

Econ 700 Problem Set 1 (Solution)

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Sep 26, 2019

1 The Language of Game Theory

1. Suppose Table 1 is the payoff matrix for the row player in a two-person symmetrical Hawk Dove, Prisoners' Dilemma, and Assurance Game. Indicate the restrictions on the values of these payoffs which are necessary and sufficient in each case for the game to be properly defined as Hawk Dove, Assurance, and Prisoners' Dilemma Games.

	Cooperate (Dove)	Defect (Hawk)
Cooperate (Dove)	b	d
Defect (Hawk)	a	c

Table 1: Symmetrical Hawk Dove, Prisoners' Dilemma, and Assurance Game (row payoff)

2. North and South are selecting environmental policies. The well-being of each is interdependent, in part due to global environmental effects. Each has a choice of two strategies: Emit or Restrict emission. Suppose this is just a two-person game. It may clarify things to let the representative citizen in each region have a reduced form utility function $u^i = u^i(e^i, e^j)$, where i is the level of emissions (0 or 1) and the superscripts i and j refer to North and South. (It is a reduced form because the citizens' well-being is proximately affected not by emissions per se but by the things with which emissions are associated positively (consumption) or negatively (health status).) Some have modeled this problem as a prisoners' dilemma, while others have proposed the Assurance Game or even the Chicken (Hawk Dove) Game ¹. Illustrate each of these possibilities with a payoff matrix and explain why it might be a reasonable depiction of the interaction. Suppose North's utility function has the form

$$u^i = \alpha e^i + \beta e^j + \gamma e^j e^i \quad (1)$$

and South's is identical (with appropriate substitution of superscripts). What values of the parameters of these utility functions would make each of these three games the appropriate model of the North-South Emissions Game?

Answer.

1. *Hawk Dove Game*, when $a > b > d > c$. Note that we usually assume $c < 0$ in the Hawk Dove game.

¹ Michael Taylor. *The possibility of cooperation*. Cambridge University Press, 1987

Assurance Game, when $b > a > c > d$.

Prisoner's Dilemma, when $a > b > c > d$. Note that we may also assume $2b > a + d$ to exclude the case that playing Cooperate and Defect alternatively (with expected utility $(a + d)/2$) is better than both playing Cooperate.

2. By the utility function (1), we have the payoff matrix as shown in Table 2. Then we have:

- (a) Prisoner's Dilemma, when $\alpha > 0 > \alpha + \beta + \gamma > \beta$ and $\alpha + \beta < 0$.

This is possible in the following case. Both regions produce the same profitable product with two different techniques. The first one cost less with emission, and the second one cost more without emission. Also assume that both regions will suffer from pollution no matter which one choose to emit. If the North choose Restrict and South choose Emit, the citizens in the North are affected negatively not only by the environmental pollution, but also the asymmetric competition of the production. The disadvantage of asymmetric competition is so large that the citizens would prefer to lower the cost and suffer more pollution.

- (b) Assurance Game, when $\beta < \alpha < \alpha + \beta + \gamma < 0$. Assume that both regions produce something with emission, and now they want to replace this industry with tourism. If both of them restrict emission, they will gain more from tourism and clear environment. However, if one of them keeps the old industry, and the other restricts emission to develop tourism, then the old industry will be negatively affected because of the increasing return to scale, and the tourism will be destroyed due to the pollution. Therefore, it is better for both to switch from the lower equilibrium to the new one.

- (c) Chicken (Hawk Dove) Game, when $\alpha > 0 > \beta > \alpha + \beta + \gamma$. Consider the following situation. If only one region Emission, the benefits from production is larger than the negative effect of the pollution in the Emission region, while the Restrict player will be affected negatively but still acceptable. If both region Emission, the pollution will beyond the threshold and lead to disaster.

Table 2: The Environmental Policies Game

	Emit	Restrict
Emit	$\alpha + \beta + \gamma$	α
Restrict	β	0

□

2 An Offer You can Refuse and Inequality Aversion

Two players i and j are playing the Ultimatum Game dividing one unit between proposer and responder. The proposer offers a certain

portion to the responder. If the responder accepts, the responder gets the proposed portion and the proposer keeps the rest. If the responder rejects the offer, both get nothing. Now suppose that individual i 's preferences are given by

$$u_i = \pi_i - \delta_i \max(\pi_j - \pi_i, 0) - \alpha_i \max(\pi_i - \pi_j, 0) \quad (2)$$

with $\alpha = \frac{1}{2}$ and $\delta = \frac{3}{4}$, where π_j and π_i are the material payoffs to the two individuals.

