Notes: Indepedent Component Analysis

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1 sphering

This section talks about the reason that ICA use sphering as a preprocessing step, and if there is any relationship between this sphering and Riemannian Manifold.

Sphering is sometimes called whitening. After sphering a zero-mean random vector $\boldsymbol{x}$, the components are uncorrelated and their variances equal unity.

$$\boldsymbol{z} = \mathbf{V}\boldsymbol{x} \tag{1}$$

$$\mathbb{E}[\boldsymbol{z}\boldsymbol{z}^\top] = \mathbf{I} \tag{2}$$

Sphering transfer the mixing matrix into a new one,

$$\boldsymbol{z} = \mathbf{V}\boldsymbol{x} = \mathbf{V}\mathbf{A}\boldsymbol{s} = \widetilde{\mathbf{A}}\boldsymbol{s}$$

Reason one: Sphering is used as processing step, because after sphering the new mixing matrix $\widetilde{\mathbf{A}}$ is orthogonal, as

$$\mathbb{E}[\boldsymbol{z}\boldsymbol{z}^\top] = \widetilde{\mathbf{A}}\mathbb{E}[\boldsymbol{s}\boldsymbol{s}^\top]\widetilde{\mathbf{A}}^\top = \widetilde{\mathbf{A}}\widetilde{\mathbf{A}}^\top = \mathbf{I}$$

That means we can search the space of orthogonal matrix instead of searching in the whole space. For orthogonal matrix there are $n(n-1)/2$ parameters to estimate, which is significantly less than original n^2 parameters.