$$\begin{cases} i(z,t) = I(z)\,e^{j\omega t} \\ v(z,t) = V(z)\,e^{j\omega t} \end{cases}$$

$$\begin{cases} V(z) = V_R(z) + j\,V_I(z) \\ I(z) = I_R(z) + j\,I_I(z) \end{cases}$$

$$v(z,t) = Re\left\{ V(z)\,e^{j\omega t} \right\}$$

$$= Re\left\{ (V_R + jV_I)(\cos\omega t + j\sin\omega t) \right\}$$

$$= V_R\cos\omega t - V_I\sin\omega t$$

同理 $i(z,t) = I_R\cos\omega t - I_I\sin\omega t$

$v(z,t)\,i(z,t)$

$$= \left[V_R\cos\omega t - V_I\sin\omega t\right]\left[I_R\cos\omega t - I_I\sin\omega t\right]$$

$$= V_R I_R\cos^2\omega t - (V_I I_R + V_R I_I)\cos\omega t\sin\omega t + V_I I_I\sin^2\omega t$$

$$\frac{1}{T}\int_0^T v\,i\,dt = \frac{1}{2}\left(V_R I_R + V_I I_I\right)$$ ①

$$VI^* = (V_R + jV_I)(I_R - jI_I)$$

$$= \left[V_R I_R + V_I I_I\right] + j\left[\cdots\right]$$

$$Re\left[\frac{1}{2}VI^*\right] = \frac{1}{2}\left(V_R I_R + V_I I_I\right)$$ ②

$$P_{av}(z) = \frac{1}{T}\int_0^T v\,i\,dt$$

$$= Re\left\{\frac{1}{2}VI^*\right\}$$

也就是說 $P_{av}(z) \times T$ 代表位置 z 在經過一個週期後所通過的能量

代表通過位置 z 的平均功率