Dynamical Systems for Engineers: Exercise Set 1

Exercise 1

Consider a continuous-time dynamical system, with state $x \in \mathbb{R}$, whose state equation is the non-linear ordinary differential equation

$$\frac{dx}{dt}(t) = x^3(t).$$

1. Find a solution $x(t)$ with initial condition $x(0) = 1$. Hint: the solution has the form $x(t) = (\alpha + \beta t)^\gamma$ where α, β, γ are real parameters to be defined.

2. Is this solution unique?

3. Does this solution exist for all $t \in \mathbb{R}_+$?

Exercise 2

Consider a continuous-time dynamical system, with state $x \in \mathbb{R}_+$, whose state equation is the nonlinear ordinary differential equation

$$\frac{dx}{dt} = 2\sqrt{x(t)}.$$

1. Find a solution $x(t)$ with initial condition $x(0) = 0$. Hint: the solution has the form $x(t) = (\alpha + \beta t)^\gamma$ where α, β, γ are real parameters to be defined.

2. Is this solution unique?

3. Does this solution exist for all $t \in \mathbb{R}_+$?

Exercise 3

Consider a discrete-time dynamical system over $\mathbb{N}$, with state $x \in \mathbb{R}$, whose state equation is the difference equation

$$x(t+1) = -x(t).$$

1. Find the solution $x(t)$ with initial condition $x(0) = x_0$.

2. What is the omega-limit set of the solution with initial condition $x(0) = 2$?

3. Does this dynamical system have an attractor? If so, which one?

Exercise 4

Consider a discrete-time dynamical system over $\mathbb{N}$, with state $x > 0$, whose state equation is the difference equation

$$x(t+1) = \sqrt{x(t)}.$$

1. Find the solution $x(t)$ with initial condition $x(0) = x_0 > 0$.

2. What is the omega-limit set of the solution with initial condition $x(0) = 2$?

3. Does this dynamical system have an attractor? If so, which one?