A Low-Complexity Design of Linear Precoding for MIMO Channels with Finite-Alphabet Inputs

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***Abstract*—This paper investigates linear precoding scheme that maximizes mutual information for multiple-input multiple-output (MIMO) channels with finite-alphabet inputs. In contrast with recent studies, optimizing mutual information directly with extensive computational burden, this work proposes a low-complexity and high-performance design. It derives a lower bound that demands low computational effort and approximates, with a constant shift, the mutual information for various settings. Based on this bound, the precoding problem is solved efficiently. Numerical examples show the efficacy of this method for constant and fading MIMO channels. Compared to its conventional counterparts, the proposed method reduces the computational complexity without performance loss.**

***Index Terms*—Linear precoder, finite-alphabet inputs, mutual information maximization, low-complexity design.**

I. Introduction

LINEAR precoding exploits channel state information at the transmitter and transforms the signal before transmission. Its advantages have been investigated based on various criteria, among which the information-theoretic approach (*e.g.*, capacity-achieving precoder) continues to fascinate researchers; see [1]–[3] and references therein.

The capacity-achieving precoder, however, is based on the assumption that input signal is Gaussian distribution, which is rarely realized in practical systems. Instead, finite-alphabet inputs, such as phase shift keying (PSK) modulation and quadrature amplitude modulation (QAM), are usually used. Since the significant difference between Gaussian signals and finite-alphabet signals, a substantial performance gap exists between precoding schemes designed based on Gaussian-input assumption and those from the standpoint of finite-alphabet inputs [4]–[6]. Thus, optimizing the precoder in terms of constellation-constrained mutual information is a topic of great interest, both as a theoretical tool and as a practical technique.

However, finding out the precoder maximizing the constellation-constrained mutual information requires the evaluation, at each iteration step, of mutual information and its gradient (related to minimum mean square error (MMSE) matrix [7]). Since both terms lack closed-form expressions and involve complicated computations, they are challenging and costly to obtain. Although Monte Carlo method can be used, it can be quite slow and provides a limited accuracy, especially for larger input-output dimensions and fading channels.

This paper focuses on a low-complexity precoder design. It first derives a closed-form lower bound of mutual information and proves the asymptotic optimality of maximizing the lower bound as an alternative. It then considers the concavity results, of the lower bound, which turn out to be the same as that of the mutual information. Thus, existing methods to maximize the mutual information can readily be used here to maximize the lower bound. Numerical examples show, with a constant shift, the lower bound actually serves as a very good approximation to mutual information. They also show the efficacy of optimizing the lower bound for both constant and fading MIMO channels. Compared to its conventional counterparts, the proposed method offers almost the same performance gains but demands a much lower computational complexity.

Notation: Boldface uppercase (lowercase) letters denote matrices (column vectors), and italics denote scalars. The superscripts $(\cdot)^H$ stand for Hermitian operation; $\mathsf{Tr}(\cdot)$ denotes the trace operation; $\|\cdot\|$ denotes the Frobenius norm of either a matrix or a vector; $\mathbf{I}$ represents an identity matrix of appropriate dimension. The term E denotes statistical expectation, $\mathbb{C}$ denotes the complex space, and $\log$ and $\ln$ are used for the base two logarithm and natural logarithm, respectively.

II. System Model

Consider a MIMO system with N_t transmit antennas and N_r receive antennas. Let $\mathbf{x} \in \mathbb{C}^{N_t}$ be the transmitted signal with zero mean and identity covariance (*i.e.*, $\mathsf{E}(\mathbf{x}\mathbf{x}^H) = \mathbf{I}$); the received signal $\mathbf{y} \in \mathbb{C}^{N_r}$ is then given by

$$\mathbf{y} = \mathbf{HPx} + \mathbf{n} \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix, $\mathbf{P} \in \mathbb{C}^{N_t \times N_t}$ is the linear precoder matrix, and $\mathbf{n} \in \mathbb{C}^{N_r}$ is the zero-mean circularly-symmetric Gaussian noise with covariance $\sigma^2\mathbf{I}$. Based on different channel model assumptions, the sequel considers two cases.

