

Introduction to light field processing

*Laboratoire de Communications Audio-Visuelles (LCAV)
Ecole Polytechnique Fédérale de Lausanne (EPFL)*

*Loïc Baboulaz
2014*





Jonas Ferdinand Gabriel Lippmann
(16 August 1845 – 13 July 1921)

1. La plus parfaite des épreuves photographiques actuelles ne montre que l'un des aspects de la réalité ; elle se réduit à une image unique fixée dans un plan, comme le serait un dessin ou une peinture tracée à la main. La vue directe de la réalité offre, on le sait, infiniment plus de variété. On voit les objets dans l'espace, en vraie grandeur et en relief, et non dans un plan. De plus, leur aspect change avec les positions de l'observateur ; les différents plans de la vue se déplacent alors les uns par rapport aux autres ; la perspective se modifie ; les parties cachées ne restent pas les mêmes ; enfin, si le spectateur regarde le monde extérieur par une fenêtre, il est maître de voir les diverses parties d'un paysage venir s'encadrer successivement entre les bords de l'ouverture, si bien que dans ce cas ce sont des objets différents qui lui apparaissent successivement.

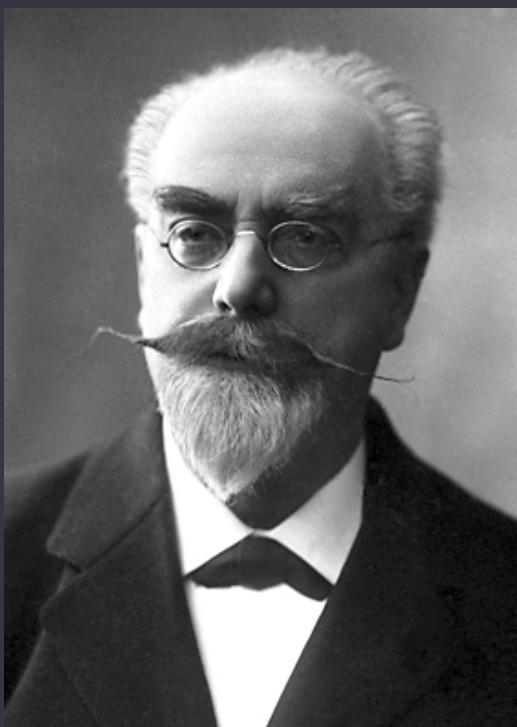
Peut-on demander à la Photographie de nous rendre toute cette variété

SÉANCE DU 2 MARS 1908.

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qu'offre la vue directe des objets ? Est-il possible de constituer une épreuve photographique de telle façon qu'elle nous représente le monde extérieur s'encadrant, en apparence, entre les bords de l'épreuve, comme si ces bords étaient ceux d'une fenêtre ouverte sur la réalité ? Il semble que oui ; on peut demander à la Photographie infiniment plus qu'à la main de l'homme. Je vais essayer d'indiquer ici une solution du problème.

2. Supposons un film comme ceux qu'on emploie couramment, formé d'une pellicule transparente de celluloid ou de collodion enduite sur l'une de ses faces d'une émulsion sensible à la lumière. Avant de coucher l'émulsion sur la pellicule, supposons que celle-ci ait été pressée à chaud dans une sorte de machine à gaufrer, de manière à faire naître sur chacune de ses faces un grand nombre de petites saillies en forme de segments sphériques. Chacune des saillies dont est couverte la face antérieure de la pellicule, celle qui restera nue, est destinée à faire office de lentille convergente. Chacune des saillies de la face postérieure est enduite d'émulsion sensible,



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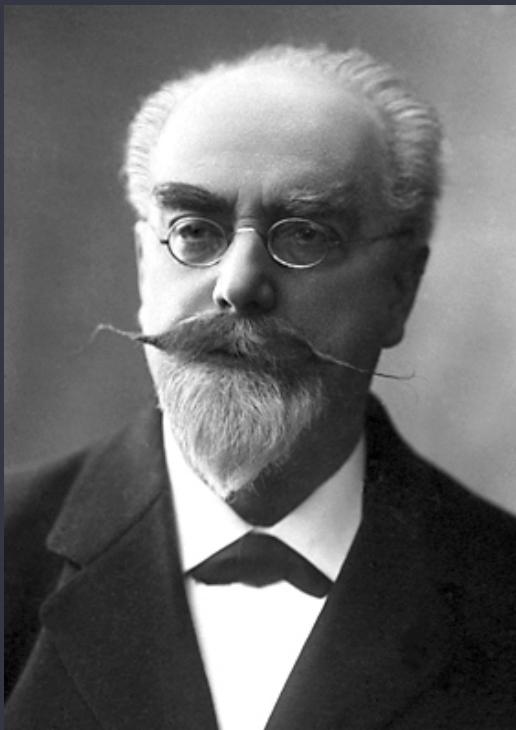
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PHOTOGRAPHIE. — *Épreuves réversibles. Photographies intégrales.*
Note de M. G. LIPPmann.

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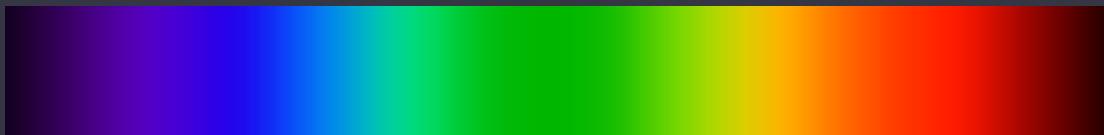
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- Part 1: Light, optics & light-field concepts
- Part 2: Traditional camera technology
- Part 3: Light field camera technology
- Part 4: Sampling & processing the light field

- **Part 1: Light, optics & light-field concepts**
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Light

Nature of light	Electromagnetic radiation (wave vs. particle)
Main light properties	Intensity Propagation direction Wavelength or color Polarization
Light propagation theories	Quantum theory (wave packets) Wave theory (Maxwell's equations) Ray-tracing (geometric optics)



Ray-tracing

Principle	Complicated nature of light is reduced to a large set of narrow beams, or rays. Approximate solution of Maxwell's equations
Assumption	Objects much bigger than wavelength of light
Pros	Computationally efficient Geometric optics Well-suited for optical design
Cons	Cannot model diffraction

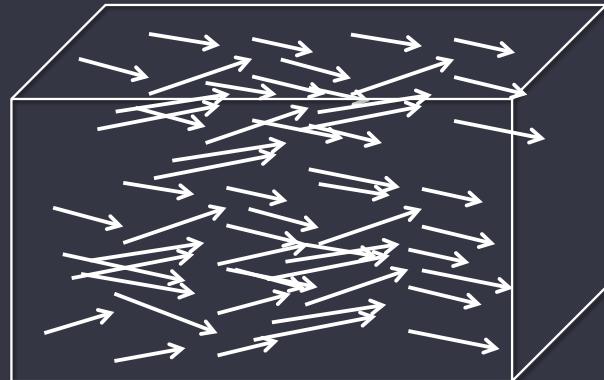


Light Phenomena

Reflection	
Refraction	
Diffraction	

Light field

- For every point in space, the light field $L(\underline{r})$ describes the amount of light that is travelling at any instant of time

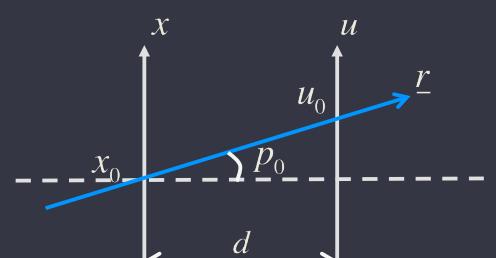


Light field parametric representation

- For every point in space, the light field $L(\underline{r})$ describes the amount of light that is travelling at any instant of time
- A 2D light ray can be parameterized as:

- Two-line intersections: $\underline{r} = \begin{bmatrix} x_0 \\ u_0 \end{bmatrix}$, or as:

- Line intersection/slope: $\underline{r} = \begin{bmatrix} x_0 \\ p_0 \end{bmatrix}$



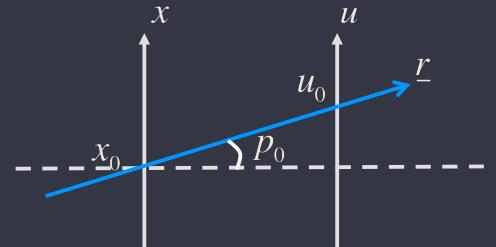
- Conversion between 2 representations is straightforward:

$$\begin{bmatrix} x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & d \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ p_0 \end{bmatrix} \quad \rightarrow \quad L(x, u) = L(x, x + d \cdot p)$$

Light field parametric representation

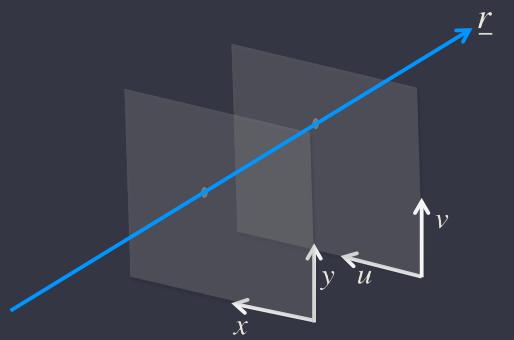
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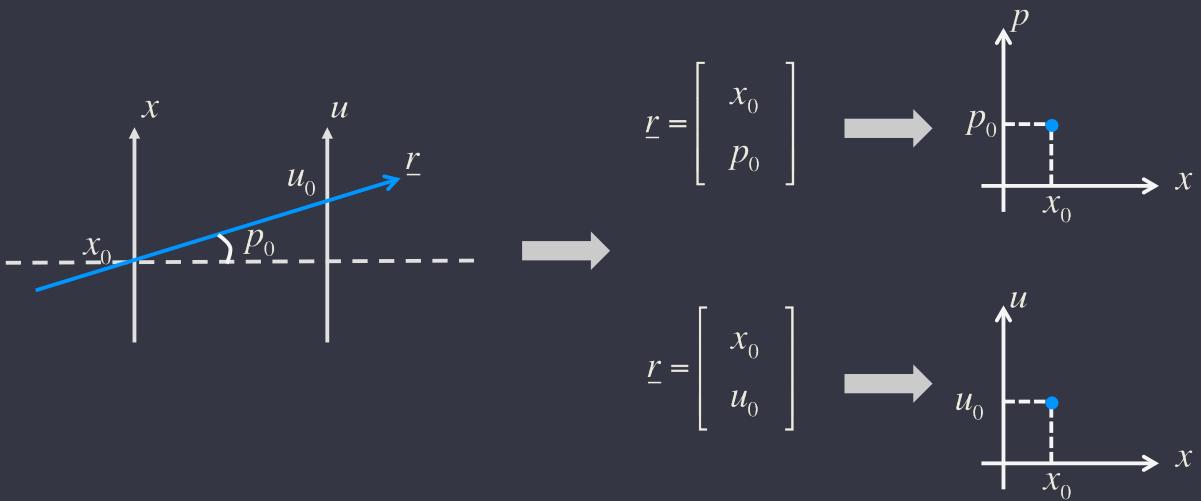
- A 3D light ray is described by 4 dimensions:

$$\underline{r} = \begin{bmatrix} x_0 \\ y_0 \\ u_0 \\ v_0 \end{bmatrix} \quad , \text{ or as: } \quad \underline{r} = \begin{bmatrix} x_0 \\ y_0 \\ p_0 \\ q_0 \end{bmatrix}$$



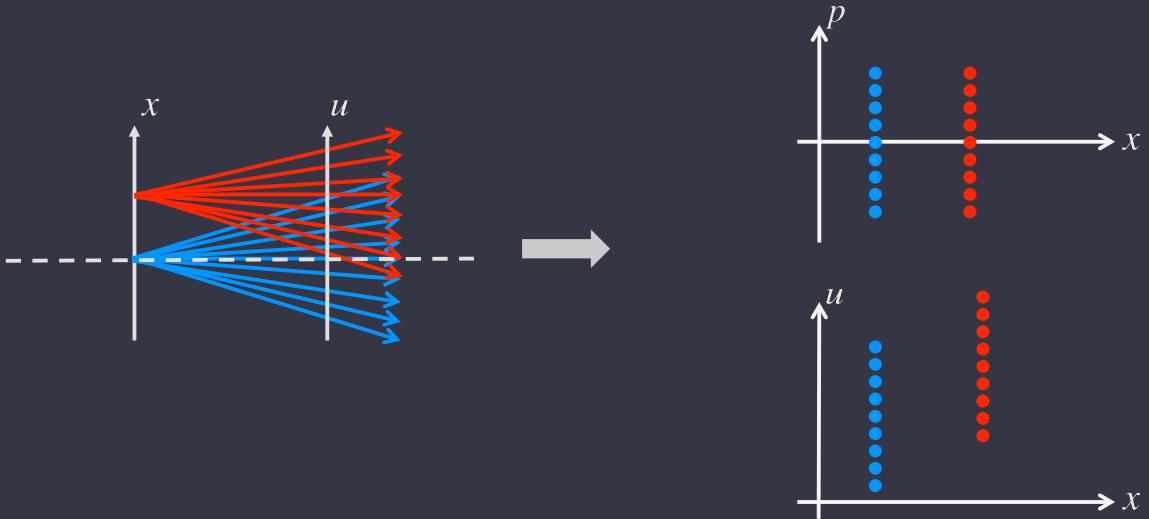
Light field parametric representation

- Cartesian ray-space diagram is used to visualize rays, beams and complete light field:



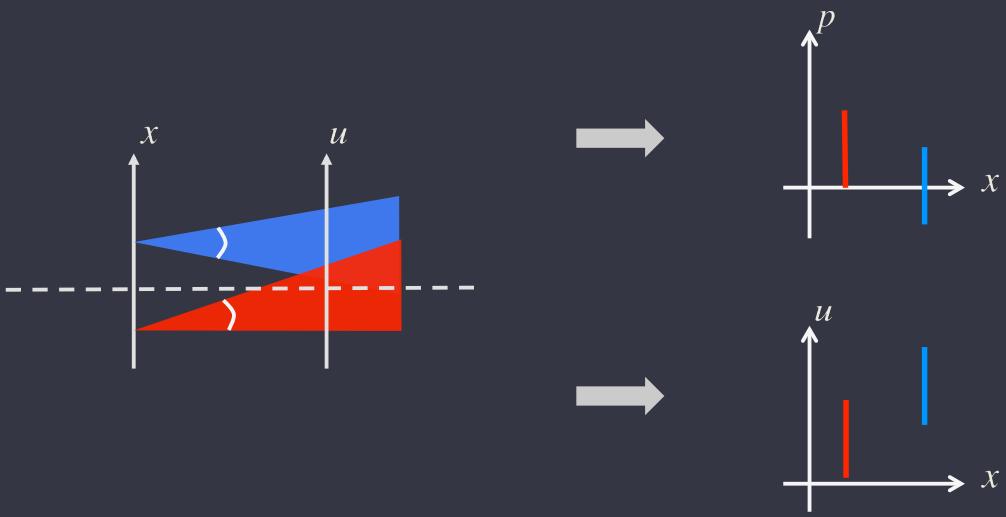
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- *Cartesian ray-space diagram is used to visualize rays, beams and complete light field:*



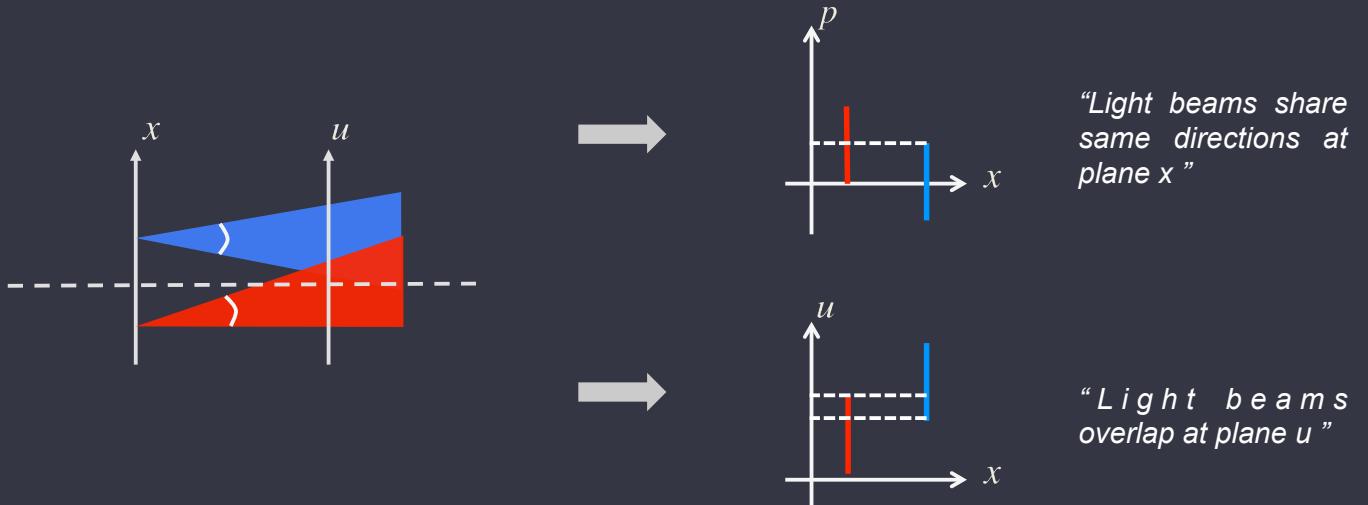
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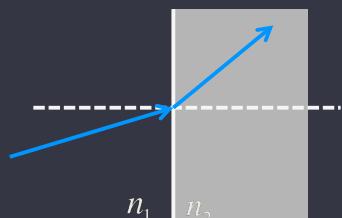
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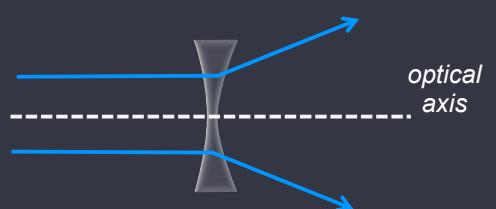
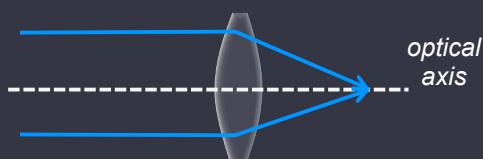
Simple optical components

- Propagation at flat interface:

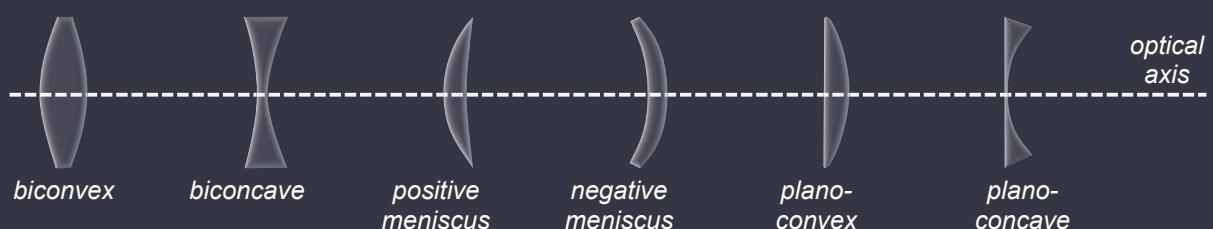
$$n_1 \cdot \sin(\theta_1) = n_2 \cdot \sin(\theta_2)$$



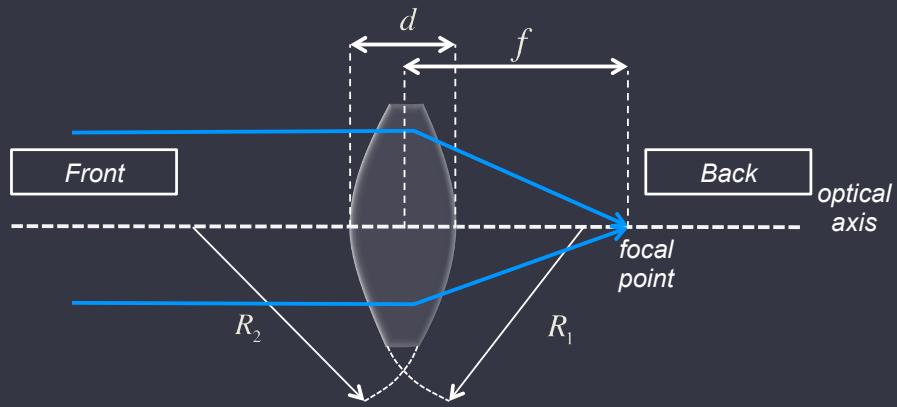
- Converging lens vs. diverging lens



- Naming convention for lenses:

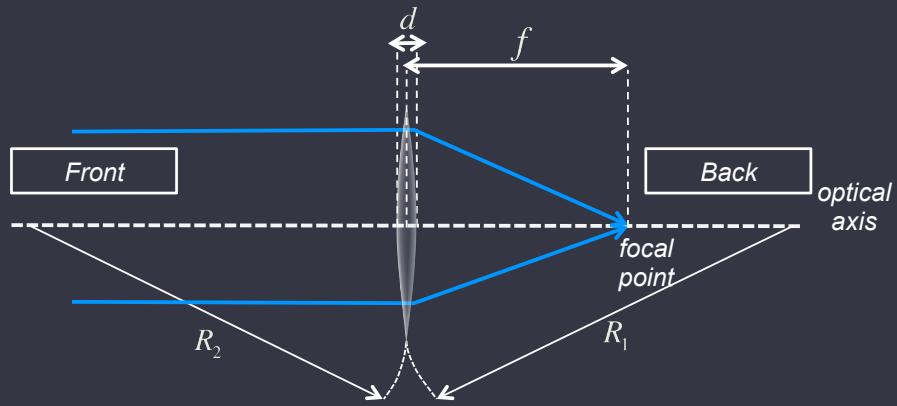


Lens parameters



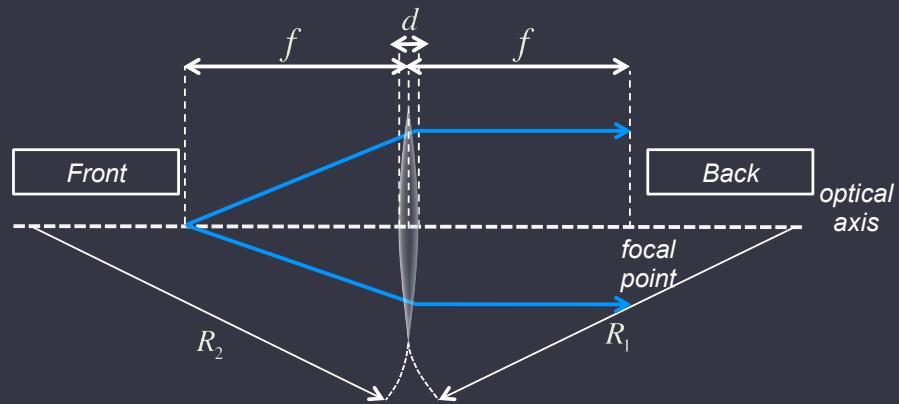
- *Lensmaker's equation:*
$$P = \frac{1}{f} = (n - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n - 1)d}{nR_1R_2} \right]$$
- Convexity/concavity \Leftrightarrow Sign of focal length \Leftrightarrow Sign of lens radii.
- P : lens power in dioptres; f : focal length in meters.

