StrategicInfluence2

June 15, 2021

1 Experimentation with Strategic Influence Network Model, Part 2

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14 June 2021

[1]:
```
import matplotlib.pyplot as plt
import numpy as np
```

This notebook implements modification for analysis of "bots".

[2]:
```
N = 5
M = 1
 = 1
Q = 0.2 * np.identity(N)
```

Starting with the baseline infinite-horizon model for comparison:

[3]:
```
A = np.array([
  [0.217,   0.2022,   0.2358,   0.1256,   0.1403],
  [0.2497,   0.0107,   0.2334,   0.1282,   0.378],
  [0.1285,   0.0907,   0.3185,   0.2507,   0.2116],
  [0.1975,   0.0629,   0.2863,   0.2396,   0.2137],
  [0.1256,   0.0711,   0.0253,   0.2244,   0.5536],
], ndmin = 2)
```

[4]:
```
B = np.array([
  0.0791,
  0,
  0,
  0,
  0,
], ndmin = 2)
B = B.T
B
```

[4]:
```
array([[0.0791],
       [0.    ],
       [0.    ],
       [0.    ],
       [0.    ]])
```

[5]:
```
x = np.array([
  -0.98,
  -4.62,
  2.74,
  4.67,
  2.15,
], ndmin = 2)
x = x.T
```

[6]:
```
K = np.zeros((N, N)) # the initial K is zero

K_t = [K, Q] # now the K_t matrices will be added on-demand
K = Q        # start here to avoid division by zero caused by inverse of zero matrix
while True:
    # iteratively construct each K_t using the discrete Riccati difference equation
    K_new =  * (A.T @ (K - (K @ B @ np.linalg.inv(B.T @ K @ B) @ B.T @ K)) @ A) + Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
    K = K_new
    print(current_difference)
    if current_difference == 0:
        break
```

```
0.10795984200000003
0.06696340809285978
0.038813937447250035
0.02038767290129101
0.010191079939001924
0.004966017862517713
0.002390172596966167
0.001143576977383542 5
0.0005455893007808577
0.0002599423287197866
0.00012376776188172123
5.891210074737696e-05
2.8037412711845455e-05
1.3342620341527667e-05
6.349359856328007e-06
3.0214256691585284e-06
1.4377740248927573e-06
6.841759506714951e-07
3.255698946547092e-07
1.549245786658382e-07
7.372185278908816e-08
3.508100937521519e-08
```

```
1.6693518456456502e-08
7.943715130132034e-09
3.780066537562021e-09
1.7987681899533925e-09
8.559550845887998e-10
4.0731157335827106e-10
1.938217919494889e-10
9.22312781703738e-11
4.388883700912061e-11
2.088479389428244e-11
9.938105893780858e-12
4.729161506844548e-12
2.250366559763961e-12
1.0708656184021947e-12
5.095923683029469e-13
2.424727085781342e-13
1.1540768340978502e-13
5.490052856771399e-14
2.6145752229922437e-14
1.2434497875801753e-14
5.88418203051333e-15
2.831068712794149e-15
1.3322676295501878e-15
6.106226635438361e-16
2.7755575615628914e-16
1.6653345369377348e-16
1.1102230246251565e-16
2.7755575615628914e-17
1.3877787807814457e-17
1.3877787807814457e-17
0.0
```

[7]:
```python
def L(t):
    return -1 * np.linalg.inv(B.T @ K_t[t+1] @ B) @ B.T @ K_t[t+1] @ A

K_t.reverse()

x_t = x
payoff = 0
r_ts = []
payoffs = []
for t in range(len(K_t) - 2): # the last entry is zero, which is not invertible␣
 ↪as a 1x1 matrix
    r_t = L(t) @ x_t
    r_ts.append(r_t)
    x_t = A @ x_t + B @ r_t
    payoff += (-1 * (x_t.T @ Q @ x_t)).item()
```

```
    payoffs.append(payoff)

old_length = len(K_t)

# division by zero is due to K_t[-1] being the zero matrix (last term)
```

