Cyber Individual Assignment 1

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1 Problem1

$$W\xi_B = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B\xi_w = W\xi_B^{-1} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{-1}$$

$$= (\begin{bmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix})^{-1}$$

$$= \begin{bmatrix} r_{11} & r_{21} & r_{31} & t_1 \\ r_{12} & r_{22} & r_{32} & t_2 \\ r_{13} & r_{23} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -t_1 \\ 0 & 1 & 0 & -t_2 \\ 0 & 0 & 1 & -t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r_{11} & r_{21} & r_{31} & -dot(r1, t) \\ r_{12} & r_{22} & r_{32} & -dot(r2, t) \\ r_{13} & r_{23} & r_{33} & -dot(r3, t) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem1 example

Figure 1 shows the example.

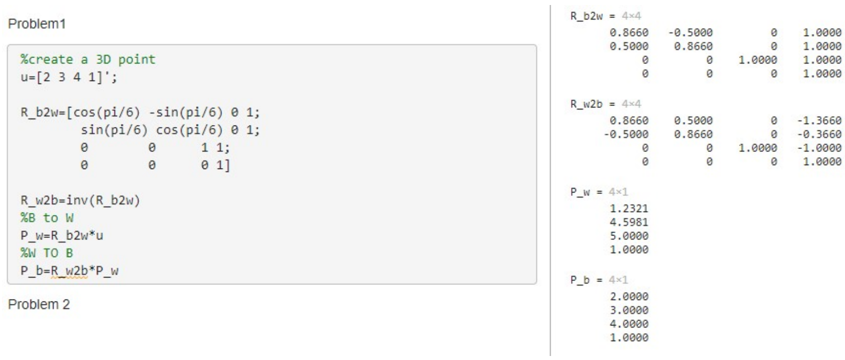

Figure 1: Example for P1

2 Problem2

$$R_{yxz} = \begin{pmatrix} \cos(\psi)\cos(\theta)+\sin(\phi)\sin(\psi)\sin(\theta) & \cos(\psi)\sin(\phi)\sin(\theta)-\cos(\theta)\sin(\psi) & \cos(\phi)\sin(\theta) \\ \cos(\phi)\sin(\psi) & \cos(\phi)\cos(\psi) & -\sin(\phi) \\ \cos(\theta)\sin(\phi)\sin(\psi)-\cos(\psi)\sin(\theta) & \sin(\psi)\sin(\theta)+\cos(\psi)\cos(\theta)\sin(\phi) & \cos(\phi) \end{pmatrix}$$

$$\mathrm{f} = \begin{pmatrix} \cos(\phi)\sin(\theta) \\ -\sin(\phi) \\ \cos(\phi)\cos(\theta) \end{pmatrix} = R_{yxz} * [0,0,1]^T$$

Thus, we have

$$\tan(\theta) = \frac{\mathrm{fx}}{\mathrm{fz}}$$
$$\Rightarrow \theta = atan2(fx, fz)$$

Besides,

$$\sqrt{(\mathrm{fx}^2 + \mathrm{fz}^2)} = \cos(\phi)$$
$$\mathrm{fy} = -\sin(\phi)$$
$$\Rightarrow \phi = atan2(-fy, \sqrt{(fx + fz^2)})$$

Check Figure 2 for mathematical deduction.

3 Problem3

Combine two equatons together: $\theta(s) = G(s)\theta_a(s) + (1 - G(s))\,\frac{1}{s}y_g$

$G(s)$ is a low-pass filter where $G(s) = \dfrac{1}{\alpha s + 1}$.

$$\theta(s) = \frac{1}{\alpha s + 1}\theta_a(s) + \left(1 - \frac{1}{\alpha s + 1}\right)\frac{1}{s}y_x$$
$$(\alpha s + 1)\theta = \theta_a + \alpha y_g$$
$$\alpha s\theta + \theta = \theta_a + \alpha y_g$$

Euler backward: $\dot{x}(t) = sx(s) \approx \frac{x_k - x_{k-1}}{h}$

Thus,

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R_x =
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$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{pmatrix}$$

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R_y =
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$$\begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

