StrategicInfluence2

June 13, 2021

1 Experimentation with Strategic Influence Network Model, Part 2

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[1]:
```
import matplotlib.pyplot as plt
import numpy as np
```

This notebook implements modification for analysis of "bots".

[2]:
```
N = 5
M = 1
 = 1
Q = 0.2 * np.identity(N)
```

Starting with the baseline infinite-horizon model for comparison:

[3]:
```
A = np.array([
  [0.217,    0.2022,   0.2358,   0.1256,   0.1403],
  [0.2497,   0.0107,   0.2334,   0.1282,   0.378],
  [0.1285,   0.0907,   0.3185,   0.2507,   0.2116],
  [0.1975,   0.0629,   0.2863,   0.2396,   0.2137],
  [0.1256,   0.0711,   0.0253,   0.2244,   0.5536],
])
```

[4]:
```
B = np.array([
  0.0791,
  0,
  0,
  0,
  0,
])
```

[5]:
```
x = np.array([
  -0.98,
  -4.62,
  2.74,
  4.67,
  2.15,
])
```

[6]:
```
K = np.zeros((N, N)) # the initial K is zero

K_t = [K, Q] # now the K_t matrices will be added on-demand
K = Q        # start here to avoid division by zero caused by inverse of zero␣
 ↪matrix

while True:
    # iteratively construct each K_t using the discrete Riccati difference␣
 ↪equation
    K_new = ( * A.T * (K - (K @ B * (1/(B.T @ K @ B)) * B.T @ K)) @ A) + Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
    K = K_new
    print(current_difference)
    if current_difference == 0:
        break
```

```
0.05735777400000003
0.018849471778646265
0.0067461258790245116
0.0024623325659086093
0.0009042138120677334
0.00033258472349328994
0.0001223868536117667
4.5042563995456586e-05
1.6577808048701126e-05
6.101484085307973e-06
2.2456655787150837e-06
8.265231525306227e-07
3.0420409807829785e-07
1.1196315397032919e-07
4.120834706800025e-08
1.5166845668268536e-08
5.582199369413843e-09
2.0545438683683415e-09
7.56180562611064e-10
2.7831431603786427e-10
1.0243433878898145e-10
3.770123102597722e-11
1.3876066962126288e-11
5.107081424426951e-12
1.8797186029928525e-12
6.918354777951663e-13
2.5462965069777965e-13
9.370282327836321e-14
3.447242491461111e-14
1.2712053631958042e-14
```

```
4.6629367034256575e-15
1.7208456881689926e-15
6.661338147750939e-16
2.220446049250313e-16
1.1102230246251565e-16
2.0816681711721685e-17
2.168404344971009e-18
0.0
```

[7]:
```python
def L(t):
    return -1 * (1/(B.T @ K_t[t+1] @ B)) * B.T @ K_t[t+1] @ A

K_t.reverse()

x_t = x
payoff = 0
r_ts = []
payoffs = []
for t in range(len(K_t) - 1):
    r_t = L(t) @ x_t
    r_ts.append(r_t)
    x_t = A @ x_t + B * r_t
    payoff += -1 * (x_t.T @ Q @ x_t)
    payoffs.append(payoff)

old_length = len(K_t)

# division by zero is due to K_t[-1] being the zero matrix (last term)
```

```
c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: divide by zero encountered in
double_scalars

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
multiply

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
matmul
```

A strategy purely using the steady-state K_t can also be used:

[8]:
```python
def steady_L(t):
    return -1 * (1/(B.T @ K_t[0] @ B)) * B.T @ K_t[0] @ A

steady_x_t = x
```

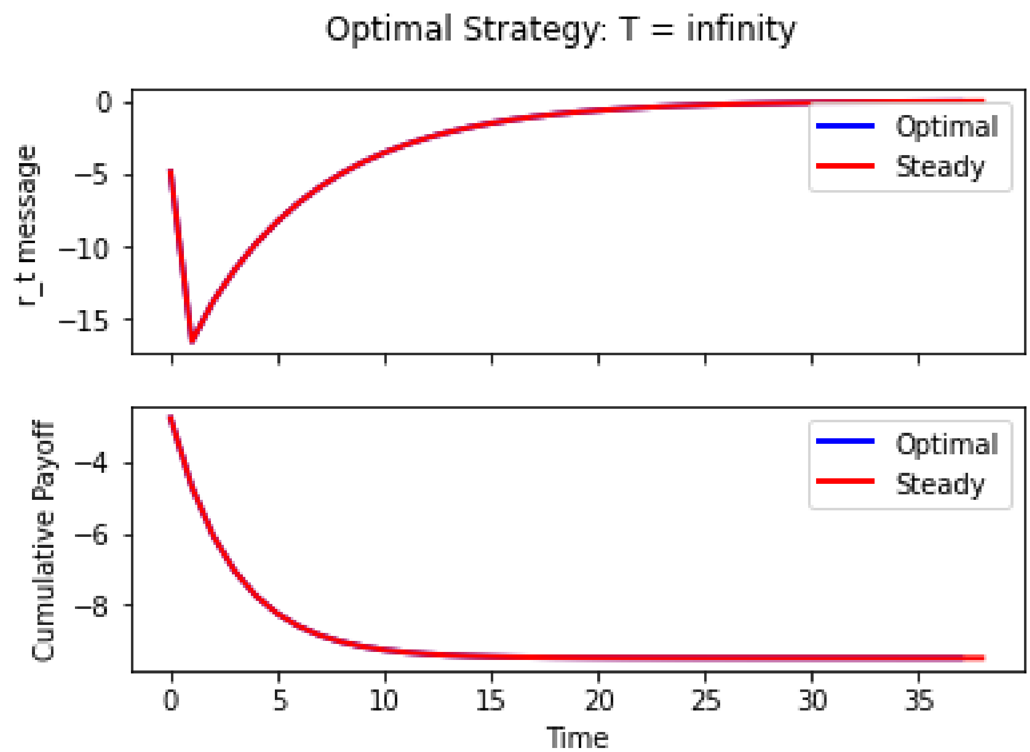
```
steady_payoff = 0
steady_r_ts = []
steady_payoffs = []
for t in range(len(K_t) - 1):
    steady_r_t = steady_L(t) @ steady_x_t
    steady_r_ts.append(steady_r_t)
    steady_x_t = A @ steady_x_t + B * steady_r_t
    steady_payoff += -1 * (steady_x_t.T @ Q @ steady_x_t)
    steady_payoffs.append(steady_payoff)
```

