Orientation estimation of smartphones

Weilong Chen

May 26, 2023

Task 1

Pros: Gyroscope measurements provided by smartphones are generally considered accurate. Using gyroscope measurements as inputs can provide more reliable and precise information than accelerator and magnetic sensors. Gyroscope measurements are usually available at a high sampling rate, allowing for real-time updates of the system state.

Cons: Gyroscopes measure angular velocity directly, but to estimate orientation or position, integration of the angular velocity is required. Unfortunately, integrating angular velocity over time can introduce errors due to drift. Over time, these errors accumulate and can lead to significant deviations from the true state. Besides, Gyroscopes cannot provide absolute orientation or position information on their own. They measure the rate of change of angular velocity, requiring an initial alignment or reference orientation to establish a starting point.

When dealing with scenarios involving sudden and impulsive changes in angular velocity due to external disturbances, relying solely on gyroscope measurements may not be a good choice.

Including angular velocities in the state vector, along with other state variables, can be beneficial in situations where the filtering task requires robust estimation of orientation or position over a longer duration.

Task 2

The device is Android IQOO Phone. With a few seconds of data collection, the measurement noise is generally Gaussian. The z-direction of the accelerometer has a bias due to gravity. The covariance for the gyroscope is very small, which is the reason why we trust the gyroscope as our measurement input. We tend to collect long-time data to avoid short-term oscillation and try to press the start and end buttons to minimize disturbance to the phone.

The mean and covariance of three sensors are shown below:

$$\begin{aligned}
\mu_{acc} &= [0.0665, 0.0866, 9.9058]^T \\
\mu_{gyr} &= \left[2.5561 \cdot 10^{-5}, -3.9716 \cdot 10^{-5}, -2.5681 \cdot 10^{-6}\right]^T \\
\mu_{\text{mag}} &= [2.9769, 18.4024, -40.5299]^T
\end{aligned}$$

$$\begin{aligned}
\text{Cov}_{\ acc} &= \begin{bmatrix} 4.0569 \cdot 10^{-5} & -3.9946 \cdot 10^{-7} & -2.7897 \cdot 10^{-6} \\ -3.9946 \cdot 10^{-7} & 4.0969 \cdot 10^{-5} & 1.8549 \cdot 10^{-6} \\ -2.7897 \cdot 10^{-6} & 1.8549 \cdot 10^{-6} & 6.2628 \cdot 10^{-5} \end{bmatrix} \\
\text{Cov}_{\text{gyr}} &= \begin{bmatrix} 5.4689 \cdot 10^{-7} & -2.2873 \cdot 10^{-9} & -1.0864 \cdot 10^{-7} \\ -2.2873 \cdot 10^{-9} & 4.1250 \cdot 10^{-7} & 4.2133 \cdot 10^{-8} \\ -1.0864 \cdot 10^{-7} & 4.2133 \cdot 10^{-8} & 3.8925 \cdot 10^{-7} \end{bmatrix} \\
\text{Cov}_{\ \text{mag}} &= \begin{bmatrix} 0.0953 & -0.0033 & 0.0095 \\ -0.0033 & 0.1026 & -0.0054 \\ 0.0095 & -0.0054 & 0.1097 \end{bmatrix}
\end{aligned}$$

