Solution to analysis in Home Assignment 4

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Analysis

In this report I will present my independent analysis of the questions related to home assignment 4. I have discussed the solution with Jing Zhang.

1 Smoothing

1.1 a

Figure 1.1 illustrates the positions of the smooth and filter. The covariance circle of the smooth is smaller than that of the filter. The smoothed position tends to be more stable and accurate, closely approaching the true position. In Figure 1.2, the smoothing error is generally lower than the filtering error. However, in certain positions, the smoothing error may be slightly larger.

The smooth operation involves incorporating information from past and future measurements to estimate the current position. While, the filter operation focuses on estimating the current position based only on the available measurements at that particular time. It relies solely on the current data point and does not consider future measurements. Consequently, the filter's covariance circle tends to be larger than that of the smooth, indicating a higher level of uncertainty or error in the estimated position.

1.2 b

As shown in Figure 1.3, there is an outlier present at time step 300. In response to this change, the smoother demonstrates a reduced reaction compared to the filtering approach. The smoother quickly returns to a position closer to the true value, indicating its ability to recover faster from the impact of the outlier. In contrast, the filter exhibits a slower adjustment to the outlier.

The smoother's superior ability to overcome the presence of the outlier is attributed to its utilization of future measurements. By considering information from both past and future data points, the smoother can effectively smooth out the effect of the outlier based on a broader knowledge of the system's behavior.

In Figure 1.4, it is observed that there is a slight increase in error at the time step corresponding to the outlier. However, overall, the error tends to be smaller for the smoother compared to the filter. This outcome is again due to the

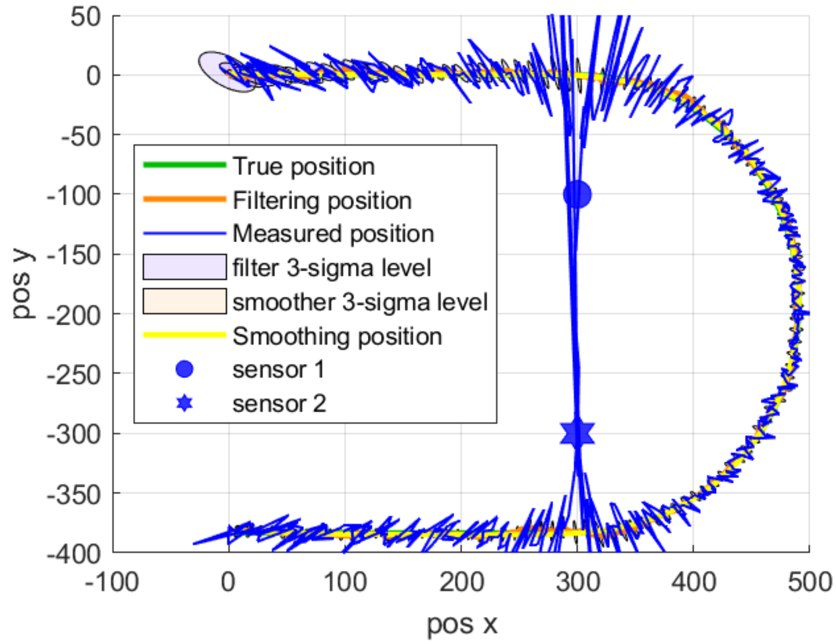


Figure 1.1: Smooth and Filter position comparison

smoother's access to future measurements, allowing it to incorporate additional information and improve the estimation accuracy.

In summary, the smoother exhibits better resilience to outliers by leveraging future measurements and incorporating more comprehensive knowledge of the system's dynamics. This results in a faster recovery from outliers and generally smaller estimation errors compared to the filtering approach.

2 Particle filters for linear/Gaussian systems

2.1 a

Figure 2.1 illustrates the mean squared error (MSE) for our problem, indicating that using 100 particles is generally sufficient. The MSE provides a measure of the accuracy of the estimated trajectory compared to the true trajectory.

In Figure 2.2, the true trajectory and the measurement sequence, along with their respective mean and covariance, are displayed for three different filters. Among these filters, the Kalman filter and the particle filter (PF) with resampling demonstrate slightly better performance compared to the PF without resampling.