[9]:
```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(len(K_t) - 1), r_ts, 'b', label = "Optimal", linewidth=2)
sub[0].plot(range(len(K_t) - 1), steady_r_ts, 'r', label = "Steady",␣
 ↪linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(len(K_t) - 1), payoffs, 'b', label = "Optimal", linewidth=2)
sub[1].plot(range(len(K_t) - 1), steady_payoffs, 'r', label = "Steady",␣
 ↪linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



First change: to maintain parity with our five-agent network, A would be modified to have a sixth row/column as follows:

[10]:
```
A = np.array([
  [1,         0,         0,         0,         0,         0     ],
  [0,         0.217,     0.2022,    0.2358,    0.1256,    0.1403],
  # proportionate shifting of agent 2's influence profile to the bot
  [0.1012,    0.8988*0.2497,    0.8988*0.0107,    0.8988*0.2334,    0.8988*0.1282,    0.8988*0.378 ],
  [0,         0.1285,    0.0907,    0.3185,    0.2507,    0.2116],
  [0,         0.1975,    0.0629,    0.2863,    0.2396,    0.2137],
  [0,         0.1256,    0.0711,    0.0253,    0.2244,    0.5536],
])
```

Likewise, B becomes:

[11]:
```
B = np.array([
  0, # the robot does not care about the strategic agent
  0.0791,
  0,
  0,
  0,
  0,
])
```

The x initial opinion vector similarly has an extra entry for the (fixed) opinions of the bot:

[12]:
```
x = np.array([
  2, # the robot, which is against the strategic agent
  -0.98,
  -4.62,
  2.74,
  4.67,
  2.15,
])
```

Additionally, Q needs to be modified. If the strategic agent's payoff was dependent on the bot, the fact that the bot's opinion never changes would result in infinite cost (i.e. a downward-sloping payoff function with no optimal solution - see the original notebook PDF file).

[13]:
```
N = 6
Q = 0.2 * np.identity(N)
Q[0, :] = 0 # strategic agent does not care about the bot
Q
```

[13]:
```
array([[0. , 0. , 0. , 0. , 0. , 0. ],
       [0. , 0.2, 0. , 0. , 0. , 0. ],
       [0. , 0. , 0.2, 0. , 0. , 0. ],
       [0. , 0. , 0. , 0.2, 0. , 0. ],
       [0. , 0. , 0. , 0. , 0.2, 0. ],
       [0. , 0. , 0. , 0. , 0. , 0.2]])
```

[14]:
```
K = np.zeros((N, N)) # initial K

K_t = [K, Q] # saved K
K = Q
i = 0
while True:
    K_new = ( * A.T * (K - (K @ B * (1/(B.T @ K @ B)) * B.T @ K)) @ A) + Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
    K = K_new
    i += 1
    if i % 300 == 0:
        print(i, current_difference)
        break
    if abs(current_difference) == 0:
        break
```

```
300 1.2484611557303776e-20
```

[15]:
```
K_t.reverse()

x_t = x
x_ts = [x]
payoff = 0
```

```
r_ts2 = []
payoffs2 = []
for t in range(len(K_t) - 1):
    r_t = L(t) @ x_t
    r_ts2.append(r_t)
    x_t = A @ x_t + B * r_t
    x_ts.append(x_t)
    payoff += -1 * (x_t.T @ Q @ x_t)
    payoffs2.append(payoff)
```

```
c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: divide by zero encountered in
double_scalars

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
multiply

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
matmul
```

[16]:
```
print("\n".join(str(x) for x in x_ts)) # it converges sometime prior to 300
```

```
[ 2.   -0.98 -4.62  2.74  4.67  2.15]
[2.         0.         1.78138488 1.953435   1.878701   1.85594   ]
[ 2.00000000e+00 -2.22044605e-16  1.47634774e+00  1.64744790e+00
  1.51806869e+00  1.62510726e+00]
[ 2.00000000e+00 -2.22044605e-16  1.28924533e+00  1.38306941e+00
  1.27554129e+00  1.38696275e+00]
[ 2.00000000e+00 -1.11022302e-16  1.12313039e+00  1.17070168e+00
  1.07907993e+00  1.18071104e+00]
[2.         0.         0.98427159 0.99510021 0.91668229 1.00526049]
[2.         0.         0.86777735 0.84873822 0.78126912 0.85737346]
[2.         0.         0.77010598 0.72631492 0.66798974 0.73313078]
[ 2.00000000e+00 -1.11022302e-16  6.88220956e-01  6.23775415e-01
  5.73104017e-01  6.28888402e-01]
[ 2.00000000e+00 -5.55111512e-17  6.19573434e-01  5.37844073e-01
  4.93585174e-01  5.41471189e-01]
[2.00000000e+00 5.55111512e-17 5.62024235e-01 4.65815754e-01
 4.26931328e-01 4.68178089e-01]
[2.         0.         0.51377935 0.40543608 0.37105678 0.40673184]
[2.         0.         0.47333457 0.35481957 0.32421687 0.35521913]
[ 2.00000000e+00 -5.55111512e-17  4.39428842e-01  3.12387017e-01
  2.84950279e-01  3.12034601e-01]
[ 2.00000000e+00 -5.55111512e-17  4.11004940e-01  2.76815018e-01
```

```
  2.52032358e-01  2.75831980e-01]
[2.         0.         0.38717658 0.24699429 0.2244366  0.24548252]
[ 2.00000000e+00 -2.77555756e-17  3.67200750e-01  2.21994952e-01
  2.01302494e-01  2.20039904e-01]
[2.         0.         0.35045459 0.20103748 0.18190869 0.19871082]
[2.         0.         0.33641593 0.18346839 0.16565045 0.18083019]
[ 2.00000000e+00 -2.77555756e-17  3.24647019e-01  1.68739840e-01
  1.52020819e-01  1.65840474e-01]
[ 2.00000000e+00 -2.77555756e-17  3.14780892e-01  1.56392587e-01
  1.40594811e-01  1.53274279e-01]
[2.00000000e+00 2.77555756e-17 3.06509907e-01 1.46041623e-01
 1.31016146e-01 1.42739770e-01]
[ 2.00000000e+00 -2.77555756e-17  2.99576165e-01  1.37364189e-01
  1.22986147e-01  1.33908468e-01]
[2.         0.         0.29376346 0.13008971 0.11625443 0.126505  ]
[ 2.00000000e+00 -1.38777878e-17  2.88890549e-01  1.23991362e-01
  1.10611085e-01  1.20298513e-01]
[ 2.00000000e+00 -1.38777878e-17  2.84805482e-01  1.18878986e-01
  1.05880151e-01  1.15095484e-01]
[ 2.00000000e+00 -4.16333634e-17  2.81380883e-01  1.14593172e-01
  1.01914107e-01  1.10733674e-01]
[ 2.00000000e+00 -1.38777878e-17  2.78509970e-01  1.11000284e-01
  9.85892890e-02  1.07077075e-01]
[ 2.00000000e+00 -1.38777878e-17  2.76103222e-01  1.07988288e-01
  9.58020229e-02  1.04011671e-01]
[ 2.00000000e+00 -1.38777878e-17  2.74085594e-01  1.05463269e-01
  9.34653985e-02  1.01441878e-01]
[ 2.00000000e+00 -1.38777878e-17  2.72394173e-01  1.03346491e-01
  9.15065565e-02  9.92875655e-02]
[ 2.00000000e+00 -1.38777878e-17  2.70976219e-01  1.01571952e-01
  8.98644176e-02  9.74815595e-02]
[2.         0.         0.26978752 0.10008432 0.08848778 0.09596755]
[2.         0.         0.268791   0.0988372  0.08733371 0.09469832]
[ 2.00000000e+00 -1.38777878e-17  2.67955607e-01  9.77917180e-02
  8.63662324e-02  9.36342945e-02]
[ 2.00000000e+00 -2.77555756e-17  2.67255275e-01  9.69152669e-02
  8.55551745e-02  9.27423021e-02]
[2.         0.         0.26666817 0.09618052 0.08487525 0.09199453]
[ 2.00000000e+00 -2.77555756e-17  2.66175990e-01  9.55645647e-02
  8.43052501e-02  9.13676492e-02]
[ 2.00000000e+00 -1.38777878e-17  2.65763384e-01  9.50481969e-02
  8.38274092e-02  9.08421251e-02]
[ 2.00000000e+00 -1.38777878e-17  2.65417488e-01  9.46153148e-02
  8.34268250e-02  9.04015670e-02]
[ 2.00000000e+00 -1.38777878e-17  2.65127515e-01  9.42524205e-02
  8.30910067e-02  9.00322379e-02]
[2.         0.         0.26488443 0.0939482  0.08280948 0.08972262]
[ 2.00000000e+00 -1.38777878e-17  2.64680638e-01  9.36931627e-02
```

