Robust Data-Driven Design of a Smart Cardiac Arrest Response System

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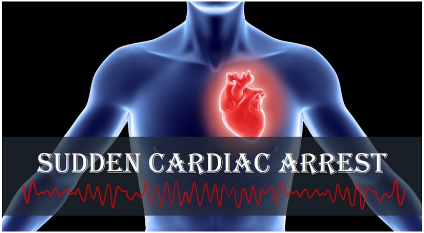
Out-of-Hospital Cardiac Arrest (OHCA)



- sudden and unexpected loss of heart function
- survival rate drops by 7-15%/minute (Larsen et al. 1993)
- 400,000 deaths/year in North America (Coute et al. 2021)
- $\sim$ productivity loss of 10.2 billions dollars

Out-of-Hospital Cardiac Arrest (OHCA)

- three critical interventions:

(a) CPR

(b) Defibrillation

(c) ACLS

- *survival rate* $= 67\% - 2.3\% t_{\text{CPR}} - 1.1\% t_{\text{AED}} - 2.1\% t_{\text{ACLS}}$

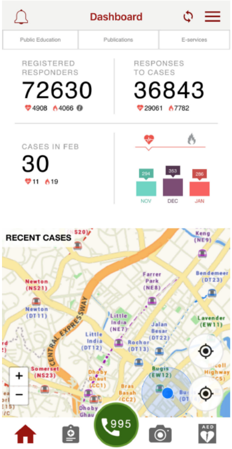
status quo:

- low bystander CPR rate: 20-40% (Virani et al. 2021)
- low AED utilization rate: $<$5% (Delhomme et al. 2019)
- heavily rely on ambulance paramedics
- $\Rightarrow$ low OHCA survival rate: 3-12%

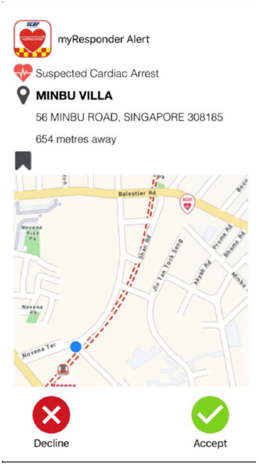
Smart Initiative I: AED-Loaded Drone

- flying height∼60m, maximum velocity∼28m$/s$
- widespread exploration of clinical trials (Schierbeck et al. 2022)

Smart Initiative II: Responder Crowdsourcing App



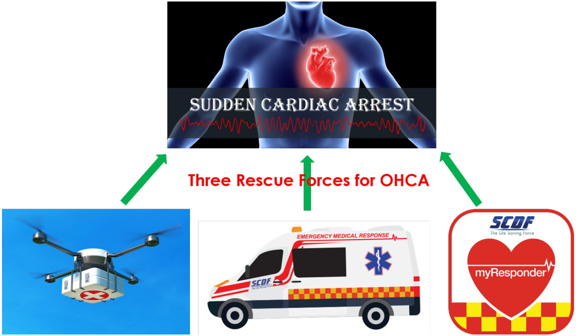
(a) app dashboard



(b) alert prompting

(c) post-acceptance

The Proposed Smart OHCA Response System



- alert nearby community responders via crowsourcing Apps
- send an AED-loaded drone from the nearest base
- dispatch an ambulance from the nearest station

Characteristics of the Smart Response System - I

Ambulance/Drone Dispatch

Controllable & Predictable

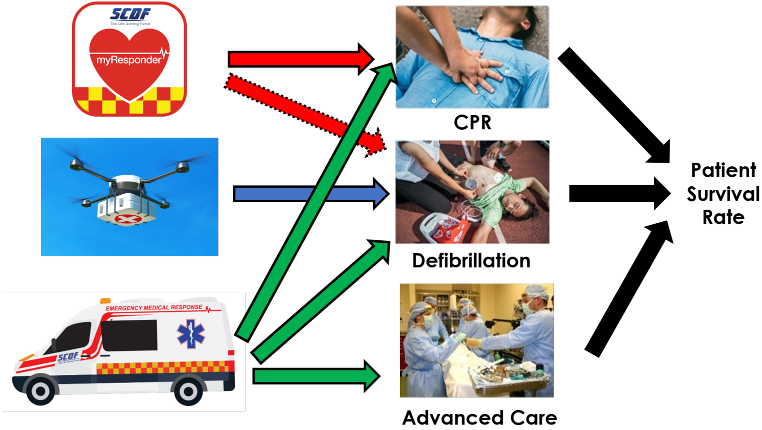
V.S.

