Utility Theory

Econ 702 Game Theory Recitations 1

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1 Preferences

- Completeness
- Transitivity

2 Utility Function

- $u : X \to \mathbb{R}$ represents $\succeq$ if $x \succeq y \Leftrightarrow u(x) \geq u(y)$
- X is finite. Then $\succeq$ is complete and transitive if and only if there exists u represents $\succeq$.
  - Lexicographic Preferences
- $X = \mathbb{R}^n$ and $\succeq$ is *continuous*. Then $\succeq$ is complete and transitive if and only if there exists a **continous** utility function represents $\succeq$.
- Monotone Transformation. If u represents $\succeq$ and f is increasing, then $v = f \circ u$ represents $\succeq$.

3 Expected Utility

- Lottery on finite set X, $l = (p_1, \ldots, p_n)$ where $p_k \geq 0, \forall k$ and $\sum_k p_k = 1$.
- Compound lottery
  - Def: $\alpha l + (1-\alpha)l'$ for any $\alpha \in [0,1]$
  - The compound lottery of two lottries is a lottery.
- Axioms:
  - Completeness
  - Transitivity

- Continunity
- Independence

- Expected Utility Theorem
  - Expected utility form: $\exists(u_1, \ldots, u_n)$ such that for all $l = (p_1, \ldots, p_n)$, $U(p_1, \ldots, p_n) = \sum_k p_k u_k$.
  - A relation $\succeq$ satisfies Completeness, Transitivity, Continunity and Independence $\Leftrightarrow \succeq$ has a representation of the expected utility form.

- The proof of Expected Utility Theorem
  - - Exercise 8. ($\Leftarrow$)
  - The proof of $\Rightarrow$
    - **Lemma** For any $l \in \Delta$, $\delta_1 \preceq l \preceq_n$ where δ_k are degenerate lotteries with $\delta_1 \preceq \cdots \preceq \delta_n$. (Exercise 9.)
    - Define $f : \Delta \to \mathbb{R}$: Let $f(l) = \alpha$ where $\alpha \in [0, 1]$ such that $l \sim (1 - \alpha)\delta_1 + \alpha\delta_n$
    - **Lemma** $f(l) \leq f(l') \Leftrightarrow l \preceq l'$
    - Define $u_k = f(\delta_k)$
    - **Lemma** $f(\alpha l + (1 - \alpha l')) = \alpha f(l) + (1 - \alpha)f(l')$
    - **Lemma** For $l = (p_1, \ldots, p_n)$, $f(l) = \sum_k p_k u_k$. (Exercise 9.)

- Affine Transformation. u and v represent the same vNM preference relation if and only if $\exists a > 0$ and b such that for all k, $v_k = au_k + b$. (Exercise 11.)

- Attitudes Towards Risk. (Exercise 12.)
  - Risk neutral
  - Risk averse
  - Risk loving