$$\begin{cases} \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = f'(x) \\ \lim_{h\to 0} \dfrac{g(x+h)-g(x)}{h} = g'(x) \end{cases} \Longrightarrow \boxed{\begin{aligned} f(x+h) &= f(x)+f'(x)h+\epsilon_1 h \\ g(x+h) &= g(x)+g'(x)h+\epsilon_2 h \end{aligned} \quad \begin{aligned} &\epsilon_1 、\epsilon_2 \to 0 \\ &\text{as } h\to 0 \end{aligned}}$$

這是前提
也就是已知條件

$$\boxed{\begin{aligned} &f(x+h)-f(x) = f'(x)h+\epsilon_1 h \\ &\epsilon_1 \to 0 \text{ as } h\to 0 \end{aligned}}$$

$$\boxed{\begin{aligned} &g(x+h)-g(x) = g'(x)h+\epsilon_2 h \\ &\epsilon_2 \to 0 \text{ as } h\to 0 \end{aligned}}$$

$$\begin{aligned} f(x+h)g(x+h) = {} & f(x)g(x)+f(x)g'(x)h+f(x)\epsilon_2 h \\ & + f'(x)g(x)h+f'(x)g'(x)h^2+f'(x)\epsilon_2 h^2 \\ & + g(x)\epsilon_1 h+g'(x)\epsilon_1 h^2+\epsilon_1\epsilon_2 h^2 \end{aligned}$$

$$f(x+h)g(x+h)-f(x)g(x) / h$$

∵ this term → 0 as $h\to 0$

$$= f(x)g'(x)+f'(x)g(x)+\underbrace{\epsilon_2 f(x)+\epsilon_1 g(x)}+\underbrace{(f'(x)g'(x)+\epsilon_2 f'(x)+\epsilon_1 g'(x)+\epsilon_1\epsilon_2)h}$$

∵這些東西

$$\lim_{h\to 0} \frac{f(x+h)g(x+h)-f(x)g(x)}{h} = f(x)g'(x)+f'(x)g(x)$$

$$(fg)' = f'g+fg'$$