

$$\begin{aligned}
 -\frac{\partial}{\partial t} \int_V u dv &= \int_{\Sigma} \vec{J} \cdot \hat{n} da + \left( \text{对 } V \text{ 内部物质所做的功} \right) \\
 &= \int_V \vec{E} \cdot \vec{J} dv \\
 &= \int_V \nabla \cdot \vec{J} dv + \int_V \vec{E} \cdot \vec{J} dv
 \end{aligned}$$

$$-\frac{\partial u}{\partial t} = \nabla \cdot \vec{J} + \vec{E} \cdot \vec{J}$$

$$\vec{E} \cdot \vec{J} = -\frac{\partial u}{\partial t} - \nabla \cdot \vec{J}$$

$$\begin{aligned}
 \nabla \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \\
 \vec{J} &= \frac{1}{\mu_0} \nabla \times \vec{B} - \epsilon_0 \frac{\partial \vec{E}}{\partial t}
 \end{aligned}$$

$$\begin{aligned}
 \vec{E} \cdot \frac{1}{\mu_0} (\nabla \times \vec{B}) - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} &\leftarrow \\
 &\rightarrow -\frac{\partial}{\partial t} \left( \frac{\epsilon_0 E^2}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 &\nabla \cdot (\vec{E} \times \vec{B}) \\
 &= \nabla_E \cdot (\vec{E} \times \vec{B}) + \nabla_B \cdot (\vec{E} \times \vec{B}) \\
 &= \vec{B} \cdot (\nabla_E \times \vec{E}) + \vec{E} \cdot (\vec{B} \times \nabla_B) \\
 &= \vec{B} \cdot (\nabla_E \times \vec{E}) - \vec{E} \cdot (\nabla_B \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{B})
 \end{aligned}$$

$$\begin{aligned}
 &\rightarrow \frac{1}{\mu_0} \vec{E} \cdot (\nabla \times \vec{B}) \\
 &= \frac{1}{\mu_0} [\vec{B} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{B})] \\
 &= \frac{1}{\mu_0} \left[ \vec{B} \cdot \left( -\frac{\partial \vec{B}}{\partial t} \right) - \nabla \cdot (\vec{E} \times \vec{B}) \right] \\
 &= -\frac{\partial}{\partial t} \left( \frac{B^2}{2\mu_0} \right) - \nabla \cdot (\vec{E} \times \vec{B})
 \end{aligned}$$

$$\begin{aligned}
 &\frac{\partial}{\partial t} (\vec{E} \cdot \vec{E}) \\
 &= \frac{\partial \vec{E}}{\partial t} \cdot \vec{E} + \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}
 \end{aligned}$$

$$\frac{\partial \vec{E}}{\partial t} \cdot \vec{E} = \frac{1}{2} \frac{\partial E^2}{\partial t}$$

$$\begin{aligned}
 &\frac{\partial}{\partial t} (\vec{B} \cdot \vec{B}) \\
 &= \frac{\partial \vec{B}}{\partial t} \cdot \vec{B} + \vec{B} \cdot \frac{\partial \vec{B}}{\partial t}
 \end{aligned}$$

$$\frac{\partial \vec{B}}{\partial t} \cdot \vec{B} = \frac{1}{2} \frac{\partial B^2}{\partial t}$$

$$\begin{aligned}
 &-\frac{\partial u}{\partial t} - \nabla \cdot \vec{J} \\
 &= -\frac{\partial}{\partial t} \left[ \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right] - \nabla \cdot (\vec{E} \times \vec{B}) \\
 &= \vec{E} \cdot \vec{J}
 \end{aligned}$$

$$\begin{aligned}
 &-\frac{\partial}{\partial t} \left[ \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right] \leftarrow \text{减少的电磁能} \\
 &= \nabla \cdot (\vec{E} \times \vec{B}) + \vec{E} \cdot \vec{J} \leftarrow \begin{array}{l} \text{转移给物质} \\ \text{的部分} \end{array} \\
 &\quad \uparrow \\
 &\quad \text{离开的部分}
 \end{aligned}$$