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R_z =
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$$\begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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R_yxz =
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$$\begin{pmatrix} \cos(\psi)\cos(\theta) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\theta)\sin(\psi) & \cos(\phi)\sin(\theta) \\ \cos(\phi)\sin(\psi) & \cos(\phi)\cos(\psi) & -\sin(\phi) \\ \cos(\theta)\sin(\phi)\sin(\psi) - \cos(\psi)\sin(\theta) & \sin(\psi)\sin(\theta) + \cos(\psi)\cos(\theta)\sin(\phi) & \cos(\phi)\cos(\theta) \end{pmatrix}$$

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f =
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$$\begin{pmatrix} \cos(\phi)\sin(\theta) \\ -\sin(\phi) \\ \cos(\phi)\cos(\theta) \end{pmatrix}$$

`theta =` $\mathrm{atan2}(\cos(\phi)\sin(\theta), \cos(\phi)\cos(\theta))$

`phi =` $\mathrm{asin}(\sin(\phi))$

Figure 2: P2 Matlab

$\alpha\frac{(\theta_k-\theta_{k-1})}{\hbar}+\theta_k=\theta_{a,k}+\alpha y_{g,k}$

$\Rightarrow \frac{(\alpha+h)}{\hbar}\theta_k=\frac{\alpha}{\hbar}\theta_{k-1}+\theta_{a,k}+\alpha y_{x,k}$

$\Rightarrow \theta_k=\frac{h}{h+\alpha}\theta_{a,k}+\frac{\alpha}{h+\alpha}(\theta_{k-1}+hy_{g,k})$
According to the lecture, $\gamma=\frac{\alpha}{h+\alpha}(0<\gamma<1)$
We have,
$\theta_k=(1-\gamma)\theta_{a,k}+\gamma(\theta_{k-1}+hy_{g,k})$ □

4 Problem4

(a)See Figure 3
(b)See Figure 4 to 7. The model can capture more accurately the behavior of the system if we choose to use enable zero-crossing. For not using zero-crossing, it cannot detect the transition mode which will have numerical error due to discretized scheme.
(c)Zeno behavior can occur in hybrid systems when the system switches back and forth between continuous and discrete modes with increasingly higher frequencies.

For our system, it is not ZENO, since the higher frequency transitions in a finite amount of time is not possible. The transition region is relatively huge ($10<l<15$). From physical perspective, if the system wants to switch the state, the water level will have to increase or decrease 5m to be able to change the state again, which prevents the ZENO behaviour.

