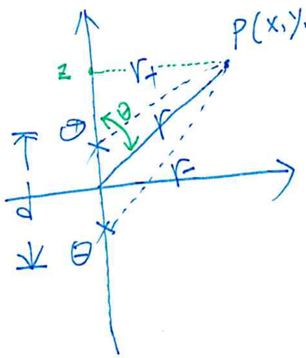
We assume $r >> d$



$$\phi(P) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_+} + \frac{-q}{r_-}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$

$$= \frac{q}{4\pi\epsilon_0} \times \frac{d}{r^2}\cos\theta$$

$$= \frac{qd\cos\theta}{4\pi\epsilon_0 r^2}$$

$$r_+ = \left[\left(z-\frac{d}{2}\right)^2 + x^2 + y^2\right]^{\frac{1}{2}}$$

$$= \left[z^2\left(1-\frac{d}{2z}\right)^2 + x^2 + y^2\right]^{\frac{1}{2}}$$

$$\cong \left[z^2\left(1-\frac{d}{z}\right) + x^2 + y^2\right]^{\frac{1}{2}}$$

$$= \left(x^2+y^2+z^2 - zd\right)^{\frac{1}{2}}$$

$$= \left(r^2 - zd\right)^{\frac{1}{2}}$$

$$= r\left(1 - \frac{zd}{r^2}\right)^{\frac{1}{2}}$$

$$\frac{1}{r_+} = \frac{1}{r}\left(1-\frac{zd}{r^2}\right)^{-\frac{1}{2}}$$

$$\cong \frac{1}{r}\left(1 + \frac{zd}{2r^2}\right)$$

$$r_- = \left[\left(z+\frac{d}{2}\right)^2 + x^2 + y^2\right]^{\frac{1}{2}}$$

$$= \left(z^2\left(1+\frac{d}{2z}\right)^2 + x^2 + y^2\right)^{\frac{1}{2}}$$

$$\cong \left[z^2\left(1+d/z\right) + x^2 + y^2\right]^{\frac{1}{2}}$$

$$= \left(x^2+y^2+z^2 + zd\right)^{\frac{1}{2}}$$

$$= \left(r^2 + zd\right)^{\frac{1}{2}}$$

$$= r\left(1 + \frac{zd}{r^2}\right)^{\frac{1}{2}}$$

$$\frac{1}{r_-} = \frac{1}{r}\left(1+\frac{zd}{r^2}\right)^{-\frac{1}{2}}$$

$$\cong \frac{1}{r}\left(1 - \frac{zd}{2r^2}\right)$$

because $r >> d$

so $z >> d$

$\because r >> d$

$\therefore r^2 >> zd$

$$\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \cancel{\frac{1}{r}} + \frac{zd}{2r^3} - \cancel{\frac{1}{r}} + \frac{zd}{2r^3}$$

$$= \frac{zd}{r^3} = \frac{d}{r^2} \times \frac{z}{r} = \frac{d}{r^2}\cos\theta$$

$\because \frac{z}{r} = \cos\theta$

If we defined $qd = p$

$\vec{p} = p\hat{a}_z$

$$\phi(P) = \frac{\vec{p}\cdot\hat{a}_r}{4\pi\epsilon_0 r^2}$$