MAT 653: Statistical Simulation

Instructor: Dr. Wei Li Scribe: Jianchen Wei

Oct 14th, 2021

Self-normalize Importance Sampling

Target: $E_f(h(x))$

$f \propto f_0$, g is importance pdf, and $supp(f) \subset supp(g)$.

Self-normalized importance sampling $\implies \left(x_i, \frac{f_0(x_i)}{g_0(x_i)}\right)$ importance sample. It turns out this can be recycled by multinomial resampling into a sample that is from f.

Step 1: Sample $x_i \sim g$, obtain $\left(x_i, \frac{f_0(x_i)}{g_0(x_i)}\right)_{i=1}^n$

Step 2: importance weights $w_i = \frac{f_0(x_i)}{g_0(x_i)}$. Let $\hat{w}_i = \frac{\frac{1}{n}w_i}{\frac{1}{n}\sum_j^n w_j}$. Notice that $\hat{w}_i \in (0,1), \sum \hat{w}_i = 1$

Step 3: draw a random sample of size m with replacement from $x_1, \cdots, x_n$ with weighted probabilities by $\hat{w}_1, \cdots, \hat{w}_n$: that is for $k = 1, 2, \ldots, m$,

$$X_k^* = \begin{cases} x_1 & \text{with prob} = \hat{w}_1 \\ x_2 & \text{with prob} = \hat{w}_2 \\ \vdots & \\ x_n & \text{with prob} = \hat{w}_n \end{cases}$$

It turns out that $(X_1^*, X_2^*, \cdots, X_m^*)$ is a random sample from f.

Remark: for this sampling to work, the target sample size m should be no more than 10% of the original sample size n.