=\frac{d}{\varepsilon_0\varepsilon_r}$,

so the function $\int_0^{\infty}G_2(k)\cdot\sin c(\delta_l k)\cdot\sin c(\delta_n k)\cdot\cos(k\left|\xi^l-\xi^n\right|)$ is bounded at the k=0.

So we could choose a small value *kdmin*=$\left|k\right|d$ such as 0.001 as the low limit of above integration and could come to enough accuracy.

By now, we have got the relation between potential part caused by $G_1(k)$ and $G_2(k)$.

From relation (3.24), the physical potential at the interface of the substrate is:

$$\varphi(x)=\varphi_1(x)+\varphi_2(x)=A_{\ln}\cdot q_n \qquad (6.3)$$

Where: $A_{\ln} = AI_{\ln} + AII_{\ln}$ (6.4)

From equation (6.3) and (6.4), we could get the every section's charge quantity in electrode from the potential given on electrodes. The charge density in that section could get from:

$$\rho_n = \frac{q_n}{(x_n^e - x_n^b)} \tag{6.5}$$

Notice, the approach to $\varphi_1(x)$ is same as for the case when the depth of the substrate is infinity, so the item $\varphi_2(x)$ is the modification due to the limited depth of the substrate.

7. Simulation:

Based on above results, we developed a Matlab program to simulate the charge density distribution at the surface of electrode. The assumption for this simulation is:

Electrode 1: $x_{ini}^{(1)}$=0, $x_{fin}^{(1)}$ =1; Electrode 2: $x_{ini}^{(2)}$=3, $x_{fin}^{(2)}$=5; ε_r=3; substrate depth D=0.1.

The potential at electrode 1 is: -1(v), the potential in electrode 2 is: +1(v).
Kdmin=0.01, kdmax=15. The numerical integration is use Matlab function quadl().

The figure 7.1 is the simulation results with section width as 0.1. So in this simulation, there are 10 sections in electrode 1, and 20 sections in electrode 2.

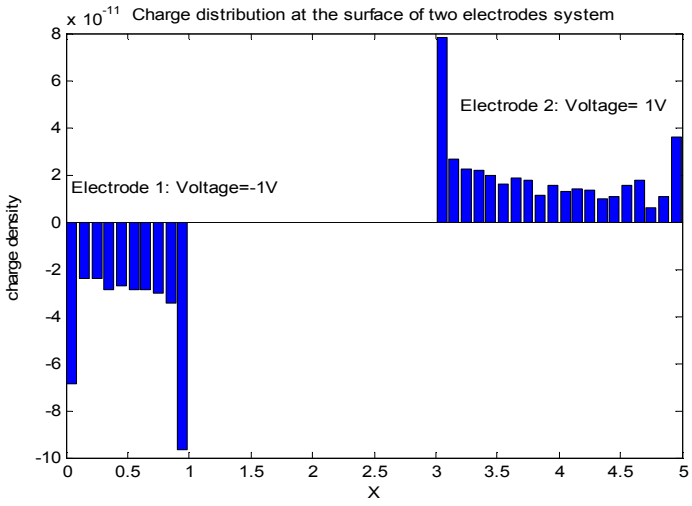


Figure 7.1 Simulation results with section width=0.1

From above simulation result, we could find that the charge density at the edge is much higher than the middle point of the electrode. This result meets our intuitive expectation. This could be shown more clear with more delicate diction division. The results from section width=0.02 shown in following graph.

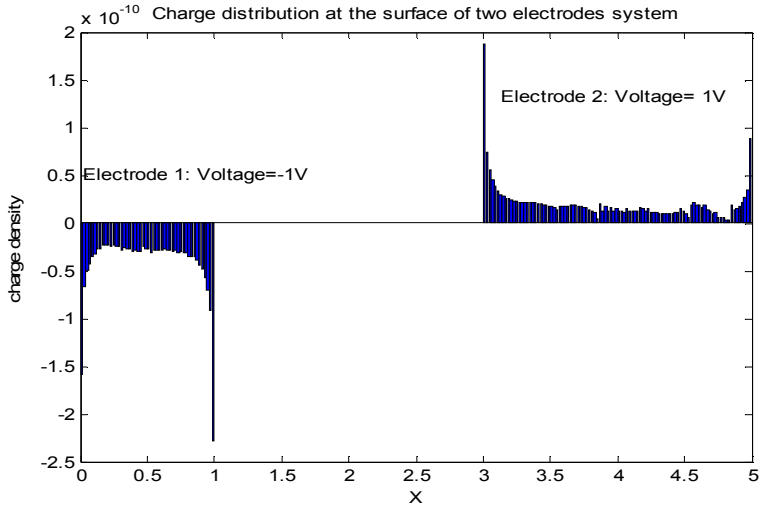


Figure 7.2 Simulation results with section width=0.02

To further improve simulation accuracy, we could use non-equal section division. In this simulation, for the reason of limited time, we use the Matlab numerical integration function quadl(). This function may not be so accurate and robust in this oscillated integration function. If we develop some better integration function in future, we may get a better result.