Thin Lens approximation



- *Thin lens equation:*
$$P = \frac{1}{f} \approx (n - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \quad \text{if } \begin{cases} d \ll R_1 \\ d \ll R_2 \end{cases}$$

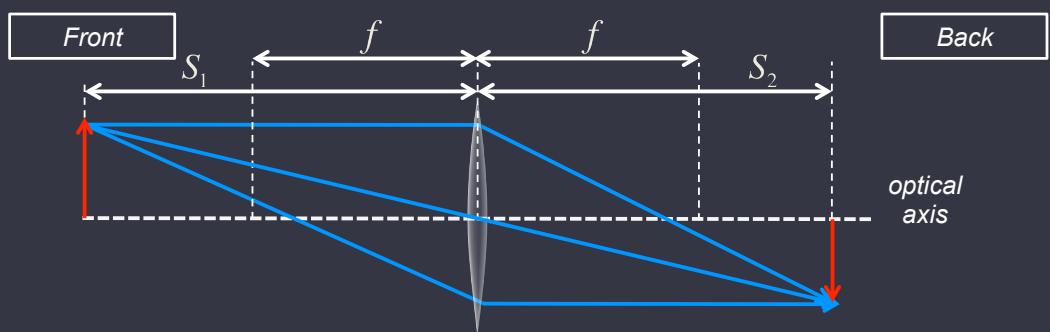
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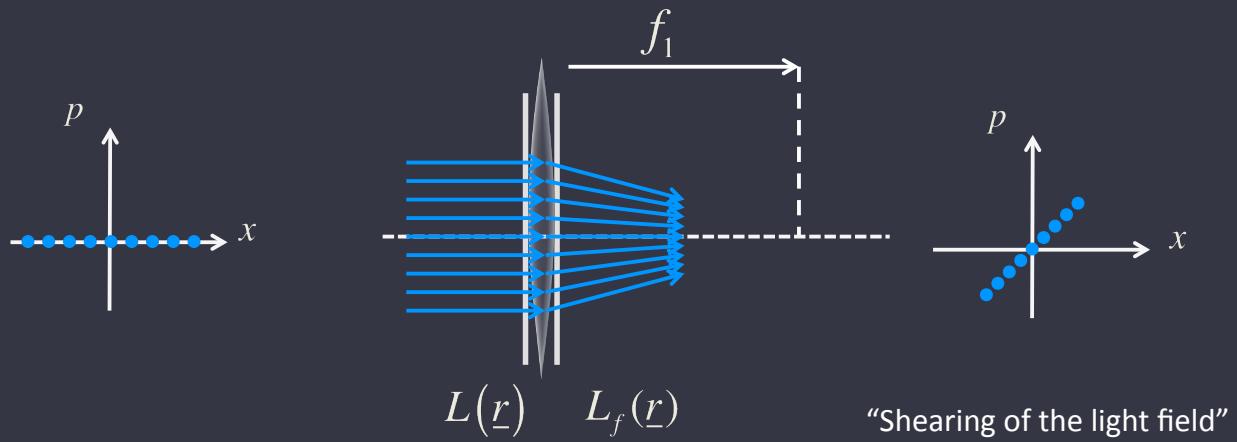
Thin Lens imaging



- *Thin lens formula:*

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

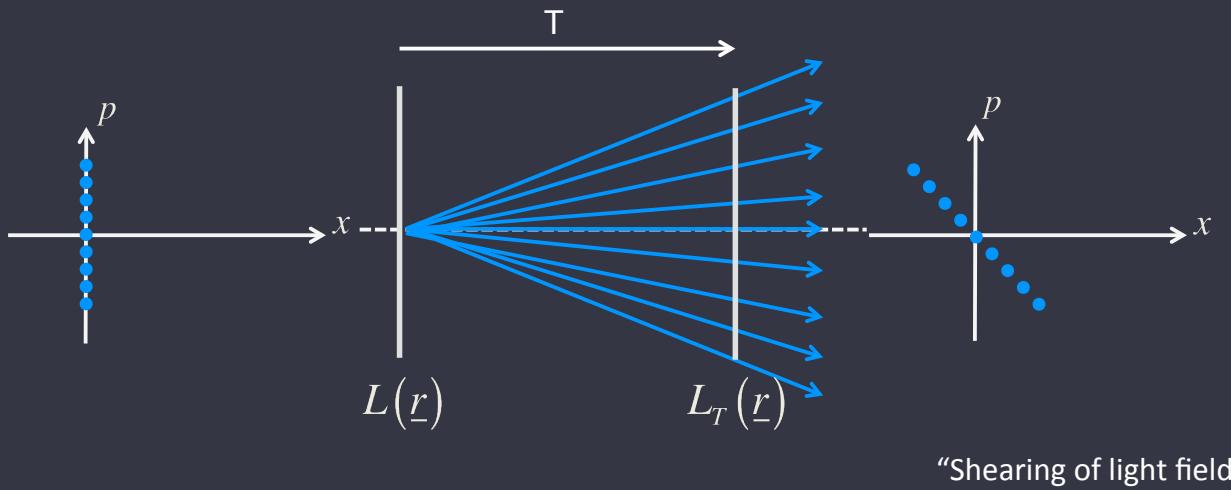
Example of propagation in thin lens



$$L_f(\underline{r}) = L \left(\begin{pmatrix} 1 & 0 \\ -1/f_1 & 0 \end{pmatrix} \cdot \underline{r} \right)$$

“Ray transfer matrix”

Example of propagation in free space



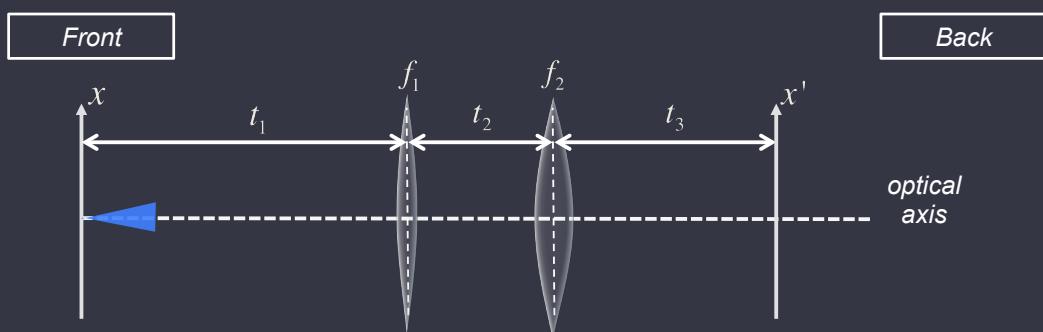
$$L_T(\underline{r}) = L \left(\begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \cdot \underline{r} \right)$$

Ray transfer matrix

- Ray transfer matrices can be used to propagate light field forward and backward

<i>Propagation in free space with no scattering</i>	t : propagation distance	$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}$	Shear matrix
<i>Refraction at flat interface</i>	n_1 : initial refractive index n_2 : final refractive index	$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}$	Scaling matrix
<i>Reflection at flat mirror</i>		$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}$	Identity matrix
<i>Refraction at thin lens</i>	f : focal length	$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}$	Shear matrix
<i>Refraction at thick lens</i>	n_1 : refractive index outside lens n_2 : refractive index inside lens R_1, R_2 : lens radii d : thickness of lens	$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{n_2-n_1}{R_2n_1} & n_1/n_2 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{n_1-n_2}{R_1n_2} & n_1/n_2 \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}$	

Ray transfer matrix: example with concatenated optical components



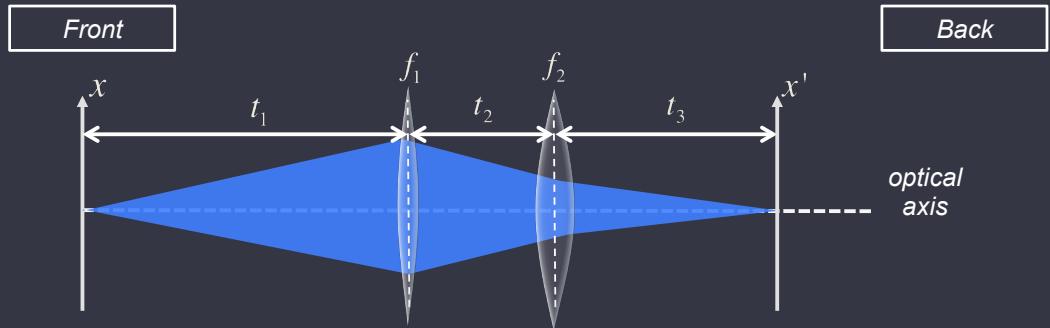
Ray transfer
matrices

$$\begin{pmatrix} x \\ p \end{pmatrix}$$

Operations
on light field



Ray transfer matrix: example with concatenated optical components



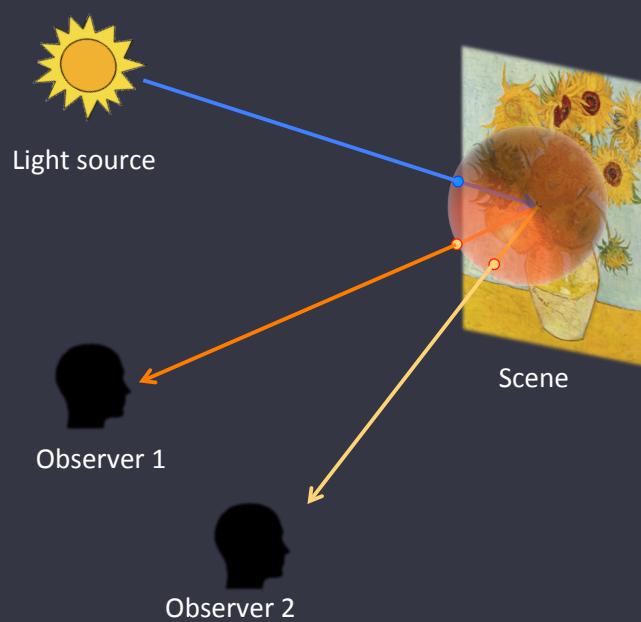
*Ray transfer
matrices*

$$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & t_3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & t_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & t_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}$$

*Operations
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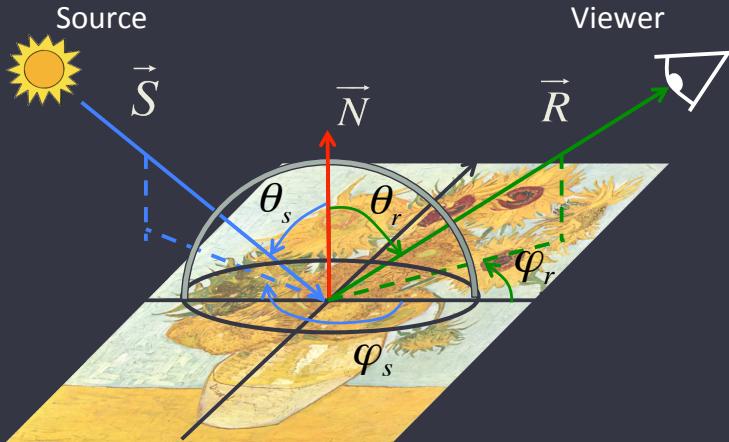


Surface reflectances



Surface reflectances

- The BRDF describes how light is reflected. It is material-specific.

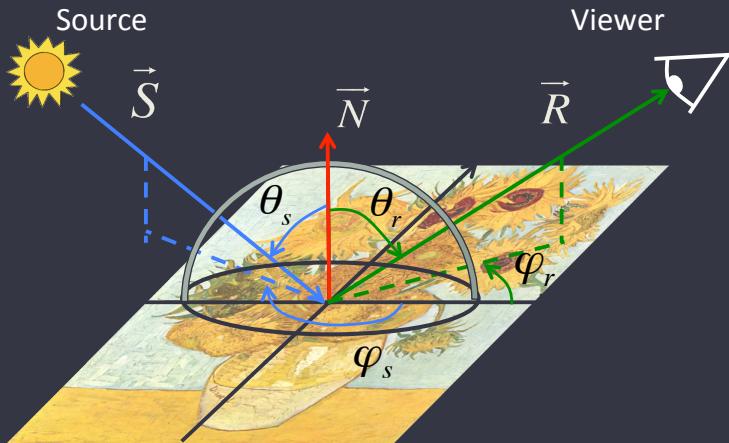


θ_s : light source elevation angle
 φ_s : light source azimuth angle
 θ_r : receiver elevation angle
 φ_r : receiver azimuth angle

S : set of light sources
 $L_\lambda(\vec{R})$: reflected radiance at point of reflection
 $L_\lambda(\vec{S})$: incident radiance at point of reflection
 $f_\lambda(\vec{S}, \vec{R})$: bidirectional reflectance distribution function (BRDF)
 \vec{N} : surface normal vector
 λ : wavelength

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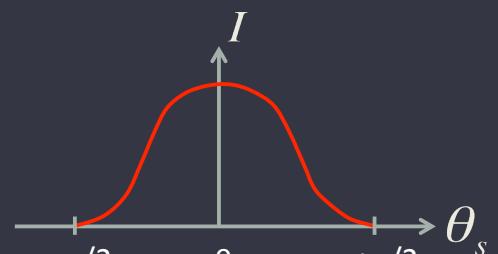
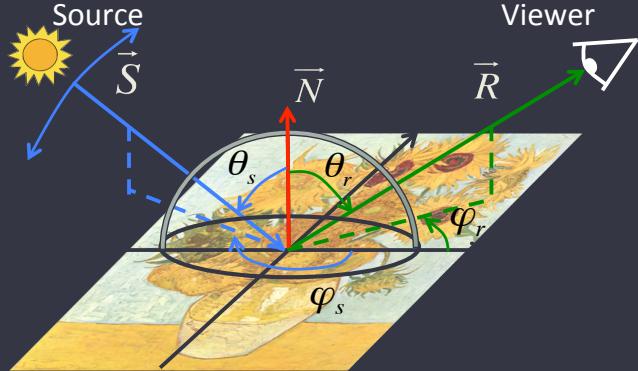
θ_s : light source elevation angle
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Rendering equation:

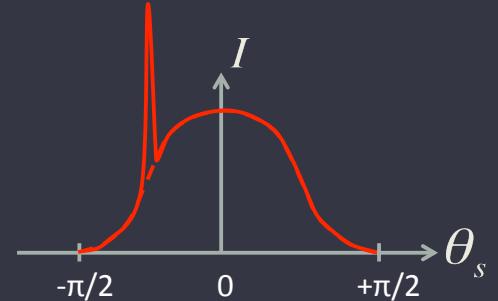
$$L_\lambda(\vec{R}) = \int_{\vec{S} \in \mathbf{S}} L_\lambda(\vec{S}) f_\lambda(\vec{S}, \vec{R})(\vec{N} \cdot \vec{S}) d\vec{S}$$

S : set of light sources
 $L_\lambda(\vec{R})$: reflected radiance at point of reflection
 $L_\lambda(\vec{S})$: incident radiance at point of reflection
 $f_\lambda(\vec{S}, \vec{R})$: bidirectional reflectance distribution function (BRDF)
 \vec{N} : surface normal vector
 λ : wavelength

Surface reflectance: illustration



Diffuse (Lambertian)



Diffuse + specular

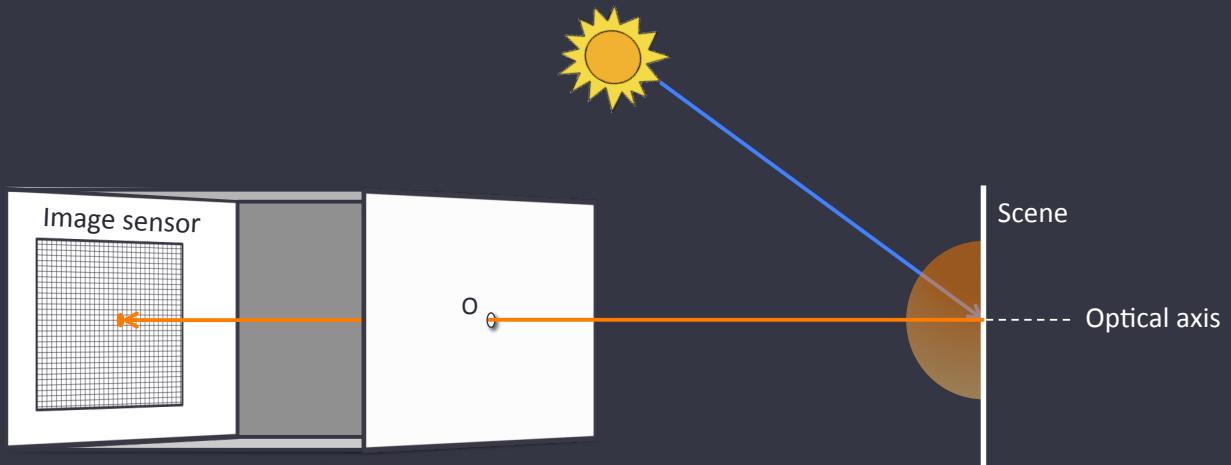
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Camera technology: the pinhole camera



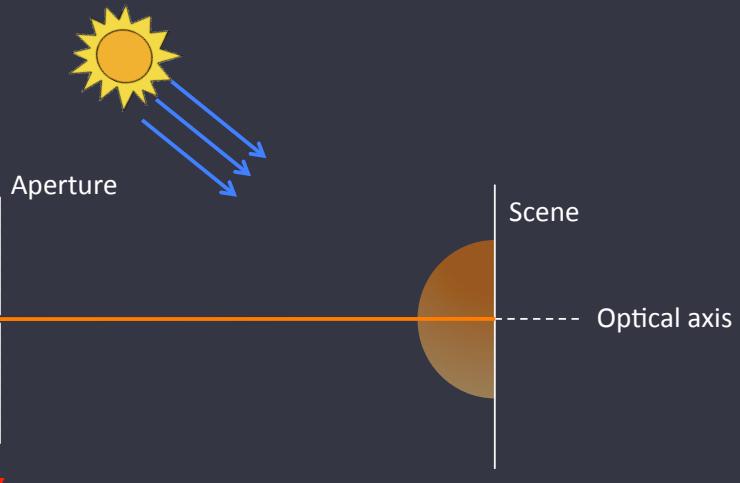
Anthony Browne Rosy at Elkington Park 4/75

Pinhole camera

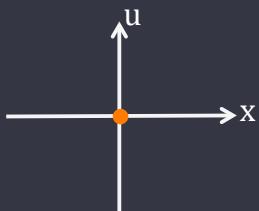


Pinhole camera

Image sensor

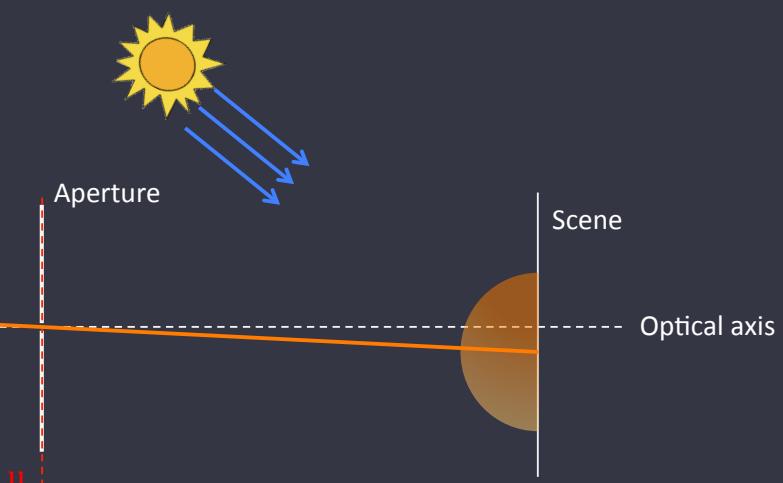


- Ray-space parameterization:

