Solution to analysis in Home Assignment 3

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Analysis

In this report I will present my independent analysis of the questions related to home assignment 3. I have discussed the solution with no one.

1 Approximations of mean and covariance

1.1 a

Using 10,000 samples, we can start from one of three distributions. After applying a transformation function h(x) and adding some noise, we can obtain the mean and variance of these samples. The results are represented by the blue spot and closed curve.

1.2 b

The EKF method uses first-order Taylor approximation to estimate the mean and covariance directly as a linear transformation. On the other hand, UKF and CKF methods calculate different sigma points which contain the relevant information. After transforming these sigma points, they are averaged to obtain the mean and covariance. Check Figure 1.1 for all values.

1.3 c

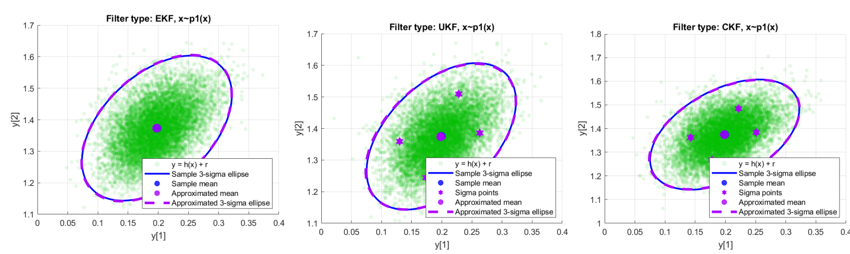
Figure 1.2a to Figure 1.4c show all 9 cases.

1.4 d

It is evident that the EKF method performs worse for the second distribution, and there is a significant difference compared to the UKF and CKF methods. The reason for this is that at the standing point, the first-order approximation of the EKF method is highly nonlinear, making it challenging to approximate with a linear function. In contrast, the UKF and CKF methods are better suited for non-linear systems because they use a set of sigma points to approximate the distribution, which provides a better representation of the nonlinearity. This

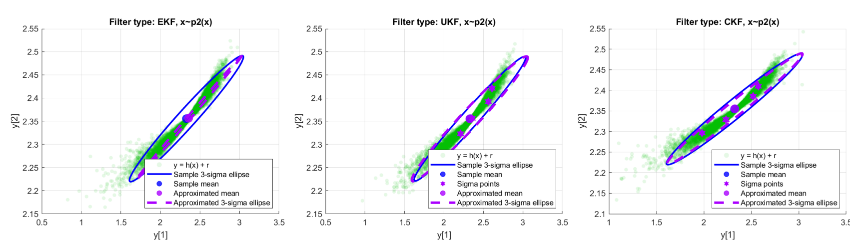
	mean1	mean2	cov1	cov2	cov3	cov4
Sample1	0.1982	1.3743	0.0017	0.0014	0.0014	0.0058
EKF1	0.1974	1.3734	0.0017	0.0015	0.0015	0.0060
UKF1	0.1983	1.3743	0.0017	0.0015	0.0015	0.0059
CKF1	0.1983	1.3743	0.0017	0.0015	0.0015	0.0059
Sample2	2.3284	2.3554	0.0569	0.0105	0.0105	0.0020
EKF2	2.3562	2.3562	0.0500	0.0100	0.0100	0.0020
UKF2	2.3269	2.3550	0.0600	0.0108	0.0108	0.0020
CKF2	2.3265	2.3550	0.0566	0.0105	0.0105	0.0020
Sample3	-0.5942	2.1521	0.0096	-0.0110	-0.0110	0.0148
EKF3	-0.5880	2.1588	0.0092	-0.0111	-0.0111	0.0158
UKF3	-0.5949	2.1524	0.0099	-0.0112	-0.0112	0.0151
CKF3	-0.5948	2.1523	0.0097	-0.0112	-0.0112	0.0150

Figure 1.1: Mean and covariance Table for different methods



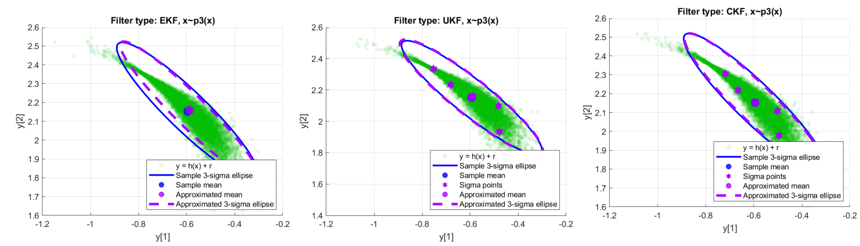
(a) EKF (b) UKF (c) CKF

Figure 1.2: Comparison of three figures in distribution 1



(a) EKF (b) UKF (c) CKF

Figure 1.3: Comparison of three figures in distribution 2



(a) EKF (b) UKF (c) CKF

Figure 1.4: Comparison of three figures in distribution 3

enables the UKF and CKF methods to capture the nonlinearity of the system more accurately than the EKF method.

