$$\left.\begin{aligned} \nabla\cdot\vec{E} &= \rho \\ \nabla\times\vec{B} &= \vec{J} + \frac{\partial\vec{E}}{\partial t} \end{aligned}\right\} \Rightarrow \boxed{\partial_\nu F^{\mu\nu} = J^\mu}$$

$$\left.\begin{aligned} \nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{B} &= 0 \end{aligned}\right\} \Rightarrow \boxed{\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$\partial_\nu = \frac{\partial}{\partial x^\nu}$$

$$\nu, \mu, \lambda = 0, 1, 2, 3$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$F_{\alpha\beta} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$\nabla \times \vec{E} =$$

$$\begin{bmatrix} \frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y \\ \frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z \\ \frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x \end{bmatrix}$$

$$F_{23} = B_x = F^{23} = B_1 = \epsilon^{231} B_1$$

$$F_{31} = B_y = F^{31} = B_2 = \epsilon^{312} B_2$$

$$F_{12} = B_z = F^{12} = B_3 = \epsilon^{123} B_3$$

$$\frac{\partial}{\partial x^2} F^{03} + \frac{\partial}{\partial x^3} F^{20} + \frac{\partial}{\partial x^0} F^{23} = 0$$

$$\boxed{\frac{\partial}{\partial x^2} F_{03} + \frac{\partial}{\partial x^3} F_{20} + \frac{\partial}{\partial x^0} F_{32} = 0}$$ 正確

$$\Rightarrow \frac{\partial}{\partial x^2} F_{30} + \frac{\partial}{\partial x^3} F_{02} = -\frac{\partial}{\partial x^0} F_{23}$$

$$\frac{\partial}{\partial x^2} F_{30} + \frac{\partial}{\partial x^3} F_{02} + \frac{\partial}{\partial x^0} F_{23} = 0$$

比較好

$$-\frac{\partial}{\partial x^2} E_z + \frac{\partial}{\partial x^3} E_y \pm \frac{\partial}{\partial t} B_x = 0$$

$$\frac{\partial}{\partial z} E_y - \frac{\partial}{\partial y} E_z = \frac{\partial}{\partial t} B_x$$

$$(\nabla \times \vec{E})^1 = \frac{\partial}{\partial x^2} E^3 - \frac{\partial}{\partial x^3} E^2$$

$$= \frac{\partial}{\partial x^2} F^{03} - \frac{\partial}{\partial x^3} F^{02}$$

$$= \frac{\partial}{\partial x^2} F^{03} + \frac{\partial}{\partial x^3} F^{20}$$

$$\cancel{\frac{\partial}{\partial x^0} F_{32} + \frac{\partial}{\partial x^2} F_{03} + \frac{\partial}{\partial x^3} F_{20} = 0}$$

$$\cancel{\frac{\partial}{\partial x^0} F}$$

$$-\frac{\partial B^1}{\partial t} = -\frac{\partial}{\partial x^0} B^1$$

$$= -\frac{\partial}{\partial x^0} \epsilon^{231} B_1 = -\frac{\partial}{\partial x^0} F^{23}$$

$$\nabla \cdot \vec{E} = \rho$$

$$\nabla \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\partial_j E^j = J^0 \Rightarrow \partial_j F^{0j} = J^0$$

$$\partial_j F^{ij} = J^i + \partial_0 F^{0i}$$

$$\partial_j F^{ij} - \partial_0 F^{0i} = J^i$$

$$\boxed{\partial_j F^{ij} + \partial_0 F^{i0} = J^i}$$

$$\cancel{\partial_\nu F^{\mu\nu} = J^\mu}$$

$$\partial_\nu F^{i\nu} = J^i$$

$$\boxed{\partial_\nu F^{\mu\nu} = J^\mu}$$

$$\partial_i = \frac{\partial}{\partial x^i}$$

$$\partial'_i = \frac{\partial}{\partial x'^i}$$

$$\frac{\partial}{\partial x'^i} = \frac{\partial x^j}{\partial x'^i} \frac{\partial}{\partial x^j}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$F^{0i} = E^i, \quad F^{i0} = -E^i$$