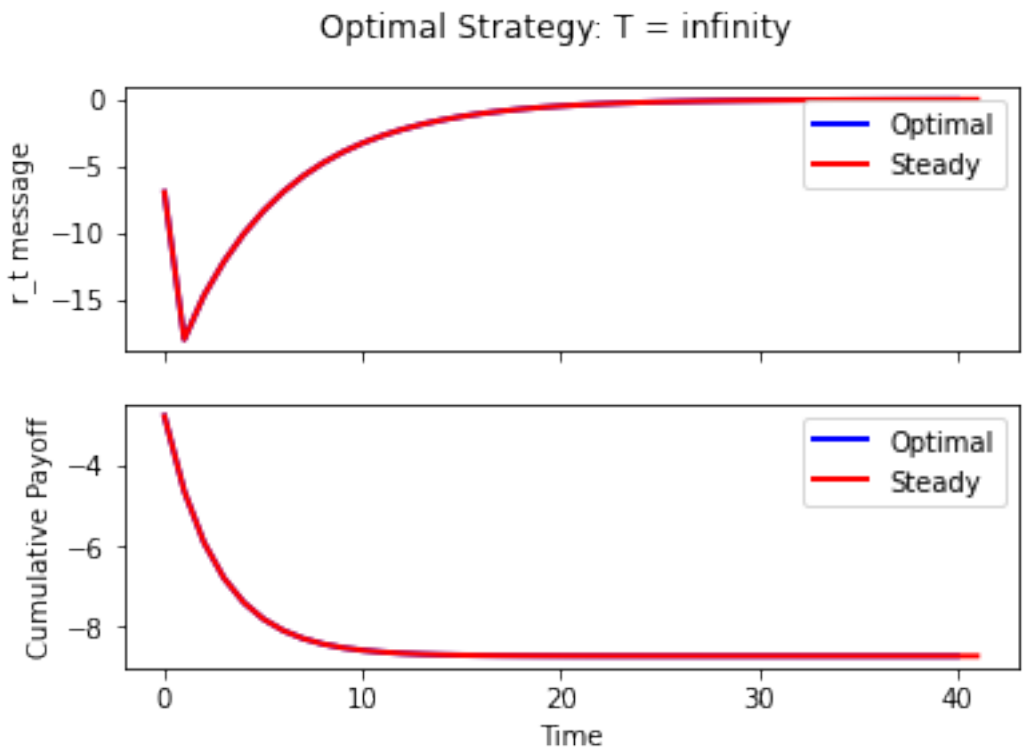
2 INCORRECT PLOT: Has properties of myopic setup:

[10]:
```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(len(K_t) - 1), r_ts, 'b', label = "Optimal", linewidth=2)
sub[0].plot(range(len(K_t) - 1), steady_r_ts, 'r', label = "Steady",␣
 ↪linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(len(K_t) - 1), payoffs, 'b', label = "Optimal", linewidth=2)
sub[1].plot(range(len(K_t) - 1), steady_payoffs, 'r', label = "Steady",␣
 ↪linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



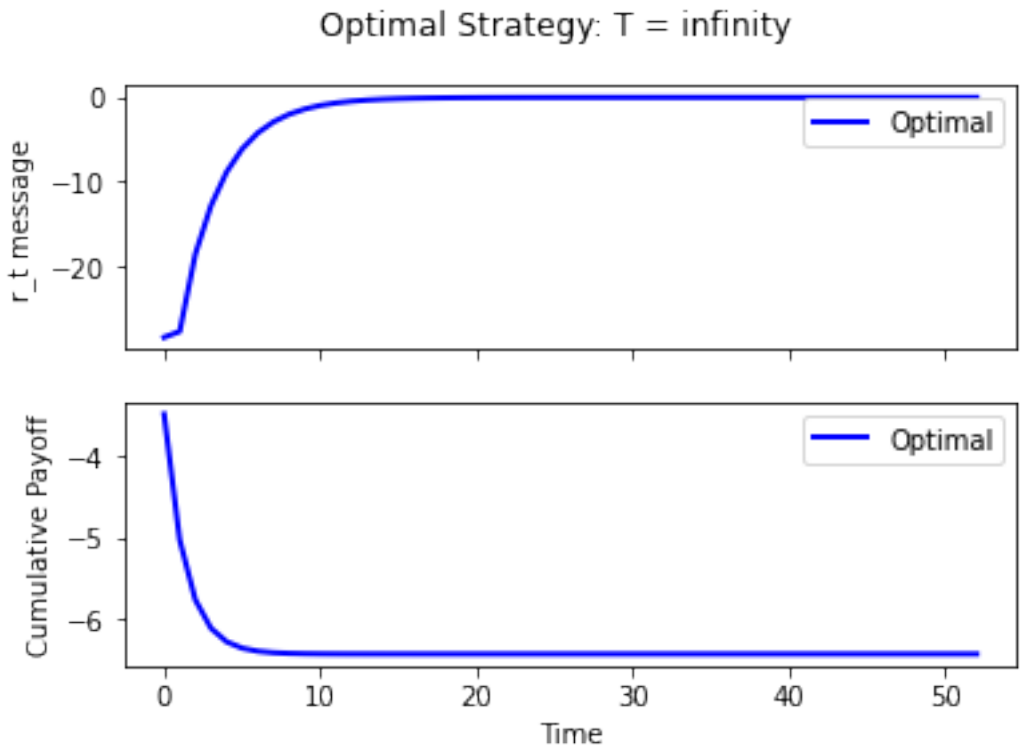
3 CORRECT PLOT:

[8]:
```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(len(K_t) - 2), [a.item() for a in r_ts], 'b', label =␣
 ↪"Optimal", linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(len(K_t) - 2), payoffs, 'b', label = "Optimal", linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



