

$$d\vec{r} = \hat{x} dx + \hat{y} dy + \hat{z} dz$$

$$dz = dx dy dz$$

$$\vec{\nabla} V = \hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\vec{\nabla}_x \vec{E} = \begin{pmatrix} \hat{x} \left(\frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial z} \right) \\ \hat{y} \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \\ \hat{z} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \end{pmatrix} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

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$$d\vec{r} = \hat{r} dr + \hat{\theta} r d\theta + \hat{\phi} r \sin\theta d\phi$$

$$dz = r^2 \sin\theta dr d\theta d\phi$$

$$\vec{\nabla} V = \hat{r} \frac{\partial V}{\partial r} + \hat{\theta} \frac{\partial V}{r \partial \theta} + \hat{\phi} \frac{\partial V}{r \sin\theta \partial \phi}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta E_\theta) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} E_\phi$$

$$\vec{\nabla}_x \vec{E} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r \hat{\theta} & r \sin\theta \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ E_r & r E_\theta & r \sin\theta E_\phi \end{vmatrix} = \begin{pmatrix} \hat{r} \left[\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r} (r^2 E_r) - \frac{\partial E_\theta}{r \sin\theta \partial \phi} \right] \\ + \hat{\theta} \left[\frac{\partial E_r}{r \sin\theta \partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \right] \\ + \hat{\phi} \left[\frac{1}{r} \frac{\partial}{\partial r} (r E_\theta) - \frac{\partial E_r}{r \partial \theta} \right] \end{pmatrix}$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$

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$$d\vec{r} = \hat{s} ds + \hat{\phi} s d\phi + \hat{z} dz, \quad dz = s ds d\phi dz$$

$$\vec{\nabla} V = \hat{s} \frac{\partial V}{\partial s} + \hat{\phi} \frac{\partial V}{s \partial \phi} + \hat{z} \frac{\partial V}{\partial z}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{s} \frac{\partial}{\partial s} (s E_s) + \frac{\partial E_\phi}{s \partial \phi} + \frac{\partial E_z}{\partial z}$$

$$\vec{\nabla}_x \vec{E} = \frac{1}{s} \begin{vmatrix} \hat{s} & s \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ E_s & s E_\phi & E_z \end{vmatrix} = \begin{pmatrix} \hat{s} \left(\frac{\partial E_s}{\partial s} - \frac{\partial E_\phi}{\partial z} \right) \\ + \hat{\phi} \left(\frac{\partial E_s}{\partial z} - \frac{\partial E_z}{\partial s} \right) \\ + \hat{z} \left(\frac{1}{s} \frac{\partial}{\partial s} (s E_\phi) - \frac{\partial E_s}{s \partial \phi} \right) \end{pmatrix}$$

$$\nabla^2 V = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V}{\partial s} \right) + \frac{\partial^2 V}{s^2 \partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

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$$\vec{\nabla} \cdot \vec{E} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta E_\theta) + \frac{1}{r \sin \theta} \frac{\partial E_\phi}{\partial \phi}$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

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$$\vec{\nabla} \cdot \vec{E} = \frac{1}{s} \frac{\partial}{\partial s} (s E_s) + \frac{1}{s} \frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z}$$

$$\nabla^2 V = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

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$$\begin{aligned}\vec{\nabla} \times \vec{E} = & \hat{r} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} (\sin\theta E_\phi) - \frac{\partial}{\partial\phi} (E_\theta) \right] \\ & + \hat{\theta} \left[\frac{\partial}{\partial\phi} (E_r) - \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \right] \\ & + \hat{\phi} \left[\frac{1}{r} \frac{\partial}{\partial r} (r E_\theta) - \frac{\partial}{\partial\theta} (E_r) \right]\end{aligned}$$

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$$\begin{aligned}\vec{\nabla} \times \vec{E} = & \hat{s} \left[\frac{\partial}{\partial\phi} (E_z) - \frac{\partial}{\partial z} (E_\phi) \right] \\ & + \hat{\phi} \left[\frac{\partial}{\partial z} (E_s) - \frac{\partial}{\partial s} (E_z) \right] \\ & + \hat{z} \left[\frac{1}{s} \frac{\partial}{\partial s} (s E_\phi) - \frac{\partial}{\partial\phi} (E_s) \right]\end{aligned}$$