
Dynamic simulation of the inflation gas of a tire under operational conditions

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Multi-physics aspects of tire simulation

■ Mechanical model

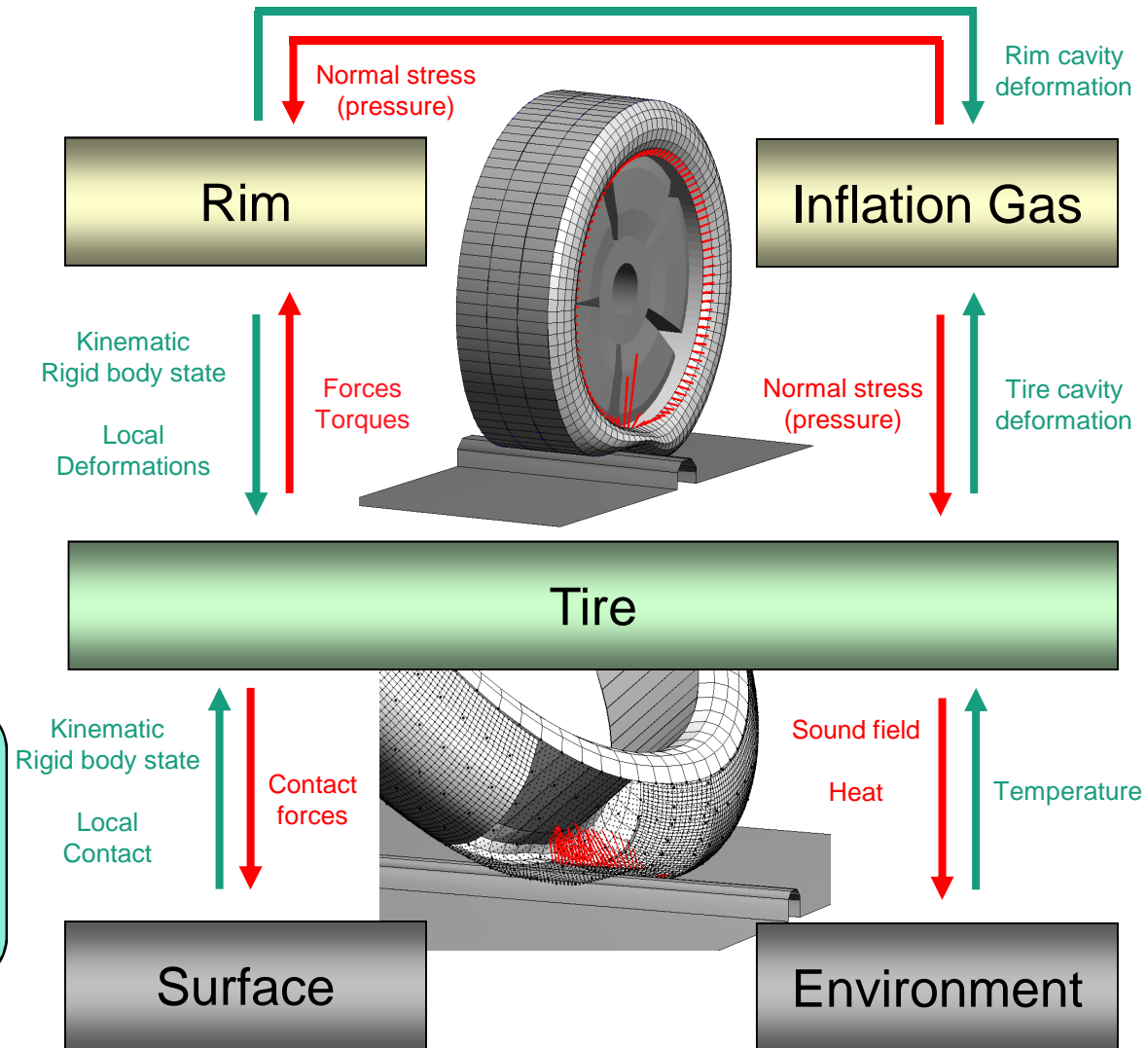
- Large deformations
- Cords, filled rubber
- Frictional contact

■ Temperature model

- Heat propagation
- Structural effects
- Frictional effects

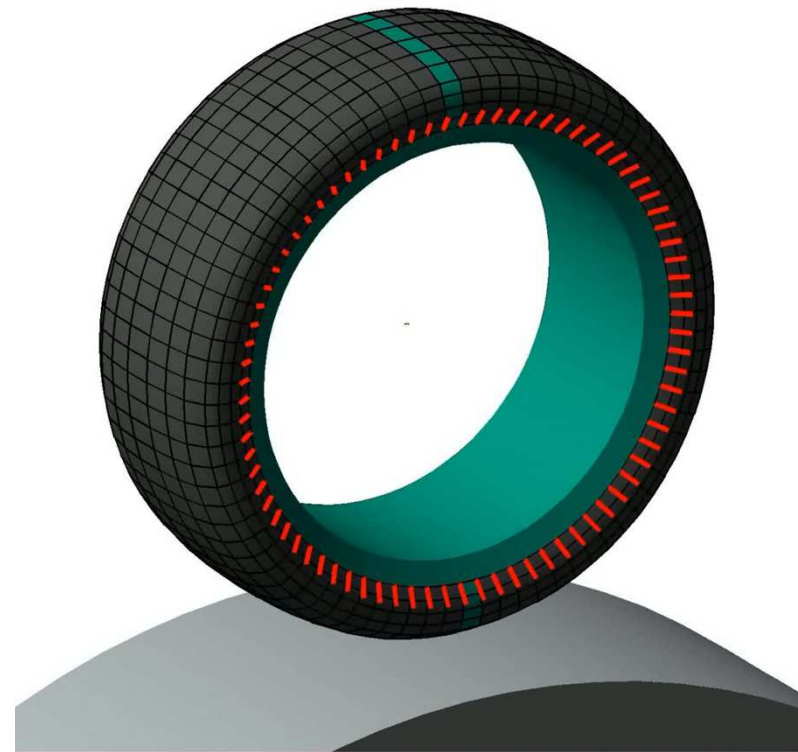
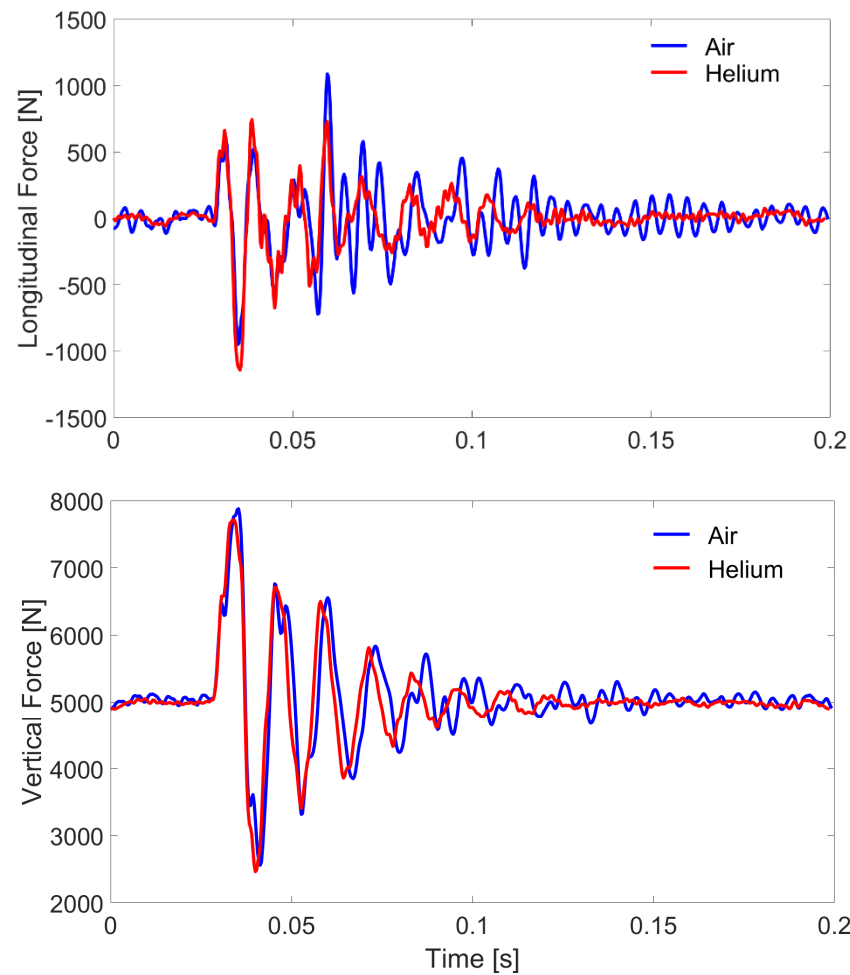
■ Cavity model