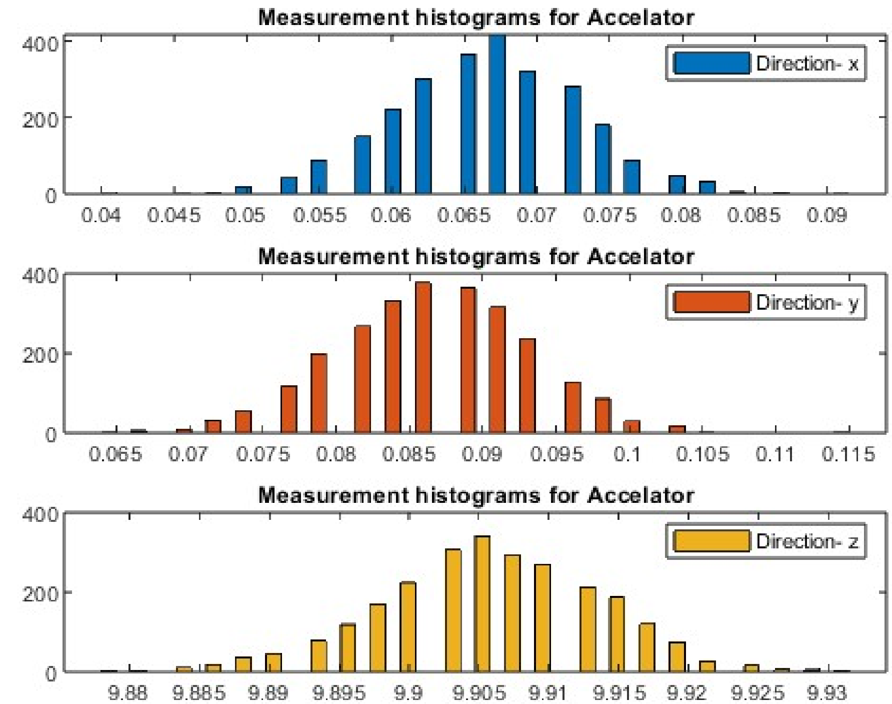

Figure 1: Accelarator Measurement

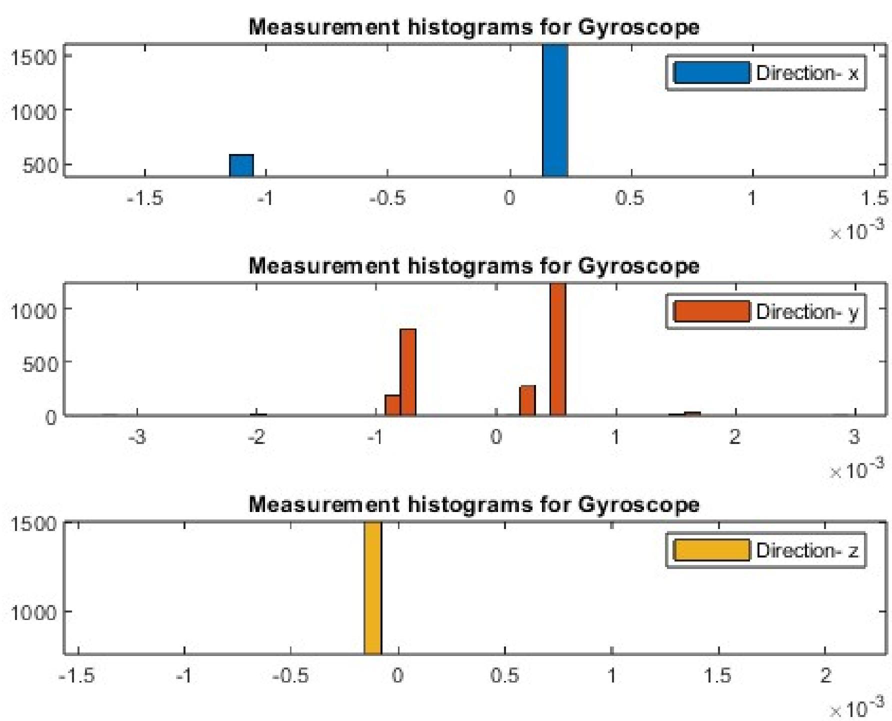

Figure 2: Gyroscope Measurement

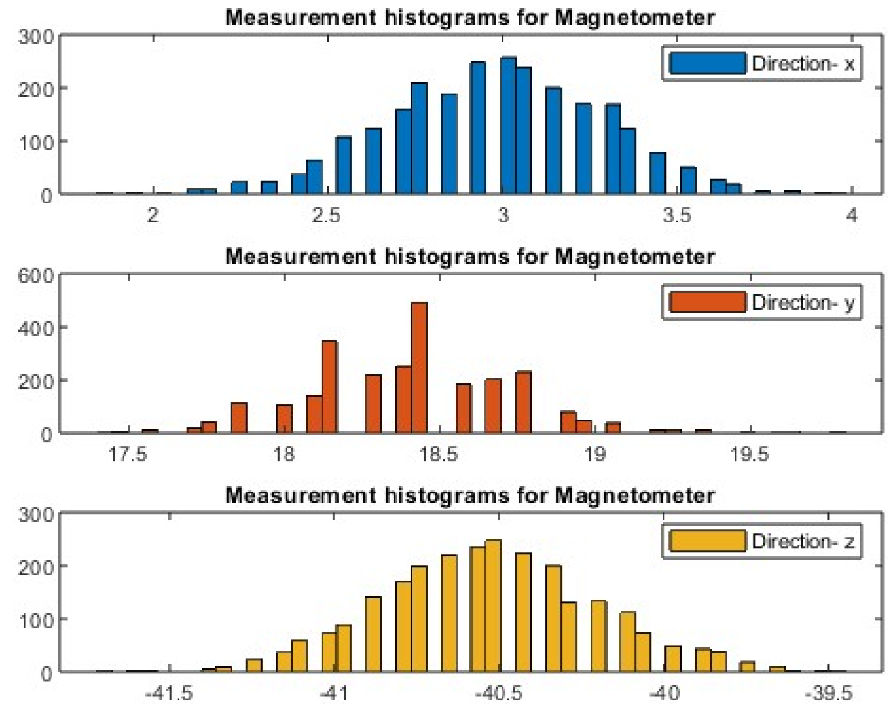

Figure 3: Magnetometer Measurement

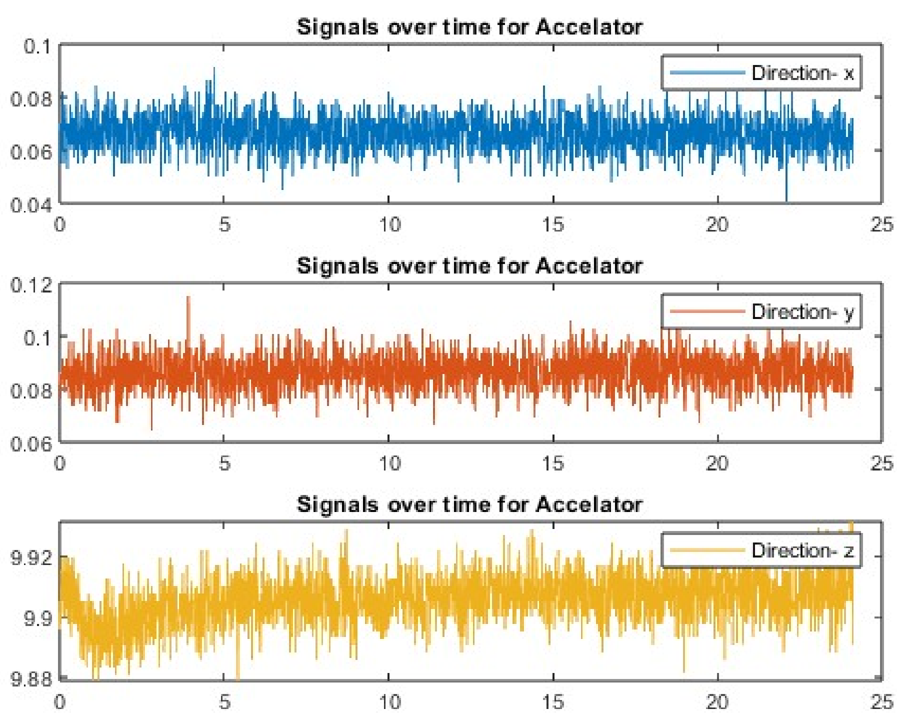

Figure 4: Accelator signals over time

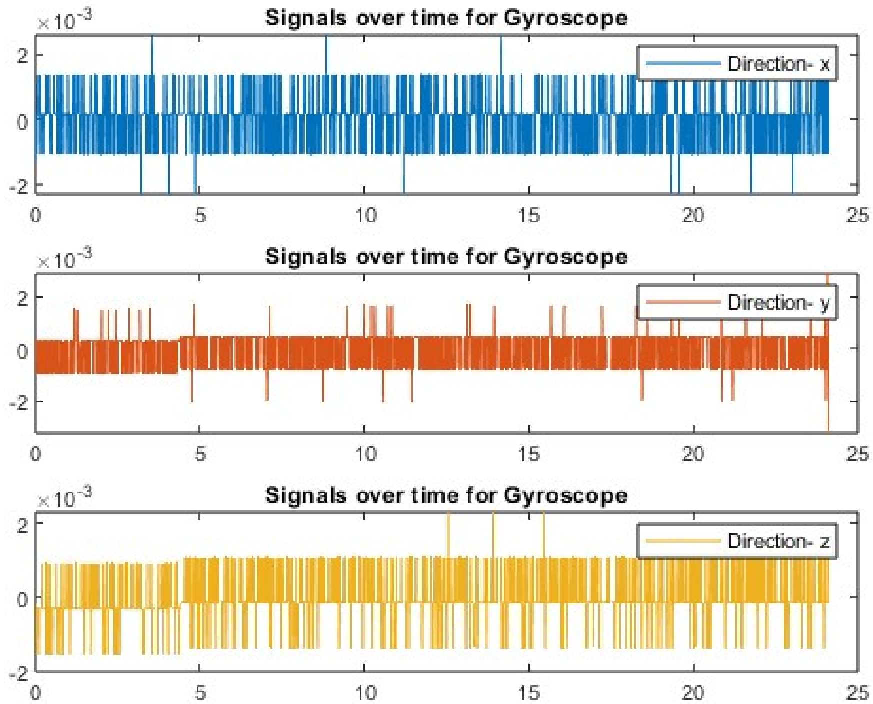

Figure 5: Gyroscope signals over time

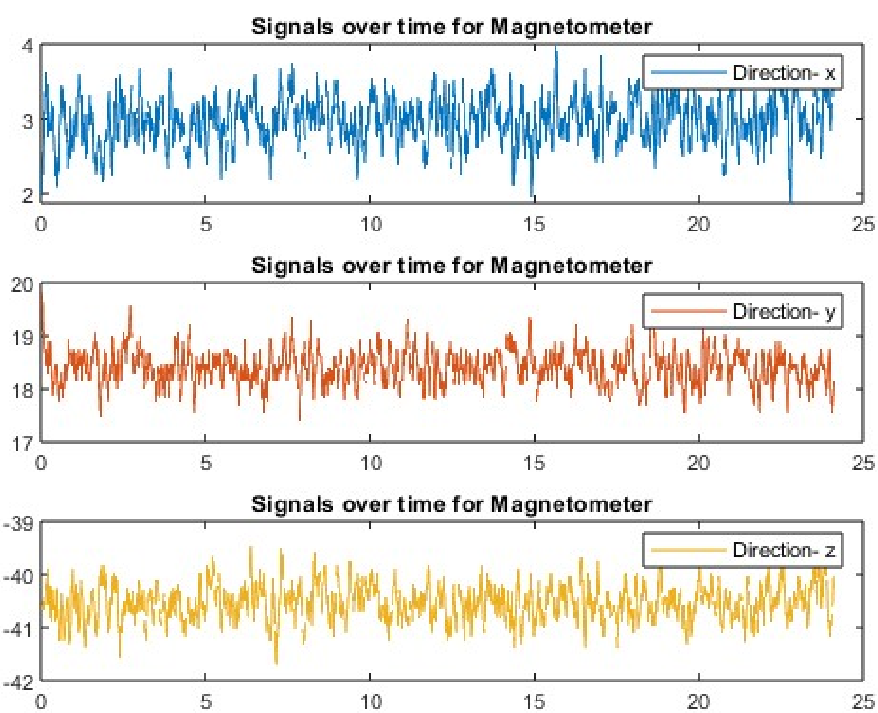

Figure 6: Magnetometer signals over time

Task 3

