Outage Performance of Half-Duplex Cooperative Relay Networks Using Rateless Codes

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***Abstract*—We propose a new cooperative transmission scheme, *Opportunistic Fountain* (OF), based on rateless codes for half-duplex relaying. In the proposed scheme, only the most appropriate relay is permitted to forward the message using rateless codes, thus as an orthogonal transmission protocol, our scheme does not need to synchronize among distributed relays. Furthermore, the transmitted information can also be heard and used even by relay nodes that fail to accumulate sufficient mutual information. As a result these nodes can decode the information faster, enlarging the transmitter candidate set (TCS), in which the best relay is selected. We first analyze the simplified one-relay version of the proposed protocol, and present closed-form expressions for the outage probability. The analysis reveals that the proposed scheme can achieve a gain of about 2 dB over the traditional method, which is supported by extensive simulations. We then evaluate the protocol performance for larger network sizes through Monte Carlo simulation, which confirms the benefits of the scheme.**

I. Introduction

Cooperative communications, where a collection of terminals work together in order to assist the source in passing its message to destination, promise enhancements to the ability of combating fading induced by multipath propagation even with single-antenna terminals in wireless networks [1].

In this scenario, the use of rateless codes, exhibiting the property of naturally adapting its rate to the channel realization [2], alleviates the practical issues of transmitting data efficiently without channel side information (CSI) at the transmitter and coordinating the various transmissions reliably with limited feedback, see [3]–[5] and reference therein. Rateless codes have been introduced as an alternative to the traditional automatic repeat request (ARQ) scheme for reliable communication over lossy networks [6], [7]. They enable the transmitters to generate a potentially infinite stream of encoding packets as random and equally important descriptions of the message block of finite length. In that way a receiver can recover the original message from unordered subsets of the codestream, once the total obtained mutual information marginally exceeds the entropy of the source information [4]. All these features above render rateless codes quite attractive for cooperative communication networks.

There have been some works related to cooperative communications employing rateless codes. In [3], a space-time collaboration coding framework for half-duplex wireless relay channels based on rateless codes is proposed, where Alamouti scheme is used as inner code to provide diversity gains. In particular, it is pointed out in [4] that mutual information accumulation in relay networks with the help of rateless codes leads to a lower transmission time as compared to the traditional energy accumulation method. Additionally, in a recent work [5] achievable rates and fairness are studied for several protocols using rateless codes.

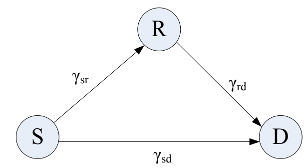

Fig. 1. System consists of the source (S), the destination (D) and the relay (R). γ_{sr}, γ_{rd} and γ_{sd} denote the channel gain of source-relay, relay-destination and source-destination, respectively.

The main contribution of this paper is that a novel protocol using rateless codes, *Opportunistic Fountain*, is proposed for half-duplex cooperative transmission. The protocol, requiring no synchronization among distributed relays, is designed to accumulate mutual information from the best relay selected from the TCS, which is referred to as a transmitter candidate set that decodes the message and is ready for cooperation. Another special contribution is that although previous work about rateless coded schemes focuses primarily on ergodic settings and characterizes performance via Shannon capacity or capacity regions, in this paper we focus on nonergodic or delay-constrained scenarios and characterize performance by outage probability. In this way, we present closed-form expressions of the outage probability for the simplified one-relay version of the proposed scheme. The outage performance improvement between the new scheme and the conventional one is about 2 dB, which is confirmed both theoretically and experimentally. For larger relay network sizes, experiment results prove the benefits of our proposed scheme.

The rest of the paper is organized as follows. In Sec. II, the relay channel model is given, and the difference of energy accumulation and mutual information accumulation is described. Section III analyzes the outage performance of the baseline schemes: direct transmission, repetition coding scheme and the rateless coding space-time collaboration scheme, while Sec. IV concentrates on our proposed protocol, including the description and the outage analysis for the simplified one relay case. Sec. V presents some numerical results to show the advantage of the new scheme. A summary and conclusions in Sec. VI wrap up this paper.

II. System Model

The three node relay network model is shown in Fig.1. The source S wants to transmit an information codeword with bandwidth normalized entropy H_t, given in bps/Hz, to the destination D, and the relay R tries to help by decoding and forwarding. Considering the practical hardware constraint, only half-duplex protocols are taken into account in the sequel [8], i.e., transmitting and receiving at the same time in the same frequency band are precluded.