- Euler equation
- Coupled with tire



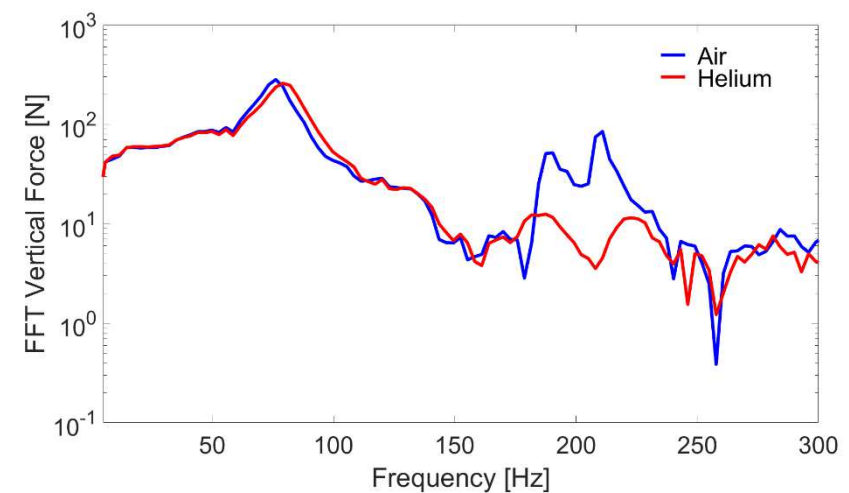
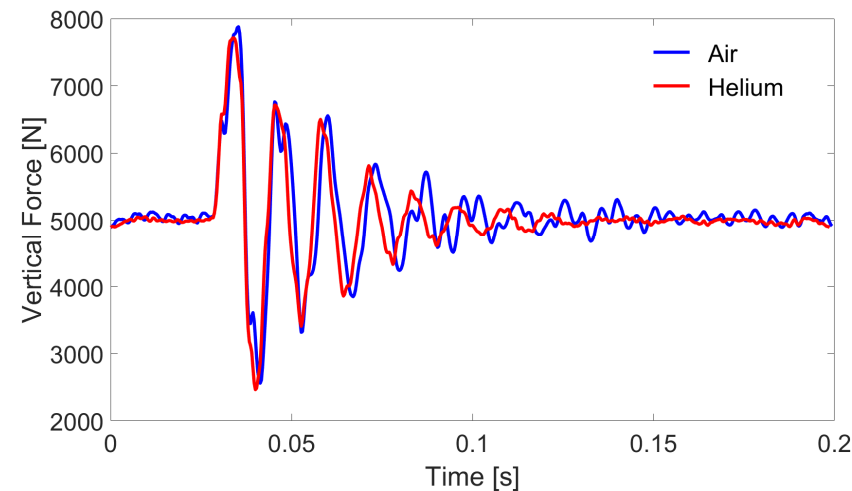
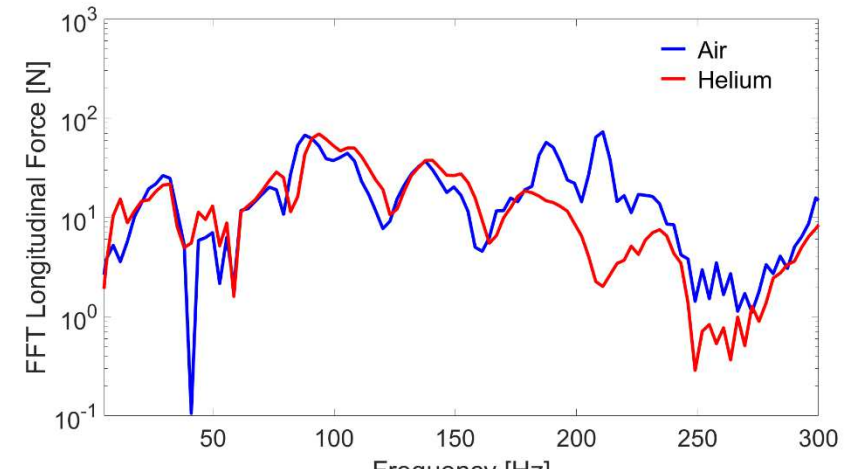
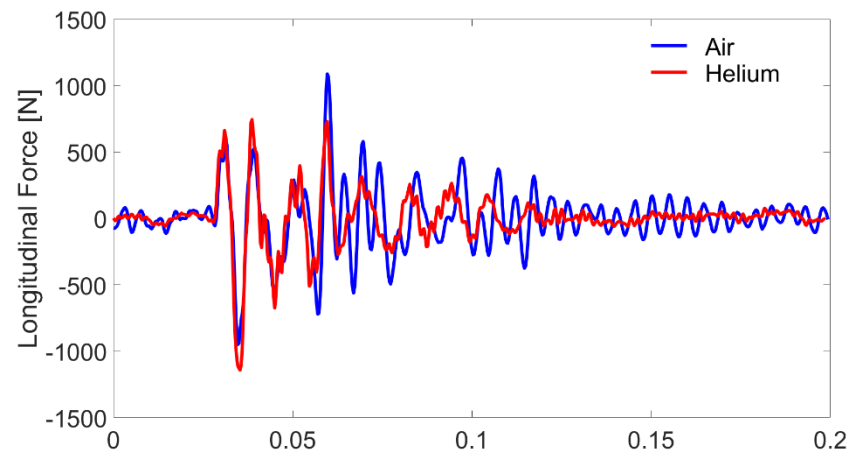
Air (black) vs. Helium (red)

■ 9 Cleat runs (3 preloads a 3 velocities), measurements courtesy fka



Motivation: Inflation Gas Air (black) vs. Helium (red)

■ Cleat run (5000 N, 90 km/h, 10x20 mm), measurements courtesy fka



CDTire model family

- **CDTire/3D** – detailed shell-based 3D structural model
- **CDTire/Realtime** – physical tire model for applications from comfort to durability under soft and hard realtime conditions
- **CDTire/MF++** – enhanced Magic Formula for coupling to CDTire/Thermal in advanced handling applications

