Samuel Bowles & Weikai Chen

Post-Walrasian Microeconomics

Notes, Problems and Solutions

OCTOBER 9, 2019

[T]he age of chivalry is gone. That of Sophisters, economists, and calculators has succeeded.

Edmund Burke, *Reflections on the Revolution in France (1790)*

Mathematicians are a kind of Frenchmen. Whenever you say anything or talk to them, they translate it into their own language, and right away it is something completely different.

Johann Wolfgang von Goethe (1749-1832), *Maxims and Reflections*

If economists could manage to get themselves thought of as humble, competent people, on a level with dentists, that would be splendid!

John Maynard Keynes, *Economic Possibilities for our Grandchildren (1930)*

Contents

(b) There are three Pareto efficient outcomes in this game. They are (Fish 4 hours, Fish 4 hours), (Fish 4 hours, Fish 9 hours), (Fish 9 hours, Fish 4 hours), as shown in Figure 4.1.

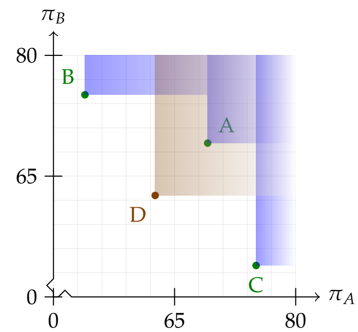

Figure 4.1: Denote the four ordered pairs of payoffs in the payoff matrix by A, B, C, and D. D is Parato dominanted by A, since A locates in the upper-right conner of D. All the outcomes are Pareto efficient except D.

2. Given $\delta = 0.3$, we have the payoff matrix as Table 4.2. The game is no longer a PD game since the dominant strategy equilibrium (Fish 4 hours, Fish 4 hours) is Pareto efficient.

Table 4.2: The Game of Fishermen's Tragedy, with $\delta = 0.3$.

	Bob	
Alfredo	4 hours	9 hours
4 hours	**65.9**, **65.9**	**50.6**, 58.9
9 hours	58.9, **50.6**	46.4, 46.4

3. To keep the game a PD game, by Problem 1.1, we should have

$$U_n(4,9) < U_n(9,9) < U_n(4,4) < U_n(9,4)$$
$$U_n(4,9) + U_n(9,4) < 2U_n(4,4)$$

for $n = A, B$. Given the utility function (4.1), we can solve that $0 < \delta < 0.192$.

4. Suppose Bob chose p, Alfredo's best response is to fish 9 hours all the time since Fish 9 hours is the dominant strategy as shown in Table 4.1. Then the payoff to Bob by choosing p is

$$\pi_B(p) = 53.9(1-p) + 62.6p = 8.7p + 53.9$$

Therefore, the Bob will choose to fish 9 hours, that is $p^* = 1$.

5. Suppose that Bob can offer (p, q) where p is the fraction of time to fish 9 hours himself and q is that for Alfredo. Then Given the payoff matrix in Table 4.1, the payoff to them is

$$\begin{aligned}\pi_A(p,q) &= 69.1(1-p)(1-q) + 53.9p(1-q) + 75.1(1-p)q + 62.6pq \\ &= 2.7pq - 15.2p + 6q + 69.1 \\ \pi_B(p,q) &= 2.7pq + 6p - 15.2q + 69.1\end{aligned}$$

Since Bob has the TIOLI power and Alfredo's fallback position is back to play the Nash equilibrium (Fish 9 hours, Fish 9 hours), Bob's programming problem is

$$\begin{aligned}\max_{0 \le p,q \le 1} \quad & \pi_B = 2.7pq + 6p - 15.2q + 69.1 \\ \text{s.t.} \quad & \pi_A = 2.7pq - 15.2p + 6q + 69.1 \ge 62.6\end{aligned}$$

Let[1]

$$\mathcal{L} = \pi_B + \lambda(\pi_A - 62.6) + \mu(1-p) + \gamma(1-q)$$

The Kuhn-Tucker condition is

$$\mathcal{L}_p = 2.7q + 6 + \lambda(2.7q - 15.2) - \mu \le 0 \tag{4.2}$$
$$\mathcal{L}_q = 2.7p - 15.2 + \lambda(2.7p + 6) - \gamma \le 0 \tag{4.3}$$
$$p\mathcal{L}_p = q\mathcal{L}_p = \lambda(\pi_A - 62.6) = \mu(1-p) = \gamma(1-q) = 0 \tag{4.4}$$
$$p, q, \lambda, \mu, \gamma \ge 0, \quad \pi_A \ge 62.6 \tag{4.5}$$

[1] Figure 4.2 provides an alternative solution by looking the π_A-π_B planes.