First change: to maintain parity with our five-agent network, A would be modified to have a sixth row/column as follows:

[9]:
```
A = np.array([
  [1,        0,        0,        0,        0,        0     ],
  [0,        0.217,    0.2022,   0.2358,   0.1256,   0.1403],
  # proportionate shifting of agent 2's influence profile to the bot
  [0.1012,   0.8988*0.2497,   0.8988*0.0107,   0.8988*0.2334,   0.8988*0.1282,
 ↪ 0.8988*0.378 ],
  [0,        0.1285,   0.0907,   0.3185,   0.2507,   0.2116],
  [0,        0.1975,   0.0629,   0.2863,   0.2396,   0.2137],
  [0,        0.1256,   0.0711,   0.0253,   0.2244,   0.5536],
], ndmin = 2)
```

Likewise, B becomes:

[10]:
```
B = np.array([
  0, # the robot does not care about the strategic agent
  0.0791,
  0,
  0,
  0,
  0,
], ndmin = 2)
B = B.T
```

```
B
```

[10]:
```
array([[0.    ],
       [0.0791],
       [0.    ],
       [0.    ],
       [0.    ],
       [0.    ]])
```

The x initial opinion vector similarly has an extra entry for the (fixed) opinions of the bot:

[11]:
```
x = np.array([
  10, # the robot, which is against the strategic agent
  -0.98,
  -4.62,
  2.74,
  4.67,
  2.15,
], ndmin = 2)
x = x.T
x
```

[11]:
```
array([[10.  ],
       [-0.98],
       [-4.62],
       [ 2.74],
       [ 4.67],
       [ 2.15]])
```

Additionally, Q needs to be modified. If the strategic agent's payoff was dependent on the bot, the fact that the bot's opinion never changes would result in infinite cost (i.e. a downward-sloping payoff function with no optimal solution - see the original notebook PDF file).

[12]:
```
N = 6
Q = 0.2 * np.identity(N)
Q[0, :] = 0 # strategic agent does not care about the bot
Q
```

[12]:
```
array([[0. , 0. , 0. , 0. , 0. , 0. ],
       [0. , 0.2, 0. , 0. , 0. , 0. ],
       [0. , 0. , 0.2, 0. , 0. , 0. ],
       [0. , 0. , 0. , 0.2, 0. , 0. ],
       [0. , 0. , 0. , 0. , 0.2, 0. ],
       [0. , 0. , 0. , 0. , 0. , 0.2]])
```

[13]:
```
K = np.zeros((N, N)) # initial K

K_t = [K, Q] # saved K
K = Q
i = 0
while True:
```

```
    K_new =  * (A.T @ (K - (K @ B @ np.linalg.inv(B.T @ K @ B) @ B.T @ K)) @ A)␣
↪+ Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
    K = K_new
    i += 1
    if i == 200:
        print(i, current_difference)
        break
    if abs(current_difference) == 0:
        break
```

```
200 0.002067215919870191
```

[14]:
```
def L(t):
    return -1 * np.linalg.inv(B.T @ K_t[t+1] @ B) @ B.T @ K_t[t+1] @ A

K_t.reverse()

x_t = x
x_ts = [x]
payoff = 0
r_ts2 = []
payoffs2 = []
for t in range(len(K_t) - 2):
    r_t = L(t) @ x_t
    r_ts2.append(r_t)
    x_t = A @ x_t + B @ r_t
    x_ts.append(x_t)
    payoff += (-1 * (x_t.T @ Q @ x_t)).item()
    payoffs2.append(payoff)
```

