Class 3, Bivariate Regression

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this version: Monday 9th March, 2015 17:40

outline

misc

hypothesis testing

measurement

outline

misc

hypothesis testing

measurement

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today and looking ahead

- much of today's class is the repetition of the last class
- and we are adding hypothesis testing
- next week we will start multiple regression
- · class material will get more applied
- · we will have more examples
- and more programming

misc 4/5

paper?

- how is the paper going ?
- again, start early
- ♦ if you do not know where to start email me...

nisc 5/

basic calculations again

- let's do some basic calculations again
- here we will cover some of the ps1
- and later demonstrate hypothesis testing on ps1 data

misc 6/5

basic calculations [blackboard;dofile:ps1]

<u>Y</u>	X	У	y2	X	x2	ху
1	17					
3	13					
5	8					
7	10					
9	2					

Sum:

25 50

$$\overline{Y} = 5$$
 $\overline{X} = 10$

misc 7/59

predicted values and residuals

[hlackhoard·dofile·ns1]

$$\diamond \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

- \diamond for obs 1:
- $\hat{Y}_1 = 10.24 + (-0.524)(17) = 1.332$
- $\diamond e_1 = 1 1.33 = -0.33$
- let's calculate TSS and Rsq and p-value! (this is your ps)

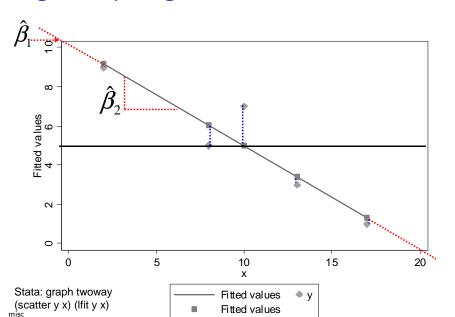
misc

the coefficients-interpretation [dofile:ps1]

 Beta hat two is the slope coefficient. Thus, a one unit change in X leads to a 0.524 decrease in Y. Beta hat one is the intercept term. It is the predicted value for Y when X is equal to zero.

nisc 9/5

regression plot again



sum of squared residuals

$$\diamond \sum e_i = 0$$

$$\Rightarrow \sum e_i^2 = 5.42$$

$$\diamond s = \sqrt{\frac{\sum e_i^2}{n-2}} =$$

The sum of the squared residuals is the quantity that was minimized. s is the estimate of sigma, the standard deviation of the disturbance terms under the assumption of homoskedastic (constant variance) disturbance terms.

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standard error of the slope

$$\diamond$$
 $s_{\hat{\beta}_2} = \frac{s}{\sqrt{\sum x_i^2}}$

♦ This is the standard error of the slope coefficient, and gives us information about the reliability (like sd or se) of our estimate of the slope. We can use it to construct confidence intervals or conduct hypothesis tests.

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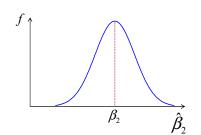
coefficient distribution

- we know the mean and variance of the estimator
- but what about the shape of the sampling distribution of the slope coefficient?
- \diamond If the disturbance term is normally distributed, then the coefficients also have a normal distribution. That is: $u_i \sim N(0,\sigma^2) \rightarrow$ If the distribution of the disturbance term is not normal or unknown, the distribution of the OLS estimators is still normal in large samples "asymptotically".

⋄ so you want to use big samples!

sampling distribution of the slope

The probability distribution $\hat{\beta}_2$ is centered on the true value of the parameter (i.e. unbiased) and is normally distributed with variance estimated by:



 $\diamond H_0: \beta_2 = 0 H_A: \beta_2 \neq 0$

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outline

misc

hypothesis testing

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confidence intervals

 \diamond In general, a confidence interval is the point estimator plus or minus a margin of error, which consists of a distribution parameter (z or t) times the standard error of the estimator. In this case (small sample, σ unknown, we use the t distribution

 $\diamond PE \pm (t_{\frac{\alpha}{2},DOF})(SE) = \hat{\beta}_2 \pm t_{0.025,3} s_{\hat{\beta}_2}$

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hypothesis test

the null is that slope ("the onobserved true parameter") is zero (i.e. no effect)

$$\diamond H_0: \beta_2=0$$

$$\diamond \ H_A: \beta_2 \neq 0$$

$$\diamond t = \frac{\hat{\beta}_2 - \beta_2}{s_{\hat{\beta}_2}}$$

What if our null hypothesis was that the slope was -1?
Can you think of an example where a null hypothesis other than 0 would be appropriate?

