$$\begin{cases} V(z) = V_{i0}\, e^{-rz} + V_{r0}\, e^{rz} \\ Z_0 \times I(z) = \dfrac{V_{i0}}{\cancel{Z_0}}\, e^{-rz} - \dfrac{V_{r0}}{\cancel{Z_0}}\, e^{rz} \end{cases}$$

$$\Rightarrow \begin{cases} V_L = V_{i0}\, e^{-r\ell} + V_{r0}\, e^{r\ell} \\ Z_0 \times I_L = \dfrac{V_{i0}}{\cancel{Z_0}}\, e^{-r\ell} - \dfrac{V_{r0}}{\cancel{Z_0}}\, e^{r\ell} \end{cases}$$

$$\Rightarrow \begin{cases} V_{i0} = \dfrac{1}{2}(V_L + Z_0 I_L)\, e^{r\ell} \\ V_{r0} = \dfrac{1}{2}(V_L - Z_0 I_L)\, e^{-r\ell} \end{cases}$$

代入 V_{i0} 和 V_{r0}

並有 $V_L = Z_L \times I_L$

$$\Rightarrow \begin{cases} V(z) = \dfrac{1}{2} I_L (Z_L + Z_0)\, e^{r(\ell - z)} + \dfrac{I_L}{2}(Z_L - Z_0)\, e^{-r(\ell - z)} \\ Z_0 \times I(z) = \dfrac{I_L}{2}(Z_L + Z_0)\, e^{r(\ell - z)} - \dfrac{I_L}{2}(Z_L - Z_0)\, e^{-r(\ell - z)} \end{cases}$$

$$\boxed{\text{定義：} \dfrac{V(z)}{I(z)} = Z(z)}$$

$$\frac{Z(z)}{\cancel{Z_0}} = \frac{(Z_L + Z_0)\, e^{r(\ell - z)} + (Z_L - Z_0)\, e^{-r(\ell - z)}}{(Z_L + Z_0)\, e^{r(\ell - z)} - (Z_L - Z_0)\, e^{-r(\ell - z)}} \times Z_0$$

$$= \frac{Z_L\left[e^{r(\ell - z)} + e^{-r(\ell - z)}\right]/\cancel{2} + Z_0\left[e^{r(\ell - z)} - e^{-r(\ell - z)}\right]/\cancel{2}}{Z_L\left[e^{r(\ell - z)} - e^{-r(\ell - z)}\right]/\cancel{2} + Z_0\left[e^{r(\ell - z)} + e^{-r(\ell - z)}\right]/\cancel{2}} \times Z_0$$

$$= \frac{Z_L \cosh r(\ell - z) + Z_0 \sinh r(\ell - z)}{Z_L \sinh r(\ell - z) + Z_0 \cosh r(\ell - z)} \times Z_0$$

$$\boxed{Z(z) = Z_0\, \frac{Z_L + Z_0 \tanh r(\ell - z)}{Z_0 + Z_L \tanh r(\ell - z)}}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} \times (2n+1)$$

$$= \frac{\pi}{2}(2n+1)$$

$$\tan \beta l \to \infty$$

约分

$$Z_i = R_0 \frac{+jR_0 \cancel{\tan\beta l}}{R_0 + jZ_L \cancel{\tan\beta l}}$$

$$\boxed{Z_i = \frac{R_0^2}{Z_L}}$$

$$\leftarrow l = \frac{\lambda}{4} \times (2n+1)$$

$$Z_i = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

$$\rightarrow l = \frac{\lambda}{2} \times n$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} n = \pi n$$

$$\tan \pi n = 0$$

约分

$$Z_i = \cancel{R_0} \frac{Z_L + \cancel{jR_0 \tan\beta l}}{\cancel{R_0} + \cancel{jZ_L \tan\beta l}}$$

忽略

$$\boxed{Z_i = Z_L}$$

$$\uparrow r = j\beta$$

$$Z_i = Z_0 \frac{Z_L + Z_0 \tanh rl}{Z_0 + Z_L \tanh rl}$$

$$\swarrow Z_L \to \infty \qquad\qquad Z_L \to 0 \searrow$$

约分

$$Z_i = Z_0 \frac{Z_L + \cancel{Z_0 \tanh rl}}{\cancel{Z_0} + Z_L \tanh rl}$$

可忽略

$$\boxed{Z_{i\,open} = \frac{Z_0}{\tanh rl}}$$

if lossless $r = j\beta$ and $Z_0 = \sqrt{\frac{L}{C}} \triangleq R_0$

$$Z_i = \frac{R_0}{\tanh j\beta l}$$

$$\boxed{Z_{i\,open} = \frac{R_0}{j\tan \beta l}}$$

约分

$$Z_i = Z_0 \frac{\cancel{Z_L} + Z_0 \tanh rl}{\cancel{Z_0} + \cancel{Z_L \tanh rl}}$$

可忽略

$$\boxed{Z_{i\,short} = Z_0 \tanh rl}$$

if lossless

$$\boxed{Z_{i\,short} = jR_0 \tan \beta l}$$

$$Z_{i\,open} \times Z_{i\,short} = Z_0^2$$

$$\sqrt{\frac{Z_{i\,short}}{Z_{i\,open}}} = (\tanh rl)^{\cancel{2}}$$

$$Z_0 = \sqrt{Z_{i\,open} \times Z_{i\,short}}$$

$$r = \frac{1}{l} \tanh^{-1}\left[\sqrt{\frac{Z_{i\,short}}{Z_{i\,open}}}\right]$$

$$\boxed{Z_{i_{open}} = \frac{R_o}{j\tan\beta l}}$$

$$\downarrow\ l \ll \lambda$$

$\beta l = \frac{2\pi}{\lambda} l$

if $l \ll \lambda$

then $\beta l \ll 1$

$\tan \beta l \cong \beta l$

$\beta = \frac{2\pi}{\lambda}$

$\omega = \frac{2\pi}{T}$

$\frac{\omega}{\beta} = V_p = \frac{1}{\sqrt{LC}}$

$\beta = \omega\sqrt{LC}$

$$Z_{i_{open}} \cong \frac{R_o}{j\beta l}$$

$$= \frac{\sqrt{\frac{\cancel{L}}{C}}}{j\omega\sqrt{\cancel{L}C}\ l}$$

$$= \frac{1}{j\omega C l}$$

$$\boxed{Z_{i_{short}} = jR_o \tan\beta l}$$

$$\downarrow\ l \ll \lambda$$

$$Z_{i_{short}} = jR_o\beta l$$

$$= j\sqrt{\frac{L}{\cancel{C}}} \times \omega\sqrt{L\cancel{C}}\ l$$

$$= j\omega L l$$