SympyFOC

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[1]:
```python
from sympy import *
```

0.1 A small example: K_t recursive equation

[2]:
```python
K = Matrix(([[symbols("K11"), symbols("K12")], [symbols("K12"),␣
↪symbols("K22")]]))
Q = eye(2)
A = Matrix(([[symbols("a11"), symbols("a12")], [symbols("a21"), 1 -␣
↪symbols("a21")]]))
B = Matrix(([[1 - symbols("a11") - symbols("a12")], [0]]))
K
```

[2]:
$$\begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix}$$

[3]:
```python
A
```

[3]:
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 1 - a_{21} \end{bmatrix}$$

[4]:
```python
B
```

[4]:
$$\begin{bmatrix} -a_{11} - a_{12} + 1 \\ 0 \end{bmatrix}$$

[5]:
```python
K_sol = simplify(A.T *(K - K*B*(B.T * K * B).inv() * B.T * K.T) * A + Q)
K_sol
```

[5]:
$$\begin{bmatrix} K_{22}a_{21}^2 + 1 - \frac{K_{12}^2 a_{21}^2}{K_{11}} & \frac{a_{21}\left(-K_{11}K_{22}a_{21} + K_{11}K_{22} + K_{12}^2 a_{21} - K_{12}^2\right)}{K_{11}} \\ \frac{a_{21}\left(-K_{11}K_{22}a_{21} + K_{11}K_{22} + K_{12}^2 a_{21} - K_{12}^2\right)}{K_{11}} & K_{22}a_{21}^2 - 2K_{22}a_{21} + K_{22} + 1 - \frac{K_{12}^2 a_{21}^2}{K_{11}} + \frac{2K_{12}^2 a_{21}}{K_{11}} - \frac{K_{12}^2}{K_{11}} \end{bmatrix}$$

So this says that K_{t+1}, which is:

[6]:
```python
K
```

[6]:
$$\begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix}$$

leads to K_t via the following expression for K_t as a function of K_{t+1}:

[7]:
```python
K_sol
```

[7]:

$$\begin{bmatrix} K_{22}a_{21}^2 + 1 - \frac{K_{12}^2 a_{21}^2}{K_{11}} & \frac{a_{21}\left(-K_{11}K_{22}a_{21}+K_{11}K_{22}+K_{12}^2 a_{21}-K_{12}^2\right)}{K_{11}} \\ \frac{a_{21}\left(-K_{11}K_{22}a_{21}+K_{11}K_{22}+K_{12}^2 a_{21}-K_{12}^2\right)}{K_{11}} & K_{22}a_{21}^2 - 2K_{22}a_{21} + K_{22} + 1 - \frac{K_{12}^2 a_{21}^2}{K_{11}} + \frac{2K_{12}^2 a_{21}}{K_{11}} - \frac{K_{12}^2}{K_{11}} \end{bmatrix}$$

(Note that indeed this is a symmetric matrix).

Then for example I can do comparative statics with respect to variables:

[8]:
```
K_sol[0, 0]
```

[8]:

$$K_{22}a_{21}^2 + 1 - \frac{K_{12}^2 a_{21}^2}{K_{11}}$$

Remember that K here is K_{t+1}, which is

[9]:
```
K
```

[9]:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix}$$

and A is:

[10]:
```
A
```

[10]:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 1 - a_{21} \end{bmatrix}$$

If we factor out a_{21}^2, we get that the upper left entry of K_t is:

$$a_{21}^2(K_{22} - \frac{K_{12}^2}{K_{11}}) + 1$$

From this (where K is K_{t+1}), we have that the upper left entry is increasing in K_{22}, increasing in K_{11} and decreasing in K_{12}. If $K_{22} > \frac{K_{12}^2}{K_{11}}$, then it is increasing in a_{21}, otherwise it is decreasing.

[11]:
```
simplify(expand(K_sol[0, 1]))
```

[11]:

$$\frac{a_{21}\left(K_{11}K_{22}\left(1 - a_{21}\right) + K_{12}^2 a_{21} - K_{12}^2\right)}{K_{11}}$$

This is the bottom left and upper right corner of K_t. Without going into too many details, the effect of a_{21} appears to be increasing. The only decreasing term is $(1 - a_{21})$ which expands out as $K_{11}K_{22}(a_{21} - a_{21}^2)$ and $a_{21} \geq a_{21}^2$ since $a_{21} \leq 1$.

[12]:
```
simplify(expand(K_sol[1, 1]))
```

[12]:

$$\frac{K_{11}\left(K_{22}a_{21}^2 - 2K_{22}a_{21} + K_{22} + 1\right) - K_{12}^2 a_{21}^2 + 2K_{12}^2 a_{21} - K_{12}^2}{K_{11}}$$

This is the bottom right corner of K_t.

[13]:
```
diff(simplify(expand(K_sol[1, 1])), symbols("a21"))
```

[13]:

$$\frac{K_{11}\left(2K_{22}a_{21} - 2K_{22}\right) - 2K_{12}^2 a_{21} + 2K_{12}^2}{K_{11}}$$

This is its derivative with respect to a_{21}, which shows the condition required for the original function to be increasing.