$$F^{ij} = \epsilon^{ijk} B_k, \quad F^{ji} = \epsilon^{jik} B_k = -F^{ij}$$

$$F^{\mu\nu} = -F^{\nu\mu}$$

$$F^{00} = 0, \quad F^{11} = 0, \quad F^{22} = 0, \quad F^{33} = 0$$

$$J^\mu = (J^0, J^1, J^2, J^3) = (\rho, \vec{J})$$

$$\epsilon^{ijk} \Rightarrow \begin{cases} \epsilon^{123} = 1, \ \epsilon^{231}, \ \epsilon^{312} \\ \epsilon^{213} = -1, \ \epsilon^{132}, \ \epsilon^{321} \\ \text{else} = 0 \end{cases}$$

$$F^{12} = \epsilon^{123} B_3 = B_z$$

$$F^{13} = \epsilon^{132} B_2 = -B_y$$

$$F^{23} = \epsilon^{231} B_1 = B_x$$

$$\nabla \times \vec{B}$$

$$= \begin{bmatrix} \frac{\partial}{\partial y} B_z - \frac{\partial}{\partial z} B_y \\ \frac{\partial}{\partial z} B_x - \frac{\partial}{\partial x} B_z \\ \frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x \end{bmatrix}$$

$$(\nabla \times \vec{B})^1 = \frac{\partial}{\partial y} B_z - \frac{\partial}{\partial z} B_y$$

$$= \frac{\partial}{\partial x^2} B_3 - \frac{\partial}{\partial x^3} B_2$$

$$= \frac{\partial}{\partial x^2} \epsilon^{123} B_3 + \frac{\partial}{\partial x^3} \epsilon^{132} B_2$$

$$= \frac{\partial}{\partial x^2} F^{12} + \frac{\partial}{\partial x^3} F^{13}$$

$$= \partial_2 F^{12} + \partial_3 F^{13}$$

$$\boxed{(\nabla \times B)^i = \partial_j F^{ij}}$$

$$(\nabla \times \vec{E})^i = \left(\frac{\partial}{\partial t}\vec{B}\right)^i$$

$$\frac{\partial}{\partial x^2} F_{30} + \frac{\partial}{\partial x^3} F_{02} + \frac{\partial}{\partial x^0} F_{23} = 0$$

$$\frac{\partial}{\partial x^3} F_{10} + \frac{\partial}{\partial x^1} F_{03} + \frac{\partial}{\partial x^0} F_{31} = 0$$

$$\frac{\partial}{\partial x^1} F_{20} + \frac{\partial}{\partial x^2} E_{01} + \frac{\partial}{\partial x^0} F_{12} = 0$$

$$\boxed{\partial_\mu F_{\alpha\beta} + \partial_\alpha F_{\beta\mu} + \partial_\beta F_{\mu\alpha} = 0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\frac{\partial}{\partial x^1} B_x + \frac{\partial}{\partial x^2} B_y + \frac{\partial}{\partial x^3} B_z = 0$$

$$\boxed{\frac{\partial}{\partial x^1} F_{23} + \frac{\partial}{\partial x^2} F_{31} + \frac{\partial}{\partial x^3} F_{12} = 0}$$

&ZZDGÆk=└l" BEBt └H@B └$B∓Đ └╫%CBBD └AC#«* └%A*¦KC∓fA └B"Ł=@BHDDEBBD=└C=B?⌈$?B" K?=e≈=D "' C?=Q" H=E?=D?@b≈└' =ZZZGòN└H?└. ² $B?" (#?à" ?="=*² /" | ?B" C
* ' C└ū?=p" D?└A?└b └² ≈ @ └┼ pā≈≈=σ? └ ² "ū c "ú "τ? ≈ └╧ └┼? ≡= −? ■" à ²²=Ω? "ZZZG_X └n └h C A=$ e ≡

$$\cancel{\frac{\partial x^\beta}{\partial x'_\mu} \frac{\partial}{\partial x^\beta} \frac{\partial x'^\nu}{\partial x^\alpha} A^\alpha - \frac{\partial x^\alpha}{\partial x'_\nu} \frac{\partial}{\partial x^\alpha} \frac{\partial x'^\mu}{\partial x^\alpha} A^\beta}$$