```
  8.25734759e-02  8.94630633e-02]
[ 2.00000000e+00 -1.38777878e-17  2.64509798e-01  9.34793608e-02
  8.23756261e-02  8.92454702e-02]
[2.         0.         0.26436658 0.09330013 0.08220976 0.08906306]
[2.         0.         0.26424652 0.09314987 0.08207072 0.08891014]
[2.         0.         0.26414587 0.09302391 0.08195415 0.08878194]
[ 2.00000000e+00 -1.38777878e-17  2.64061488e-01  9.29183093e-02
  8.18564354e-02  8.86744701e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63990752e-01  9.28297847e-02
  8.17745157e-02  8.85843758e-02]
[2.         0.         0.26393145 0.09275557 0.08170584 0.08850885]
[2.         0.         0.26388174 0.09269336 0.08164827 0.08844553]
[2.         0.         0.26384007 0.0926412  0.08160001 0.08839245]
[ 2.00000000e+00 -1.38777878e-17  2.63805129e-01  9.25974815e-02
  8.15595450e-02  8.83479533e-02]
[2.         0.         0.26377584 0.09256083 0.08152563 0.08831065]
[2.         0.         0.26375129 0.0925301  0.08149719 0.08827938]
[2.         0.         0.2637307  0.09250434 0.08147335 0.08825316]
[ 2.00000000e+00 -2.77555756e-17  2.63713449e-01  9.24827462e-02
  8.14533702e-02  8.82311835e-02]
[2.00000000e+00 1.38777878e-17 2.63698984e-01 9.24646428e-02
 8.14366176e-02 8.82127592e-02]
[2.         0.         0.26368686 0.09244947 0.08142257 0.08819731]
[ 2.00000000e+00 -1.38777878e-17  2.63676691e-01  9.24367437e-02
  8.14108001e-02  8.81843654e-02]
[ 2.00000000e+00 -2.77555756e-17  2.63668168e-01  9.24260780e-02
  8.14009301e-02  8.81735105e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63661024e-01  9.24171367e-02
  8.13926560e-02  8.81644107e-02]
[2.         0.         0.26365503 0.09240964 0.08138572 0.08815678]
[2.         0.         0.26365001 0.09240336 0.0813799  0.08815039]
[2.         0.         0.2636458  0.09239809 0.08137503 0.08814503]
[2.         0.         0.26364228 0.09239367 0.08137094 0.08814053]
[ 2.00000000e+00 -1.38777878e-17  2.63639317e-01  9.23899711e-02
  8.13675173e-02  8.81367634e-02]
[ 2.00000000e+00 -2.77555756e-17  2.63636837e-01  9.23868676e-02
  8.13646452e-02  8.81336048e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63634758e-01  9.23842658e-02
  8.13622376e-02  8.81309569e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63633015e-01  9.23820846e-02
  8.13602192e-02  8.81287370e-02]
[2.         0.         0.26363155 0.09238026 0.08135853 0.08812688]
[2.00000000e+00 1.38777878e-17 2.63630329e-01 9.23787232e-02
 8.13571086e-02 8.81253161e-02]
[ 2.00000000e+00 -2.77555756e-17  2.63629302e-01  9.23774382e-02
  8.13559194e-02  8.81240082e-02]
[2.         0.         0.26362844 0.09237636 0.08135492 0.08812291]
[ 2.00000000e+00 -1.38777878e-17  2.63627720e-01  9.23754578e-02
```