1. Were she the respondent in an Ultimatum Game, what is the smallest offer she would accept?
2. If she were the proposer and knew that the respondent had identical preferences to hers, can you say what she would offer?

Answer.

1. Suppose $\pi_i \leq \pi_j$, with $\alpha = \frac{1}{2}$, $\delta = \frac{3}{4}$, we have

$$u_i = \pi_i - \frac{3}{4}(\pi_j - \pi_i) = \pi_i - \frac{3}{4}(1 - 2\pi_i) = \frac{5}{2}\pi_i - \frac{3}{4}$$

Let $u_i \geq 0$, we have $\pi_i \geq 0.3$ as shown in Figure 1.

2. Since j has the same preference as i , the smallest offer j would accept is $\pi_j^{min} = 0.3$. For $\pi_i > \pi_j$, we have

$$u_i = \pi_i - \frac{1}{2}(\pi_i - \pi_j) = \frac{1}{2}$$

That is, i is indifferent with any $\pi_i \geq 0.5$. Therefore, she would offer $\pi_j \in [0.3, 0.5]$ as shown in Figure 1.

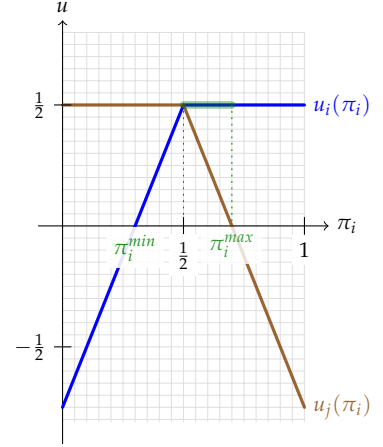


Figure 1: Given the parameters $\alpha = \frac{1}{2}$, $\delta = \frac{3}{4}$, and $\pi_j = 1 - \pi_i$ we have

$$u_i(\pi_i) = \begin{cases} \frac{5}{2}\pi_i - \frac{3}{4}, & 0 \leq \pi_i \leq \frac{1}{2} \\ \frac{1}{2}, & \frac{1}{2} < \pi_i \leq 1 \end{cases}$$

If i is the responder, then $\pi_i^{min} = 0.3$ is the smallest offer she would accept to make sure $u_i \geq 0$. Suppose i is the proposer and the responder has same utility, then to make sure that j would accept the offer, we have $\pi_i^{max} = 0.7$. Therefore, she would set

$$0.5 \leq \pi_i \leq 0.7.$$

□

3 Reciprocity

Where individuals have social preferences there may be a large number of equilibria even in simple interactions. This is particularly the case if preferences are endogenous or if reciprocity is a strong motive. Here is an example concerning reciprocity. Two individuals are considering contributing effort e_i and e_j , both in $[0, 1]$, to a common project, the output of which, $e_i + e_j$, will be shared equally between the two. The two have preferences as described below.

$$u_i = \pi_i + \beta_{ij}\pi_j \quad (3)$$

where

$$\beta_{ij} = \frac{b_i + \lambda_i a_j}{1 + \lambda_i} \quad (4)$$

and $a_j \in [-1, 1]$ and $\lambda_i \geq 0$. The parameter b_i is i 's level of unconditional good will or ill will (altruism or spite) toward others, and $a_j \in [-1, 1]$ is i 's belief about j 's good will, while λ_i indicates the extent to which i conditions his evaluations of others' payoffs on (beliefs about) the other's type. Suppose the subjective cost of effort is

$$c(e) = \frac{3}{4}e$$

and

$$b = \lambda = \frac{1}{2}$$

for each person. The belief about the goodwill of the other is simply the amount that each believes other will contribute to the project (so, for example, if believes will contribute 1 to the project, then $a_j = 1$).