[15]:
```
print("\n".join(str(x.T) for x in x_ts[:20]))
print("... continues ...")
print("\n".join(str(x.T) for x in x_ts[100:])) # for some reason the last␣
↪entry, no matter what it is, corrupts
```

```
[[10.   -0.98 -4.62  2.74  4.67  2.15]]
[[10.         -2.38323439  2.59098488  1.953435    1.878701    1.85594   ]]
[[10.         -1.81611759  1.75886364  1.414633    1.09830373  1.38333558]]
[[10.         -1.41927658  1.51462155  0.94477699  0.71573112  0.94501499]]
[[10.         -1.14582353  1.30976923  0.63530957  0.4588911   0.63710167]]
[[10.         -0.95682287  1.17004312  0.42375855  0.2840724   0.42095714]]
[[10.         -0.82613323  1.07315967  0.27842975  0.16396756  0.27052192]]
[[10.         -0.73573563  1.00612254  0.17820644  0.08115203  0.16613885]]
[[10.         -0.67319915  0.95973468  0.10897183  0.02394572  0.09382052]]
[[10.         -0.62993389  0.92763806  0.06110499 -0.01560404  0.04375277]]
```

```
[[ 1.00000000e+01 -6.00000287e-01  9.05430407e-01  2.79983589e-02
  -4.29579124e-02  9.10131275e-03]]
[[ 1.00000000e+01 -5.79290044e-01  8.90065145e-01  5.09626746e-03
  -6.18803192e-02 -1.48768444e-02]]
[[10.         -0.56496109  0.87943413 -0.01074804 -0.07497133 -0.03146803]]
[[10.         -0.55504717  0.87207871 -0.02171002 -0.08402842 -0.04294754]]
[[10.         -0.54818792  0.86698961 -0.02929429 -0.09029474 -0.05089013]]
[[10.         -0.54344213  0.86346856 -0.03454166 -0.09463026 -0.0563855 ]]
[[10.         -0.5401586   0.8610324  -0.03817221 -0.09762992 -0.06018767]]
[[10.         -0.53788679  0.85934687 -0.04068412 -0.09970532 -0.06281832]]
[[10.         -0.53631496  0.85818068 -0.04242207 -0.10114126 -0.06463842]]
[[10.         -0.53522744  0.85737381 -0.04362452 -0.10213475 -0.06589772]]
... continues ...
[[10.         -0.53278541  0.85556199 -0.04632463 -0.10436566 -0.06872549]]
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[[10.         -0.5327854   0.85556199 -0.04632463 -0.10436566 -0.06872549]]
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[[10.         -0.53278539  0.85556199 -0.04632463 -0.10436566 -0.06872548]]
[[10.         -0.53278538  0.85556199 -0.04632463 -0.10436565 -0.06872548]]
[[10.         -0.53278537  0.855562   -0.04632462 -0.10436565 -0.06872548]]
[[10.         -0.53278535  0.855562   -0.04632462 -0.10436564 -0.06872547]]
[[10.         -0.53278532  0.85556201 -0.04632461 -0.10436564 -0.06872547]]
[[10.         -0.53278529  0.85556202 -0.0463246  -0.10436563 -0.06872546]]
[[10.         -0.53278523  0.85556203 -0.04632459 -0.10436561 -0.06872545]]
[[10.         -0.53278515  0.85556206 -0.04632457 -0.10436559 -0.06872543]]
[[10.         -0.53278504  0.85556209 -0.04632454 -0.10436556 -0.0687254 ]]
[[10.         -0.53278487  0.85556213 -0.0463245  -0.10436551 -0.06872536]]
[[10.         -0.53278463  0.8555622  -0.04632444 -0.10436545 -0.0687253 ]]
[[10.         -0.53278429  0.85556229 -0.04632436 -0.10436535 -0.06872522]]
[[10.         -0.53278379  0.85556243 -0.04632424 -0.10436521 -0.0687251 ]]
[[10.         -0.53278307  0.85556262 -0.04632406 -0.10436501 -0.06872493]]
[[10.         -0.53278202  0.8555629  -0.04632381 -0.10436472 -0.06872468]]
[[10.         -0.53278052  0.85556331 -0.04632345 -0.1043643  -0.06872432]]
[[10.         -0.53277834  0.8555639  -0.04632292 -0.10436369 -0.0687238 ]]
[[10.         -0.53277519  0.85556475 -0.04632215 -0.10436282 -0.06872304]]
[[10.         -0.53277063  0.85556599 -0.04632105 -0.10436155 -0.06872195]]
[[10.         -0.53276405  0.85556777 -0.04631945 -0.10435973 -0.06872038]]
[[10.         -0.53275454  0.85557034 -0.04631714 -0.10435708 -0.0687181 ]]
[[10.         -0.5327408   0.85557406 -0.04631381 -0.10435326 -0.06871481]]
[[10.         -0.53272093  0.85557944 -0.04630899 -0.10434774 -0.06871006]]
[[10.         -0.53269221  0.85558721 -0.04630202 -0.10433976 -0.06870319]]
[[10.         -0.53265071  0.85559845 -0.04629196 -0.10432822 -0.06869326]]
[[10.         -0.53259072  0.85561469 -0.0462774  -0.10431155 -0.06867891]]
[[10.         -0.53250402  0.85563815 -0.04625637 -0.10428745 -0.06865816]]
[[10.         -0.53237871  0.85567207 -0.04622597 -0.10425263 -0.06862818]]
[[10.         -0.53219759  0.8557211  -0.04618204 -0.10420229 -0.06858485]]
```