Figure 2.3 shows the posterior densities without resampling, while Figure 2.4 depicts the posterior densities with resampling. It is observed that the mean and covariance of the Kalman filter and the PF with resampling are more stable

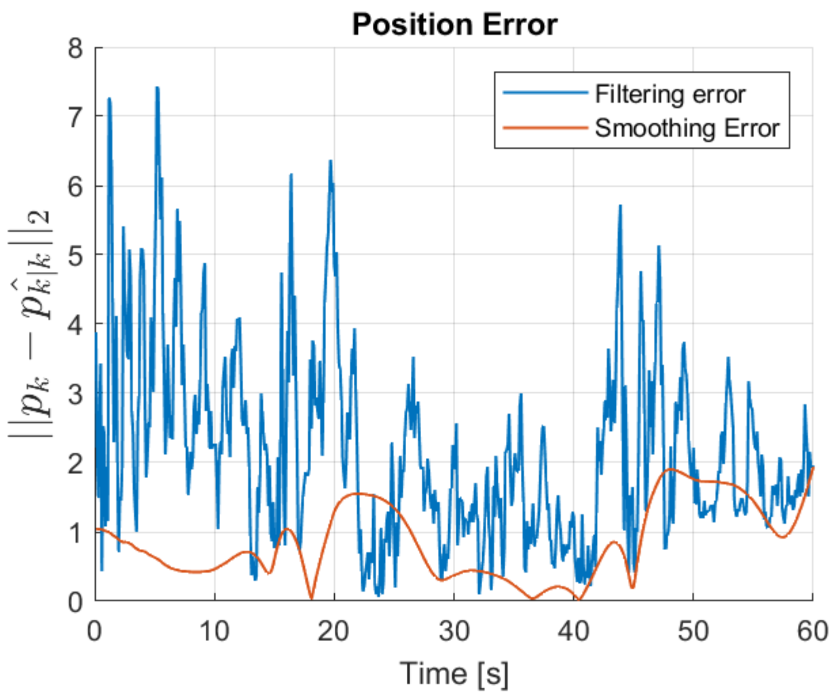


Figure 1.2: Smooth and Filter position error

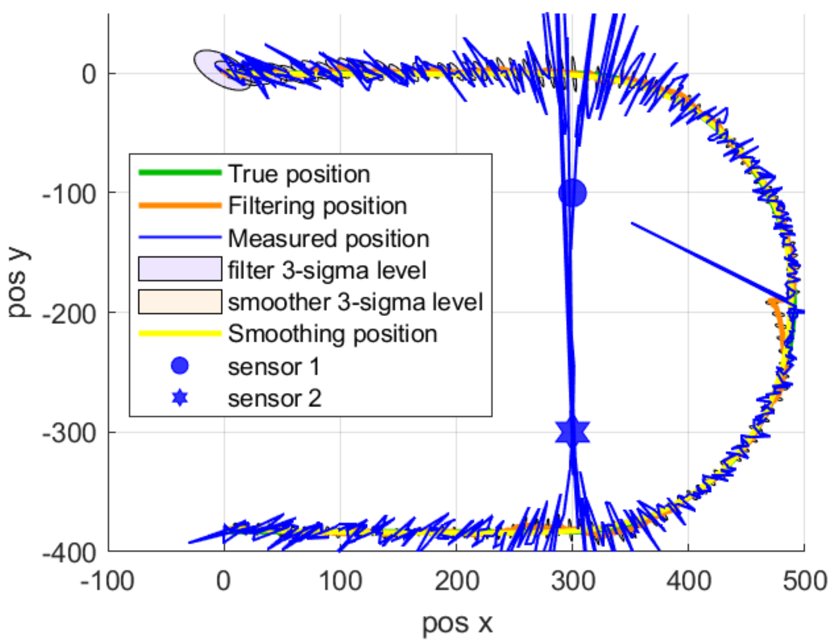


Figure 1.3: Smooth and Filter position comparison with changes at step 300

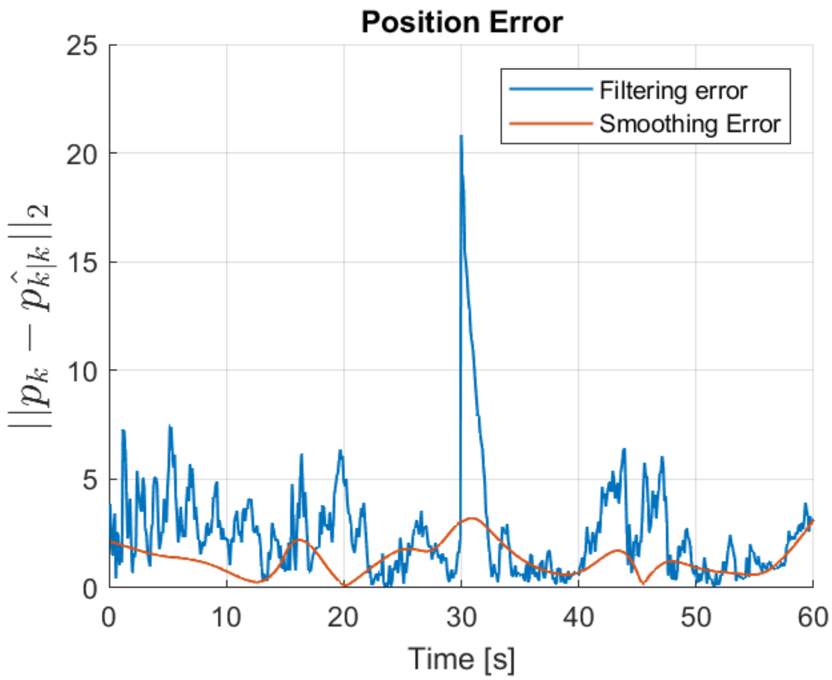


Figure 1.4: Smooth and Filter position error with changes at step 300

and exhibit similarities. This indicates that these filters provide consistent and reliable estimates of the state. However, the PF without resampling occasionally produces estimates that are far from the true state, suggesting less reliable performance in certain situations.