1. Identify the three pure strategy Bayesian equilibria of this game.
2. Indicate which are stable, and give the critical values of the initial beliefs a_i and a_j , such that the Pareto-superior outcome can be sustained as a Nash equilibrium.

Answer.

1. The material payoff is

$$\begin{aligned}\pi_i &= \frac{e_i + e_j}{2} - \frac{3}{4}e_i = \frac{1}{2}e_j - \frac{1}{4}e_i \\ \pi_j &= \frac{e_i + e_j}{2} - \frac{3}{4}e_j = \frac{1}{2}e_i - \frac{1}{4}e_j\end{aligned}$$

Given the parameters $b = \lambda = \frac{1}{2}$ we have

$$\beta_{ij} = \frac{0.5 + 0.5a_j}{1 + 0.5} = \frac{1 + a_j}{3}$$

then ²

$$u_i = \pi_i + \beta_{ij}\pi_j = \frac{2a_j - 1}{12}e_i + \frac{5 - a_j}{12}e_j \implies \frac{du_i}{de_i} = \frac{1}{6}(a_j - \frac{1}{2})$$

Therefore, we have

$$e_i^* = \begin{cases} 1 & a_j > \frac{1}{2}, \\ [0, 1] & a_j = \frac{1}{2}, \\ 0 & a_j < \frac{1}{2}. \end{cases}$$

At any Bayesian equilibrium, the belief a_j should be consistent with the behavior. Therefore, the three pure Bayesian Nash equilibria are $e_i = e_j = a_i = a_j = 1$, $e_i = e_j = a_i = a_j = 0$ and $e_i = e_j = a_i = a_j = \frac{1}{2}$, as shown in Figure 2.

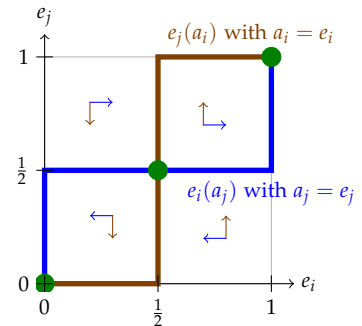


Figure 2: Any point (e_i, e_j) in the region $[0, 1] \times [0, 1]$ can be supported as a Nash equilibrium by some belief system (a_i, a_j) . With consistent belief $a_j = e_j$ and $a_i = e_i$, $e_i(e_j)$ and $e_j(e_i)$ are the best reply for i and j . There are three (Bayesian) Nash equilibria:

$$(0, 0), (1, 1), (\frac{1}{2}, \frac{1}{2}).$$

The arrows in the figure show that $(0, 0)$, $(1, 1)$ are stable while $(\frac{1}{2}, \frac{1}{2})$ is not.

² Symmetrically

$$u_j = \frac{2a_i - 1}{12}e_j + \frac{5 - a_i}{12}e_i.$$

2. Both $e_i = e_j = a_i = a_j = 1$ and $e_i = e_j = a_i = a_j = 0$ are stable in the sense that a small disturbances in the beliefs would always bring the system back to the equilibrium. The Pareto-superior outcome is $e_i = e_j = 1$, which is supported as an equilibrium when a_i and a_j is greater than $\frac{1}{2}$.

□

4 The Tragedy of Fishers

Right now, my only incentive is to go out and kill as many fish as I can [...] any fish I leave is just going to be picked by the next guy.

— John Sorlien, Rhode Island lobsterman

SUPPOSE the fishing technology and abundance of fish is such that, letting H_A and H_B be the number of hours worked by the Alfredo and Bob respectively, the number of kilos of fish caught respectively by Alfredo and Bob is

$$F_n = 100 \frac{\sqrt{H_n}}{\sqrt{H_A + H_B}}, \quad n = A, B$$

Let the dis-utility of working H_n be δH_n^2 , $n = A, B$. Taking both the value they place on the fish and the fact that effort is onerous into account we can write the utility of the two as the fish caught minus the subjective cost of effort. That is

$$U_n = 100 \frac{\sqrt{H_n}}{\sqrt{H_A + H_B}} - \delta H_n^2, \quad n = A, B \quad (5)$$

Suppose $\delta = 0.1$ and there are only two choices Alfredo and Bob can make: fish 4 hours or fish 9 hours.