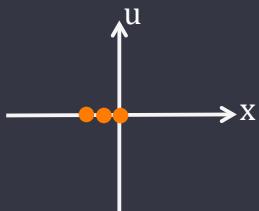


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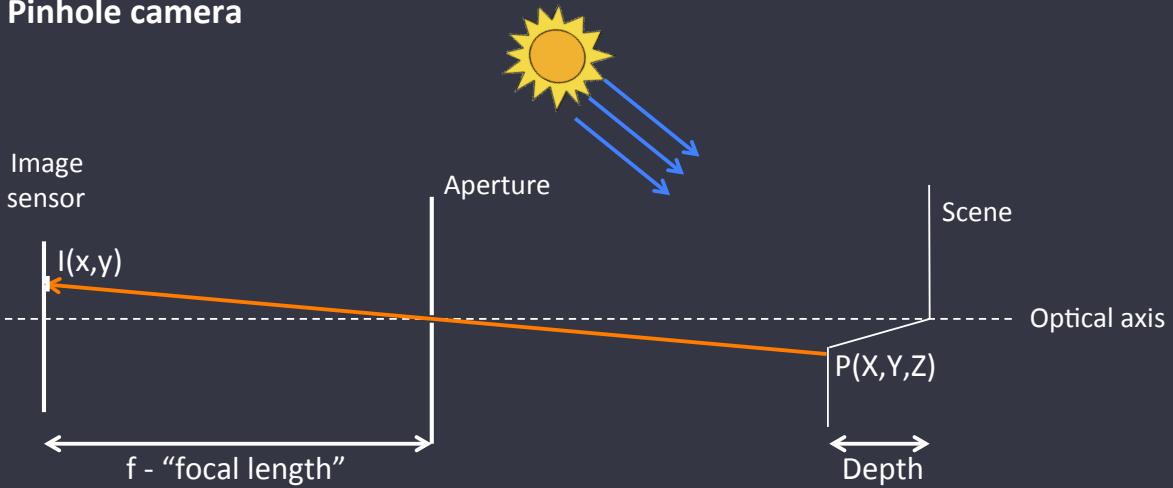


- Ray-space parameterization:



- Each light ray gives one image point

Pinhole camera

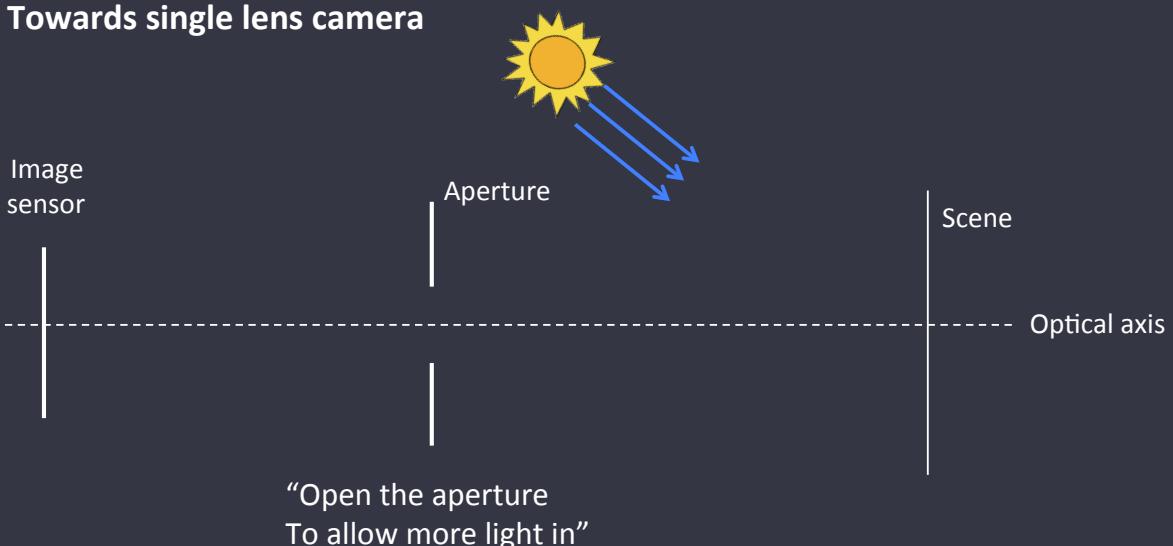


- Change of depth in the scene
- All-in-focus image (\sim infinite depth of field)
- Mapping from 3D to 2D:

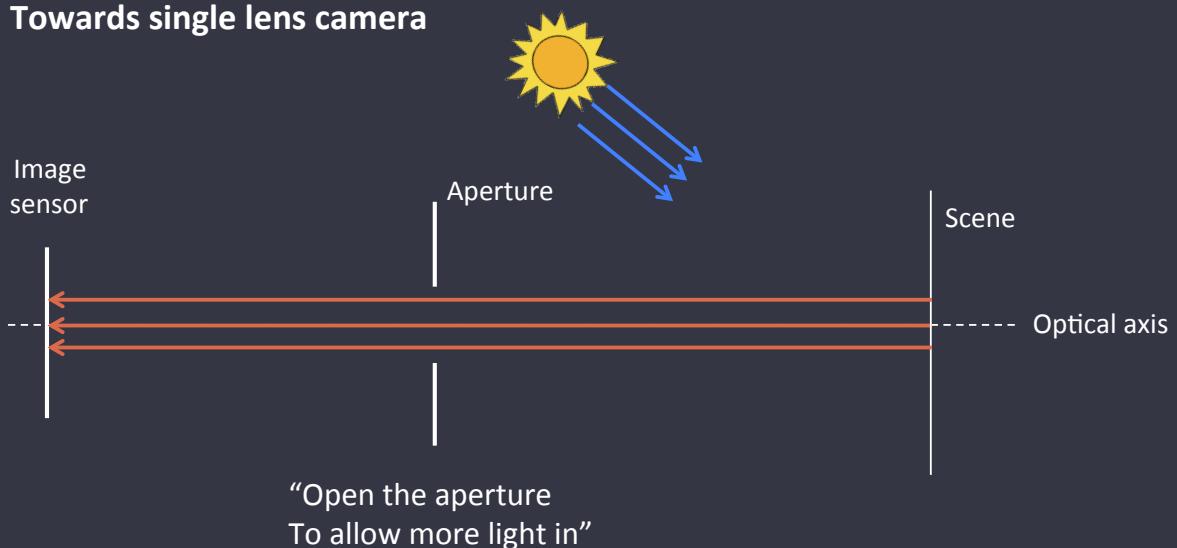
$$x = \frac{f \cdot X}{Z}; y = \frac{f \cdot Y}{Z}$$

Problem: the amount of light hitting the sensor is very low, i.e. long exposure time

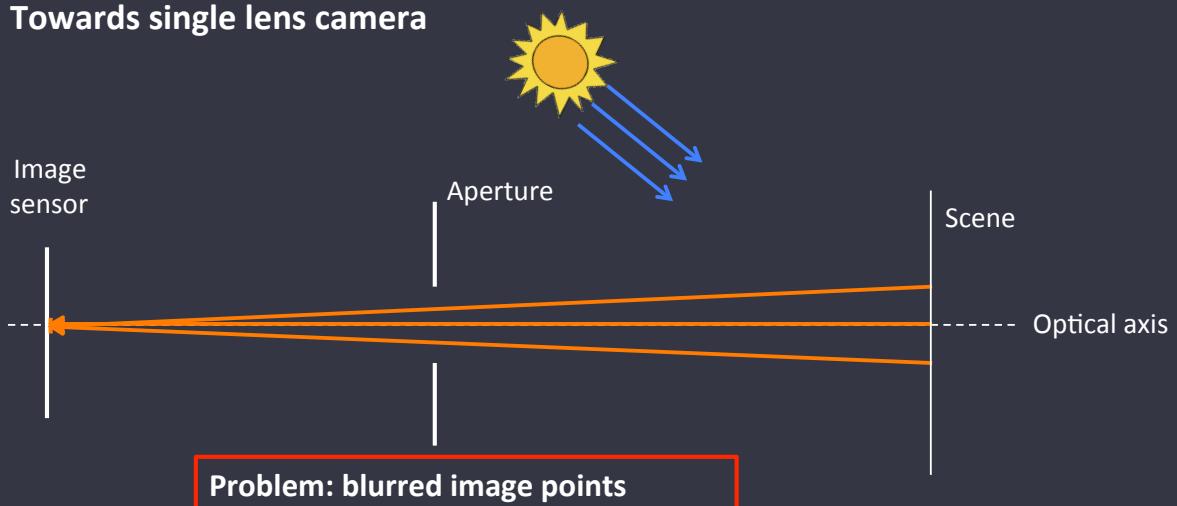
Towards single lens camera



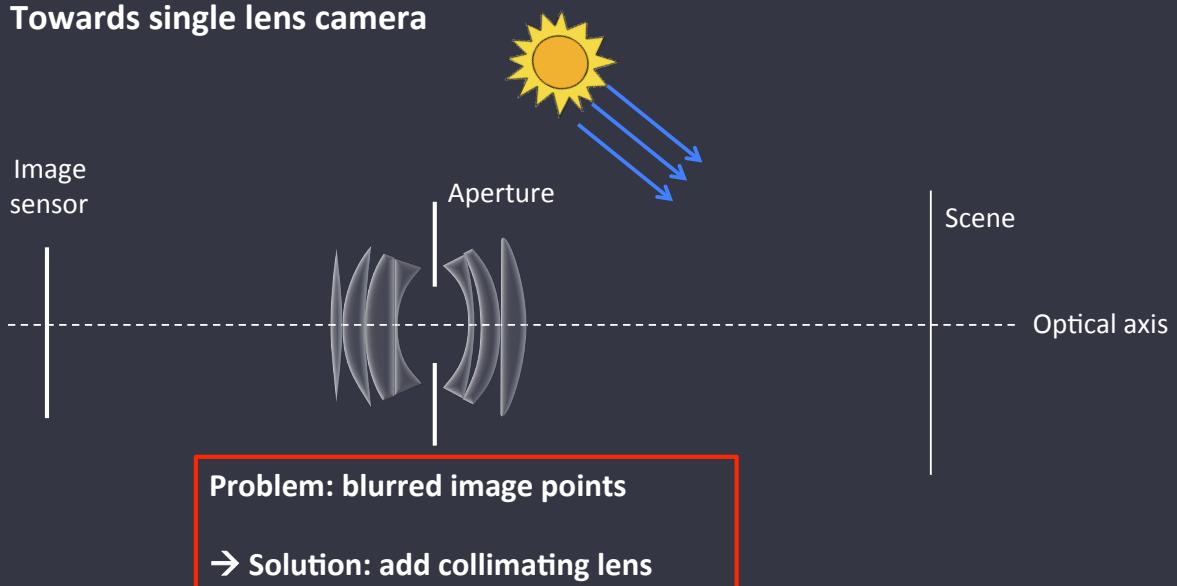
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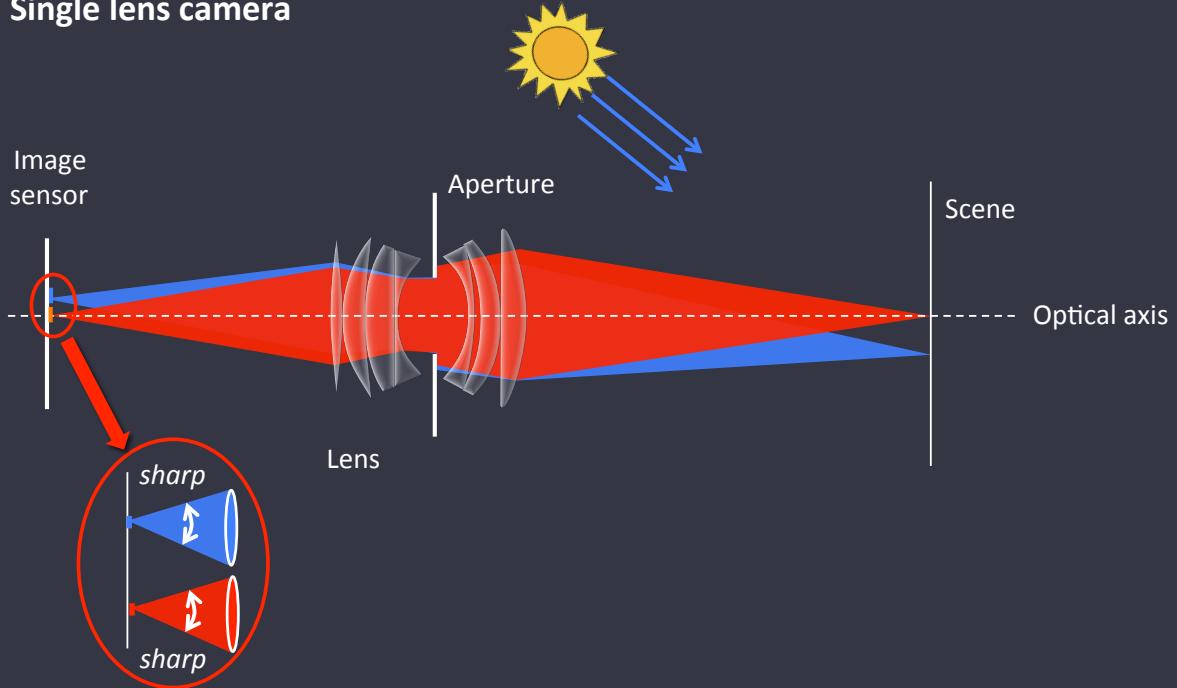
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Towards single lens camera

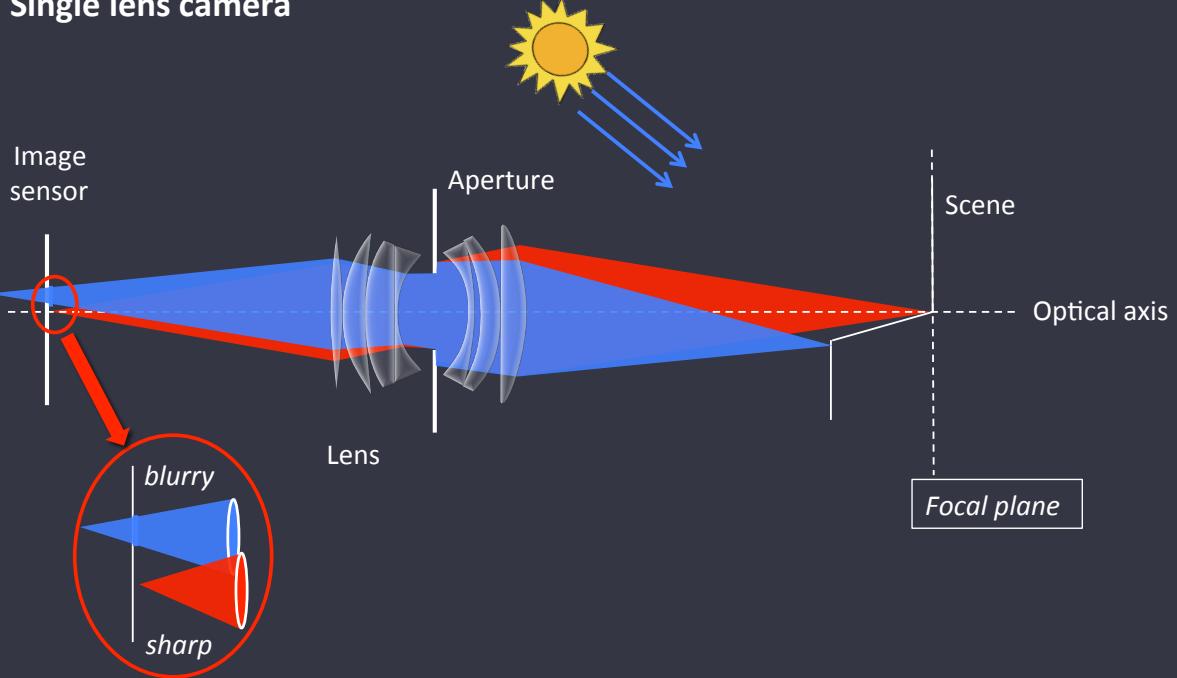


Single lens camera



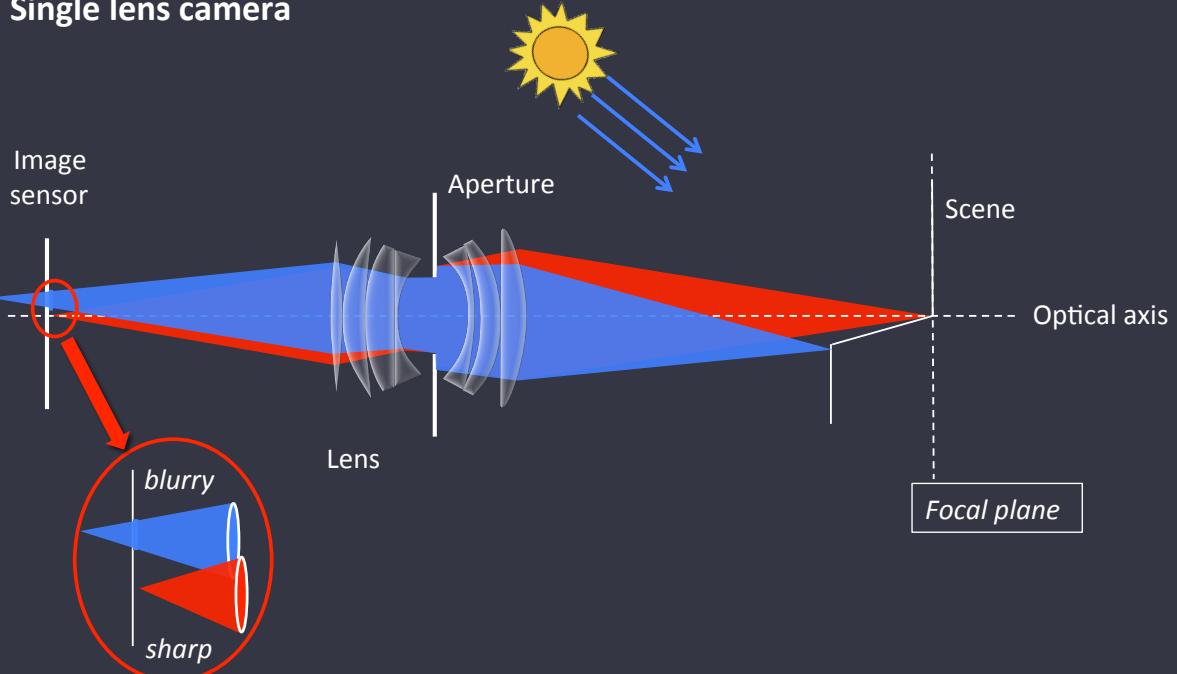
- Each image point receives more light → lower exposure time
- Image sensor integrates the incoming light cone

Single lens camera



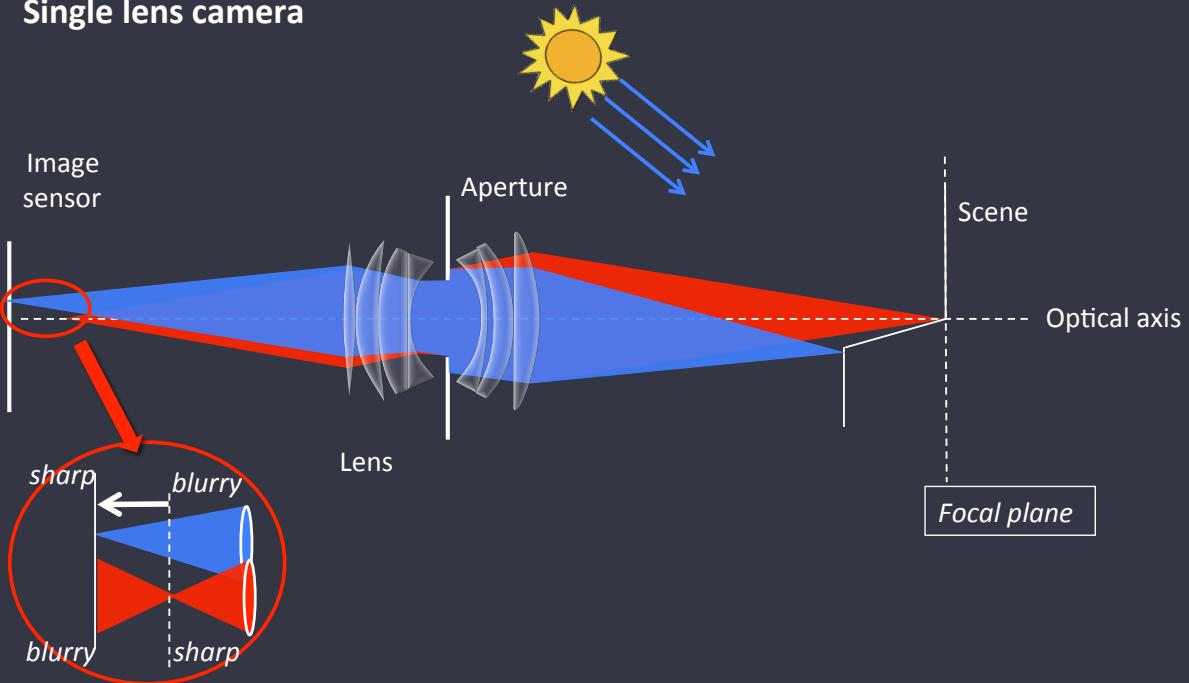
→ *Focal plane*: points in object space that are focused by lens on image sensor

Single lens camera



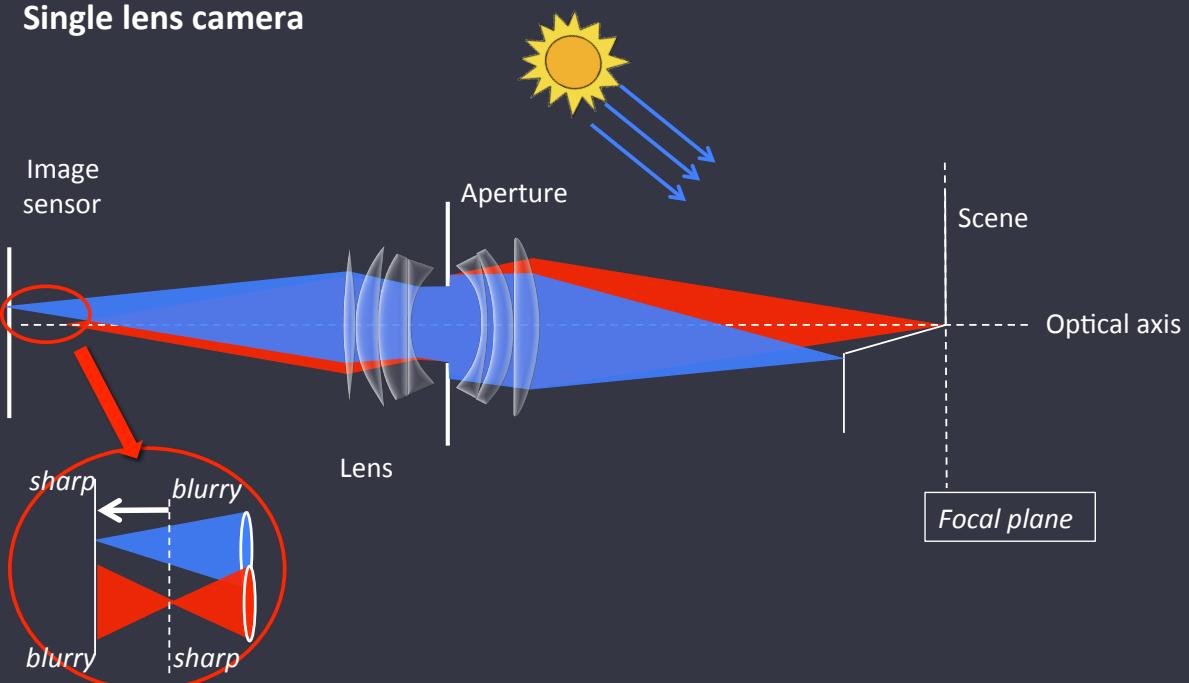
How to change the focal plane?

