Multi-view Subspace Clustering via An Adaptive Consensus Graph Filter

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Abstract

Most existing MVSC methods first collect complementary information from different views and consequently derive a consensus reconstruction coefficient matrix to indicate the subspace structure of a multi-view data set. In this paper, we initially assume the existence of a consensus reconstruction coefficient matrix and then use it to build a consensus graph filter. In each view, the filter is employed for smoothing the data and designing a regularizer for the reconstruction coefficient matrix. Finally, the obtained reconstruction coefficient matrices from different views are used to create constraints for the consensus reconstruction coefficient matrix.

Multi-view subspace clustering

The generalized framework of MVSC could be expressed as follows:

$$\min_{\mathbf{C}^i}\left\{\gamma^i\left[\mathcal{L}\left(\mathbf{X}^i,\mathbf{C}^i\mathbf{X}^i\right)+\lambda\mathcal{R}\left(\mathbf{C}^i\right)\right]\right\}_{i=1}^{v}$$

Where $\mathbf{X}^i\in\mathbb{R}^{n\times d_i}$ and $\mathbf{C}^i\in\mathbb{R}^{n\times n}$ is the feature matrix and the reconstruction coefficient matrix in the i-th view. Once the reconstruction coefficient matrix $\mathbf{C}^i$ in each view is obtained, a consensus matrix will be defined as $\mathbf{C}=\mathcal{F}\left(\left\{\mathbf{C}^i\right\}_{i=1}^{v}\right)$, where $\mathcal{F}(\cdot)$ is some kind fusion function

Graph filter

- Suppose an undirected graph $\mathbb{G}$ has n vertices with $\mathbf{X}$ being the feature matrix corresponding to these vertices and $\mathbf{W}\in\mathbb{R}^{n\times n}$ represents the edge weights matrix. The normalized adjacency matrix and the normalized Laplacian matrix of this graph are defined as $\widetilde{\mathbf{A}}=\mathbf{W}+\mathbf{I}$ and $\mathbf{L}=\mathbf{I}-\widetilde{\mathbf{D}}^{-\frac{1}{2}}\widetilde{\mathbf{A}}\widetilde{\mathbf{D}}^{-\frac{1}{2}}$ respectively. $\widetilde{\mathbf{D}}$ is a diagonal matrix satisfying $\left[\widetilde{\mathbf{D}}\right]_{ii}=\sum_{i=1}^{n}\left[\widetilde{\mathbf{A}}\right]_{ij}$. Then the low-pass graph filter $\mathbf{G}=\mathbf{I}-\frac{\mathbf{L}}{2}$.
- By using the defined low-pass graph filter, we can get the smooth data matrix of $\mathbf{Y}$, namely $\mathbf{Y}=\mathbf{G}\mathbf{X}$.