The trade-off when using UKF or CKF compared to EKF is computational complexity versus accuracy. EKF is computationally simpler, as it only requires the first-order Taylor series approximation, but it may not be suitable for highly nonlinear systems. On the other hand, UKF and CKF can handle highly nonlinear systems, but they are computationally more complex, as they require calculating multiple sigma points and their corresponding weights.

As an engineer, the choice of filter would depend on the specific application requirements. If the system is highly nonlinear, then UKF or CKF may be the better choice, despite the higher computational cost. However, if the system is relatively linear or only slightly nonlinear, then EKF may be sufficient and preferred due to its lower computational complexity.

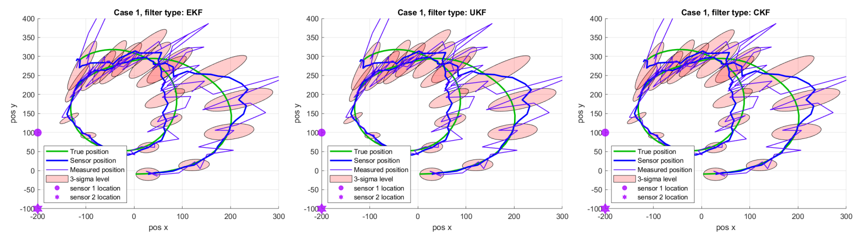
2 Non-linear Kalman filtering

2.1 a

In Case 1, the measurement is very noisy in both directions, which results in a measured position that is not very smooth. However, the filtered position is still reasonable, as shown in the figure 2.1. It follows the green curve, which represents the accurate position, and it always remains inside the 3-sigma circle. This indicates that the filtered position is acceptable, despite the noisy measurements.

2.2 b

From figure 2.4 to figure 2.5, The figure shows Case 2, which involves one noisy sensor, and Case 3, which involves two accurate sensors. In Case 2, the uncertainty circle points towards Sensor 2, indicating that the measurements have more certainty towards the direction of Sensor 2. The filter generally solves the problem accurately in Case 2, but CKF tends to perform better than the other two filters.

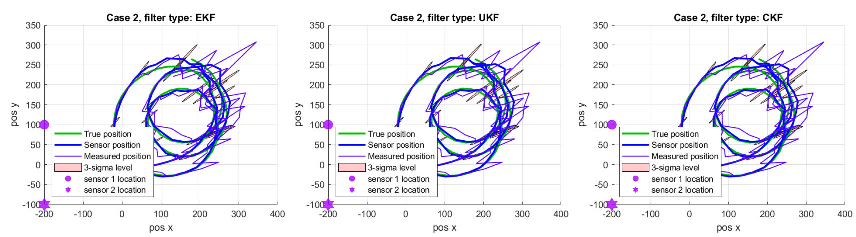


(a) EKF (b) UKF (c) CKF

Figure 2.1: Comparison of three figures in case 1

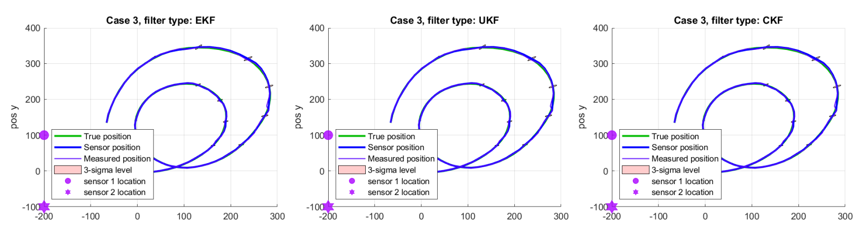
In Case 3, the uncertainty is much smaller, and the filter works well. This is because the measurements from the two sensors provide redundant information that allows the filter to estimate the position more accurately. In such cases, the choice of filter may not be critical, and all three filters could work equally well.

From case 1 to case 3, the sizes of 3-sigma ellipses are becoming smaller and smaller, which shows the less uncertainty from sensors.



(a) EKF (b) UKF (c) CKF

Figure 2.2: Comparison of three figures in case 2



(a) EKF (b) UKF (c) CKF

Figure 2.3: Comparison of three figures in case 3

2.3 c

From figure 2.4 to figure 2.6, it seems that the EKF method has a larger error covariance compared to the UKF and CKF methods. Additionally, the CKF method seems to be performing better than the UKF method.

When comparing the error histograms with fitted Gaussian distributions, it appears that the error distribution is not Gaussian.

In terms of the differences between the histograms of the errors in the x and y positions, it seems that the y direction is generally more accurate than the x direction for cases 1 and 2. However, for case 3, there does not appear to be a significant difference in accuracy between the x and y directions. This is because the two sensors are distributed along y direction.

3 Tuning non-linear filters

3.1 a

Increasing the value of σ_v, which is the measurement noise for speed, results in a noisy filtered position and excessive velocity oscillations that are not realistic for a vehicle driving on a road. Decreasing σ_v too much can improve the filter output to a certain extent.

On the other hand, increasing σ_w, which is the measurement noise for yaw rate, causes the vehicle's yaw to change frequently at a high frequency, resulting in unrealistic driving behavior as well. Conversely, decreasing σ_w too much can prevent the filter from changing the yaw rate quickly enough when the vehicle leaves a straight line and enters a curve, leading to a larger turning radius than the actual one.

