Opportunistic Cooperation for Multi-Antenna Multi-Relay Networks

Weiliang Zeng, *Student Member, IEEE*, Chengshan Xiao, *Fellow, IEEE*, Youzheng Wang, and Jianhua Lu, *Senior Member, IEEE*

***Abstract*—A low-complexity, near-optimal transmit antenna selection algorithm is proposed for multi-relay networks where all nodes are equipped with multiple antennas. We first establish a system model and a unified capacity maximization framework for a two-hop opportunistic relaying scheme where the source node (S) transmits signals to multiple relay nodes (R) in the first time slot, and the selected relay antennas and their corresponding relay nodes receive, decode and forward the messages to the destination (D) in the second time slot. Based on the system model, we develop a transmit antenna selection algorithm that maximizes the network capacity assuming that the channel state information is available at the receivers but not available at the transmitters, and total transmit power constraints are imposed on source/relay transmitters. The proposed algorithm first constructs a sorted list of relay antennas with decreasing S-R capacities, then iteratively maximizes the R-D capacity over a candidate antenna set using a low-complexity, near-optimal antenna selection scheme. The candidate set is reduced in the next iteration according to the selected antenna set of the current iteration. The overall network capacity is computed for the selected antenna sets of all iterations, and the set yielding the highest S-R-D capacity is the solution to the maximization problem. We show that this novel iterative algorithm achieves near-optimal solution and has a polynomial-time complexity. We also derive the lower and upper bounds of the achievable network capacity for both average capacity and outage capacity. Numerical examples show the significant performance gains obtained via the proposed scheme compared to its conventional counterparts.**

***Index Terms*—Opportunistic relaying, relay networks, multiple-input multiple-output (MIMO), non-deterministic polynomial-time hard (NP-hard).**

I. Introduction and Related Work

COOPERATIVE relaying is an emerging technology that can compensate for the effects of multipath fading and shadowing by employing multiple relay nodes between sources and destinations. It has the potential to provide reliable transmission, high throughput, and broad coverage, thus drawing great attention in recent years, see [1]–[3] and references therein.

The performance of a cooperative network is often influenced by two major factors: network configuration and cooperative strategy. Network configuration refers to the numbers of source, relay, and destination nodes, the number of antennas employed by each node, and the channel state information (CSI) available at each node. This study focuses on one source-destination pair with multiple relay nodes, where nodes may be equipped with *different* numbers of antennas. This generalized configuration includes several special cases investigated in the literature: the single-antenna single-relay network [1]–[4], single-antenna multiple-relay network [5]–[8], multiple-antenna single-relay network [9], [10], and the multiple-antenna multiple-relay network in [11], [12] with the same number of antennas for all relay nodes. Furthermore, we assume that CSI is available at receivers but not at transmitters. This means that the source does not know the channel, relay nodes know their corresponding source-relay (S-R) channel, and the destination has full knowledge of all channels. This CSI assumption is practically feasible through pilot-assisted channel estimation and S-R channel knowledge forwarding.

We fully exploit the advantage of available CSI by adopting the decode-and-forward (DF) strategy [3] rather than the amplify-and-forward (AF) scheme [3], because the DF scheme has the potential to vary the communication rate and prevent error propagation [2]–[4], [8]. However, the analysis of DF scheme is more challenging due to the nonlinear processing at relay nodes.

Selection of transmit antennas and relay nodes is an important research topic in the DF cooperative strategy. For single-antenna multiple-relay network, only relay selection is needed. Opportunistic relaying was introduced by Bletsas *et al.* in [5], [6] for both DF and AF schemes, in which the best relay node is selected to forward the information, while other nodes simply keep silent. Besides its low complexity and high power efficiency, it is proved to be optimal in terms of outage probability and network capacity for single-antenna networks. It was extended to partial and imperfect CSI cases for single-antenna multiple-relay network in [14], [15]. For multiple-antenna single-relay network, transmit antenna selection is considered for the AF scheme, and a heuristic method is proposed to maximize the end-to-end performance [16]. In multiple-antenna multiple-relay network, both transmit antenna selection and relay selection can be considered, but

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W. Zeng and J. Lu are with the Tsinghua National Laboratory for Information Science and Technology, Department of Electronic Engineering, Tsinghua University, Beijing 100084, China (e-mail: {zengwl, lujh}@wmc.ee.tsinghua.edu.cn). This work has been carried out while W. Zeng is a visiting scholar at Missouri University of Science and Technology.

C. Xiao is with the Department of Electrical and Computer Engineering, Missouri University of Science and Technology, Rolla, MO 65409, USA (e-mail: xiaoc@mst.edu).

Y. Wang is with the Department of Aeronautics and Astronautics Engineering, Tsinghua University, Beijing 100084, China (e-mail: yzhwang@mail.tsinghua.edu.cn).

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limited work has been done. In [11], the opportunistic relaying scheme in [5] was extended to multiple-antenna relay network such that only one relay is selected at a time using all antennas of the selected node for DF. This scheme is shown to provide larger performance gains than several AF based schemes as well as single-antenna multiple-relay schemes. However, this straightforward extension does not fully exploit the spatial degrees of freedom of the multiple-antenna multiple-relay channels. Therefore, there exhibits a big gap between the performance of [11] and the achievable one [17].

In this paper, we propose a flexible scheme for both antenna and relay selection in DF multi-antenna multi-relay networks. The proposed scheme allows multiple relays to be selected. The selected relays use all their antennas for reception and decoding, but may only use a few antennas for transmission. We assume that relay nodes share the same bandwidth and have a total transmit power constraint. Since CSI is not available at the transmitters, equal power is allocated to every selected transmit antenna. To select multiple transmit antennas distributed on multiple relay nodes to forward messages simultaneously, we choose the network capacity as the utility function, which is the maximal amount of information that can be reliably transmitted between the source and destination for a given transmit antenna set. The maximization of this utility function over all possible sets falls in the framework of combinatorial optimization. However, this problem can be shown to be nonconvex and non-deterministic polynomial-time hard (NP-hard). An exhaustive search is to evaluate all the transmit antenna subsets for optimal solution, but its computational complexity grows exponentially with the total number of transmit antennas, N_T, at all relay nodes. That is intractable when the size of the network is large.

We propose a low-complexity iterative algorithm to solve this NP-hard maximization problem. The proposed algorithm first computes the capacity of each source-relay MIMO channel and sorts all nodes into a list with decreasing S-R link capacity. Then for a given size of transmit antenna selection, it selects transmit antennas to maximize the R-D capacity over a candidate transmit antenna set. The candidate set is reduced through iterations and the overall network capacities of the selected antenna sets at all iterations are compared. The set yielding the highest network capacity is the final solution.

The novelty of the algorithm is the iterative approach enabled by the new candidate set reduction technique. We prove that the maximal network capacity can be achieved on one of these important sets provided by different iterations. Although finding out these important sets is still a NP-hard problem, there exists several low-complexity algorithms with performance very close to the exact solution. For practical applications, the proposed algorithm utilizes a near-optimal MIMO antenna selection algorithm [33] to reduce the computational complexity to $\mathcal{O}\left(N_T^5\right)$ comparing to that of the exact search $\mathcal{O}\left(2^{N_T}N_T^3\right)$.

We also derive the lower and upper bounds of the network capacity in terms of average capacity and outage capacity, and investigate the relationship between the proposed selection method and the existing literature. Several simulation examples are provided to demonstrate the performance gains of our transmit antenna selection scheme. By selecting multiple transmit antennas involved in the message forwarding judiciously, better performance is achieved with the same transmit power while the network complexity is kept low because not all nodes nor all antennas are utilized for transmission.

The remainder of the paper is organized as follows. In Sec. II, we introduce the signal model for the new two-hop relaying scheme. We also develop the capacity tradeoff between the source-relay and relay-destination phases and formulate the transmit antenna selection problem for multi-antenna multi-relay networks. Section III and IV proposes the transmit antenna selection algorithm and analyzes the performance of the proposed method, respectively. Section IV derives the lower and upper bounds of the network capacity, and Sec. V presents several numerical examples to demonstrate the performance gain of the proposed approach over the existing ones. Finally, conclusions are drawn in Sec. VI.