The propagation channels between the different nodes are modeled as quasi-static flat-fading channel, which is applicable for scenarios of narrow-band transmissions in low-mobility environment [4]. The channel gains of any two nodes are assumed to be independent and exponentially distributed corresponding to Rayleigh fading of their amplitudes. For example, the channel gain, γ_{sr}, between the source and the relay has the probability density function (pdf)

$$f_{\gamma_{sr}}\left(\gamma_{sr}\right)=\frac{1}{\sigma_{sr}^{2}}\exp\left[-\frac{\gamma_{sr}}{\sigma_{sr}^{2}}\right],\ \gamma_{sr}>0 \tag{1}$$

where σ_{sr}^2 denotes the mean channel gain from source to relay. Similarly, the channel gains of source-destination and relay-destination are written as γ_{sd} and γ_{rd} with mean σ_{sd}^2 and σ_{rd}^2, respectively. In addition, we assume that transmitting nodes have a sum power constraint ρ per channel use, and the noise is normalized to unity.

According to the system model, if we suppose that the relay has known source's message a priori, three kinds of cooperation method as the fundamental building blocks for the protocols throughout the sequel are considered [4]:

1) Energy Accumulation with Repetition Coding: In this case the decode-and-forward (DF) protocol is based on repetition coding at the relay [8]. Given maximum-ratio combining (MRC) is applied at the receiver, message can be reliably decoded within L channel uses once

$$\frac{L}{2}\log_2\left(1+\rho\gamma_{rd}+\rho\gamma_{sd}\right)\geq H_t \tag{2}$$

where $L/2$ captures that only half of the available degrees of freedom is used, and $\rho\gamma_{rd}+\rho\gamma_{sd}$ represents receive SNR at destination via MRC. The outage event occurs if (2) fails to hold, i.e. , the transmission fails to complete within the limited channel uses.

2) Energy Accumulation with Space-Time Coding: If distributed space-time code is utilized for transmission, e.g. Alamouti code [9], both source and relay transmit in all the degrees of freedom of the channel, and the transmit power is $\rho/2$ limited by sum power constraint. We consider the scenario in which the distributed source and relay are symbol synchronous. In this way message can be reliably decoded at the receiver once

$$L\log_2\left(1+\frac{\rho}{2}\gamma_{rd}+\frac{\rho}{2}\gamma_{sd}\right)\geq H_t \tag{3}$$

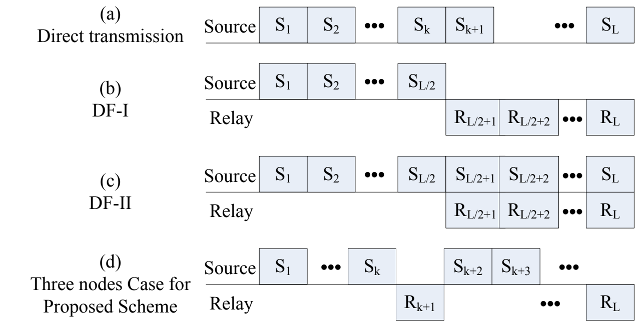

Fig. 2. Illustration of transmission protocols for every channel use limited by the maximum value L: (a) Direct transmission. Here the relay does not participate in the transmission. (b) DF with repetition coding (DF-I). Here the relay repeats the message after $L/2$, and the receiver accumulates energy via MRC. (c) Distributed space-time rateless coding scheme (DF-II). The relay aligns its output symbols in time to correspond to the source's output symbols after $L/2$ if the relay could decode the message. Energy accumulation occurs at the second phase with the help of Space-Time coding, and mutual information accumulation occurs between the two phases with the help of rateless codes. (d) The simplified one-relay version of the proposed scheme. Choose the most appropriate transmitter and devote all available power to it. Mutual information accumulation occurs for every channel uses.

3) Mutual Information Accumulation with Rateless Coding: The concept of mutual information accumulation for cooperative diversity could be realized by passing distinguished information streams from various transmitters, thus the mutual information transmitted by different nodes can be accumulated. This idea can be realized, without coordinating, by rateless codes via employing the same rateless code book at the various transmitters [4]. Message is reliably decoded at the receiver once

$$k\log_2\left(1+\rho\gamma_{sd}\right)+(L-k)\log_2\left(1+\rho\gamma_{rd}\right)\geq H_t \tag{4}$$

where k and $L-k$ represent the channel uses of source and relay, respectively.

