Brief Introduction to Game Theory

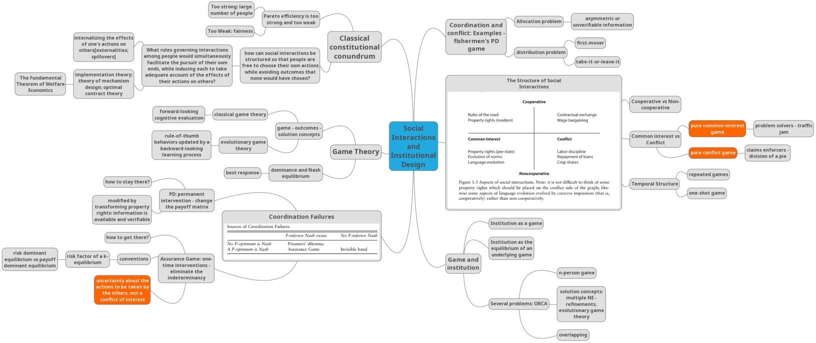
Econ 700 Tutorial

Weikai Chen

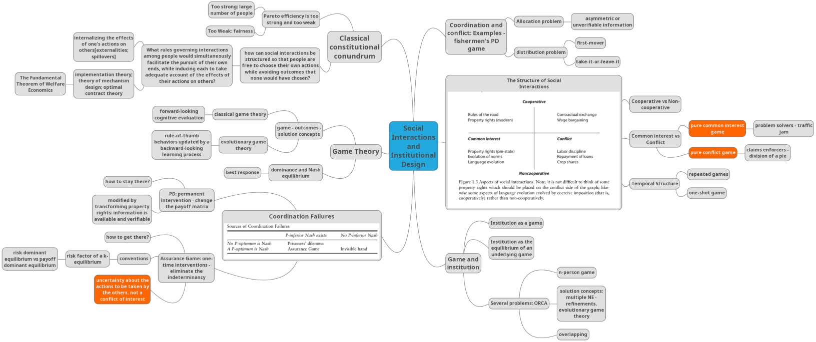
University of Massachusetts, Amherst

Sep 11, 2019

Map for Chapter 1



Map for Chapter 1



Introduction

Game Theory

What is it?

Game Theory models situation in which multiple agents make *strategically interdependent* decision.

- **classical** & evolutionary
- **noncooperative** & cooperative
- **normal form** & extensive form (strategic & dynamic)

Game Theory

What is it?

Game Theory models situation in which multiple agents make *strategically interdependent* decision.

- **classical** & evolutionary
- **noncooperative** & cooperative
- **normal form** & extensive form (strategic & dynamic)

Game Theory

What is it?

Game Theory models situation in which multiple agents make *strategically interdependent* decision.

- **classical** & evolutionary
- **noncooperative** & cooperative
- **normal form** & extensive form (strategic & dynamic)

Game Theory

What is it?

Game Theory models situation in which multiple agents make *strategically interdependent* decision.

- **classical** & evolutionary
- **noncooperative** & cooperative
- **normal form** & extensive form (strategic & dynamic)

History of (Noncooperative) Game Theory

1. Cournot (1838), Bertrand (1883)
2. John von Neumann & Oskar Morgenstern (1944)
3. John Nash (1950)
4. Selten (1965)
5. Harsanyi (1967–68)

History of (Noncooperative) Game Theory

1. Cournot (1838), Bertrand (1883)
2. John von Neumann & Oskar Morgenstern (1944)
3. John Nash (1950)
4. Selten (1965)
5. Harsanyi (1967-68)

History of (Noncooperative) Game Theory

1. Cournot (1838), Bertrand (1883)
2. John von Neumann & Oskar Morgenstern (1944)
3. John Nash (1950)
4. Selten (1965)
5. Harsanyi (1967-68)

History of (Noncooperative) Game Theory

1. Cournot (1838), Bertrand (1883)
2. John von Neumann & Oskar Morgenstern (1944)
3. John Nash (1950)
4. Selten (1965)
5. Harsanyi (1967–68)

History of (Noncooperative) Game Theory

1. Cournot (1838), Bertrand (1883)
2. John von Neumann & Oskar Morgenstern (1944)
3. John Nash (1950)
4. Selten (1965)
5. Harsanyi (1967–68)

Normal Form Games

The game in normal (or strategic) form

$$\Gamma = (\mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$$

has three elements:

