

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\lim_{h \rightarrow 0} \frac{x+h-x}{h} = \lim_{h \rightarrow 0} 1 = 1$$

$$n=1, \quad \frac{d}{dx} x = 1 \quad \text{顯然成立}$$

$$\text{設 } n=k, \quad \frac{d}{dx} x^k = k x^{k-1} \quad \text{成立}$$

$$\text{則 } n=k+1 \quad \text{時}$$

$$\begin{aligned} \frac{d}{dx} (x \cdot x^k) &= \left(\frac{d}{dx} x \right) x^k + x \frac{d}{dx} (x^k) \\ &= x^k + x \cdot k \cdot x^{k-1} \\ &= x^k + k x^k \\ &= (k+1) x^{(k+1)-1} \end{aligned}$$

原式亦成立

故 $\forall n \in \mathbb{N}$

原式皆成立 QED