The proposed method: MVSC²GF

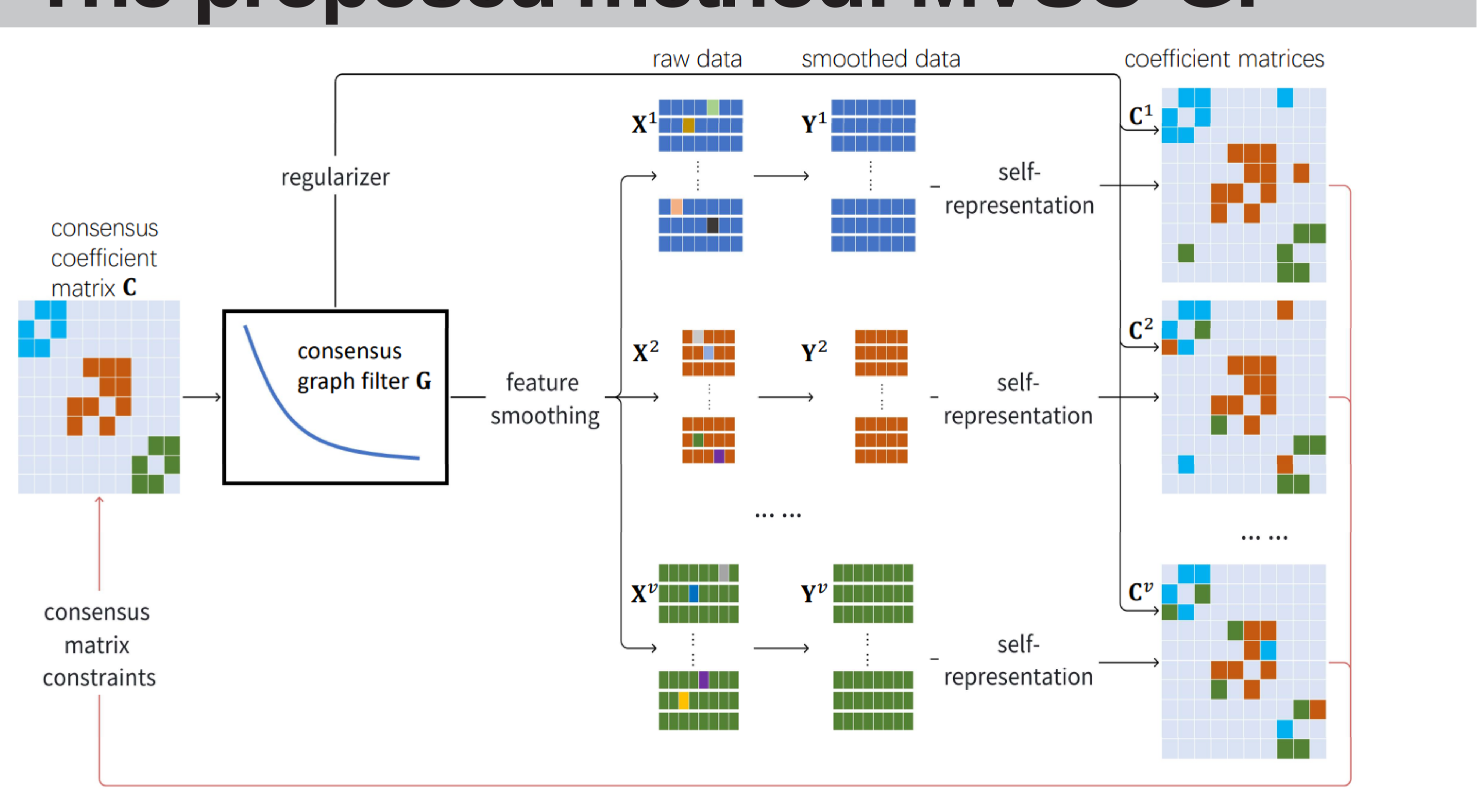

Figure 1: The overview of the proposed method. The consensus graph filter G is used to obtain the smoothed data for each view. Meanwhile, in each view, a regularizer of the coefficient matrix is devised by $\mathbf{G}$. Then a coefficient matrix could be obtained in each view. These coefficient matrices are used to construct constraints for obtaining the consensus coefficient matrix $\mathbf{C}$. Finally, $\mathbf{G}$ is derived by $\mathbf{C}$.

The proposed method contains three main steps:

A. Designing a consensus graph filter. We first assume that a consensus coefficient matrix $\mathbf{C}$ from a multi-view data set has been gotten. Then a low-pass graph filter could be defined as $\mathbf{G}=\mathbf{I}-\frac{\mathbf{L}}{2}=\frac{3\mathbf{I}}{4}+\frac{3\mathbf{C}}{4}$.

Here, we require $\mathbf{C}\geq\mathbf{0},\mathbf{C}=\mathbf{C}^{\top},\mathbf{C1}=\mathbf{C},\mathrm{diag}(\mathbf{C})=\mathbf{0}$. Then the affinity graph $\mathbf{W}=\mathbf{C}$, hence the graph filter will have the above formulation.

B. Computing the reconstruction coefficient matrix in each view. The defined graph filter $\mathbf{G}$ should be consistent across different views.

- The smoothed features of a multi-view data set in each view could be computed, i.e., $\mathbf{Y}^i=\mathbf{G}\mathbf{X}^i$.
- $\mathbf{Y}^i$ has the self-expressive property, namely, $\mathbf{Y}^i\approx\mathbf{C}^i\mathbf{Y}^i$, where $\mathbf{C}^i$ is the reconstruction coefficient matrix.
- $\mathbf{C}^i$ should also satisfies self-expressive property $\mathbf{C}^i\approx\mathbf{C}\mathbf{C}^i$.