Single lens camera



How to change the focal plane? By translating the position of the image sensor

Single lens camera

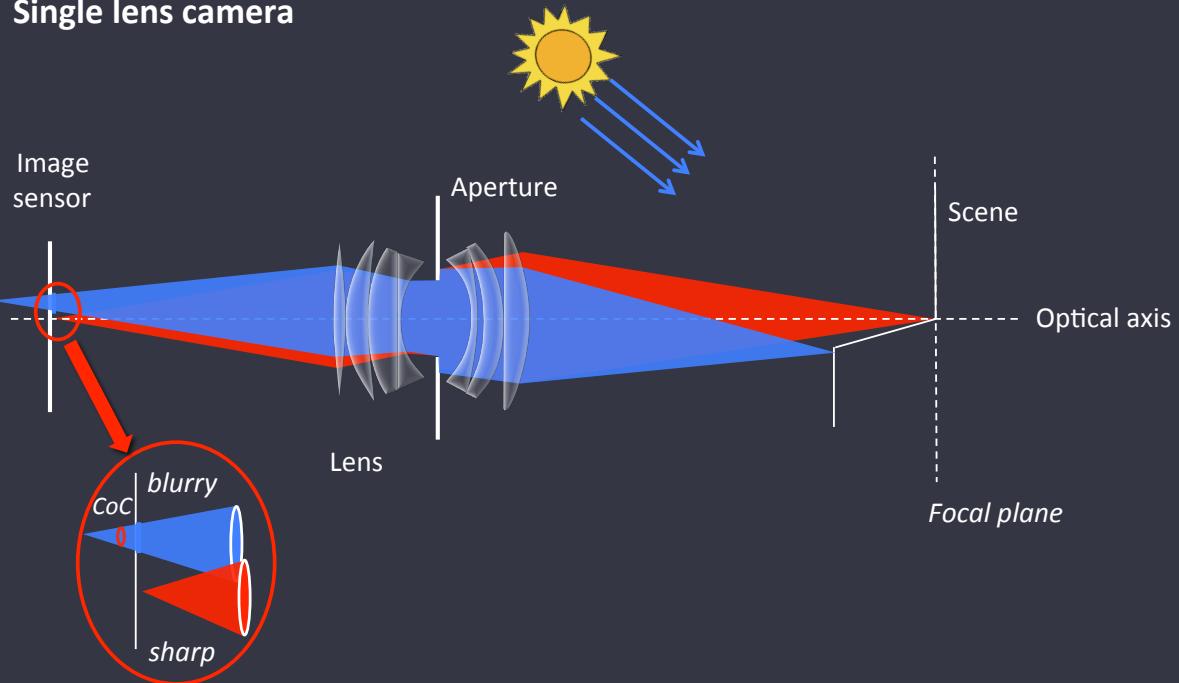


How to change the focal plane? By translating the position of the image sensor

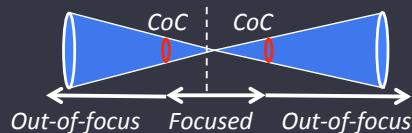
In other words:

- When refocusing, the light field propagates differently in the camera before acquisition,
- The image sensor captures the same light field, sheared differently for each focal plane.

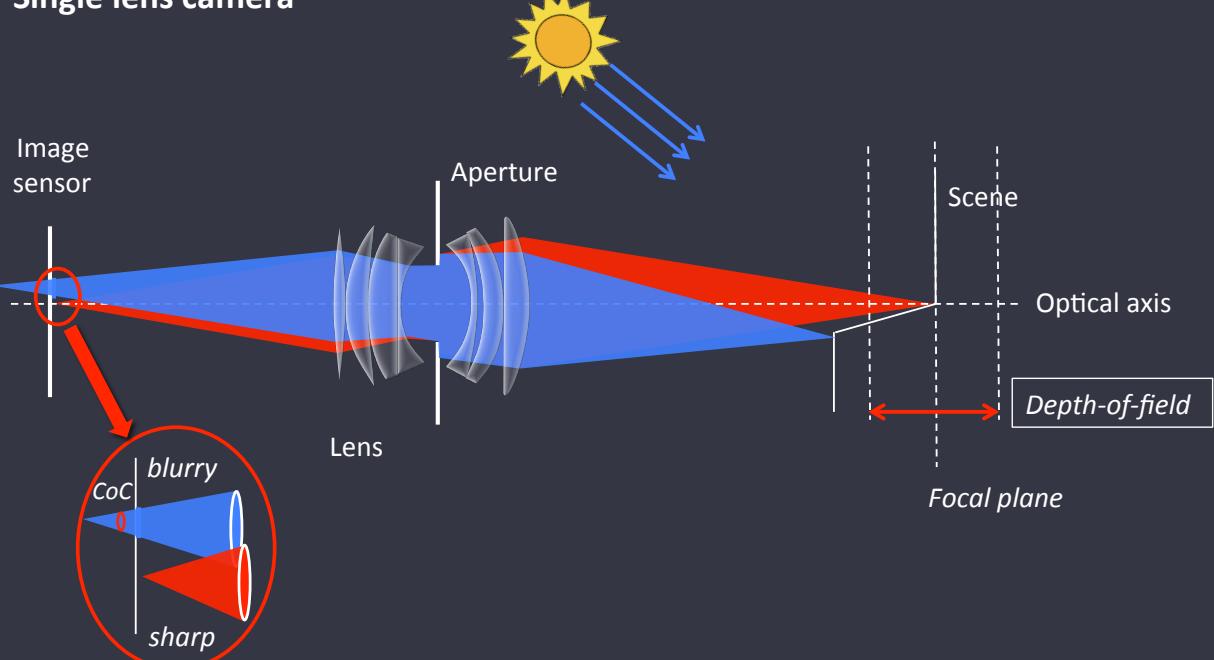
Single lens camera



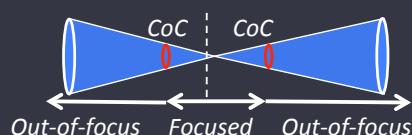
→ *Circle of Confusion (CoC)*: amount of acceptable blur for a point to be focused .



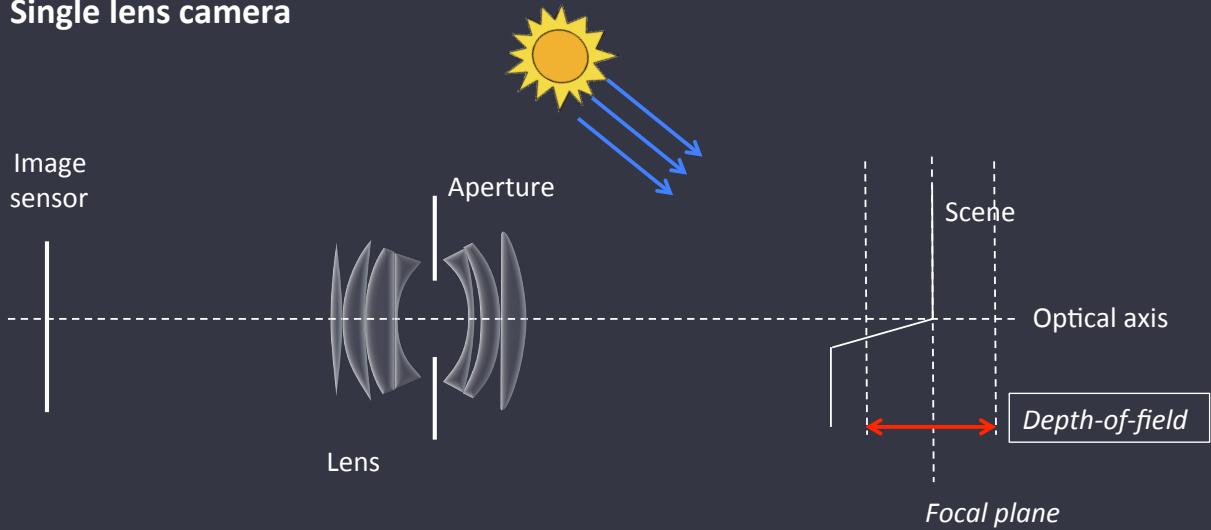
Single lens camera



→ *Depth-of-field*: part of the scene that is focused well enough on image sensor.

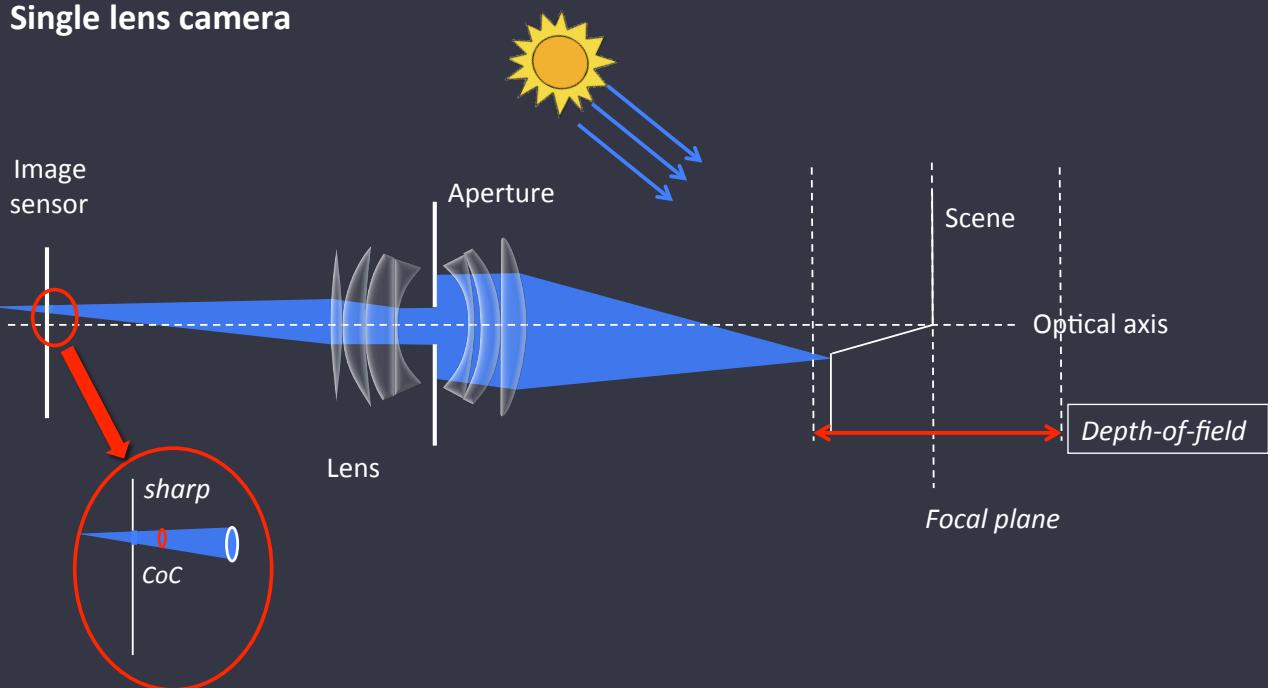


Single lens camera



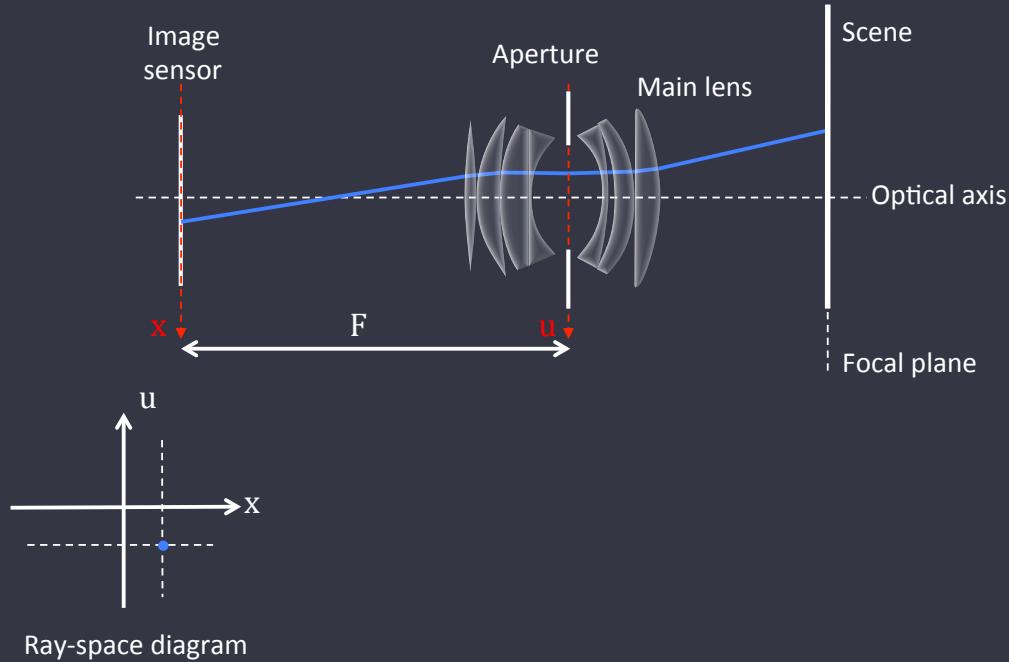
How to change the depth-of-field?

Single lens camera

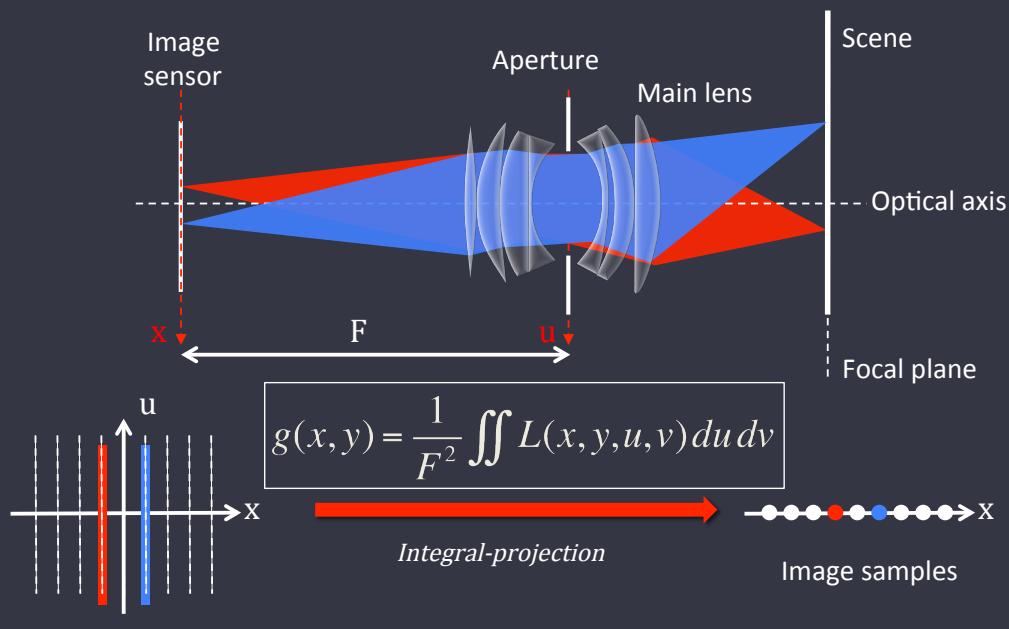


- Closing the aperture:
 1. Extends the depth of field,
 2. Reduces the amount of light,
 3. Increases the exposure time.

Single lens camera: ray-space analysis



Single lens camera: ray-space analysis



Ray-space diagram

Integral Projection Operator

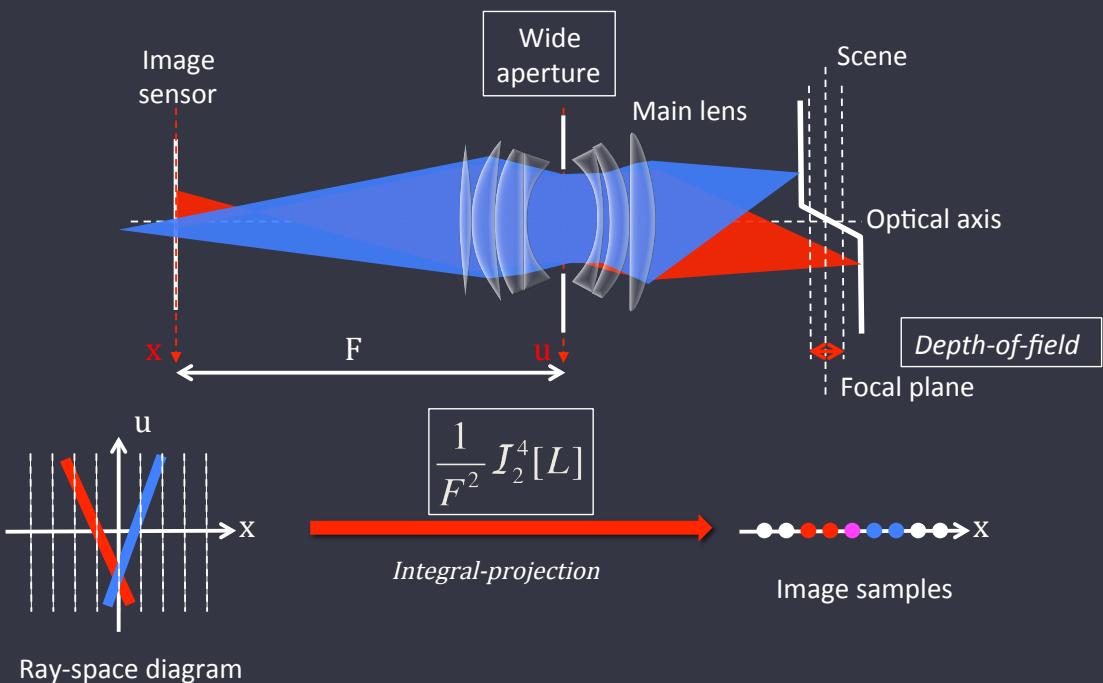
- Let \mathcal{I}_M^N be the canonical projection operator that reduces an N -dimensional function down to M -dimensions by integrating the last $N-M$ dimensions [Ng06]:

$$\boxed{\mathcal{I}_M^N[f](x_1, \dots, x_N) = \int f(x_1, \dots, x_N) dx_{M+1} \dots dx_N}$$

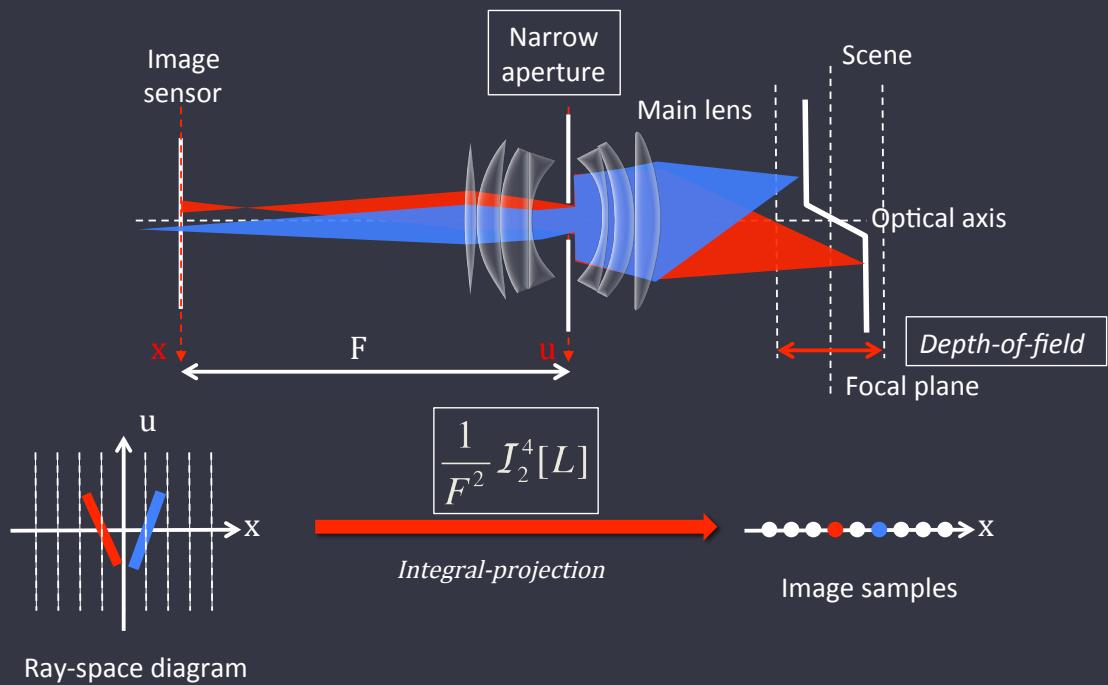
- The photographic imaging operator of single lens camera can thus be written as:

$$\boxed{g(x, y) = \frac{1}{F^2} \iint L(x, y, u, v) du dv \\ = \frac{1}{F^2} \mathcal{I}_2^4[L]}$$