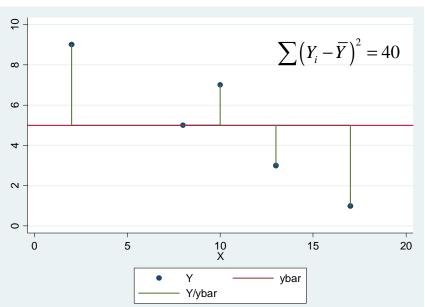
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partitioning the variance in Y

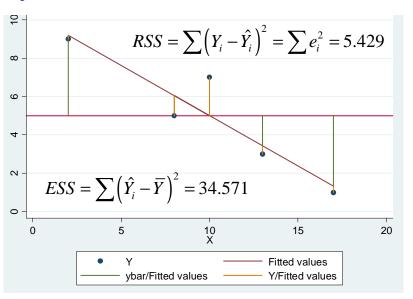
- \diamond before regression $E[Y_i] = \bar{Y}$
- $TSS = \sum (Y_i \bar{Y})^2 = \sum y_i^2 = 40$
- \diamond after regression $E[Y_i|X_i] = \hat{Y}_i$
- $RSS = \sum (Y_i \hat{Y}_i)^2 = \sum e_i^2 = 5.43$
- $\cdot ESS = TSS RSS = 40 5.4 = 34.57$

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yet another look at tss



yet another look at rss



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R^2 again

- $\diamond R^2 = 1 \frac{\sum e_i^2}{\sum y_i^2}$
- The R2 is the proportion of the total variance in the Y variable that is explained by the model. However, do not overestimate the importance of this statistic. We will see (when we move on to multiple regression) that it can be misleading. Notes:
- $0 < R^2 < 1$
- $\diamond R^2 = (r_{xy})^2$ for bivariate case only

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exercise 1

you regressed car's price on its weight

- interpret the coefficient
- is it significant ?
- ♦ calculate 95% CI
- (check with answer in the dofile)

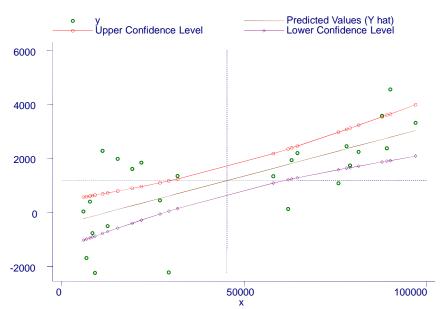
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reliability of predict. val. (se of of E(Y|X)

- We have discussed the fact that parameter estimates are random variables, and so they have standard errors.
 Predicted values are also random variables because they are linear combinations of the coefficients.
- ♦ The further from the mean of X, the wider the confidence interval around the predicted value.
- $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$ why no residual in this equation?
- $\Rightarrow var(\hat{Y}_0|X_0) = \sigma^2(\frac{1}{n} + \frac{(X_0 \bar{X})^2}{\sum (X_1 \bar{X})^2})$

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another illustration...



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outline

misc

hypothesis testing

measurement

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intuition

- what happens to the regression estimates if we change the measurement of the variables
- but first let's look at the relationship between correlation coefficient and regression coefficient

measurement 26/59

correlation vs regression

$$\diamond r = \frac{\sum y_i x_i}{\sqrt{(\sum x_i^2)(\sum y_i^2)}} \quad \hat{\beta}_2 = \frac{\sum y_i x_i}{\sum x_i^2}$$

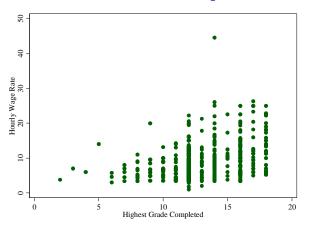
⋄ bivariate slope equals corr coef scaled by std dev of Y and