The complete continuous time model is:

$$\dot{q}(t) = \frac{1}{2} S\left(w_{k-1} + v_{k-1}\right) q(t)$$

Condier analytical solution for linear systems, for ODE $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + b$, then

$$x(t+T) = \exp(\mathbf{A}T) x(t) + \left(\mathbf{I}T + \frac{\mathbf{A}T^2}{2} + \frac{\mathbf{A}^2T^3}{3} + \dots\right) \mathbf{b}$$

for our problem:

$$q(t+T) = \exp(\mathbf{A}T) \cdot q(t)$$

where by definition $\mathbf{A} = \frac{1}{2} S\left(w_{k-1} + v_{k-1}\right)$. Use $\exp(\mathbf{A}) = \mathbf{I} + \mathbf{A}$, we have

$$\begin{aligned}
q(t+T) &= \exp\left(\frac{1}{2} S\left(w_{k-1} + v_{k-1}\right) T\right) \cdot q(t) \\
&= \left[\mathbf{I} + \frac{T}{2} S\left(w_{k-1} + v_{k-1}\right)\right] \cdot q(t) \\
q_k &= \mathbf{I} \cdot q_{k-1} + \frac{T}{2} S\left(w_{k-1} + v_{k-1}\right) \cdot q_{k-1} \\
&= \left(\mathbf{I} + \frac{1}{2} S\left(w_{k-1}\right)\right) \cdot q_{k-1} + \frac{T}{2} S\left(v_{k-1}\right) \cdot q_{k-1}
\end{aligned}$$