III. Baseline Transmission Schemes

In this section, we consider a variety of DF based cooperative diversity protocols illustrated in Fig. 2. Some of them are inspired by the cooperation schemes [3], [8] and [10], but here we primarily focus on rateless codes based protocols and nonergodic or delay-constrained scenarios, where performance is characterized in terms of outage events $I<H_t$ and associated outage probabilities $\Pr\left[I<H_t\right]$. Throughout the sequel the smallest number of channel uses, after which the relay can decode the message is denoted as T_r that captures the time when relay is ready for cooperative transmission.

A. Direct Transmission

For direct transmission using rateless codes, the mutual information between source and destination is

$$I_{sd}^{(\mathrm{d})}=\log_2\left(1+\rho\gamma_{sd}\right) \tag{5}$$

where ρ is the transmit SNR and γ_{sd} is the channel gain. For quasi-static channels, the outage probability after L channel uses is

$$P_{out}^{(\mathrm{d})} = \Pr\left[LI_{sd}^{(d)} < H_t\right] = 1 - \exp\left(\frac{1-2^{H_t/L}}{\rho\sigma_{sd}^2}\right) \quad (6)$$

where rateless codes are used as a practical mean to naturally adapt its rate to the channel realization [2].

B. Traditional DF with Repetition Coding(DF-I)

In this part we consider repetition coding based cooperative transmission protocol where the source and relay are given equal channel uses $L/2$ [8]. If the relay decodes the message during $L/2$, it forwards the message using the same codes as source. Otherwise the source simply continues its transmission, in the form of repetition codes. Therefore, the destination always receives two copies of the message, and MRC can be employed to accumulate energy at the receiver. The outage probability is given by:

$$P_{out}^{(\mathrm{I})} = P_{out}^{(\mathrm{I})}\Big|_{T_r \le \frac{L}{2}} \cdot \Pr\left[T_r \le \frac{L}{2}\right] + P_{out}^{(\mathrm{I})}\Big|_{T_r > \frac{L}{2}} \cdot \Pr\left[T_r > \frac{L}{2}\right] \quad (7)$$

where the first part represents the relay decodes the message during $L/2$, receiving two copies of the message from source and relay, respectively, while the second part represents the relay fails to decode, receiving both copies of the message from source.

For the first case, the channel capacity of the network is

$$I_{srd}^{(\mathrm{I})} = \log_2\left(1 + \rho\gamma_{sd} + \rho\gamma_{rd}\right) \quad (8)$$

Therefore, the conditioned probability $P_{out}^{(\mathrm{I})}|_{T_r \le k}$ is

$$\begin{aligned} P_{out}^{(\mathrm{I})}\Big|_{T_r \le \frac{L}{2}} &= \Pr\left[\frac{L}{2} I_{srd}^{(\mathrm{I})} < H_t\right] \\ = 1 - e^{-\frac{2^{2H_t/L}-1}{\rho\sigma_{sd}^2}} &- \frac{1}{\sigma_{sd}^2}\int_{\gamma_{sd}=0}^{\frac{2^{2H_t/L}-1}{\rho}} \exp\left[f_1^{(\mathrm{I})}\right] d\gamma_{sd} \end{aligned} \quad (9)$$

and $f_1^{(\mathrm{I})}$can be expressed as

$$f_1^{(\mathrm{I})} = -\frac{\gamma_{sd}}{\sigma_{sd}^2} + \frac{1+\rho\gamma_{sd}}{\rho\sigma_{rd}^2} - \frac{2^{2H_t/L}}{\rho\sigma_{rd}^2} \quad (10)$$

using the result in Appendix, with

$$\begin{gathered} a = 0,\ R = 2H_t \\ \rho_1 = 0,\ \rho_2 = \rho_3 = \rho \end{gathered}$$

For the second case, $P_{out}^{(\mathrm{I})}\Big|_{T_r > \frac{L}{2}}$ is expressed as

$$\begin{aligned} P_{out}^{(\mathrm{I})}\Big|_{T_r > \frac{L}{2}} &= \Pr\left[\frac{L}{2}\log_2\left(1 + 2\rho\gamma_{sd}\right) < H_t\right] \\ &= 1 - \exp\left(-\frac{2^{2H_t/L}-1}{2\rho\sigma_{sd}^2}\right) \end{aligned} \quad (11)$$

In addition the probability $\Pr\left[T_r \le \frac{L}{2}\right]$ in (7) is given by

$$\begin{aligned} \Pr\left[T_r \le \frac{L}{2}\right] &= 1 - \Pr\left[T_r > \frac{L}{2}\right] \\ &= \exp\left(-\frac{2^{2H_t/L}-1}{\rho\sigma_{sr}^2}\right) \end{aligned} \quad (12)$$

Thus the outage probability of DF-I (7) can be expressed by (9), (11) and (12). As we might expect, and the analysis in the sequel confirms, rateless codes improve the performance via mutual information accumulation, so we focus in the sequel on rateless codes as an alternative for efficient transmission.