Overall, the results suggest that both the Kalman filter and the PF with resampling offer improved estimation accuracy compared to the PF without resampling. The stability and similarity of the mean and covariance for the Kalman filter and the PF with resampling indicate their effectiveness in providing robust and accurate estimates of the state.

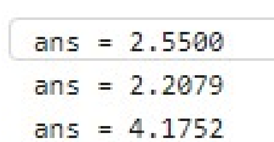


Figure 2.1: Result of MSE

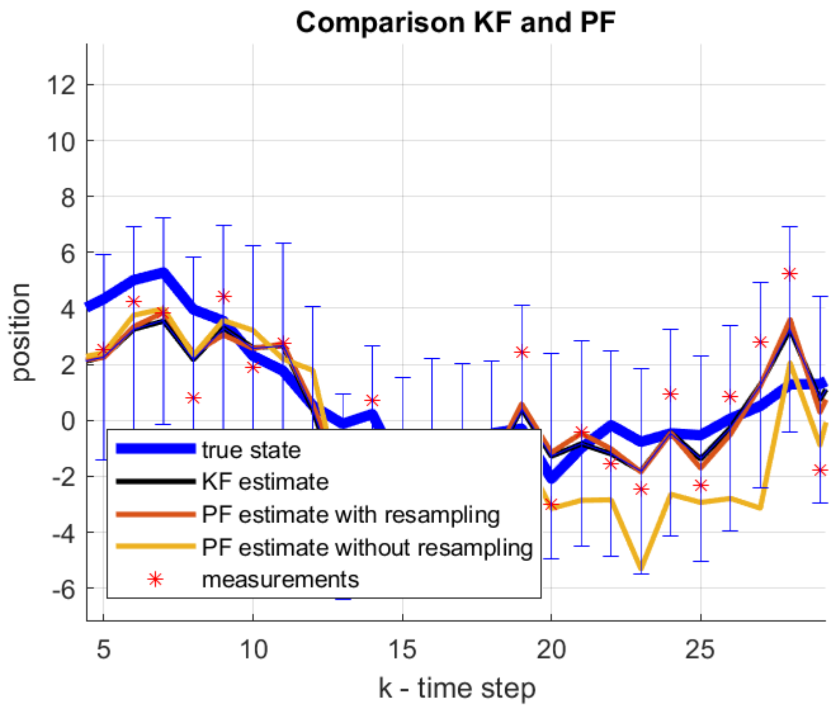
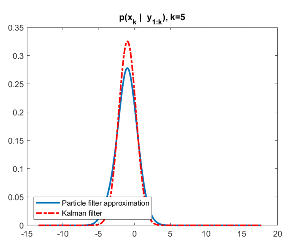
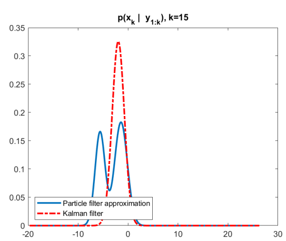


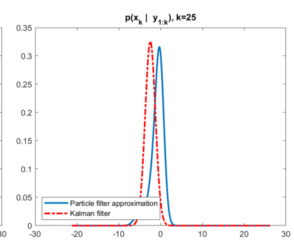
Figure 2.2: True trajectory and the measurement sequence for three filters