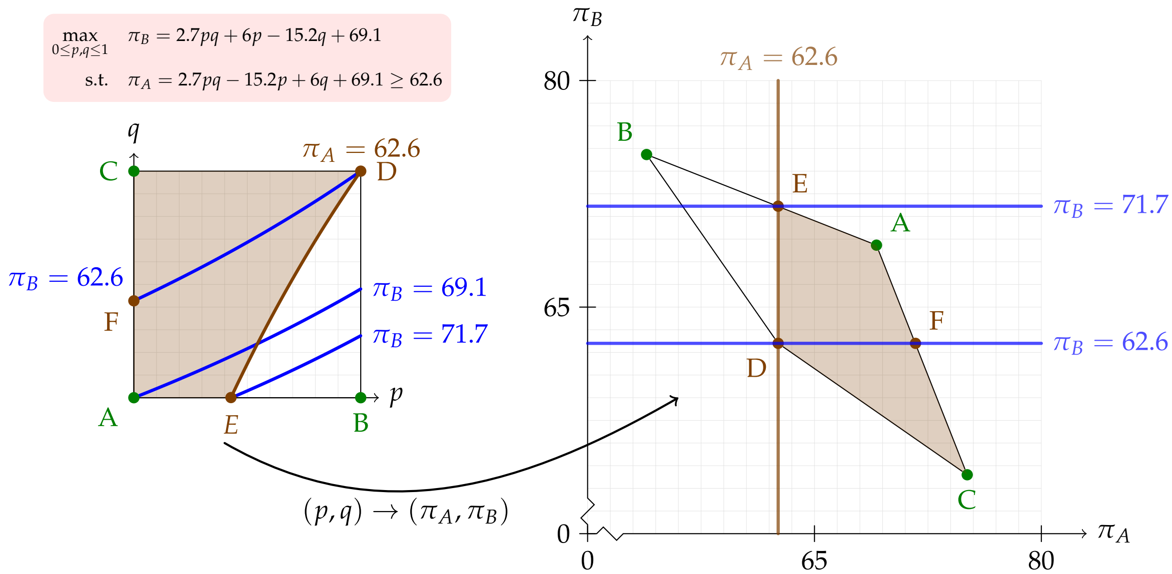

Figure 4.2: Denote the four ordered pairs of payoffs in the payoff matrix by A, B, C, and D. Bob has the TIOLI power and offers (p,q). The expected payoff function

$$(p,q) \to (\pi_A, \pi_B)$$

maps any point in left plane to the point in the right. $\pi_A = c$ and $\pi_B = c$ are indifferences curves, and the shadow region

$$\{(p,q) \mid \pi_A(p,q) \geq 62.6\}$$

is the feasible set. Point E maximizes π_B, representing the offer he will make $(p^*, q^*) = (0.43, 0)$.

We will show that $0 < p < 1$ and $q = 0$ by contradiction. Suppose $p = 1$, by $\pi_A(1,q) = 53.9(1-q) + 62.6q \geq 62.6$, we have $q = 1$. Then (4.2) and (4.3) become

$$\begin{cases} 8.7 - 12.5\lambda - \mu = 0 \\ -12.5 + 8.7\lambda - \gamma = 0 \end{cases} \Rightarrow \begin{cases} \lambda = \frac{8.7-\mu}{12.5} \leq \frac{8.7}{12.5} \\ \lambda = \frac{12.5+\gamma}{12.5} \geq \frac{12.5}{8.7} \end{cases}$$

contradicted. Therefore, $p < 1$ and then $\mu = 0$.

Suppose $p = 0$, then

$$\pi_A(0,q) = 69.1(1-q) + 75.1q > 62.6$$

so we have $\lambda = 0$. Then (4.2) becomes $2.7q + 6 \leq 0$, contradicted. Therefore, we have $0 < p < 1$ and $\mathcal{L}_p = 0$. Therefore, $\lambda > 0$ otherwise $\mathcal{L}_p = 2.7q + 6 > 0$ contradicted.

