Algorithms

December 25, 2018

1 Linked List

We can use array to initialize the linked list as the following code. While building up the linked list from the given array, we use the **two-pointer technique** to maintain the linking between two nodes.

```
#Definition for singly-linked list.
class ListNode(object):
    def __init__(self, x):
        self.val = x
        self.next = None


class List(object):
    def __init__(self, array):
        if(array):
            self.head = ListNode(array[0])
            prev=self.head
            cur = None
            for i in range(1,len(array)):
                cur=ListNode(array[i])
                prev.next=cur
                prev=cur
        else:
            self.head = None

    def __str__(self):
        if(not self.head):
            return "[]"
        else:
```

```
            s="["
            cur=self.head
            while(cur):
                s=s+str(cur.val)+" "
                cur=cur.next
            s=s.strip()+"]"
            return s


array1=[1,4,5]
list1=List(array1)
print(list1)
```

2 Sliding Widow Technique

2.1 Count distinct elements in every window of size k

Tag: Sliding Window Technique, Hashtable. See [1].

```
Input:  arr[] = {1, 2, 1, 3, 4, 2, 3}, k = 4
Output: [3, 4, 4, 3]
```

We use the sliding window to update a hashtable, which maintains the distinct elements. And the time complexity is $O(n)$.

```
class Solution():
    '''2018-12-21
    '''
    def distinct(self,nums,k):
        if(not nums or len(nums)<k):
            return []
        d=dict()
        res=[]
        #init the first window
        for j in range(0,k):
            if(nums[j] not in d):
                d[nums[j]]=1
            else:
                d[nums[j]]+=1
```

[1] https://www.geeksforgeeks.org/count-distinct-elements-in-every-window-of-size-k/

```
        res.append(len(d))


        #update the remaining windows
        for i in range(1,len(nums)-k+1):
            #remove the first in the window, and add the last
            #to the window.
            first=nums[i-1]
            last=nums[i+k-1]
            if(d[first]==1):
                d.pop(first)
            else:
                d[first]-=1

            if(last not in d):
                d[last]=1
            else:
                d[last]+=1

            res.append(len(d))

        return res

    def testAll(self):
        testcase1={"nums":[1, 2, 1, 3, 4, 2, 3],"k":4,"expected":[3,4,4,3]}
        testcase2={"nums":[1, 2, 1],"k":4,"expected":[]}
        testcase3={"nums":[1, 2, 1, 3, 4, 2, 3, 5],"k":4,"expected":[3,4,4,3,4]}
        testcases=[testcase1,testcase2,testcase3]
        for testcase in testcases:
            self.test(testcase["nums"],testcase["k"],testcase["expected"])


    def test(self,nums,k,expected):
        res=self.distinct(nums,k)
        print("Test on nums={0}, k={1}. And {2} is expected, and {3} is got."\
                .format(nums,k,expected,res))


a=Solution()
a.testAll()
```

2.2 Sliding Window Maximum (Maximum of all subarrays of size k)

See [2].

```
Input :
arr[] = {1, 2, 3, 1, 4, 5, 2, 3, 6}
k = 3
Output :
3 3 4 5 5 5 6

Input :
arr[] = {8, 5, 10, 7, 9, 4, 15, 12, 90, 13}
k = 4
Output :
10 10 10 15 15 90 90
```

We use the priority queue to .

```
class Solution():
    """2018-12-21
    """
    def maxSlidingWindow(self,nums,k):
        pass
```

3 Heap

Heap can be viewed as a complete tree, but stored as the array. Suppose the current node's index is idx, then the left child's index is $2 * idx + 1$, and the right child $2 * idx + 2$, while the parent $floor((idx - 1)/2)$.

We take the binary max heap as an example. The basic external function is **insert** and **extractMax**, which is implemented by **siftup** and **siftdown**. The **siftup** function check the current node's value with its parent's value, then swap them if the current node's value is bigger than the parent's, and do the check-swap operation recursively to meet the guarantee of the binary max heap.

The python source code is as following.

