## Assignment 1.

1. Consider stochastic gradient descent method to learn the house price model

$$h(x_1, x_2) = \sigma(b + w_1x_1 + w_2x_2),$$

where  $\sigma$  is the sigmoid function.

Given one single data point  $(x_1,x_2,y)=(1,2,3)$ , and assuming that the current parameter is  $\theta^0=(b,w_1,w_2)=(4,5,6)$ , evaluate  $\theta^1$ .

$$h(X_1,X_2) = \sigma(z)$$

$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial \theta} = (h-y) \sigma'(z) \frac{\partial z}{\partial \theta} = (h-y) \sigma(z) (1-\sigma(z)) \frac{\partial z}{\partial \theta}$$

$$W_2' = W_2 - \alpha (h-y) \sigma(x) (1- \Gamma(x)) \chi_2$$

$$\Rightarrow h(x_1, x_2) = \sigma(z) = \sigma(z)$$

$$W_2^2 = 6 - \alpha(\sigma(21) - 3)) \sigma(21) (1 - \sigma(21)) \cdot 2$$

- 2. (a) Find the expression of  $\frac{d^k}{dx^k}\sigma$  in terms of  $\sigma(x)$  for  $k=1,\cdots,3$  where  $\sigma$  is the sigmoid function.
  - (b) Find the relation between sigmoid function and hyperbolic function.

$$\frac{d\sigma}{dx} = \frac{1}{(1 + e^{-x})^2} = \frac{1}{1 + e^{-x}} \cdot \frac{e^{-x}}{1 + e^{-x}} = \sigma(x) (1 - \sigma(x))$$

$$\frac{d^2\sigma}{dx^2} = \sigma'(x) (1 - \sigma(x)) - \sigma(x) \cdot \sigma'(x)$$

$$= \sigma(x) (1 - \sigma(x)) (1 - \sigma(x)) - \sigma(x) \sigma(x) (1 - \sigma(x))$$

$$= \sigma(x)(1-\sigma(x))(1-2\sigma(x)) ,$$

$$\frac{d^3\sigma}{dx^3} = \sigma'(x)(1-\sigma(x))(1-2\sigma(x)) - \sigma(x) \sigma'(x)(1-2\sigma(x))$$

$$= \sigma(x) (1 - \sigma(x))^{2} (1 - 2\sigma(x)) - \sigma(x)^{2} (1 - \sigma(x)) (1 - 2\sigma(x))$$

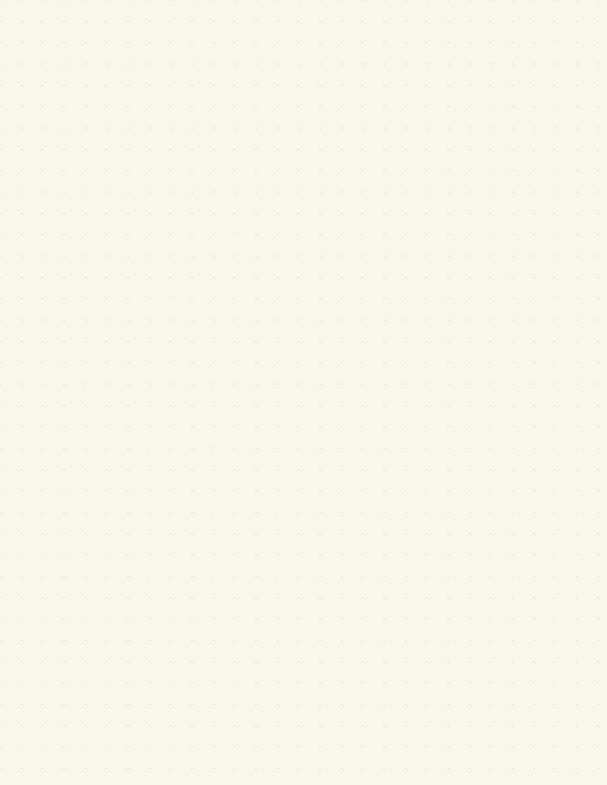
$$-2 \sigma(x)^{2} (1 - \sigma(x))^{2}$$

$$= \sigma(x)(1-\sigma(x)) [(1-\sigma(x))(1-2\sigma(x)) - \sigma(x)(1-2\sigma(x))$$

$$- 2\sigma(x)(1-\sigma(x)) ]$$

$$= \sigma(x) (1 - \sigma(x)) (1 - 6 \sigma(x) + 6 \sigma^{2}(x))$$

(b) 
$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$
  
 $\Rightarrow \tanh(\frac{x}{2}) = \frac{e^{x} - 1}{e^{x} + 1} = \frac{1 - e^{-x}}{1 + e^{-x}} = \frac{2 - (1 + e^{-x})}{1 + e^{-x}} = \sum \sigma(x) - 1$   
 $\Rightarrow \sigma(x) = \frac{1}{2} (1 + \tanh(x))$ 



3. There are unanswered questions during the lecture, and there are likely more questions we haven't covered. Take a moment to think about them and write them down here.															n					