C. Distributed Space-Time Rateless Coding Scheme(DF-II)

Distributed space-time coding is introduced in [10] to exploit cooperative diversity with repetition coded between the two phases. Employing rateless codes as an alternative, the following protocol is given.

During both the listening and collaboration phase, the source simply passes through the rateless encoded packets while the relay, once decoding the message during $L/2$ channel uses, aligns its output symbols in time to correspond to source's output for collaboration transmission, acting as the secondary antenna in which consecutive pairs of input symbols are transformed according to Alamouti scheme [9]. In other words, if relay accumulates sufficient entropy during $L/2$, standard Alamouti decoding is performed at the receiver, else destination receives symbols only from source. The protocol is illustrated in Fig. 2 (c), and the outage probability for this setting is given by:

$$P_{out}^{(\mathrm{II})} = P_{out}^{(\mathrm{II})}\Big|_{T_r \le \frac{L}{2}} \cdot \Pr\left[T_r \le \frac{L}{2}\right] + P_{out}^{(\mathrm{II})}\Big|_{T_r > \frac{L}{2}} \cdot \Pr\left[T_r > \frac{L}{2}\right] \quad (13)$$

where the first part represents the outage probability that cooperative transmission occurs, whereas the second part represents relay fails to decode, thus source transmits the message alone.

For the first case, the channel capacity for the cooperative transmission is

$$I_{srd}^{(\mathrm{II})} = \log_2\left(1 + \frac{\rho}{2}\gamma_{sd} + \frac{\rho}{2}\gamma_{rd}\right) \quad (14)$$

where the transmit power $\frac{\rho}{2}$ comes from sum power constraint, since the source and relay transmit at the same time. Mutual information accumulation occurs between the two transmission phase. Therefore, the conditioned probability $P_{out}^{(\mathrm{II})}|_{T_r \le k}$ is

$$\begin{aligned} P_{out}^{(\mathrm{II})}\Big|_{T_r \le \frac{L}{2}} &= \Pr\left[\frac{L}{2} I_{sd}^{(\mathrm{II})} + \frac{L}{2} I_{srd}^{(\mathrm{II})} < H_t\right] \\ = 1 - e^{-\frac{2^{R/L}-1}{\rho\sigma_{sd}^2}} &- \frac{1}{\sigma_{sd}^2}\int_{\gamma_{sd}=0}^{\frac{2^{R/L}-1}{\rho}} \exp\left[f_1^{(\mathrm{II})}\right] d\gamma_{sd} \end{aligned} \quad (15)$$

and $f_1^{(\mathrm{II})}$can be expressed as

$$f_1^{(\mathrm{II})} = -\frac{\gamma_{sd}}{\sigma_{sd}^2} + \frac{2+\rho\gamma_{sd}}{\rho\sigma_{rd}^2} - \frac{2^{2H_t/L}}{\frac{\rho}{2}\sigma_{rd}^2\left(1+\rho\gamma_{sd}\right)} \quad (16)$$

where we have utilized the result in Appendix with

$$a = \frac{L}{2},\ R = H_t$$
$$\rho_1 = \rho,\ \rho_2 = \rho_3 = \frac{\rho}{2}$$

For the second case, $P_{out}^{(\mathrm{II})}\Big|_{T_r > \frac{L}{2}}$ is expressed as

$$\begin{aligned} P_{out}^{(\mathrm{II})}\Big|_{T_r > \frac{L}{2}} &= \Pr\left[LI_{sd}^{(\mathrm{II})} < H_t\right] \\ &= 1 - \exp\left(\frac{1 - 2^{H_t/L}}{\rho\sigma_{sd}^2}\right) \end{aligned} \tag{17}$$

Similar to DF-I, inserting (15), (17) and (12) to (13), outage probability for DF-II, in which synchronizing at the symbol level between source and relay is required, can be expressed. This protocol can be easily extended to multiple relay nodes scenario, which is also used as the baseline system for performance comparison in the sequel.