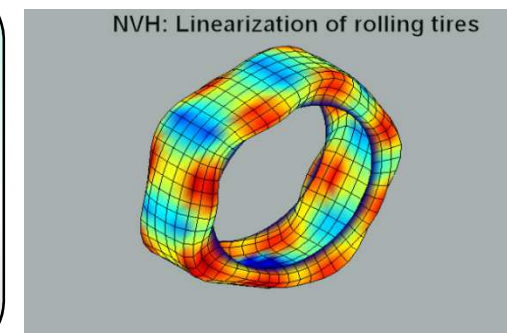
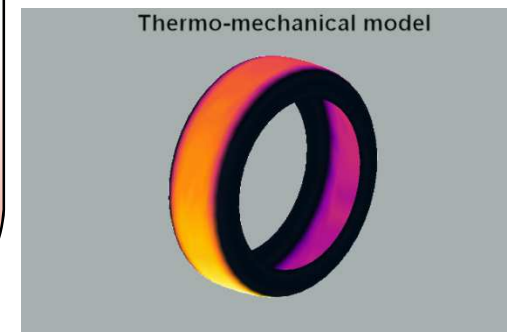
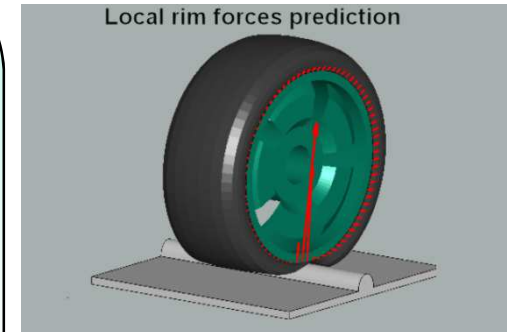
Mechanical models

- **CDTire/Thermal** – 3D thermo-dynamical model to predict temperature creation and propagation in a tire

Thermo-dynamical model

- **CDTire/NVH** – a software toolbox to derive linear models out of CDTire/3D with rotation
- **CDTire/PI** – a software tool for perform parameter identification for the CDTire submodels

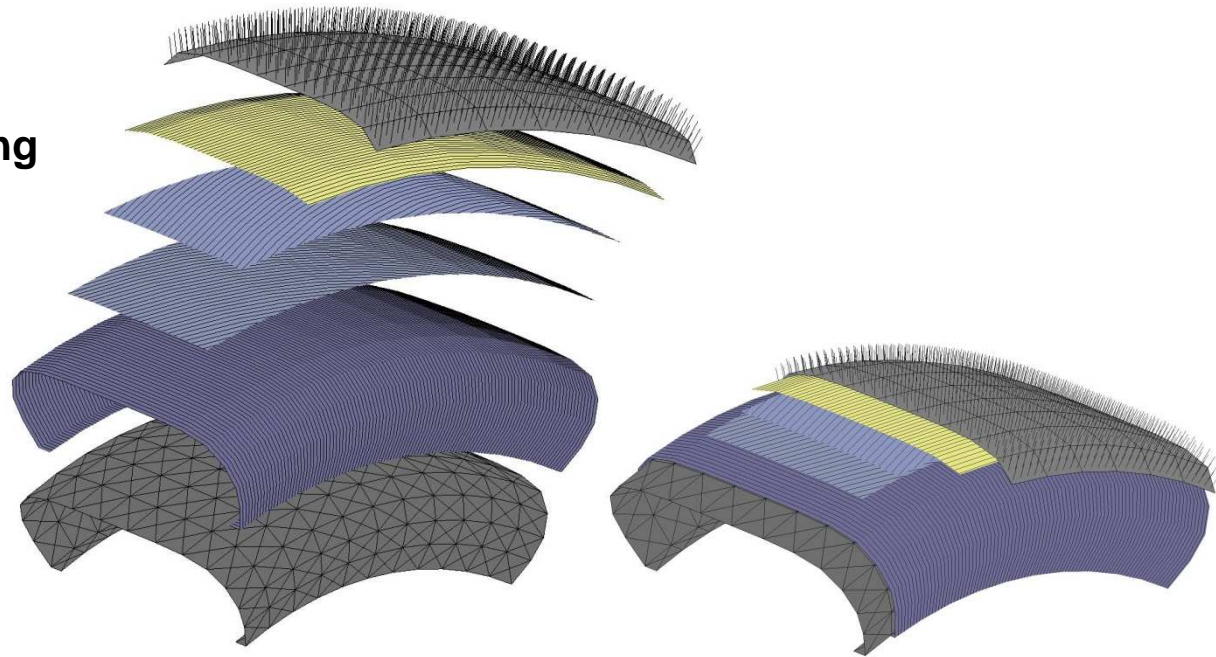
Stand-alone tools



CDTire/3D

■ Functional layer modeling

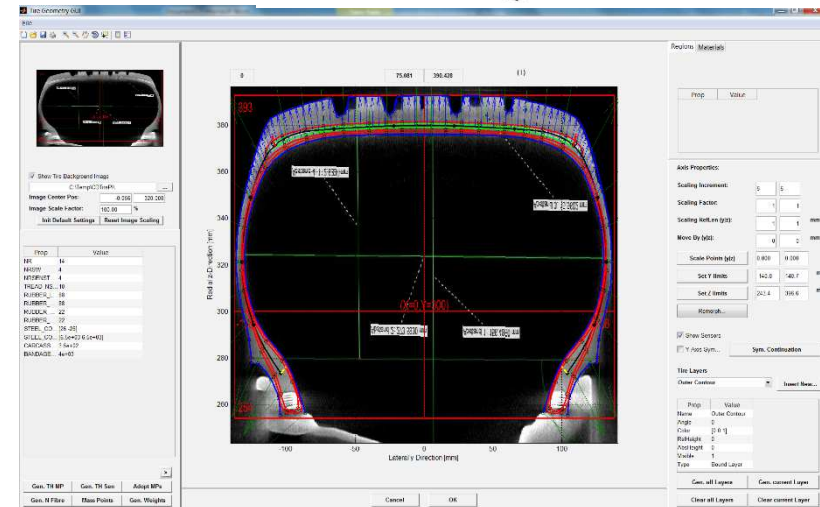
- Tread (brush type)
- Cap ply
- Belt 1
- Belt 2
- Carcass
- Innerliner + matrix



■ Condensed into one shell

■ Highlights

- Strict separation of material and geometry
- Re-mounting on different rim width
- Innerliner geometry fully available for inflation pressure application



Cavity models

■ Time dependently prescribed pressure

- Inflation pressure given

$$p = p(t)$$

■ Ideal gas equation

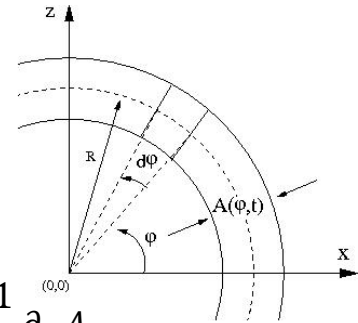
- Inflation pressure statically calculated

$$p = \frac{p_0 V_0}{V}$$

■ Isentropic compressible Euler equation

- Pipe flow with variable cross area
- Conservation of mass
- Conservation of momentum
- Isentropic process

$$\begin{aligned}\partial_t(\rho A) + \frac{1}{R} \partial_\varphi(\rho A v) &= 0 \\ \partial_t(\rho A v) + \frac{1}{R} \partial_\varphi(\rho A v^2 + p A) &= p \frac{1}{R} \partial_\varphi A \\ p &= p(\rho)\end{aligned}$$



- with $\alpha = \rho A$
- and $\mu = \rho A v$
- and $p(\rho) = c^2 \rho$ rewrite as

