Recoverability from Input-Output Data

Foundation of Empirical Multisectoral Analysis

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Introduction

Motivation

- Theoretical Multisectoral Analysis: non-aggregated technical coefficients in physical unit.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}, l = (l_1, \ldots, l_n)$$

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- Empirical Studies: aggregated and nominal data

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- Price of production(e.g., Froehlich, 2013, CJE)
- Capital-reverse and reswitching: wage-profit frontier (e.g., Han & Schefold, 2006, CJE)

  ...

Mehtods

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Assume that there is no joint production and fixed capital for simplicity.

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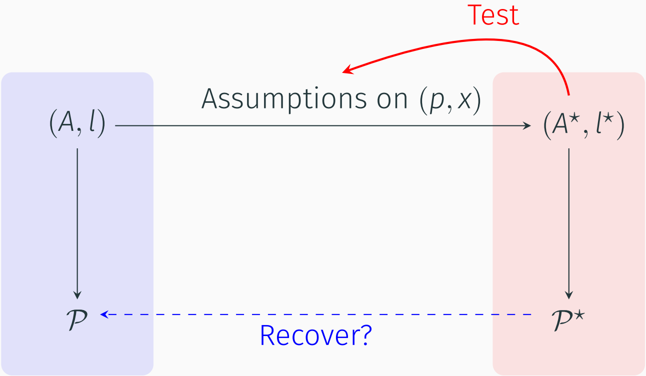

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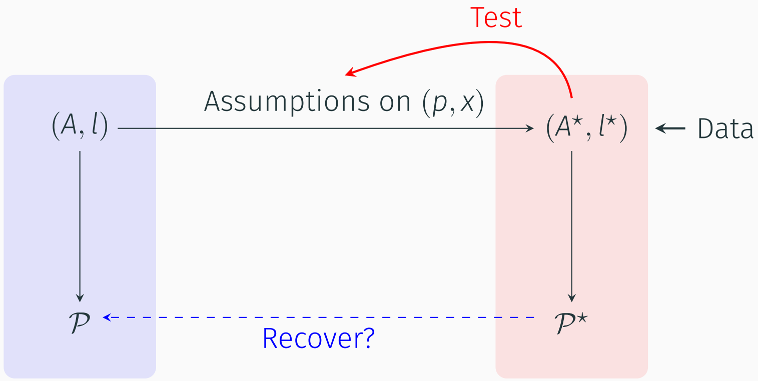

Three Properties to Recover

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- Wage-profit frontier with reference to consumption bundle b:

$$w(r; b) = \frac{1}{(1 + r)l[I - (1 + r)A]^{-1}b}$$

Non-aggregated Nominal System

The non-aggregated nominal system

Let p be the vector of prices and $\hat{p} = \mathrm{diag}\{p\}$. Denote the nominal coefficients by (A^n, l^n).

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What is a^n_{ij}?

The cost of good i used in the production of \$1 of good j.

Results: Labor Value

Proposition
Given the technique (A, l) and the nominal data (A^n, l^n) with prices $\hat{p}$, we have

$$v^n = v\hat{p}^{-1} = (v_1/p_1, \ldots, v_n/p_n)$$

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- What is the labor value in \$$x_i^n$ of good i? $x_i^n v_i^n$

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Given (A, l) and the nominal data (A^n, l^n) with prices $\hat{p}$, we have $r^n = r^*$ and $p^n = p^* \hat{p}^{-1}$ where (r^n, p^n) is defined by

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- The uniform rate of profit can be recover no matter what the actual prices are.

Results: Wage-Profit Frontier

Define the following function

$$w^n(r; b^n) = \frac{1}{(1+r)l^n[I - (1+r)A^n]^{-1}b^n}$$

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when $b^n = \hat{p}b$, the vector of worker's expenditure on each commodity.

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Summary
With the nonaggregate nominal system, we could recover almost everything no matter what the actual prices are.

Aggregated Nominal System

The aggregated system

To aggregate the data, we need both the prices and output levels, and the composition of each sectors.

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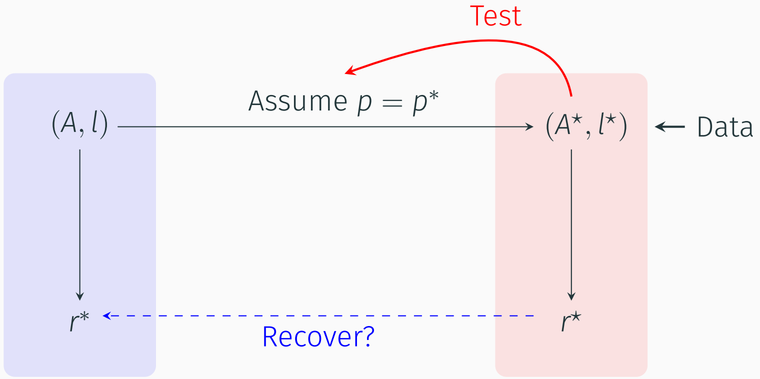
NOT correct to interpret $A^\star$ as the nonaggregated nominal system with composite commodities.

Preliminary Result

Theorem
Given the underlying technical (A, l) and the aggregate system $(A^\star, l^\star)$ with (p, x) and some aggregator $S_{k\times n}$, if $p = p^*$, then $r^\star = r^*$ and $p^\star = \mathbf{1} \in \mathbb{R}^k$, where $(p^\star, r^\star)$ is defined by $p^\star = (1 + r^\star)(p^\star A^\star + l^\star)$



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(A, l) \arrow[rr, "\text{Assume } p = p^*"] \arrow[d] & & (A^\star, l^\star) \arrow[d] \arrow[l, bend right, red, "p^\star = \mathbf{1}?"'] & \longleftarrow \text{Data} \\
r^* & & r^\star \arrow[ll, dashed, blue, "\text{Recover?}"]
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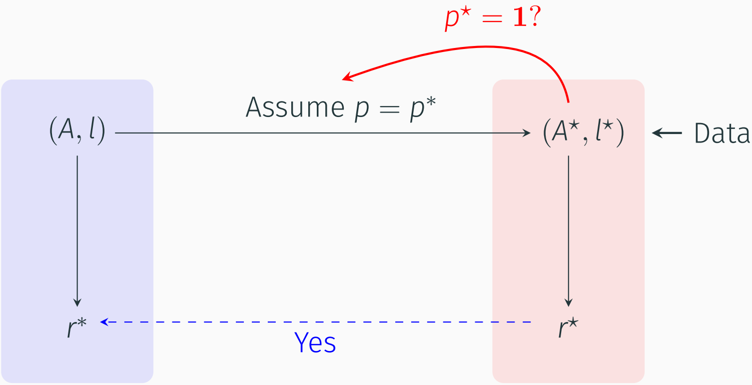

Questions?

References

Flaschel, P., Franke, R., & Veneziani, R. (2010). *Labor Productivity and the Law of Decreasing Labor Content*. University of Massachusetts. Amherst.

Froehlich, N. (2013). Labour Values, Prices of Production and the Missing Equalisation Tendency of Profit Rates: Evidence from the German Economy. *Cambridge Journal of Economics*, *37*(5), 1107–1126.

Han, Z., & Schefold, B. (2006). An empirical investigation of paradoxes: Reswitching and reverse capital deepening in capital theory. *Cambridge Journal of Economics*, *30*(5), 737–765.