Notation: We use $\mathbb{R}$ and $\mathbb{C}$ to denote the real and complex spaces. Boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, and italics denote scalars. The superscripts $(\cdot)^T$ and $(\cdot)^H$ stand for transpose and conjugate transpose, respectively. The subscripts $[\mathbf{A}]_{i,j}$ and $[\mathbf{A}]_{:,j}$ denote the (i,j)th element and jth column of matrix $\mathbf{A}$, respectively, and $[\mathbf{c}]_i$ denotes the ith element of vector $\mathbf{c}$. Furthermore, $\mathbf{I}_M$ denotes the $M \times M$ identity matrix; $\oplus_{i=1}^n \mathbf{A}_i$ denotes a block diagonal matrix with block elements given by $\mathbf{A}_i$; and $\det(\mathbf{A})$ denotes the determinant. For a set $\mathcal{X}$, the operator $|\mathcal{X}|$ denotes the cardinality of the set, *i.e.*, the number of the elements in the set. Besides, all logarithms are base 2.

II. Problem Formulation

A two-hop multiple-antenna multiple-relay network is considered as shown in Fig. 1, where the source node (S) with N_s transmit antennas attempts to communicate to the destination node (D) with N_d receive antennas through the assistance of relay nodes (R). The size of the relay network, *i.e.*, the total number of relay nodes, is K, comprised of either cooperative relays [1] or fixed relay stations [2], [8], [18]. The number of transmit antennas at the jth relay is denoted by N_j, for $j = 1, 2, \cdots, K$, and the total number of transmit antennas is $N_T = \sum_{j=1}^{K} N_j$. The direct link between the source and destination is ignored assuming large distance and significant pathloss in comparison to the relay links in practical deployment [2].

A. Channel and Signal Model

We model the S-R and R-D links as quasi-static flat-fading channels, which are applicable to scenarios of narrow-band transmissions in a low-mobility environment. We also use the standard assumption that perfect channel state information (CSI) is available at the receiver side but not at the transmitter side. This means that the source node does not know the channel; the jth relay node, $j = 1, \cdots, K$, knows the CSI of the corresponding channel from source to itself; and the destination node obtains full knowledge of both S-R and R-D channels. In slow varying channels, the necessary CSI can be obtained with high precision at reasonably low cost through pilot assisted channel estimation.

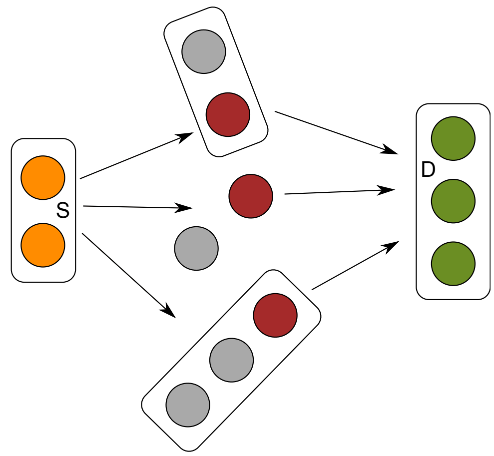

Fig. 1. System model of a dual-hop multiple-antenna multiple-relay network. Some transmit antennas of the relay nodes are allowed to forward messages while others keep silent.

Moreover, the data transmission is over two time slots at the same bandwidth. In the first time slot, the source transmits signals to all relay nodes (henceforth called the S-R phase). Then, only N_L of the N_T transmit antennas are selected to forward messages in the second time slot (R-D phase). This is one of the main differences between the proposed scheme and the method of optimal selection routing (OSR) in [11]. The latter assumes all the selected transmit antennas are from one and only one relay node.

Without CSI at the source node, a reasonable strategy is to utilize uniform power allocation across the antennas at the source. Consequently, the complex baseband equivalent signal received at the jth relay can be written as

$$\mathbf{r}_j = \sqrt{\frac{P_s}{N_s}}\mathbf{H}_j\mathbf{x}_s + \mathbf{n}_j,\ j = 1, 2, \cdots, K, \tag{1}$$

where P_s denotes the total transmit power of the source node; $\mathbf{x}_s \in \mathbb{C}^{N_s\times 1}$ is the transmitted signal vector; $\mathbf{n}_j \in \mathbb{C}^{N_j\times 1}$ is the channel noise vector, assuming to be independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance $\sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{N_j}\right)$; and $\mathbf{H}_j \in \mathbb{C}^{N_j\times N_s}$ is the channel matrix from the source node to the jth relay, which can be further expressed as $\mathbf{H}_j = \sqrt{\alpha_j}\widetilde{\mathbf{H}}_j$, where the elements of $\widetilde{\mathbf{H}}_j$ are i.i.d. complex Gaussian random variables with unit variance, and each factor α_j captures the effects of path-loss and shadowing.

Each relay node decodes its received signal independently, and N_L transmit antennas are selected to forward the signal $\mathbf{x}_r \in \mathbb{C}^{N_L\times 1}$ to the destination simultaneously in the R-D phase. We place a total transmit power constraint, P_r, across the selected transmit antennas, which can control the received power at the destination and reduce the interference to neighboring transceiver pairs that share the same frequency [12], [19], [20]. This setting can be easily generalized to networks whose nodes have individual power constraint. Without CSI of the overall R-D channels at transmitters, uniform power is also allocated across the selected transmit antennas. The signal received at the destination can then be written as:

$$\mathbf{y} = \sqrt{\frac{P_r}{N_L}}\mathbf{G}_L\mathbf{x}_r + \mathbf{n}_d, \tag{2}$$

where $\mathbf{y} \in \mathbb{C}^{N_d\times 1}$ is the received signal vector; $\mathbf{G}_L \in \mathbb{C}^{N_d\times N_L}$ is the channel matrix from the N_L selected transmit antennas to the destination, which can also be written as $\mathbf{G}_L = \widetilde{\mathbf{G}}_L\mathbf{W}_d$, where the entries of $\widetilde{\mathbf{G}}_L \in \mathbb{C}^{N_d\times N_L}$ are i.i.d. complex Gaussian variables with zero mean and unit covariance matrix, and $\mathbf{W}_d = \oplus_{i=1}^{N_L}\left(\sqrt{\beta_i}\right)$ captures the effects of path-loss and shadowing between the selected transmit antennas and the receive antennas at the destination; and $\mathbf{n}_d \in \mathbb{C}^{N_d\times 1}$ is the complex additive white Gaussian noise at the destination with zero mean and unit covariance matrix.

B. Capacity Tradeoff of Relaying

From the signal model (1), we can express the Shannon capacity of the MIMO channel from the source to the jth relay as

$$C_{s,j} = \log\det\left(\mathbf{I}_{N_s} + \frac{P_s}{N_s}\mathbf{H}_j^H\mathbf{H}_j\right),\ j = 1, 2, \cdots, K. \tag{3}$$

Assume a subset of the relay nodes, denoted as $\mathcal{R}$, are active, and they listen to the signal transmitted from the source and decode it independently using all their antennas based on the known channel matrix $\mathbf{H}_j$ at the relay receivers. The S-R channels can be viewed as a multicast channel from the information theoretic standpoint [21], [22] and the maximal achievable rate is given as

$$C_{s,r}\left(\mathcal{R}\right) = \min_{j\in\mathcal{R}}\left(C_{s,j}\right). \tag{4}$$

In the second time slot, N_L transmit antennas are selected to forward messages by uncorrelated and independently encoded substreams. The distributed encoding scheme can be realized via the parallel channel coding [23] or rateless coding with different random generator seeds at each transmit antenna (see [7], [8], [24], [25] and references therein). Assuming that all antennas at the destination are used to receive and decode the substreams, the Shannon capacity from the selected transmit antennas to the destination is given by

$$C_{r,d}\left(\mathcal{T}\right) = \log\det(\mathbf{I}_{N_d} + \frac{P_r}{|\mathcal{T}|}\mathbf{G}_L\mathbf{G}_L^H), \tag{5}$$

where $\mathcal{T}$ is the selected transmit antenna set, and $|\mathcal{T}| = N_L$.