1. Give the payoff matrix for the two and
 - (a) Confirm that fishing 9 hours is the dominant strategy, and that fishing 4 hours is a Pareto improvement over the dominant strategy.
 - (b) Identify the Pareto efficient outcomes in this game.
2. Now suppose the World Cup is on, so the subjective cost of the time they spend fishing increases to $\delta = 0.3$. They would rather be watching the giant screen in the town square rather than peering at the radar screens on their boats, looking for signs of fish. The opportunity cost of their time fishing is no longer sitting in the square just having a coffee; it is missing the World Cup! Is the game still a Prisoners' Dilemma? Explain why or why not.

John Tierney. A Tale of Two Fisheries, aug 2000. URL <http://www.nytimes.com/2000/08/27/magazine/a-tale-of-two-fisheries.html>

3. Find the restriction on δ such that the game is still a Prisoners' Dilemma.

Answer.

1. Given the utility function and $\delta = 0.1$, we have the payoff matrix as Table 3.

- (a) It is a symmetric game, so we only need to confirm that fishing 9 hours is a dominant strategy for Alfredo. As we can see in the payoff matrix, $75.1 > 69.1$, $62.6 > 53.9$. That is, for Alfredo, the payoff to fishing 9 hours is higher than that to fishing 4 hours, no matter what Bob plays. Therefore, fishing 9 hours is the dominant strategy. However, the payoff to both fishing 4 hours (69.1) is greater than the dominant strategy (both fishing 9 hours, 62.6). Therefore, fishing 4 hours is a Pareto improvement over the dominant strategy.
- (b) There are three Pareto efficient outcomes in this game. They are (Fish 4 hours, Fish 4 hours), (Fish 4 hours, Fish 9 hours), (Fish 9 hours, Fish 4 hours), as shown in Figure 3.

2. Given $\delta = 0.3$, we have the payoff matrix as Table 4. The game is no longer a PD game since the dominant strategy equilibrium (Fish 4 hours, Fish 4 hours) is Pareto efficient.
3. To keep the game a PD game, by Problem 1, we should have

$$U_n(4,9) < U_n(9,9) < U_n(4,4) < U_n(9,4)$$

$$U_n(4,9) + U_n(9,4) < 2U_n(4,4)$$

for $n = A, B$. Given the utility function (5), we can solve that $0 < \delta < 0.192$.

□

5 Nash's "American Way" and Collective Action

It seemed to von Neumann that the solution concept Nash equilibrium is "too individualistic" as it considers only the potential gains that are to be had by deviating from a strategy profile singly, ignoring the possibility that more than one individual might deviate collectively. But if deviations by more than one individual are possible, why not everyone at once (and then we would have a cooperative game, and Pareto-inferior equilibria would be impossible). Or why not some set of feasible deviations from a status quo strategy profile

Table 3: The Game of Fishermen's Tragedy, with $\delta = 0.1$.

Alfredo	Bob	
	4 hours	9 hours
4 hours	69.1, 69.1	53.9, 75.1
9 hours	75.1, 53.9	62.6, 62.6

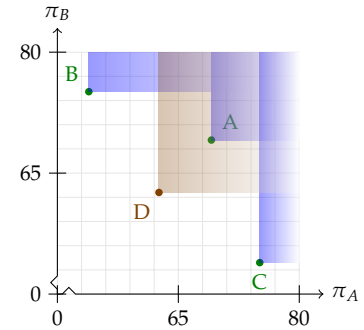


Figure 3: Denote the four ordered pairs of payoffs in the payoff matrix by A, B, C, and D. D is Pareto dominated by A, since A locates in the upper-right corner of D. All the outcomes are Pareto efficient except D.

Table 4: The Game of Fishermen's Tragedy, with $\delta = 0.3$.

Alfredo	Bob	
	4 hours	9 hours
4 hours	65.9, 65.9	50.6, 58.9
9 hours	58.9, 50.6	46.4, 46.4

intermediate between the fully cooperative game and Nash's "American way" in which only solo deviations are considered in defining equilibrium?