1) Constant MIMO Channels: Let the input signal $\mathbf{x}$ draw from equiprobable constellation set with cardinality M. When the channel is constant and is known at the receiver, the mutual information between $\mathbf{x}$ and $\mathbf{y}$ is given by [6]

$$\mathcal{I}(\mathbf{H}, \mathbf{P}) = N_t \log M - \frac{1}{M^{N_t}} \sum_{m=1}^{M^{N_t}} \mathsf{E}_{\mathbf{n}} \log \sum_{k=1}^{M^{N_t}} \exp(-d_{mk}) \tag{2}$$

Manuscript received September 18, 2011. The associate editor coordinating the review of this letter and approving it for publication was I. Lee.

This work was supported in part by the China National Science Foundation under Grant 61101071, and the U.S. National Science Foundation under Grant CCF-0915846.

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Digital Object Identifier 10.1109/WCL.2012.121411.110025

where d_{mk} is $(\|\mathbf{HPe}_{mk}+\mathbf{n}\|^2-\|\mathbf{n}\|^2)/\sigma^2$, and $\mathbf{e}_{mk}$ is the difference between $\mathbf{x}_m$ and $\mathbf{x}_k$, which contain N_t symbols and are taken independently from the M-ary signal constellation.

Assuming the channel matrix $\mathbf{H}$ known at the transmitter, the objective is to find a linear precoder $\mathbf{P}$ that maximizes the mutual information. The optimization is carried out over a feasible set, in which transmit power constraint is satisfied: $\mathsf{Tr}(\mathbf{PP}^H)\le N_t$. Since $\mathcal{I}(\mathbf{H},\mathbf{P})$ is an increasing function of the transmit power, the power constraint can be replaced with an equality constraint: $\mathsf{Tr}(\mathbf{PP}^H)=N_t$. Thus, the limit on the mutual information with finite-alphabet inputs is given by

$$\mathcal{I}(\mathbf{H})=\max_{\mathbf{P}:\mathsf{Tr}(\mathbf{PP}^H)=N_t}\mathcal{I}(\mathbf{H},\mathbf{P}). \tag{3}$$

2) Fading MIMO Channels: When channels change slow enough to be reliably fed back to the transmitter with negligible delay, the ergodic limit on mutual information is the average of $\mathcal{I}(\mathbf{H})$ over channel realizations [2]. It is given by

$$\mathcal{I}=\mathsf{E}_{\mathbf{H}}\left[\max_{\mathbf{P}:\mathsf{Tr}(\mathbf{PP}^H)=N_t}\mathcal{I}(\mathbf{H},\mathbf{P})\right] \tag{4}$$

which resorts to $\mathcal{I}(\mathbf{H})$ in (3) for each channel realization.

The computational efficiency of solving (3) depends on the efficient calculation of $\mathcal{I}(\mathbf{H},\mathbf{P})$ and optimization over $\mathbf{P}$, which are prevented by the following barriers: first, closed-form expression of $\mathcal{I}(\mathbf{H},\mathbf{P})$ is challenging, if not impossible, to obtain (see [8] for high SNR approximation when $\mathbf{H}$ is diagonal). Second, the expectation over $\mathbf{n}$ in (2) implies the evaluation of $2N_r$ integrals, which are computationally prohibitive. Third, the gradient of mutual information involves MMSE matrix, which is helpful to solve (3) but is even more challenging to obtain. Although Monte Carlo method can be used to estimate mutual information and MMSE matrix, it can be quite slow and provides a limited accuracy, especially for larger input-output dimensions.

III. Lower Bound on Mutual Information and Precoder Design

This section investigates a lower bound, which helps to solve the precoding problem in (3) and (4) with high performance but low complexity.

