$$-\frac{\partial}{\partial t}\int u\,dv = \int_\Sigma \vec{S}\cdot\hat{n}\,da + \left(\text{对V内部物質所做的功}\right)$$

$$= \int \vec{E}\cdot\vec{J}\,dv$$

$$= \int \nabla\cdot\vec{S}\,dv + \int \vec{E}\cdot\vec{J}\,dv$$

$$\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) = \frac{\partial\vec{E}}{\partial t}\cdot\vec{E} + \vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\Downarrow$$

$$\frac{\partial\vec{E}}{\partial t}\cdot\vec{E} = \frac{1}{2}\frac{\partial}{\partial t}E^2$$

$$\frac{\partial}{\partial t}(\vec{B}\cdot\vec{B}) = \frac{\partial\vec{B}}{\partial t}\cdot\vec{B} + \vec{B}\cdot\frac{\partial\vec{B}}{\partial t}$$

$$\Downarrow$$

$$\frac{\partial\vec{B}}{\partial t}\cdot\vec{B} = \frac{1}{2}\frac{\partial}{\partial t}B^2$$

$$-\frac{\partial u}{\partial t} = \nabla\cdot\vec{S} + \vec{E}\cdot\vec{J}$$

$$\vec{E}\cdot\vec{J} = -\frac{\partial u}{\partial t} - \nabla\cdot\vec{S}$$

$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{J} = \frac{1}{\mu_0}\nabla\times\vec{B} - \epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{E}\cdot\frac{1}{\mu_0}(\nabla\times\vec{B}) - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\hookrightarrow -\frac{\partial}{\partial t}\left(\frac{\epsilon_0 E^2}{2}\right)$$

$$\frac{1}{\mu_0}\vec{E}\cdot(\nabla\times\vec{B})$$

$$= \frac{1}{\mu_0}\left[\vec{B}\cdot(\nabla\times\vec{E}) - \nabla\cdot(\vec{E}\times\vec{B})\right]$$

$$= \frac{1}{\mu_0}\left[\vec{B}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) - \nabla\cdot(\vec{E}\times\vec{B})\right]$$

$$= -\frac{\partial}{\partial t}\left(\frac{B^2}{2\mu_0}\right) - \nabla\cdot(\vec{E}\times\vec{H})$$

$$-\frac{\partial u}{\partial t} - \nabla\cdot\vec{S}$$

$$= -\frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right] - \nabla\cdot(\vec{E}\times\vec{H})$$

$$= \vec{E}\cdot\vec{J}$$

$$\nabla\cdot(\vec{E}\times\vec{B})$$

$$= \nabla_E\cdot(\vec{E}\times\vec{B}) + \nabla_B\cdot(\vec{E}\times\vec{B})$$

$$= \vec{B}\cdot(\nabla_E\times\vec{E}) + \vec{E}\cdot(\vec{B}\times\nabla_B)$$

$$= \vec{B}\cdot(\nabla_E\times\vec{E}) - \vec{E}\cdot(\nabla_B\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{B})$$

減少的電磁能

$$-\frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right]$$

$$= \nabla\cdot(\vec{E}\times\vec{H}) + \vec{E}\cdot\vec{J}$$

離開的部份

轉移給物質的部份