If we know the social relationships among the individuals then we may be able to say plausibly who might have the capacity to act collectively to disrupt a status quo strategy profile. For example, the individuals making up a family might find it easy to coordinate a deviation, or a members of trade union. Network theory and data can help us answer the question³.

To see how a network of these relationships might help determine if a particular strategy profile is an equilibrium, let's consider the social network with an adjacency matrix

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

The matrix G describes the social relationships among 5 people (called nodes of the network) where the entries $g_{ij} = 1$ if individual i and j are connected by an edge and $g_{ij} = 0$ otherwise.

1. Draw the social network represented by G , with an edge depicted as a line between the points representing the nodes.

Now consider the following game called Plant in the village of Palanpur⁴, in which the players has the social relationships represented by G and each player has two strategies, Plant Early (E) and Plant Late (L). Suppose that the payoffs to adopting strategy $s_i = E, L$ is given by

$$u_i(s_i, n_i) = \begin{cases} 2 + 0.25n_i & s_i = L \\ n_i & s_i = E \end{cases} \quad (6)$$

where n_i is the number of *others* adopting E .

2. Identify any pure strategy Nash equilibria of this game.

Now define an m -equilibrium an outcome for which no person can do better by deviating, when the individuals making up a feasible deviation group must be connected by not more than m edges. That is, players can deviate together as a group in which one could reach another by at most m edges. This definition include Nash equilibrium as a special case with $m = 0$.

3. Is all planting late a 1-equilibrium of the game? Explain your reasoning.

³ Mathew Jackson. *Social and economic networks*. Princeton University Press, Princeton, 2008; and Willemien Kets, Garud Iyengar, Rajiv Sethi, and Samuel Bowles. Inequality and network structure. *Games and Economic Behavior*, 73(1):215–226, sep 2011

⁴ Peter Lanjouw and Nicholas Stern. *Economic development in Palanpur over five decades*. Clarendon Press, 1998

4. What is the least value of m such that only all planting early is equilibrium of this game?
5. Denote the set of m -equilibrium by $E(m)$. What is the relationship between $E(m)$ and $E(m - 1)$ in general? Explain your reasoning.

Answer.

1. The social network representing by G is shown in Figure 4.
2. Denote the total number of players playing E by N , then $N = 0, 1, \dots, 5$. All plant late and all plant early are the two pure Nash equilibrium as discussed below and shown in Figure 5.

- (a) For $N = 0$, i.e., all players play L . For any players i , we have $n_i = 0$ and

$$u_i(L, 0) = 2 > u_i(E, 0) = 0$$

which means no one has incentive to deviate. Therefore, all players playing L is a Nash equilibrium.

- (b) For $N = 1$, i.e., all but one play L . For the one playing E , we have $u_i = 0$. By

$$u_i(E, 0) = 0 < u_i(L, 0) = 2$$

she has incentive to deviate. Therefore, $N = 1$ is not a Nash equilibrium. By similar reasoning we find that $N = 2, 3, 4$ are not Nash equilibria.

- (c) For $N = 5$, i.e., three players play E . For any player, we have $n_i = 4$. By

$$u_i(E, 4) = 4 > u_i(L, 4) = 3$$

no one has incentive to deviate. Therefore, all players playing is a Nash equilibrium.

3. When $m = 1$, the largest deviation group is $\{3, 4, 5\}$ as shown in Figure 6. Consider the case that $N = 0$, i.e., all playing L . We can show that the group $\{3, 4, 5\}$ has no incentive to deviate, since for any $i \in \{3, 4, 5\}$, the payoff to deviate together is

$$u_i(E, 2) = 2$$

which is not greater than $u_i(L, 0) = 2$. Therefore, all playing late is a (weakly) 1-equilibrium.