Theorem 1: The mutual information of a constant MIMO channel with finite-alphabet inputs can be lower bounded by

$$\begin{aligned}\mathcal{I}_L(\mathbf{H},\mathbf{P})&=N_t\log M-(1/\ln 2-1)N_r\\&\quad-\frac{1}{M^{N_t}}\sum_{m=1}^{M^{N_t}}\log\sum_{k=1}^{M^{N_t}}\exp\left(-\frac{\mathbf{c}_{mk}^H\mathbf{c}_{mk}}{2\sigma^2}\right)\end{aligned} \tag{5}$$

where $\mathbf{c}_{mk}$ equals $\mathbf{HPe}_{mk}$.

Proof: The mutual information in (2) is reformulated as

$$\begin{aligned}\mathcal{I}(\mathbf{H},\mathbf{P})&=N_t\log M-\mathsf{E}_{\mathbf{n}}\log\exp\left(\|\mathbf{n}\|^2/\sigma^2\right)\\&\quad-\frac{1}{M^{N_t}}\sum_{m=1}^{M^{N_t}}\mathsf{E}_{\mathbf{n}}\log\sum_{k=1}^{M^{N_t}}\exp\left(-f_{mk}\right)\end{aligned} \tag{6}$$

where f_{mk} denotes $(\|\mathbf{c}_{mk}+\mathbf{n}\|^2)/\sigma^2$, and the second term of the right-hand side in (6) equals $N_r/\ln 2$. Since $\log(x)$ is a concave function, a lower bound of (6) can be derived by taking the expectation of the variable first (Jensen's inequality):

$$N_t\log M-\frac{N_r}{\ln 2}-\frac{1}{M^{N_t}}\sum_{m=1}^{M^{N_t}}\log\sum_{k=1}^{M^{N_t}}\mathsf{E}_{\mathbf{n}}\exp\left(-f_{mk}\right). \tag{7}$$

Let $c_{mk,i}$ and n_i be the i-th element of $\mathbf{c}_{mk}$ and $\mathbf{n}$, respectively. The expectation over $\mathbf{n}$ in (7) is given by

$$\begin{aligned}\mathsf{E}_{\mathbf{n}}\exp\left(-f_{mk}\right)&=\frac{1}{(\pi\sigma^2)^{N_r}}\int_{\mathbf{n}}\exp\left(-f_{mk}-\frac{\|\mathbf{n}\|^2}{\sigma^2}\right)d\mathbf{n}\\&=\prod_{i=1}^{N_r}\frac{1}{\pi\sigma^2}\int_{n_i}\exp\left(-\frac{\|c_{mk,i}+n_i\|^2+\|n_i\|^2}{\sigma^2}\right)dn_i.\end{aligned} \tag{8}$$

Extending the integrals of exponential function [9, eq. (2.33.1)] to the complex value and considering the integral interval $(-\infty,+\infty)$, (8) is rewritten as $\frac{1}{2^{N_r}}\exp\left(-\frac{\mathbf{c}_{mk}^H\mathbf{c}_{mk}}{2\sigma^2}\right)$. Substituting this result into (7), the bound (5) is obtained.

Note that a different bound for high SNR has been derived in [5] for two-user single-antenna multiple access channels. □

The sequel shows that maximizing the proposed lower bound helps maximizing mutual information. The problem of maximizing $\mathcal{I}_L(\mathbf{H},\mathbf{P})$ is equivalent to the following problem:

$$\begin{aligned}&\text{minimize}\quad\sum_{m=1}^{M^{N_t}}\log\sum_{k=1}^{M^{N_t}}\exp\left(-\frac{\mathbf{c}_{mk}^H\mathbf{c}_{mk}}{2\sigma^2}\right)\\&\text{subject to}\quad\mathsf{Tr}(\mathbf{PP}^H)=N_t\end{aligned} \tag{9}$$

which is asymptotically optimal, at low and high SNR regions, compared to maximizing mutual information directly.