Responder Crowdsourcing

SCDF
The Life Saving Force
myResponder

Exogenous & Uncertain

Characteristics of the Smart Response System - II



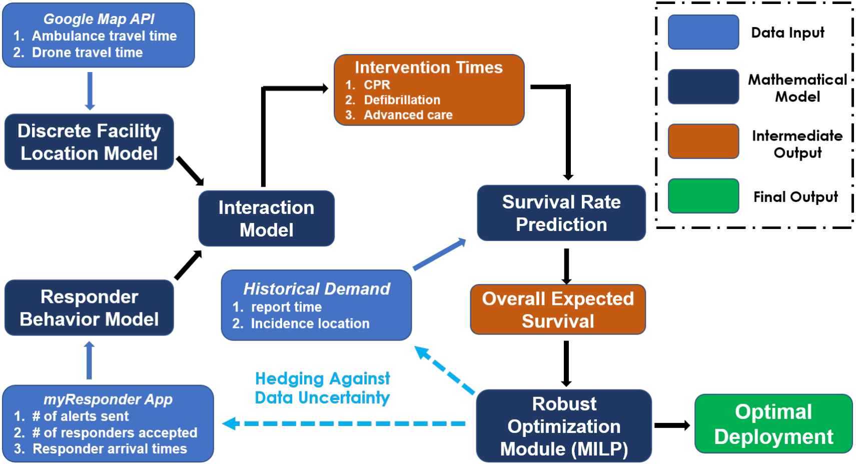
- rescue forces interact by competition
- first come first intervene

Our Interest: Optimal Resource Deployment

How to optimally deploy drones and ambulances in the system?

- go beyond existing facility location models... (Siddiq et al. 2013, Chan et al. 2018, Sun et al. 2016, Derevitskii et al. 2020, Pulver et al. 2016, Claesson et al. 2016, Boutilier and Chan 2022)
  - describe and quantify responder's behavior
  - capture rescue forces' interactions
- alleviate data uncertainty in model primitive estimation (Atamturk and Zhang 2007, Baron et al. 2011, Chan et al. 2018)
  - OHCA demand
  - responder's acceptance probability
  - responder's response time (right-censored)

Research Framework



The Robust Deployment Model

deployment and assignment optimization

expected survival churn rate

hedging against data uncertainty

$$
\begin{aligned}
\boxed{\min_{\boldsymbol{y}^{\mathrm{D}},\boldsymbol{y}^{\mathrm{A}},\boldsymbol{x}^{\mathrm{D}},\boldsymbol{x}^{\mathrm{A}}}}\ \boxed{\max_{\boldsymbol{d}\in\mathcal{D},\boldsymbol{p}\in\mathcal{P}(\boldsymbol{d})}\ \max_{\mathbb{Q}\in\mathcal{Q}}}\ & \mathbb{E}_{\mathbb{Q}}\Bigg[\sum_{i\in[I]} d_i\Bigg(p_i\min\{\tilde{u}_i,\sum_{k\in[K]} x^{\mathrm{A}}_{ik}t^{\mathrm{A}}_{ik}\} \\
&+\rho_1\min\{\sum_{j\in[J]} x^{\mathrm{D}}_{ij}t^{\mathrm{D}}_{ij},\sum_{k\in[K]} x^{\mathrm{A}}_{ik}t^{\mathrm{A}}_{ik}\}+(1-p_i+\rho_2)\sum_{k\in[K]} x^{\mathrm{A}}_{ik}t^{\mathrm{A}}_{ik}\Bigg)\Bigg] \\
\text{s.t.}\quad & \boldsymbol{y}^{\mathrm{D}}\in\mathcal{Y}^{\mathrm{D}},\ \boldsymbol{y}^{\mathrm{A}}\in\mathcal{Y}^{\mathrm{A}} \\
& \boldsymbol{x}^{\mathrm{D}}\in\mathcal{X}^{\mathrm{D}}(\boldsymbol{y}^{\mathrm{D}}),\ \boldsymbol{x}^{\mathrm{A}}\in\mathcal{X}^{\mathrm{A}}(\boldsymbol{y}^{\mathrm{A}}).
\end{aligned}
$$

- $\boldsymbol{d} = (d_1, \dots, d_I)$: OHCA demand rate
- $\boldsymbol{p} = (p_1, \dots, p_I)$: responder's acceptance probability
- $\tilde{\boldsymbol{u}} = (\tilde{u}_1, \dots, \tilde{u}_I)$: responder's random response time
- $\tilde{\boldsymbol{u}} \sim \mathbb{Q}$: responder's response time distribution

