

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$f(x+h) = f(x) + f'(x)h + \epsilon h$$

$\epsilon \rightarrow 0$  as  $h \rightarrow 0$

$$\frac{1}{f(x+h)} - \frac{1}{f(x)}$$

$$= \frac{f(x) - f(x+h)}{f(x+h)f(x)}$$

$$\frac{\frac{1}{f(x+h)} - \frac{1}{f(x)}}{h}$$

$$= \frac{-\frac{f(x+h) - f(x)}{h}}{f(x+h)f(x)}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{f(x+h)} - \frac{1}{f(x)}}{h}$$

$$= \frac{-f'(x)}{[f(x)]^2}$$

$$\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$$

$$\therefore (fg)' = f'g + fg'$$

$$\therefore \left(\frac{f}{g}\right)' = \left(f \cdot \frac{1}{g}\right)'$$

$$= f' \left(\frac{1}{g}\right) + f \left(\frac{1}{g}\right)'$$

$$= f' \left(\frac{1}{g}\right) + f \left(\frac{-g'}{g^2}\right)$$

$$= \frac{f'g - fg'}{g^2}$$