```
 8.13540868e-02  8.81219927e-02]
[2.         0.         0.26362711 0.0923747  0.08135339 0.08812122]
[ 2.00000000e+00 -2.77555756e-17  2.63626608e-01  9.23740661e-02
  8.13527989e-02  8.81205763e-02]
[2.         0.         0.26362618 0.09237353 0.08135231 0.08812003]
[ 2.00000000e+00 -1.38777878e-17  2.63625826e-01  9.23730879e-02
  8.13518937e-02  8.81195808e-02]
[2.         0.         0.26362553 0.09237271 0.08135155 0.0881192 ]
[2.         0.         0.26362528 0.0923724  0.08135126 0.08811888]
[ 2.00000000e+00 -2.77555756e-17  2.63625067e-01  9.23721378e-02
  8.13510144e-02  8.81186138e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63624891e-01  9.23719174e-02
  8.13508106e-02  8.81183896e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63624743e-01  9.23717328e-02
  8.13506397e-02  8.81182016e-02]
[2.         0.         0.26362462 0.09237158 0.0813505  0.08811804]
[2.         0.         0.26362452 0.09237145 0.08135038 0.08811791]
[ 2.00000000e+00 -1.38777878e-17  2.63624429e-01  9.23713393e-02
  8.13502756e-02  8.81178012e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63624356e-01  9.23712481e-02
  8.13501912e-02  8.81177084e-02]
[ 2.00000000e+00 -2.77555756e-17  2.63624295e-01  9.23711717e-02
  8.13501204e-02  8.81176306e-02]
[2.         0.         0.26362424 0.09237111 0.08135006 0.08811757]
[ 2.00000000e+00 -1.38777878e-17  2.63624201e-01  9.23710538e-02
  8.13500114e-02  8.81175106e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63624165e-01  9.23710088e-02
  8.13499697e-02  8.81174648e-02]
[2.         0.         0.26362413 0.09237097 0.08134993 0.08811743]
[ 2.00000000e+00 -1.38777878e-17  2.63624109e-01  9.23709393e-02
  8.13499054e-02  8.81173941e-02]
[2.         0.         0.26362409 0.09237091 0.08134988 0.08811737]
[ 2.00000000e+00 -1.38777878e-17  2.63624070e-01  9.23708905e-02
  8.13498603e-02  8.81173445e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63624055e-01  9.23708719e-02
  8.13498430e-02  8.81173255e-02]
[ 2.00000000e+00 -1.38777878e-17  2.63624043e-01  9.23708562e-02
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[2.         0.         0.26362403 0.09237084 0.08134982 0.0881173 ]
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[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[2.         0.         0.26362398 0.09237078 0.08134975 0.08811723]
[nan nan nan nan nan nan]
```
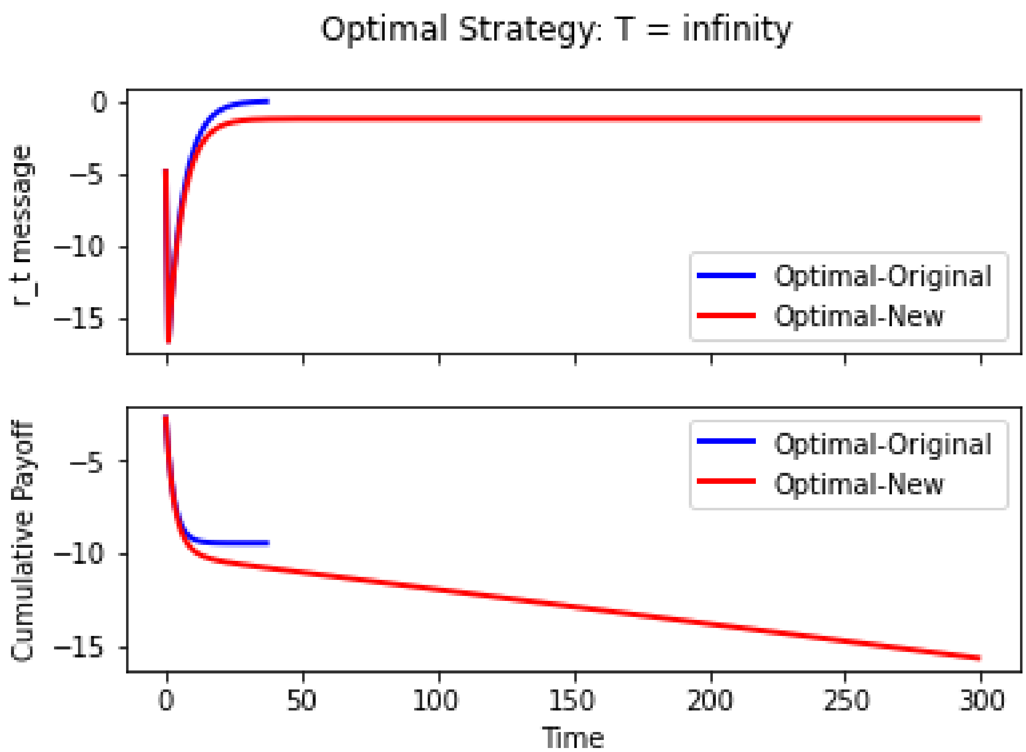

[17]:
```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(old_length - 1), r_ts, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[0].plot(range(len(K_t) - 1), r_ts2, 'r', label = "Optimal-New", linewidth=2)
sub[0].set(ylabel = "r_t message")
```

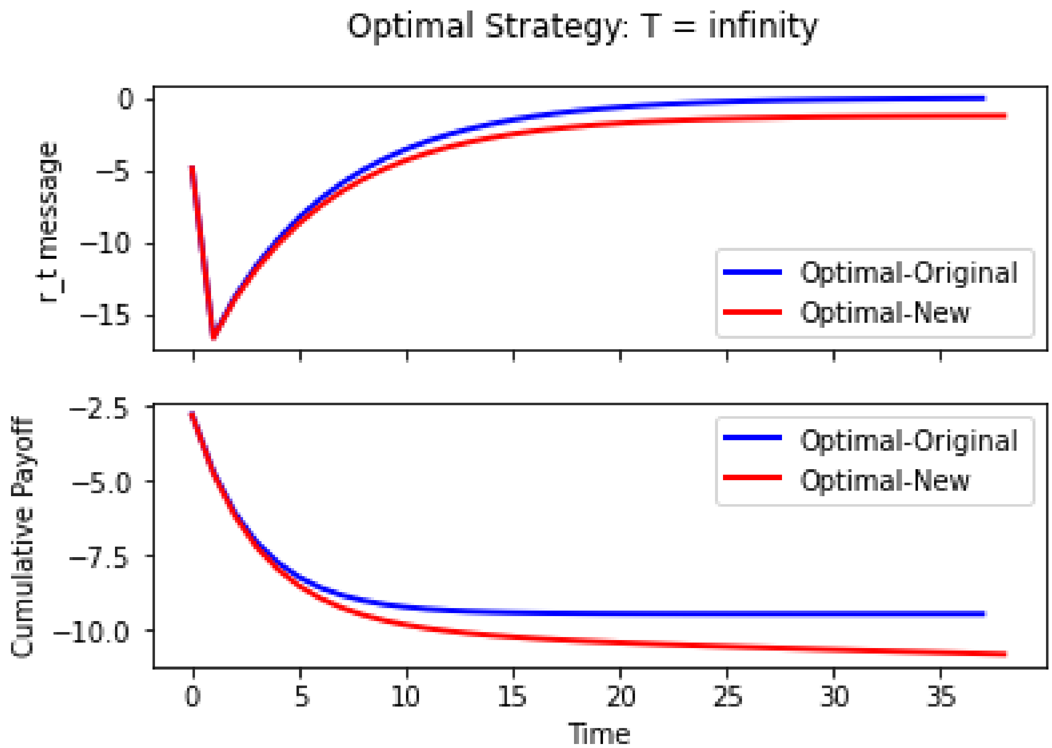
```
sub[1].plot(range(old_length - 1), payoffs, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[1].plot(range(len(K_t) - 1), payoffs2, 'r', label = "Optimal-New",␣
 ↪linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



We still see eternally decreasing cumulative payoff.