Asymptotic Optimality at Low SNR Region: As $\sigma^2\to+\infty$, the objective function of (9) is re-expressed, based on Taylor expansion, as

$$M^{N_t}\log M^{N_t}-\frac{\sum_{m=1}^{M^{N_t}}\sum_{k=1}^{M^{N_t}}\mathbf{c}_{mk}^H\mathbf{c}_{mk}}{2\ln(2)M^{N_t}\sigma^2}+\mathcal{O}(1/\sigma^4) \tag{10}$$

where $\mathcal{O}(1/\sigma^4)$ denotes the least significant terms on the order of $1/\sigma^4$. For a equiprobable zero-mean constellation set, the vectors $\mathbf{e}_{mk},\ m,k\in\{1,\cdots,M^{N_t}\}$ satisfy $\sum_m\sum_k\mathbf{e}_{mk}\mathbf{e}_{mk}^H=\alpha\cdot\mathbf{I}$, where α is determined by the constellations and the number of antennas. Thus, recalling the definition of $\mathbf{c}_{mk}$, the numerator of the second term in (10) is re-expressed as $\alpha\mathsf{Tr}(\mathbf{P}^H\mathbf{H}^H\mathbf{HP})$.

The optimization problem in (9) is then equivalent, at low SNR region, to maximizing $\mathsf{Tr}(\mathbf{P}^H\mathbf{H}^H\mathbf{HP})$ with power constraint $\mathsf{Tr}(\mathbf{PP}^H)=N_t$. This problem is solved by the beamforming strategy. Consider singular value decomposition of the precoding matrix $\mathbf{P}=\mathbf{U_P\Sigma_PV_P}^H$ and channel matrix $\mathbf{H}=\mathbf{U_H\Sigma_HV_H}^H$, where $\mathbf{U_P}$, $\mathbf{V_P}$, $\mathbf{U_H}$, and $\mathbf{V_H}$ are unitary matrices, and $\mathbf{\Sigma_P}$ and $\mathbf{\Sigma_H}$ are nonnegative diagonal matrices containing the singular values. The beamforming solution suggests that $\mathbf{U_P}$ equals $\mathbf{V_H}$ and that the diagonal elements of $\mathbf{\Sigma_P}$ are zero except for the element corresponding to the maximum singular value of $\mathbf{H}$.

The solution of maximizing $\mathcal{I}_L(\mathbf{H},\mathbf{P})$ at low SNR region thus yields the same result as maximizing $\mathcal{I}(\mathbf{H},\mathbf{P})$ directly [4], [8]; that is, maximize $\mathcal{I}_L(\mathbf{H},\mathbf{P})$ is asymptotically optimal.

Asymptotic Optimality at High SNR Region: Let's start from the optimization problem (9). As $\sigma^2 \to 0$, the function $\log \sum_{k=1}^{M^{N_t}} \exp(f_k)$, known as a soft version of maximization [10], is dominated—with the help of exponential operator—by the most significant term. The idea here is to approximate the soft maximization of the objective function in (9) by its hard version and consider a lower bound of it:

$$\sum_{m=1}^{M^{N_t}} \max_k \left(-\frac{\mathbf{c}_{mk}^H \mathbf{c}_{mk}}{2\sigma^2} \right) \geq \max_{\substack{m,k \\ m \neq k}} \left(-\frac{\mathbf{c}_{mk}^H \mathbf{c}_{mk}}{2\sigma^2} \right).$$

The problem (9) corresponds to minimizing the right-hand side, which leads to the following problem

$$\begin{aligned} &\text{maximize} && \min_{\substack{m,k \\ m \neq k}} \left(\mathbf{e}_{mk}^H \mathbf{P}^H \mathbf{H}^H \mathbf{H} \mathbf{P} \mathbf{e}_{mk} \right) \\ &\text{subject to} && \mathrm{Tr}(\mathbf{P}\mathbf{P}^H) = N_t. \end{aligned} \tag{11}$$

The objective function of (11) can be identified as the minimum distance of the input vector: $d_{\min} = \min_{\substack{m,k \\ m \neq k}} \|\mathbf{HP}(\mathbf{x}_m - \mathbf{x}_k)\|^2$. Hence, the optimization of $\mathcal{I}_L(\mathbf{H}, \mathbf{P})$ at high SNR region is equivalent to maximizing the minimum distance, which implies the asymptotic optimality because maximizing $\mathcal{I}(\mathbf{H}, \mathbf{P})$ at high SNR directly yields the same result [4], [8].