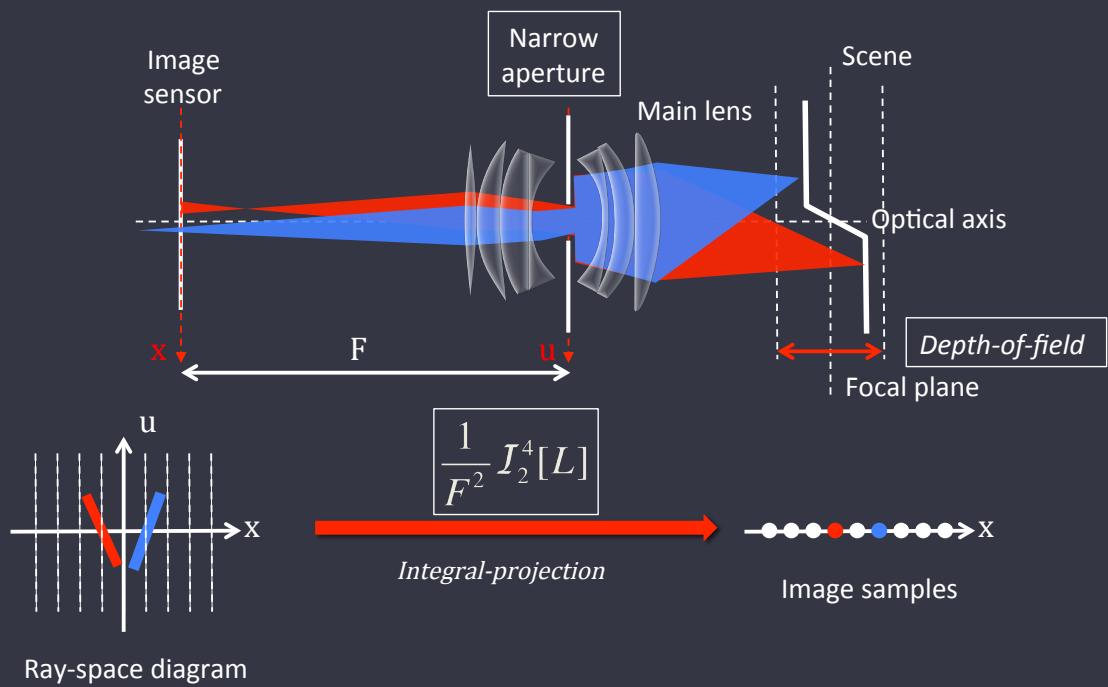
Single lens camera: ray-space analysis



Single lens camera: ray-space analysis



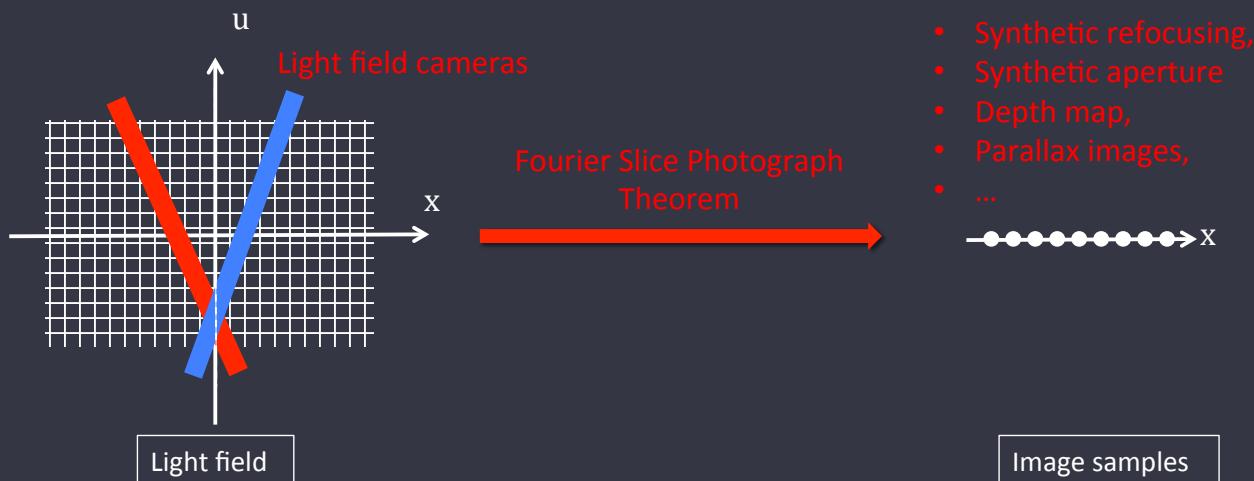
Single lens camera: ray-space analysis



Can we actually capture the light field itself?

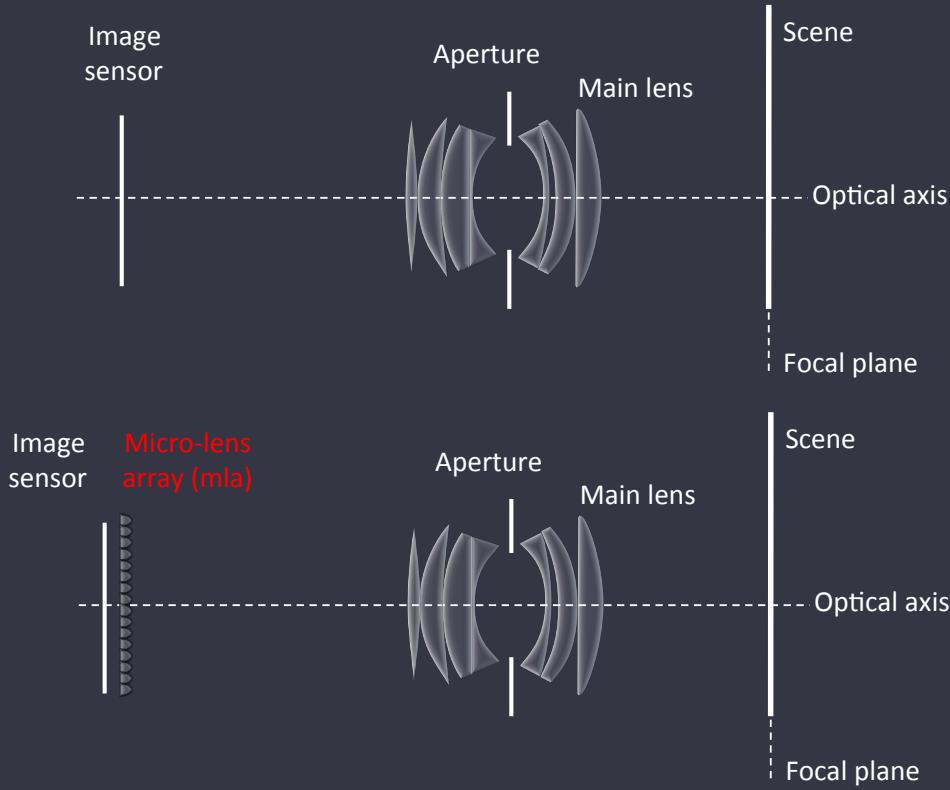
- Part 1: Light, optics & light-field concepts
- Part 2: Traditional camera technology
- Part 3: Light field camera technology
- Part 4: Sampling & processing the light field

Beyond single lens camera



- How to acquire directly the light-field?
- What is an efficient image generation process?
- What advantages can be gained in terms of rendered images?

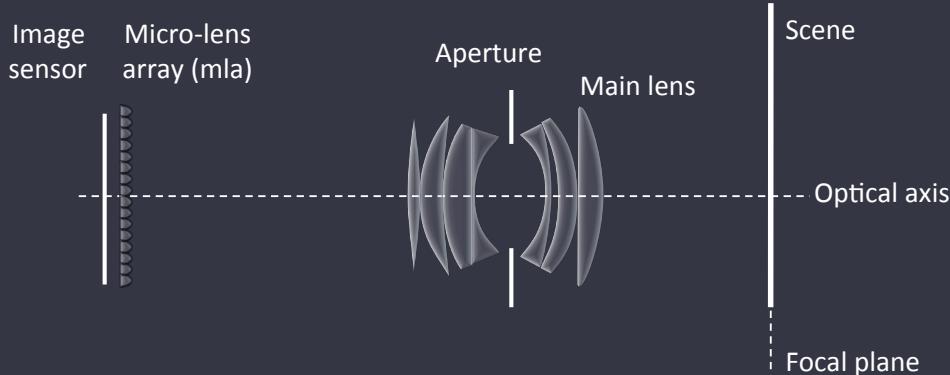
Single lens camera vs. Light field camera



Light field camera

Same user-settings as in single lens cameras:

- Aperture
- Focus
- Zoom (if available)
- Exposure time
- Sensor sensitivity (ISO)



Micro-lens array and light field camera



Microlens array (Ng06)