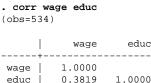
X:

$$\hat{\beta}_2 = \frac{\sum y_i x_i}{\sum x_i^2} = r(\frac{s_Y}{s_X})$$

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education and wages [dofile:wages]





. sum wage educ									
Variable	0bs	Mean	Std. Dev.	Min	Max				
+									
wage	534	9.023939	5.138876	1	44.5				
educ	534	13.01873	2.615373	2	18				

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education and wages [dofile:wages]

. regress wage educ

Source	ss	df	MS		Number of obs	
Model Residual	2053.22494	1 2053 532 22.5	.22494 982396		Prob > F R-squared Adj R-squared	= 0.0000 = 0.1459
Total					Root MSE	= 4.7538
wage	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
educ _cons	.7504488	.07873 1.045404	9.532 -0.714	0.000 0.476	.5957891 -2.799576	.9051086 1.307678

The estimated regression line:

$$\widehat{wage}_i = \hat{\beta}_1 + \hat{\beta}_2 educ_i = -0.75 + 0.75 educ_i$$

Interpret the coefficients.

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anatomy of stata output

. regress DV IV

Source	 	SS	df	MS	Number of o	obs = n
Model	ESS =	$\sum (\hat{Y}_i - \overline{Y})$	1	••••	F(1, n-2)	=
Residual	RSS	$I = \sum e_i^2$	n-2	$2 s^2 = \frac{RSS}{n-2}$	Prob > F R-squared	
Total	TSS =	$\sum (Y_i - \overline{Y})$	n-1	$s_Y^2 = \frac{TSS}{n-1}$	Adj R-Squa Root MSE	red = $= s$
DV	Coef.	Std.Err.	t	P> t	[95% Conf.	Interval]
IV	$\hat{oldsymbol{eta}}_2$	$s_{\hat{eta}_2}$	$\left(rac{\hat{oldsymbol{eta}}_2}{s_{\hat{eta}_2}} ight)$	p val. for H_0 that $\beta_2 = 0$	$\hat{eta}_2 - t_{0.025} s_{\hat{eta}_2}$	$\hat{eta}_2 + t_{0.025} s_{\hat{eta}_2}$
Intercept	$\hat{eta}_{_{1}}$	$s_{\hat{eta}_{ m l}}$	$ \frac{\left(\frac{\hat{\beta}_{1}}{s_{\hat{\beta}_{1}}}\right)}{\left(\frac{s_{\hat{\beta}_{1}}}{s_{\hat{\beta}_{1}}}\right)} $	p val. for H_0 that $\beta_1 = 0$	$\hat{eta}_{1} - t_{0.025} s_{\hat{eta}_{1}}$	$\hat{\beta}_{1} + t_{0.025} s_{\hat{\beta}_{1}}$

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some examples

- $\diamond \hat{\beta}_2 = \frac{\sum y_i x_i}{\sum x_i^2} = r(\frac{s_Y}{s_X})$
- \diamond if x is packs instead of cigarettes per capita, how does s_X change? how does the slope change?
- \diamond if y is deaths per 1,000 instead of deaths per million, how does s_Y change? how does the slope change?
- ⋄ if both x and y have a variance of 1 (e.g. z scores), what is the slope coefficient?

measurement 31/

shifting X by a constant c $\diamond Y_i = \beta_1 + \beta_2 X_i' + u_i \quad X_i' = X_i + c$

♦ The mean of X changes, but the deviations from the mean do not.