Precoder Design for an Arbitrary SNR: Since the left singular vector matrix of the optimal precoder, $\mathbf{U}_\mathbf{P}$, should coincide to the right singular vector matrix of the channel matrix, $\mathbf{V}_\mathbf{H}$ [6], the system model (1) can be simplified, without loss of generality, as $\tilde{\mathbf{y}} = \mathbf{\Sigma}_\mathbf{H} \tilde{\mathbf{P}} \mathbf{x} + \tilde{\mathbf{n}}$, where $\tilde{\mathbf{P}} = \mathbf{\Sigma}_\mathbf{P} \mathbf{V}_\mathbf{P}^H$ is the remaining part of the linear precoder, and $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{n}}$ are the results of $\mathbf{y}$ and $\mathbf{n}$ transformed by a unitary matrix $\mathbf{U}_\mathbf{H}^H$.

The problem (9) can thus be reformulated as

$$\begin{aligned} &\text{minimize} && \sum_{m=1}^{M^{N_t}} \log \sum_{k=1}^{M^{N_t}} \exp \left(-\frac{\mathbf{e}_{mk}^H \tilde{\mathbf{P}}^H \mathbf{\Sigma}_\mathbf{H}^2 \tilde{\mathbf{P}} \mathbf{e}_{mk}}{2\sigma^2} \right) \\ &\text{subject to} && \mathrm{Tr}(\tilde{\mathbf{P}} \tilde{\mathbf{P}}^H) = N_t. \end{aligned}$$

The objective function fails to be convex, with respect to $\tilde{\mathbf{P}}$, which can be confirmed by a counterexample (*e.g.*, $\mathbf{e}_{mk}$, $\tilde{\mathbf{P}}$, and $\mathbf{\Sigma}_\mathbf{H} \in \mathbb{C}^1$).

However, the convexity with respect to $\mathbf{W} = \tilde{\mathbf{P}}^H \mathbf{\Sigma}_\mathbf{H}^2 \tilde{\mathbf{P}}$ can be identified because $\log \sum_k \exp(f_k)$ is convex whenever f_k is convex and f_k, in this case, is an affine function [10]. At the same time, given a specific $\mathbf{V}_\mathbf{P}$, the convexity with respect to the squared singular value of the precoder $\mathbf{\Sigma}_\mathbf{P}^2$ can be identified. Interestingly, these above-mentioned convexity results of $\mathcal{I}_L(\mathbf{H}, \mathbf{P})$ are exactly the same as those of $\mathcal{I}(\mathbf{H}, \mathbf{P})$; thus, the optimization methods developed for maximizing $\mathcal{I}(\mathbf{H}, \mathbf{P})$ (*e.g.*, [6]) can be readily used to maximize $\mathcal{I}_L(\mathbf{H}, \mathbf{P})$. Details about these methods are omitted here for brevity.

IV. Simulations

This section offers examples to illustrate the relationship between the lower bound and mutual information and to show the efficacy of the precoder designed based on maximizing the lower bound for both constant and fading MIMO channels.

Lower Bound with a Constant Shift as an Approximation to Mutual Information: Consider the limits of the mutual information and lower bound in (2) and (5), respectively. When the SNR approaches 0 and $+\infty$, the limits of $\mathcal{I}(\mathbf{H}, \mathbf{P})$ are 0 and $N_t \log M$, whereas the limits of $\mathcal{I}_L(\mathbf{H}, \mathbf{P})$ are $-N_r(1/\ln(2) - 1)$ and $N_t \log M - N_r(1/\ln(2) - 1)$. This means, at low and high SNR regions, a constant gap, $N_r(1/\ln(2) - 1)$, between $\mathcal{I}(\mathbf{H}, \mathbf{P})$ and $\mathcal{I}_L(\mathbf{H}, \mathbf{P})$ exists. Since adding a constant value to $\mathcal{I}_L(\mathbf{H}, \mathbf{P})$ keeps the precoder solution maximizing $\mathcal{I}_L(\mathbf{H}, \mathbf{P})$ unchanged, we show that the lower bound with a constant shift provides a very good approximation to the mutual information for various settings.