IV. Opportunistic Fountain Scheme

In practice, distributed space-time code design is quite difficult due to the distributed nature of cooperative links. For simplicity, the distributed transmitters are assumed to be synchronized at the cooperative phase, which is hard to achieve in practice [11]. Moreover, when scaling cooperation to more than one relay, the space-time coding scheme is really involved, since the number of elements in the TCS is generally unknown and varying, instead of fixed as in typical space-time codes. Our proposed scheme deals with these problems and provides better outage performance as shown here.

A. The proposed protocol

In the protocols of DF-II, the relay nodes receive their information only from the source, and transmit with partial power to the destination at the same time. However, in our proposed scheme, called *Opportunistic Fountain*, relay nodes receive their information not only from the source, but also from other relays. As a result they can accumulate mutual information faster, enlarging the TCS. Furthermore, at each step only the best relay is chosen, which transmits the message with all power available. Thus, this is an orthogonal transmission protocol which does not need to synchronize between distributed transmitters.

With no loss in generality, the protocol is described in terms of multiple relay nodes as the following steps (see also Fig. 2 (d) for a simplified version of the protocol and Fig. 3 for multiple relay nodes illustration):

1) The source starts to transmit information to the relay nodes and destination using rateless codes.
2) All relay nodes constantly receive information from the transmitter, and accumulate the mutual information.
3) The best transmitter for the next transmission is selected from the TCS, comprised of source and relays with sufficient entropy. Then it switches from reception mode to transmission mode, and transmits information utilizing rateless codes with the same codebook as the source.

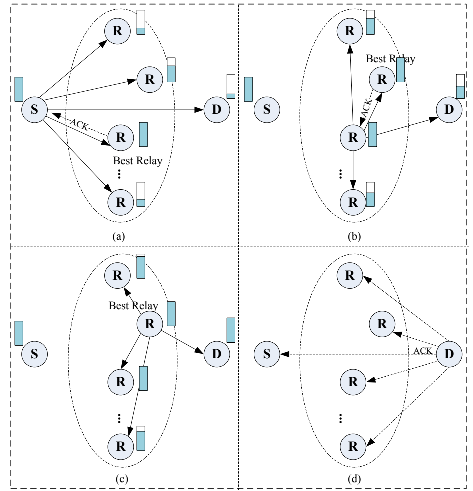

Fig. 3. A multiple relay cooperation scenario for our proposed scheme. The shaded block indicates the amount of entropy accumulated at each nodes. (a) The source transmits the packets encoded by rateless codes until certain relay, as one element of the TCS, accumulates sufficient entropy. Then the best transmitter for destination is selected from the TCS. The selected relay transmits an acknowledgment to the former transmitter, at the same time the former ceases transmission, and the selected relay switch from reception to transmission. (b) The selected relay transmits to other nodes with the same codebook as the source until some one has accumulated sufficient entropy, and the best transmitter for destination is selected among the enlarged TCS. (c) The new selected relay transmits the information encoded by rateless codes. (d) The destination accumulates sufficient entropy and decodes the message, then signals its finish.

4) The destination continues to receive all the transmitted information until sufficient mutual information is accumulated to decode the message or the maximum channel use limit is reached.

The key idea here is that the best relay chosen from the TCS transmits information to the destination with all power available. Moreover, the transmission codeword can also be heard and used by relay nodes that are still receiving.

The method of relay selection, distributed timers, based on instantaneous channel measurements can be found in [11], and an implementation example is described in [12].

B. Outage performance analysis for only one relay

The most tractable model, as described in Fig. 1, is the three-node reduction of the proposed scheme, which greatly simplifies the analysis. In this extreme case, the transmitter is selected between the source and the sole relay; there is no mutual information accumulation among relay nodes; and selection is processed only once for the quasi-static channel assumption. Consequently, the outage probability is given by:

$$P_{out}^{(\mathrm{OF})} = \sum_{k=1}^{L} P_{out}^{(\mathrm{OF})}|_{T_r = k} \cdot \Pr\left[T_r = k\right] \tag{18}$$

which is due to the total probability theorem over disjoint sets T_r that partition the space.