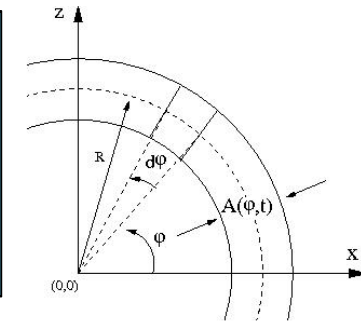
$$\begin{aligned}\partial_t \alpha + \frac{1}{R} \partial_\varphi \mu &= 0 \\ \partial_t \mu + \frac{1}{R} \partial_\varphi \left(\frac{\mu^2}{\alpha} + c^2 \alpha \right) &= c^2 \alpha \frac{1}{R} \frac{\partial_\varphi A}{A} \\ p &= c^2 \frac{\alpha}{A}\end{aligned}$$

Cavity models: Euler flow model in detail

■ (Main) model parameter

- Velocity of sound c
- Effective tube radius R
- Sub-division $\Delta x = N \Delta s$
- Courant-Friedrichs-Lewy $CFL = c \frac{\Delta t}{\Delta x} < 1$

$$\begin{aligned}\partial_t \alpha + \frac{1}{R} \partial_\varphi \mu &= 0 \\ \partial_t \mu + \frac{1}{R} \partial_\varphi \left(\frac{\mu^2}{\alpha} + c^2 \alpha \right) &= c^2 \alpha \frac{1}{R} \frac{\partial_\varphi A}{A} \\ p &= c^2 \frac{\alpha}{A}\end{aligned}$$

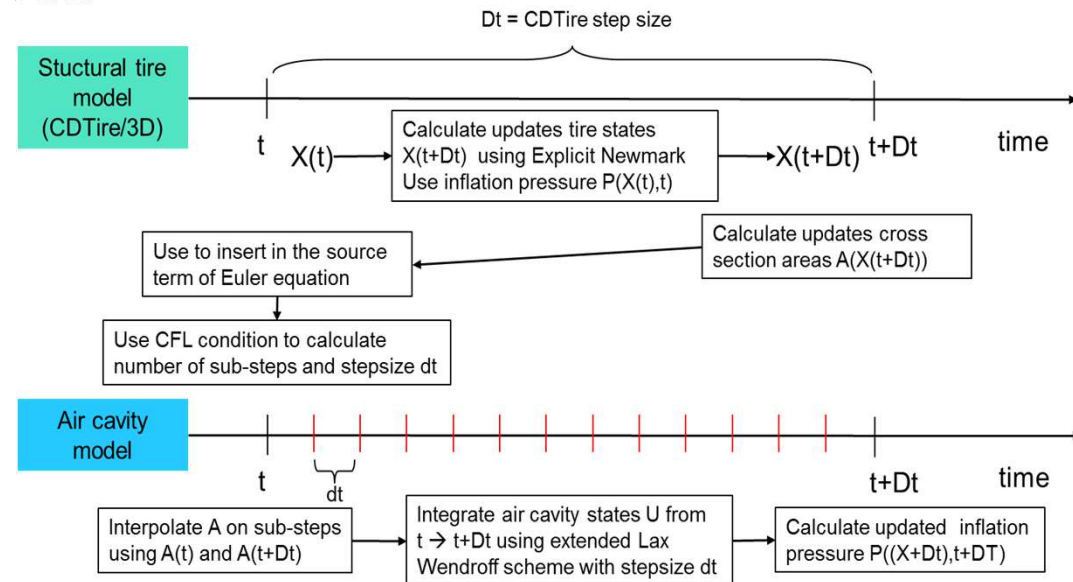


■ Interaction with mechanical model

- Local pressure p
- Cross section area A

■ Co-simulation scheme

- Top level: MBS solver
- Int. level: tire model
- Bot.level: cavity mode
- Scheme: Lax-Wendroff with source term