Contributions

- the first joint deployment model of ambulance and drone incorporating responders' behavior
- ambiguity set tailored to the right-censoring feature
- reformulation technique that exploits program constraint
- a customized row-and-column generation algorithm with convergence speed analysis

Contributions

Proposition 1. *Let the ambiguity set $\mathcal{Q}$ be defined as in (11), where the parameters satisfy $0 \le \underline{q}_i \le \bar{q}_i \le 1$, $0 \le \check{u}_i \le \tau$, and $\varepsilon_i \ge 0, i \in [I]$. Suppose that $\sum_{k\in[K]} x^{\mathrm{A}}_{ik} t^{\mathrm{A}}_{ik} \le \tau$ for $i \in [I]$. Then the maximization problem over $\mathbb{Q} \in \mathcal{Q}$ in (R–JD), i.e.,*

$$\max_{\mathbb{Q}\in\mathcal{Q}} \mathbb{E}_{\mathbb{Q}} \sum_{i\in[I]} d_i p_i \min\{\tilde{u}_i, \sum_{k\in[K]} x^{\mathrm{A}}_{ik} t^{\mathrm{A}}_{ik}\},$$

is independent of the dispersion upper bounds $\{\varepsilon_i\}_{i\in[I]}$ and evaluates to

$$\sum_{i\in[I]} d_i p_i \left((1-\bar{q}_i) \min\{\check{u}_i, \sum_{k\in[K]} x^{\mathrm{A}}_{ik} t^{\mathrm{A}}_{ik}\} + \bar{q}_i \sum_{k\in[K]} x^{\mathrm{A}}_{ik} t^{\mathrm{A}}_{ik} \right).$$

Proposition 3. *Consider $\mathcal{P}(\boldsymbol{d})$ in (13) such that $0 \le \underline{p}_{(v)} \le \bar{p}_{(v)} \le 1$ for $v \in [V]$. Suppose that the clusters $\{I_{(v)}\}_{v\in[V]}$ are non-overlapping, i.e., $I_{(v)} \cap I_{(v')} = \emptyset$ for all $v \ne v'$, and that the coefficients satisfy $d_i w_i \le 0$, $i \in [I]$. Then the maximization problem (14) over $\boldsymbol{p} \in \mathcal{P}(\boldsymbol{d})$ is equivalent to the following minimization problem*

$$\begin{aligned} \min_{\boldsymbol{\lambda},\boldsymbol{\mu}} \quad & \sum_{v\in[V]} \sum_{i\in I_{(v)}} d_i \underline{p}_{(v)} \lambda_{(v)} + \sum_{i\in[I]} \mu_i \\ \text{s.t.} \quad & d_i \lambda_{(v)} + \mu_i \ge d_i w_i, \ i \in [I], \\ & \boldsymbol{\lambda} \in \mathbb{R}^V, \ \boldsymbol{\mu} \in \mathbb{R}^I_+. \end{aligned}$$

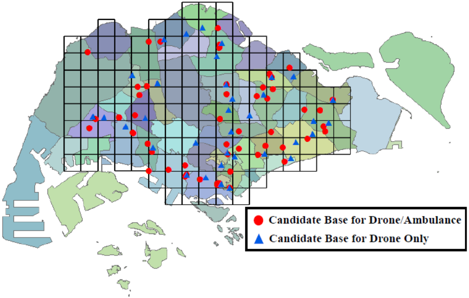
Moreover, for each cluster $v \in [V]$, let $i^v_1, i^v_2, \ldots, i^v_{|I_{(v)}|}$ be a permutation of the regions in the index set $I_{(v)}$ so that $d_{i^v_1} w_{i^v_1} \le d_{i^v_2} w_{i^v_2} \le \cdots \le d_{i^v_{|I_{(v)}|}} w_{i^v_{|I_{(v)}|}}$. Then there exists a worst-case alert response probability vector $\boldsymbol{p}^\star \in \mathcal{P}(\boldsymbol{d})$ attaining the maximum of (14) which satisfies $p^\star_{i^v_1} \le p^\star_{i^v_2} \le \cdots \le p^\star_{i^v_{|I_{(v)}|}}$ and $\sum_{i\in I_{(v)}} d_i p^\star_i / \sum_{i\in I_{(v)}} d_i = \underline{p}_{(v)}$ for each cluster $v \in [V]$.

