



$$r_+ \approx r - \frac{d}{2} \cos \theta$$

$$\frac{1}{r_+} \approx \left(r - \frac{d}{2} \cos \theta \right)^{-1}$$

$$= \frac{1}{r} \left(1 - \frac{d}{2r} \cos \theta \right)^{-1}$$

$$\approx \frac{1}{r} \left(1 + \frac{d}{2r} \cos \theta \right)$$

$$r_- \approx r + \frac{d}{2} \cos \theta$$

$$\frac{1}{r_-} \approx \left(r + \frac{d}{2} \cos \theta \right)^{-1}$$

$$= \frac{1}{r} \left(1 + \frac{d}{2r} \cos \theta \right)^{-1}$$

$$\approx \frac{1}{r} \left(1 - \frac{d}{2r} \cos \theta \right)$$

\therefore we assume
 $r \gg d$

$$\phi(P) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_+} - \frac{1}{r_-} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \times \frac{d \cos \theta}{r^2}$$

$$= \frac{\vec{p} \cdot \hat{a}_r}{4\pi\epsilon_0 r^2}$$

$$\left(\frac{1}{r_+} - \frac{1}{r_-} \right) = \cancel{\frac{1}{r}} + \frac{d}{2r^2} \cos \theta - \cancel{\frac{1}{r}} + \frac{d}{2r^2} \cos \theta$$

$$= \frac{d}{r^2} \cos \theta$$