In practice, the capacity in (5) can be approached with MMSE-SIC detector, a combination of minimum mean-square error (MMSE) estimation and successive interference cancellation (SIC) at the destination [26], [27], and also by the strategy of rateless coding and acknowledgment [24].

Based on (4) and (5), the capacity between the source and destination of the two-hop relay scheme is defined as [9]:

$$C_{srd}\left(\mathcal{R}, \mathcal{T}\right) = \frac{1}{2}\min\left\{C_{s,r}\left(\mathcal{R}\right), C_{r,d}\left(\mathcal{T}\right)\right\}, \tag{6}$$

where the factor $1/2$ is the penalty for the half multiplexing loss. This capacity is also named as the network capacity.

The objective of relay node and antenna selection is to maximize the network capacity (6) over possible selection sets $\mathcal{R}$ and $\mathcal{T}$ with some practical constraints. Intuitively, there exists an optimal number of transmit antenna, N_L, that maximizes the R-D link capacity due to the total power constraint, P_r. This in turn plays an important role in maximizing the overall network capacity. Therefore, we constrain the selection of the nodes, $\mathcal{R}$, to those containing the selected transmit antennas. That is $\mathcal{R} = \mathcal{N}(\mathcal{T})$, where the operator $\mathcal{N}(x)$ denotes the relay nodes containing the antenna set x. Note that each selected relay node utilizes all its antennas to receive in the S-R phase, but may use only some of its antennas to transmit in the R-D phase. If we let $\mathcal{A}(\mathcal{R})$ denote all antennas provided by the node set $\mathcal{R}$, then $\mathcal{T} \subseteq \mathcal{A}(\mathcal{R})$.

The antenna selection problem is then expressed as:

$$\begin{aligned} \text{maximize}_{\mathcal{T}} \quad & \frac{1}{2}\min\left\{C_{s,r}\left(\mathcal{R}\right), C_{r,d}\left(\mathcal{T}\right)\right\} & (7)\\ \text{subject to} \quad & \mathcal{T} \subseteq \{1, \cdots, N_T\} & (8)\\ & \mathcal{R} = \mathcal{N}(\mathcal{T}). & (9) \end{aligned}$$

The integer programming (7) is NP-hard as will be shown in Sec. IV-A. Exhaustive search is often prohibitively expensive with exponential computational complexity. In addition, separately maximizing $C_{s,r}\left(\mathcal{R}\right)$ or $C_{r,d}\left(\mathcal{T}\right)$ often does not achieve a good solution, because there exists a fundamental *tradeoff* between the capacities of the S-R and R-D links. In general, the optimal solution of maximizing $C_{s,r}\left(\mathcal{R}\right)$ leads to bad performance on $C_{r,d}\left(\mathcal{T}\right)$ and vice versa. Therefore, in the next section, we develop a low complexity algorithm to iteratively solve (7) to achieve a good balance between $C_{s,r}\left(\mathcal{R}\right)$ and $C_{r,d}\left(\mathcal{T}\right)$.

III. Transmit Antenna Selection Algorithm

Based on the unified framework established in Sec. II-B, we now propose an efficient transmit antenna selection algorithm to maximize the network capacity in (7). The proposed algorithm uses iterative antenna selection over a sorted list to balance the S-R and R-D channels and to maximize the overall network capacity. For a given number of transmit antennas, N_L, the proposed antenna selection algorithm consists of four steps:

1) *Sort S-R link capacity:* construct a sorted list of antennas according to the decreasing order of $C_{s,j}$ for all relay nodes;
2) *Maximize R-D capacity over a transmit antenna candidate set:* in the i-th iteration, consider a candidate set of transmit antennas, $\mathcal{V}_i$, then solve for the optimal or near-optimal transmit antenna set that maximizes the R-D capacity $C_{r,d}$. Denote the solution as $\mathcal{T}^{(i)}$;
3) *Iterate with reduced candidate set:* repeat step 2 by reducing the candidate set of transmit antennas $\mathcal{V}_{i+1}$ according to $\mathcal{T}^{(i)}$, and maximizing $C_{r,d}$ over the new candidate set;
4) *Find the optimal solution:* stop the iterations when the candidate set is irreducible, then compute the overall network capacity $C_{srd}^{(i)}$ for all selections of $\mathcal{T}^{(i)}$. The one that yields the maximal network capacity is the solution to (7) for a given N_L.

We now present each step of the algorithm in details.

A. Step 1: sorting S-D link capacity

To construct the sorted list, we first sort all relay nodes with decreasing S-R capacity, *i.e.*, $C_{s,1} \geq C_{s,2} \geq \cdots \geq C_{s,K}$. Then

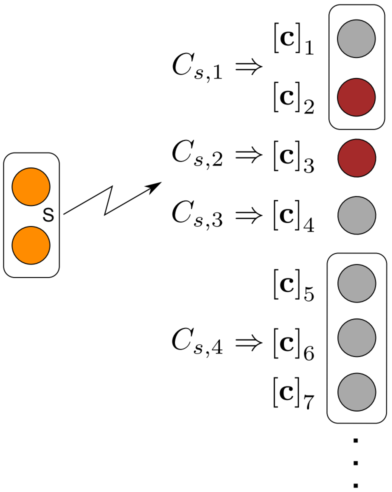

Fig. 2. The S-R phase of the two-hop multiple-antenna multiple-relay network. For a given transmit selection set $\mathcal{T}$, the capacity of the S-R phase is limited by the worst transmit antenna, that is, $C_{s,r}\left(\mathcal{T}\right) = \min_{t\in\mathcal{T}}\left([\mathbf{c}]_t\right)$.

we define the S-R capacity for each antenna by assigning the capacity of the corresponding relay node and form a vector

$$\mathbf{c} = \text{diag}\left(\oplus_{j=1}^{K}\left(C_{s,j}\mathbf{I}_{N_j}\right)\right), \tag{10}$$

where the vector length is $N_T = \sum_{j=1}^{K} N_j$, and the element $[\mathbf{c}]_t$ denotes the capacity from the source to the relay node containing the t-th antenna. This is illustrated by the example shown in Fig. 2. Note we assume that if transmit antenna t is selected to forward messages, the relay node containing transmit antenna t will use all the equipped antennas of the node to decode messages from the source node. Therefore, the S-R capacity for every antenna of the node equals to the capacity from the source to the corresponding relay node. Moreover, $[\mathbf{c}]_t \geq [\mathbf{c}]_{t+1}$ for all $t < N_T$.

B. Step 2: maximizing R-D link capacity

For a given candidate set of transmit antennas, $\mathcal{V}_i$, and a given number of selected antennas, N_L, maximizing the R-D capacity $C_{r,d}\left(\mathcal{T}^{(i)}\right)$ via antenna selection can be expressed as

$$\begin{aligned} \text{maximize} \quad & C_{r,d}\left(\mathcal{T}^{(i)}\right) & (11)\\ \text{subject to} \quad & \mathcal{T}^{(i)} \subseteq \mathcal{V}_i. & (12) \end{aligned}$$

This problem is solved by adopting the antenna selection algorithms developed for MIMO systems [28], [29]. Without loss of generality, we denote the antenna selection scheme for solving (11) as *AntSel*. We also define the optimal value and the optimal set of (11) as $C_{r,d^\star}^{(i)}$ and $\mathcal{T}_\star^{(i)}$, respectively.

