Taylor Expansion

$$f(z) = \frac{1}{2\pi i}\oint_C \frac{f(z')\,dz'}{z' - z}$$

$$= \frac{1}{2\pi i}\oint_C \frac{f(z')\,dz'}{(z'-z_0)-(z-z_0)}$$

$$= \frac{1}{2\pi i}\oint_C \frac{f(z')\,dz'}{(z'-z_0)\left(1 - \frac{z-z_0}{z'-z_0}\right)}$$

$$= \frac{1}{2\pi i}\oint_C \frac{f(z')}{z'-z_0}\left[1 + \frac{z-z_0}{z'-z_0} + \left(\frac{z-z_0}{z'-z_0}\right)^2 + \cdots\right] dz'$$

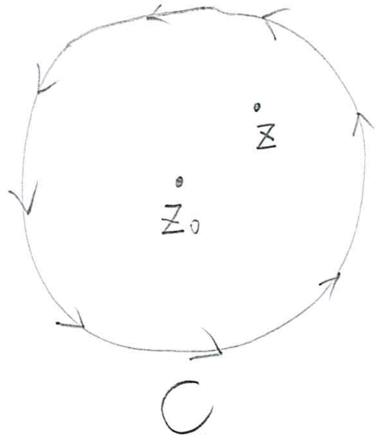
$$= \frac{1}{2\pi i}\oint_C \left[\frac{f(z')}{z'-z_0} + \frac{f(z')}{(z'-z_0)^2}(z-z_0) + \frac{f(z')}{(z'-z_0)^3}(z-z_0)^2\right] dz'$$

$$= \left[\frac{1}{2\pi i}\oint_C \frac{f(z')\,dz'}{z'-z_0}\right] + \left[\frac{1}{2\pi i}\oint_C \frac{f(z')\,dz'}{(z'-z_0)^2}\right](z-z_0) + \left[\frac{1}{2\pi i}\oint_C \frac{f(z')\,dz'}{(z'-z_0)^3}\right](z-z_0)^2 + \cdots$$

$$= \sum_{n=0}^{\infty}\left[\frac{1}{2\pi i}\oint_C \frac{f(z')\,dz'}{(z'-z_0)^{n+1}}\right](z-z_0)^n$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{n!}{2\pi i}\oint \frac{f(z')\,dz'}{(z'-z_0)^{n+1}}\right](z-z_0)^n$$

$$= \sum_{n=0}^{\infty}\frac{f^{(n)}(z_0)}{n!}(z-z_0)^n$$



勞倫展開 Laurent Expansion

$$f(z)=\frac{1}{2\pi i}\oint_C \frac{f(z')\,dz'}{z'-z} = \frac{1}{2\pi i}\oint_{C_1}\frac{f(z')\,dz'}{(z'-z_0)\left(1-\frac{z-z_0}{z'-z_0}\right)} - \frac{1}{2\pi i}\oint_{C_2}\frac{f(z')\,dz'}{(z-z_0)\left(\frac{z'-z_0}{z-z_0}-1\right)}$$

$$=\sum_{n=0}^{\infty}\left[\frac{1}{2\pi i}\oint_{C_1}\frac{f(z')\,dz'}{(z'-z_0)^{n+1}}\right](z-z_0)^n + \frac{1}{2\pi i}\oint_{C_2}\frac{f(z')}{z-z_0}\left[1+\frac{z'-z_0}{z-z_0}+\left(\frac{z'-z_0}{z-z_0}\right)^2+\cdots\right]dz'$$

$$= \cdots + \left\{\left[\frac{1}{2\pi i}\oint_{C_2} f(z')\,dz'\right](z-z_0)^{-1} + \left[\frac{1}{2\pi i}\oint_{C_2} f(z')(z'-z_0)\,dz'\right](z-z_0)^{-2} + \left[\frac{1}{2\pi i}\oint_{C_2}\frac{f(z')\,dz'}{(z'-z_0)^{-2}}\right](z-z_0)^{-3}+\cdots\right\}$$

$$=\sum_{n=0}^{\infty}\left[\frac{1}{2\pi i}\oint_{C_1}\frac{f(z')\,dz'}{(z'-z_0)^{n+1}}\right](z-z_0)^n$$

$$+\sum_{n=-1}^{-\infty}\left[\frac{1}{2\pi i}\oint_{C_2}\frac{f(z')\,dz'}{(z'-z_0)^{n+1}}\right](z-z_0)^n$$