- Players $i \in \mathcal{N} = \{1, \ldots, N\}$. Denote all players other than player i by $-i$
- Pure-strategy space for each player i, S_i.
- Payoff functions $u_i(s) : S \to \mathbb{R}$ where $s \in S = S_1 \times \cdots \times S_n$

Normal Form Games

The game in normal (or strategic) form

$$\Gamma = (\mathcal{N}, (S_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$$

has three elements:

- Players $i \in \mathcal{N} = \{1, \ldots, N\}$. Denote all players other than player i by $-i$
- Pure-strategy space for each player i, S_i.
- Payoff functions $u_i(s) : S \to \mathbb{R}$ where $s \in S = S_1 \times \cdots \times S_n$

Normal Form Games

The game in normal (or strategic) form

$$\Gamma = (\mathcal{N}, (S_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$$

has three elements:

- Players $i \in \mathcal{N} = \{1, \ldots, N\}$. Denote all players other than player i by $-i$
- Pure-strategy space for each player i, S_i.
- Payoff functions $u_i(s) : S \to \mathbb{R}$ where $s \in S = S_1 \times \cdots \times S_n$

Normal Form Games

The game in normal (or strategic) form

$$\Gamma = (\mathcal{N}, (S_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$$

has three elements:

- Players $i \in \mathcal{N} = \{1, \ldots, N\}$. Denote all players other than player i by $-i$
- Pure-strategy space for each player i, S_i.
- Payoff functions $u_i(s) : S \to \mathbb{R}$ where $s \in S = S_1 \times \cdots \times S_n$

Finite Two-Player Games

- We focus our attention on finite games, that is, games where S is finite.
- Strategic forms for finite two-player games are often depicted as matrices.

Example (Prison Dilemma)

		Player 2	
		C	D
Player 1	C	1, 1	−1, 2
	D	2, −1	0, 0

Then $\mathcal{N} = \{1, 2\}$, $S_1 = S_2 = \{C, D\}$, and for $(C, D) \in S_1 \times S_2$, we have $u_1(C, D) = -1$.

Finite Two-Player Games

- We focus our attention on finite games, that is, games where S is finite.
- Strategic forms for finite two-player games are often depicted as matrices.

Example (Prison Dilemma)

Player 2

		C	D
Player 1	C	1, 1	−1, 2
	D	2, −1	0, 0

Then $\mathcal{N} = \{1, 2\}$, $S_1 = S_2 = \{C, D\}$, and for $(C, D) \in S_1 \times S_2$, we have $u_1(C, D) = -1$.

Mixed Strategy: Definition

- A *mixed strategy* σ_i is a probability distribution over pure strategies. Denote the set of mixed strategies by Σ_i and

$$\Sigma = \Sigma_1 \times \cdots \times \Sigma_N$$

with element σ.

- The support of a mixed strategy σ_i is the set of pure strategies to which σ_i assigns positive probability.
- Player i's payoff to profile σ is

$$\sum_{s \in S} (\Pi_{j=1}^{N} \sigma_j(s_j)) u_i(s)$$

- Note that the set of mixed strategies contains the pure strategies.

Mixed Strategy: Definition

- A *mixed strategy σ_i* is a probability distribution over pure strategies. Denote the set of mixed strategies by Σ_i and

$$\Sigma = \Sigma_1 \times \cdots \times \Sigma_N$$

  with element σ.
- The support of a mixed strategy σ_i is the set of pure strategies to which σ_i assigns positive probability.
- Player i's payoff to profile σ is

$$\sum_{s \in S} (\Pi_{j=1}^{N} \sigma_j(s_j)) u_i(s)$$

- Note that the set of mixed strategies contains the pure strategies.

Mixed Strategy: Definition

- A *mixed strategy σ_i* is a probability distribution over pure strategies. Denote the set of mixed strategies by Σ_i and

$$\Sigma = \Sigma_1 \times \cdots \times \Sigma_N$$

  with element σ.
- The support of a mixed strategy σ_i is the set of pure strategies to which σ_i assigns positive probability.
- Player i's payoff to profile σ is

$$\sum_{s \in S} (\Pi_{j=1}^{N} \sigma_j(s_j)) u_i(s)$$

- Note that the set of mixed strategies contains the pure strategies.

Mixed Strategy: Definition

- A *mixed strategy* σ_i is a probability distribution over pure strategies. Denote the set of mixed strategies by Σ_i and

$$\Sigma = \Sigma_1 \times \cdots \times \Sigma_N$$

  with element σ.
- The support of a mixed strategy σ_i is the set of pure strategies to which σ_i assigns positive probability.
- Player i's payoff to profile σ is

$$\sum_{s \in S} (\Pi_{j=1}^{N} \sigma_j(s_j)) u_i(s)$$

- Note that the set of mixed strategies contains the pure strategies.