With the definition $S(w)q = \bar{S}(q)w$ and the suggested approximation $G\left(q_{k-1}\right) v_{k-1} \approx G\left(\hat{q}_{k-1}\right) v_{k-1}$, we have:

$$\begin{aligned}
q_k &= \left(\mathbf{I} + \frac{T}{2} S\left(w_{k-1}\right)\right) \cdot q_{k-1} + \frac{T}{2} \bar{S}\left(q_{k-1}\right) \cdot v_{k-1} \\
&= \underbrace{\left(\mathbf{I} + \frac{T}{2} S\left(w_{k-1}\right)\right)}_{F\left(w_{k-1}\right)} \cdot q_{k-1} + \underbrace{\frac{T}{2} \bar{S}\left(q_{k-1}\right)}_{G\left(q_{k-1}\right)} \cdot v_{k-1}
\end{aligned}$$

Task 4

%Mean update

$$\begin{aligned}
&F = \text{ eye } (\text{size}(x, 1)) + (T/2) * \text{ Somega(omega);} \\
&x = F * x;
\end{aligned}$$

%Cov update

$$\begin{aligned}
&G = (T/2) * Sq(x); \\
&P = F * P * F' + G * Rw * G';
\end{aligned}$$

Besides, if the data is not available, a random walk is implemented as follows:

$$P = P + 0.1 * \text{eye}(4);$$

Task 5

Since the EKF lacks absolute orientation, it will provide reactive estimates of the rotations but won't be able to determine the initial reference orientation.

If we start the filter with the phone on its side instead of laying face up on the desk, the initial estimates of the rotations will be incorrect. The filter will estimate the rotations based on the gyroscope measurements but won't have an absolute reference to determine the correct orientation. As a result, the estimated rotations will deviate from the true rotations, leading to inaccurate orientation estimates.

When you shake the phone, the gyroscope measurements will capture the angular velocity caused by the shaking motion. However, without absolute orientation information, the filter won't be able to differentiate between the shaking motion and actual changes in orientation. The integration of angular velocity measurements over an extended period can accumulate errors, leading to a gradual degradation in the accuracy of the estimated rotations.

Task 6

```
function [x, P] = mu_g(x, P, yacc, Ra, g0)
% Calculate predicted accelerometer measurement using the current
    state estimate
hx = Qq(x)'*g0;

% Calculate the Jacobian matrix of the predicted accelerometer
    measurement with respect to the quaternion
[dq1, dq2, dq3, dq4] = dQqdq(x);
dhx = [dq1'*g0 dq2'*g0 dq3'*g0 dq4'*g0];
% Calculate the innovation covariance matrix
sk = dhx*P*dhx'+Ra;
% Calculate the Kalman gain
kk = P*dhx'/sk;
% Update the state estimate using the Kalman gain and accelerometer
    measurement
x = x+kk*(yacc-hx);
% Update the covariance matrix
P = P-kk*sk*kk';

end
```

Task 7

When the device undergoes back and force motion, the total specific force measured by the accelerometer will include both the gravitational acceleration and the acceleration due to motion. This can lead to errors in the orientation estimation because the EKF assumes that the measured acceleration is solely due to gravity and does not account for other acceleration.

As a result, when f_a^k is large, the assumption that f_a^k is zero is violated, and the EKF's accelerometer update function will not perform well. The errors in the specific force measurement will propagate through the orientation estimation process, leading to inaccurate results.

Task 8

Compare the magnitude of the measured specific force with the expected value of g. If the measured value deviates significantly from the expected value, it can be considered an outlier, indicating a disturbed measurement. In such cases, we can skip the update step or implement a random walk for the accelerometer measurement in the EKF. The outlier parameter should be adjusted in the final step. Till task 8, personally I set it as 15%.