Suppose $q = 1$, by $p < 1$ we have

$$\pi_A(p,1) = 75.1(1-p) + 62.6p > 62.6$$

contradicted with $\lambda > 0$. Therefore, we have $q < 1$ and then $\gamma = 0$.

Suppose $q > 0$, then $\mathcal{L}_q = 0$, which is inconsistent with $\mathcal{L}_p = 0$[2]. Therefore, we have $q = 0$. Then by $\pi_A(p,0) = 62.5$ we have

$$p = \frac{69.1 - 62.6}{15.2} \approx 0.43$$

Therefore, Bob will offer $(p^*, q^*) = (0.43, 0)$ if he has TIOLI power. As a result, $p_A = 62.5$ and $\pi_B = 71.7$.

[2] Since

$$\mathcal{L}_p = 0 \Rightarrow \lambda = \frac{2.7q + 6}{15.2 - 2.7q} < 8.7/12.5$$

while

$$\mathcal{L}_q = 0 \Rightarrow \lambda = \frac{15.2 - 2.7p}{2.7p + 6} > 12.5/8.7.$$

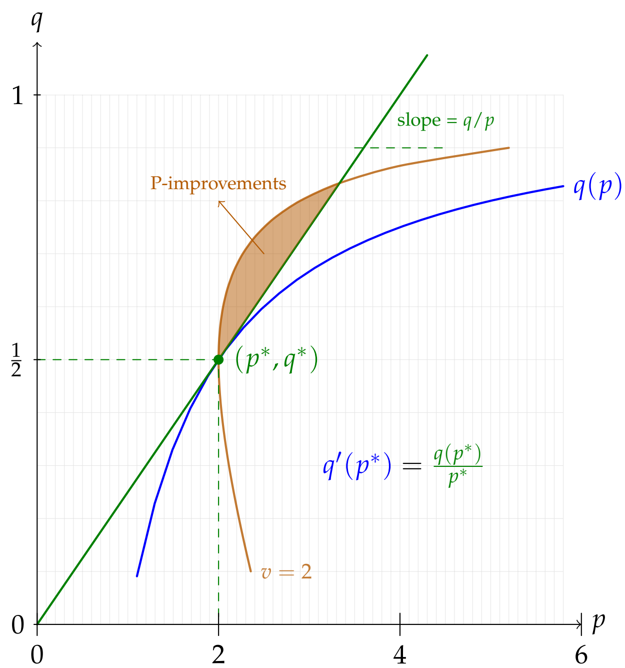

Figure 6.5: Maximizing p/q subject to the supplier's best-response function. $v = 2$ is the supplier's indifference curve and $q(p)$ is her best response function. The slope of the green ray is $\frac{q}{p}$. The first order condition to $\min_p p/q$, or $\max_p q/p$, is

$$q'(p) = \frac{q(p)}{p},$$

the green ray is tangent with the best response function. The price and level of quality supplied at equilibrium is

$$(p^*, q^*) = (4\delta, \frac{1}{2}),$$

with $\delta = 0.5$. It is not Pareto efficient and any point in the shadow region is ay Pareto-improvement.

5. We know that the the optimal quality level is $q^* = \frac{1}{2}$ for any $\delta > 0$. Therefore, the change in δ affects the buyer's payoff only through the price. Since the equilibrium price is $p^* = 4\delta$, the benifit from reducing δ is

$$\Delta p^* = 4\Delta\delta.$$

Therefore, the largest cost (per unit of δ) the buyer would be willing to pay to reduce δ for a single period is 4.

□

6.4 *Truck and Barter*

Where the care of a capital good is not verifiable, conventional rental contracts are often unattractive to the owner. This a reason why instead of renting bicycles, some companies sell the bikes to the user and then buy them back at the end of the contracted period, with the price depending on the condition of the bike.

Here is another vehicular example: P owns a truck worth \$1 which is to be used by A; it may be run at speed f, resulting in a probability $\phi(f) \in [0, 1]$ that the truck will be wrecked (in which case its scrap value is zero) with $\phi' > 0$ and $\phi'' > 0$. If the truck is not wrecked, its value at the end of the period is undiminished and still worth \$1. The benefits to the agent are βD, where D is the distance traveled in the period (which, normalizing the hours of work of the agent