Mixed Strategy: Example

Example (Prison Dilemma)

		Player 2	
		C	D
Player 1	C	1, 1	−1, 2
	D	2, −1	0, 0

Then $\mathcal{N} = \{1, 2\}$, $S_1 = S_2 = \{C, D\}$, and suppose

$$\sigma_1 = (\frac{1}{2}, \frac{1}{2}), \sigma_2 = (1, 0)$$

Then

$$u_1(\sigma) =$$

Mixed Strategy: Example

Example (Prison Dilemma)

Player 2

		C	D
Player 1	C	1, 1	−1, 2
	D	2, −1	0, 0

Then $\mathcal{N} = \{1, 2\}$, $S_1 = S_2 = \{C, D\}$, and suppose

$$\sigma_1 = (\frac{1}{2}, \frac{1}{2}), \sigma_2 = (1, 0)$$

Then

$$u_1(\sigma) =$$

Mixed Strategy: Example

Example (Prison Dilemma)

		Player 2	
		C	D
Player 1	C	$1, 1$	$-1, 2$
	D	$2, -1$	$0, 0$

Then $\mathcal{N} = \{1, 2\}$, $S_1 = S_2 = \{C, D\}$, and suppose

$$\sigma_1 = (\frac{1}{2}, \frac{1}{2}), \sigma_2 = (1, 0)$$

Then

$$u_1(\sigma) = \frac{1}{2}(1 \cdot 1 + 0 \cdot (-1)) + \frac{1}{2}(1 \cdot 2 + 0 \cdot 0) = \frac{3}{2}$$

Solution Concepts

- Solution concepts are the rules for predicting how a game will be play.
  - **Dominance**
  - **Nash equilibrium** and its refinements
  - Maximin solution
  - Rationalizability
- Different assumptions on rationality and information are used with different solution concepts.
- Which one is better depends on the context.

Solution Concepts

- Solution concepts are the rules for predicting how a game will be play.
  - **Dominance**
  - **Nash equilibrium** and its refinements
  - Maximin solution
  - Rationalizability
- Different assumptions on rationality and information are used with different solution concepts.
- Which one is better depends on the context.

Solution Concepts

- Solution concepts are the rules for predicting how a game will be play.
  - **Dominance**
  - **Nash equilibrium** and its refinements
  - Maximin solution
  - Rationalizability
- Different assumptions on rationality and information are used with different solution concepts.
- Which one is better depends on the context.

Dominated Strategy

We would like to vary the strategy of a single player i while holding the strategies of others fixed.

- Let $s_{-i} \in S_{-i}$ denote a strategy selection for all players but i, and write (s_i', s_{-i}) for the profile

$$(s_1, \ldots, s_{i-1}, s_i', s_{i+1}, \ldots, s_N)$$

- For the mixed strategies, we let

$$(\sigma_i', \sigma_{-i}) = (\sigma_1, \ldots, \sigma_{i-1}, \sigma_i', \sigma_{i+1}, \ldots, \sigma_N)$$

Dominated Strategy

Pure strategy s_i is strictly dominated for player i if there exists $\sigma_i' \in \Sigma_i$ such that

$$u_i(\sigma_i', s_{-i}) > u_i(s_i, s_{-i}), \quad \forall s_{-i} \in S_{-i}$$

It is weakly dominated if the inequality holds with weak inequality, and the inequality is strictly for at least one s_{-i}.

Prison Dilemma: continued

Example (Prison Dilemma)

No matter how player 2 plays, D gives player 1 a strictly higher payoff than C does.

		Player 2	
		C	D
Player 1	C	1, 1	−1, 2
	D	2, −1	0, 0

Then strategy C is said to be strictly dominated.

Dominated Strategy: Example

Example

Player 2

		L	R
Player 1	U	$2, -1$	$-1, 0$
	M	$0, 0$	$0, 0$
	D	$-1, 0$	$2, 0$

Strategy M strictly dominated and L is weakly dominated, why?

Dominated Strategy: Example

Example

Player 2

		L	R
Player 1	U	2, −1	−1, 0
	M	0, 0	0, 0
	D	−1, 0	2, 0

Strategy M strictly dominated and L is weakly dominated, why?