Cavity models: Vertical stiffness comparisons

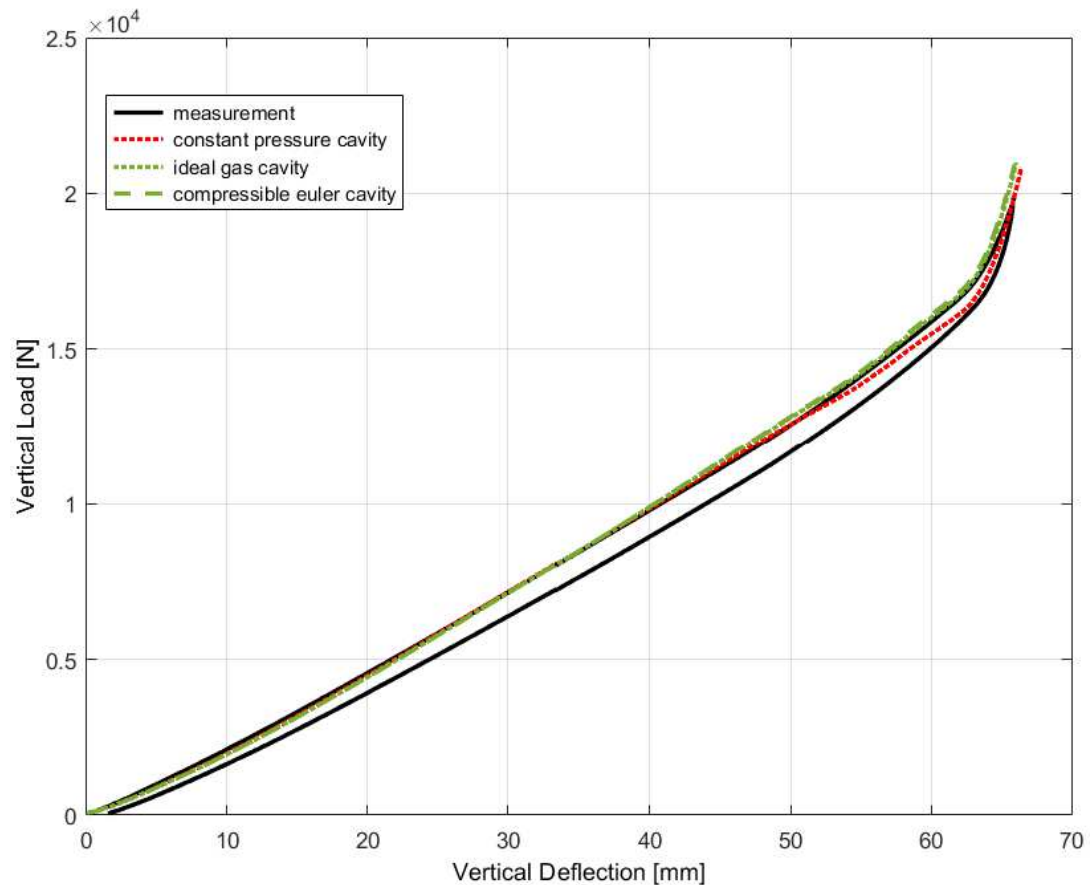
■ Vertical stiffness

- flat surface
- Preload 0 N .. 21000 N
- Velocity 0 km/h

■ Constant pressure **does not** capture decrease in volume

■ Ideal gas and compressible Euler equation models **do**

■ For quasi-static scenarios ideal gas equation model **seems to suffice**



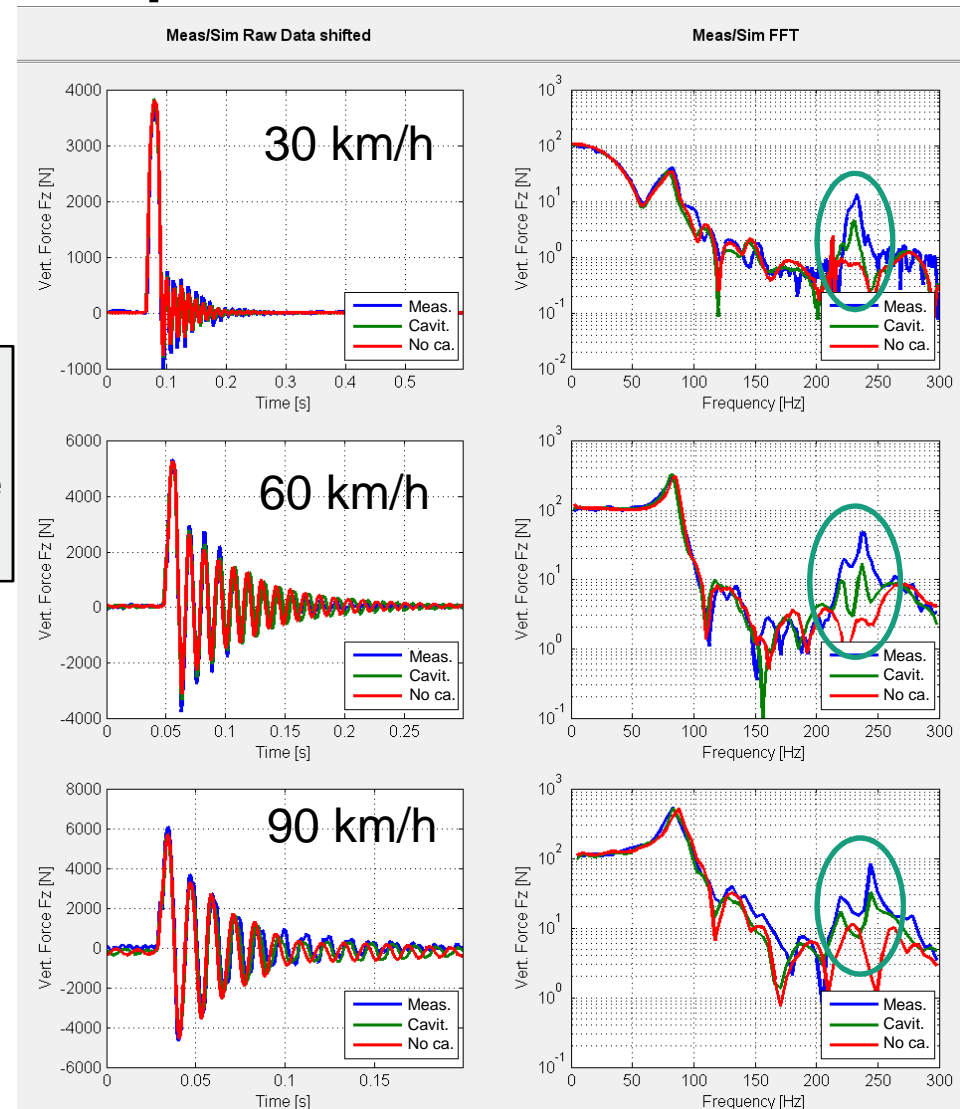
Cavity models: Cleat run comparisons

Vertical force

- 20x20 mm cleat
- Preload 2400 N
- 30 / 60 / 90 km/h

— Measurement
 — Euler equation
 — Constant pressure
 (= Ideal gas)

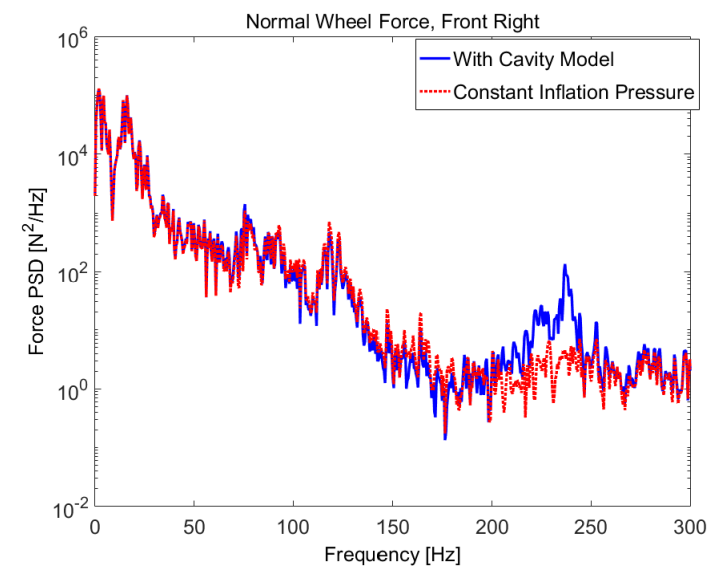
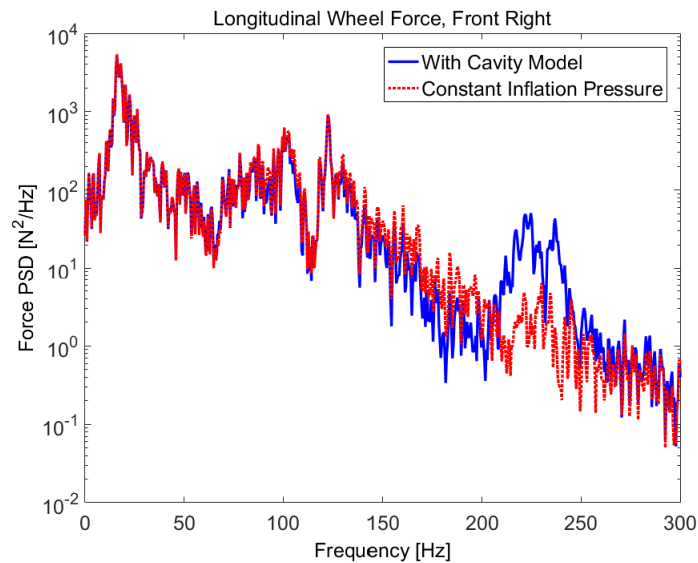
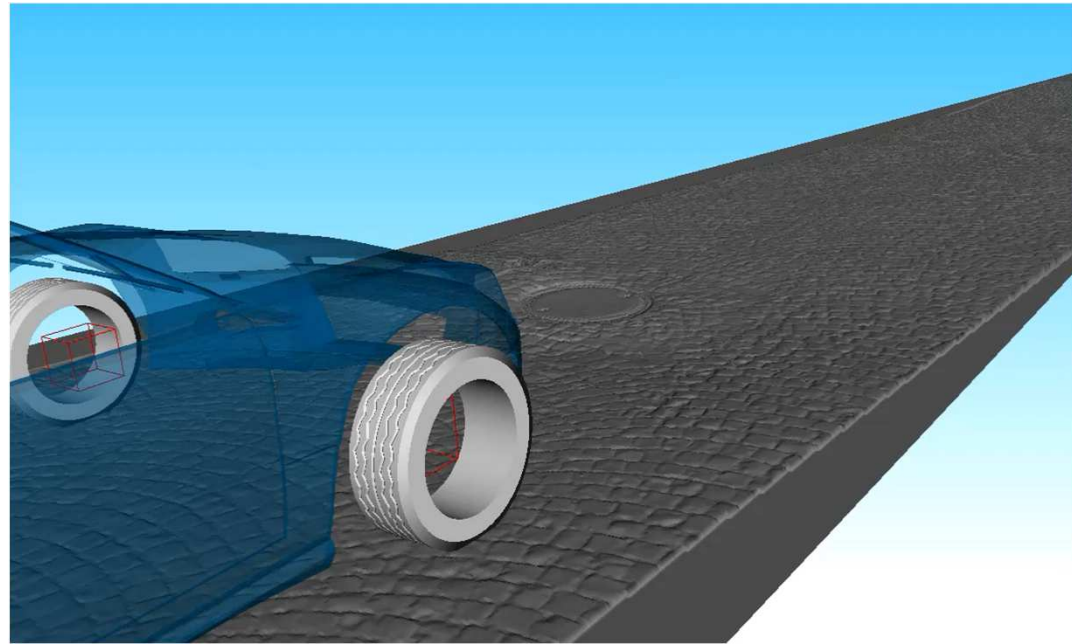
- Observe **cavity mode** around 230 Hz
- Observe **split** due to rotational vel. increase (Doppler effect)
- Only Euler equation model captures this correctly