$$\Lambda^\nu_\alpha = \begin{bmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{\frac{\partial x^\beta}{\partial x'_\mu} = \left(-\frac{\partial x^\beta}{\partial x'^0}, \frac{\partial x^\beta}{\partial x'^1}, \frac{\partial x^\beta}{\partial x'^2}, \frac{\partial x^\beta}{\partial x'^3} \right)}$$

$$F'^{\mu\nu} = \partial'^\mu A'^\nu - \partial'^\nu A'^\mu$$

$$= \underset{\frac{\partial}{\partial x'_\mu}}{\partial'^\mu} \Lambda^\nu_\alpha A^\alpha - \underset{\frac{\partial}{\partial x'_\nu}}{\partial'^\nu} \Lambda^\mu_\beta A^\beta$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

~~if~~ $A'^\mu = \Lambda^\mu_\nu A^\nu$

[illegible]

[illegible]

ZZZGÉN?ⁿH?└C≡) G └(ⁿC7└e └"≡G=└H E ⁿC≡' d ≡θ?ⁿd ⁿC?ⁿE ╧#?C "└A└Cⁿb ≡B└B K?≡d ≡eⁿE└C ⁿQⁿH?≡E $ d? └%ⁿZZZG αX=└. B? D≈ 'ⁿBⁿ% é └└G?└HⁿEⁿ

[illegible]

$$A^\mu = (A^0, A^1, A^2, A^3)$$

$$F^{\mu\nu} \triangleq \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial x_0}, \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$$

$$= \left(-\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right)$$

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$

$$\eta_{\mu\nu} x^\nu \triangleq x_\mu$$

$$F'^{\mu\nu} = \partial'^\mu A'^\nu - \partial'^\nu A'^\mu = \frac{\partial x'^\mu}{\partial x^\alpha} \frac{\partial x'^\nu}{\partial x^\beta} F^{\alpha\beta}$$

$$= \frac{\partial}{\partial x'_\mu} \frac{\partial x'^\nu}{\partial x^\alpha} A^\alpha - \frac{\partial}{\partial x'_\nu} \frac{\partial x'^\mu}{\partial x^\beta} A^\beta$$

$$= \frac{\partial x_\beta}{\partial x'_\mu} \frac{\partial}{\partial x_\beta} \frac{\partial x'^\nu}{\partial x^\alpha} A^\alpha - \frac{\partial x_\alpha}{\partial x'_\nu} \frac{\partial}{\partial x_\alpha} \frac{\partial x'^\mu}{\partial x^\beta} A^\beta$$

$$= \frac{\partial x_\beta}{\partial x'_\mu} \frac{\partial x'^\nu}{\partial x^\alpha} \partial^\beta A^\alpha - \frac{\partial x_\alpha}{\partial x'_\nu} \frac{\partial x'^\mu}{\partial x^\beta} \partial^\alpha A^\beta$$

$$= \frac{\partial x'^\mu}{\partial x^\beta} \frac{\partial x'^\nu}{\partial x^\alpha} \partial^\beta A^\alpha - \frac{\partial x'^\nu}{\partial x^\alpha} \frac{\partial x'^\mu}{\partial x^\beta} \partial^\alpha A^\beta$$

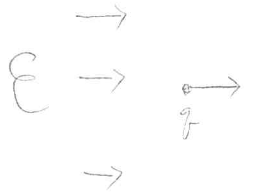
$$\cancel{\frac{\partial x^\nu}{\partial x_\mu} = \left(-\frac{\partial x^\nu}{\partial x^0}, \frac{\partial x^\nu}{\partial x^1}, \frac{\partial x^\nu}{\partial x^2}, \frac{\partial x^\nu}{\partial x^3}\right)}$$

$$=$$

[illegible] ZZZG [illegible] ⁿA? D≡K? f■ â ⁿ?≡β< ⁿ≡ ≡≡D?@" D≡Bⁿ B≡ñ?ⁿ ≡?A? h?
d [illegible] ≈ZZZGûO? G? D≡J 1ⁿⁿ ! A ┬ⁿ≡<≡' # D‖Bⁿ D?≡D≡Bⁿ D? #0β ■e?≤
0? G? D≡1 [illegible]

