$$R = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

$$\phi(x, y, z)$$

$$= \int \frac{d\vec{p}\cdot\hat{a}_R}{4\pi\epsilon_0 R^2}$$

$$= \int \frac{\vec{p}\,dv\cdot\hat{a}_R}{4\pi\epsilon_0 R^2}$$

$$= \frac{1}{4\pi\epsilon_0}\int \vec{p}\cdot\frac{\hat{a}_R}{R^2}\,dv$$

$$= \frac{1}{4\pi\epsilon_0}\int \vec{p}\cdot\nabla'\left(\frac{1}{R}\right)dv$$

$$\nabla'\cdot\left(\frac{1}{R}\vec{P}\right)$$

$$= \nabla'\left(\frac{1}{R}\right)\cdot\vec{P} + \frac{1}{R}\nabla'\cdot\vec{P}$$

$$\vec{P}\cdot\nabla'\left(\frac{1}{R}\right) = \frac{-1}{R}\nabla'\cdot\vec{P} + \nabla'\cdot\left(\frac{\vec{P}}{R}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left[\int \frac{-1}{R}\nabla'\cdot\vec{p}\,dv + \int \nabla'\cdot\left(\frac{\vec{P}}{R}\right)dv\right]$$

$$\boxed{\oint\!\!\oint_S \frac{\vec{p}\cdot\hat{a}_n\,ds}{R}}$$

$$= \iiint_V \frac{(-\nabla'\cdot\vec{p})\,dv}{4\pi\epsilon_0 R} + \oint\!\!\oint_S \frac{(\vec{p}\cdot\hat{a}_n)\,ds}{4\pi\epsilon_0 R}$$