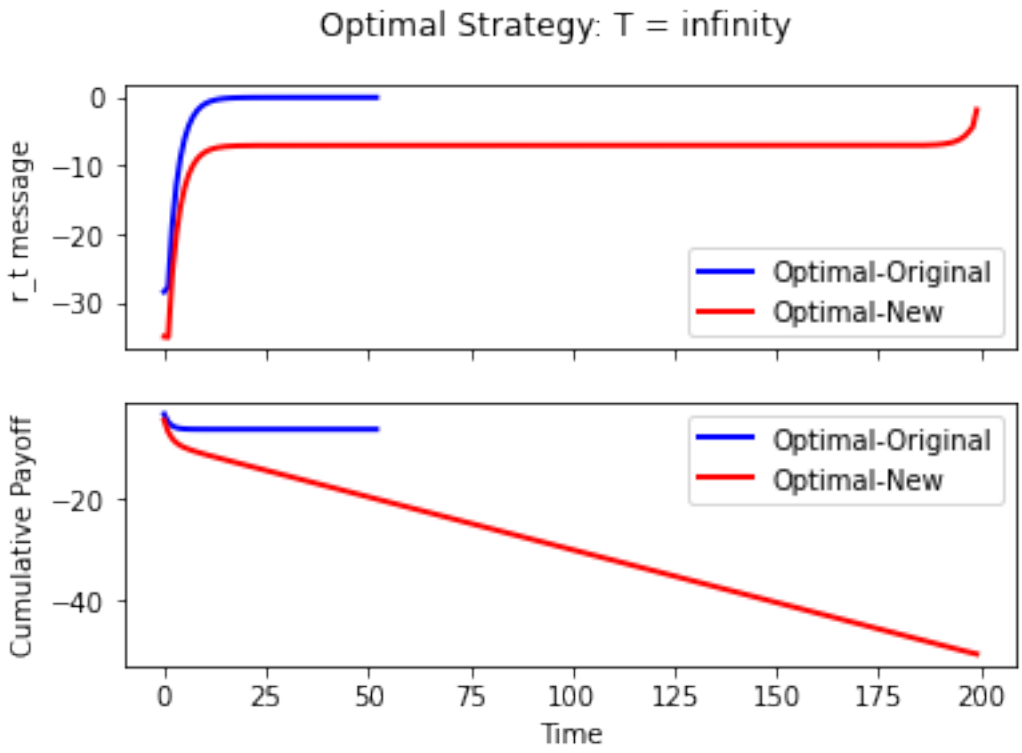
```
[[10.         -0.53193582  0.85579196 -0.04611853 -0.10412954 -0.06852222]]
[[10.         -0.53155747  0.85589437 -0.04602675 -0.10402438 -0.0684317 ]]
[[10.         -0.53101062  0.8560424  -0.04589409 -0.1038724  -0.06830086]]
[[10.         -0.53022025  0.85625634 -0.04570236 -0.10365273 -0.06811176]]
[[10.         -0.52907791  0.85656556 -0.04542524 -0.10333524 -0.06783845]]
[[10.         -0.52742683  0.85701249 -0.04502472 -0.10287636 -0.06744343]]
[[10.         -0.52504048  0.85765844 -0.04444582 -0.10221313 -0.06687248]]
[[10.         -0.52159139  0.85859206 -0.04360912 -0.10125453 -0.06604728]]
[[10.         -0.5166063   0.85994146 -0.04239982 -0.09986904 -0.06485459]]
[[10.         -0.50940113  0.86189179 -0.04065196 -0.09786654 -0.06313074]]
[[10.         -0.49898708  0.86471068 -0.03812572 -0.09497225 -0.0606392 ]]
[[10.         -0.48393475  0.86878497 -0.03447442 -0.09078899 -0.05703806]]
[[10.         -0.46217776  0.87467381 -0.02919697 -0.08474264 -0.05183312]]
[[10.         -0.43072305  0.88318553 -0.02156903 -0.07600329 -0.04430996]]
[[10.         -0.38529837  0.89548994 -0.01054273 -0.06337007 -0.03343515]]
[[ 1.00000000e+01 -3.18881033e-01  9.13266400e-01  5.39047954e-03
  -4.51170586e-02 -1.77208184e-02]]
[[ 1.00000000e+01 -2.34448583e-01  9.39128009e-01  2.85133457e-02
  -1.85882393e-02  5.08364941e-03]]
[[ 1.00000000e+01 -2.77555756e-17  9.73981192e-01  6.05493966e-02
   1.75635611e-02  3.66897544e-02]]
```

[16]:
```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(old_length - 2), [a.item() for a in r_ts], 'b', label =␣
 ↪"Optimal-Original", linewidth=2)
sub[0].plot(range(len(K_t) - 2), [a.item() for a in r_ts2], 'r', label =␣
 ↪"Optimal-New", linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(old_length - 2), payoffs, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[1].plot(range(len(K_t) - 2), payoffs2, 'r', label = "Optimal-New",␣
 ↪linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



We still (still) see eternally decreasing cumulative payoff.

3.1 Testing Delta = 0.9

[17]:
```
K = np.zeros((N, N)) # initial K

K_t = [K, Q] # saved K
K = Q
i = 0
while True:
    K_new = 0.9 * (A.T @ (K - (K @ B @ np.linalg.inv(B.T @ K @ B) @ B.T @ K)) @␣
 ↪A) + Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
    K = K_new
    i += 1
    if i == 300:
        print(i, current_difference)
        break
    if abs(current_difference) == 0:
        break
```