$$E = mc^2 \quad (v = 0)$$

$$W = \int F\,dx$$



$$F = q\mathcal{E} = \frac{dp}{dt}$$

$$= \frac{d}{dt}\left(\frac{mv}{\sqrt{1-v^2}}\right)$$

$$m\int_0^t \frac{d}{dt}\frac{v}{\sqrt{1-v^2}}\,dt = \int_0^t q\mathcal{E}\,dt$$

$$\boxed{\frac{mv}{\sqrt{1-v^2}} = q\mathcal{E}t}$$

$$\frac{m^2v^2}{1-v^2} = q^2\mathcal{E}^2t^2$$

$$m^2v^2 = q^2\mathcal{E}^2t^2 - v^2q^2\mathcal{E}^2t^2$$

$$v^2\left[m^2 + q^2\mathcal{E}^2t^2\right] = q^2\mathcal{E}^2t^2$$

$$v = \frac{q\mathcal{E}t}{\sqrt{m^2 + q^2\mathcal{E}^2t^2}} \Rightarrow \lim_{t\to\infty} v = 1$$

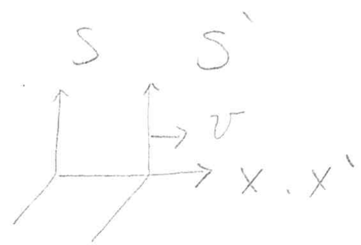
$$v(t=0) = 0$$

$$\Lambda^{\mu}{}_{\nu} = \begin{bmatrix} r & -rv & 0 & 0 \\ -rv & r & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cosh\theta & -\sinh\theta & 0 & 0 \\ -\sinh\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r = \frac{1}{\sqrt{1-v^2}}$$

$$\left\{ \begin{aligned} r &= \cosh\theta \\ rv &= \sinh\theta \\ \tanh\theta &= v \end{aligned} \right.$$

