Homework 3

Response by: **TODO: replace this with your name (and computing id)**

Due: **10:00pm, Friday, February 13**

This problem set focuses on regular languages (Chapter 6 in TCS). Write your answers in the hw3.tex LaTeX template. You will submit your solutions in GradeScope as a PDF file with your answers to the questions in this template.

> **Collaboration Policy:** You may discuss the problems with anyone you want. You are permitted to use any resources you find for this assignment **other than *solutions* from previous/concurrent CS3120 courses**. You must write up your own solutions and understand everything in them, and submit only your own work. You should note in the *Collaborators and Resources* box below the people you collaborated with and any external resources you used (you do not need to list resources you used for help with LaTeX).

> **Collaborators and Resources:** TODO: replace this with your collaborators and resources (if you did not have any, replace this with *None*)

To do this assignment:

1. Open this read-only Overleaf project located at https://www.overleaf.com/read/mfrcqdfkqjgd#e2d3d2, then select the "File" button at the top-left, and then select "Make a copy". You will have an opportunity to rename the project, and then Overleaf will create a new copy of the project which you can edit.

2. Open your copy of the project and in the left side of the browser, you should see a file directory containing hw3.tex. Click on hw3.tex to see the LaTeX source for this file, and enter your solutions in the marked places. (You will also see the uvatoc.sty file, a "style" file that defines some useful macros. You are welcome to look at this file but should not need to modify it.)

3. The first thing you should do in hw3.tex is set up your name as the author of the submission by replacing the line, \submitter{TODO: your name}, with your name and UVA id, e.g., \submitter{Wei-Kai Lin (twc7zv)}.

4. Write insightful and clear answers to all of the questions. As typical people, we prefer short, precise, and comprehensive answers.

5. There are *optional* problems. There are no points, but if you wrote your solutions, we will tell you how you did.

6. Before submitting your hw3.pdf file, also remember to:

   – List your collaborators and resources, replacing the TODO in \collaborators{TODO: replace ...} with your collaborators and resources. (Remember to update this before submitting if you work with more people.)

Problem 1 (6,6,9,9pt) *Regular Languages*

Consider each of the following languages (that is, subsets of binary strings). State if the given language is regular or not. If it is regular, provide a regular expression or DFA. Otherwise, it is not regular; provide an argument. You may use the equivalence between regular expressions and DFAs, and you may use Pumping Lemma.

For any string $x, y \in \{0,1\}^*$, $|x|$ denotes the bit-length of x, and for any $n \in \mathbb{N}$, x^n denotes the string repeating x for n times, and xy denotes the concatenation of x and y.

Note: We describe a language in English and/or math notations as accurate as possible. If the description is ambiguous, it is ambiguous to claim and prove regularity. So, if you find something is ambiguous, write your clarification or ask for clarification.

(a) $L = \{w \in \{0,1\}^* \mid w$ contains an even number of 0s$\}$

(b) $L = \{w \in \{0,1\}^* \mid w = 0^{k_1} 1^{k_2}$ for some $k_1, k_2 \in \mathbb{N}\}$

(c) $L = \{w \in \{0,1\}^* \mid$ every '0' must be immediately followed by a '1' in $w\}$

(d) $L = \{w \in \{0,1\}^* \mid \sum_{i=0}^{k-1} w[i] \leq k/4$ where $k = |w|$ and $w[i]$ denotes the ith bit of w $\}$

Answer:

(a)

(b)

(c)

(d)

Problem 2 (Optional) *Regular Functions*

Suppose that $F : \{0,1\}^* \to \{0,1\}$ is regular. Let

$$H(x) := \begin{cases} 1 & \text{if } x = uu \text{ and } F(u) = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Is H regular? Either prove it is regular by constructing a regular expression or a DFA, or prove it is not regular by giving a counterexample.

Note: This is similar to Exercise 6.1 (6) in TCS. Recall that we say a boolean function $f : \{0,1\}^* \to \{0,1\}$ is regular iff the corresponding language $L = \{x \in \{0,1\}^* \mid f(x) = 1\}$ is a regular language.

Answer:

Problem 3 (Optional challenge) *Half of Regular Language*

Consider any regular language L. Define $\mathsf{Half}(L) := \{w \in \{0,1\}^* \mid ww \in L\}$. Notice that there may exists $x \in L$ but $x \neq ww$ for all w, and such x shall contribute nothing in $\mathsf{Half}(L)$.

Is $\mathsf{Half}(L)$ regular? We might guess that $\mathsf{Half}(L)$ is not regular. Wrong! Indeed, consider the set $L' = \{ww \mid ww \in L\}$. If $|L'|$ is finite, then clearly $\mathsf{Half}(L)$ is finite, and thus regular. Otherwise, $|L'|$ is infinite, and there are infinitely many ww's such that are longer than any finite length $n_0 \in \mathbb{N}$. Because w is too long to remember in the finite memory of any DFA, we shall have a DFA or regular expression to match it; otherwise, L cannot be regular.

The challenge is to construct a DFA / regular expression that matches exactly $\mathsf{Half}(L)$ (but no other strings).

Hint: It is easier to build a *non-deterministic* finite automata from the given DFA of L. The source is cited in the footnote,[1] but it is more fun to figure it out yourself using non-deterministic techniques.

Answer:

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[1]Introduction to automata theory, languages, and computation, 3rd ed. Exercise 4.2.9. John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman.

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This is the end of the problems for HW3. Remember to follow the last step in the directions on the first page to prepare your PDF for submission.