

$$p(x_s|\cdot) = \frac{1}{Z_s} \exp\left\{-\frac{1}{T} \left(\alpha \sum_{\tilde{s}} \psi(x_s, x_{\tilde{s}}) + \beta \sum_r \psi(x_r, x_s)\right)\right\}$$

$$\log p(x_s|\cdot) = -\frac{1}{T} \left(\alpha \sum_{\tilde{s}} \psi(x_s, x_{\tilde{s}}) + \beta \sum_r \psi(x_r, x_s)\right) - \log Z_s$$

$$Z_s = \sum_{s \in \cdot} \exp\left\{-\frac{1}{T} \left(\alpha \sum_{\tilde{s}} \psi(x_s, x_{\tilde{s}}) + \beta \sum_r \psi(x_r, x_s)\right)\right\}$$

$$a = -\frac{1}{T} \sum_{\tilde{s}} \psi(x_s, x_{\tilde{s}}) \quad b = -\frac{1}{T} \sum_r \psi(x_r, x_s)$$

$$\log p(x_s|\cdot) = a\alpha + b\beta - \log \sum_s e^{a\alpha + b\beta}$$

$$\frac{\partial \log p(x_s|\cdot)}{\partial \beta} = b - \frac{\sum_s e^{a\alpha + b\beta} \cdot b}{\sum_s e^{a\alpha + b\beta}}$$

$$\frac{\partial^2 \log p(x_s|\cdot)}{\partial \beta^2} = - \frac{(\sum_s e^{a\alpha + b\beta} \cdot b^2)(\sum_s e^{a\alpha + b\beta}) - (\sum_s e^{a\alpha + b\beta})^2}{(\sum_s e^{a\alpha + b\beta})^2}$$

$$\left(\sum_s e^{a\alpha + b\beta} \cdot b\right)^2$$

$$\frac{\partial \log p(x_s|\cdot)}{\partial \alpha} = a - \frac{\sum_s e^{a\alpha + b\beta} \cdot a}{\sum_s e^{a\alpha + b\beta}}$$

$$\frac{\partial^2 \log p(xs|\cdot)}{\partial \alpha^2} = - \frac{\left( \sum_s e^{a\alpha + b\beta} \cdot a^2 \right) \left( \sum_s e^{a\alpha + b\beta} \right) - \left( \sum_s e^{a\alpha + b\beta} \right)^2}{\left( \sum_s e^{a\alpha + b\beta} \right)^2}$$

" "

$\left( \sum_s e^{a\alpha + b\beta} \cdot a \right)^2$

Let  $M_0 = \sum_s e^{a\alpha + b\beta}$ ,  $M_1 = \sum_s (e^{a\alpha + b\beta} \cdot a)$   
 $M_2 = \sum_s (e^{a\alpha + b\beta} \cdot a^2)$

$$\log p(xs|\cdot) = a\alpha + b\beta - \log \sum M_0$$

$$\frac{\partial \log p(xs|\cdot)}{\partial \beta} = b - \frac{M_1}{M_0}$$

$$\frac{\partial^2 \log p(xs|\cdot)}{\partial \beta^2} = - \frac{M_2 M_0 - M_1^2}{M_0^2} = \frac{M_1}{M_0} - \frac{M_2}{M_0}$$