♦ Thus, the slope is unaffected. The intercept shifts to reflect in the opposite direction of the constant.

$$\hat{\beta}_{2} = \frac{\sum y_{i}x_{i}'}{\sum x_{i}'^{2}} = \frac{\sum y_{i}x_{i}}{\sum x_{i}^{2}}$$

$$\hat{\beta}_{1} = \bar{\mathbf{Y}} - \hat{\beta}_{2}\bar{\mathbf{X}}' = \bar{\mathbf{Y}} - \hat{\beta}_{2}(\bar{\mathbf{X}} + \mathbf{c}) = \bar{\mathbf{Y}} - \hat{\beta}_{2}\bar{\mathbf{X}} - \hat{\beta}_{2}\mathbf{c}$$

 $\diamond \ \hat{\beta}_1 = \overline{\bar{Y}} - \hat{\beta}_2 \overline{X}' = \overline{Y} - \hat{\beta}_2 (\overline{X} + c) = \overline{Y} - \hat{\beta}_2 \overline{X} - \hat{\beta}_2 c$ \diamond If X is shifted by subtracting \bar{X} (e.g. $\bar{X}=c$): $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} - \hat{\beta}_2 (\bar{X}) = \bar{Y}$

shifting Y by a constant d

$$\diamond Y_i' = \beta_1 + \beta_2 X_i + u_i \quad Y_i' = Y_i + d$$

♦ The mean of Y changes, but the deviations from the mean do not.

♦ Again, the slope is unaffected. The intercept shifts in the same direction and amount as the constant.

$$\hat{\beta}_2 = \frac{\sum y_i' x_i}{\sum x_i^2} = \frac{\sum y_i x_i}{\sum x_i^2}$$

$$\hat{\beta}_1 = \bar{Y}' - \hat{\beta}_2 \bar{X} = (\bar{Y} + d) - \hat{\beta}_2 \bar{X} = \bar{Y} - \hat{\beta}_2 \bar{X} + d$$

neasurement 33/59

regression on Z scores

⋄ z scores always have a mean of 0 and a variance (and standard deviation) of 1

$$\hat{\beta}_2 = r_{Z_Y Z_X} \frac{s_{Z_Y}}{s_{Z_X}} = r_{YX}$$

$$\hat{\beta}_1 = \bar{z}_Y - \hat{\beta}_2 \bar{z}_X = 0 - r(0) = 0$$

Thus, a regression of the z scores of Y on the z scores of X produces a slope equal to the correlation coefficient of X and Y and a zero intercept.

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exercise 2 [do it at home; see dofile for ans]

- confirm the above by using data from ps1 in stata
- ⋄ run regression of Y on X
- ⋄ add 3 to X (gen new var) and repeat regression
- load original data, substruct 3 from Y, and repeat regression
- ♦ load original data and run regression on Z scores

measurement 35/9

interpretation: transforming variables \diamond Linear: One unit change in X leads to a β_2 unit change in

- Y. \diamond Log-Lin: One unit change in X leads to a $100 * \beta_2$ % change in Y. (guj ed4:p180 fig6.4; ed5:p163 ex6.4)
- \diamond Lin-Log: One percent change in X leads to a $\beta_2/100$ unit change in Y. (guj: ed4:p182 fig6.5; ed5:p165-6 ex6.5)
- Log-Log (aka log-linear or "linear in logs"): One percent change in X leads to a β₂ % change in Y (elasticity).
 These interpretations are valid for small changes

(coefficients less than 0.10, i.e 10%). of for interpretation and examples see

meahttp://www.ats.ucla.edu/stat/mult_pkg/faq/general/log_36/59

review of logarithms

$$\diamond y = log_b(x) \rightarrow x = b^y$$

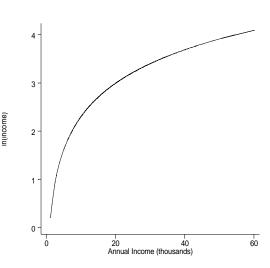
- e.g. $log_2(8) = 3$ because $2^3 = 8$
- $log_{10}100 = 2 ext{ because } 10^2 = 100$
- $\diamond \ \ \textit{In} \ x = \textit{log}_{e}x; \ \ e = 2.72...$
- e.g. $e^{0.5} = 1.65$; In 1.65 = 0.5
- $\Diamond \ln(xy) = \ln(x) + \ln(y)$
- $\Rightarrow \ln(\frac{x}{y}) = \ln(x) \ln(y)$
- $\diamond \ln(x^n) = n \ln(x)$
- $\Diamond \ln(e^x) = x \ln(e) = x$

measurement

logarithms and relative change

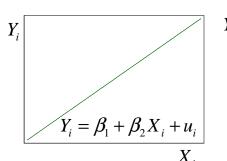
For small changes, the change in ln(x) is the percentage change in x. E.g. a 0.05 increase in ln(x) is a 5 percent increase in x. Example:

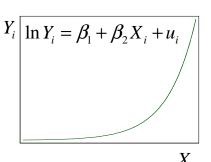
x 100 105	In(x) 4.6052 4.6540	chg 0.0488
12,345 12,962	9.4210 9.4698	0.0488

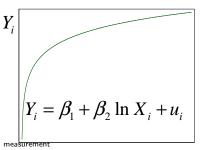


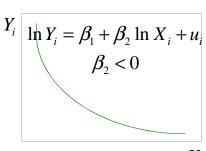
measurement 38/59

it makes a difference









lin-lin [dofile: measurment]

- ♦ e.g. people with more education earn higher wages...
- $\diamond Y_i = \beta_1 + \beta_2 X_i + u_i \ \beta_2 = \frac{\Delta Y_i}{\Delta X_i}$
- \diamond This model specifies that the change is constant regardless of the level of X (because β is constant)
- \diamond wage_i = $\beta_1 + \beta_2$ educ_i + u_i
- $\cdot \widehat{wage}_i = -0.75 + 0.75 educ_i$
- $\widehat{wage}_{10} = \$6.75 \quad \widehat{wage}_{11} = \$7.50 \quad \widehat{\Delta wage} = \0.75 The change is the same for any 1 year change in educ.

measurement 40/

relative change: log-lin

We estimate this model by taking the natural log of Y first, then regression InY on X. The regression treats InY the same as any other variable.

$$\diamond InY_i = \beta_1 + \beta_2 X_i + u_i$$

$$\phi \ \beta_2 = \frac{\Delta(\ln Y_i)}{\Delta X_i} = \frac{\Delta Y_i/Y_i}{\Delta X_i} = \frac{\text{relative change in } Y_i}{\text{unit change in } X_i}$$

...for small changes. The percent change in Y per unit change in X is $100*\beta_2$ times the unit change in X. This is still a linear regression, but with a new dependent variable (lnY). It is not linear in terms of Y.

measurement 41/

e.g. log-lin [dofile: measurment]

- \diamond $ln(wage_i) = \beta_1 + \beta_2 educ_i + u_i$
- $\phi \ \widehat{ln(wage)_i} = 1.06 + 0.08educ_i$
- $\diamond \ \textit{In}(\textit{wage})_{10} = 1.06 + 0.08(10) = 1.86$
- This is the predicted In(wage). But what about the predicted wage?
- $\Rightarrow \widehat{wage}_{10} = e^{1.86} = \6.42
- $\Rightarrow \widehat{wage}_{11} = e^{1.94} = \6.96
- $\Diamond \ \% \Delta \widehat{wage}_{10 \to 11} = \frac{\$6.96 \$6.42}{\$6.42} = 0.08 = 8\%$

measurement 42/

the change varies in dollar terms

- But let's examine the change in wage for an additional year of graduate school, e.g. master's degree years.
- $\Rightarrow \widehat{ln(wage)_i} = 1.06 + 0.08educ_i$
- $\Rightarrow \widehat{ln(wage)}_{17} = 1.06 + 0.08(17) = 2.42 \quad \widehat{wage}_{17} = \11.25
- $\phi \ \widehat{ln(wage)}_{17} = 1.06 + 0.08(18) = 2.50 \ \widehat{wage}_{18} = \12.18
- $\diamond~\% \Delta \widehat{\textit{wage}}_{17 \rightarrow 18} = 0.08 = 8\%$
- ♦ The change in relative (percentage) terms is constant at 0.08 (8 percent), but the dollar change is larger.

measurement 43/

lin-log [dofile: measurment]

