f is conti on $[a, b]$

and $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$ at some pt $x \in (a, b)$

g is defined on an interval I which contains the range of f, and g 在 $f(x)$ 可微分

If $h(t) \triangleq g(f(t))$, $t \in (a,b)$

Then h 在 x 可微分，並且

$h'(x) = g'(f(x))\, f'(x)$

$$f(x+h) = f(x) + f'(x)h + \epsilon_1 h$$

$\epsilon_1 \to 0$ as $h \to 0$

$$g(y+k) = g(y) + g'(y)k + \epsilon_2 k$$

Here, $y=f(x)$

$\epsilon_2 \to 0$ as $k \to 0$

$$g(f(x+h)) = g(\underbrace{f(x)+f'(x)h+\epsilon_1 h}_{})$$

$$= g(f(x)) + g'(f(x))[f'(x)h+\epsilon_1 h] + \epsilon_2[f'(x)h+\epsilon_1 h]$$

$$= g(f(x)) + g'(f(x))f'(x)h + \epsilon_1 g'(f(x))h + \epsilon_2 f'(x)h + \epsilon_1\epsilon_2 h$$

注意：f 在 $[a,b]$ conti 是必要的

當 $h\to 0$, $f(x+h) \to f(x)$

$k \to 0$

$$\frac{g(f(x+h)) - g(f(x))}{h} = g'(f(x))f'(x) + \underbrace{\epsilon_1 g'(f(x)) + \epsilon_2 f'(x) + \epsilon_1\epsilon_2}_{\text{this terms } \to 0 \text{ as } h\to 0}$$

$$\lim_{h\to 0} \frac{g(f(x+h))-g(f(x))}{h} = g'(f(x))f'(x)$$