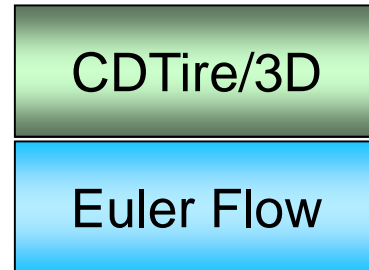
CDTire/3D

■ Application example

- Rough road
- 50 km/h
- PSD of spindle forces



Short summery of Euler flow cavity model



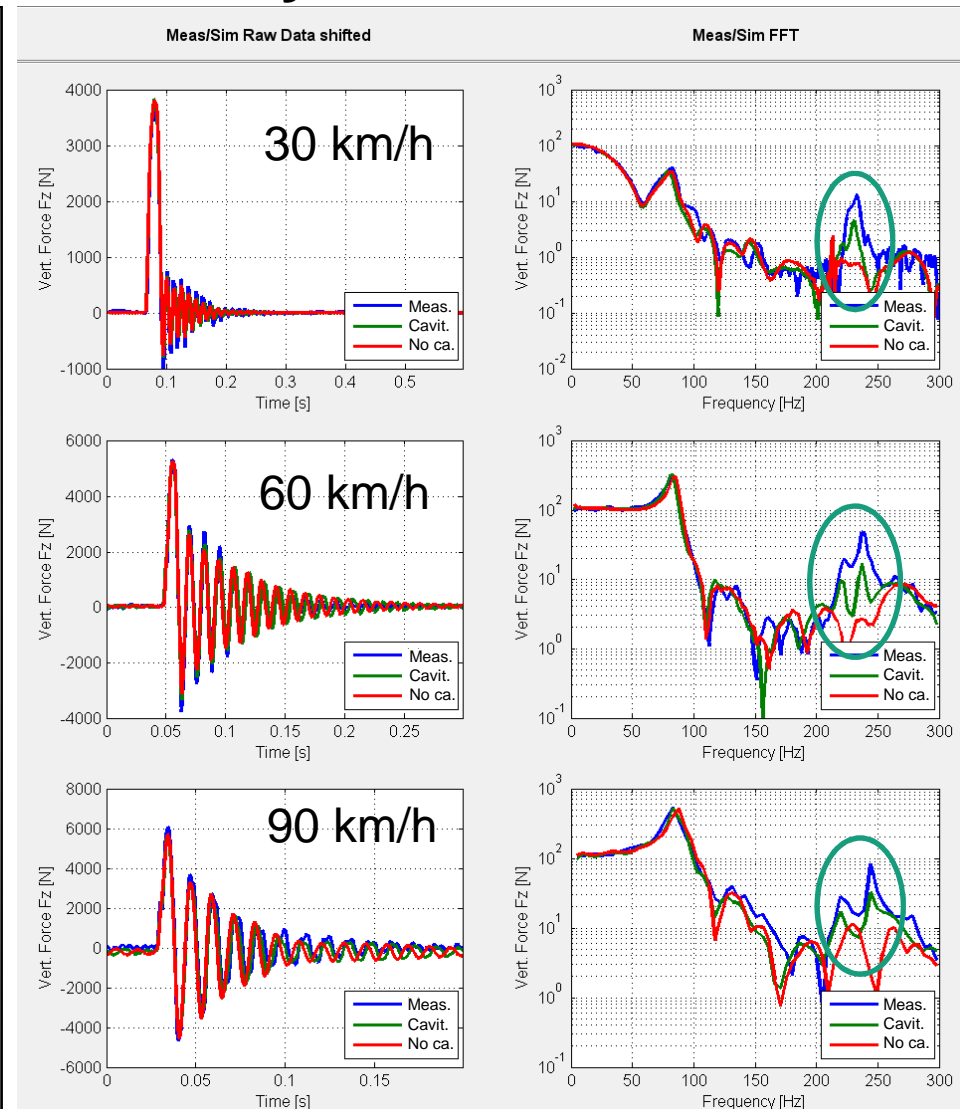
Transient
Non-
Linear:
MBS
Solvers

Modeling explained

$$\begin{aligned}\partial_t \alpha + \frac{1}{R} \partial_\varphi \mu &= 0 \\ \partial_t \mu + \frac{1}{R} \partial_\varphi \left(\frac{\mu^2}{\alpha} + c^2 \alpha \right) &= c^2 \alpha \frac{1}{R} \frac{\partial_\varphi A}{A} \\ p &= c^2 \frac{\alpha}{A}\end{aligned}$$

Numerics explained

- Conservation form
- Lax-Wendroff
- CFL condition



Linearization including rotational effects

CDTire/3D



Linearization:
CDTire/NVH

- M. Baecker, A. Gallrein, M. Roller:
Noise, vibration, harshness model
of a rotating tyre, Vehicle System
Dynamics Vol.54 – Issue 4, pp.
474-491, 2016, doi:
10.1080/00423114.2016.1158844