Task 9

It's almost the same as Task 6, just change the parameters from accelerometer to magnetometer. R_{mag} is caculated in Task2, $m0 = [0, 18.6417, -40.5299]$;

Task 10

After adding the magnetometer update to the EKF, the orientation behaves much better, almost the same as the Google filter. The results will be shown in task 12. However, when attaching two phones together, which means introducing a disturbance, the filter becomes very noisy, as shown in the figure below.

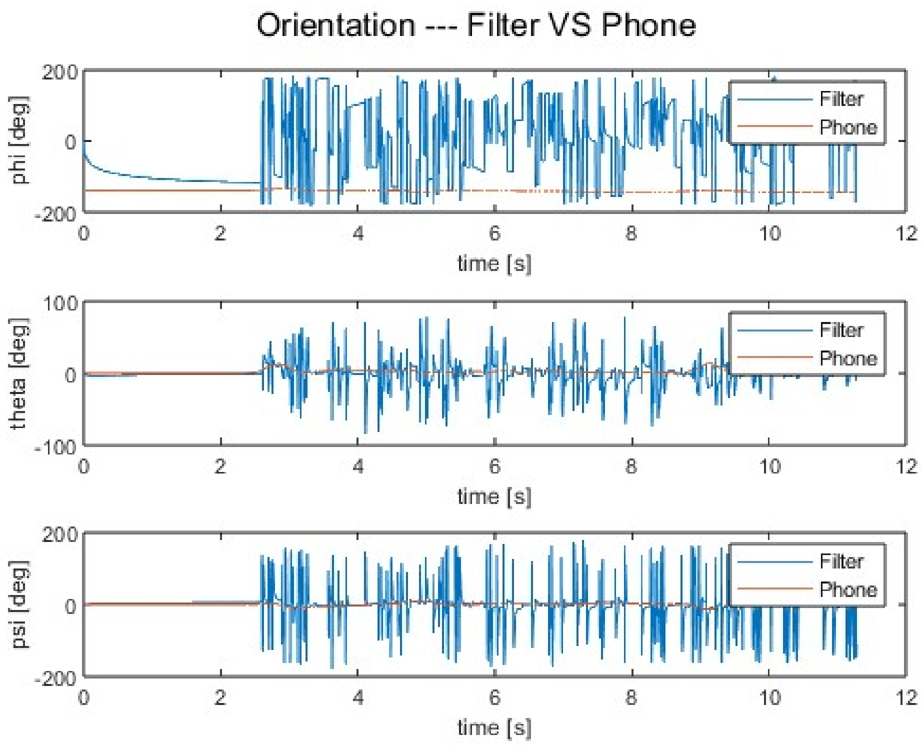

Figure 7: Orientation estimation with disturbance

Task 11

After realizing that disturbances can significantly impact the results, it is better to set up an outlier rejection technique. The approach involved estimating the expected magnetic field strength and comparing it with the actual measurements. If the measured field strength deviated too much from our expectations, we rejected those particular measurements. We set $\alpha = 0.01$ and allow 10% changes. As a result, our measurements became more reliable and unaffected by outlier values, ultimately improving the accuracy of our magnetic field analysis. The final result is shown in Task 12.

Task 12

First situation: three sensors fused together, as shown in Figure 8, work well. The noise is due to my working environment being close to the computer, with several electronic devices nearby.

Second situation: Accelerometer and magnetometer used, without gyro, as shown in Figure 9. Generally, it performs well, but when the phone is moved up and down dramatically, it can lose some accuracy.

Third situation: Accelerometer and gyro utilized, without magnetometer, as depicted in Figure 10. The performance is not satisfactory, as at certain positions, the estimated result undergoes repetitive rotations for a certain period of time. The reference point is unknown when for this case, resulting wrong absolute positions.

Fourth situation: Mag and gyro utilized, without accelerometer. Shown in figure 11. It performs worse than Phone filter since acceleration is not considered in this case.