For the first case $k = 1, \cdots, L-1$, which means relay decodes the message before L, $\Pr[T_r = k]$ is [13]

$$\begin{aligned}\Pr\left[T_r = k\right] &= \Pr\left[\left(k-1\right) I_{sr}^{(\mathrm{OF})} < H_t, kI_{sr}^{(\mathrm{OF})} > H_t\right] \\ &= \Pr\left[\left(k-1\right) I_{sr}^{(\mathrm{OF})} < H_t\right] - \Pr\left[kI_{sr}^{(\mathrm{OF})} < H_t\right] \\ &= \exp\left(\frac{1-2^{H_t/k}}{\rho\sigma_{sr}^2}\right) - \exp\left(\frac{1-2^{H_t/(k-1)}}{\rho\sigma_{sr}^2}\right)\end{aligned} \tag{19}$$

Furthermore, the channel capacity in the setting is

$$I_{srd}^{(\mathrm{OF})} = \log_2\left(1+\rho\gamma_{max}\right) \tag{20}$$

where $\gamma_{max} = \max\left(\gamma_{sd}, \gamma_{rd}\right)$. Therefore, the conditioned probability $P_{out}^{(\mathrm{OF})}|_{T_r=k}$ for $k < L$, denoting the outage probability while cooperation occurs after k, is

$$\begin{aligned}P_{out}^{(\mathrm{OF})}|_{T_r=k} &= \Pr\left[kI_{sd}^{(\mathrm{OF})} + \left(L-k\right) I_{srd}^{(\mathrm{OF})} < H_t\right] \\ &= \Pr\left[LI_{sd}^{(\mathrm{OF})} < H_t\right] \Pr\left[\gamma_{rd} < \gamma_{sd}\right] \\ &+ \Pr\left[kI_{sd}^{(\mathrm{OF})} + \left(L-k\right) I_{rd}^{(\mathrm{OF})} < H_t\right] \left(1-\Pr\left[\gamma_{rd} < \gamma_{sd}\right]\right)\end{aligned} \tag{21}$$

where the first term have the same results with (6), and the third term is given by

$$\begin{aligned}&\Pr\left[kI_{sd}^{(\mathrm{OF})} + \left(L-k\right) I_{rd}^{(\mathrm{OF})} < H_t\right] \\ &= 1 - e^{-\frac{2^{R/L}-1}{\rho\sigma_{sd}^2}} - \frac{1}{\sigma_{sd}^2}\int_{\gamma_{sd}=0}^{\frac{2^{R/L}-1}{\rho}} \exp\left[f_1^{(\mathrm{OF})}\right] d\gamma_{sd}\end{aligned} \tag{22}$$

and $f_1^{(\mathrm{OF})}$can be expressed as

$$f_1^{(\mathrm{OF})} = -\frac{\gamma_{sd}}{\sigma_{sd}^2} + \frac{1}{\rho\sigma_{rd}^2} - \frac{2^{H_t/(L-k)}}{\rho\sigma_{rd}^2\left(1+\rho\gamma_{sd}\right)^{k/(L-k)}} \tag{23}$$

where we have utilized the result in Appendix with

$$a = k,\ R = H_t$$
$$\rho_1 = \rho_3 = \rho,\ \rho_2 = 0$$

In addition the probability $\Pr\left[\gamma_{rd} < \gamma_{sd}\right]$ in (21) is

$$\begin{aligned}\Pr\left[\gamma_{rd} < \gamma_{sd}\right] &= \int_{\gamma_{sd}=0}^{\infty}\int_{\gamma_{rd}=0}^{\gamma_{sd}} \frac{\exp\left[-\frac{\gamma_{sd}}{\sigma_{sd}^2} - \frac{\gamma_{rd}}{\sigma_{rd}^2}\right]}{\sigma_{sd}^2\sigma_{rd}^2} d\gamma_{rd} d\gamma_{sd} \\ &= \frac{\sigma_{sd}^2}{\sigma_{sd}^2+\sigma_{rd}^2}\end{aligned} \tag{24}$$

For the case of $k = L$, it means that relay fails to decode before the maximum channel uses L, i.e., there is no cooperation at all. The probability $\Pr[T_r = L]$ is

$$\begin{aligned}\Pr\left[T_r = L\right] &= \Pr\left[\left(L-1\right) I_{sr}^{(\mathrm{II})} < H_t\right] \\ &= 1 - \exp\left(\frac{1-2^{H_t/(L-1)}}{\rho\sigma_{sr}^2}\right)\end{aligned} \tag{25}$$