- $\diamond Y_i = \beta_1 + \beta_2 \ln X_i + u_i$
- In this model, we generate the natural log of education and regress dollar wage on the log of education

$$\diamond \ \beta_2 = \frac{\Delta Y_i}{\Delta \ln X_i} = \frac{\Delta Y_i}{\Delta X_i / X_i} = \frac{\text{unit change in } Y_i}{\text{relative change in } X_i}$$

measurement 44/5

relative change in education

educ	%change		educ	%change	
	1			11	10%
	2	100%		12	9%
	3	50%		13	8%
	4	33%		14	8%
	5	25%		15	7%
	6	20%		16	7%
	7	17%		17	6%
	8	14%		18	6%
	9	13%		19	6%
	10	11%		20	5%

The relative

change in education per year is declining because the base is getting larger. So the lin-log model will predict a smaller impact on wage each year (see graph few slides back)

measurement 45/

e.g.: lin-log

 Wage as a function of relative change in education \diamond wage_i = $\beta_1 + \beta_2 ln(educ_i) + u_i$

$$\Rightarrow \widehat{wage}_i = -10.15 + 7.54 \ln(educ_i)$$

 $\Rightarrow \widehat{wage}_{10} = -10.15 + 7.54 \ln(10) = 7.21$

$$\Rightarrow \widehat{wage}_{11} = -10.15 + 7.54 \ln(11) = 7.93$$

 $\Rightarrow \% \widehat{\Delta wage}_{10 \to 11} = \0.72

$$\widehat{wage}_{18} = -10.15 + 7.54 \ln(17) = 11.21$$
 $\widehat{wage}_{18} = -10.15 + 7.54 \ln(18) = 11.64$

 $\Rightarrow \widehat{wage}_{18} = -10.15 + 7.54 \ln(18) = 11.64$ \diamond % $\Delta \widehat{wage}_{17\rightarrow 18} = \0.43

$$\Rightarrow \widehat{wage}_{18} = -10.15 + 7.54 \ln(18) = 11.64$$

 $\Rightarrow \% \Delta \widehat{wage}_{17 \to 18} = \0.43
 $\Rightarrow \frac{1}{10} = 0.1 \quad \frac{1}{17} = 0.06 \quad 0.06 * 7.54 = .44$

 \diamond For a 1% (0.01) change in X, the change in Y is $\beta_2/100$,

meain this case 0.0754.

log-log (elasticity model)

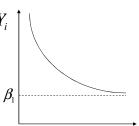
$$\diamond InY_i = \beta_1 + \beta_2 InX_i + u_i$$

- \diamond $\beta_2 = \frac{\Delta \ln Y_i}{\Delta \ln X_i} = = \frac{\text{relative change in } Y_i}{\text{relative change in } X_i}$
- \diamond Thus, for small changes, the relative (percentage) change in Y is β_2 time the relative (percentage) change in X.

measurement 47/

reciprocal models

- In the case of diminishing marginal returns, more of X produces less and less effect on Y. Eventually Y reaches a max/min value and more of X has no effect.
 - · Example: effect of GDP on infant mortality rate. β_1 is the value that Y approaches as X $\to \infty$



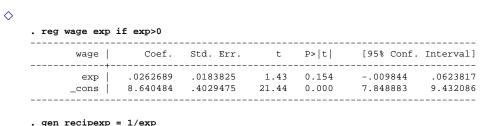
$$Y_i = \beta_1 + \beta_2(\frac{1}{X_i}) + u_i$$

neasurement 48,

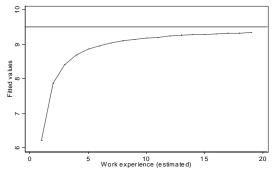
e.g.: wage and experience

. reg wage recipexp

 After a point, you are experienced enough, and more experience has less and less effect on wage. In this model, X can not be equal to 0, because 1/0 is undefined. Thus, you must either drop zeros or add a positive constant to all X.



interpreting reciprocal model [dofile: measurment]



$$\diamond Y_i = \beta_1 + \beta_2(\frac{1}{X_i}) + u_i$$

 \diamond changes in Y are smaller and smaller for larger Xs; yhats:

♦ di 9.5 -3.3*1

♦ di 9.5 -3.3*1/3

and now quadratic regression

measures to positive or negative

- not bivariate regression anymore, but trivariate—see next week
 we introduce it here because the third var is just a sq of
- 2nd var
 and it does what logs or reciprocals do-fits a curve as opposed to a line
- and i think it is more intuitive than logs or reciprocals!
 the idea is that quadratic coef is smaller than linear, and
 - opposite sign

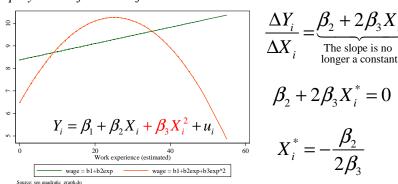
 but as X gets bigger, its square get huge, and so quadrati

 but as X gets bigger, its square get huge, and so quadratic coef with opposite sign overpowers fiorst term and curve

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quadratic model

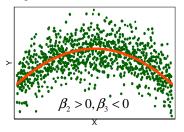
If a *non-linear relationship* between *X* and *Y* is suspected, a *polynomial function of X* can be used to model it.

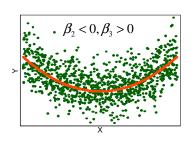


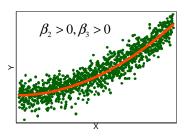
This curve reaches a maximum wage at the point were the marginal effect of experience is zero.

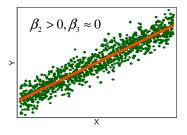
measurement 52/59

quadratic model









measurement 53/59

quadratic: interpretation

dofile: quadratic

- the slope changes with X, it is not constant
- the best way to show the quadratic relationship is to graph it
- there is always a tipping point, but it may be outside the range of the data; in fact, the estimated line may be approximately linear for the observed data range even if the quadratic term is significant!
- the t test on squared term has a null hypothesis of linearity
- · if it is not significant, only linear term is left

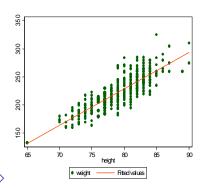
♦ more practice http://www.ats.ucla.edu/stat/mult_pkg/faq/general/curves.htm

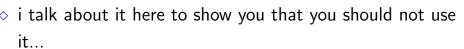
54/59 measurement

regression through the origin (RTO)

- often the intercept has to be zero, for reasons of logic or theory.
- rto seems logical ? be careful!
- rto is virtually always a bad idea
- just don't use it

measurement 55/5





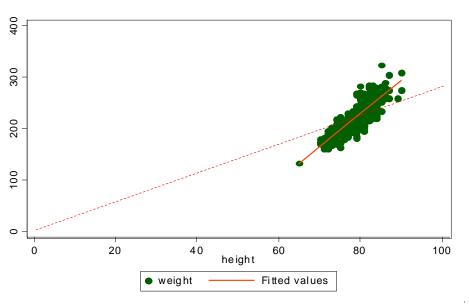
measurement 56/5

very different answers... why?

. reg weight h	neight						
Source	SS		MS		Number of obs		
Model Residual	253964.038 104720.96 +	1 25 423 24	7.567281		F(1, 423) Prob > F R-squared Adj R-squared Root MSE	= 0.00 = 0.70 = 0.70	000 080 074
10tai		424 64	 		ROOL MSE	= 15./	
weight	Coef.	Std. Err		P> t	[95% Conf.	Interva	al]
	!		32.03	0.000	6.057885 -318.7081		
. reg weight h	neight, <mark>nocons</mark>						
	SS		MS		Number of obs		
Model	21544113 184643.978	1 424 43	5.481079		F(1, 424) Prob > F R-squared Adj R-squared	= 0.00 = 0.99	000 915
Total	21728757				Root MSE		
weight	Coef.	Std. Err	. t	P> t	[95% Conf.	Interva	11]
height	2.837525	.0127573	222.42	0.000	2.81245	2.8626	501

neasurement 57

oops!



measurement

bonus

http:
//www.ats.ucla.edu/stat/stata/webbooks/reg/default.htm

♦ dofile:bonus

measurement 59/59