Advanced Algorithms, Fall 2011

Homework #8, due Monday, December 5

1. Devise and analyze (in worst-case terms) a divide-and-conquer algorithm for the following problem. Given a set of rectangles, all bases of which lie on the x-axis, determine the upper envelope of the collection of rectangles.

   Would your algorithm still work if the set contained triangles rather than rectangles (still with their base on the x-axis, of course)? How far can you generalize the types of shapes for which your algorithm will work?

2. Recall that Fibonacci numbers are defined by the recurrence $F(n) = F(n-1) + F(n-2)$, with initial conditions $F(0) = 0$ and $F(1) = 1$. Computing the nth Fibonacci number is easily done in $n-1$ additions. However, we can use divide-and-conquer techniques to compute the nth Fibonacci number with $\Theta(\log n)$ arithmetic operations. Derive such an algorithm.

3. Convexity is a very strong property, but a weaker version often suffices. Consider this version: a polygon is "locally convex" if there exists some point p in the interior of the polygon such that, for each vertex v of the polygon, the line segment $\overline{pv}$ lies entirely within the polygon. (This is much as in the geometric definition of a convex polygon, except that only one endpoint of the segment is arbitrary.)

   Design and analyze a randomized algorithm that, given just the polygon (by a counterclockwise list of its vertices along the perimeter), decides in linear expected time whether the polygon is locally convex.

4. You are in a crowd of n people; over half of them belong to the same secret party. You go around asking questions; the only question you can ask is "are these two people (pointing at one, then at another) from the same party?" Everyone in the crowd is truthful.

   Use a divide-and-conquer approach to identify with no possibility of error one person who belongs to the secret party. (Note that it is possible to solve this problem in linear time by other techniques; the divide-and-conquer algorithm will impose an extra factor of $\log n$, but it is what is asked of you here.)

5. What can you do for the previous problem if some fixed (small) percentage of the people are systematic liars, who lie every single time? (Assume that the percentage of liars is reasonably small.) And is that similar to handling some small percentage of errors in the answers, this time distributed evenly among all answers?