Nash Equilibrium

A mixed-strategy profile σ^* is a Nash equilibrium if, for all players i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(s_i, \sigma_{-i}^*) \quad \forall s_i \in S_i$$

- A *pure-strategy Nash equilibrium* is a pure-strategy profile that satisfies the same conditions.
- If a player uses a mixed strategy, she must be indifferent between all pure strategies in the *support* of mixed strategy (to which she assigns positive probability). (Why?)

Nash Equilibrium

A mixed-strategy profile σ^* is a Nash equilibrium if, for all players i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(s_i, \sigma_{-i}^*) \quad \forall s_i \in S_i$$

- A *pure-strategy Nash equilibrium* is a pure-strategy profile that satisfies the same conditions.
- If a player uses a mixed strategy, she must be indifferent between all pure strategies in the *support* of mixed strategy (to which she assigns positive probability). (Why?)

Nash Equilibrium

A mixed-strategy profile σ^* is a Nash equilibrium if, for all players i,

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(s_i, \sigma_{-i}^*) \quad \forall s_i \in S_i$$

- A *pure-strategy Nash equilibrium* is a pure-strategy profile that satisfies the same conditions.
- If a player uses a mixed strategy, she must be indifferent between all pure strategies in the *support* of mixed strategy (to which she assigns positive probability). (Why?)

Strict Nash Equilibrium

A pure-strategy profile s^* is a strict Nash equilibrium if, for all players i,

$$u_i(s^*, s^*_{-i}) > u_i(s_i, s^*_{-i}) \quad \text{for all } s_i \neq s_i^*$$

Nash equilibria are "consistent" predictions of how the game will be played, in the sense that *if all players predict that a particular NE will occur then no player has an incentive to play differently*.

Strict Nash Equilibrium

A pure-strategy profile s^* is a strict Nash equilibrium if, for all players i,

$$u_i(s^*, s^*_{-i}) > u_i(s_i, s^*_{-i}) \quad \text{for all } s_i \neq s_i^*$$

Nash equilibria are "consistent" predictions of how the game will be played, in the sense that *if all players predict that a particular NE will occur then no player has an incentive to play differently*.

Best Response Correspondence/Function

Player i's best response correspondence, $\boldsymbol{b}_i$ maps each strategy profile σ to the set of mixed strategies that maximize player i's payoff when her opponents play σ_{-i}. That is

$$\boldsymbol{b}_i(\sigma) = \arg\max_{\sigma_i' \in \Sigma_i} u_i(\sigma_i', \sigma_{-i})$$

- The profile σ^* such that $\sigma_i^* \in \boldsymbol{b}_i(\sigma^*)$ for all player i is a Nash equilibrium.
- Define the correspondence $\mathbf{b} : \Sigma \to \Sigma$ to be the Cartesian product of the b_i. A fixed point of $\mathbf{b}$ is a σ^* such that $\sigma^* \in \mathbf{b}(\sigma^*)$. Thus a NE is a fixed point of $\mathbf{b}$.
- Therefore, every finite strategy-form game has a mixed-strategy equilibrium, guaranteed by the so-called **Kakutani's fixed point theorem.**

Best Response Correspondence/Function

Player i's best response correspondence, $\mathbf{b}_i$ maps each strategy profile σ to the set of mixed strategies that maximize player i's payoff when her opponents play σ_{-i}. That is

$$\mathbf{b}_i(\sigma) = \arg\max_{\sigma_i' \in \Sigma_i} u_i(\sigma_i', \sigma_{-i})$$

- The profile σ^* such that $\sigma_i^* \in \mathbf{b}_i(\sigma^*)$ for all player i is a Nash equilibrium.
- Define the correspondence $\mathbf{b} : \Sigma \to \Sigma$ to be the Cartesian product of the b_i. A fixed point of $\mathbf{b}$ is a σ^* such that $\sigma^* \in \mathbf{b}(\sigma^*)$. Thus a NE is a fixed point of $\mathbf{b}$.
- Therefore, every finite strategy-form game has a mixed-strategy equilibrium, guaranteed by the so-called **Kakutani's fixed point theorem.**

Best Response Correspondence/Function

Player i's best response correspondence, $\boldsymbol{b}_i$ maps each strategy profile σ to the set of mixed strategies that maximize player i's payoff when her opponents play σ_{-i}. That is

$$\boldsymbol{b}_i(\sigma) = \arg\max_{\sigma_i' \in \Sigma_i} u_i(\sigma_i', \sigma_{-i})$$