```
300 5.551115123125783e-17
```

[18]:
```
def L(t):
    return -1 * np.linalg.inv(B.T @ K_t[t+1] @ B) @ B.T @ K_t[t+1] @ A

K_t.reverse()

x_t = x
x_ts = [x]
payoff = 0
r_ts2 = []
payoffs2 = []
for t in range(len(K_t) - 2):
    r_t = L(t) @ x_t
    r_ts2.append(r_t)
    x_t = A @ x_t + B @ r_t
    x_ts.append(x_t)
    payoff += (-1 * 0.9**t * (x_t.T @ Q @ x_t)).item()
    payoffs2.append(payoff)
```

[19]:
```
print("\n".join(str(x.T) for x in x_ts[:20]))
print("... continues ...")
print("\n".join(str(x.T) for x in x_ts[100:]))
```

```
[[10.   -0.98 -4.62  2.74  4.67  2.15]]
[[10.         -1.99917049  2.59098488  1.953435    1.878701    1.85594   ]]
[[10.         -1.56350497  1.84505924  1.46398521  1.17415635  1.431574  ]]
[[10.         -1.24910223  1.60762659  1.02999783  0.81365619  1.02784637]]
[[10.         -1.0256862   1.40615878  0.7348323   0.56391319  0.73507415]]
[[10.         -0.86644017  1.26419754  0.52669674  0.3884558   0.52322213]]
[[10.         -0.75288019  1.16271602  0.37917774  0.26407595  0.37121024]]
[[10.         -0.67187436  1.09030253  0.27423327  0.17559981  0.26246118]]
[[10.         -0.61408272  1.03862918  0.19945754  0.1125595   0.1847743 ]]
[[10.         -0.57285013  1.00175825  0.14613817  0.06760866  0.12931343]]
[[10.         -0.54343109  0.97545002  0.10810545  0.03554536  0.08973163]]
[[10.         -0.52244063  0.95667873  0.08097244  0.01267107  0.06148643]]
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  -3.64888324e-03  4.13321921e-02]]
[[10.         -0.49677784  0.93372883  0.04780205 -0.01529296  0.02695165]]
[[10.         -0.4891533   0.92691028  0.03794723 -0.02360098  0.01669091]]
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  -3.37582925e-02  4.14605078e-03]]
[[ 1.00000000e+01 -4.77062006e-01  9.16097123e-01  2.23191713e-02
  -3.67760783e-02  4.18894787e-04]]
[[ 1.00000000e+01 -4.75085916e-01  9.14329921e-01  1.97650726e-02
  -3.89292890e-02 -2.24045931e-03]]
[[ 1.00000000e+01 -4.73675962e-01  9.13069009e-01  1.79427052e-02
  -4.04656200e-02 -4.13792813e-03]]
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... continues ...
[[ 1.00000000e+01 -4.70164486e-01  9.09928721e-01  1.34041192e-02
  -4.42918355e-02 -8.86355507e-03]]
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  -4.42918355e-02 -8.86355507e-03]]
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[[ 1.00000000e+01 -4.70164486e-01  9.09928721e-01  1.34041192e-02
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```