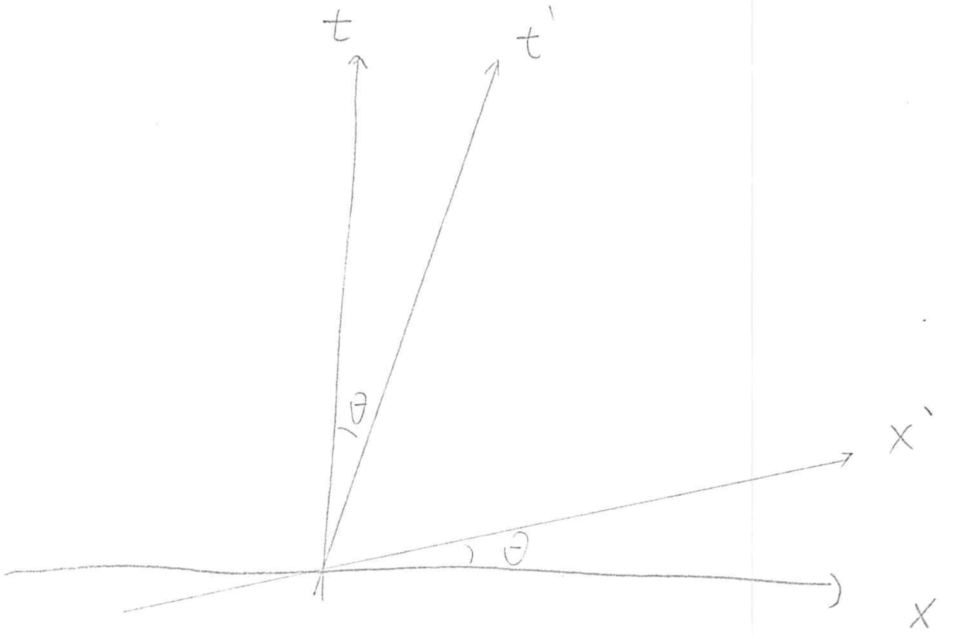
θ : rapidity



$$r^2 = \frac{1}{1-v^2}$$

$$r^2 - r^2v^2 = 1$$

$$\cosh^2\theta - \sinh^2\theta = 1$$



馬克斯威爾方程式

$$
\left\{
\begin{array}{l}
\left.
\begin{aligned}
&\nabla \cdot \vec{E} = 4\pi k_1 \rho \\
&\nabla \times \vec{B} = 4\pi k_2 \alpha \vec{J} + \frac{k_2}{k_1} \alpha \frac{\partial \vec{E}}{\partial t}
\end{aligned}
\right\} \text{with source} \\[2ex]
\left.
\begin{aligned}
&\nabla \times \vec{E} + k_3 \frac{\partial \vec{B}}{\partial t} = 0 \\
&\nabla \cdot \vec{B} = 0
\end{aligned}
\right\} \text{sourceless}
\end{array}
\right.
$$

	k_1	k_2	k_3	α
最熟悉的 SI	$\frac{1}{4\pi\epsilon_0}$	$\frac{\mu_0}{4\pi}$	1	1
Gaussian	1	$1/c^2$	c	c
Heaviside-Lorentz	$\frac{1}{4\pi}$	$\frac{1}{4\pi c^2}$	$\frac{1}{c}$	c

H-L

$$
\left\{
\begin{aligned}
&\nabla \cdot \vec{E} = \rho \\
&\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \vec{J}/c \\
&\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \\
&\nabla \cdot \vec{B} = 0
\end{aligned}
\right.
$$

$\Downarrow$ choose $c = 1$

$$
\boxed{
\left\{
\begin{aligned}
&\nabla \cdot \vec{E} = \rho \\
&\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} \\
&\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \\
&\nabla \cdot \vec{B} = 0
\end{aligned}
\right.
}
$$

$$F^{\mu\nu}$$

$$\vec{E} = (E_x, E_y, E_z) = (E^1, E^2, E^3) = (E_1, E_2, E_3)$$

$$\vec{B} = (B_x, B_y, B_z) = (B^1, B^2, B^3) = (B_1, B_2, B_3)$$

$$F^{0i} = E^i$$

$$F^{ij} = \epsilon^{ijk} B_k$$

$$F^{12} = \epsilon^{123} B_3 = B_3$$

$$F^{\mu\nu} = -F^{\nu\mu}$$

$$F^{23} = \epsilon^{231} B_1 = B_1$$

$$\epsilon^{ijk} = \begin{cases} 1, & \text{if } i=1, j=2, k=3 \text{ and } (1,2,3) \text{ cyclic permutation} \\ -1, & \text{if } i=1, j=3, k=2 \text{ and } (1,3,2) \text{ cyclic permutation} \\ 0, & \text{else} \end{cases}$$

$$\epsilon^{ijk} = -\epsilon^{jik}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{bmatrix}$$

$$\epsilon^{123} = 1, \quad \epsilon^{231}, \epsilon^{312}$$

$$\epsilon^{132} = -1, \quad \epsilon^{321}, \epsilon^{213}.$$

total entire asymmetric symbo

$$F^{13} = \epsilon^{132} B_2 = -B_2$$

$$F'^{\mu\nu}(x') = \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} F^{\alpha\beta}(x)$$

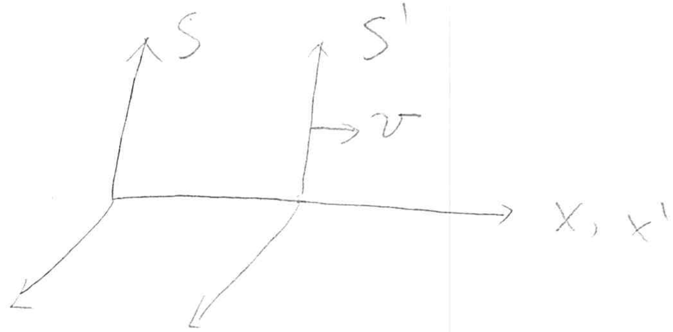
in S frame

$$\boxed{\partial_\mu F^{\nu\mu} = J^{\nu}}$$

$$\Updownarrow$$

$$\boxed{\partial'_\mu F'^{\nu\mu} = J'^{\nu}}$$

in S' frame



$$F'_{\mu\nu}(x) = \Lambda^{\alpha}_{\mu} \Lambda^{\beta}_{\nu} F_{\alpha\beta}(x)$$