(a) 5 step

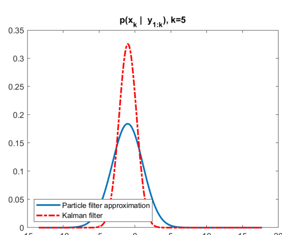


(b) 15 step

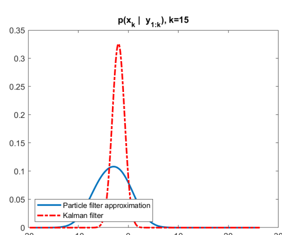


(c) 25 step

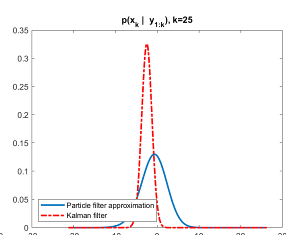
Figure 2.3: Comparison of different steps without resampling



(a) 5 step



(b) 15 step



(c) 25 step

Figure 2.4: Comparison of different steps with resampling

2.2 b

As it is shown in Figure 2.5, when using an incorrect prior at time 0, the Kalman filter demonstrates the fastest recovery towards the true trajectory. The PF with resampling follows with relatively slower speed. Meanwhile, the PF without resampling exhibits the slowest convergence due to the lack of an effective mechanism to update the particle set.

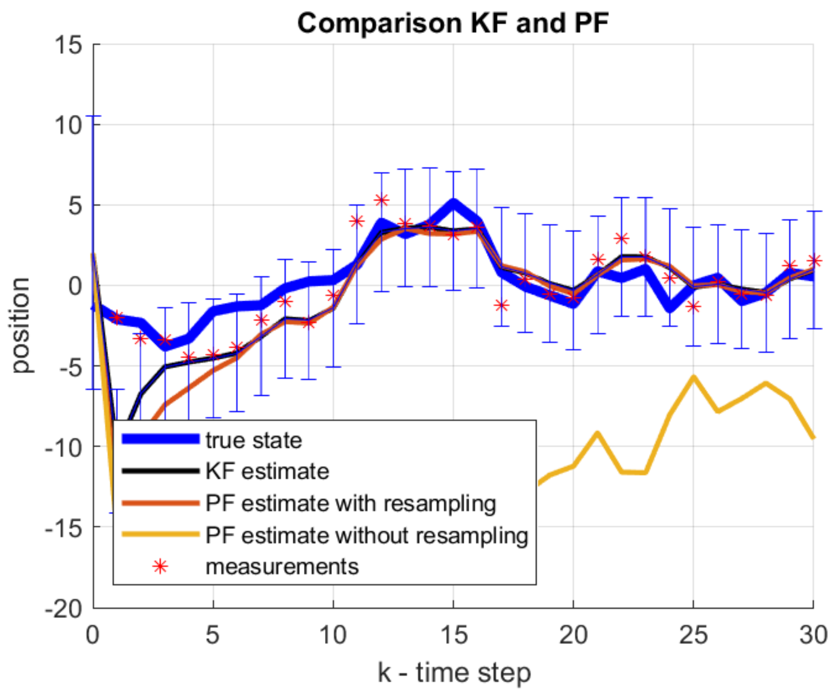


Figure 2.5: True trajectory and the measurement sequence for three filters with wrong prior information

2.3 c,d

The particle filter without resampling suffers from the problem of particle spreading, where particles tend to spread out and lose accuracy over time due to the uncertainty in the motion model, which is the degeneracy problem. Consequently, the filter's performance is greatly reduced as it relies on a single particle for estimation. The result is shown in Figure 2.7.

Resampling generally solves this problem, the result is shown in Figure 2.6. By resampling, particles with low weights are more likely to be replaced by particles with high weights. As a result, the resampled particles remain close to the true state and provide accurate representation of the true state probability. Resampling prevents excessive spreading of particles and improves the overall performance of the particle filter by maintaining a diverse set of particles that capture the true state effectively.