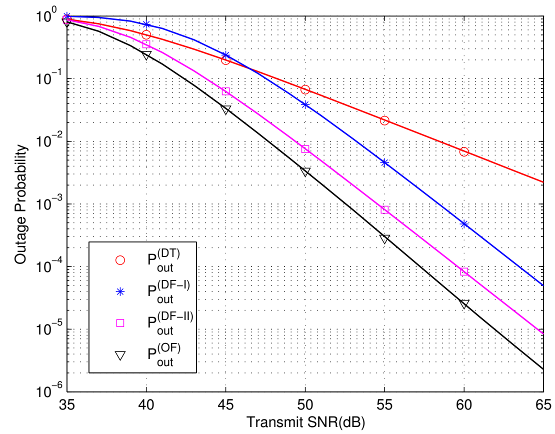

Fig. 4. Outage Probability vs. Transmit SNR for one relay case. $H_t = 12$ bps/Hz, $L = 4$ channel uses, $d_{sr} = 5$ m, $d_{rd} = 5$ m and $d_{sd} = 10$ m. The solid lines are the analytical results, and marks denote the simulated results.

And the conditioned probability $P_{out}^{(\mathrm{OF})}|_{T_r=k}$ for $k = L$ is

$$\begin{aligned}P_{out}^{(\mathrm{OF})}|_{T_r=L} &= \Pr\left[LI_{sd}^{(\mathrm{OF})} < H_t\right] \\ &= 1 - \exp\left(\frac{1-2^{H_t/L}}{\rho\sigma_{sd}^2}\right)\end{aligned} \tag{26}$$

Inserting (19), (21) and (25), (26) to (18), the outage probability for this scheme $P_{out}^{(\mathrm{OF})}$ can be expressed.

For the multiple relay problem, analytic calculation of the outage probability becomes really involved, which motivates a future work.

V. Numerical Results and Discussion

The following distance-dependent path loss model is used to compute the mean of the channel gain [13]:

$$\sigma_{ij}^2 = \left(\frac{d_{ij}}{d_0}\right)^{-n}, (ij) \in \left\{(rd), (rd), (sr), (rr)\right\} \tag{27}$$

where $d_0 = 1$ m is the reference distance, d_{ij} is the distance from terminal i to j, and the path loss exponent $n = 3$. Specifically, σ_{rr}^2 is only valid for the multiple relay case, where mutual help among relays exists.

In Fig. 4, the outage probability is plotted against the transmit SNR ρ for one relay case with the parameter listed below the figure. It is clear that the proposed scheme is advantageous over the baseline schemes. In fact our protocol outperforms DF-II with a gain of about 2 dB, and the simulations denoted by marks confirm our analytical results.

Fig. 5 compares the outage probability, along with network sizes for cooperative transmission protocols of DF-II and the proposed scheme. Successive curves from right to left correspond to larger networks. Clearly, our scheme provides better outage performance for various network sizes as the simulation shown.

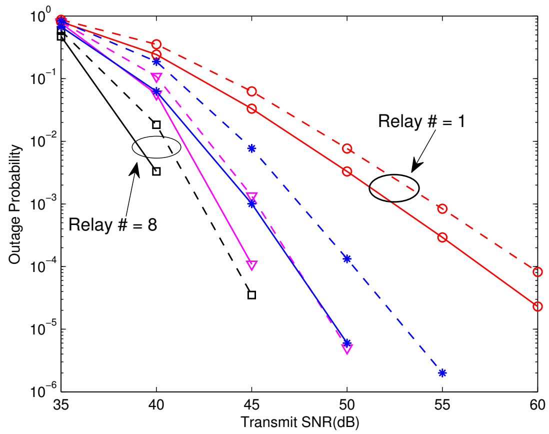

Fig. 5. Outage Probability vs. Transmit SNR for multiple relay nodes. $H_t = 12$ bps/Hz, $L = 4$ channel uses, $d_{rr} = 3$ m, $d_{sr} = 5$ m, $d_{rd} = 5$ m and $d_{sd} = 10$ m. Comparison of DF-II scheme (dashed lines) to Opportunistic Fountain scheme (solid lines) for different network sizes, relay number = 1, 3, 5, 8. Successive curves from right to left correspond to larger networks.

VI. Conclusion

We have proposed a new cooperative transmission protocol, *Opportunistic Fountain*, for half-duplex relaying, based on rateless codes. Our method has two major advantages which distinguish it from other schemes. First, this orthogonal transmission protocol does not need to synchronize between distributed transmitters. Second, with the help of rateless codes, relays in the cooperative networks accumulate mutual information not only from the source but also from other relays, with only the most appropriate one permitted to forward the message.

We have presented closed-form expressions of outage probability, and presented extensive simulations for the simplified one-relay version of the protocol. For larger network sizes, Monte Carlo simulation confirms the benefits of our proposed scheme. The promising results obtained here motivate further study of analytical results for larger networks, and represent a useful direction for continued research.