If both σ_v and σ_w are increased, the filter will rely more on the noisy measurements, which can lead to an unrealistic and noisy vehicle trajectory. If both v and w are decreased too much, the vehicle may take a longer time to change the yaw rate, resulting in a larger turning radius than it should be.

3.2 b

The true trajectory is constant velocity. Thus, it is reasonable to make σ_v smaller (around 0.001) and sufficient to follow the change of accelerations. For yaw noise, it is generally good around $\pi/180$.

3.3 c

As it is shown in figure 3.1, When the noise level is too small, the filter finds it difficult to follow the acceleration of the vehicle when it is about to rotate. This happens because the filter may not be able to distinguish between the actual movement and the noise in the measurements.

On the other hand, when the noise level is too large, the filter will give more importance to the measurements, which are full of fluctuations in this case. As a

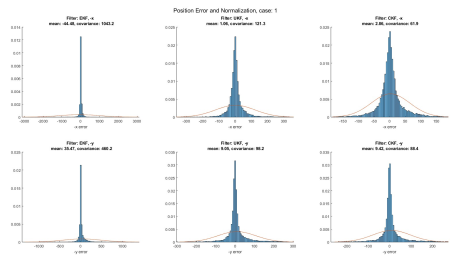


Figure 2.4: Case 1 position error distribution

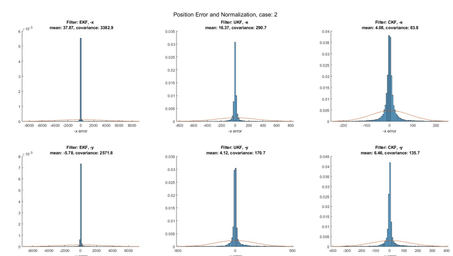


Figure 2.5: Case 2 position error distribution

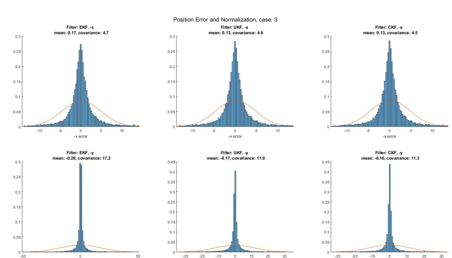
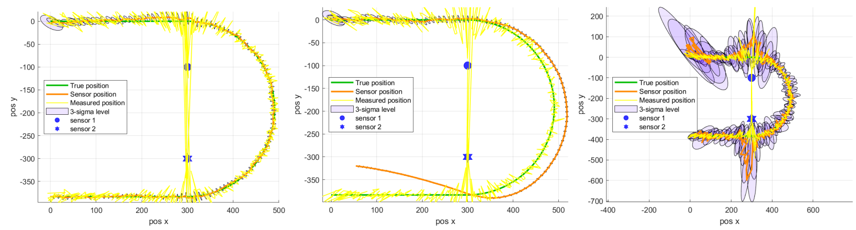


Figure 2.6: Case 3 position error distribution

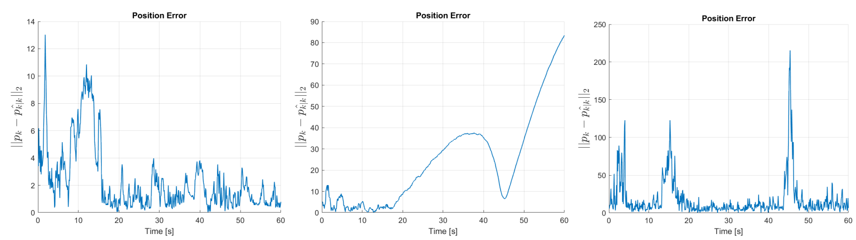
result, the filter may produce an unrealistic trajectory that does not accurately represent the actual movement of the vehicle.

The position errors are shown in figure 3.2.



(a) Normal (b) Too Small (c) Too Large

Figure 3.1: Comparison of three figures



(a) Normal (b) Too Small (c) Too Large

Figure 3.2: Comparison of three figures in position errors

3.4 d

It is possible to tune the filter to have accurate estimates of all stages. However, this requires a very good understanding of general and detailed trajectory, as well as the properties of the sensor measurements. It might be better to divide one trajectory into different models.

There could be conflicting requirements when tuning the filter for different parts of the trajectory. For example, the optimal parameter settings for a straight line may differ from a turn.

The vehicle's actual motion differs in each situation. When the vehicle is moving in a straight line, its velocity and heading remain constant, while the turn-rate is zero. In contrast, during a turn, the velocity remains constant while the heading and turn-rate change. Additionally, during a transition from a straight line to a turn, the vehicle's velocity, heading, and turn-rate may change rapidly as it begins to enter the turn. This rapid change in motion can result in a higher level of noise in the sensor measurements, making it more difficult for the filter to accurately estimate the vehicle's state.

It is necessary to adjust the filter parameters to provide accurate estimates of velocity, heading, and turn-rate for the entire sequence or different stages.