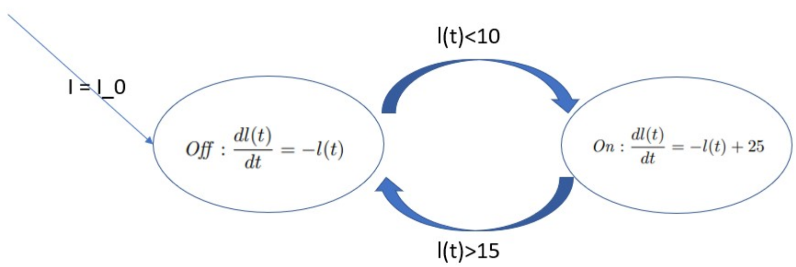

Figure 3: P4 drawing

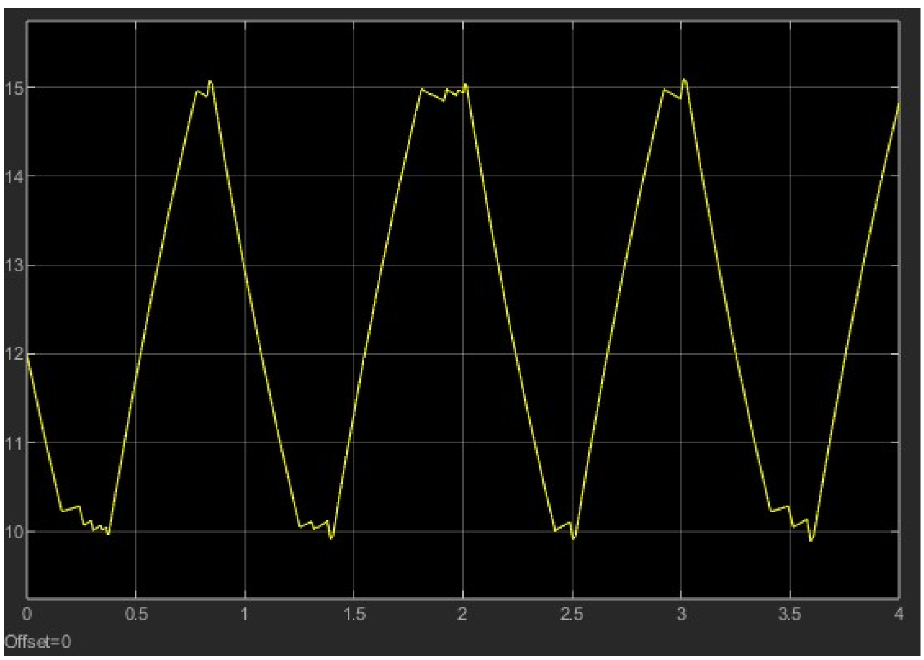

Figure 4: No zero-crossing-1

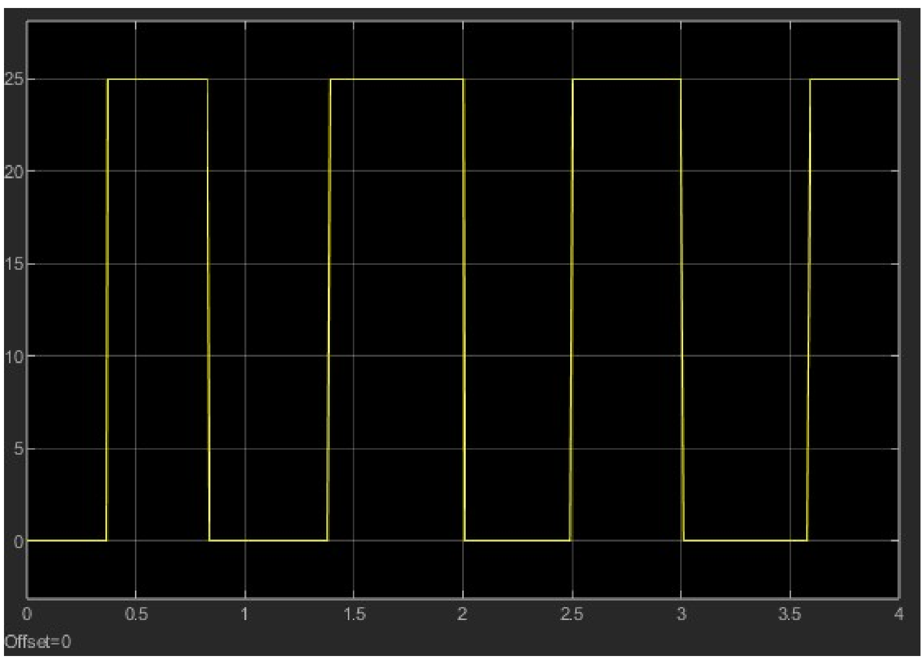

Figure 5: No zero-crossing-2

