Examples and Exercises for bifurcations

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19 March 2025

Set all interval in Integrator to 100 or more. Remember to open Window/Output → Graphic → 2D plot. Choose axis by clicking MatCont → Layout, Abscissa (x-axis) and Ordinate (y-axis).

1. Saddle node normal form
   `x'=a+x^2`

   for $a \in [-0.5, 0.5]$.

   Starts from $a = -0.3, x = 0$.

2. Global average temperature model (saddle-node bifurcations) from this source
   `T'=Q*(1-(0.5+0.2*tanh(0.1*(265-T))))-EPS*SIGMA*T^4`

   for (atmospheric emissivity factor) $\varepsilon = 0.6$, (constant of proportionality) $\sigma = 5.67 * 10^-8$, and (annual global mean incoming solar radiation) $Q \in [250, 450]$.

   Change `RelTolarence = 1e-5`, in the integrator window for better convergence and `MaxStepsize = 2`, in Continuer for faster computation. Starts from $Q = 250, T = 300$.

3. Discretization of Bratu-Gelfand equation (Pitchfork bifurcation)
   `U1'=-2*U1+U2+LAMBDA*exp(U1)`

   `U2'=U1-2*U2+LAMBDA*exp(U2)`

   for LAMBDA $\in [0, 0.5]$.

   Start from equilibrium point $U1 = U2 = 0$ at LAMBDA$= 0$.

4. Alternative predator-prey model (supercritical Hopf bifurcation)
   `F'=r*F*(1-F/K)-a*F*C/(1+a*h*F)`

   `C'=EPS*a*F*C/(1+a*h*F)-muu*C`

   for $\mu = 0.1, a = 5, h = 3, \varepsilon = 0.5, r = 0.5$ and $K, C \in [0, 0.4]$.

   Starts from $F = C = -0.3, K = 0.25$, set integrator `RelTolerance = AbsTolerance = 1e-10`, and `interval = 10000`. Continuer `MinStepsize = MaxStepsize = 0.5`, and `MaxNumPoints = 100`.

5. Aircraft Wing Flutter reformulated (subcritical Hopf bifurcation) from this source

   `x'= y`

   `y'=(a+x^2-0.4*x^4)*y-x`

   for $a \in [-1, 1], x \in [-3, 3]$.

   Starts from equilibrium point $x = y = 0$ at $a = -0.5$, or from $x = y = 0.1$ to see that $x = y = 0$ is a stable fixed point at $a = -0.5$. In the Continuer window set `MinStepsize = MaxStepsize = 0.5`.