[18]:
```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(old_length - 1), r_ts, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[0].plot(range(old_length - 1), r_ts2[:old_length - 1], 'r', label =␣
 ↪"Optimal-New", linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(old_length - 1), payoffs, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
```

```
sub[1].plot(range(old_length - 1), payoffs2[:old_length - 1], 'r', label =␣
 ↪"Optimal-New", linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



Locally the two trajectories are extremely similar, but the solution does not converge to exactly zero; rather it stops short and payoff is continuously decreasing (albeit at a very slow rate). A possible explanation is that, due to the bot essentially having a "proxy" through the agent that listens to it, the fact that this "proxy" agent is in fact part of the Q payoff matrix results in this infinitely decreasing payoff.

Next: what happens if the bot has a lesser agenda? This becomes:

[19]:
```
x = np.array([
  0.5, # the robot, which is against the strategic agent
  -0.98,
  -4.62,
  2.74,
  4.67,
  2.15,
])
```

[20]:
```python
K = np.zeros((N, N)) # initial K

K_t = [K, Q] # saved K
K = Q
i = 0
while True:
    K_new = ( * A.T * (K - (K @ B * (1/(B.T @ K @ B)) * B.T @ K)) @ A) + Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
    K = K_new
    i += 1
    if i % 300 == 0:
        print(i, current_difference)
        break
    if abs(current_difference) == 0:
        break
```

```
300 1.2484611557303776e-20
```

[21]:
```python
K_t.reverse()

x_t = x
x_ts = [x]
payoff = 0
r_ts2 = []
payoffs2 = []
for t in range(len(K_t) - 1):
    r_t = L(t) @ x_t
    r_ts2.append(r_t)
    x_t = A @ x_t + B * r_t
    x_ts.append(x_t)
    payoff += -1 * (x_t.T @ Q @ x_t)
    payoffs2.append(payoff)
```

```
c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: divide by zero encountered in
double_scalars

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
multiply

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
matmul
```

[22]:
```python
print("\n".join(str(x) for x in x_ts))
```

```
[ 0.5  -0.98 -4.62  2.74  4.67  2.15]
[0.5        0.         1.62958488 1.953435   1.878701   1.85594   ]
[ 5.00000000e-01 -2.22044605e-16  1.32308785e+00  1.63367964e+00
  1.50852047e+00  1.61431428e+00]
[ 5.00000000e-01 -2.22044605e-16  1.12831602e+00  1.36010602e+00
  1.25736517e+00  1.36760002e+00]
[ 5.00000000e-01 -1.11022302e-16  9.56292670e-01  1.14013764e+00
  1.05389025e+00  1.15389007e+00]
[0.5        0.         0.81244053 0.95824301 0.88567063 0.9721244 ]
[0.5        0.         0.6917617  0.8066279  0.74539715 0.81892063]
[0.5        0.         0.59058171 0.67980844 0.62804988 0.69021352]
[ 5.00000000e-01 -5.55111512e-17  5.05755215e-01  5.73586035e-01
  5.29756127e-01  5.82226111e-01]
[0.5        0.         0.43464175 0.48456806 0.44738097 0.49166857]
[0.5        0.         0.37502529 0.40995241 0.37833286 0.41574261]
[ 5.00000000e-01 -5.55111512e-17  3.25047361e-01  3.47403821e-01
  3.20451215e-01  3.52089097e-01]
[0.5        0.         0.28314974 0.29496908 0.27192874 0.29872596]
[0.5        0.         0.24802606 0.25101228 0.23125163 0.25399017]
[ 5.00000000e-01 -2.77555756e-17  2.18581126e-01  2.14162480e-01
  1.97151246e-01  2.16487086e-01]
[ 5.00000000e-01 -2.77555756e-17  1.93896807e-01  1.83270543e-01
  1.68564200e-01  1.85047419e-01]
[ 5.00000000e-01 -2.77555756e-17  1.73203417e-01  1.57373187e-01
  1.44599081e-01  1.58690866e-01]
[ 5.00000000e-01 -1.38777878e-17  1.55855707e-01  1.35662887e-01
  1.24508616e-01  1.36595602e-01]
[0.5        0.         0.14131275 0.11746268 0.10766635 0.11807267]
[ 5.00000000e-01 -1.38777878e-17  1.29121084e-01  1.02205062e-01
  9.35471257e-02  1.02544502e-01]
[ 5.00000000e-01 -1.38777878e-17  1.18900550e-01  8.94142758e-02
  8.17106771e-02  8.95269087e-02]
[0.5        0.         0.11033246 0.07869149 0.07178793 0.07861398]
[ 5.00000000e-01 -1.38777878e-17  1.03149644e-01  6.97023455e-02
  6.34694807e-02  6.94654449e-02]
[5.00000000e-01 1.38777878e-17 9.71281384e-02 6.21665567e-02
 5.64959473e-02 6.17960308e-02]
[0.5        0.         0.09208018 0.05584914 0.05064989 0.0553666 ]
[ 5.00000000e-01 -6.93889390e-18  8.78483730e-02  5.05531235e-02
  4.57490081e-02  4.99766672e-02]
[ 5.00000000e-01 -1.38777878e-17  8.43007574e-02  4.61133564e-02
  4.16404981e-02  4.54581737e-02]
[0.5        0.         0.08132672 0.04239141 0.03819625 0.04167022]
[0.5        0.         0.07883351 0.03927121 0.03530886 0.03849471]
[ 5.00000000e-01 -6.93889390e-18  7.67434085e-02  3.66554917e-02
```

```
  3.28882975e-02  3.58326014e-02]
[ 5.00000000e-01 -6.93889390e-18  7.49912293e-02  3.44626759e-02
  3.08590907e-02  3.36009024e-02]
[ 5.00000000e-01 -6.93889390e-18  7.35223402e-02  3.26243917e-02
  2.91579634e-02  3.17300216e-02]
[ 5.00000000e-01 -6.93889390e-18  7.22909392e-02  3.10833190e-02
  2.77318722e-02  3.01616224e-02]
[ 5.00000000e-01 -3.46944695e-18  7.12586295e-02  2.97914050e-02
  2.65363496e-02  2.88468001e-02]
[ 5.00000000e-01 -1.04083409e-17  7.03932223e-02  2.87083659e-02
  2.55341176e-02  2.77445565e-02]
[ 5.00000000e-01 -6.93889390e-18  6.96677331e-02  2.78004312e-02
  2.46939251e-02  2.68205222e-02]
[0.5        0.         0.06905954 0.02703929 0.02398957 0.02604588]
[ 5.00000000e-01 -3.46944695e-18  6.85496788e-02  2.64012096e-02
  2.33991013e-02  2.53964895e-02]
[ 5.00000000e-01 -3.46944695e-18  6.81222513e-02  2.58662930e-02
  2.29040956e-02  2.48520877e-02]
[0.5        0.         0.06776393 0.02541786 0.02248912 0.0243957 ]
[ 5.00000000e-01 -3.46944695e-18  6.74635412e-02  2.50419310e-02
  2.21412403e-02  2.40131080e-02]
[0.5        0.         0.06721172 0.02472678 0.0218496  0.02369237]
[0.5        0.         0.06700061 0.02446258 0.02160512 0.02342349]
[ 5.00000000e-01 -3.46944695e-18  6.68236350e-02  2.42411017e-02
  2.14001620e-02  2.31980783e-02]
[ 5.00000000e-01 -3.46944695e-18  6.66752720e-02  2.40554285e-02
  2.12283422e-02  2.30091128e-02]
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[0.5        0.         0.06644663 0.02376929 0.02096355 0.0227179 ]
[ 5.00000000e-01 -3.46944695e-18  6.63592202e-02  2.36598963e-02
  2.08623211e-02  2.26065669e-02]
[0.5        0.         0.06628594 0.02356819 0.02077746 0.02251324]
[ 5.00000000e-01 -6.93889390e-18  6.62245136e-02  2.34913138e-02
  2.07063168e-02  2.24349951e-02]
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  2.06466770e-02  2.23694040e-02]
[0.5        0.         0.06612984 0.02337284 0.02059668 0.02231442]
[ 5.00000000e-01 -3.46944695e-18  6.60936525e-02  2.33275438e-02
  2.05547659e-02  2.22683211e-02]
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[ 5.00000000e-01 -3.46944695e-18  6.60378773e-02  2.32577423e-02
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[0.5        0.         0.06598369 0.02318993 0.02042742 0.02212827]
```