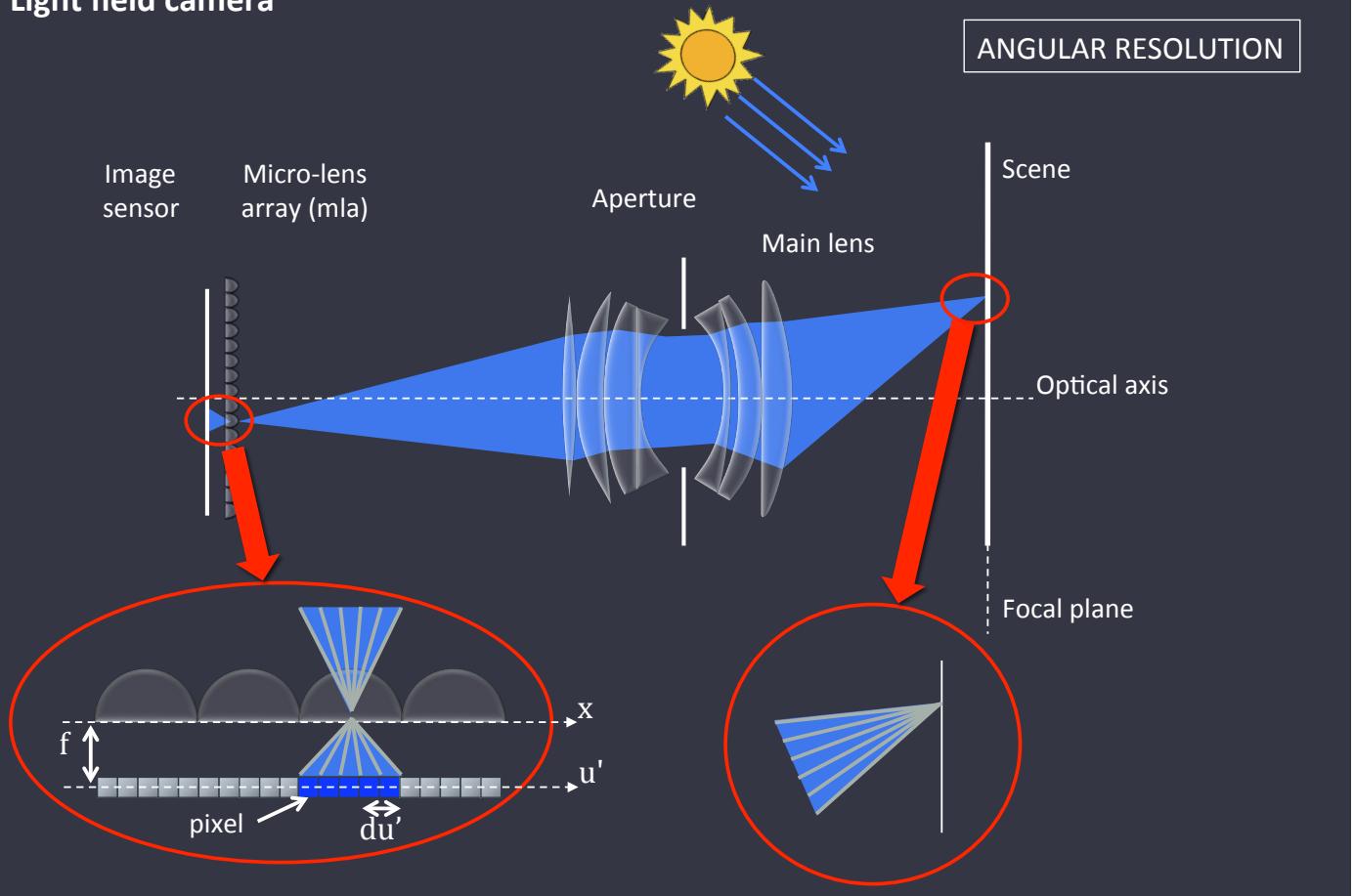


Lytro

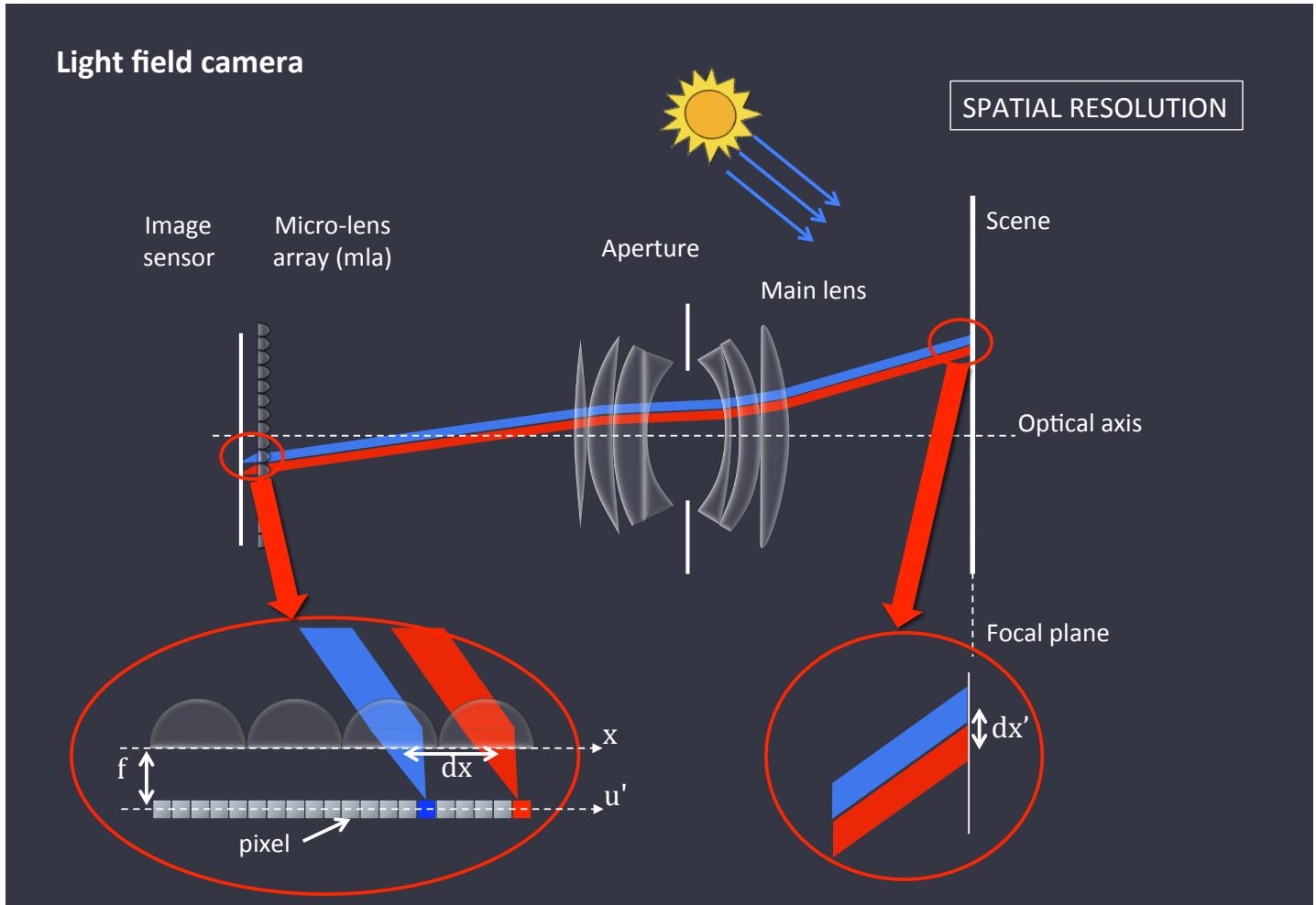


Raytrix

Light field camera

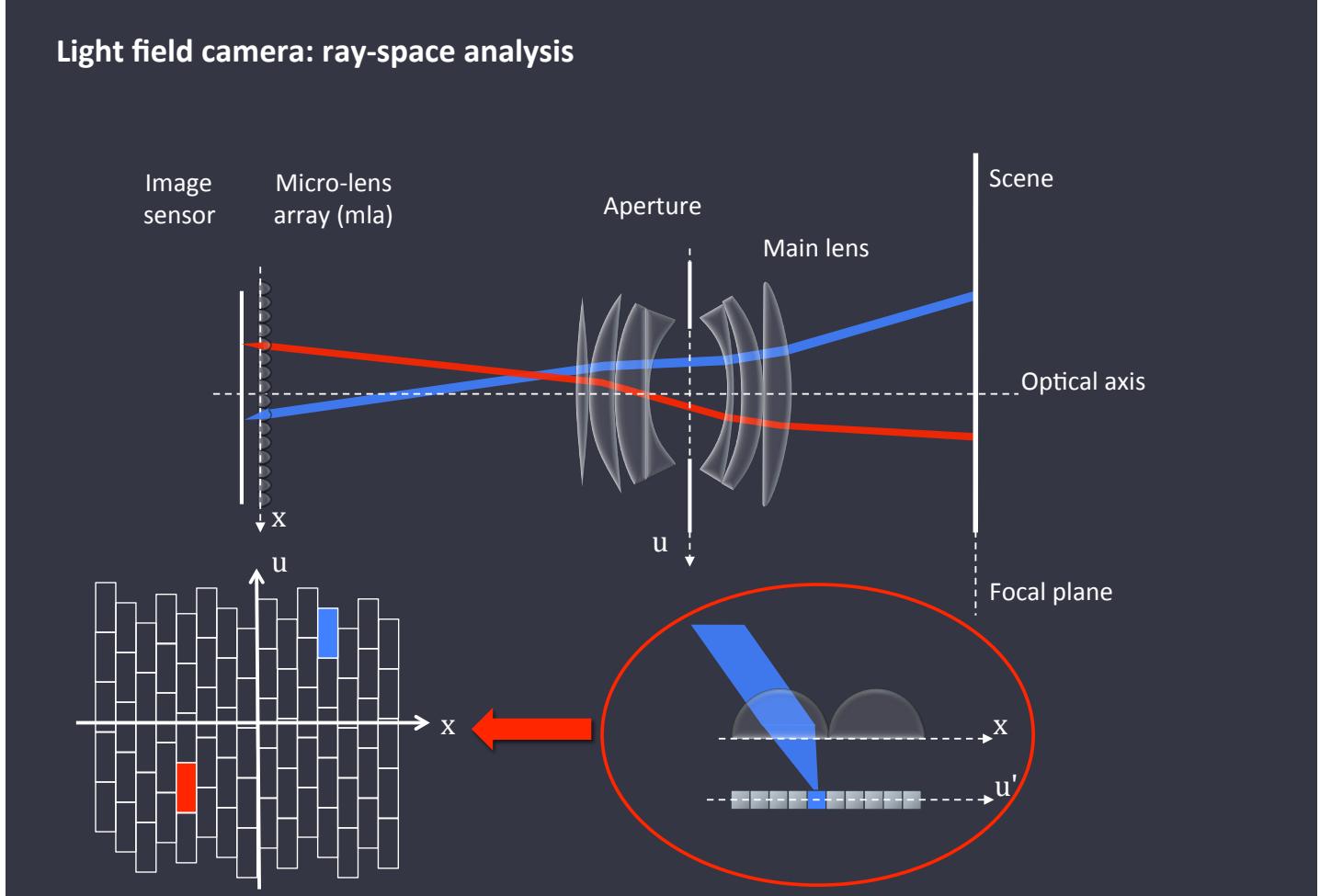


Light field camera

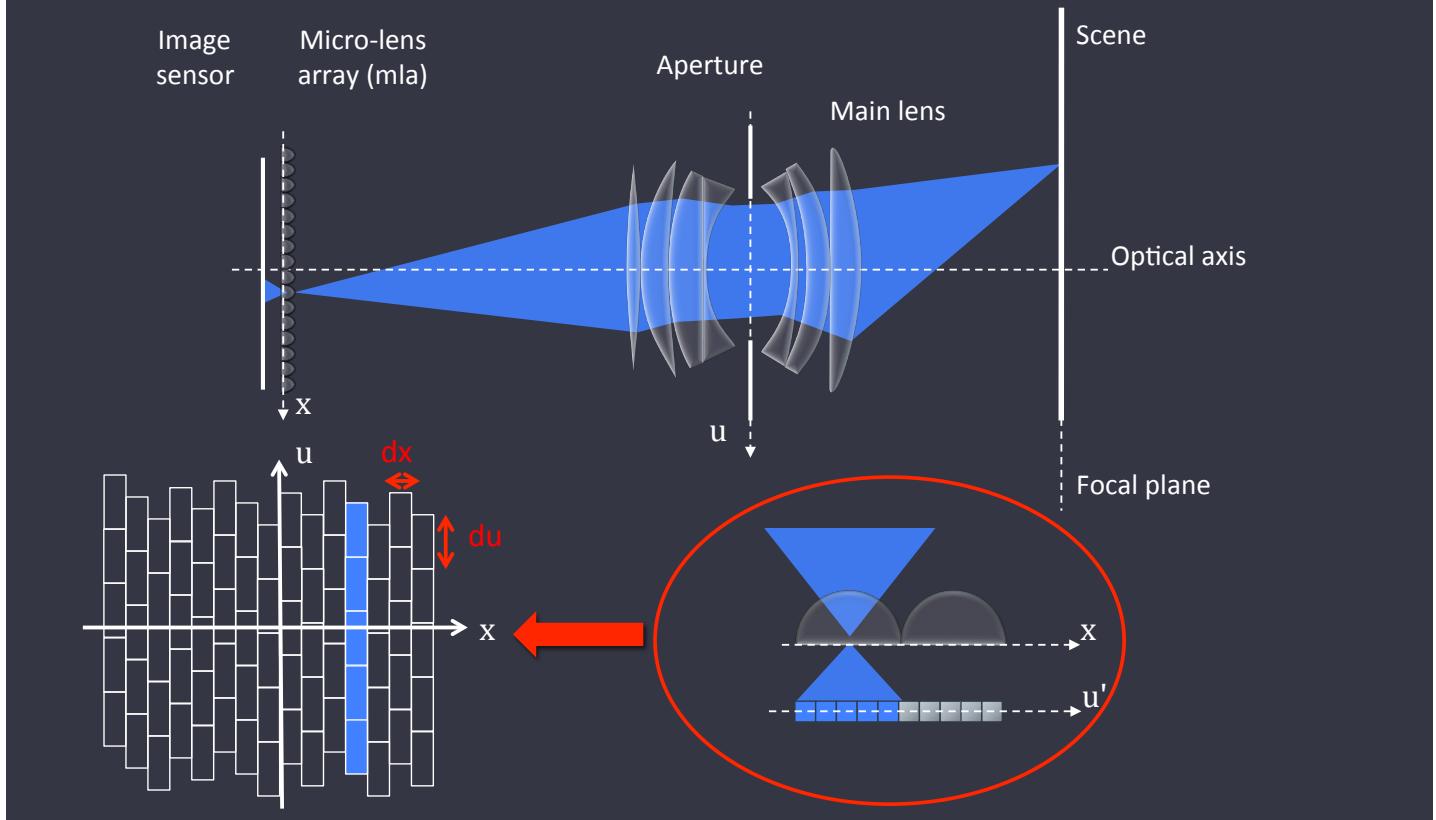


SPATIAL RESOLUTION

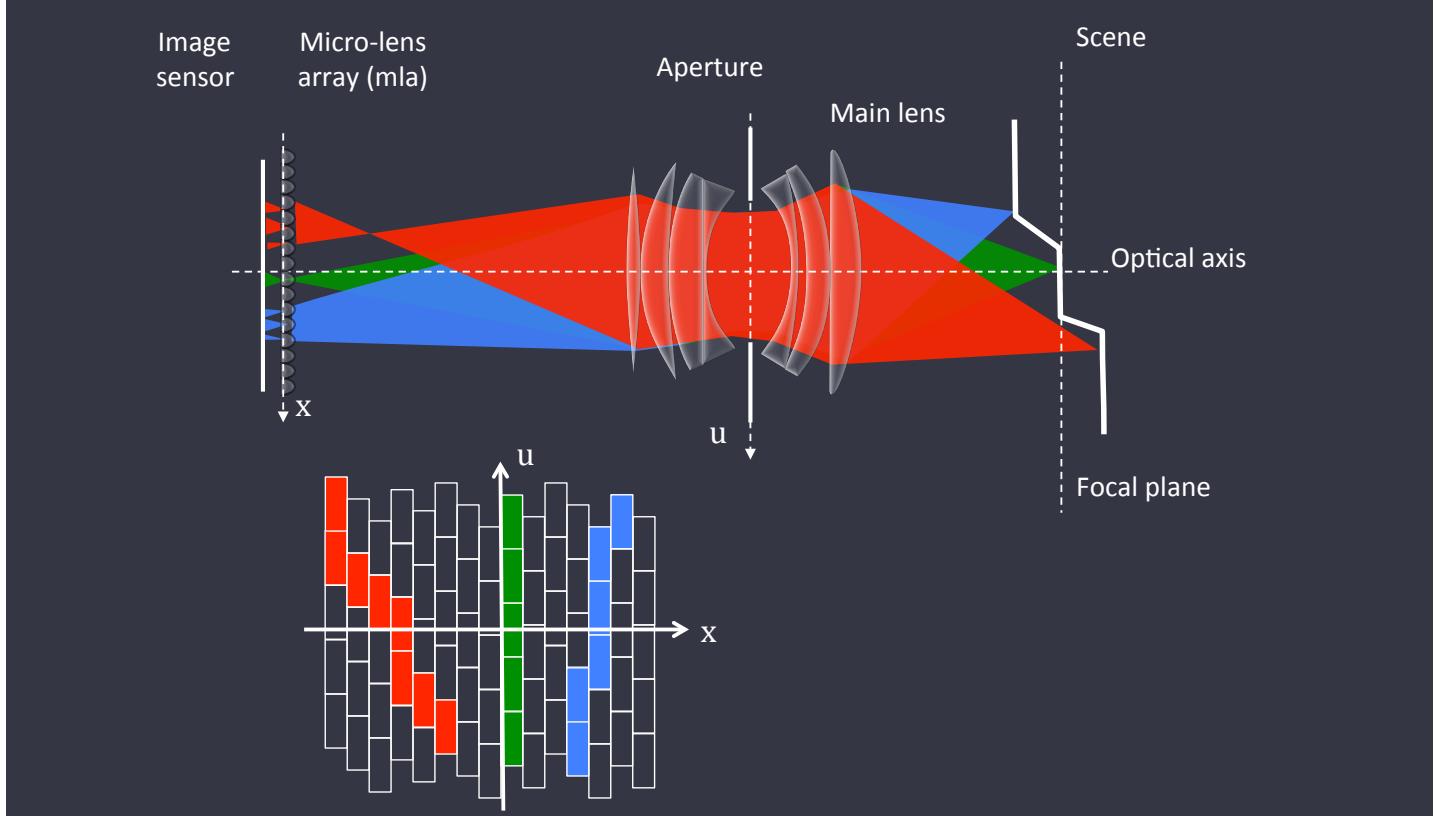
Light field camera: ray-space analysis



Light field camera: ray-space analysis

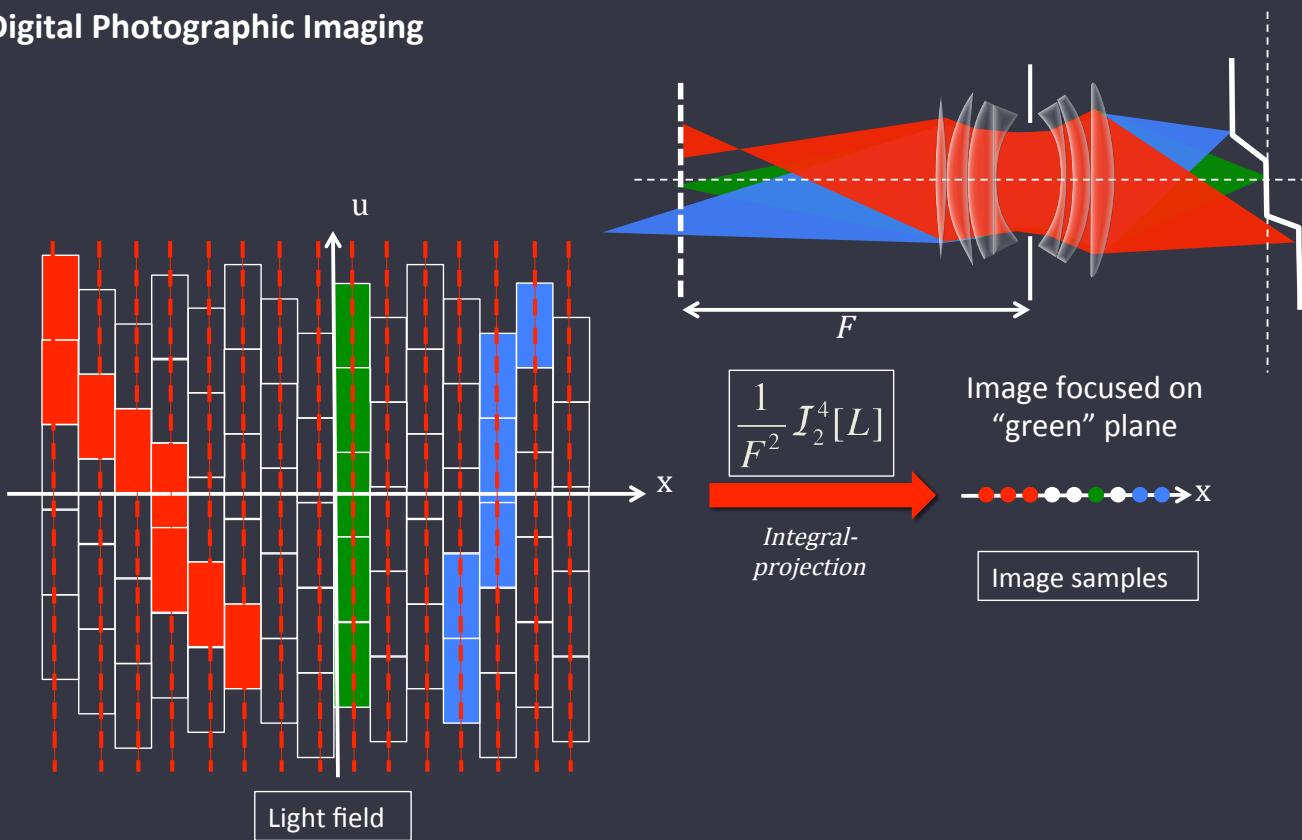


Light-field camera: ray-space analysis

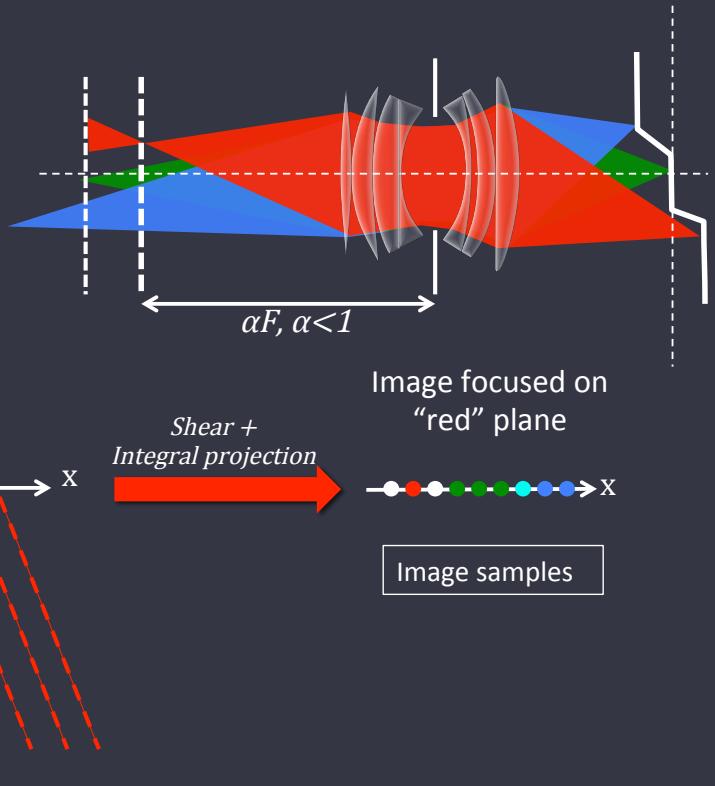
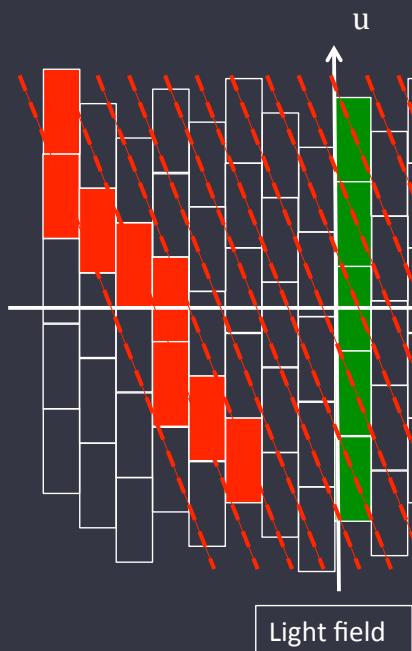


- Part 1: Light, optics & light-field concepts
- Part 2: Traditional camera technology
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- Part 4: Sampling & processing the light field

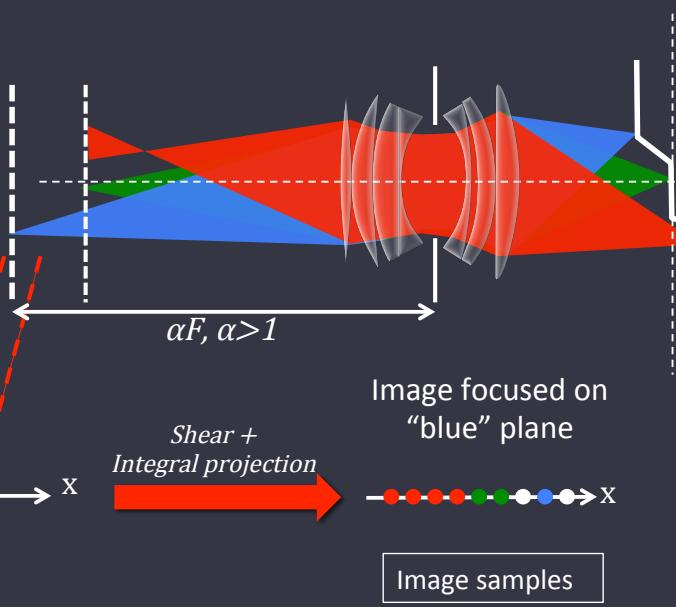
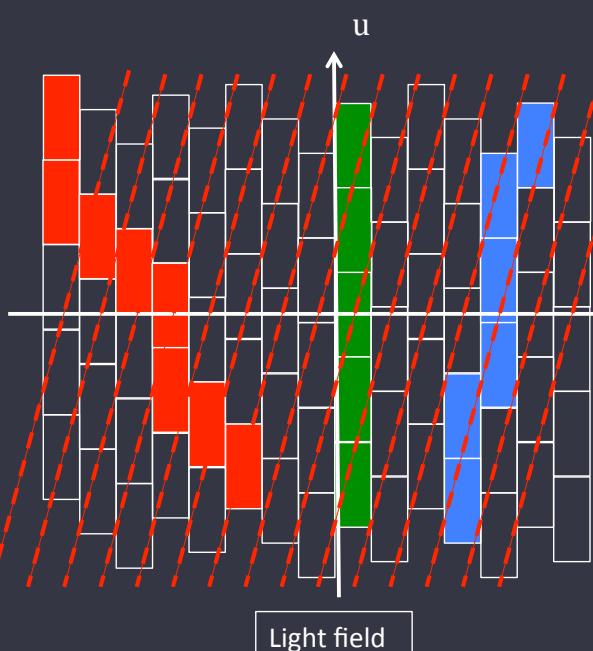
Digital Photographic Imaging



Digital Photographic Imaging



Digital Photographic Imaging



Imaging Operator of a light field camera

- Let \mathcal{B}_α be the shear operator that propagates the light-field at αF :

$$\boxed{\mathcal{B}_\alpha[f](\mathbf{x}) = f(\mathcal{B}_\alpha^{-1}\mathbf{x})}$$

- The photographic imaging operator \mathcal{P}_α of a light field camera can thus be written as:

$$\boxed{\mathcal{P}_\alpha[L] = \frac{1}{\alpha^2 F^2} I_2^4 \circ \mathcal{B}_\alpha[L]}$$

- How about in the Fourier domain?

Fourier and Slicing Operators

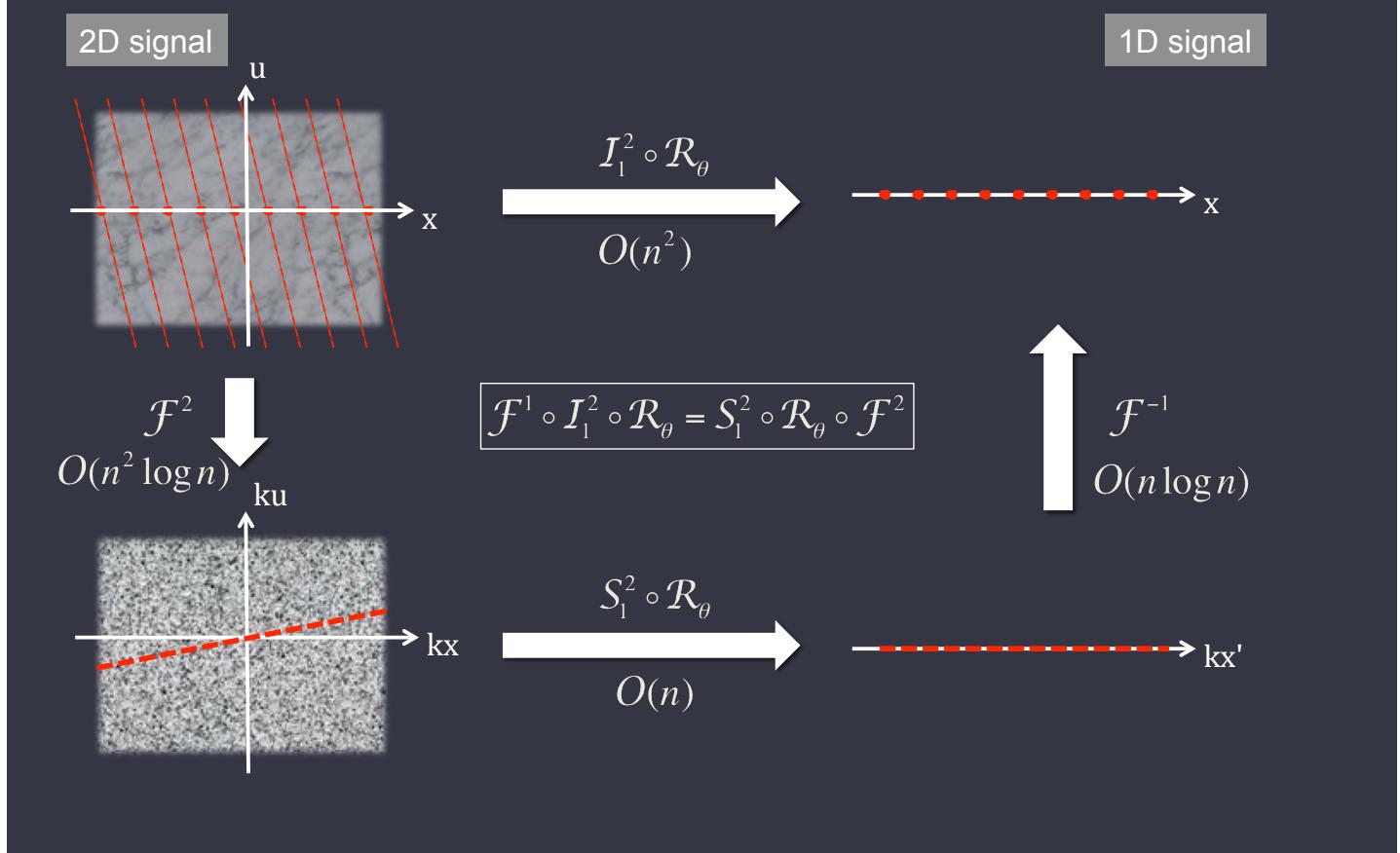
- Let \mathcal{F}^N be the N -dimensional Fourier transform operator of a function and let \mathcal{F}^{-N} be its inverse:

$$\boxed{\mathcal{F}^N[f](\mathbf{u}) = \int f(\mathbf{x}) e^{-2\pi i \mathbf{x} \cdot \mathbf{u}} d\mathbf{x}}$$

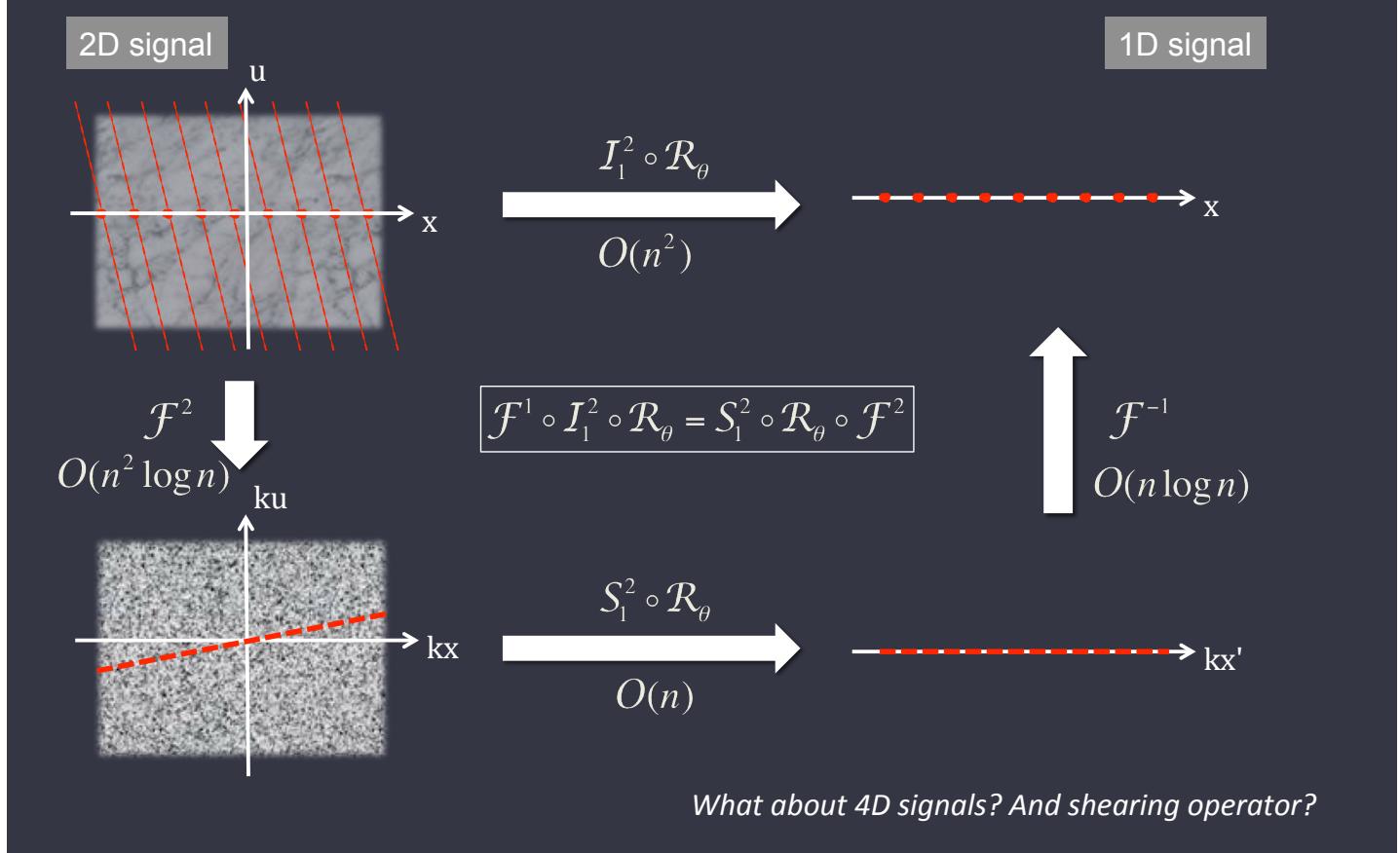
- Let S_M^N be the slicing operator that reduces an N -dimensional function down to an M -dimensional one by setting to zero the last $N-M$ dimensions:

$$\boxed{S_M^N[f](x_1, \dots, x_N) = f(x_1, \dots, x_M, 0, \dots, 0)}$$

Classical Fourier Slice Theorem: illustration



Classical Fourier Slice Theorem: illustration



The Generalized Fourier Slice Theorem

Let f be an N -dimensional function. Then:

- changing the basis of f ,
- integral-projecting it down to M dimensions and
- applying the Fourier transform

is equivalent to:

- applying the Fourier transform to f ,
- changing basis with the normalized inverse transpose of the original basis and
- slicing it down to M dimensions.