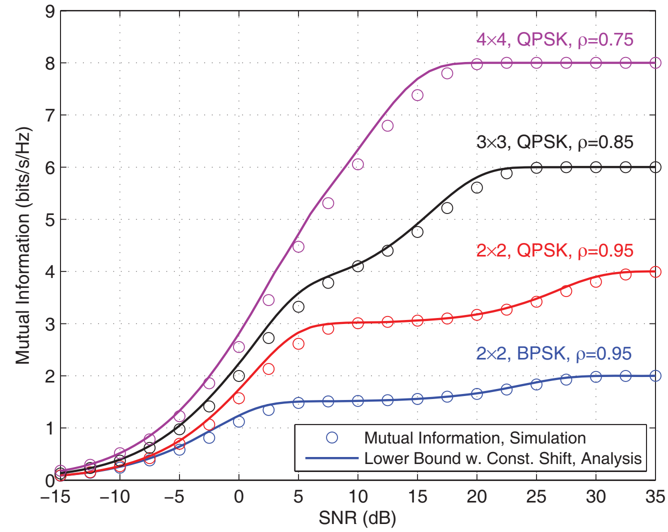

Fig. 1. Mutual information for various numbers of transmit and receive antennas and various input types and channel parameters.

To exemplify the results, this example considers constant exponential MIMO channels with the (i, j)-th element defined by $[\mathbf{H}(\rho)]_{ij} = \rho^{|i-j|}$, where $\rho \in [0, 1)$, $i = 1, \cdots, N_r$, and $j = 1, \cdots, N_t$. Figure 1 shows the relationship between the lower bound plus $N_r(1/\ln(2) - 1)$ and the simulated mutual information by Monte Carlo method for the case of without precoding. It considers various numbers of transmit and receive antennas and various input types and channel parameters. From Fig. 1, the lower bound with a shift and the simulated mutual information match exactly at low and high SNR, and they close to each other at medium SNR; that is, with a constant shift, the lower bound serves as a very good approximation.

Precoding for Constant MIMO Channels: Considering the relationship revealed in the above example, the precoder designed based on maximizing the lower bound, with much lower computational complexity, can be expected to provide almost the same performance as that designed based on maximizing the mutual information directly. This statement is verified in this example. Let's take a 2×2 MIMO channel with channel matrix $\mathbf{H} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ for instance.

When the input signal $\mathbf{x}$ is drawn from QPSK constellation set, the problems of maximizing the lower bound and maximizing the mutual information directly can be solved with the method proposed in [6], and the performance of both cases (denoted as Max Lower Bound and Max MI Directly, respectively) is shown in Fig. 2. For comparison, Fig.

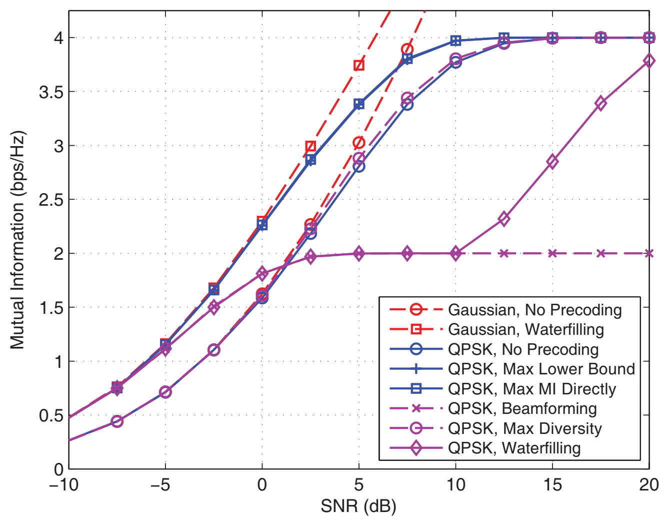

Fig. 2. Mutual information versus SNR for 2×2 constant MIMO channels with Gaussian and QPSK inputs.