C. Step 3: iterating with reduced candidate set

In the first iteration, the candidate subset includes all antennas, $\mathcal{V}_1 = \{1, 2, \cdots, N_T\}$. Using the MIMO antenna selection algorithm, *AntSel*, N_L elements is selected from the set $\mathcal{V}_1$ to obtain the maximal R-D capacity $C_{r,d^\star}^{(1)}$ and the optimal transmit antenna set $\mathcal{T}_\star^{(1)}$. This selection maximizes

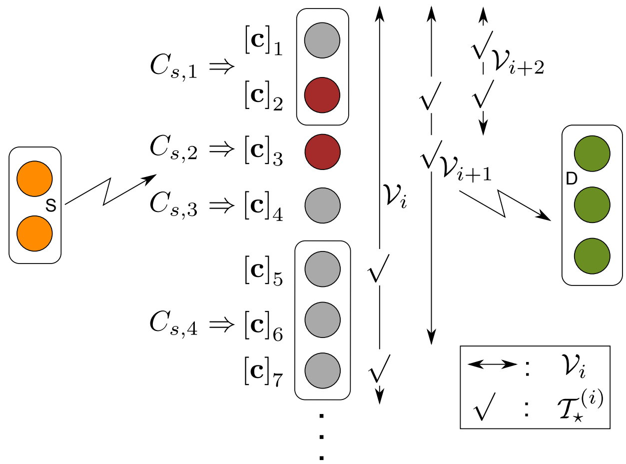

Fig. 3. Illustration of the proposed transmit antenna selection algorithm for multi-antenna multi-relay networks through an example. The algorithm starts with the ordered relay nodes and selects the transmit antennas from a decreasing subset to maximize $C_{r,d}$ at each iteration.

the capacity of the R-D channel, but often leads to low S-R capacity, denoted as $C_{s,r}^{(1)}$, and we have

$$C_{s,r}^{(i)} = [\mathbf{c}]_{n(i)}, \tag{13}$$

$$n(i) = \arg\max_{t\in\mathcal{T}_\star^{(i)}} t, \tag{14}$$

where $i = 1$ for the first iteration. The result, $n(1)$, may be way down at the bottom of the sorted list, as illustrated in the example of Fig. 3. Therefore, the overall network capacity for this selection can be quite low because

$$C_{srd}^{(i)} = \frac{1}{2}\min\left(C_{s,r}^{(i)}, C_{r,d^\star}^{(i)}\right). \tag{15}$$

We try to balance the capacities of the S-R and R-D channels through iterative maximization of (11). In the $(i+1)$-th iteration, the transmit antenna in $\mathcal{T}_\star^{(i)}$ with the lowest S-R capacity is identified and discarded along with the ones lower in the ordered list. The candidate transmit antenna set is reduced from $\mathcal{V}_i$ to $\mathcal{V}_{i+1}$, where $\mathcal{V}_{i+1} = \{1, 2, \cdots, n(i) - 1\}$. Then, the integer programming of (11) is solved again with the antenna selection algorithm *AntSel*,

D. Step 4: finding the optimal set

Denoting the number of iterations as m, we have $m \le (N_T - N_L + 1)$ and

$$C_{s,r}^{(1)} \le C_{s,r}^{(2)} \le \cdots \le C_{s,r}^{(m)}, \tag{16}$$

$$C_{r,d^\star}^{(1)} \ge C_{r,d^\star}^{(2)} \ge \cdots \ge C_{r,d^\star}^{(m)}. \tag{17}$$

This is because the cardinality of the candidate set is decreasing from one iteration to the next. Computing the network capacity $C_{srd}^{(1)}, \cdots, C_{srd}^{(m)}$ for the selected set of each iteration using (15), we find the set that yields the maximal network capacity

$$\mathcal{T}_\star = \arg\max_{\mathcal{T}_\star^{(i)}}\left(C_{srd}^{(1)}, \cdots, C_{srd}^{(m)}\right). \tag{18}$$

We would like to point out that the candidate set reduction technique in the algorithm reduces the number of possible selection subsets in the search space, but it guarantees to find the optimal solution to maximize the network capacity. This is presented in Proposition 1.

Proposition 1: For a given size of selected transmit antennas, the set that maximizes the network capacity will be achieved by one of the sets, $\mathcal{T}_\star^{(1)}, \cdots, \mathcal{T}_\star^{(m)}$, found by each iteration.

Proof: We prove this proposition by contradiction. Assume that there exists a transmit antenna set $\mathcal{K} \notin \left\{\mathcal{T}_\star^{(1)}, \cdots, \mathcal{T}_\star^{(m)}\right\}$ such that $C_{srd}(\mathcal{K}) > \max_i\left(C_{srd}^{(i)}\right)$. Thus, we have $C_{srd}(\mathcal{K}) > C_{srd}^{(i)}$ for all $i = 1, \cdots, m$.

The S-R capacity corresponding to the set $\mathcal{K}$ is $C_{s,r}(\mathcal{K}) = [\mathbf{c}]_n$, where $n = \arg\max_{t\in\mathcal{K}} t$.

1) When $C_{s,r}(\mathcal{K}) < C_{s,r}^{(1)}$, we obtain from (11) that $C_{r,d}(\mathcal{K}) \le C_{r,d}(\mathcal{T}_\star^{(1)})$, since $\mathcal{K} \subseteq \mathcal{V}_1 = \{1, 2, \cdots, N_T\}$. Hence, we have $C_{srd}(\mathcal{K}) \le C_{srd}^{(1)}$, which contradicts to the assumption.

2) Consider the case that there exists an iteration i such that $C_{s,r}^{(i)} < C_{s,r}(\mathcal{K}) \le C_{s,r}^{(i+1)}$.

Since the candidate set that transmit antennas are selected from at the $(i+1)$th iteration is $\mathcal{V}_{i+1} = \{1, 2, \cdots, n(i) - 1\}$, We have $\mathcal{K} \subseteq \mathcal{V}_{i+1}$, that is, $C_{r,d}(\mathcal{K}) \le C_{r,d^\star}^{(i+1)}$.

Because $C_{s,r}^{(i+1)} \ge C_{s,r}(\mathcal{K})$ and $C_{r,d^\star}^{(i+1)} \ge C_{r,d}(\mathcal{K})$, we have $C_{srd}^{(i+1)} \ge C_{srd}(\mathcal{K})$, which also contradicts to the assumption.

3) The case $C_{s,r}(\mathcal{K}) > C_{s,r}^{(m)}$ does not exist because the candidate set $\mathcal{V}_m$ is irreducible, that is, there cannot exist a set $\mathcal{K}$ such that $\mathcal{K} \subset \mathcal{V}_m$.

Consequently, we can conclude that the set that maximizes the network capacity is one of the solutions given by different iterations, *i.e.*, $\mathcal{K} \in \left\{\mathcal{T}_\star^{(1)}, \cdots, \mathcal{T}_\star^{(m)}\right\}$. □

The transmit antenna selection algorithm for multi-antenna multi-relay networks is summarized in Algorithm 1.

Algorithm 1: Transmit Antenna Selection for Multi-Antenna Multi-Relay Networks

1) Initialization:
   $i = 1$, $\mathcal{V}_i = \left\{1, 2, \cdots, \sum_{j=1}^{K} N_j\right\}$;
   arrange $C_{s,j}$, $j = 1, \cdots, K$, in descending order; and $\mathbf{c} = \mathrm{diag}\left(\oplus_{j=1}^{K}\left(C_{s,j}\mathbf{I}_{N_j}\right)\right)$.
2) For different selected transmit antenna number N_L, in each iteration i, select transmit antenna subset $\mathcal{T}^{(i)}$ as
   $$\left(C_{r,d}^{(i)}, \mathcal{T}^{(i)}\right) = \mathrm{AntSel}\left(P_r, \mathbf{G}_T, \mathcal{V}_i, N_L\right).$$
   Let $n = \arg\max_{t\in\mathcal{T}^{(i)}} t$, $C_{s,r}^{(i)} = [\mathbf{c}]_n$, and $C_{srd}^{(i)} = \frac{1}{2}\min\left(C_{s,r}^{(i)}, C_{r,d}^{(i)}\right)$.
3) If $\mathcal{V}_i = \{1, 2, \cdots, N_L\}$, calculate $J_{N_L} = \arg\max_i C_{srd}^{(i)}$. The transmit antenna set and network capacity for N_L selected transmit antennas is $\mathcal{T}_{N_L} = \mathcal{T}^{(J_{N_L})}$ and $C_{N_L} = C_{srd}^{(J_{N_L})}$, respectively;
   else, $i = i + 1$, $\mathcal{V}_i = \{1, 2, \cdots, n - 1\}$ and go to step 2.
   If $N_L = N_T$, go to step 4; else, let $N_L = N_L + 1$ and go to step 1.
4) Calculate $J = \arg\max_{N_L} C_{N_L}$. The selected transmit antenna set and the network capacity are $\mathcal{T} = \mathcal{T}_J$ and $C = C_J$, respectively.