- The profile σ^* such that $\sigma_i^* \in \boldsymbol{b}_i(\sigma^*)$ for all player i is a Nash equilibrium.
- Define the correspondence $\mathbf{b} : \Sigma \to \Sigma$ to be the Cartesian product of the $\boldsymbol{b}_i$. A fixed point of $\mathbf{b}$ is a σ^* such that $\sigma^* \in \mathbf{b}(\sigma^*)$. Thus a NE is a fixed point of $\mathbf{b}$.
- Therefore, every finite strategy-form game has a mixed-strategy equilibrium, guaranteed by the so-called **Kakutani's fixed point theorem.**

Best Response Correspondence/Function

Player i's best response correspondence, $\boldsymbol{b}_i$ maps each strategy profile σ to the set of mixed strategies that maximize player i's payoff when her opponents play σ_{-i}. That is

$$\boldsymbol{b}_i(\sigma) = \arg\max_{\sigma_i' \in \Sigma_i} u_i(\sigma_i', \sigma_{-i})$$

- The profile σ^* such that $\sigma_i^* \in \boldsymbol{b}_i(\sigma^*)$ for all player i is a Nash equilibrium.
- Define the correspondence $\mathbf{b} : \Sigma \to \Sigma$ to be the Cartesian product of the $\boldsymbol{b}_i$. A fixed point of $\mathbf{b}$ is a σ^* such that $\sigma^* \in \mathbf{b}(\sigma^*)$. Thus a NE is a fixed point of $\mathbf{b}$.
- Therefore, every finite strategy-form game has a mixed-strategy equilibrium, guaranteed by the so-called **Kakutani's fixed point theorem.**

Best Response Correspondence: Example

Example (Matching Pennies)

Player 2

		H	T
Player 1	H	$1, -1$	$-1, 1$
	T	$-1, 1$	$1, -1$

- Example of the nonexistence of a pure-strategy equilibrium.
- The Monitor and Work Game (Problem 3 on p.503) has the same structure.

Best Response Correspondence: Example

Player 2

		H	T
Player 1	H	1, −1	−1, 1
	T	−1, 1	1, −1

Suppose $\sigma_1 = (q, 1-q)$, $\sigma_2 = (p, 1-p)$, then

$$u_1(H, \sigma_2) = p \cdot 1 + (1-p) \cdot (-1) = 2p - 1$$
$$u_1(T, \sigma_2) = p \cdot (-1) + (1-p) \cdot 1 = 1 - 2p$$

Best Response Correspondence: Example

		Player 2	
		H	T
Player 1	H	1, −1	−1, 1
	T	−1, 1	1, −1

Suppose $\sigma_1 = (q, 1-q), \sigma_2 = (p, 1-p)$, then

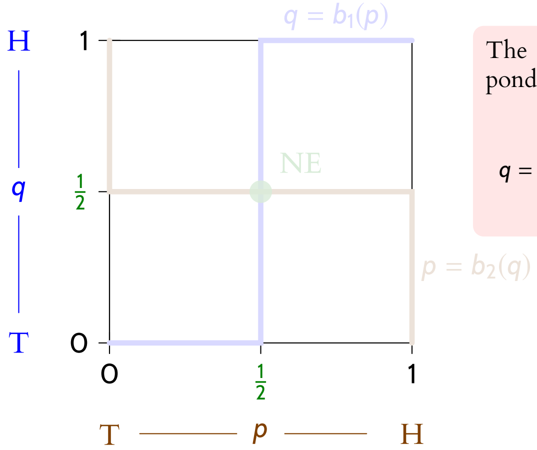
$$u_1(H, \sigma_2) = p \cdot 1 + (1-p) \cdot (-1) = 2p - 1$$
$$u_1(T, \sigma_2) = p \cdot (-1) + (1-p) \cdot 1 = 1 - 2p$$

Therefore, 1's best response correspondence is

$$q = b_1(\sigma_2) = \begin{cases} 1, & p > \frac{1}{2} \\ [0, 1] & p = \frac{1}{2} \\ 0, & p < \frac{1}{2} \end{cases}$$

Best Response Correspondence: Example

Figure: Best response correspondences in the Matching Pennies Game

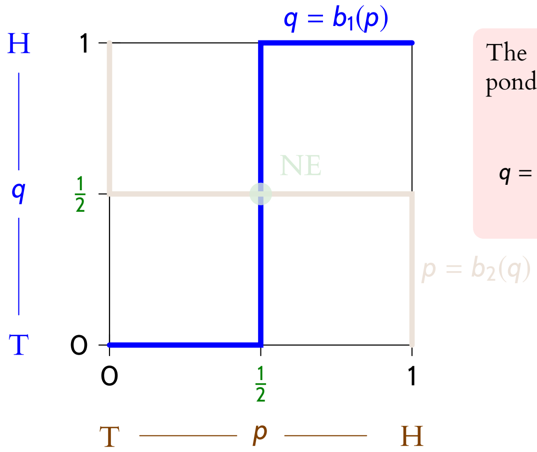


The Best Response Correspondence

$$q = b_1(\sigma_2) = \begin{cases} 1, & p > \frac{1}{2} \\ [0,1] & p = \frac{1}{2} \\ 0, & p < \frac{1}{2} \end{cases}$$