Wheel Specification

Select Parameter File

C:\Temp\CDTire\FN\unlop_SP_Sport_FastResponse_225_45_R_17_91W

Inertia [kg m ²]	0.25	Take Inertia/Mass as is	Mass [kg]
	0		0.4
	0		0.25
			10

Operating Conditions

PreTime [s] 0.2

End Time [s] 0.3

Pressure [bar] 2.5

☐ Gravity

Preload [N] 3000

Velocity [m/s] 20

Slip Angle [deg] 0

Inclination [deg] 0

Drive Torque [Nm] 0

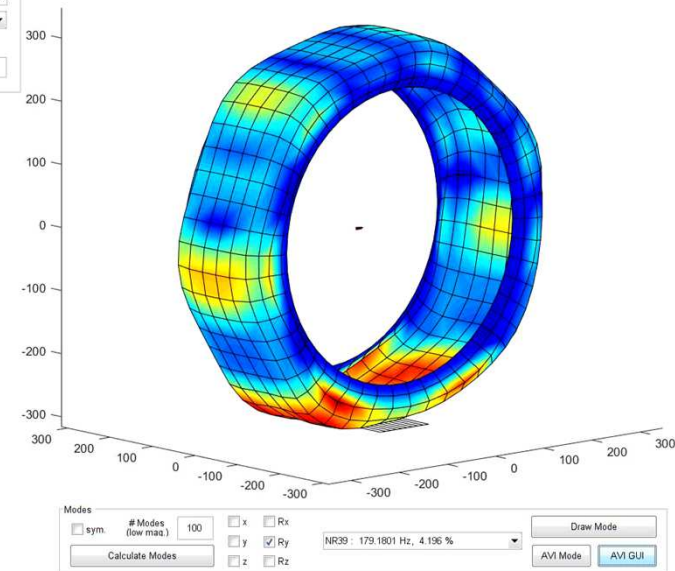
Contact LX [m] 0.1

Contact NX 10

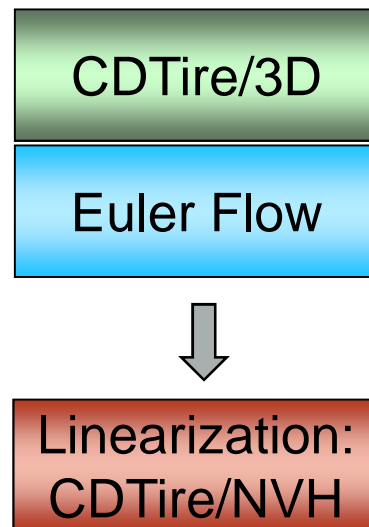
Contact LY [m] 0.2

Contact NY 2

Linearize



Linearization including rotational effects with cavity



$$\begin{aligned}\partial_t \alpha + \frac{1}{R} \partial_\varphi \mu &= 0 \\ \partial_t \mu + \frac{1}{R} \partial_\varphi \left(\frac{\mu^2}{\alpha} + c^2 \alpha \right) &= c^2 \alpha \frac{1}{R} \frac{\partial_\varphi A}{A} \\ p &= c^2 \frac{\alpha}{A}\end{aligned}$$

↓ Differentiation

$$\partial_{tt} \alpha = \frac{c^2}{R^2} \partial_{\varphi\varphi} \alpha - \frac{c^2}{R} \partial_\varphi \left(\alpha \frac{1}{R} \frac{\partial_\varphi A}{A} \right)$$

↓ Arbitrary Lagrangian Eulerian Method of Lines

$$\begin{aligned}M_T \ddot{x}_T &= f_{str}(x_T, \dot{x}_T, \alpha) + f_{gyr}(x_T, \dot{x}_T, \omega) \\ \ddot{\alpha} &= g_{cav}(\alpha, x_T) + g_{gyr}(\alpha, x_T, \omega)\end{aligned}$$

