Markov Random Field modeling, inference & learning in computer vision & image understanding: A survey

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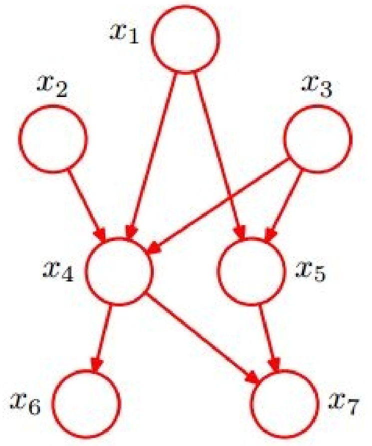
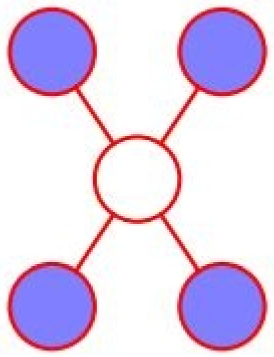
Probabilistic Graphical Model:

1) Bayesian Networks.
2) Markov Random Field.

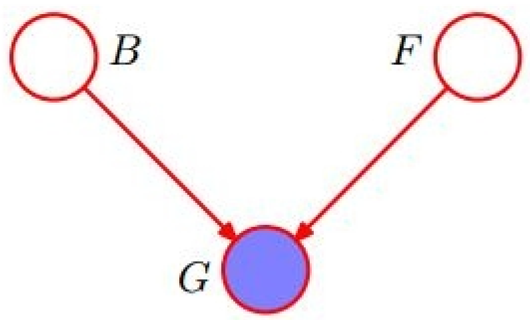




Markov random field find important application in many vision problems. Some examples:

1) Image denoising.
2) image segmentation.

Conditional Independence: We can define intrinsic conditional independence between random variables on any probabilistic graphical model using graph structure of the model.



A probability density function that can be factorised in factors that depends only on local property of graphical model are said to be satisfying factorization property.

Pairwise Markov Property: Two nodes that are not directly connected can be made independent given all other node.

Local Markov Property: A set of the nodes(variables) can be made independent form the rest of the node variables given its immediate neighbours.

Global Markov Property: A vertex set A is independent of the vertex set B(A and B are disjoint) given set C if all chains between A and B intersect C.

Hammersley-Clifford theorem: A probability density function can be represented as a Markov Random Field if and only if it is a Gibbs random field, that is its density can be factorised over the cliques of the graph.

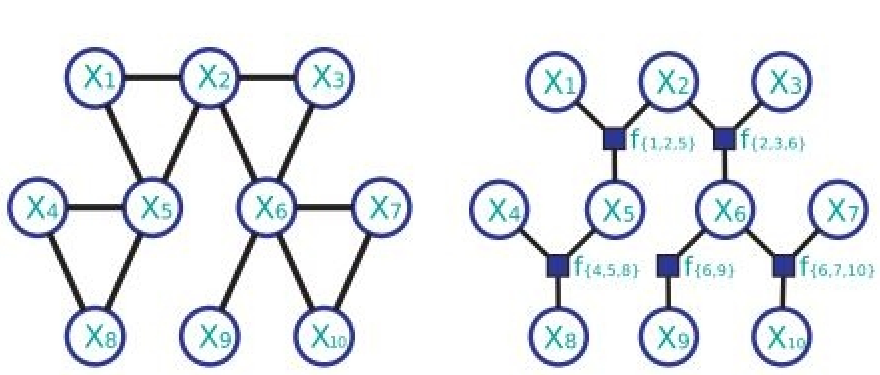
Probability Density function can be written as:

$$p(\mathbf{x}) = \frac{1}{Z}\prod_{c \in C}\psi_c(x_c), \qquad p(\mathbf{x}) = \frac{1}{Z}\exp\{-E(\mathbf{x})\},$$

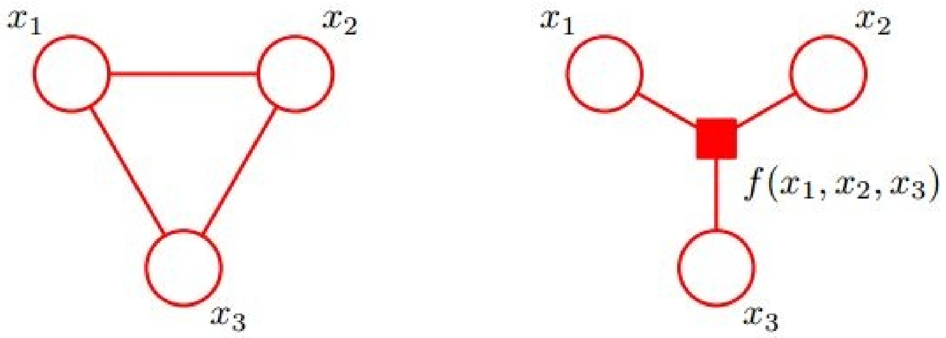
Each factor can be symbolized by a factor node thus giving rise to factor graph.