```
[0.5        0.         0.06597113 0.02317421 0.02041287 0.02211227]
[0.5        0.         0.0659606  0.02316103 0.02040068 0.02209886]
[ 5.00000000e-01 -3.46944695e-18  6.59517719e-02  2.31499832e-02
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  2.03818820e-02  2.20781853e-02]
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  2.03746964e-02  2.20702827e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59329646e-02  2.31264463e-02
  2.03686726e-02  2.20636578e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59286041e-02  2.31209892e-02
  2.03636227e-02  2.20581039e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59249487e-02  2.31164145e-02
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[0.5        0.         0.06592188 0.02311258 0.02035584 0.02204954]
[ 5.00000000e-01 -3.46944695e-18  6.59193152e-02  2.31093643e-02
  2.03528650e-02  2.20462728e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59171615e-02  2.31066690e-02
  2.03503709e-02  2.20435298e-02]
[0.5        0.         0.06591536 0.02310441 0.02034828 0.02204123]
[ 5.00000000e-01 -3.46944695e-18  6.59138425e-02  2.31025153e-02
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[0.5        0.         0.06591257 0.02310093 0.02034506 0.02203769]
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[0.5        0.         0.06591062 0.02309848 0.02034279 0.0220352 ]
[0.5        0.         0.06590987 0.02309754 0.02034193 0.02203424]
[ 5.00000000e-01 -3.46944695e-18  6.59092440e-02  2.30967604e-02
  2.03412016e-02  2.20334455e-02]
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  2.03405931e-02  2.20327763e-02]
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  2.03400831e-02  2.20322154e-02]
[ 5.00000000e-01 -6.93889390e-18  6.59079090e-02  2.30950897e-02
  2.03396555e-02  2.20317451e-02]
[5.00000000e-01 3.46944695e-18 6.59075995e-02 2.30947023e-02
 2.03392970e-02 2.20313509e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59073400e-02  2.30943776e-02
  2.03389965e-02  2.20310204e-02]
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[ 5.00000000e-01 -6.93889390e-18  6.59066591e-02  2.30935254e-02
  2.03382080e-02  2.20301531e-02]
[ 5.00000000e-01 -6.93889390e-18  6.59065516e-02  2.30933910e-02
  2.03380835e-02  2.20300163e-02]
```

```
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[ 5.00000000e-01 -6.93889390e-18  6.59063860e-02  2.30931837e-02
  2.03378918e-02  2.20298054e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59063227e-02  2.30931045e-02
  2.03378185e-02  2.20297248e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59062697e-02  2.30930381e-02
  2.03377570e-02  2.20296572e-02]
[0.5        0.         0.06590623 0.02309298 0.02033771 0.0220296 ]
[ 5.00000000e-01 -6.93889390e-18  6.59061879e-02  2.30929358e-02
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[ 5.00000000e-01 -3.46944695e-18  6.59061566e-02  2.30928967e-02
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[ 5.00000000e-01 -3.46944695e-18  6.59061304e-02  2.30928639e-02
  2.03375958e-02  2.20294798e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59061085e-02  2.30928364e-02
  2.03375703e-02  2.20294519e-02]
[ 5.00000000e-01 -6.93889390e-18  6.59060900e-02  2.30928133e-02
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  2.03375035e-02  2.20293784e-02]
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[0.5        0.         0.06590603 0.02309274 0.02033748 0.02202935]
[ 5.00000000e-01 -3.46944695e-18  6.59060223e-02  2.30927286e-02
  2.03374706e-02  2.20293422e-02]
[ 5.00000000e-01 -6.93889390e-18  6.59060178e-02  2.30927229e-02
  2.03374654e-02  2.20293364e-02]
[ 5.00000000e-01 -3.46944695e-18  6.59060141e-02  2.30927182e-02
  2.03374610e-02  2.20293316e-02]
[0.5        0.         0.06590601 0.02309271 0.02033746 0.02202933]
[0.5        0.         0.06590601 0.02309271 0.02033745 0.02202932]
[ 5.00000000e-01 -3.46944695e-18  6.59060061e-02  2.30927082e-02
  2.03374517e-02  2.20293214e-02]
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  2.03374495e-02  2.20293190e-02]
[ 5.00000000e-01 -6.93889390e-18  6.59060026e-02  2.30927039e-02
  2.03374477e-02  2.20293171e-02]
[0.5        0.         0.065906   0.0230927  0.02033745 0.02202932]
[0.5        0.         0.065906   0.0230927  0.02033744 0.02202931]
[5.00000000e-01 3.46944695e-18 6.59059993e-02 2.30926998e-02
 2.03374439e-02 2.20293128e-02]
```