4. When $m = 2$, the largest deviation group is $\{2, 3, 4, 5\}$ as shown in Figure 7. Consider all playing late, the payoff to any players in this group is $u_i(L, 0) = 2$. However, if they deviate together, then for any $i \in \{2, 3, 4, 5\}$, we have $n_i = 3$ and

$$u_i(E, 3) = 3 > u_i(L, 0) = 2.$$

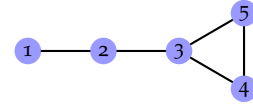


Figure 4: The social network representing by the adjacency matrix

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

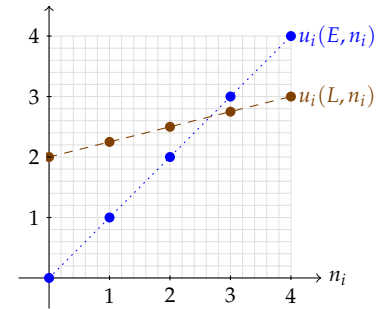


Figure 5: The payoff functions in the game "Plant in Palampur"

$$u_i(E, n_i) = n_i$$

$$u_i(L, n_i) = 2 + 0.25n_i$$

where n_i is the number of others adopting E . Note that for any player i , n_i remains the same when she deviates alone.

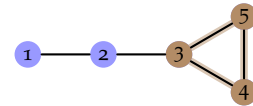


Figure 6: The largest deviation group, $\{3, 4, 5\}$ when $m = 1$, in which one could reach another by at most 1 edge. The possible deviation groups with 2 members include $\{1, 2\}$, $\{2, 3\}$, $\{3, 4\}$, $\{3, 5\}$ and $\{4, 5\}$. All singletons are deviation groups with 1 member.

That is, the group $\{2, 3, 4, 5\}$ has incentive to deviate together. Therefore, planting late is not a 2-equilibrium.

Next we check that all playing early is a 2-equilibrium. For any player, the payoff after deviate is

$$u_i(L, n_i) = 2 + 0.25n_i \leq 3 < 4$$

since $n_i \leq 4$. Therefore, no one has the incentive to deviate within any possible deviation groups. In sum, $m = 2$ is the least value such that only planting early is equilibrium of this game.

5. Any feasible deviation group for $(m - 1)$ -equilibrium, with all members connected by at most $m - 1$ edges, is also a feasible deviation group for m -equilibrium. Hence, any m -equilibrium must survive from the deviation tests for $(m - 1)$ -equilibrium. Therefore, $E(m) \subseteq E(m - 1)$. In other words, a larger m destabilizes the equilibrium.

□

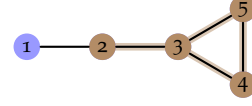


Figure 7: The largest deviation group, $\{2, 3, 4, 5\}$ when $m = 2$, in which one could reach another by at most 2 edge. The possible deviation groups with 3 members include $\{1, 2, 3\}$, $\{2, 3, 4\}$, $\{2, 3, 5\}$ and $\{3, 4, 5\}$.

6 Incentive and Social Preference

Consider an individual who may bear a cost to take an action that confers benefits on others, which may be encouraged by a subsidy implemented by a social planner. Citizens also have values that may motivate such pro-social actions even in the absence of the subsidy. We study a single member of a community of identical citizens who may contribute to a public project by taking an action a at a cost $g(a)$ that is increasing and convex in its argument, and that may be offset partially by a subsidy s , that is proportional to the individual's level of contribution. The output of the project is available in equal measure to all, and it varies positively and linearly with A , the sum of the n members' contributions, according to ϕA where ϕ is a positive constant.

We express the individual's social preferences as v , the effect of an increase in the contribution level on the individual's utility that is unrelated to material payoffs. Thus we have the individual's utility

$$u = \phi A - g(a) + as + av \quad (7)$$

we make explicit the sources of non-separability by the value function

$$v(s; \lambda_o; \lambda_c, \lambda_m) = \lambda_0(1 + \mathbf{1}_{\{s > 0\}}\lambda_c + s\lambda_m) \quad (8)$$

where the indicator $\mathbf{1}_{\{s > 0\}} = 1$ if $s > 0$ and zero otherwise. In equation (8) $\lambda_0 \geq 0$ measure the citizen's baseline social preferences,

namely the citizen's values in the absence of a subsidy or $v(s; \mathbf{0})$, λ_c measures the categorical effect of the presence of an incentive, and λ_m measures the marginal effect of variations in s on values for $s > 0$. The crowding effects represented by λ_c and λ_m may arise because of any of the three mechanisms by which state dependent preferences arise: bad news, moral disengagement or control aversion.