IV. Performance of The Proposed Algorithm

A. Complexity Analysis

We would like to point out several important aspects on the complexity of the proposed algorithm.

Intuitively speaking, if $C_{s,r}(\mathcal{R})$ is larger than the maximal $C_{r,d}(\mathcal{T})$ for all possible selection set $\mathcal{T}$, the transmit antenna selection problem for relay networks in (7) will be simplified into the antenna selection problem of MIMO systems (11). From this point of view, we can conclude that the antenna selection in MIMO system is a *specific case* of the transmit antenna selection problem considered in this study. In other words, the problem of transmit antenna selection in (7) is *much more difficult* than that of antenna selection in MIMO system (11).

However, a simple exact solution to MIMO antenna selection problem (11) does not exist [28], and the problem (11) is claimed to be NP-hard [30]–[32]. Hence, the problem of transmit antenna selection for relay networks considered in this study is also a NP-hard problem.

For a given N_L, the exact solution to transmit antenna selection problem (7) involves an exhaustive search over all possible $\binom{N_T}{N_L}$ transmit antenna sets. For each possible set, it requires about N_L^3 additions/multiplications to compute the matrix multiplication and determinant. Thus the overall complexity is approximately $\binom{N_T}{N_L}N_L^3$, which grows exponentially with N_T for $N_L \approx N_T/2$. Moreover, the complexity of exact search for the case of unknown N_L will be

$$\sum_{N_L=1}^{N_T}\binom{N_T}{N_L}N_L^3 = \frac{1}{2}\cdot 2^{N_T}\cdot N_T + \frac{3}{4}\cdot 2^{N_T}\cdot N_T\cdot(N_T-1) + \frac{1}{8}\cdot 2^{N_T}\cdot N_T\cdot(N_T-1)\cdot(N_T-2), \quad (19)$$

that is, its asymptotic expression is $\mathcal{O}\left(2^{N_T}N_T^3\right)$.

In contrast, the proposed algorithm solves this selection by calling the antenna selection subroutine $AntSel$ for less than $(N_T - N_L + 1)$ times. Several near-optimal algorithms for MIMO antenna selection are available in the literature. In the sequel, we adopt the fast antenna selection algorithm in [33] to solve (11). The algorithm is originally design for selecting MIMO receive antennas assuming all transmit antennas are used. We modify the algorithm by replacing the received SNR with transmit power constraint P_r and replacing the receive channel matrix with the forward R-D channel, $\mathbf{G}_T \in \mathbb{C}^{N_d\times N_T}$. It can achieve near-optimal solution with a low computational complexity of $\mathcal{O}(N_T^3)$ [33] in comparison to $\binom{N_T}{N_L}$ required by the exhaustive search. As a consequence, the proposed scheme has $\mathcal{O}\left((N_T-N_L)\,N_T^3\right)$ complexity for practical applications.

Moreover, the optimal number of selected transmit antennas, N_L, can also be found if additional computing complexity is paid. The most intuitive method is to find the network capacity for $N_L = 1$ to N_T, as shown by Algorithm 1. In this way, the problem of how many and which transmit antennas should be selected can be solved at the same time, expecting to have less than $\mathcal{O}\left(N_T^5\right)$ complexity comparing to the complexity $\mathcal{O}\left(2^{N_T}N_T^3\right)$ of the exact search.

Finally, the selection algorithm is executed at the destination, which has the full CSI knowledge of S-R and R-D channels. After the calculation, the transmit antenna set will be transmitted via a feedback channel. Specifically, the bits that convey to the relay nodes depend on the size of the network. If there are N_T transmit antennas in the network, at most $\left\lceil \log\binom{N_T}{N_L}\right\rceil$ bits are needed to identify all the different selection results for N_L selected transmit antennas, where $\lceil x\rceil$ denotes the ceiling operator.

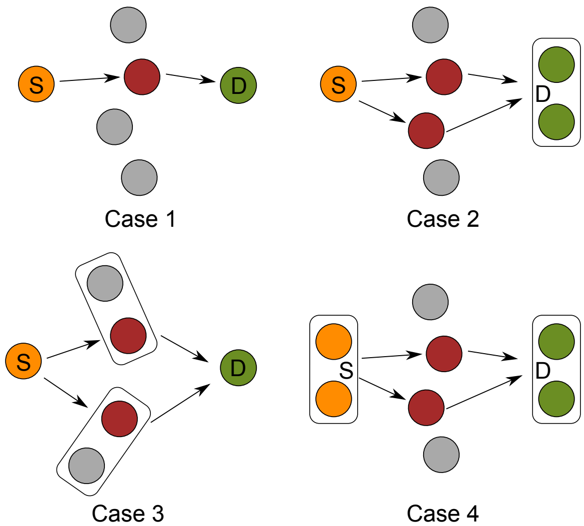

Fig. 4. Several important cases. Case 1: single antenna at every node of the network. Case 2: only the destination has multiple antennas. Case 3: only the relay nodes have multiple antennas. Case 4: both the source and destination have multiple antennas, but all the relay nodes have only single antenna.

B. Special Cases

The relationship between the proposed method and the existing work is elaborated for five special cases. In the first four cases as shown in Fig. 4, different network configurations are considered. In the last case, the channel is assumed to be ill-conditioned.

Single antenna at every node of the network: This model has been analyzed extensively, in which the transmission is from the source to the destination with the help of K single-antenna relay nodes. It has been proved that the opportunistic relaying scheme with one best relay forwarding messages can maximize the instantaneous capacity under an aggregate power constraint [6]. For this network configuration, our proposed selection approach is simplified to the optimal single relay selection method with $\mathcal{R} = \mathcal{N}(\mathcal{T}) = \mathcal{T}$. That is, the transmit antenna is selected via

$$C = \frac{1}{2}\max_{|\mathcal{T}|=1}\min\left(C_{s,r}(\mathcal{T}), C_{r,d}(\mathcal{T})\right), \quad (20)$$

which is the same as the optimal single relay selection method in [6].

Only the destination has multiple antennas: This model corresponds to the uplink case in cellular networks, where the base station has multiple antennas, while the source node and relay nodes (*e.g.*, cooperative users) are equipped with single antenna due to the size limitation [34].

Intuitively, the scheme that only one relay is selected to forward signals does not fully exploit the degrees of freedom of the R-D channel. For example, when S-R channel is good enough so that every relay node can decode messages from the source, it is obvious that the system performance will be limited if we choose one relay to forward messages. Our proposed algorithm can select transmit antennas from multiple relay nodes to maximize the network capacity and utilize the potential degrees of freedom of the channel.

Only the relay nodes have multiple antennas: This model corresponds to the networks, where both the source and destination node have single antenna due to the size limitation, and multiple fixed relay stations equipped with multiple antennas try to help the transmission.

The scheme in [18] uses only one transmit antenna from each relay node for messages forwarding. Our scheme selects transmit antenna adaptively to maximize the network capacity according to the instantaneous channel state.

Both the source and destination have multiple antennas: The similar model is considered in [13] for the AF opportunistic relaying scheme, in which multiple relay nodes with single antenna are allowed to retransmit information signals. A heuristic method was proposed to select the same number of relay nodes as the antenna number at the destination, *i.e.*, $N_L = N_d$.