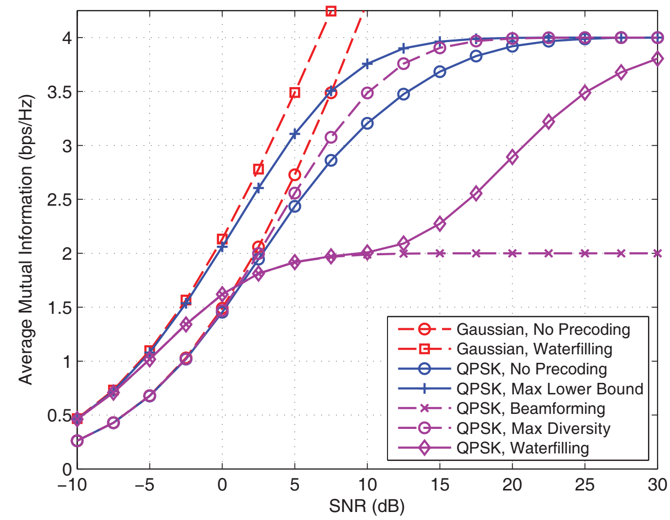

Fig. 3. Average mutual information versus SNR for 2×2 fading MIMO channels with Gaussian and QPSK inputs when $\rho_r = 0.5$ and $\rho_t = 0.95$.

2 also provides the performance of several additional cases: no precoding and capacity-achieving precoder (by waterfilling method [2]) with both QPSK inputs and Gaussian inputs, beamforming, and maximum diversity precoding [1].

The results in Fig. 2 imply two observations. First, the curves for maximizing lower bound and maximizing mutual information directly are virtually the same, but the complexity is radically different. Typically, the computing effort for maximizing lower bound is *several orders of magnitude less* than that for maximizing mutual information directly. This is because the method of maximizing mutual information requires, at each iteration step, to estimate the mutual information and its gradient by extensive calculation. In contrast, the lower bound and its gradient are easy to evaluate; thus, it is able to be used for practical systems. Second, the performance gain between no precoding and the proposed method with finite-alphabet inputs is almost the same as the gain between no precoding and waterfilling with Gaussian inputs in a large range of SNR regions. This implies the proposed method is actually optimal in a large range of SNR regions.

Precoding for Fading MIMO Channels: The lower bound is optimized for each channel realization, which is generated based on the correlated MIMO model: $\mathbf{H} = \mathbf{H_r}^{1/2}(\rho_r)\mathbf{H_w}\mathbf{H_t}^{1/2}(\rho_t)$, where $\mathbf{H_w}$ is a complex matrix with i.i.d. zero-mean and unit variance Gaussian entries, and $\mathbf{H_r}(\rho_r)$ and $\mathbf{H_t}(\rho_t)$ are receive and transmit correlation matrices with their entries defined by exponential parameters. More than 5,000 realizations are considered, and the average mutual information is evaluated as (4).

Figure 3 shows the performance of various strategies considered in Fig. 2, except for the method of maximizing mutual information directly due to the computational complexity, for fading MIMO channels when $\rho_r = 0.5$ and $\rho_t = 0.95$. This result verifies that maximizing the lower bound is near optimal because it makes QPSK inputs perform almost the same as the waterfilling method with Gaussian inputs, which is the upper bound for all possible precoders, for a large SNR regions.

V. Conclusion

This paper has investigated the linear precoding design that maximizes mutual information with finite-alphabet constraints. Unlike previous studies that estimate mutual information and its gradient with extensive computations, this study has made a step towards exploring a low complexity method. It has derived a lower bound and has proved the asymptotic optimality of maximizing it. Numerical examples have revealed, with a constant shift, the lower bound offers a very good approximation and thus provides a good alternative formula to mutual information for various settings. Since the concavity of the lower bound is the same as that of the mutual information, existing methods of maximizing mutual information can readily be used here. Numerical examples have shown the performance gains of the proposed method for both constant and fading MIMO channels.

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