Intraclass Correlation (ICC) in fMRI Testing

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This is a summary of the reading for the intraclass correlation (ICC) used in fMRI. Assuming there are N samples, and each sample have K measurements, the NxK matrix X is the data we have. the measurement can be obtained from different raters or observers (we take raters for example in the following text). The ICC is used to measure whether the measurement from different raters are consistent. There are multiple ways of computing ICC depending on the experiment condition McGraw and Wong [1996], Weir [2005], Shrout and Fleiss [1979]. Here we take two case of that. Before going into details of the definition, we give some definitions of the sum of squares (SS) that are often used in ANOVA. These SS will be used for estimation of ICC.

$$
\begin{aligned}
x_{\cdot\cdot} &= \frac{1}{NK}\sum_{i=1}^{N}\sum_{j=1}^{K} x_{ij}, \qquad x_{i\cdot} = \frac{1}{K}\sum_j x_{ij}, \qquad x_{\cdot j} = \frac{1}{N}\sum_i x_{ij} \\
SS_b &= K\sum_{i=1}^{N}(x_{i\cdot} - x_{\cdot\cdot})^2 \\
SS_w &= \sum_i\sum_j (x_{ij} - x_{i\cdot})^2 \\
SS_t &= N\sum_j (x_{\cdot j} - x_{\cdot\cdot})^2 \\
SS_e &= \sum_i\sum_j (x_{ij} - x_{i\cdot} - x_{\cdot j} + x_{\cdot\cdot})^2 \\
SS &= \sum_i\sum_j (x_{ij} - x_{\cdot\cdot})^2 \\
SS &= SS_b + SS_w = SS_b + SS_t + SS_e \\
SS_w &= SS_t + SS_e
\end{aligned}
$$

SS is called total variance. by variance decomposition, it can be decomposed into a few separate parts. SS_b is between-subject variance, SS_w is with-in subject variance. SS_w can be further decomposed into two variance (in two-way ANOVA model): SS_t (between-rater variance) and SS_e (residual errors).

The first case is similar to the one-way ANOVA model. Define $x_{ij} = \mu + r_i + w_{ij}$. μ is a fixed parameter for the population mean, r_i is row effects, and are i.i.d from $\mathcal{N}(0, \sigma_b^2)$. w_{ij} is noise term and also i.i.d. from $\mathcal{N}(0, \sigma_w^2)$. The variance decomposition is $SS = SS_b + SS_w$.

The definition of ICC in this case is

$$\rho = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_w^2}.$$

Because

$$MS_b = \frac{1}{N-1} SS_b, \quad \mathbb{E}[MS_b] = K\sigma_b^2 + \sigma_w^2$$
$$MS_w = \frac{1}{N(K-1)} SS_w, \quad \mathbb{E}[MS_w] = \sigma_w^2$$

We can estimate ICC as

$$ICC = \frac{MS_b - MS_w}{MS_b + (K-1)MS_w}$$

The second case is the two-way ANOVA model where $x_{ij} = \mu + r_i + c_j + w_{ij}$. c_j is i.i.d variable in $\mathcal{N}(0, \sigma_c^2)$. We call c_j a systematic error. Based on the first case, we can further decompose $SS_w = SS_t + SS_e$. The definition of ICC with and without systematic error taken into account are

$$\rho_u = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_t^2 + \sigma_e^2}, \quad \rho_c = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_e^2}.$$

Because

$$MS_b = \frac{1}{N-1} SS_b, \quad \mathbb{E}[MS_b] = K\sigma_b^2 + \sigma_e^2$$
$$MS_w = \frac{1}{N(K-1)} SS_w, \quad \mathbb{E}[MS_w] = \sigma_c^2 + \sigma_e^2$$
$$MS_t = \frac{1}{K-1} SS_t, \quad \mathbb{E}[MS_t] = N\sigma_c^2 + \sigma_e^2$$
$$MS_e = \frac{1}{(N-1)(K-1)} SS_e, \quad \mathbb{E}[MS_e] = \sigma_e^2$$

We can estimate ICC as

$$ICC_u = \frac{MS_b - MS_e}{MS_b + (K-1)MS_e + (K/N)(MS_t - MS_e)}, \quad ICC_c = \frac{MS_b - MS_e}{MS_b + (K-1)MS_e}$$

Most formulas are from McGraw and Wong [1996]. But the definition of the sum of squares (SS) are missing. The SS_e is tricky and I got the definition from Zuo et al. [2010]'s appendix. An example in Shrout and Fleiss [1979] confirmed the variance decomposition and SS_e definition is correct.

References

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