

$\hat{r}, \hat{\theta}, \hat{\phi}$ are associated with a particular point P ,
and change direction as P moves around.

I claimed on very first page that vectors have no location,
and I'll stand by that.

The vectors themselves live out there, completely independent of
our choice of coordinates.

But the notation we use to represent them
does depend on the point in question, in curvilinear
coordinates.