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```

```
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```

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  -4.38372594e-02 -8.48100624e-03]]
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[[ 1.00000000e+01 -4.62940518e-01  9.11620157e-01  1.48715201e-02
```

```
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  -4.16185589e-02 -6.61385817e-03]]
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  -3.78089964e-02 -3.40792503e-03]]
[[ 1.00000000e+01 -4.27679253e-01  9.19875926e-01  2.20337540e-02
  -3.41963061e-02 -3.67680741e-04]]
[[ 1.00000000e+01 -4.03997098e-01  9.25419397e-01  2.68428979e-02
  -2.85702013e-02  4.36691897e-03]]
[[10.         -0.36714143  0.9340534   0.03433287 -0.01980763  0.01174078]]
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  -6.16587747e-03  2.32217190e-02]]
[[10.         -0.22839535  0.96855193  0.06422606  0.01518889  0.04117016]]
[[10.          0.          0.99926678  0.09147432  0.04663908  0.06800269]]
```
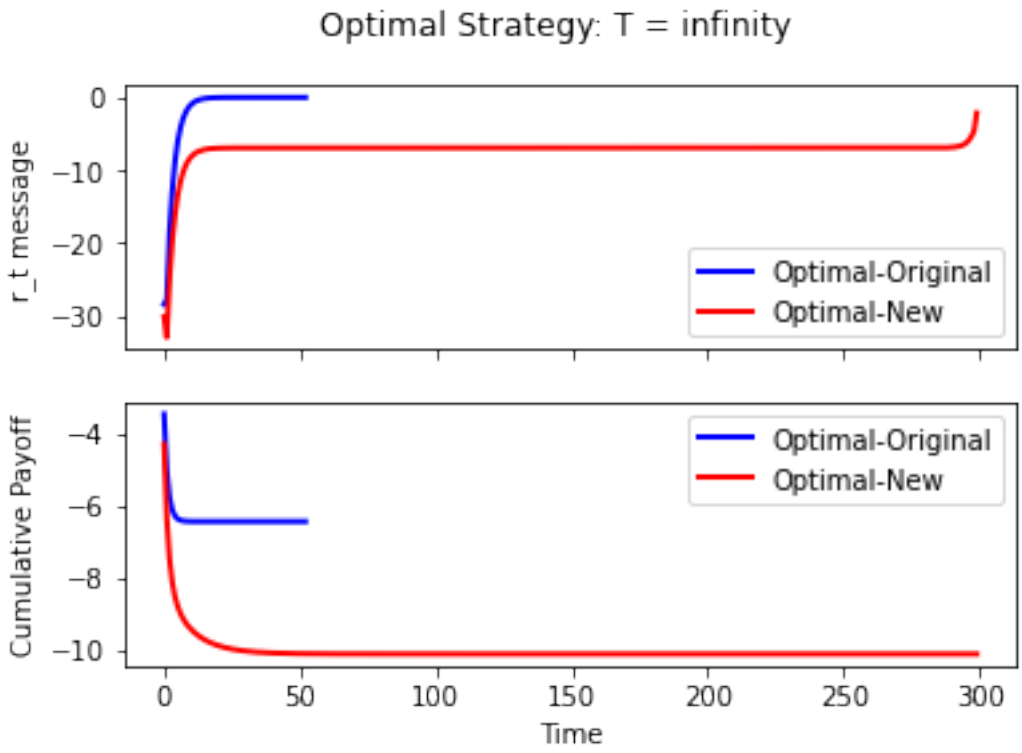

[20]:
```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(old_length - 2), [a.item() for a in r_ts], 'b', label =␣
 ↪"Optimal-Original", linewidth=2)
sub[0].plot(range(len(K_t) - 2), [a.item() for a in r_ts2], 'r', label =␣
 ↪"Optimal-New", linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(old_length - 2), payoffs, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[1].plot(range(len(K_t) - 2), payoffs2, 'r', label = "Optimal-New",␣
 ↪linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



3.2 Trying delta = 0.5:

[21]:
```
K = np.zeros((N, N)) # initial K

K_t = [K, Q] # saved K
K = Q
i = 0
while True:
    K_new = 0.5 * (A.T @ (K - (K @ B @ np.linalg.inv(B.T @ K @ B) @ B.T @ K)) @␣
 ↪A) + Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
    K = K_new
    i += 1
    if i == 300:
        print(i, current_difference)
        break
    if abs(current_difference) == 0:
        break
```

```
300 6.938893903907228e-18
```

[22]:
```
def L(t):
    return -1 * np.linalg.inv(B.T @ K_t[t+1] @ B) @ B.T @ K_t[t+1] @ A

K_t.reverse()

x_t = x
x_ts = [x]
payoff = 0
r_ts2 = []
payoffs2 = []
for t in range(len(K_t) - 2):
    r_t = L(t) @ x_t
    r_ts2.append(r_t)
    x_t = A @ x_t + B @ r_t
    x_ts.append(x_t)
    payoff += (-1 * 0.5**t * (x_t.T @ Q @ x_t)).item()
    payoffs2.append(payoff)
```

[23]:
```
print("\n".join(str(x.T) for x in x_ts[:20]))
print("... continues ...")
print("\n".join(str(x.T) for x in x_ts[100:]))
```

