Advanced Algorithms, Fall 2012

Prof. Bernard Moret

Homework Assignment #6

due Sunday night, Nov. 4

Write your solutions in LaTeX using the template provided on the Moodle and web sites and upload your PDF file through Moodle by 4:00 am Monday morning, Nov. 5.

Question 1. (Matching)
Consider the following simple model for mobile phones. We have n base stations and n phones, all of which are specified as points in the plane. The phones are said to be *fully connected* if each phone can be assigned to a distinct base station, and the straight-line distance between the assigned pair of phone and station is no more than a given constant c.

Suppose the user of the first phone drives from its original point along a line for k units of distance. As this phone moves, we have to update the assignment (possible several times) to keep all phones fully connected. Give an $O(n^3)$ running time algorithm to decide whether it is possible to keep all phones fully connected at all times during the driving. (Hint: a straight line cuts a circle in at most two points.)

Question 2. (Stable Matching)
Prove that a stable matching is both man-optimal and woman-optimal if and only if it is the unique stable matching for the problem.

Question 3. (Stable Matching)
Consider the stable matching problem in the case where ties are allowed in the preference lists.

1. A *strong instability* in a perfect matching S consists of a man m and a woman w such that m and w prefer each other to their partners in S. Does there always exist a perfect matching with no strong instability?

   Give a polynomial-time algorithm that is guaranteed to find a perfect matching without strong instability and prove the correctness of your algorithm, or give an instance for which every perfect matching has a strong instability.

2. A *weak instability* in a perfect matching S consists of a man m and a woman w such that m prefers w to his partner in S and w either prefers m to his partner in S or likes the two men equally; or w prefers m to her partner in S and m either prefers w to her partner in S or is likes them equally. Does there always exist a perfect matching with no weak instability?

   Give a polynomial-time algorithm that is guaranteed to find a perfect matching without weak instability, or give an instance for which every perfect matching has a weak instability.

Question 4. (Network Flow)
We are given a directed network $G = (V,E)$ with a single specified *good* node $g \in V$ and a set

of *bad* nodes $B \subset V$ (naturally, we have $g \notin B$). We want to disconnect bad nodes from g by removing edges, but face a tradeoff: we want to remove as few edges as possible and yet we want to remove as many bad nodes as possible.

In consequence, we want to maximize the objective function $|f(S)| - \alpha \cdot |S|$, where S is the subset of E to remove, $f(S)$ is the set of bad nodes that cannot be reached from g in the subgraph $(V, E - S)$, and α is a positive constant. Give a polynomial-time algorithm to find a subset $S \subset E$ to maximize this objective function, and prove the correctness of your algorithm.