1. Suppose (here and below) that $g(a) = \frac{1}{2}a^2$, what is the best response function of the citizen?

Let θ be the marginal benefits of contributing, i.e., the returns from the public goods plus the subsidy plus the effect on the individual's values.

2. Consider the case in which there initially is no incentive, find the effect of an incentive on the θ , i.e., $\frac{\Delta\theta}{\Delta s}$ at $s = 0$.

We say that a particular change in incentive Δs has crowded out social preferences if $\frac{\Delta\theta}{\Delta s} < 1$, that is, if the total effect of the incentive is less than the direct effect, and conversely for the case of crowding-in. What we term strong crowding out holds if $\frac{\Delta\theta}{\Delta s} < 0$.

3. If baseline values $\lambda_0 = 1$, and $\Delta s = 1$, what are the values of the two crowding parameters λ_c and λ_m such that strong crowding out occurs.

The planner wishes to maximize the citizens benefits of public good project, net of the cost of their contributing and the administrative cost of the subsidy $c(s)$. Because citizens are identical we can just let the planner consider a single individual, and assume the planner's objective function

$$\omega(a, s) = \phi A - g(a) - c(s) \quad (9)$$

where $c(s) = \frac{1}{2}s^2$.

The sophisticated planner knows that explicit economic incentives may crowd out social preferences, while the naive planner does not. In other words, the sophisticated planner knows the true value of λ_m and λ_c , while the naive planner does not and takes them as zeros.

4. If $\lambda_0 = 1, \lambda_m = -0.5, \lambda_c = 0, n = 15, \phi = 0.1$, what levels of s will be selected by the sophisticated and naive planner respectively? why does the sophisticated planner implement a lesser subsidy than the naive planner?

Answer.

1. The utility function for individual i is

$$u_i = \phi \sum a_i - g(a_i) + a_i[s + \lambda_0(1 + \mathbf{1}_{\{s>0\}}\lambda_c + s\lambda_m)] \quad (10)$$

Then the first order condition of a_i to maximize u_i is

$$\phi - g'(a_i) + [s + \lambda_0(1 + \mathbf{1}_{\{s>0\}}\lambda_c + s\lambda_m)] = 0$$

That is

$$a_i = \phi + s + \lambda_0(1 + \mathbf{1}_{\{s>0\}}\lambda_c + s\lambda_m) \quad (11)$$

which is the best response function of the citizen.

2. From the utility function (10), we have the marginal benefit

$$\theta = \phi + s + \lambda_0(1 + \mathbf{1}_{\{s>0\}}\lambda_c + s\lambda_m)$$

and thus

$$\frac{\Delta\theta}{\Delta s} = 1 + \lambda_0\left(\frac{\lambda_c}{\Delta s} + \lambda_m\right) \quad (12)$$

3. The condition of strong crowding-out is

$$\frac{\Delta\theta}{\Delta s} = 1 + \lambda_0\left(\frac{\lambda_c}{\Delta s} + \lambda_m\right) = 1 + \lambda_c + \lambda_m < 0$$

4. The social planner would maximize the object function subject to the implementation technology given by the individual's best response function she believes. Therefore, for the sophisticated planner, the problem is

$$\begin{aligned} \max_s \quad & \omega(a, s) = \phi A - g(a) - c(s) \\ \text{s.t.} \quad & a = \phi + s + \lambda_0(1 + \mathbf{1}_{\{s>0\}}\lambda_c + s\lambda_m) \end{aligned} \quad (13)$$

The first order condition requires equating the marginal rate of substitute (MRS) to the marginal rate of transformation of the subsidy into action (MRT), i.e.,

$$MRT = \frac{1 + \lambda_0\lambda_m}{g''(a)} = \frac{c'(s)}{n\phi - g'(a)} = MRS \quad (14)$$

Given the parameters, we have

$$\frac{1 - 0.5}{1} = \frac{s}{15 * 0.1 - (0.1 + s + 1 - 0.5s)} \Rightarrow s = 0.16$$

For the naive planner who does not consider the crowding effect and takes $\lambda_c = \lambda_m = 0$, we have

$$\frac{1 - 0.5}{1} = \frac{s}{15 * 0.1 - (0.1 + s + 1)} \Rightarrow s = 0.2$$

□

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