Our proposed algorithm aims to achieve the maximal network capacity, and selects the most effective relay set to forward messages. The number of relay nodes that maximizes the network capacity does not need to be N_d, and our proposed algorithm will find out how many and which relays should transmit at the same time.

The channel matrix is ill-conditioned: This is the case that the MIMO channel with multiple co-located antennas fails to provide the potential degree-of-freedom gain, for example, in the environments of insufficient scatterers/reflectors [26] or the degenerate channel phenomena called pin-hole (or keyhole) [35], [36]. Although there are multiple antennas at every node, the transmitted signals are all projected onto a low-dimensional space and thus only partial spatial degree of freedom is available. In this sense, the performance of single relay selection [11] gets worse, because it is limited to utilize all the transmit antennas of a certain relay node, while our proposed scheme selects transmit antennas from more general set to maximize the end-to-end capacity. The performance is shown in Sec. VI via an example.

V. Capacity Analysis

This section analyzes the average capacity and outage capacity of the proposed transmit antenna selection scheme. Due to the complex nature of the capacity formula (7), direct calculations of the average capacity and outage capacity are complicated, if not intractable. Instead, we seek lower and upper bounds on the capacity formula (7), which results in lower and upper bounds on performance measures: average capacity and outage capacity. For simplicity and clarity of presentation, we assume that the number of transmit antennas at each relay node is the same, and the channels of S-R and R-D links are i.i.d. random matrices.

For capacity lower bound, due to the optimization approach in Algorithm 1, the instantaneous capacity of the proposed transmit selection method is higher than the relay selection method in [11], in which all the transmit antennas of the relay node with the best worse capacity are chosen, *i.e.*, $\max_{j\in 1,\cdots,K}\min\left(C_{s,j}, C_{j,d}\right)$, where $C_{s,j}$ and $C_{j,d}$ are the capacity from the source node to the jth relay and the capacity from the jth relay to the destination node, respectively. Hence, we have

$$C_{\text{lower}} = \frac{1}{2}\max_{j\in 1,\cdots,K}\min\left(C_{s,j}, C_{j,d}\right). \tag{21}$$

For the capacity upper bound, we have

$$\begin{aligned} C_{\text{upper}} &= \frac{1}{2}\max_{j\in 1,\cdots,K}\left(C_{s,j}\right) \\ &\geq \frac{1}{2}\max_{\mathcal{T}}\min\left\{C_{s,r}\left(\mathcal{R}\right), C_{r,d}\left(\mathcal{T}\right)\right\}. \end{aligned} \tag{22}$$

Actually, C_{upper} equals the capacity of the proposed method when $C_{r,d}\left(\mathcal{T}\right) \geq C_{s,r}\left(\mathcal{R}\right)$, for $\forall\mathcal{T}$.

Since the channel values are random variables, the instantaneous capacity is also a random variable. We will derive the cumulative distribution functions (CDFs) of the lower and upper bounds given in (21) and (22) for channel matrices with zero-mean complex Gaussian entries. Then exact expressions of the CDFs enable exact evaluation of the average capacity and outage capacity.

Let's start by introducing the CDF of the capacity for uncorrelated MIMO Rayleigh-fading channels with N_T transmit antennas and N_R receive antennas. The characteristic function of the capacity is [37]

$$\psi_C\left(z\right) = \zeta\cdot\det\left(\mathbf{U}\right), \tag{23}$$

where ζ is a constant given by

$$\zeta = \frac{\pi^{N_{\min}(N_{\min}-1)}}{\Gamma\left(N_{\min}\right)\Gamma\left(N_{\max}\right)},$$

with $N_{\min} = \min\left(N_T, N_R\right)$, $N_{\max} = \max\left(N_T, N_R\right)$, and $\Gamma\left(n\right) = \pi^{N_{\min}(N_{\min}-1)/2}\prod_{i=1}^{N_{\min}}\left(n-i\right)!$; $\mathbf{U}$ is an $N_{\min}\times N_{\min}$ Hankel matrix with (i,j)th element given by

$$\left[\mathbf{U}\right]_{i,j} = \int_0^{\infty} x^{N_{\max}-N_{\min}+j-i-2}e^{-x}\left(1+\frac{P}{N_T}x\right)^{\frac{\jmath 2\pi z}{\ln 2}}dx,$$

where P is the transmit power, and $\jmath$ is the imaginary unit.

From the characteristic function (23), the CDF of MIMO channels can be simply obtained by means of a single integral

$$F_C\left(x\right) = \int_{-\infty}^{+\infty}\psi_C\left(z\right)\left[\frac{1-e^{-\jmath 2\pi zx}}{\jmath 2\pi z}\right]dz. \tag{24}$$

Proposition 2: The CDFs of the capacity lower and upper bounds are given by

$$F_{\text{lower}}\left(x\right) = \left(1-\left(1-F_{C_{s,r}}(2x)\right)\left(1-F_{C_{r,d}}(2x)\right)\right)^K,$$

and

$$F_{\text{upper}}\left(x\right) = \left(F_{C_{s,r}}(2x)\right)^K,$$

where K is the number of the relay nodes; $F_{C_{s,r}}(x)$ and $F_{C_{r,d}}(x)$ are the CDFs of the capacity from the source to the relay and the relay to the destination, respectively.

Proof: The CDF of the capacity lower bound is

$$\begin{aligned} F_{\text{lower}}(x) &\overset{(a)}{=} \Pr\left(C_{\text{lower}} \le x\right) \\ &= \Pr\left(\max_{j\in 1,\cdots,K} \min\left(C_{s,j}, C_{j,d}\right) \le 2x\right) \\ &\overset{(b)}{=} \left(\Pr\left(\min\left(C_{s,j}, C_{j,d}\right) \le 2x\right)\right)^K \\ &= \left(1 - \Pr\left(\max\left(C_{s,j}, C_{j,d}\right) \ge 2x\right)\right)^K \\ &= \left(1 - \Pr\left(C_{s,j} \ge 2x\right) \cdot \Pr\left(C_{j,d} \ge 2x\right)\right)^K \\ &= \left(1 - \left(1 - F_{C_{s,r}}(2x)\right)\left(1 - F_{C_{r,d}}(2x)\right)\right)^K, \end{aligned} \tag{25}$$

where (a) follows from the definition of CDF, and (b) is from the largest order statistic for K i.i.d. variables.

Also, the CDF of the capacity upper bound is

$$\begin{aligned} F_{\text{upper}}(x) &= \Pr\left(C_{\text{upper}} \le x\right) \\ &= \Pr\left(\max_{j\in 1,\cdots,K} \left(C_{s,j}\right) \le 2x\right) \\ &= \left(\Pr\left(C_{s,j} \le 2x\right)\right)^K \\ &= \left(F_{C_{s,r}}(2x)\right)^K. \end{aligned} \tag{26}$$

Thus, the CDF of the capacity bounds are proved. □

According to the CDF of capacity bounds, exact expression of the average capacity and outage capacity can be obtained. To give an example, we provide the average capacity for the lower bound

$$\begin{aligned} \mathbb{E}\left(C_{\text{lower}}\right) &\overset{(a)}{=} \int_0^{+\infty} x \cdot f_{C_{\text{lower}}}(x)\, dx \\ &= \int_0^{+\infty}\int_0^{x} f_{C_{\text{lower}}}(x)\, dt\, dx \\ &\overset{(b)}{=} \int_0^{+\infty}\int_t^{+\infty} f_{C_{\text{lower}}}(x)\, dx\, dt \\ &= \int_0^{+\infty} \left(1 - F_{\text{lower}}(x)\right) dx, \end{aligned} \tag{27}$$

where (a) follows from the definition of expectation value, and $f_{C_{\text{lower}}}(x)$ denotes the probability density function of the capacity lower bound; (b) is from interchanging the order of integration.

The outage capacity with outage rate ϑ is given by

$$C_{\text{lower}}^{\text{outage}}(\vartheta) = F_{\text{lower}}^{-1}(\vartheta), \tag{28}$$

where $F_{\text{lower}}^{-1}(y) = \inf_x \left(F_{\text{lower}}(x) \ge y\right)$.