Theorem 1. *The robust joint deployment problem (R–JD) with ambiguity set $\mathcal{Q}$ defined by (11), uncertainty set $\mathcal{D}$ defined by (12), and uncertainty set $\mathcal{P}(\boldsymbol{d})$ defined by (13) is equivalent to the following MILP, where $M \triangleq \max\{\max_{i\in[I],j\in[J]} t^{\mathrm{D}}_{ij}, \max_{i\in[I],k\in[K]} t^{\mathrm{A}}_{ik}\}$ is a large enough positive constant:*

$$\begin{aligned} \min_{\substack{\boldsymbol{y}^{\mathrm{D}},\boldsymbol{y}^{\mathrm{A}},\boldsymbol{x}^{\mathrm{D}},\boldsymbol{x}^{\mathrm{A}}, \\ \boldsymbol{\phi},\boldsymbol{\psi},\boldsymbol{\nu},\boldsymbol{\lambda}^{(b)},\bar{\boldsymbol{\lambda}}^{(b)},\boldsymbol{\mu}^{(b)},\zeta}} \quad & \zeta \\ \text{s.t.} \quad & \zeta \ge \sum_{i\in[I]} d^{(b)}_i \Big(\rho_1 \phi_i + (1+\rho_2) \sum_{k\in[K]} x^{\mathrm{A}}_{ik} t^{\mathrm{A}}_{ik}\Big) + \sum_{v\in[V]} \sum_{i\in I_{(v)}} d^{(b)}_i \big(\bar{p}_{(v)} \bar{\lambda}^{(b)}_{(v)} - \underline{p}_{(v)} \lambda^{(b)}_{(v)}\big) + \sum_{i\in[I]} \mu^{(b)}_i, \ b \in [B], \\ & d^{(b)}_i \sum_{v\in[V]} \mathbb{1}_{\{i\in I_{(v)}\}} \big(\bar{\lambda}^{(b)}_{(v)} - \lambda^{(b)}_{(v)}\big) + \mu^{(b)}_i \ge d^{(b)}_i (1-\bar{q}_i)\Big(\psi_i - \sum_{k\in[K]} x^{\mathrm{A}}_{ik} t^{\mathrm{A}}_{ik}\Big), \ i \in I_{(v)}, v \in [V], b \in [B], \\ & \boldsymbol{\lambda}^{(b)} \in \mathbb{R}^V_+, \ \bar{\boldsymbol{\lambda}}^{(b)} \in \mathbb{R}^V_+, \ \boldsymbol{\mu}^{(b)} \in \mathbb{R}^I_+, \ b \in [B], \\ & \phi_i \ge \textstyle\sum_{j\in[J]} x^{\mathrm{D}}_{ij} t^{\mathrm{D}}_{ij} - M\nu_{i1}, \ \phi_i \ge \sum_{k\in[K]} x^{\mathrm{A}}_{ik} t^{\mathrm{A}}_{ik} - M\nu_{i2}, \ i \in [I], \\ & \psi_i \ge \check{u}_i - M\nu_{i3}, \ \psi_i \ge \textstyle\sum_{k\in[K]} x^{\mathrm{A}}_{ik} t^{\mathrm{A}}_{ik} - M\nu_{i4}, \ i \in [I], \\ & \nu_{i1} + \nu_{i2} = 1, \ \nu_{i3} + \nu_{i4} = 1, \ i \in [I], \\ & (\nu_{i1}, \nu_{i2}, \nu_{i3}, \nu_{i4}) \in \{0,1\}^4, \ i \in [I], \\ & \boldsymbol{y}^{\mathrm{D}} \in \mathcal{Y}^{\mathrm{D}}, \ \boldsymbol{y}^{\mathrm{A}} \in \mathcal{Y}^{\mathrm{A}}, \\ & \boldsymbol{x}^{\mathrm{D}} \in \mathcal{X}^{\mathrm{D}}(\boldsymbol{y}^{\mathrm{D}}), \ \boldsymbol{x}^{\mathrm{A}} \in \mathcal{X}^{\mathrm{A}}(\boldsymbol{y}^{\mathrm{A}}). \end{aligned}$$

Proposition 5. *The scenario generation algorithm terminates in not greater than $S+1$ iterations, where S is the cardinality of $\mathcal{D}^\star$.*

Case Study on Singapore



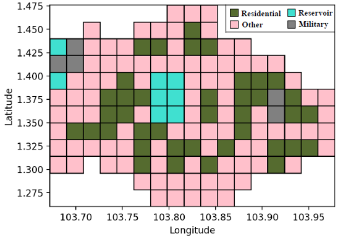


Figure 1 Discretization of the main island of Singapore using square regions (left) and the identification of region types based on Google Map information (right).