Best Response Correspondence: Example

Figure: Best response correspondences in the Matching Pennies Game

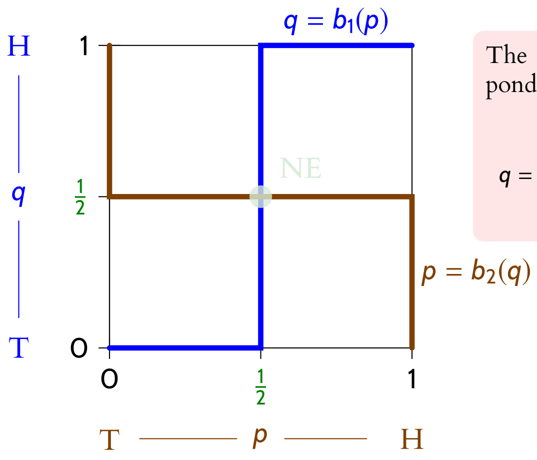


The Best Response Correspondence

$$q = b_1(\sigma_2) = \begin{cases} 1, & p > \frac{1}{2} \\ [0,1] & p = \frac{1}{2} \\ 0, & p < \frac{1}{2} \end{cases}$$

Best Response Correspondence: Example

Figure: Best response correspondences in the Matching Pennies Game

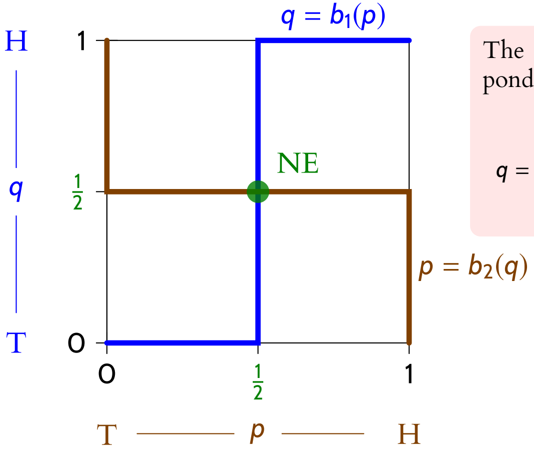


The Best Response Correspondence

$$q = b_1(\sigma_2) = \begin{cases} 1, & p > \frac{1}{2} \\ [0,1] & p = \frac{1}{2} \\ 0, & p < \frac{1}{2} \end{cases}$$

Best Response Correspondence: Example

Figure: Best response correspondences in the Matching Pennies Game



The Best Response Correspondence

$$q = b_1(\sigma_2) = \begin{cases} 1, & p > \frac{1}{2} \\ [0,1] & p = \frac{1}{2} \\ 0, & p < \frac{1}{2} \end{cases}$$

Exercise

Find all the Nash equilibria.

Planting in Palanpur

		Player 2	
		Early	Late
Player 1	Early	4, 4	0, 3
	Late	3, 0	2, 2

What would be the Maximin Solution?

Exercise

Find all the Nash equilibria.

Planting in Palanpur

Player 2

		Early	Late
Player 1	Early	4, 4	0, 3
	Late	3, 0	2, 2

What would be the Maximin Solution?

Application: the household public goods problem

- The action, a_i, is a contribution of the public good, e.g. cleaning the house.
- Individual utility function $u_i = f \ln(\sum_i a_i) - g a_i$
- FOC:

$$\frac{\partial u_i}{\partial a_i} = \frac{f}{\sum_i a_i} - g = 0$$

- Best Response:

$$a_i^{BR} = \frac{f}{g} - \sum_{j \neq i} a_j$$

- Two individuals in a couple, $i = 1, 2$

$$a_1^{BR} = \frac{f}{g} - a_2$$

$$a_2^{BR} = \frac{f}{g} - a_1$$

Application: the household public goods problem

- The action, a_i, is a contribution of the public good, e.g. cleaning the house.
- Individual utility function $u_i = f \ln(\sum_i a_i) - g a_i$
- FOC:

$$\frac{\partial u_i}{\partial a_i} = \frac{f}{\sum_i a_i} - g = 0$$

- Best Response:

$$a_i^{BR} = \frac{f}{g} - \sum_{j \neq i} a_j$$

- Two individuals in a couple, $i = 1, 2$

$$a_1^{BR} = \frac{f}{g} - a_2$$

$$a_2^{BR} = \frac{f}{g} - a_1$$

Application: the household public goods problem

- The action, a_i, is a contribution of the public good, e.g. cleaning the house.
- Individual utility function $u_i = f \ln(\sum_i a_i) - g a_i$
- FOC:

$$\frac{\partial u_i}{\partial a_i} = \frac{f}{\sum_i a_i} - g = 0$$

- Best Response:

$$a_i^{BR} = \frac{f}{g} - \sum_{j \neq i} a_j$$

- Two individuals in a couple, $i = 1, 2$

$$a_1^{BR} = \frac{f}{g} - a_2$$

$$a_2^{BR} = \frac{f}{g} - a_1$$

Application: the household public goods problem

- The action, a_i, is a contribution of the public good, e.g. cleaning the house.
- Individual utility function $u_i = f \ln(\sum_i a_i) - g a_i$
- FOC:

$$\frac{\partial u_i}{\partial a_i} = \frac{f}{\sum_i a_i} - g = 0$$

- Best Response:

$$a_i^{BR} = \frac{f}{g} - \sum_{j \neq i} a_j$$

- Two individuals in a couple, $i = 1, 2$

$$a_1^{BR} = \frac{f}{g} - a_2$$

$$a_2^{BR} = \frac{f}{g} - a_1$$

Application: the household public goods problem

The Nash Equilibria?

Figure: Best response function in the Household Public Good Problem

Application: the household public goods problem

The Nash Equilibria?

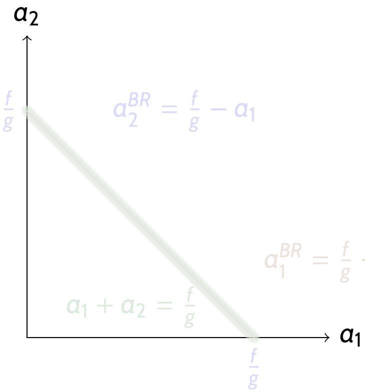


Figure: Best response function in the Household Public Good Problem

Application: the household public goods problem

The Nash Equilibria?

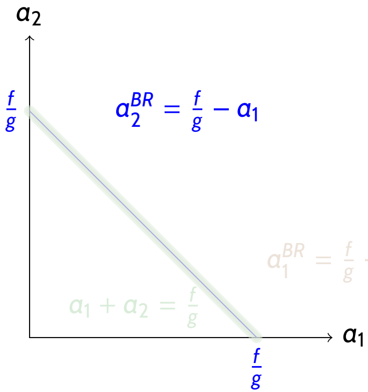


Figure: Best response function in the Household Public Good Problem

Application: the household public goods problem

The Nash Equilibria?

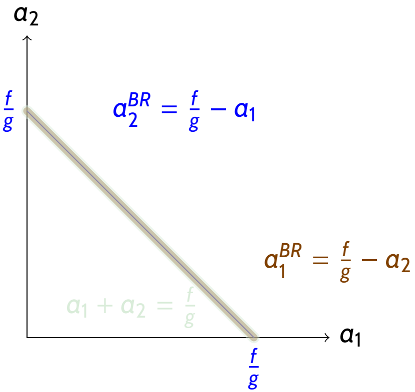


Figure: Best response function in the Household Public Good Problem

Application: the household public goods problem

The Nash Equilibria?

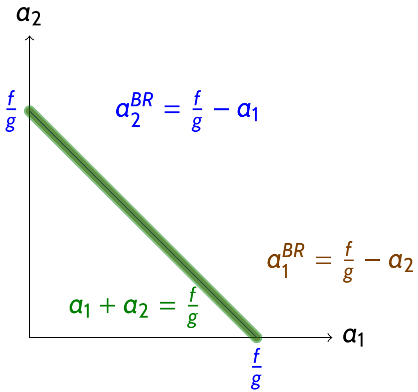


Figure: Best response function in the Household Public Good Problem

Allocation and Distribution: First Mover Advantage in the Household Public Good Problem

Suppose Player 1 is the first mover, what would be the outcome?