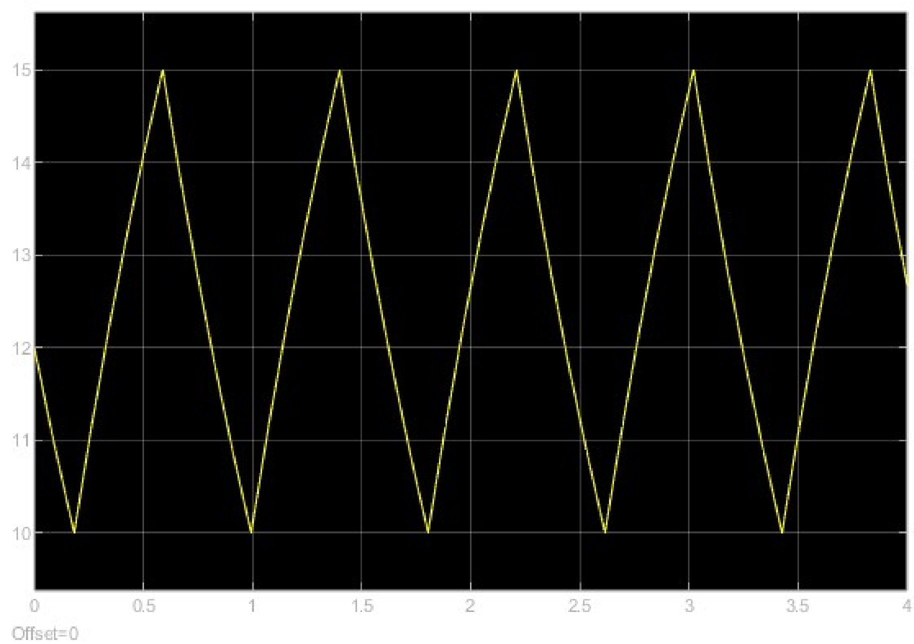

Figure 6: zero-crossing-1

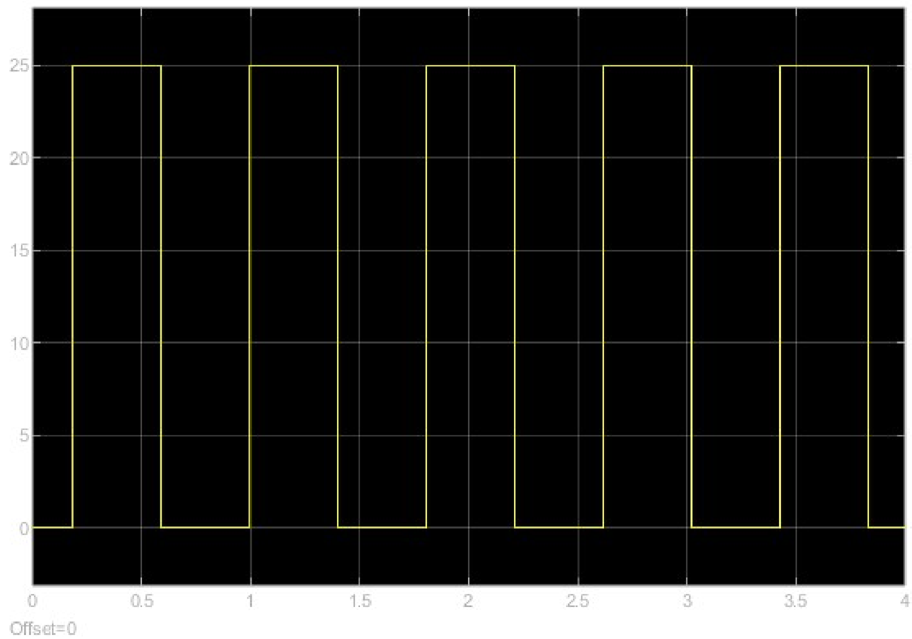

Figure 7: zero-crossing-2

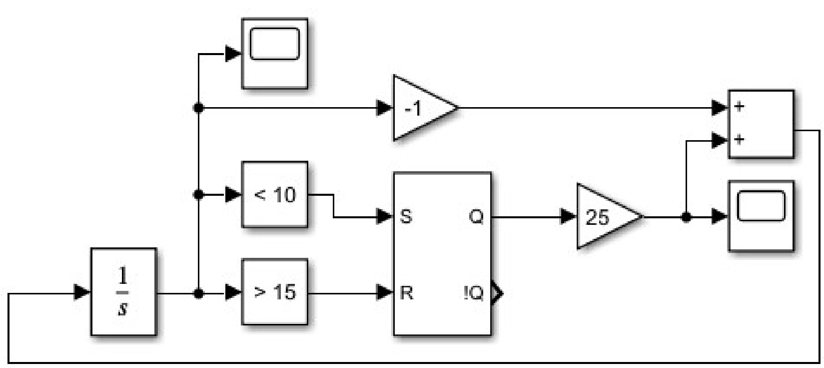

Figure 8: simulink