Dynamical Systems for Engineers: Exercise Set 3

Exercise 1

Consider the autonomous linear system

$$\begin{aligned}\dot{x_1}(t) &= -x_1(t) - x_2(t)\\ \dot{x_2}(t) &= x_1(t) - 3x_2(t).\end{aligned}$$

1. Characterize the stability of this system.
2. Find the solution of the system subject to the initial conditions $(x_1(0), x_2(0)) = (1, 0)$.
3. Find the solution of the system subject to the initial conditions $(x_1(0), x_2(0)) = (1, 1)$.
4. Sketch the phase portrait of this system.

Exercise 2 (Optional Matlab exercise)

This exercise allows you to use Matlab to get a feeling of the behavior of a few two-dimensional autonomous systems. Don't worry, most of the Matlab code is already written for you.

We provide on Moodle a Matlab script named `orbitPlot.m`. It implements a function that computes and plots the orbit of a solution for a 2-D autonomous system, given its initial condition. It also plots the temporal evolution of the two state variables $x_1(t)$ and $x_2(t)$.

The function `orbitPlot(A, x0, tstart, tend)` takes the following arguments:

- `A` is the 2×2 matrix describing the system.
- `x0` is the 2×1 initial condition vector $(x_1(0), x_2(0))^T$.
- [`tstart`, `tend`] is the time interval during which the solution is to be evaluated and plotted.

Try the script for the following systems, initial conditions, and time intervals. In each case, characterize the stability.

- **System 1:**

$$\begin{aligned}\dot{x_1}(t) &= -3x_1(t) + x_2(t)\\ \dot{x_2}(t) &= x_1(t) - 2x_2(t),\end{aligned}$$

  for the time interval $t \in [0, 3]$. Try with initial conditions $(x_1(0), x_2(0)) = (2, 1)$, and then with initial conditions $(x_1(0), x_2(0)) = (-1, 5)$. What do you observe?

- **System 2:**

$$\begin{aligned}\dot{x_1}(t) &= x_1(t) + x_2(t)\\ \dot{x_2}(t) &= -2x_2(t),\end{aligned}$$

  for the time interval $t \in [0, 3]$. Try with initial conditions $(x_1(0), x_2(0)) = (1, -3)$, and then with initial conditions $(x_1(0), x_2(0)) = (1.1, -3)$. What do you observe?

- **System 3:**

$$
\begin{aligned}
\dot{x_1}(t) &= x_2(t) \\
\dot{x_2}(t) &= -x_1(t),
\end{aligned}
$$

for the time interval $t \in [0, 60]$. Try with initial conditions $(x_1(0), x_2(0)) = (1, 1)$. Then, try again after having slightly perturbed the system as follows:

$$
\begin{aligned}
\dot{x_1}(t) &= x_2(t) \\
\dot{x_2}(t) &= -x_1(t) - 0.1x_2(t).
\end{aligned}
$$