- ambulance bases: public hospitals and fire stations
- drone bases: plus police centers

Deployment Results and Benchmarks

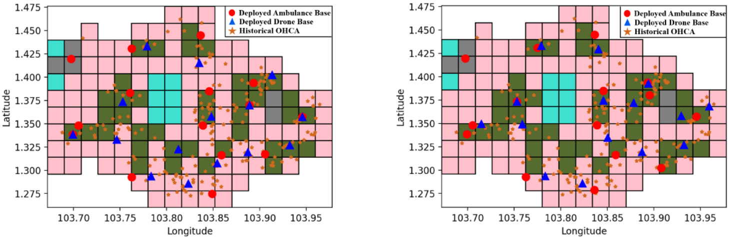


Figure 5 Deployment solutions of 15 ambulances/drones on the main island of Singapore produced by the proposed robust model (left) and the SAA model (right) based on the October 2017 data (cf. Section 7.1). The grid map in the background is adopted from Figure 1.

- benchmark I: a sample average approximation (SAA)
- benchmark II: an ex post model having perfect foresight
- evaluate models on multiple sets of test data

Values of Accounting for Data Uncertainty

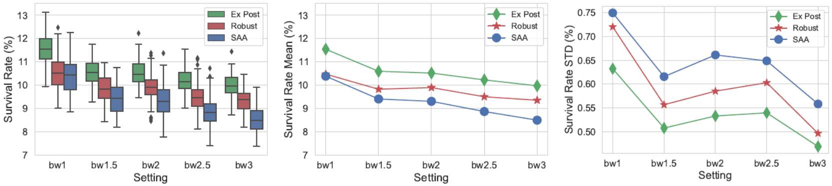
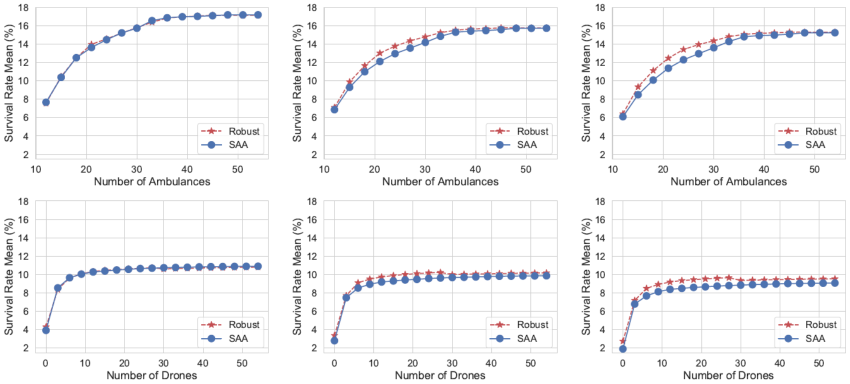


Figure 6 The survival rates (left), means (middle) and standard deviations (right) of the survival rates under each test data setting for the robust, SAA and ex post deployments. The bw-1 setting mimics the October 2017 data, while the other settings simulate deviations from the October 2017 data.

- bw1 $\rightarrow$ bw3: high similarity $\rightarrow$ low similarity

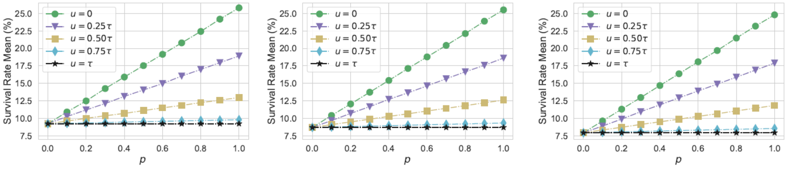
Impacts of Resource Level



(a) bw-1 (b) bw-2 (c) bw-3

Figure 8 Mean survival rates for the robust and SAA deployments of different numbers of ambulances/drones under the test data settings bw-1 (a), bw-2 (b) and bw-3 (c). The number of ambulances (resp. drones) is fixed at 15 when varying the number of drones (resp. ambulances).

Impacts of Responders' Behavior



(a) $h=1$ (b) $h=2$ (c) $h=3$

Figure 9 **Mean survival rates for the robust deployment of 15 ambulances/drones derived in Section 8.2 under the test data indexed by (h,p,u): When generating the test data, each OHCA incident is responded with probability p by a responder whose response time is exactly u. Larger h indicates higher dissimilarity between the test data and the October 2017 data in terms of OHCA spatial distribution.**

- almost triples the largest possible improvement obtained by simply adding drones/ambulances

Summary

- a robust data-driven joint deployment model of ambulance and drone incorporating responders' behavior
- robust deployment leads to higher survival rate
- adding drones/ambulances exhibits diminishing return
- a few drones can dramatically increase the survival rate
- the benefit of improved responder response is significant

Q&A

Thank You!

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