The Generalized Fourier Slice Theorem

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- changing the basis of f ,
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is equivalent to:

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- slicing it down to M dimensions.

$$\mathcal{F}^M \circ \mathcal{I}_M^N \circ \mathcal{B} = S_M^N \circ \frac{\mathcal{B}^{-T}}{|\mathcal{B}^{-T}|} \circ \mathcal{F}^N$$

The Fourier Slice Photograph Theorem [Ng06]

- Let recall the photographic operator of a light field camera:

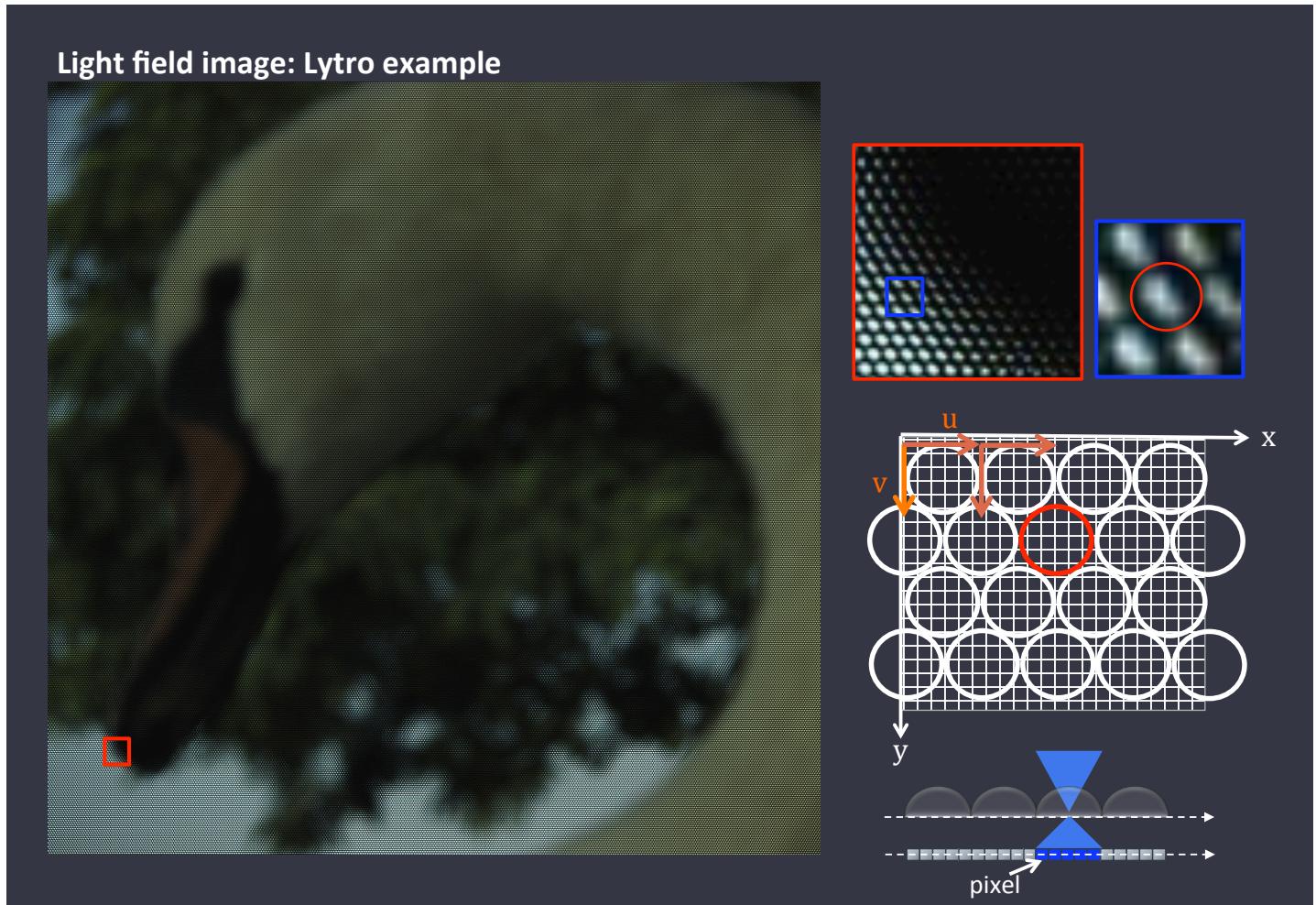
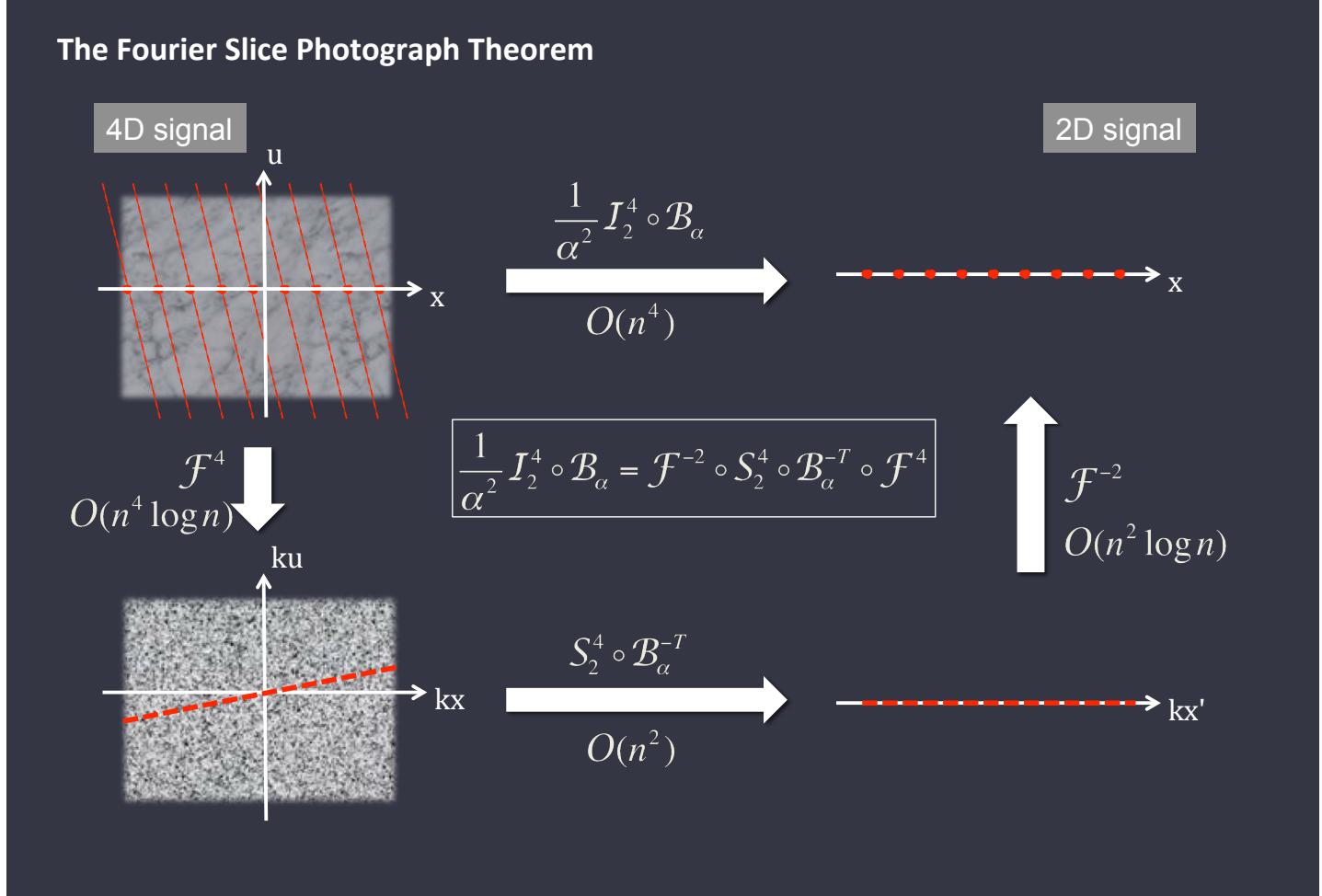
$$\begin{aligned}\mathcal{P}_\alpha[L] &= \frac{1}{\alpha^2 F^2} I_2^4 \circ \mathcal{B}_\alpha[L] \\ &= \frac{1}{\alpha^2 F^2} \mathcal{F}^{-2} \circ \mathcal{F}^2 \circ I_2^4 \circ \mathcal{B}_\alpha[L] \\ &= \frac{1}{\alpha^2 F^2} \mathcal{F}^{-2} \circ S_2^4 \circ \frac{\mathcal{B}_\alpha^{-T}}{|\mathcal{B}_\alpha^{-T}|} \circ \mathcal{F}^4 \\ &= \frac{1}{F^2} \mathcal{F}^{-2} \circ S_2^4 \circ \mathcal{B}_\alpha^{-T} \circ \mathcal{F}^4\end{aligned}$$
$$\mathcal{F}^M \circ I_M^N \circ \mathcal{B} = S_M^N \circ \frac{\mathcal{B}^{-T}}{|\mathcal{B}^{-T}|} \circ \mathcal{F}^N$$
$$|\mathcal{B}_\alpha^{-T}| = |\mathcal{B}_\alpha^{-1}| = 1/\alpha^2$$

The Fourier Slice Photograph Theorem [Ng06]

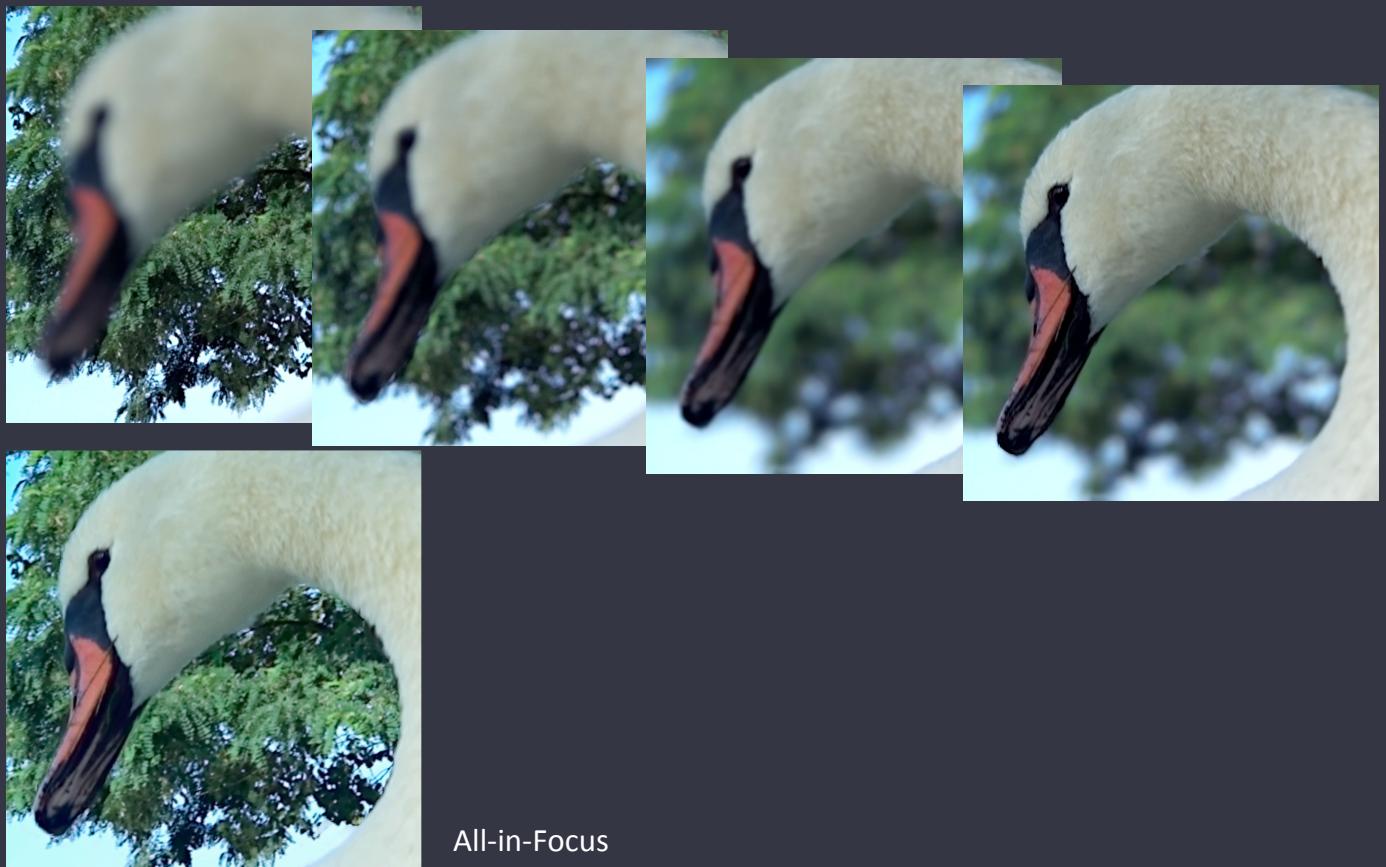
A Photograph is the inverse 2D Fourier transform of a dilated 2D slice in the 4D Fourier transform of the light field

$$\boxed{\mathcal{P}_\alpha = \frac{1}{F^2} \mathcal{F}^{-2} \circ S_2^4 \circ \mathcal{B}_\alpha^{-T} \circ \mathcal{F}^4}$$

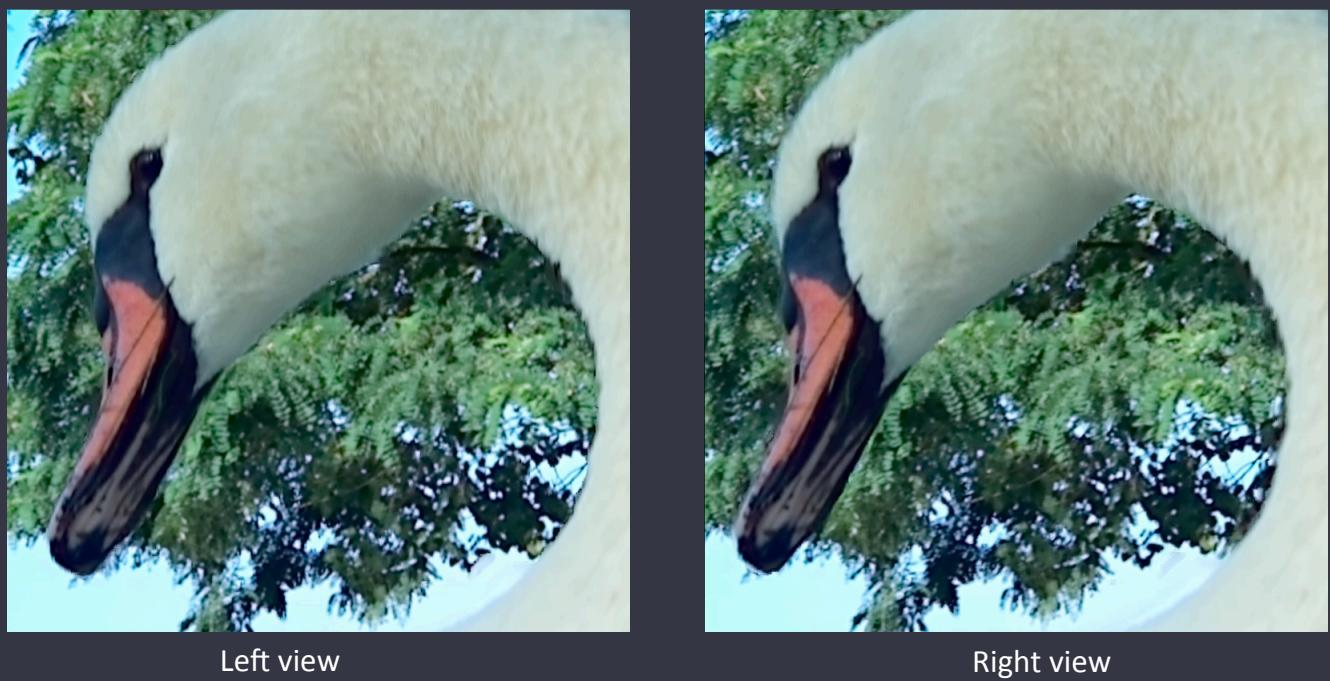
The Fourier Slice Photograph Theorem



Light field image: digital refocusing



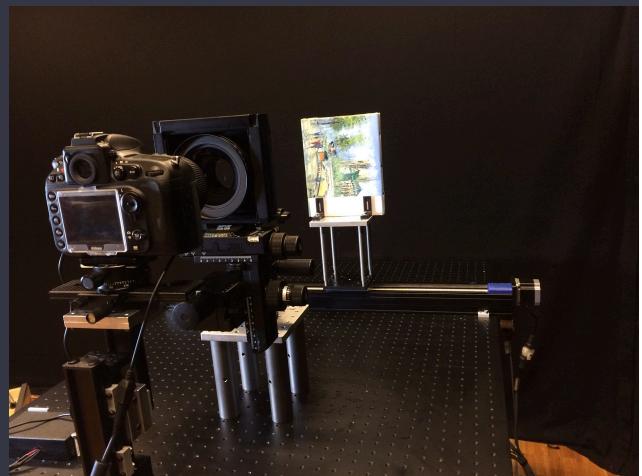
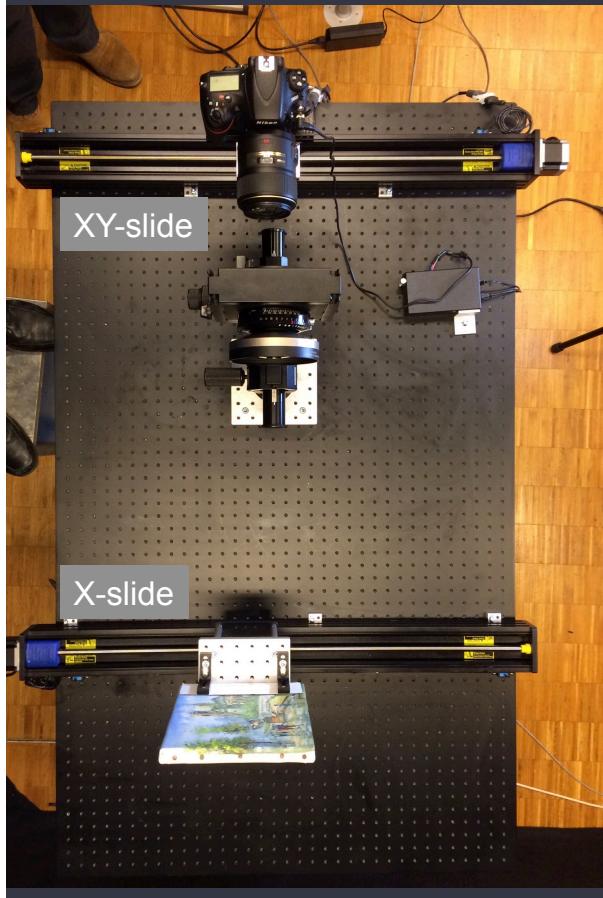
Light field image: perspective shift