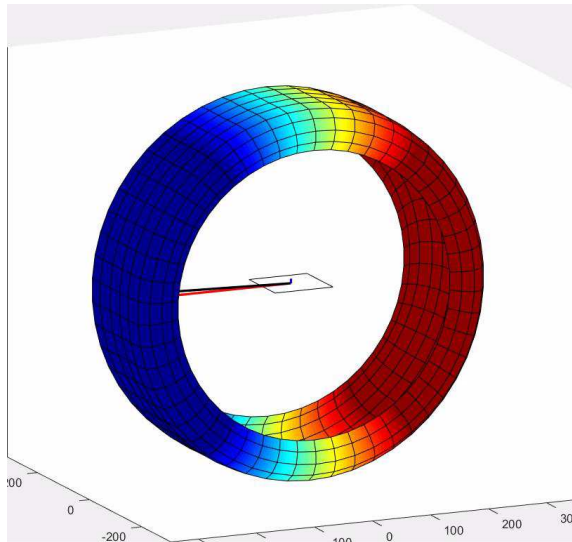
↓ Finite Differencing

M, C, K matrices

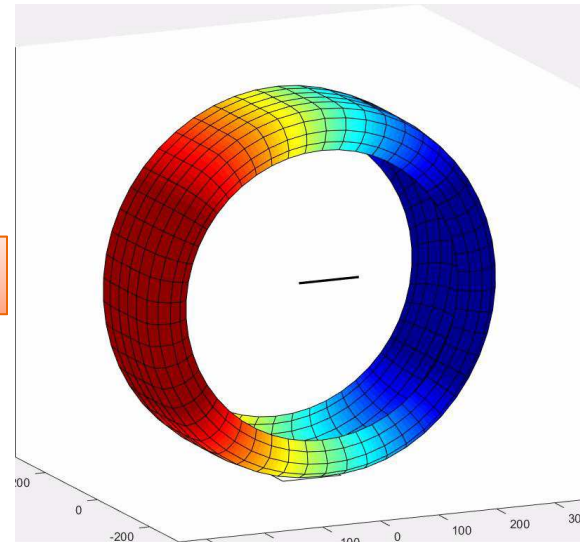
free tire

w = 0 km/h / r

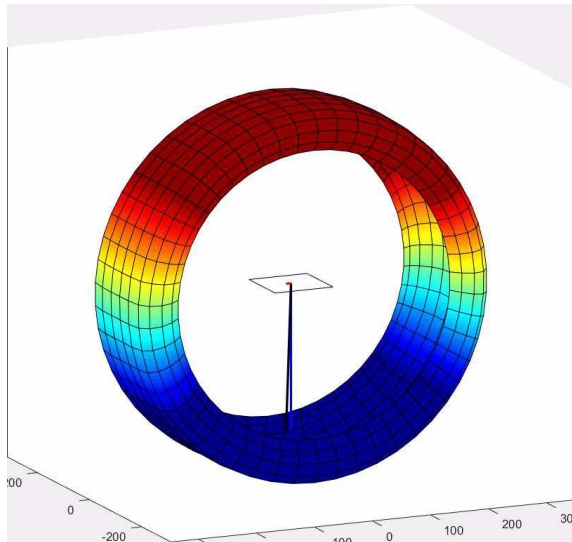
loaded tire 2400 N



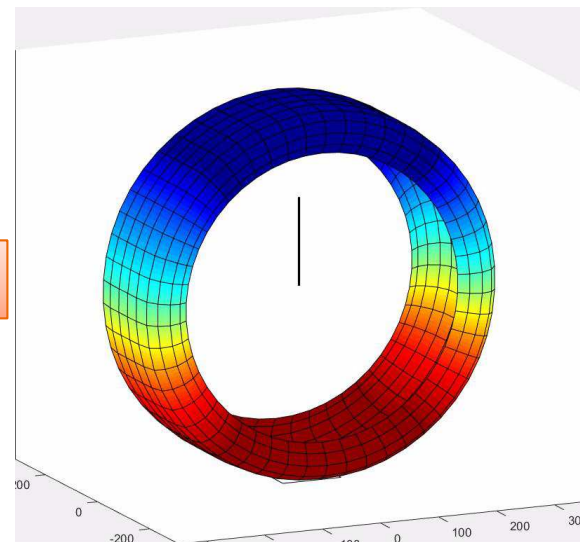
225.4 Hz



226.5 Hz



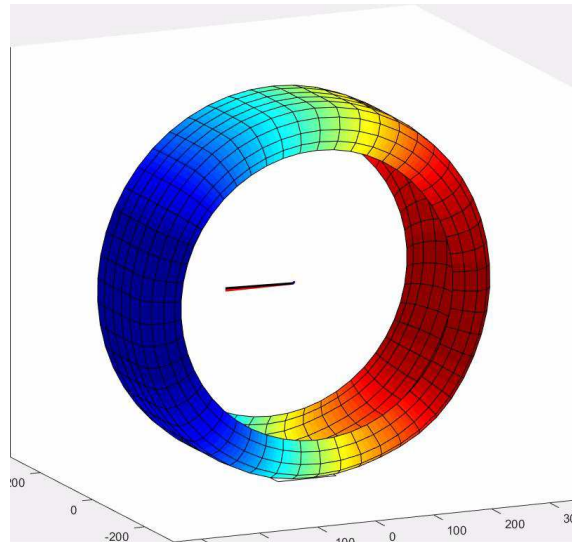
228.4 Hz



w = 90 km/h / r

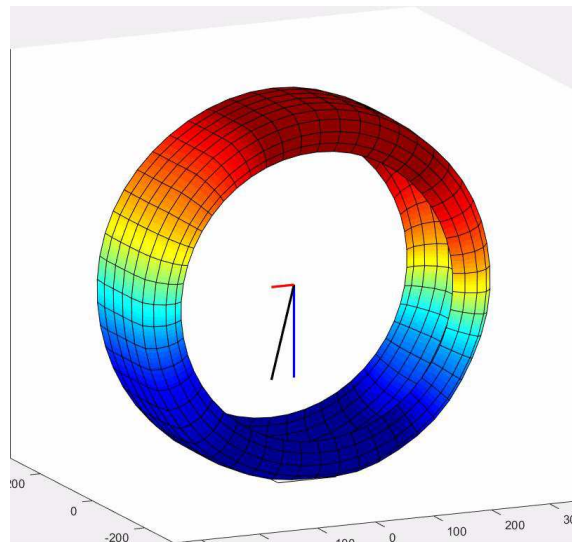
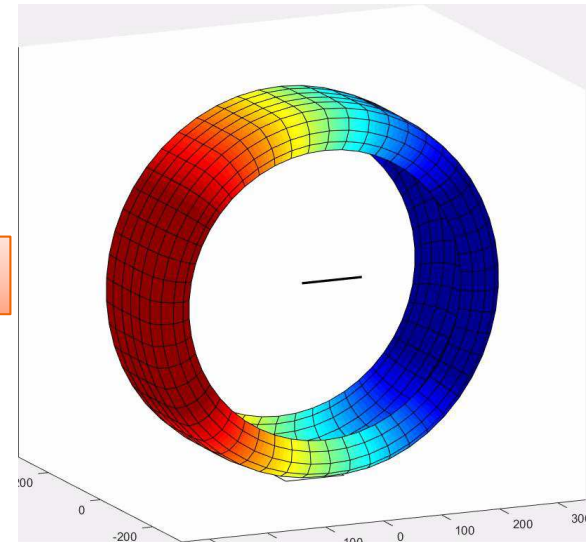
loaded tire 2400 N

w = 0 km/h / r



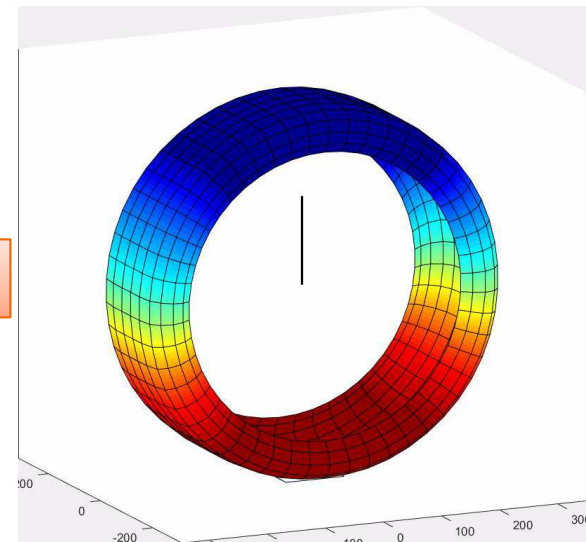
213.1 Hz

225.4 Hz



238.9 Hz

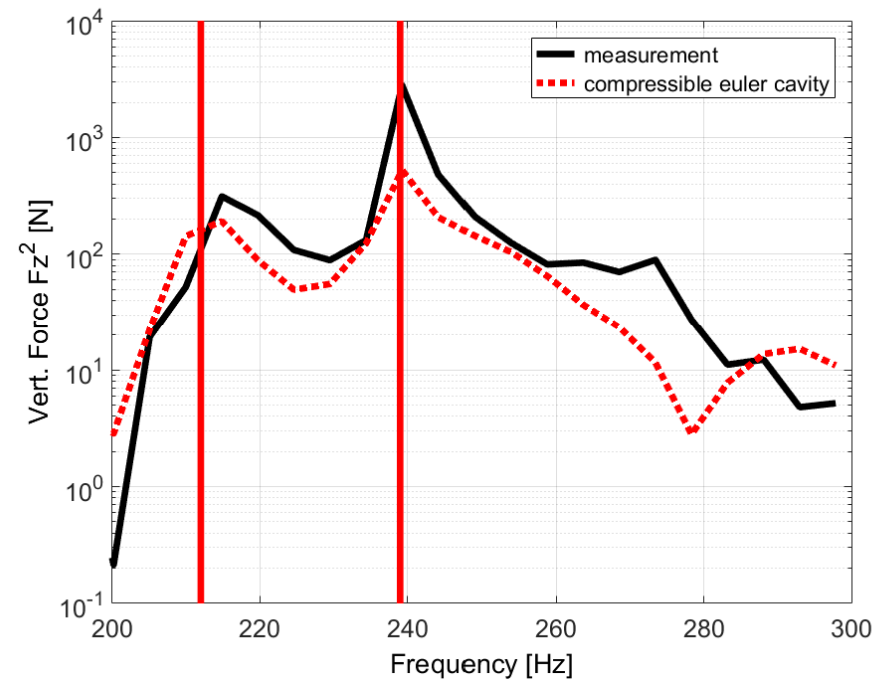
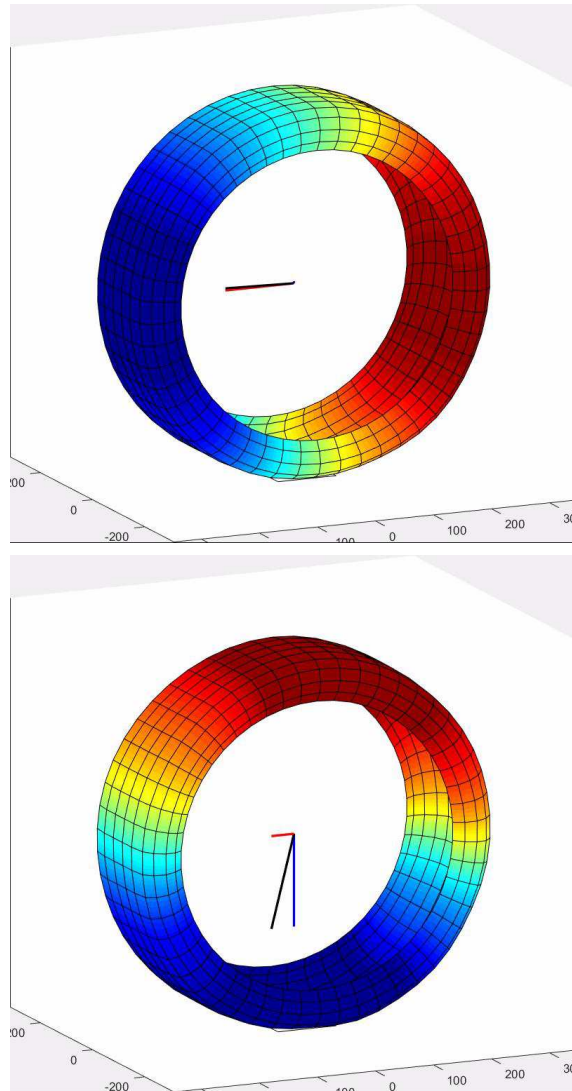
228.4 Hz



w = 90 km/h / r

loaded tire 2400 N

w = 0 km/h / r



Summary

■ Compared cavity models

- Prescribed inflation pressure
- Ideal gas equation
- Euler flow equation

■ Observations

- Prescribed IP only for moderate loads and frequency range below cavity modes
- Ideal gas equation only for frequency range below cavity modes

■ Validations

- Transient Euler flow with cleat runs
- Derived linearized Euler flow with cleat runs

■ Outlook

- Temperature dependency
- Pressure regulation

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Thank you for your attention !

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