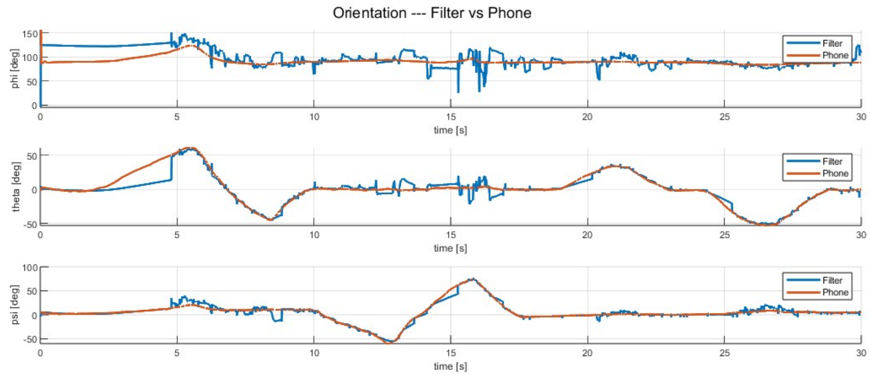

Figure 8: Orientation estimation with three sensors together

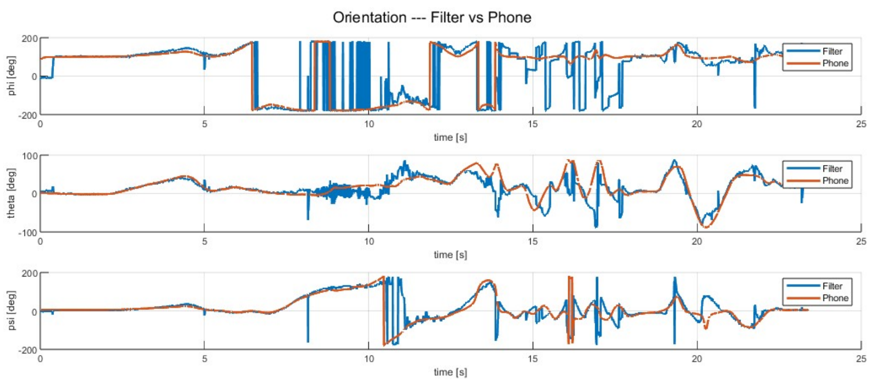

Figure 9: Orientation estimation with acc and mag without gyro.

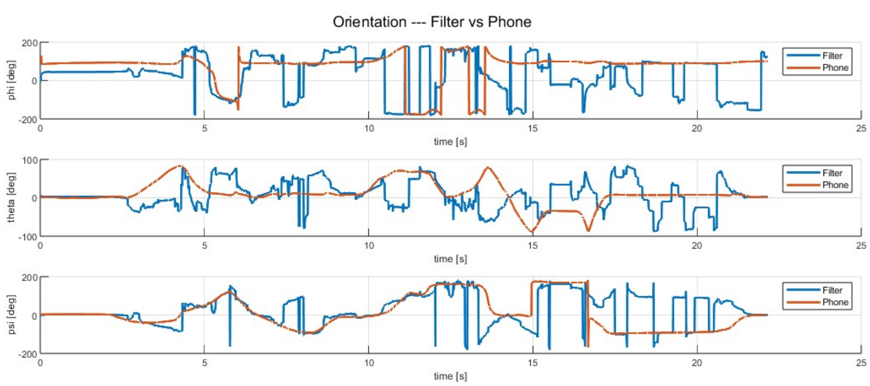

Figure 10: Orientation estimation with acc and gyro without mag.

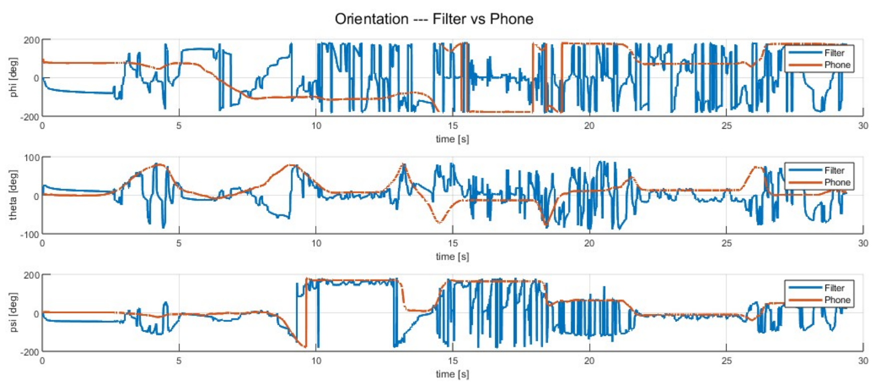

Figure 11: Orientation estimation with Mag and Gyro, without acc.