8. Conclusion:

In this paper, we simulated the surface charge density distribution of the electrodes which place at the interface of two domains. The upper domain is a semi-infinite free space, the lower domain is a substrate domain has a limited depth. The simulation is based on the Green's function and the Method of Moments.

Because the difficult to get Green's function's closed-form in the real space domain, we made a sectioned process for the deduced Green's function. At the high wave number section, the Green's function's approximation from could have a real space closed-form which has the same form with the substrate depth is infinite. So the matrix coefficients in MOM could be calculated analytically.

In the lower wave number section, we follow the frequency domain process method shown in the reference [1]. The MOM matrix coefficients in this section could only calculated with numerical integration. We have shown that the final potential at the surface of the electrodes could be the addition of above two parts. The physical means for it is that the limited depth substrate could be regards as the infinite depth case add a modification item. In this way, we could reduce computing load significantly.

In the final part, we simulated a two-electrode system at the same structure with Matlab program. The simulation results satisfy our expectation. The simulation accuracy could be increased further if we use the non-equal section division and better numerical integration program. Although the simulation is for the two-electrode system, the deduced results could apply to any number electrode system at the same structure.

9. Acknowledgment:

All the work in this paper is under the guidance of the Prof. A.R. Baghai-Wadji. The author expresses his sincere thanks to Prof. A.R. Baghai-Wadji.

10. References:

[1] A. R. Baghai-Wsdji, Oswald Manner, R. Ganb-Puchstein, “Analysis and Measurement of Transducer End Radiation in SAW Filters on Strongly Coupling Substrates”, IEEE Transactions on Microwave Theory and Techniques, VOL.37,No.1, Jan. 1989. pp150-158.
[2]Prof. A. R. Baghai-Wsdji, “EEET2147 Electronic Devices&Physical Devices Simulation” class notes.

11. Appendix--- Matlab Source Code:

```
% File: sectionedgreen.m
% Copywrite by Wei Han, Date: 19/10/2009
% Initialization
epsi0=8.854*10^(-12);epsir=3;
d=0.1; kdmax=15; kmax=kdmax/d;
kdmin=0.01; kmin=kdmin/d;
x1ini=0; x1fin=1; x2ini=3; x2fin=5; xstep=0.1; delta=xstep/2;
V1=-1; V2=1;

x1=x1ini:xstep:x1fin;    %electrode 1 section
x2=x2ini:xstep:x2fin;    %electrode 2 section

x1field=zeros([1 length(x1)-1]);
x2field=zeros([1 length(x2)-1]);

for i=1:length(x1)-1
```

```
    x1field(i)=(x1(i)+x1(i+1))/2;    %section middle point in electorde 1
end

for i=1:length(x2)-1
    x2field(i)=(x2(i)+x2(i+1))/2;    %section middle point in electrode2
end

xfield = [x1field x2field];
N=length(xfield);N1=length(x1field);N2=length(x2field); %total sections number, electrode1 and electrode2
section number
pfield = zeros([1 N]); q=zeros([1 N]);
A=zeros([N    N]); A1=zeros([N    N]); A2=zeros([N    N]);

for i=1:length(x1field)
    pfield(i)=V1;                        %electrode1 potnetial init
end
for i=length(x1field)+1:N
    pfield(i)=V2;                        %electorde2 potential init
end

beta = -1/pi/epsi0/(1+epsir);
for i=1:N                %High wave number part coefficient calculation
    xp=xfield(i);
    for j=1:N1
A1(i,j)=((xp-x1(j+1))*log(abs(xp-x1(j+1)))-(xp-x1(j))*log(abs(xp-x1(j)))+(x1(j)+x1(j+1)))*beta/(x1(j)-x1(j+1));
    End

    for j=1:N2
A1(i,N1+j)=((xp-x2(j+1))*log(abs(xp-x2(j+1)))-(xp-x2(j))*log(abs(xp-x2(j)))+(x2(j)+x2(j+1)))*beta/(x2(j)-x2(j+1
));
    end
end

for i=1:N                %Low wave number part coefficient calculation
    xm=xfield(i);
    for j=1:N1
        xl=x1field(j);
        A2(i,j) = quadl(@(k)Kmlfun(k,xm,xl),kmin,kmax);
    end
    for j=1:N2
        xl=x2field(j);
        A2(i,N1+j)=quadl(@(k)Kmlfun(k,xm,xl),kmin,kmax);
    end
end
```

```
A=A1+A2;                %final coefficient calculation
q=pfield/A; q=q./xstep;    %charge density
bar(xfield,q,'b');
xlabel('X'); ylabel('charge density');
title('Charge distribution at the surface of two electrodes system');
gtext('Electrode 1: Voltage=-1V');
gtext('Electrode 2: Voltage= 1V');
```

```
% File: Kmlfun.m
% Copywrite by Wei Han, Date: 19/10/2009
function y=Kmlfun(k,m,l)
epsi0=8.854*10^(-12);epsir=3;
d=0.1; kdmax=20; kmax=kdmax/d;
xstep=0.1; delta=xstep/2;
y = (sinc(delta*k)).^2.*cos(k.*abs(m-l))./epsi0./k/pi./(1+epsir*coth(k*d));
```