1. write the loglikelihood for all data

(review the 2b) of assignment 2), to write posterior first:

$$\pi(Y, z|\theta)\pi(\theta) = \exp(-\lambda_1-\lambda_2)\prod_{i=0}^{9}\left(p\frac{\lambda_1^i e^{-\lambda_1}}{i!}\right)^{z_i}\left((1-p)\frac{\lambda_2^i e^{-\lambda_2}}{i!}\right)^{Y_i - z_i}$$

then write the loglikelihood

$$\log(\pi(\theta|Y,z))$$

$$= -\lambda_1-\lambda_2+\sum_{i=0}^{9} z_i(\log(p)-\lambda_1+i\log(\lambda_1)) + (Y_i - z_i)(\log(1-p)-\lambda_2+i\log(\lambda_2))$$

2. Describe the distribution $z|Y, \theta^{old}$

(review the 2e) of assignment 2)

the distribution z_i gives all parameters is binomial.

So we can write

$$z|Y, \theta^{old} \sim \text{binomial}(Y_i, p)$$

$$\boxed{\pi(z_i|Y, \text{all } z_i, p, \lambda_1, \lambda_2) \propto z_i \frac{Y_i}{(Y_i - z_i)z_i}(C_i p\lambda_1^i e^{\lambda_1})^{z_i}(C_i(1-p)(\lambda_2^i \cdot e^{-\lambda_2}))^{Y_i - z_i}}$$

answer from 2e of Assign 2

$$\boxed{B(n, p) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}}$$

3、do the E-step Algoritm

compute the Expectation of the loglikelihood under the conditional distribution for z above.

$$Q(\theta,\theta^{old}) = E[\log(\pi(\theta|Y,z))|Y,\theta^{old}]$$

$$= -\lambda_1-\lambda_2+E\Big[\sum_{i=0}^{9} z_i(\log(p)-\lambda_1+i\log(\lambda_1))$$

$$+(Y_i-z_i)(\log(1-p)-\lambda_2+i\log(\lambda_2))\Big|Y,\theta^{old}$$

$$= -\lambda_1-\lambda_2+\sum_{i=0}^{9} E[z_i(\log(p)-\lambda_1+i\log(\lambda_1))+$$

$$(Y_i-z_i)|Y,\theta^{old}(\log(1-p)-\lambda_2+i\log(\lambda_2))$$

4. do the M-step Algoritm

Maximize $Q(\theta,\theta^{old})$

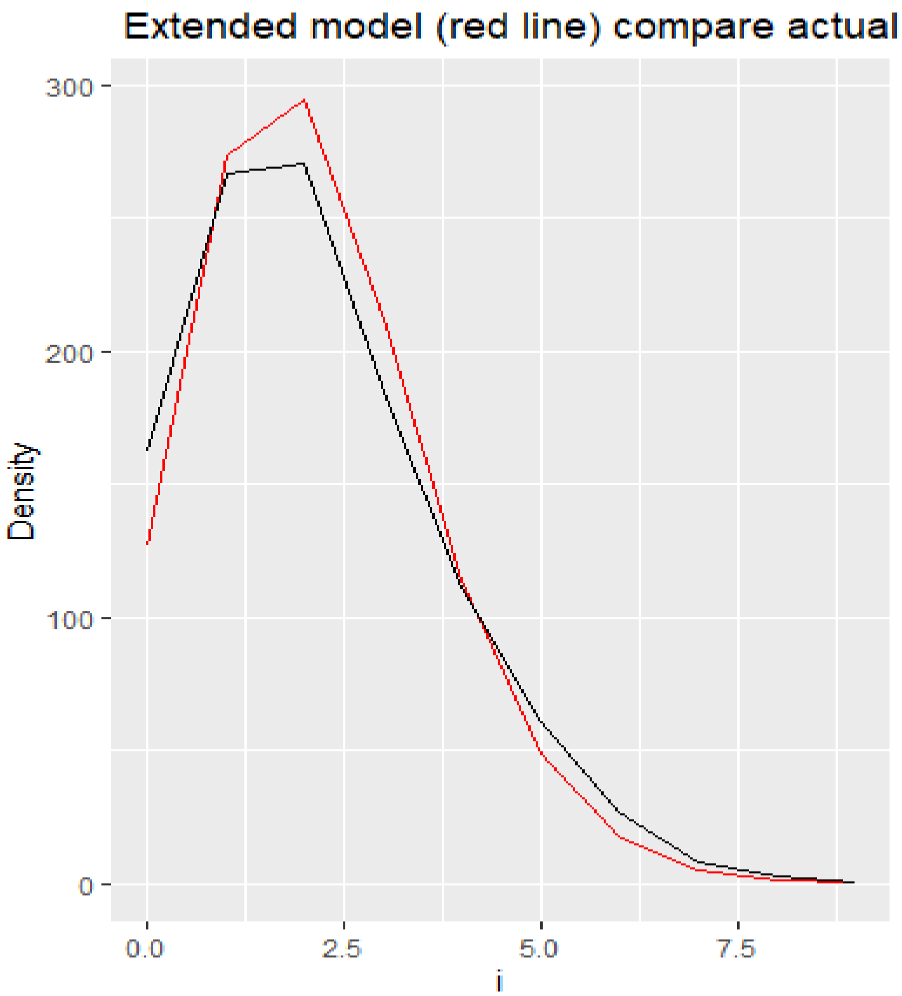
$$= \max\Big(-\lambda_1-\lambda_2+\sum_{i=0}^{9} Ez_i(\log(p)-\lambda_1+i\log(\lambda_1))$$

$$+(Y_1-E[z_i|Y,\theta^{old}])(\log(1-p)-\lambda_2+i\log(\lambda_2))\Big)$$

To find the maximize solution, set $Q=0$ then we can compute λ_1, λ_2 and p.

5.

The plot are presented below:



You can find the R code in the r.file.