[2] https://www.geeksforgeeks.org/sliding-window-maximum-maximum-of-all-subarrays-of-size-k/

```python
class MaxHeap():
    '''
    2018-12-24
    The root is bigger than its left child and right child.
    '''
    def __init__(self):
        self.array=[]

    def insert(self,num):
        if(not self.array):
            self.array.append(num)
        else:
            self.array.append(num)
            self.siftup(len(self.array)-1)


    def siftup(self,idx):
        if(idx==0):
            return
        parentIdx=(idx - 1) // 2
        if(self.array[parentIdx]<self.array[idx]):
            self.array[parentIdx], self.array[idx] = \
                    self.array[idx], self.array[parentIdx]
            return self.siftup(parentIdx)

    def extractMax(self):
        #swap the head (max) and the last one in self.array,
        ↪  then pop out the max
        if(not self.array):
            raise ValueError("pop out from an empty heap")


        ↪  self.array[0],self.array[-1]=self.array[-1],self.array[0]
        max=self.array.pop()
        self.siftdown(0)
        return max


    def siftdown(self,idx):
        '''
```

```
        move the current node down
        '''
        if(not self.array or len(self.array)==1):
            return
        left=idx*2+1
        right=idx*2+2
        maxIdx=idx
        if(left<len(self.array) and
        ↪  self.array[maxIdx]<self.array[left]):
            maxIdx=left
        if(right<len(self.array) and
        ↪  self.array[maxIdx]<self.array[right]):
            maxIdx=right

        self.array[idx], self.array[maxIdx] =
        ↪  self.array[maxIdx], self.array[idx]
        #sift down the smaller number recursively
        if(idx!=maxIdx):
            self.siftdown(maxIdx)


    def __str__(self):
        s=""
        for i in range(len(self.array)):
            if(i!=len(self.array)-1):
                s+=str(self.array[i])+" "
            else:
                s+=str(self.array[i])
        return s


heap=MaxHeap()
for i in range(1,10):
    heap.insert(i)
print("After inserting 1,2,3,4,5,6,7,8,9, the array of the
↪  heap is {0}.".format(heap))

maxInHeap=heap.extractMax()
print("Pop out from the heap, we'll get the maximum number
↪  {0}, "
```

```
74        "and the array of the heap becomes
          ↪  {1}.".format(maxInHeap,heap))
```

3.1 Python's heapq

We can use the library **heapq** in python. Since the default **heapq** is the min heap, so we need a trick to reimplement **MaxHeap** by overriding the comparison function.

```
1  import heapq
2  '''2018-12-24
3  Use python's heapq to implement a binary max heap.
4  '''
5
6  class MaxHeapObj(object):
7      def __init__(self, val):
8          self.val = val
9
10     def __lt__(self, other):
11         return self.val > other.val
12
13     def __eq__(self, other):
14         return self.val == other.val
15
16     def __str__(self):
17         return str(self.val)
18
19
20 class MaxHeap(object):
21   def __init__(self):
22       self.h = []
23
24   def heappush(self,x):
25       heapq.heappush(self.h,MaxHeapObj(x))
26
27   def heappop(self):
28       return heapq.heappop(self.h).val
29
30   def __getitem__(self,i):
31       return self.h[i].val
32
```

```
  def __str__(self):
      s=""
      for e in self.h:
          s=s+str(e)+" "
      return s


heap=MaxHeap()

for i in range(1,10):
    heap.heappush(i)

print("After inserting 1,2,3,4,5,6,7,8,9, the array of the heap is {0}.".format(heap))

maxInHeap=heap.heappop()
print("Pop out from the heap, we'll get the maximum number {0}, "
      "and the array of the heap becomes {1}.".format(maxInHeap,heap))
```

Or we can implement **MaxHeap** by multiplying -1 to each item in an array directly when using **heapq**.

```
import heapq
'''2018-12-24
Use python's heapq to implement a binary max heap.
Since heapq is the min heap, so we need to reimplement
MaxHeap by multiplying -1 to the item in min heap, and
multiply -1 as well when using heappop() funciton.
'''

class MaxHeap(object):
  def __init__(self):
      self.h = []

  def heappush(self,x):
      heapq.heappush(self.h,-x)

  def heappop(self):
      return heapq.heappop(self.h)*(-1)
```

```
18
19    def __getitem__(self, i):
20        return -1*self.h[i]
21
22    def __str__(self):
23        s=""
24        for i in range(len(self.h)):
25            s=s+str(self[i])+" "
26        return s
27
28
29  heap=MaxHeap()
30
31  for i in range(1,10):
32      heap.heappush(i)
33
34  print("After inserting 1,2,3,4,5,6,7,8,9, the array of the
    ↪  heap is {0}.".format(heap))
35
36  maxInHeap=heap.heappop()
37  print("Pop out from the heap, we'll get the maximum number
    ↪  {0}, "
38        "and the array of the heap becomes
          ↪  {1}.".format(maxInHeap,heap))
```

All the above three heap codes generate the following output.

```
After inserting 1,2,3,4,5,6,7,8,9, the array of the heap
↪  is 9 8 6 7 3 2 5 1 4 .
Pop out from the heap, we'll get the maximum number 9,
↪  and the array of the heap becomes 8 7 6 4 3 2 5 1 .
```

3.2 The application of heap

3.2.1 Merge k Sorted Lists

Merge k sorted linked lists and return it as one sorted list. Analyze and describe its complexity[3].