VI. Numerical Results

Computer simulation was carried out to validate the performance of the proposed algorithm. For the sake of completeness, unless specified otherwise, we also show the performance corresponding to the case of exact search (when possible), the lower and upper bounds in Proposition 2, optimal selection routing (OSR) of [11], R-D link based relay selection (with only the CSI knowledge of R-D channels at the destination) and random selection strategy (relay nodes are chosen at random). The cooperative system with N_s antennas at the source, N_d antennas at the destination and K relay nodes is called as the $N_s \times K \times N_d$ network in the sequel.

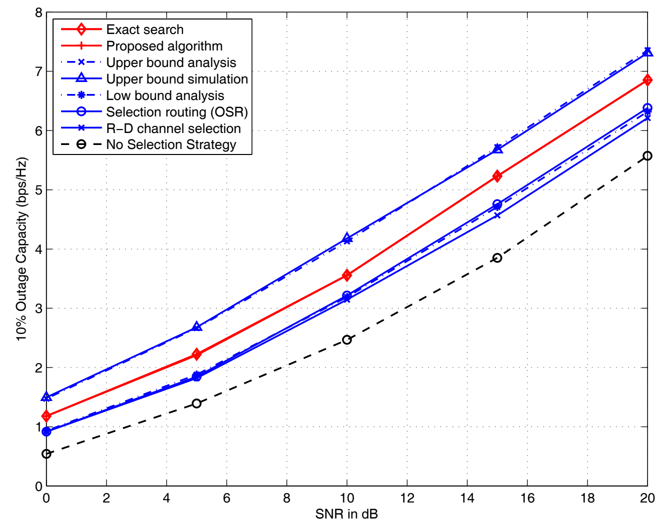

Fig. 5. 10%-Outage capacity versus SNR for the $2 \times 4 \times 2$ multiple-antenna multiple-relay network with 4 transmit antennas equipped at each relay node.

Through our simulation, we assume that the links of S-R and R-D have the same average SNR, *i.e.*, $\alpha_k = \beta_k = \alpha$; the total transmit power for the source and the relay nodes is the same $P_s = P_r = P$. The selected performance metric is the outage capacities with a certain outage probability, *e.g.*, 10%, which are obtained by averaging over 5,000 channel realizations. The fast antenna selection technique [33] is used as the subroutine $AntSel$ to solve the problem of (11), which has $\mathcal{O}(N_T^3)$ complexity and provides a performance close to that of the exact selection.

In the first example, we consider the $2 \times 4 \times 2$ multiple-antenna multiple-relay network with four transmit antenna at each relay node, *i.e.*, $N_T = 16$. Figure 5 plots the outage capacity versus the average SNR for different relay selection strategies. From Fig. 5, we have several observations. First, the curves for the proposed algorithm and the exact search are virtually the same, but the complexity is different. Typically, the computing time for the proposed method is less than 1% of that for the exact search when $N_T = 16$. Moreover, the proposed method is a *polynomial-time algorithm*, rather than a *exponential-time algorithm*. Therefore, it is practical to use even for large-size networks. Second, this figure also shows the advantage of the proposed algorithm over the existing methods. For the cases that multiple antennas are equipped at each node, the proposed algorithm selects more useful transmit antennas to forward the message, while the OSR chooses all the transmit antennas of the best relay node. Therefore, the performance gains come from the more efficient use of the power. Furthermore, if only R-D channels are known at the destination, the selection method based on R-D channels is a good choice to improve the network capacity compared with the case of random selection strategy.

It is shown in Fig. 6 that the 10%-outage capacity versus the antenna number at each relay N_j for the $2 \times 4 \times 2$ multiple-antenna multiple-relay network. The transmit power is fixed to $P = 10$ dB, and each relay is equipped with the same number of antennas. This figure illustrates that the outage capacity achieved by the method of [11] is very close to the proposed

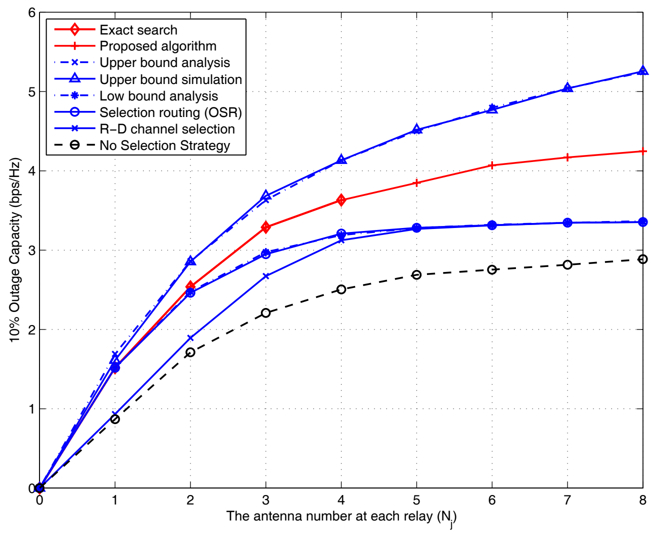

Fig. 6. 10%-Outage capacity versus the antenna number at each relay for the $2 \times 4 \times 2$ multiple-antenna multiple-relay network. Each relay is equipped with the same number of transmit antennas N_j from 1 to 8; and the transmit power is $P_s = 10$ dB.

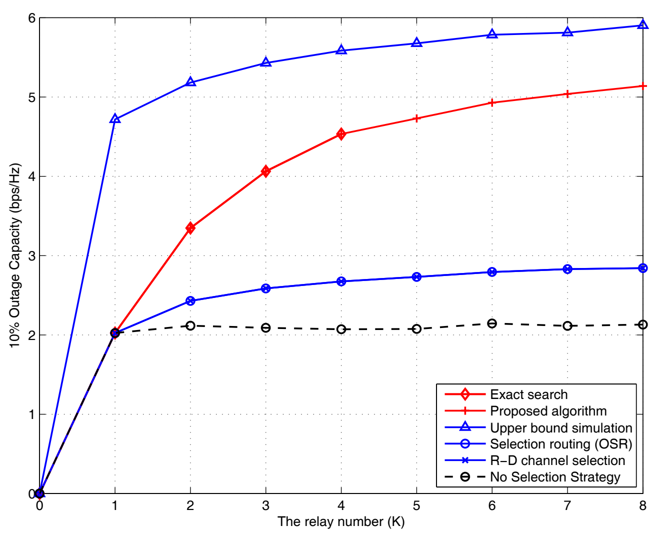

Fig. 7. 10%-Outage capacity versus the number of relay nodes for the $4 \times K \times 4$ multiple-relay network with ill-conditioned R-D channels. Each relay node is equipped with 4 transmit antennas; and the transmit power is fixed to $P_s = 10$ dB.

scheme at low N_j, and is close to R-D based selection at high N_j. The reason for such performance is that the power gains provided by the proposed scheme is more evident for higher N_j, while the OSR method equally splits the power at multiple transmit antennas of a certain relay node. When the number of the antennas at the destination is fixed, the OSR fails to provide more power gain as N_j increases, and the overall performance is limited by the R-D channel. Therefore, the performance of OSR scheme is merged with R-D based selection in this case at high N_j.

