Strategic Games I

Econ 702 Game Theory Recitations 2

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1 Strategic Game

- players $i \in N$
- actions $a \in A_i$
- preference (payoff) over all action proflie $(a_1, \ldots, a_n)$

2 Nash Equilibrium

2.1 Definition

- Notation.
  - $a = (a_1, \ldots, a_n)$, $a_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$
  - $a = (a_i, a_{-i})$
- An action profile a^* is a **Nash Equilibrium** if for each player i,

$$u_i(a^*) \geq u_i(a_i, a^*_{-i}), \forall a_i \in A_i \tag{1}$$

- It is possible for there to be one, many, or no Nash equilibria.

2.2 Interpretation

- evolutionary argument
- public and correct prediction
- self-enforcing agreements

3 Pareto Optimality

3.1 Definition

- An action profile a P-dominates $a' \Leftrightarrow ?$
- An action profile is P-optimal if there does not exits a' that P-dominates a.

3.2 Relationship with Nash Equilibrium

- P-optimal $\not\Rightarrow$ NE

- NE $\not\Rightarrow$ P-optimal

4 Best Responses

4.1 Definition

- $B_i(a_i) = \{a_i \in A_i \mid u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \forall a_i' \in A_i\}$

4.2 How to Find the Best Responses

- 2×2 games with the payoff matrix

- games with continuous action: first-order condition

4.3 Relationship with Nash Equilibrium

- a^* is NE $\Leftrightarrow a_i^* \in B_i(a_{-i}^*) \forall i \in N$

- if B_i is single value, then the condition becomes $a_i^* = b_i(a_{-i}^*)$

5 Games and Nash Equilibrium

5.1 Examples

- Meetng in NYC

- Prisoner's Dilemma (Give and Keep, Duopoly)

- Stag Hunt

- Matching Pennies

- Tragedy of Commons

5.2 How to find NE

- Guess and Prove

- 2×2 Games (payoff matrix)

- Best responses