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Appendix

Note that

$$I_{sd} = \log_2(1 + \rho_1\gamma_{sd})$$
$$I_{srd}^{(1)} = \log_2(1 + \rho_2\gamma_{sd} + \rho_3\gamma_{rd})$$

Consider

$$\begin{aligned} I_{coop}^{(1)} &= aI_{sd} + (L-a)\, I_{srd}^{(1)} \\ &= \log_2\left[(1+\rho_1\gamma_{sd})^a \cdot (1+\rho_2\gamma_{sd}+\rho_3\gamma_{rd})^{L-a}\right] \end{aligned} \quad (28)$$

Hence [13],

$$\begin{aligned} \Pr\left[I_{coop}^{(1)} < R\right] &= \Pr\left[aI_{sd} + (L-a)\, I_{srd}^{(1)} < R\right] \\ &= \Pr\left[\gamma_{rd} < f_1^{(1)}(\gamma_{sd})\right] \\ &= \int_{\gamma_{sd}=0}^{\frac{2^{R/L}-1}{\rho}} \int_{\gamma_{rd}=0}^{f_1^{(1)}} \frac{e^{-\frac{\gamma_{sd}}{\sigma_{sd}^2}-\frac{\gamma_{rd}}{\sigma_{rd}^2}}}{\sigma_{sd}^2\sigma_{rd}^2} d\gamma_{rd} d\gamma_{sd} \\ &= 1 - e^{-\frac{2^{R/L}-1}{\rho\sigma_{sd}^2}} - \frac{1}{\sigma_{sd}^2}\int_{\gamma_{sd}=0}^{\frac{2^{R/L}-1}{\rho}} \exp\left[f_1^{(2)}\right] d\gamma_{sd} \end{aligned}$$

where $\exp\left[-\frac{\gamma_{sd}}{\sigma_{sd}^2} - \frac{\gamma_{rd}}{\sigma_{rd}^2}\right]/\sigma_{sd}^2\sigma_{rd}^2$ is the joint distribution of γ_{sd} and γ_{rd}, and

$$f_1^{(1)} = \frac{2^{R/(L-a)}}{\rho_3\,(1+\rho_1\gamma_{sd})^{a/(L-a)}} - \frac{1+\rho_2\gamma_{sd}}{\rho_3}$$
$$f_1^{(2)} = -\frac{\gamma_{sd}}{\sigma_{sd}^2} + \frac{1+\rho_2\gamma_{sd}}{\rho_3\sigma_{rd}^2} - \frac{2^{R/(L-a)}}{\rho_3\sigma_{rd}^2\,(1+\rho_1\gamma_{sd})^{a/(L-a)}}$$

References

[1] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Comm. Mag.*, vol. 42, no. 10, pp. 74–80, Oct. 2004.

[2] J. Castura and Y. Mao, "Rateless coding over fading channels," *IEEE Comm. Lett.*, vol. 10, no. 1, pp. 46–48, Jan 2006.

[3] ——, "Rateless coding and relay networks," *IEEE Signal Processing Mag.*, vol. 24, no. 5, pp. 27–35, 2007.

[4] A. F. Molisch, N. B. Mehta, J. S. Yedidia, and J. Zhang, "Performance of fountain codes in collaborative relay networks," *IEEE Trans. Wireless Comm.*, vol. 6, no. 11, pp. 4108–4119, 2007.

[5] R. Nikjah and N. C. Beaulieu, "Achievable rates and fairness in rateless coded decode-and-forward half-duplex and full-duplex opportunistic relaying," in *Proc. IEEE ICC*, May 2008.

[6] D. J. MacKay, "Fountain codes," *IEE Proc. - Comm.*, vol. 152, no. 6, pp. 1062–8, 2005.

[7] M. Mitzenmacher, "Digital fountains: A survey and look forward," *Proc. IEEE Inf. Theory Workshop*, pp. 271–276, 2004.

[8] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[9] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Comm.*, vol. 16, no. 8, pp. 1451–1458, 1998.

[10] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[11] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Select. Areas Comm.*, vol. 24, no. 3, pp. 659–672, March 2006.

[12] A. Bletsas, "Intelligent antenna sharing in cooperative diversity wireless networks," Ph.D. dissertation, MIT, 2005.

[13] R. Narasimhan, "Throughput-delay performance of half-duplex hybrid-arq relay channels," in *Proc. IEEE ICC*, May 2008.