Factor graphs are useful because:

1) Simplifies calculation of common algorithms.
2) May convert cyclic graph to a tree. Thereby increasing applicability of some algorithms.



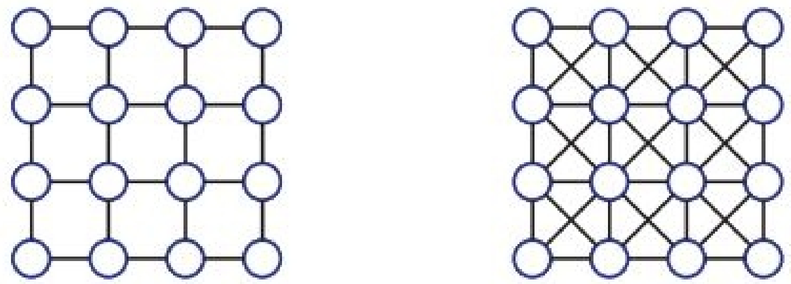
(a) (b) (c)



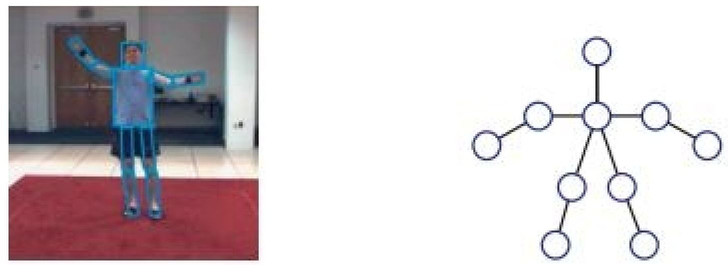
(a) (b) (c)

Types of Markov Random Field:

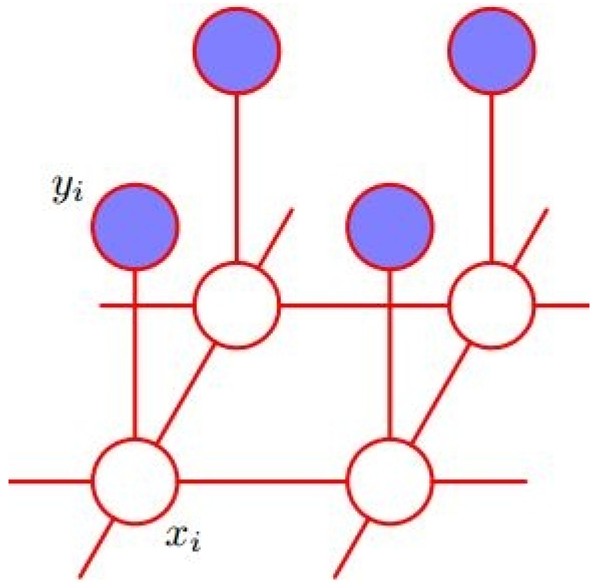
Pairwise Markov Random Field: Associated energy is factorized into a sum of potential functions defined on cliques of order strictly less then three.



(a) 4-neighborhood system (b) 8-neighborhood system



(a) Pictorial Structure (b) MRF model corresponding to (a)



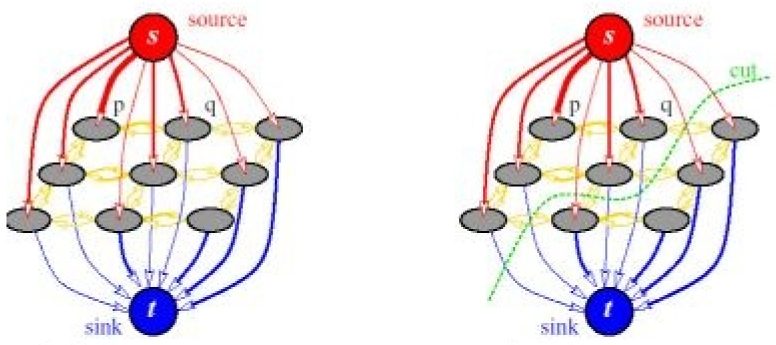
$$E(\mathbf{x}, \mathbf{y}) = h\sum_i x_i - \beta\sum_{\{i,j\}} x_i x_j - \eta\sum_i x_i y_i$$

Higher Order Models: Potential of factor term depends on more than two random variable. Difficult to train and get inference form.