$$\boxed{\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0} \Longleftrightarrow \boxed{\partial'_\mu F'_{\nu\lambda} + \partial'_\nu F'_{\lambda\mu} + \partial'_\lambda F'_{\mu\nu} = 0}$$

in S frame   in S' frame

$$F_{\mu\nu} = \eta_{\mu\alpha}\eta_{\nu\beta}F^{\alpha\beta}$$

$$F^{\alpha\beta} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$\eta = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\eta_{\nu\beta}F^{\alpha\beta} = F^{\alpha}{}_{\nu} = \begin{bmatrix} 0 & -E_x & -E_y & E_z \\ -E_x & 0 & B_z & 0 \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$\eta_{\mu\alpha}F^{\alpha}{}_{\nu} = F_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & 0 \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$\begin{cases} \nabla\cdot\vec{E} = \rho \\ \nabla\times\vec{B} - \dfrac{\partial\vec{E}}{\partial t} = \vec{J} \\ \nabla\times\vec{E} + \dfrac{\partial\vec{B}}{\partial t} = 0 \\ \nabla\cdot\vec{B} = 0 \end{cases}$$

$$(\nabla\times B)^i = \epsilon^{ijk}\partial_j B_k = \partial_j F^{ij}$$

$$\partial_i E_i = J^0$$

$$\Updownarrow$$

$$\boxed{\partial_i F^{0i} = J^0}$$

$$\boxed{\partial_j F^{ij} - \partial_0 F^{0i} = J^i}$$

$$\partial_\mu F^{\nu\mu} = J^\nu$$

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0$$

$$J^\mu = (\rho, \vec{J}) = (J^0, J^i)$$

Electromagnetic field strength tensor

$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\partial E_1}{\partial x^1} + \frac{\partial E_2}{\partial x^2} + \frac{\partial E_3}{\partial x^3} = \partial_i E_i$$

$$\partial_i \triangleq \frac{\partial}{\partial x^i}$$

$$\partial_\mu \triangleq \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right)$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$

$$\boxed{\partial^\mu = \frac{\partial}{\partial x_\mu} = \left(-\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right)}$$

(4) vector potential

$$A^{\mu} = (A^0, A^1, A^2, A^3)$$

$A^0 = \phi$: Scalar Potential

$\underbrace{A^1, A^2, A^3}_{\vec{A}}$: 3-vector Potential

$$\begin{cases} \vec{B} = \nabla \times \vec{A} \\ \vec{E} = -\nabla\phi - \dfrac{\partial \vec{A}}{\partial t} \end{cases}$$

$$\Downarrow$$

$$\boxed{F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}}$$

$$A'^{\mu}(x') = \Lambda^{\mu}{}_{\nu} A^{\nu}(x)$$

如果 A^{μ} 是 vector

then $F^{\mu\nu}$ 會是 tensor

gauge transformation

if $A^{\mu}(x) \to A'^{\mu}(x)$

$$= A^{\mu}(x) + \partial^{\mu}\alpha(x)$$

$\alpha'(x') = \alpha(x)$

某個 scalar fun

$$F^{\mu\nu} \to F'^{\mu\nu} = \partial^{\mu}A'^{\nu} - \partial^{\nu}A'^{\mu}$$

$$= \partial^{\mu}(A^{\nu} + \partial^{\nu}\alpha) - \partial^{\nu}(A^{\mu} + \partial^{\mu}\alpha)$$

$$= \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} \cancel{+ \partial^{\mu}\partial^{\nu}\alpha - \partial^{\nu}\partial^{\mu}\alpha}$$

$$= F^{\mu\nu}$$

under g.t. $F^{\mu\nu}$ 不變 [规范 gauge invariance]

gauge 規範

ㄉㄨㄟˋ ㄡˇ

$$\tilde{F}^{\mu\nu} \triangleq \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