```
[ 5.00000000e-01 -6.93889390e-18  6.59059985e-02  2.30926988e-02
  2.03374430e-02  2.20293118e-02]
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[0.5        0.         0.06590599 0.02309269 0.02033744 0.02202931]
[0.5        0.         0.06590599 0.02309269 0.02033744 0.02202931]
[nan nan nan nan nan nan]
```
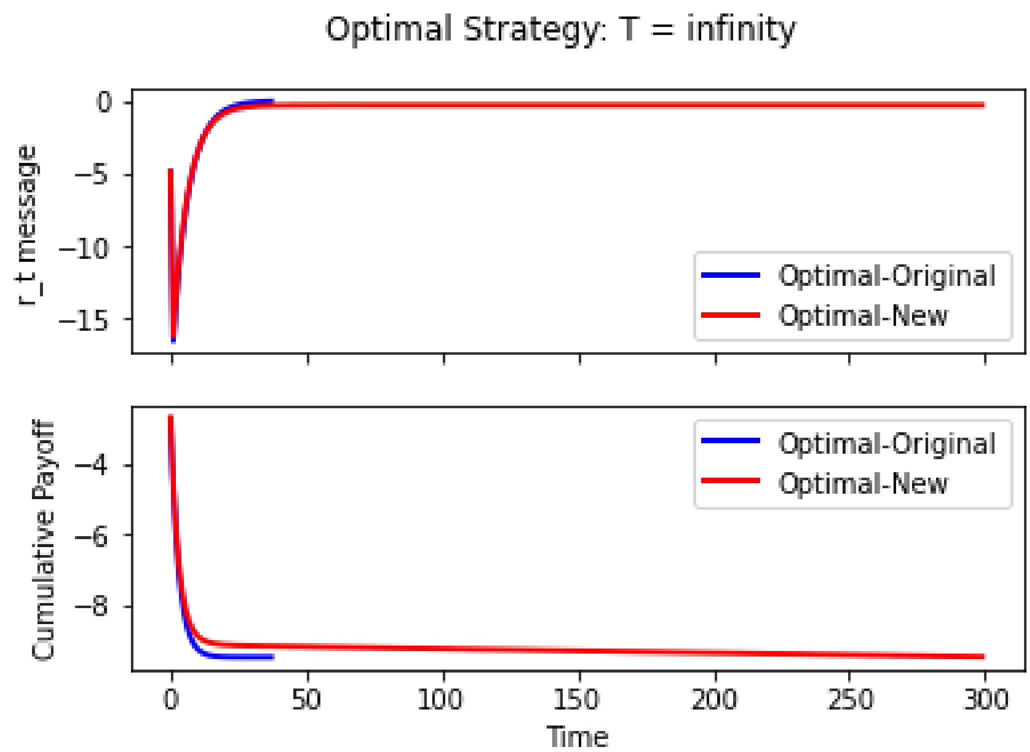

[23]:
```python
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(old_length - 1), r_ts, 'b', label = "Optimal-Original", linewidth=2)
sub[0].plot(range(len(K_t) - 1), r_ts2, 'r', label = "Optimal-New", linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(old_length - 1), payoffs, 'b', label = "Optimal-Original", linewidth=2)
sub[1].plot(range(len(K_t) - 1), payoffs2, 'r', label = "Optimal-New", linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



[24]:
```
r_ts[-2]
```

[24]: -0.03727761846790481

As expected the cost is still decreasing, but is much closer to the original problem. Just for fun, this is what happens if the superbot is too much in favour:

[25]:
```
x = np.array([
  -0.5, # the robot, which is now ahead of the strategic agent
  -0.98,
  -4.62,
  2.74,
  4.67,
  2.15,
])
```

[26]:
```
K = np.zeros((N, N)) # initial K

K_t = [K, Q] # saved K
K = Q
i = 0
while True:
    K_new = ( * A.T * (K - (K @ B * (1/(B.T @ K @ B)) * B.T @ K)) @ A) + Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
```

```
    K = K_new
    i += 1
    if i % 300 == 0:
        print(i, current_difference)
        break
    if abs(current_difference) == 0:
        break

K_t.reverse()

x_t = x
x_ts = [x]
payoff = 0
r_ts2 = []
payoffs2 = []
for t in range(len(K_t) - 1):
    r_t = L(t) @ x_t
    r_ts2.append(r_t)
    x_t = A @ x_t + B * r_t
    x_ts.append(x_t)
    payoff += -1 * (x_t.T @ Q @ x_t)
    payoffs2.append(payoff)

print("\n".join(str(x) for x in x_ts))

fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(old_length - 1), r_ts, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[0].plot(range(len(K_t) - 1), r_ts2, 'r', label = "Optimal-New", linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(old_length - 1), payoffs, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[1].plot(range(len(K_t) - 1), payoffs2, 'r', label = "Optimal-New",␣
 ↪linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```

```
300 1.2484611557303776e-20
[-0.5  -0.98 -4.62  2.74  4.67  2.15]
[-0.5         0.          1.52838488  1.953435    1.878701    1.85594   ]
[-5.00000000e-01 -2.22044605e-16  1.22091460e+00  1.62450080e+00
```

```
  1.50215499e+00  1.60711896e+00]
 [-5.00000000e-01 -2.22044605e-16  1.02102981e+00  1.34479709e+00
   1.24524776e+00  1.35469153e+00]
 [-5.00000000e-01 -2.22044605e-16  8.45067521e-01  1.11976162e+00
   1.03709713e+00  1.13600942e+00]
 [-5.00000000e-01 -1.11022302e-16  6.97886481e-01  9.33671541e-01
   8.64996182e-01  9.50033678e-01]
 [-0.5         0.          0.57441794  0.77855436  0.7214825   0.79328541]
 [-5.00000000e-01 -1.11022302e-16  4.70898860e-01  6.48804126e-01
   6.01423300e-01  6.61602015e-01]
 [-0.5         0.          0.38411139  0.54012645  0.50085753  0.55111792]
 [-0.5         0.          0.31135396  0.44905071  0.41657817  0.45846683]
 [-0.5         0.          0.25035932  0.37271018  0.34593387  0.38078563]
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   2.86714172e-01  3.15660600e-01]
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   1.31315935e-01  1.44757355e-01]
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```

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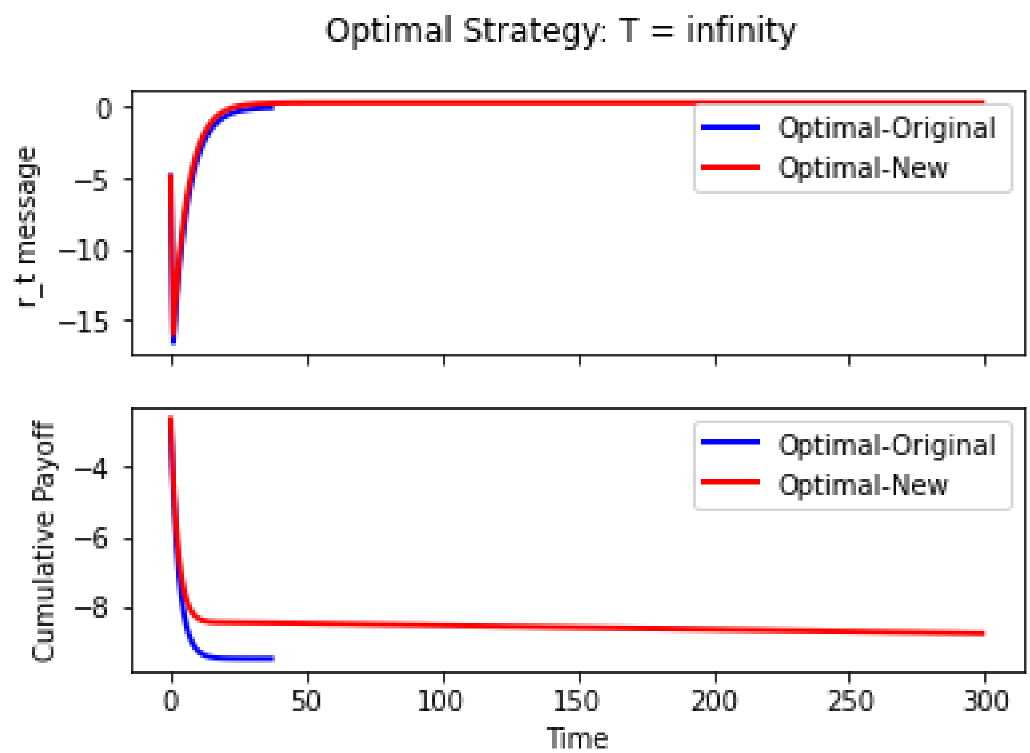
[-5.00000000e-01  3.46944695e-18 -6.59059945e-02 -2.30926938e-02
 -2.03374384e-02 -2.20293068e-02]
[nan nan nan nan nan nan]