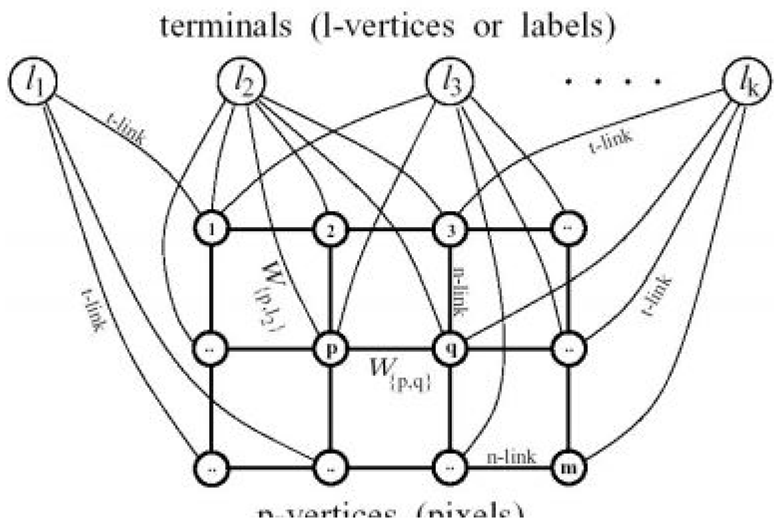
Conditional Random Field: In conditional random field observed random variable are not included in graphical model. Only unobserved variables are latent variable.

Inference Methods:

Graph Cut Algorithm: This algorithm utilizes max flow problem to draw useful inference from Markov Random field.



(a) A graph $\mathcal{G}$ (b) A cut on $\mathcal{G}$



Belief Propagation Algorithm: 1) Depends upon message passing of different nodes in graph. 2) Exact Method for Tree models. 3) Complexity can be improved using dynamic programming.

Loopy Belief Propagation: Approximate version of belief propagation. This algorithm may not converge for all the cases. Using factor graph can guarantee convergence for certain cases.

Junction Tree Algorithm:
1) Exact inference method for arbitrary graphical models.
2) Only possible if graph is triangulated.
3) May not be practically feasible due to exponential complexity.

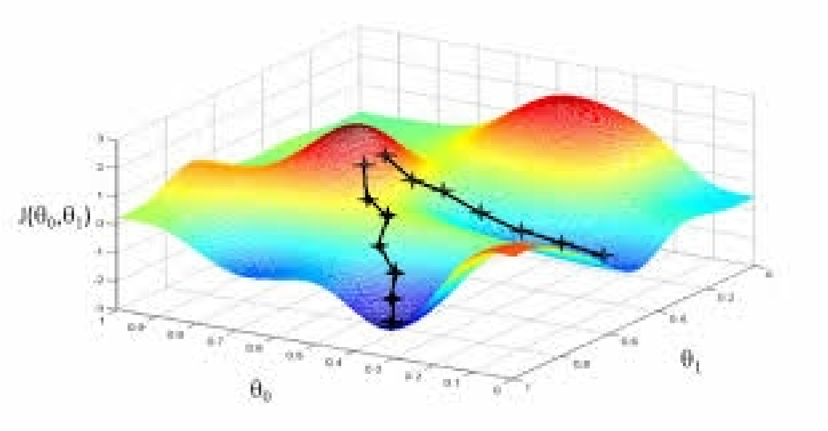
Dual Method: Dual method aims at reformulating inference problem as linear integer programming problem.

Many of the above algorithms can be extended for higher order graphical models.

Training Markov Random Field:

Gradient Ascent:

1) Climb up in steepest direction.
2) Not guaranteed to reach global maximum.



Max-margin Learning: This algorithm aims at adjusting the weight vector such that energy of desired ground truth solution is smaller then energy of any other solution by a fixed amount.

$$E(\mathbf{x}^k; \mathbf{w}) \leqslant E(\mathbf{x}; \mathbf{w}) - \Delta(\mathbf{x}, \mathbf{x}^k) + \xi_k.$$

References:

[1] Chaohui Wang, Nikos Komadakis, Nikos Paragios, Markov Random Field modeling, inference & learning in computer vision and image understanding: A survey.

[2] C.M.Bishop, Pattern Recognition and Machine Learning(Information Science and Statistics, Springer, 2006.

[3] Sudipta N. Sinha, Graph Cut Algorithms in Vision, Graphics and Machine Learning.