SSY191: Individual Assignment 1

March 2023

Instructions

Full solutions should be turned in as a PDF-file, you should turn-in the solutions in Canvas. The solutions can be handwritten or generated using LaTeX/Word. The solutions should be complete and well explained. For problems that involve programming or simulations the code and/or Matlab models should be included in a zip-file that should be attached. This is an individual assignment and cooperation is not allowed.

Problem 1

Given an arbitrary 3D homogeneous transformation matrix ${}^{W}\xi_B$,

$$ {}^{W}\xi_B = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

Derive how to calculate the homogeneous transformation matrix ${}^{B}\xi_W$. Give an example of ${}^{W}\xi_B$ containing both a non-zero translation and rotation and pick one 3D point in the 3D-space and calculate its representation in both B and W coordinate frames. (2p)

Problem 2

In the lecture we derived how to estimate the orientation from the accelerometer when using Euler-Angles with XYZ (roll-pitch-yaw) rotation order. Use the rotation matrices to derive how to estimate pitch and roll if we instead use the YXZ (pitch-roll-yaw) rotation order. (3p)

Problem 3

In the quadrotor we use a a complementary filter to estimate an angle from readings from the accelerometer and gyroscope. The complementary filter is given by

$$\theta = G(s)\theta_a(s) + (1 - G(s))\theta_g(s) \tag{1}$$

where

$$\theta_g(s) = \frac{1}{s}Y_g(s)$$

$\theta_a(t)$ is the estimated angle from the accelerometer, and $y_g(t)$ is the angular velocity from the gyroscope.

Use Euler-backward discretization to derive a difference equation that tells how to calculate the an estimated angled from readings from the accelerometer and gyroscope. Provide a derivation of the complementary filter using the Euler-backward. (2p)

Problem 4

Consider the water tank control system shown below.

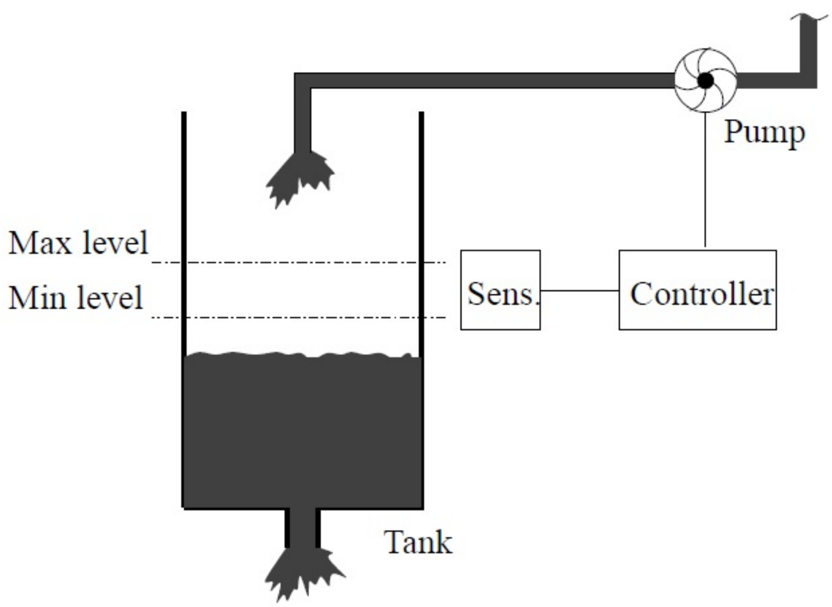

Figure 1: Water tank with level sensors and a pump.

The control objective is to maintain the water level l within the maximum and minimum levels indicated in the figure 1. The controller can either turn on the pump or turn it off. When the pump is on, the dynamics of the tank is

$$On : \frac{dl(t)}{dt} = -l(t) + 25$$

and when it is off

$$Off : \frac{dl(t)}{dt} = -l(t)$$

The controller will turn the pump on when $l < 10$ and turn it off when $l > 15$.

a) Draw a hybrid automaton of the system. Assume the initial state is $l(0) = 12$. (1p)

b) Build a model of the hybrid automaton in Simulink or Simscape (Stateflow is not allowed). Simulate the system with and without zero-crossing detection enabled, assuming the initial state is $l(0) = 12$. Is there a difference? Note, depending on how you built your model you might or might not see a difference here.) Provide plots along with an explanation of your results. (2p)

c) Is the hybrid automaton Zeno, i.e., does it have Zeno solutions, motivate your conclusion. (1p)