Figure: The first mover advantage in the Household Public Good Problem

Allocation and Distribution: First Mover Advantage in the Household Public Good Problem

Suppose Player 1 is the first mover, what would be the outcome?

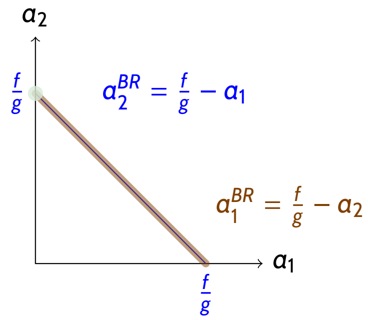


Figure: The first mover advantage in the Household Public Good Problem

Allocation and Distribution: First Mover Advantage in the Household Public Good Problem

Suppose Player 1 is the first mover, what would be the outcome?

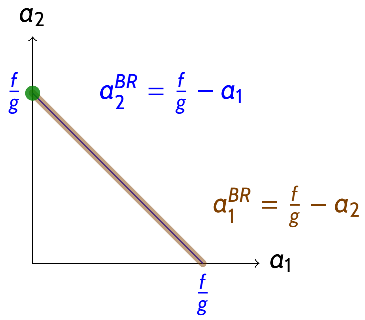


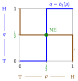
Figure: The first mover advantage in the Household Public Good Problem

Summary

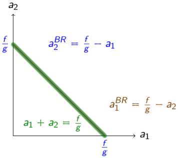
- Strategic (or normal) Game: $\Gamma = (\mathcal{N}, (S_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$
- Dominant Strategy Equilibrium and Nash Equilibrium
- Find pure-strategic NE using payoff matrix

	L	R
U	4, 3	0, 2
D	3, 4	2, 3

- Find mixed NE in finite game



- Find NE using best-response function with continuous strategic space

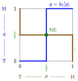


Summary

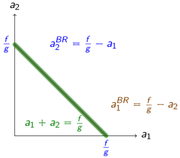
- Strategic (or normal) Game: $\Gamma = (\mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$
- Dominant Strategy Equilibrium and Nash Equilibrium
- Find pure-strategic NE using payoff matrix

	L	R
U	4, 3	0, 2
D	3, 4	2, 3

- Find mixed NE in finite game



- Find NE using best-response function with continuous strategic space

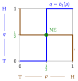


Summary

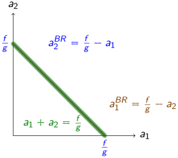
- Strategic (or normal) Game: $\Gamma = (\mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$
- Dominant Strategy Equilibrium and Nash Equilibrium
- Find pure-strategic NE using payoff matrix

	L	R
U	4, 3	0, 2
D	3, 4	2, 3

- Find mixed NE in finite game



- Find NE using best-response function with continuous strategic space

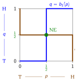


Summary

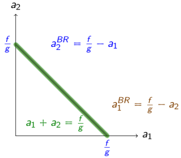
- Strategic (or normal) Game: $\Gamma = (\mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$
- Dominant Strategy Equilibrium and Nash Equilibrium
- Find pure-strategic NE using payoff matrix

	L	R
U	4, 3	0, 2
D	3, 4	2, 3

- Find mixed NE in finite game



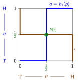
- Find NE using best-response function with continuous strategic space



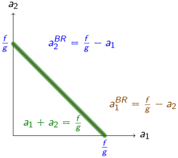
Summary

- Strategic (or normal) Game: $\Gamma = (\mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$
- Dominant Strategy Equilibrium and Nash Equilibrium
- Find pure-strategic NE using payoff matrix

	L	R
U	4, 3	0, 2
D	3, 4	2, 3

- Find mixed NE in finite game 

- Find NE using best-response function with continuous strategic space



Textbooks Recommendation

Classical Game Theory:

- Maschler, M., Zamir, S., & Solan, E. 2013. *Game Theory*. Cambridge University Press.
- Osborne, Martin J., and Ariel. Rubinstein. 1994. *A Course in Game Theory*. MIT Press.
- Myerson, Roger B. 1991. *Game Theory : Analysis of Conflict*. Harvard University Press.
- Fudenberg, Drew., and Jean. Tirole. 1991. *Game Theory*. MIT Press.

Textbooks Recommendation

Evolutionary Game Theory:

- Sandholm, William H. 2010. *Population Games and Evolutionary Dynamics*. MIT Press.
- Maynard Smith, John. 1982. *Evolution and the Theory of Games*. Cambridge University Press.
- Weibull, Jorgen W. 1995. *Evolutionary Game Theory*. MIT Press.