In the second example, we examine the performance of different relay selection methods in the case that the R-D channel is ill-conditioned. In this example, let the channel from each relay node to the destination be rank 1, *e.g.*, without sufficient scatterers/reflectors or the degenerate chan-

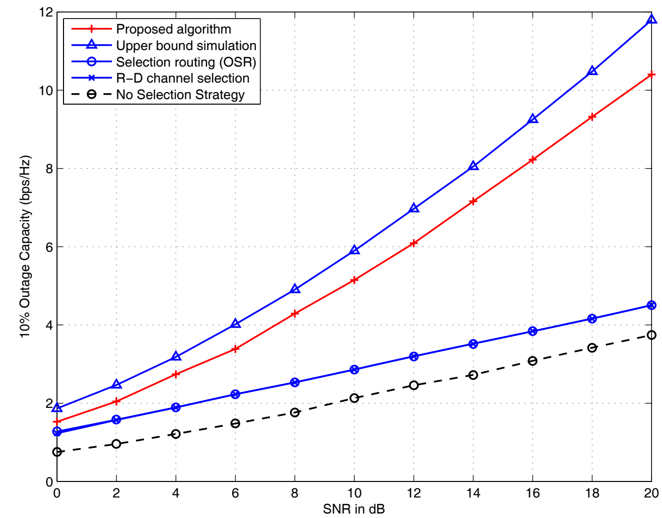

Fig. 8. 10%-Outage capacity versus the SNR for the $4 \times 8 \times 4$ multiple-antenna multiple-relay network with ill-conditioned R-D channels. Each relay node is equipped with 4 transmit antennas.

nel phenomena called pin-hole (or key-hole). In this sense, the multiple antennas do not provide the potential degree-of-freedom gain, $\min\{N_j, N_d\}$, for the R-D channels. Figure 7 shows that the 10%-outage capacity versus the relay number K, which clearly demonstrates that the performance of OSR, R-D based selection and random selection becomes saturated very quickly in comparison to that of the proposed scheme. Due to the ill-conditioned channels, the premise choosing only one relay node does not provide sufficient degrees of freedom and results in the overall network capacity is bounded by the R-D channel even though the number of the relay nodes increases. In contrast, the outage capacity of the proposed algorithm improves as the number of relay nodes increases. In the proposed scenario, the transmit antennas can be selected from different relay nodes distributed on multiple relay nodes. Furthermore, both the S-R and R-D links are considered simultaneously to improve the network capacity from the source to the destination.

Finally, in Fig. 8, we compare the 10%-outage capacity performance of the relaying schemes versus the SNR with ill-conditioned R-D channels. The simulation environment is similar to the previous one, and $K = 8$. It can be observed that the proposed relay selection approach outperforms the OSR, R-D based selection and random selection scheme for a wide range of the SNR values, and the difference in the performance becomes more substantial when the SNR increases.

VII. Summary and Conclusion

In this paper, we have studied opportunistic relaying for dual-hop multi-antenna multi-relay networks. We generalized the well-known relaying scheme for single-antenna networks, opportunistic relaying, to the multi-antenna multi-relay scenarios, in which each terminal in the network, the source, relays and the destination, is equipped with multiple antennas.

In contrast with the conventional selection, where all transmit antennas of the best relay node are selected to forward messages, the proposed solution allows multiple transmit antennas distributed on different relay nodes to forward messages simultaneously. To make the judicious selection among multiple candidates in the network, we have chosen the network capacity as the utility function. The maximization of this utility function over all possible set falls in the framework of combinatorial optimization. Unfortunately, this optimization problem of selecting the best set is nonconvex and NP-hard, *i.e.*, an exhaustive search to evaluate all the transmit antenna subsets may be needed to pick the optimal solution, which produces computational complexity growing exponentially with the total number of transmit antennas at the relay nodes. That is intractable when the size of the network is large.

We have proposed a low-complexity algorithm to solve this NP-hard problem. The proposed algorithm first ranks the relay nodes into a decreasing order based on their S-R channel capacity. Then for a given size of transmit antenna selection, we have proved that the maximal network capacity can be achieved on one of several important sets. Although finding out these sets is still a hard problem, there exists several approaches with performance very close to the exact solution. In this way, the network capacity for all these important sets and different sizes of selection are computed, and the transmit antenna set with the maximal utility function is selected. For practical purposes the proposed algorithm reduces the computational complexity to $\mathcal{O}\left(N_T^5\right)$ comparing to that of the exact search $\mathcal{O}\left(2^{N_T}N_T^3\right)$. We have also derived the lower and upper bounds of the network capacity in terms of average capacity and outage capacity, and investigated the relationship between the proposed method and the existing literature.

Numerical examples have shown the performance gains of our transmit antenna selection scheme compared to its conventional counterparts. Specifically, the achieved gains become more substantial if the channels of the relay network are ill-conditioned.

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Weiliang Zeng received the B.S. degrees in electronic engineering from the University of Electronic Sciences and Technology of China (UESTC), Chengdu, China, in July 2007 with the highest honors. Since September 2007, he has been a Ph.D. candidate at the Tsinghua National Laboratory for Information Science and Technology, Department of Electronic Engineering, Tsinghua University, Beijing, China. During his doctoral studies, he has conducted cooperative research at the Department of Electronic Engineering, Missouri University of Science and Technology, USA. His research interests include communications and information theory with special emphasis on wireless communications and signal processing.

Chengshan Xiao (M'99-SM'02-F'10) received the B.S. degree in electronic engineering from the University of Electronic Science and Technology of China, Chengdu, China, in 1987, the M.S. degree in electronic engineering from Tsinghua University, Beijing, China, in 1989, and the Ph.D. degree in electrical engineering from the University of Sydney, Sydney, Australia, in 1997.

From 1989 to 1993, he was with the Department of Electronic Engineering, Tsinghua University, where he was on the research staff and then promoted to a Lecturer. From 1997 to 1999, he was a Senior Member of Scientific Staff, Nortel, Ottawa, Canada. From 1999 to 2000, he was a Faculty Member with the University of Alberta, Edmonton, AB, Canada. From 2000 to 2007, he was with the University of Missouri, Columbia, where he was an Assistant Professor and then an Associate Professor. He is currently a Professor with the Department of Electrical and Computer Engineering, Missouri University of Science and Technology, Rolla (formerly University of Missouri, Rolla). His algorithms have been implemented into Nortel's base station radios after successful technical field trials and network integration. He is the holder of three U.S. patents. His research interests include wireless communications, signal processing, and underwater acoustic communications.

Dr. Xiao is the Editor-in-Chief of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He served as the founding Area Editor for Transmission Technology of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He has previously been an Associate Editor of the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS-I, and the *International Journal of Multidimensional Systems and Signal Processing*. He was the Lead Technical Program Co-chair of the 2010 IEEE International Conference on Communications (ICC), the Lead Co-chair of the 2008 IEEE ICC Wireless Communications Symposium, and a Phy/MAC Program Co-chair of the 2007 IEEE Wireless Communications and Networking Conference. He served as the founding Chair of the IEEE Technical Committee on Wireless Communications, and the Vice-Chair of the IEEE Technical Committee on Personal Communications. He is a Fellow of the IEEE.

Youzheng Wang received the B.S., M.S. and Ph.D. degree in electronic engineering from Tsinghua University, Beijing, China, in 1992, 1997 and 2009, respectively. Since 1997, he has been with the Department of Electronic Engineering, Tsinghua University, where he has been on the research staff and then promoted to an Associate Professor in 2004. He is currently an Associate Professor with the Department of Aeronautics and Astronautics Engineering, Tsinghua University. His research interests are resource allocation, cooperative communication, network coding, satellite communication and mobile ad hoc networks.

Jianhua Lu received the B.S.E.E. and M.S.E.E. degrees from Tsinghua University, Beijing, China, in 1986 and 1989, respectively, and Ph.D. in Electrical and Electronic Engineering from the Hong Kong University of Science and Technology. Since 1989, he has been with the Dept. of Electronic Engineering, Tsinghua Univ. Beijing, China, where he now serves as a Professor. His current research interests include broadband wireless communication, multimedia signal processing, satellite communication, and wireless networking.

Dr. Lu has been an active member of professional societies and has published more than 180 technical papers in international journals and conference proceedings. He received best paper awards at ICCCAS2002 and Chinacom2006, and awarded the National Distinguished Young Scholar Fund by the NSF committee of China in 2006. He has served in numerous IEEE Conferences as member of Technical Program Committees, and served as the lead Chair of General Symposium of the 2008 IEEE International Conference on Communications (ICC). He is now a chief scientist of the National Basic Research Program (973), China. Dr. Lu is a Senior Member of the IEEE Communication Society and the IEEE Signal Processing Society.