dual EM field strength tensor

even permitation

$$\epsilon^{0123} = 1 , \epsilon^{1032} 、 \epsilon^{2301} 、 \epsilon^{3210}$$

odd permitation

$$\epsilon^{0213} = -1 , \epsilon^{2031} 、 \epsilon^{1302} 、 \epsilon^{3120}$$

$$\epsilon^{\mu\nu\alpha\beta} = 0 \quad , \text{else}$$

$$\partial'^{\mu}\partial'_{\mu} = \partial^{\mu}\partial_{\mu}$$

$$= -\frac{\partial^2}{\partial t^2} + \nabla^2$$

ㄉㄚ ㄌㄤ ㄅ tion operation

$$\nabla \cdot \vec{E} = \rho$$

$$\nabla \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\partial_k B_k = 0$$

$$\partial_k \epsilon^{ijk} B_k = 0$$

$$\boxed{\partial_k F^{ij} = 0} \quad \partial_k F_{ij} = 0$$

$$\epsilon^{231} B_1 = B_1$$

$$F^{23} = B_1$$

$$\nabla \times \vec{E} = \begin{bmatrix} \frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y \\ \frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z \\ \frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x \end{bmatrix}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

$$\epsilon^{0123} = 1, \quad \epsilon^{1032} \cdot \epsilon^{2301} \cdot \epsilon^{3210}$$

$$\epsilon^{1023} = -1, \quad \epsilon^{0132} \cdot \epsilon^{2310} \cdot \epsilon^{3201}$$

$$0, \text{ else}$$

$$F_{\alpha\beta} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

$$\partial_0 F^{23} + \partial_2 F^{03} + \partial_3 F^{20} = 0$$

$$\partial_0 F^{31} + \partial_3 F^{01} + \partial_1 F^{30} = 0$$

$$\partial_0 F^{12} + \partial_1 F^{02} + \partial_2 F^{10} = 0$$

$$(\nabla \times \vec{E})^1 = \frac{\partial}{\partial x^2} E_3 - \frac{\partial}{\partial x^3} E_2$$

$$= \partial_2 F^{03} + \partial_3 (-F^{02})$$

$$\partial_0 B_1 = \partial_2 F^{03} + \partial_3 F^{20}$$

$$= \partial_0 \epsilon^{231} B_1$$

$$= \partial_0 F^{23} \qquad = -\partial_0 F^{23}$$

$$\partial$$

$$\cancel{\partial_0 F^{32} + \partial_2 F^{30} + \partial_3 F^{02} = 0}$$

$$\partial_0 F_{32} + \partial_2 F_{03} + \partial_3 F_{20} = 0$$

$$\partial_0 F_{13} + \partial_3 F_{01} + \partial_1 F_{30} = 0$$

$$\partial_0 F_{21} + \partial_1 F_{02} + \partial_2 F_{10} = 0$$

$$0123$$

$$\partial_1 F_{23} + \partial_2 F_{31} + \partial_3 F_{12} = 0$$

$$\partial_0 B_2 = \partial_0 \epsilon^{312} B_2 = \partial_0 F^{31}$$

$$\partial_0 B_3 = \partial_0 \epsilon^{123} B_3 = \partial_0 F^{12}$$

gravitation

WHEELER

為簡單起見
常常把張量的分量
也叫做張量

$$\bar{A}_i = \frac{\partial x^j}{\partial \bar{x}^i} A_j$$ contra

$$\bar{A}^i = \frac{\partial \bar{x}^i}{\partial x^j} A^j$$ co

contra-

$$\bar{A}^{ij} \triangleq \frac{\partial \bar{x}^i}{\partial x^a} \frac{\partial \bar{x}^j}{\partial x^b} A^{ab}$$

mix-

$$\bar{A}^i_j = \frac{\partial \bar{x}^i}{\partial x^a} \frac{\partial x^b}{\partial \bar{x}^j} A^a_b$$

$$\bar{A}_{ij} \triangleq \frac{\partial x^a}{\partial \bar{x}^i} \frac{\partial x^b}{\partial \bar{x}^j} A_{ab}$$

co-