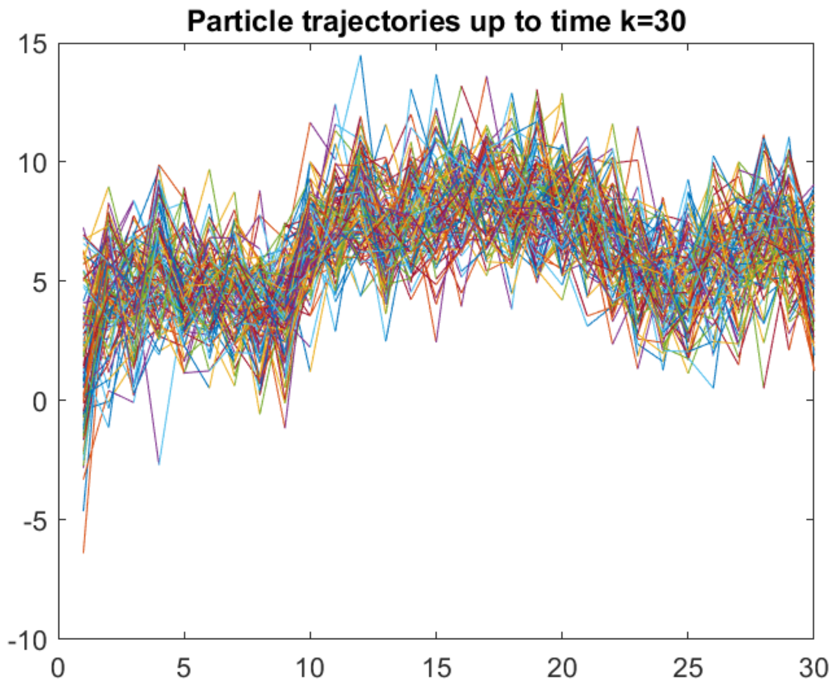


Figure 2.6: particle filter with resampling

3 Bicycle tracking in a village

3.1 a

Generated data is shown in figure 3.1

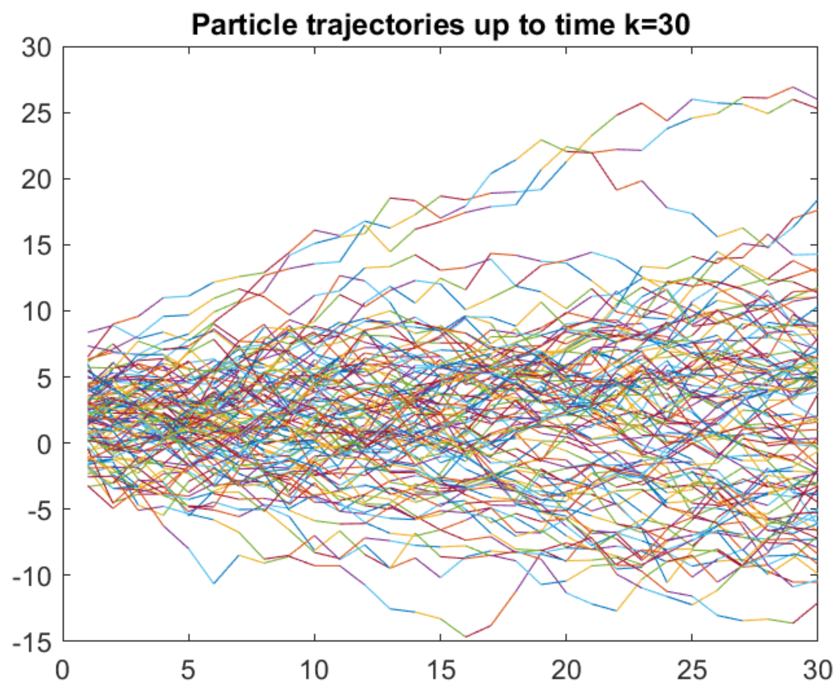


Figure 2.7: particle filter without resampling

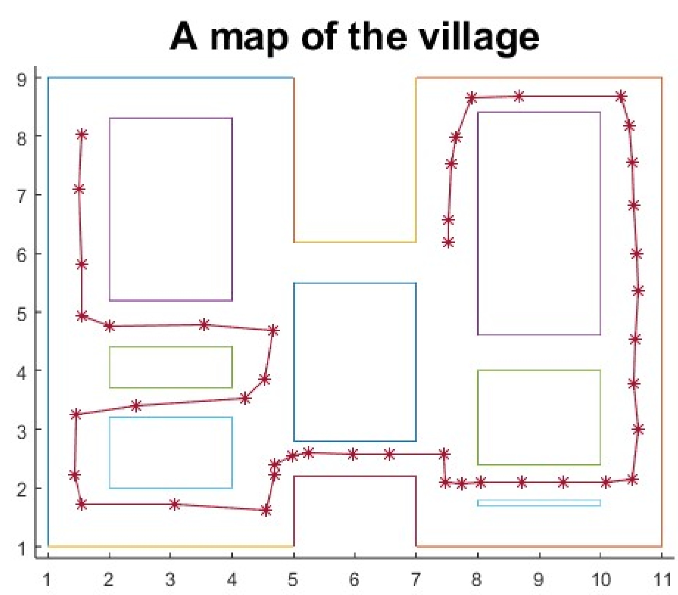


Figure 3.1: Trace in the map of the village