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: divide by zero encountered in
double_scalars

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
```

```
multiply

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
matmul
```



Also as expected, a bot pushing an agenda in the opposite direction will push all the metrics in the opposite direction - it should be noted that because this push is not exactly at zero, the long-term cumulative payoff will still dip below the original cumulative payoff, even though the local payoff is much higher.

Finally: applying the modification to Q for the superbot case (no adjustment to A as this is an entirely new network):

[27]:
```
A = np.array([
   [1,        0,         0,         0,         0],
   [0.8,      0,         0,         0,         0],
   [0.8,      0,         0,         0,         0],
   [0.2,      0,         0,         0,         0],
   [0.2,      0,         0,         0,         0],
])
```

[28]:
```
B = np.array([
   0,
   0.2,
```

```
  0.2,
  0.8,
  0.8,
])
```

[29]:
```
x = np.array([
  10,
  1,
  1,
  -1,
  -1,
])
```

[30]:
```
N = 5
Q = 0.2 * np.identity(N)
Q[0, :] = 0 # strategic agent does not care about the bot
Q
```

[30]:
```
array([[0. , 0. , 0. , 0. , 0. ],
       [0. , 0.2, 0. , 0. , 0. ],
       [0. , 0. , 0.2, 0. , 0. ],
       [0. , 0. , 0. , 0.2, 0. ],
       [0. , 0. , 0. , 0. , 0.2]])
```

[31]:
```
K = np.zeros((N, N)) # initial K

K_t = [K, Q] # saved K
K = Q

i = 0
while True:
    K_new = ( * A.T * (K - (K @ B * (1/(B.T @ K @ B)) * B.T @ K)) @ A) + Q
    K_t.append(K_new)
    current_difference = np.max(np.abs(K - K_new))
    K = K_new
    i += 1
    if i % 1000000 == 0:
        print(i, current_difference)
        break
    if current_difference == 0:
        break
```

```
1000000 0.015058823526487686
```

[32]:
```
K_t.reverse()

x_t = x
payoff = 0
r_ts2 = []
```

```
payoffs2 = []
x_ts = []
for t in range(len(K_t) - 1):
    r_t = L(t) @ x_t
    r_ts2.append(r_t)
    x_t = A @ x_t + B * r_t
    x_ts.append(x_t)
    payoff += -1 * (x_t.T @ Q @ x_t)
    payoffs2.append(payoff)
```

```
c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: divide by zero encountered in
double_scalars

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
multiply

c:\users\jbrig\appdata\local\programs\python\python37\lib\site-
packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value encountered in
matmul
```

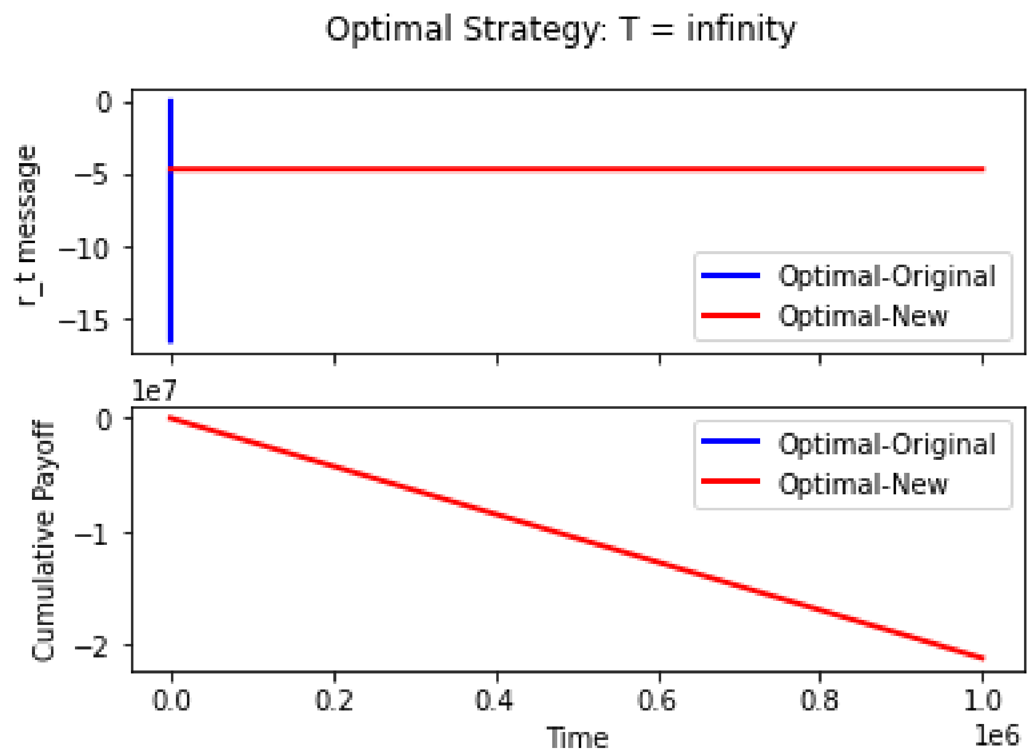
[33]:

```
fig, sub = plt.subplots(2, sharex=True)
fig.suptitle("Optimal Strategy: T = infinity")

sub[0].plot(range(old_length - 1), r_ts, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[0].plot(range(len(K_t) - 1), r_ts2, 'r', label = "Optimal-New", linewidth=2)
sub[0].set(ylabel = "r_t message")

sub[1].plot(range(old_length - 1), payoffs, 'b', label = "Optimal-Original",␣
 ↪linewidth=2)
sub[1].plot(range(len(K_t) - 1), payoffs2, 'r', label = "Optimal-New",␣
 ↪linewidth=2)
sub[1].set(xlabel = "Time", ylabel =  "Cumulative Payoff")

sub[0].legend()
sub[1].legend()
plt.show()
```



This continues to be an extreme case of the anomaly found earlier.

[34]:
```
for i in range(0, 1000000, 100000):
    print(x_ts[i])
```

```
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
[10.          7.05882353  7.05882353 -1.76470588 -1.76470588]
```