**Proof of the Analytical Solution for UVA 11150**

1. Some students give examples to show the bottles of cola one drinks is n+⎣n/2⎦. However, examples is not a proof.
2. Many students show the sum of geometrical series of (1/3)k to be 1/2 and conclude the bottles of cola one drinks is n+⎣n/2⎦. However, does not give the reason why n+⎣n/2⎦ is the solution.

Use mathematical induction to prove the solution.

**Proof:**

Let the number of bottles in n, where n≥1, at the beginning. Assume when n is even, we will borrow an empty bottle. We will show function f(n)=n+⎣n/2⎦ is the bottles of cola one drinks. Also, at the end there is one empty bottle left.

Base Case: When n=1, the person drinks only 1 bottle of cola and there is one empty bottle left. Hence, we have f(1)=1+⎣1/2⎦=1. When n=2, the person drinks two bottles at the beginning, and then use the two empty bottles and one borrowed empty bottle to exchange another cola. When he/she drinks the exchanged cola, he/she drinks 3 bottles of cola in total and there is one empty bottle which is returned to the lender in the final. Hence, we have f(2)=2+⎣2/2⎦=3.

Induction Case: For the induction hypothesis, ∀ k≥1, assume f(2k-1)=2k-1+⎣(2k-1)/2⎦, for the odd case, and f(2k)=2k+⎣2k/2⎦, for the even case. In addition, there is one empty bottle left he/she drinks all bottles of cola.

Suppose there are 2(k+1)-1 bottles of cola at the beginning, for the odd case, or 2(k+1), for the even case, at the beginning. Cola drinking will be carried in two steps as the following. For the odd case, process 2k-1 bottles at the step and then 2 bottles the second step, i.e., we will show f(2(k+1)-1)= f(2k-1)+f(2). Recall that, after finishing f(2k-1), there is one empty bottle left.

f(2k-1)+f(2) = 2k-1+⎣(2k-1)/2⎦+3

= 2k+2-1+⎣(2k+2-1)/2⎦

= 2(k+1)-1+⎣(2(k+1)-1)/2⎦

= f(2(k+1)-1).

For the even case, process 2k bottles at the step and then 2 bottles the second step, i.e., we will show f(2(k+1))= f(2k)+f(2). Recall that, after finishing f(2k), there is one empty bottle left.

f(2k)+f(2) = 2k+⎣2k/2⎦+3

= 2k+2+⎣(2k+2)/2⎦

= 2(k+1)+⎣2(k+1)/2⎦

= f(2(k+1)).

Hence, we have proved, ∀ n≥1, if there are n bottles at the beginning, he/she will drinks f(n)=n+⎣n/2⎦, finally. **Q.E.D.**