[3] https://leetcode.com/problems/merge-k-sorted-lists/description/

```
Input:
[
  1->4->5,
  1->3->4,
  2->6
]
Output: 1->1->2->3->4->4->5->6
```

```python
import heapq

# Definition for singly-linked list.
class ListNode(object):
    def __init__(self, x):
        self.val = x
        self.next = None

class List(object):
    def __init__(self,array):
        if(array):
            self.head = ListNode(array[0])
            prev=self.head
            cur = None
            for i in range(1,len(array)):
                cur=ListNode(array[i])
                prev.next=cur
                prev=cur
        else:
            self.head = None

class Solution(object):
    def mergeKLists(self, lists):
        """
        :type lists: List[ListNode]
        :rtype: ListNode
        """
        heap=[]
        #init heap, which contains the tuple of head.val and
        ↪  head of each list
        for list in lists:
```

```
            if(list):
                heapq.heappush(heap,(list.val,list))

        dummy=ListNode(0)
        tail_new_list=dummy
        #update
        while(heap):
            _,head=heapq.heappop(heap)
            if(head.next):
                heapq.heappush(heap,(head.next.val,head.next))

            tail_new_list.next=head
            tail_new_list=tail_new_list.next
        return dummy.next

def display(head):
    s=""
    cur=head
    while(cur):
        s+=str(cur.val)+" "
        cur=cur.next
    return s


array1=[1,4,5]
list1=List(array1)
print(list1)

array2=[1,3,4]
list2=List(array2)
print(list2)

array3=[2,6]
list3=List(array3)
print(list3)


solution=Solution()
mergedList=solution.mergeKLists([list1.head,list2.head,list3.head])
print(display(mergedList))
```

4 Dynamic Programming

4.1 Ugly Number II (LeetCode 264)

Write a program to find the n-th ugly number. Ugly numbers are positive numbers whose prime factors only include 2, 3, 5. Example:

```
Input: n = 10
Output: 12
Explanation: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12 is the
↪  sequence of the first 10 ugly numbers.
```

Note:

1. 1 is typically treated as an ugly number.
2. n does not exceed 1690.

Tag: Dynamic Programming.

We use the tabulation dynamic programming to compute n-th ugly number based on the previous ugly numbers. The recursive formula is defined as following.

$$F(n) = \begin{cases} 1, n = 1 & (a) \\ 2, n = 2 & (b) \\ 3, n = 3 & (c) \\ 4, n = 4 & (d) \\ 5, n = 5 & (e) \\ min\{2 * F(n-3), 3 * F(n-3), 5 * F(n-3)\}, n > 5 & (f) \end{cases} \quad (1)$$

5 Maths

5.1 Prime numbers

5.1.1 Ugly Number (LeetCode 263)

Write a program to check whether a given number is an ugly number.

Ugly numbers are positive numbers whose prime factors only include 2, 3, 5.

Example 1:

```
Input: 6
Output: true
Explanation: 6 = 2 * 3
```

Example 2:

```
Input: 8
Output: true
Explanation: 8 = 2 * 2 * 2
```

Example 3:

```
Input: 14
Output: false
Explanation: 14 is not ugly since it includes another
↪  prime factor 7.
```

Note:

1. 1 is typically treated as an ugly number.
2. Input is within the 32-bit signed integer range: $[-2^{31}, 2^{31} - 1]$.

We use the while loop to do the check and the decomposition for a given number. Since the given number is within the range $[-2^{31}, 2^{31} - 1]$, so we can do the check and the decomposition by recursion without worrying about the stack overflow (exceeding the maximum recursion depth).

```
class Solution(object):
    '''2018-12-25
    '''
    def isUgly(self, num):
        """
        :type num: int
        :rtype: bool
        """
        if (num<1):
            return False

        if (num == 1):
```

```
            return True

        while (num > 1):
            ugly = False
            for p in [2, 3, 5]:
                if (num % p == 0):
                    num = num / p
                    ugly = True
                    break
            if (ugly == False):
                return False
        return True


a=Solution()
print(a.isUgly(2147483648))
```

The recursion version is as following.

```
class Solution(object):
    '''2018-12-25
    '''
    def isUgly(self, num):
        """
        :type num: int
        :rtype: bool
        """
        def helper(num):
            if(num<=0):
                return False
            if(num==1):
                return True

            if(num%2==0):
                return helper(num//2)
            if(num%3==0):
                return helper(num//3)
            if(num%5==0):
                return helper(num//5)
            return False
```

```
22
23          return helper(num)
```