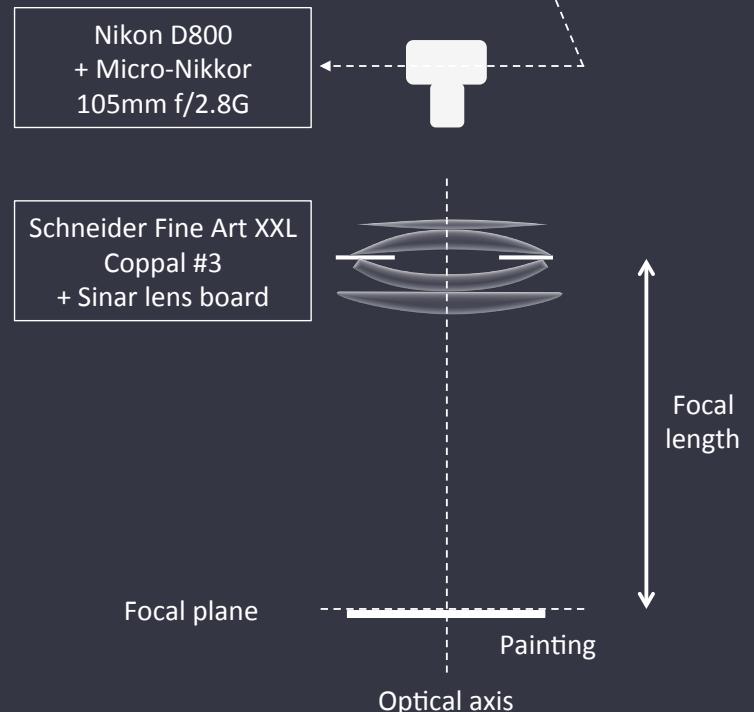
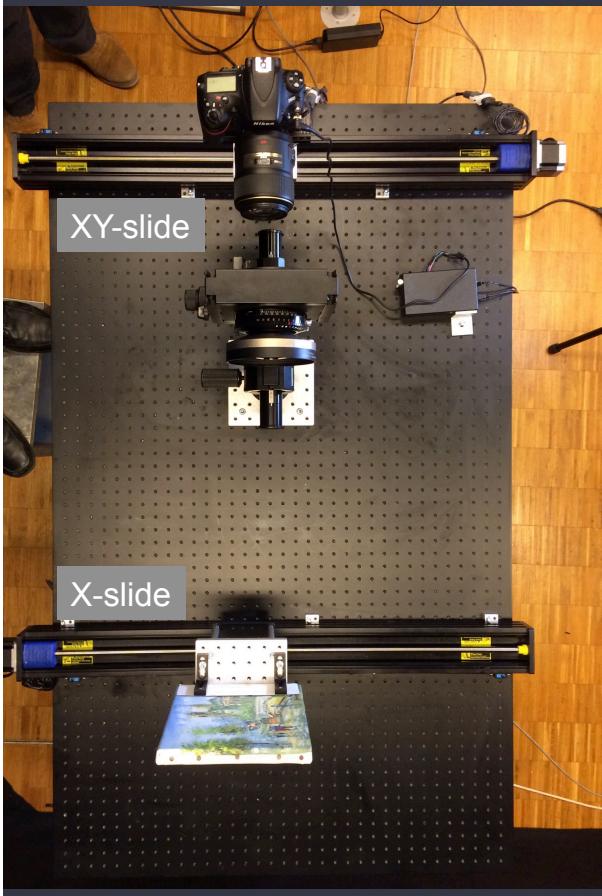
Light field image: depth map



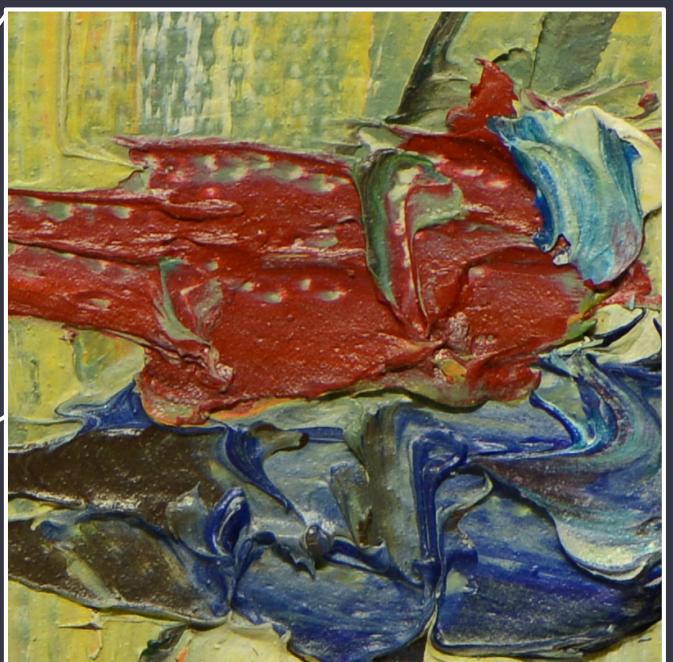
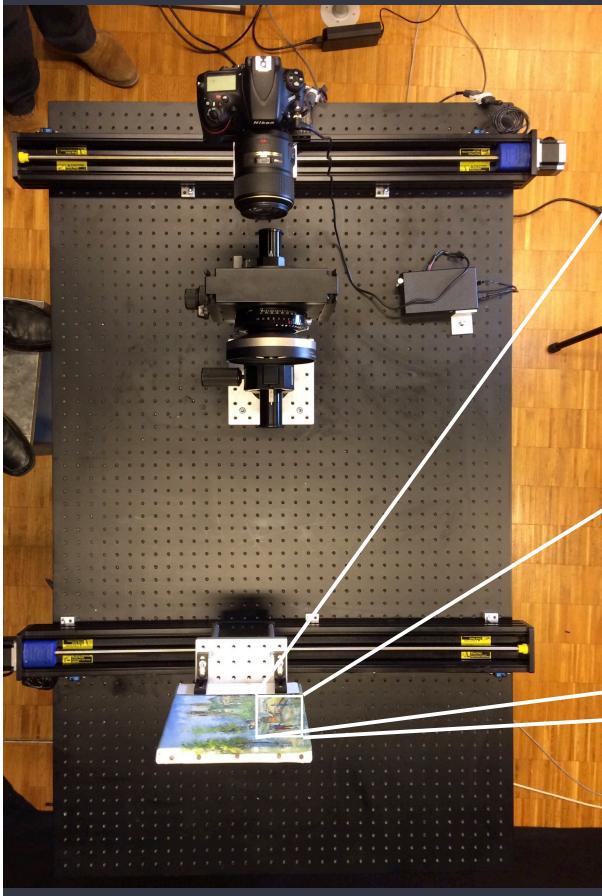
Custom light-field camera



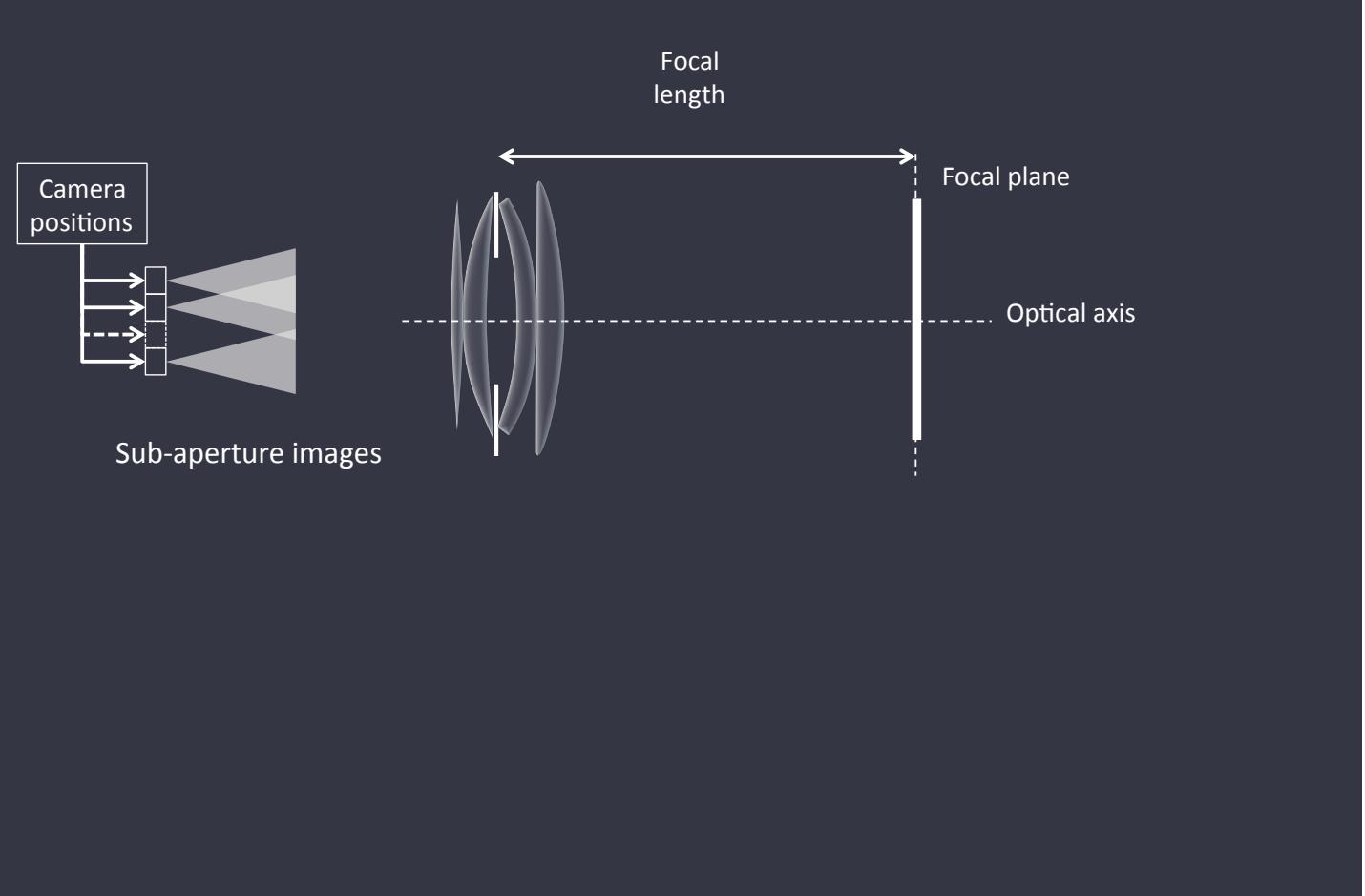
Custom light-field camera



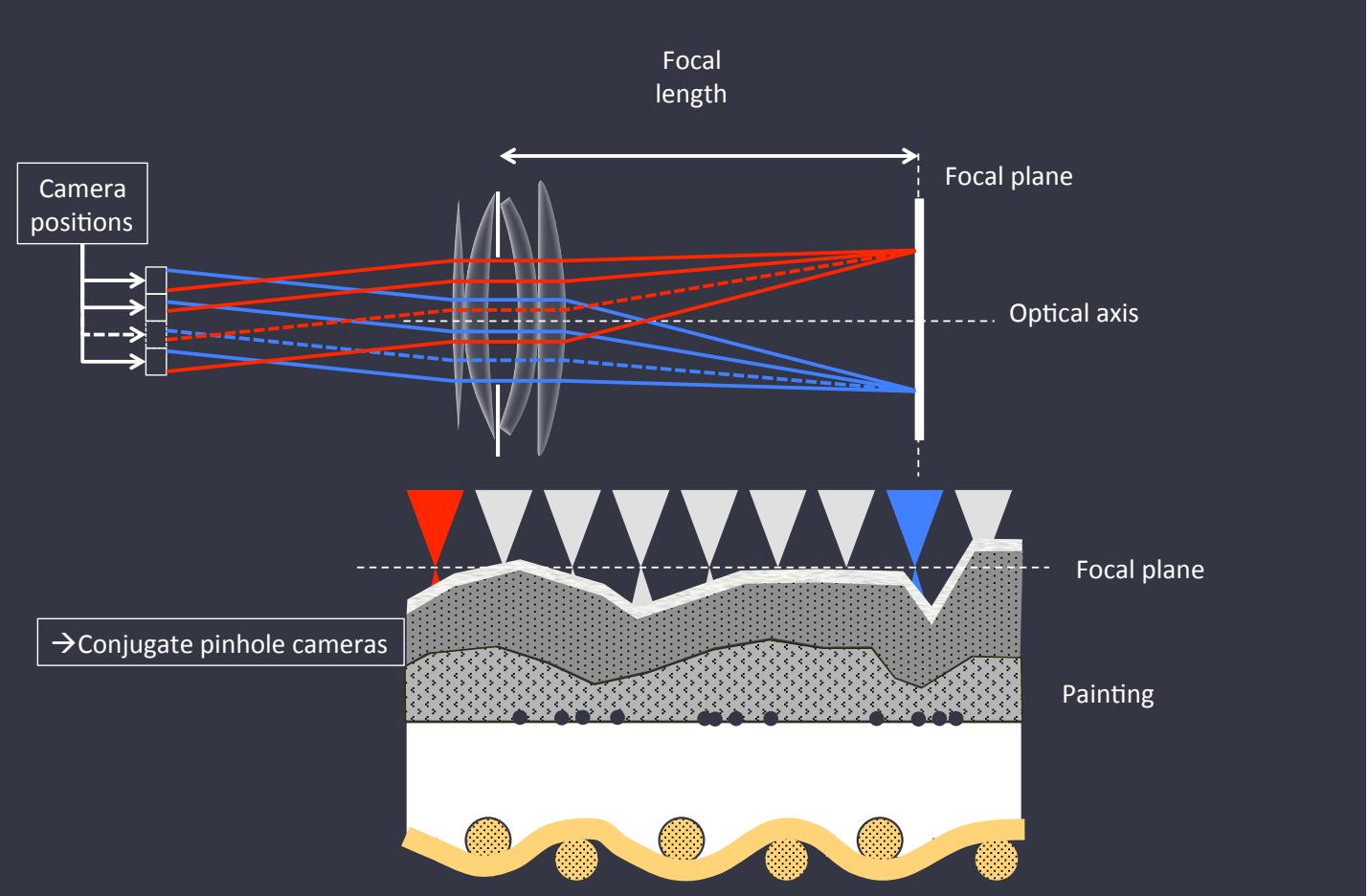
Acquiring the 4D light field of a painting



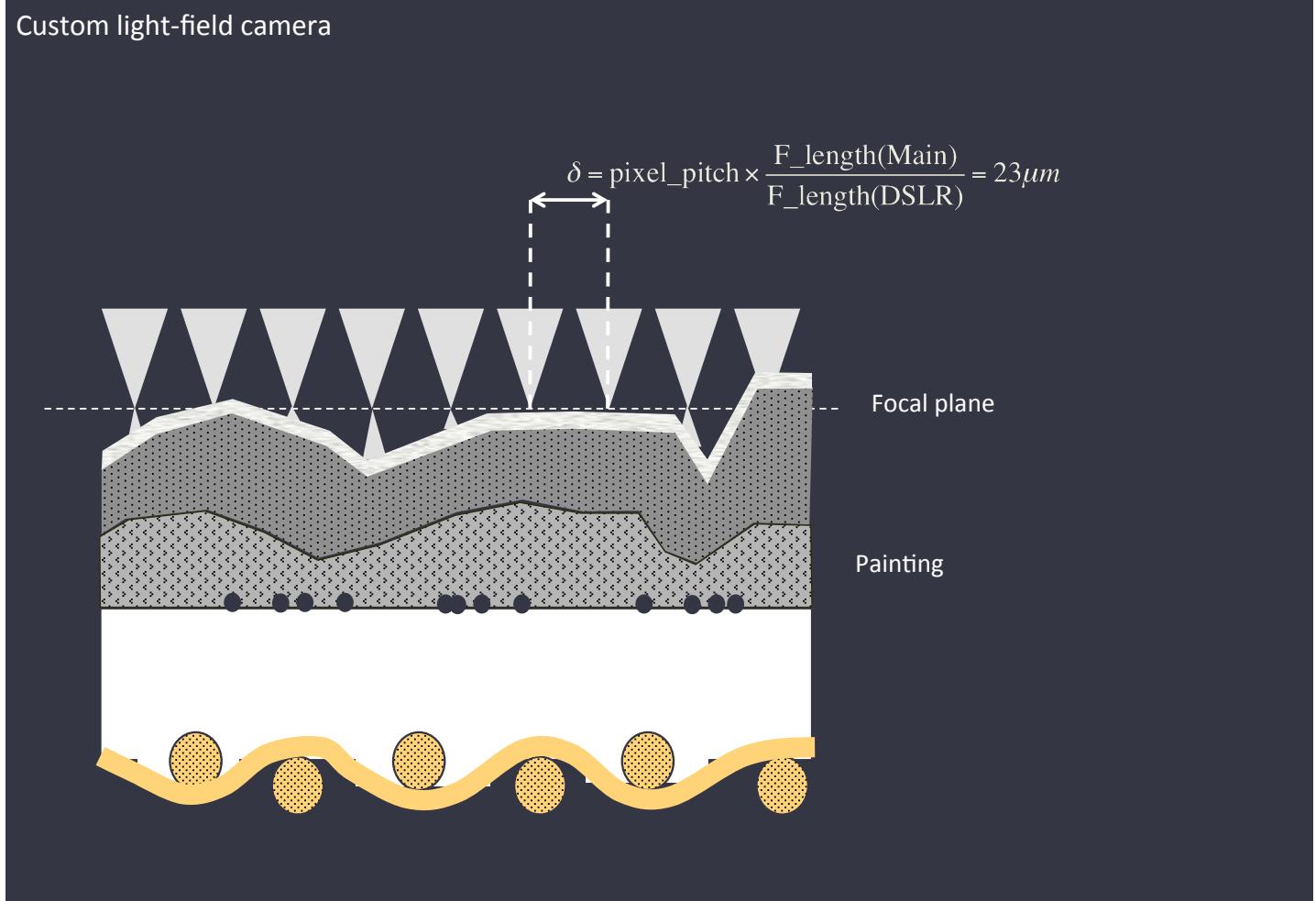
Custom light-field camera



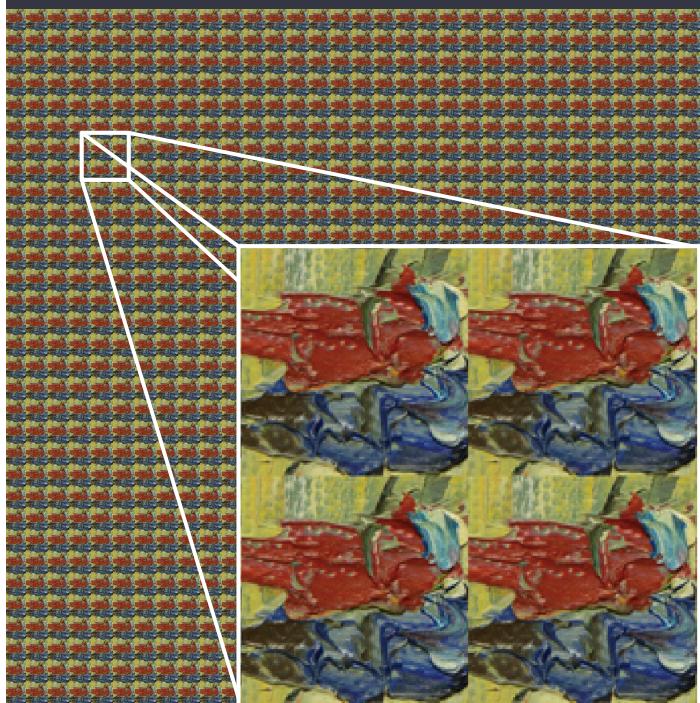
Custom light-field camera



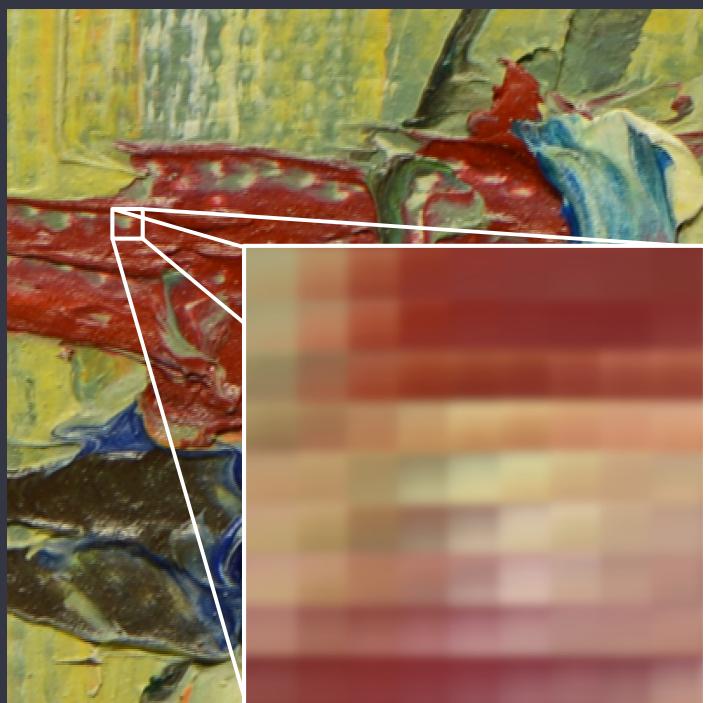
Custom light-field camera



Acquiring the 4D light field of a painting (resolution: 512x512x32x32)

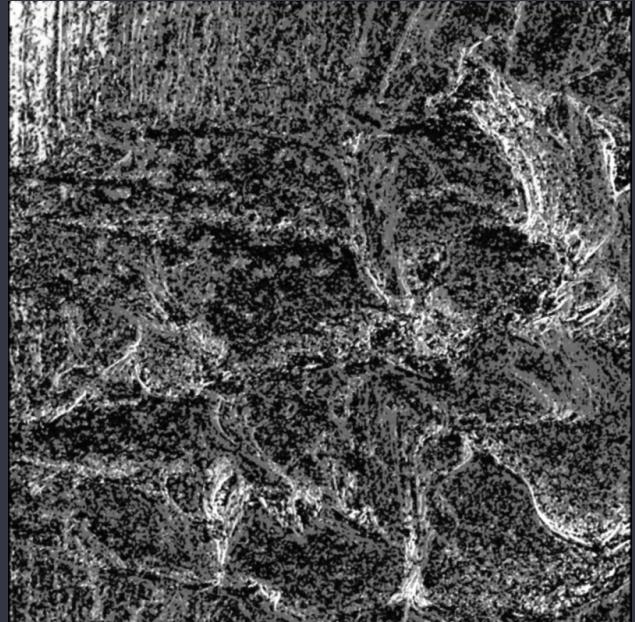


Sub-aperture image view of acquired light field
(major: UV ; minor: XY)
32x32x512x512



Microlens view of acquired light field
(major: XY ; minor: UV)
512x512x32x32

Depth map from acquired light field



What to remember

- Light field processing is at the convergence of optics and image processing.
- Ray tracing offers a convenient framework to manipulate light field.
- Optical elements such as thin lens and free space shear the light field.
- The light field is 4D signal and an image is one 2D integral-projection of it.
- Light field cameras acquire both spatial and angular information of light rays.
- From a light field, images with different focus can be generated after acquisition.
- Working in the Fourier domain is computationally more efficient.
- The Fourier Slice Theorem can be applied to the light field for image rendering.