Games with Incomplete Information

Econ 702 Game Theory Recitation 5

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1 Bayesian Games

1.1 Definition

- A set of *players*
- A set of *states* of the world $\Omega \ni \omega$
- A set of *actions* for each player, $A_i \ni a$
- A set of *signals* T_i and a *signal function* $\tau_i : \Omega \to T_i$ for each player
- A *belief* about the states consistent with the signal
- A *utility function* over (a, ω)

1.2 Baysian Nash Equilibrium

convert that into a strategic game with vMN preference

1.3 Example

Definition of the Game

- Set of players $N = \{1, 2\}$

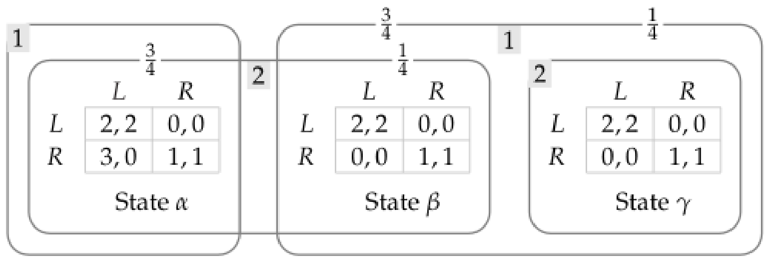

Figure 1: The Bayesian Game in Exercise 10. In the unique NE of the game, each type of each player choose R.

- Set of actions $A_1 = A_2 = \{L, R\}$
- Signal function $\tau_1(\alpha) \neq \tau_1(\beta) = \tau_1(\gamma)$, $\tau_2(\alpha) = \tau_2(\beta) \neq \tau_2(\gamma)$
- Belief $\mathbb{P}(\alpha \mid \tau_1(\alpha)) = 1$, $\mathbb{P}(\beta \mid \tau_1(\beta)) = \frac{3}{4}$
- Preference $u_1((L, L), \alpha) = 2$

Show that in the unique NE of the game, each type of each player choose R.

1. Player 1 with type $\tau_1(\alpha)$ would choose R
2. How about player 2 with type $\tau_2(\alpha) = \tau_2(\beta)$?

2 Auctions

Table 1: Types of Auctions

	Private Values	Common Values
1st price		
2nd price		

2.1 Auctions with independent private values

2.1.1 The game

- players $N = \{1, \ldots, n\}$

- States: the set of all $(v_1, \ldots, v_n)$ of valuatons, where $\underline{v} \leq v_i \leq \bar{v}$ for all i

- Action: the bid $b_i \in \mathbb{R}_+$

- Signal: $\tau_i(v_1, \ldots, v_n) = v_i$

- Belief: player i assigns $F(v_1)F(v_2)\cdots F(v_{i-1})F(v_{i+1})\cdots F(v_n)$ to the event that valuations of player j is at most v_j.

- Preference: the winners share $v_i - P(b)$

2.1.2 Nash equilibrium

Show that $\beta_i(v) = (1 - 1/n)v$ is a NE in the first price auction.

- Suppose that player 1 to n-1 use this strategy.

- For player n, by bidding b, the expected payoff is

$$
\begin{aligned}
u_n &= (v_n - b)\mathbb{P}(b > \beta_i(v), i = 1, \ldots, n-1) \\
&= (v_n - b)\mathbb{P}(v_i < \frac{n}{n+1}b, i = 1, \ldots, n-1) \\
&= \left(\frac{n}{n+1}\right)^{n-1} b^{n-1}(v_n - b)
\end{aligned}
$$

- FOC: $(n-1)b^{n-2}(v_n - b) - b^{n-1} = 0$

- Then $b = (n-1)(v_n - b) \Rightarrow b = (n-1)v_n/n$

2.1.3 Revenue

2.2 Auctions with Common Valuations

3 Decision Problem as a one person Bayesian Game

3.1 Definition of Decision Problem

3.2 Information Structure

3.3 Information is Always Good in Decision Problem

4 Knowledge

4.1 Information Partition

- An information partition $\mathscr{F}_i$ for player i is a partition of Ω.

- $\forall \omega \in \Omega$ there exists a unique $F_i(\omega) \in \mathscr{F}_i$ such that $\omega \in F_i(\omega)$

- Information partition induced by the signal function $\tau_i : \Omega \to T_i$: $\mathscr{F}_i = \{F_i(\omega) \mid \omega \in \Omega\}$ where

$$F_i(\omega) = \{\omega' \in \Omega \mid \tau_i(\omega) = \tau_i(\omega')\}$$

- Example: $\Omega = \{\alpha, \beta, \gamma\}$, if $\tau_i(\alpha) = \tau_i(\beta) \neq \tau_i(\gamma)$, then we have $\mathscr{F}_i = \{\{\alpha, \beta\}, \{\gamma\}\}$.

4.2 Knowlege

4.2.1 Definition

$K_i E = \{\omega \in \Omega \mid F_i(\omega) \subseteq E\}$

- Example, for event $E = \{\alpha, \beta\}$, at the state α, we have $F_i(\alpha) \subseteq E$, and we say i knows E, while at state γ, $F_i(\gamma) \nsubseteq E$ so i does not know E

- One more example: for event $E = \{\alpha\}$, i does not know E at all states.

4.2.2 Properties

- Logic $K_i\Omega$
- Truth $K_iE \subseteq E$
- Monotonicity
- Positive Introspection
- Negative introspection

4.3 Mutual and Common Knowlege

- mutual knowledge: $ME = \cap_{i\in I} K_iE$
- define $M^{k+1}E = M(M^kE)$
- common knowledge: $CE = \cap_{k=1}^{\infty} M^kE$

4.4 Application: Muddy Children Puzzle