```
[[10.   -0.98 -4.62  2.74  4.67  2.15]]
[[10.         -0.84053092  2.59098488  1.953435    1.878701    1.85594   ]]
[[10.         -0.71330825  2.10509313  1.6128704   1.40298767  1.57709913]]
[[10.         -0.61240985  1.90797945  1.29841424  1.1264787   1.28879874]]
[[10.         -0.5336731   1.73295177  1.06302203  0.91611755  1.05784935]]
[[10.         -0.47201783  1.59685552  0.88068584  0.75350961  0.87428016]]
[[10.         -0.42371044  1.49002984  0.73858148  0.62673362  0.72962139]]
[[10.         -0.38584899  1.40627828  0.62744712  0.52758141  0.61596663]]
[[10.         -0.35617088  1.34061761  0.54041295  0.44992841  0.52678656]]
[[10.         -0.3329061   1.28914252  0.47221267  0.38907846  0.45684827]]
[[10.         -0.31466834  1.24878896  0.4187576   0.34138427  0.40201242]]
[[10.         -0.30037124  1.21715428  0.37685544  0.30399785  0.35902183]]
[[10.         -0.28916329  1.19235472  0.34400793  0.27469024  0.32531908]]
[[10.         -0.28037701  1.17291349  0.31825798  0.2517153   0.29889805]]
[[10.         -0.27348915  1.15767284  0.29807183  0.23370456  0.2781856 ]]
[[10.         -0.26808952  1.14572517  0.28224725  0.21958535  0.26194837]]
[[10.         -0.26385657  1.13635899  0.26984184  0.20851684  0.24921944]]
[[10.         -0.26053821  1.12901652  0.26011682  0.19983986  0.2392408 ]]
[[10.         -0.25793684  1.12326051  0.25249305  0.19303768  0.2314182 ]]
[[10.         -0.25589754  1.11874818  0.24651652  0.18770522  0.2252858 ]]
... continues ...
[[10.         -0.24849851  1.10237646  0.22483237  0.1683579   0.20303613]]
[[10.         -0.24849851  1.10237646  0.22483237  0.1683579   0.20303613]]
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```

```
[[10.         -0.24849851  1.10237646  0.22483237  0.1683579   0.20303613]]
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[[10.         -0.2483886   1.10238861  0.22484132  0.16836954  0.20304489]]
[[10.         -0.24821811  1.10240744  0.22485522  0.1683876   0.20305848]]
[[10.         -0.24778314  1.1024555   0.22489066  0.16843367  0.20309317]]
[[10.         -0.24667337  1.10257811  0.22498109  0.16855119  0.20318165]]
[[10.         -0.24384158  1.10289093  0.2252118   0.16885104  0.2034074 ]]
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[[10.         -0.23661852  1.10368913  0.22580049  0.16961614  0.20398341]]
[[10.         -0.21807743  1.10572523  0.22730223  0.17156785  0.20545284]]
[[10.         -0.17389366  1.11094515  0.23114797  0.17656938  0.2092158 ]]
[[10.          0.          1.12357305  0.24057403  0.18872756  0.21843924]]
```
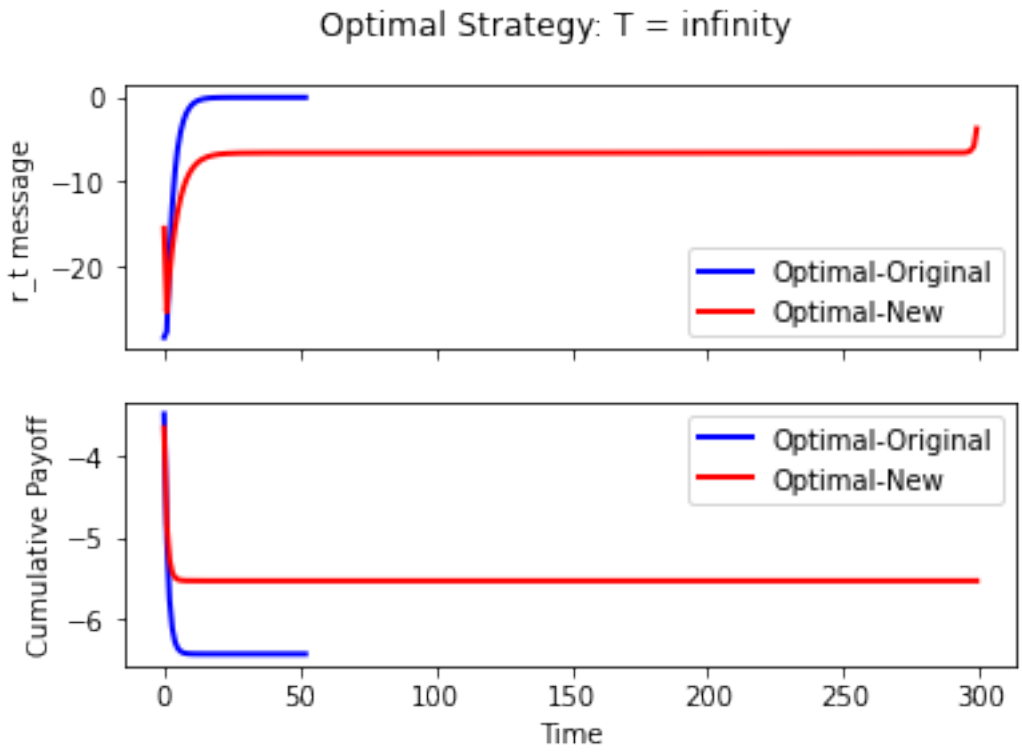

[24]:
```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(old_length - 2), [a.item() for a in r_ts], 'b', label =␣
 ↪"Optimal-Original", linewidth=2)
sub[0].plot(range(len(K_t) - 2), [a.item() for a in r_ts2], 'r', label =␣
 ↪"Optimal-New", linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(old_length - 2), payoffs, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[1].plot(range(len(K_t) - 2), payoffs2, 'r', label = "Optimal-New",␣
 ↪linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



Changing delta seems to manifest a myopic-like dip at the beginning.