C. Fusing the reconstruction coefficient matrices.

Finally, the MVSC²GF problem could be obtained:

$$\min_{\mathbf{C}^i}\ \sum_{i=1}^{v}\left\|\mathbf{Y}^i-\mathbf{C}^i\mathbf{Y}^i\right\|_F^2+\alpha\left\|\mathbf{C}^i-\mathbf{C}\mathbf{C}^i\right\|_F^2+\beta\left(\gamma^i\right)^{\eta}\left\|\mathbf{C}^i-\mathbf{C}\right\|_F^2$$

$$s.t.\quad 4\mathbf{Y}^i=3\mathbf{X}^i+\mathbf{C}\mathbf{X},\mathbf{C}\geq\mathbf{0},\mathbf{C}=\mathbf{C}^{\top},\mathbf{C1}=\mathbf{C},\mathrm{diag}(\mathbf{C})=\mathbf{0}$$

$$\sum_{i=1}^{v}\gamma^i=1$$

Experiments

Databases	Metrics	Methods										
		DiMSC	AGML	MLAN	mPAC	GMC	LMSC	LMVSC	LSRMVSC	CiMSC	SIREN	MVSC²GF
3sources	ACC	79.53(0.3)	76.43(0.4)	***81.70(1.2)***	79.76(0.1)	69.23(0.1)	78.53(0.4)	76.92(0.0)	75.15(0.0)	80.13(0.0)	76.12(1.2)	**85.80(0.0)**
	NMI	71.91(0.9)	69.32(0.0)	***75.43(0.0)***	72.30(0.1)	62.16(0.0)	65.87(0.2)	65.38(0.0)	65.03(0.2)	71.27(0.1)	62.16(0.1)	**78.66(0.0)**
	ARI	63.71(0.2)	60.43(0.2)	66.64(0.0)	64.52(0.0)	65.56(0.2)	50.56(0.5)	60.57(0.3)	***69.64(0.2)***	68.64(0.2)	64.52(0.1)	**69.71(0.0)**
	F-score	67.43(0.2)	72.43(0.1)	***82.46(0.4)***	80.54(0.1)	73.27(0.0)	70.13(0.7)	79.24(0.4)	76.73(0.6)	77.47(0.0)	77.83(0.2)	**83.75(0.0)**
ORL	ACC	79.70(2.6)	72.93(2.0)	68.50(0.0)	71.31(1.4)	63.25(0.0)	82.75(1.2)	71.33(2.3)	79.00(0.0)	***89.56(2.8)***	82.75(1.2)	**90.00(0.0)**
	NMI	90.45(1.3)	89.14(1.0)	83.12(0.0)	86.79(0.0)	85.71(0.0)	92.81(0.5)	84.19(2.5)	92.39(0.0)	***94.22(0.2)***	90.18(2.3)	**95.97(0.0)**
	ARI	73.76(3.3)	54.79(5.0)	33.16(0.0)	51.23(0.0)	33.67(0.0)	75.90(0.0)	69.77(0.1)	77.43(2.5)	***83.56(2.3)***	74.35(0.1)	**84.58(0.0)**
	F-score	74.39(3.25)	70.34(3.0)	33.47(1.1)	68.74(3.2)	35.90(0.0)	77.53(0.1)	73.12(0.0)	76.52(0.0)	***84.30(0.4)***	77.53(0.0)	**86.58(0.0)**
MSRC-v1	ACC	75.93(0.9)	76.44(0.5)	85.43(0.3)	81.43(0.1)	***89.56(0.3)***	77.02(2.2)	88.10(0.2)	68.67(1.3)	78.30(3.0)	88.18(0.8)	**91.90(0.0)**
	NMI	62.24(1.5)	77.65(2.1)	75.13(0.3)	75.08(0.0)	***82.00(1.9)***	67.95(2.5)	81.11(2.6)	55.63(0.0)	76.09(2.0)	78.49(0.9)	**85.37(0.0)**
	ARI	54.80(1.5)	59.07(0.4)	70.94(0.4)	61.27(0.0)	***76.74(2.3)***	59.88(2.8)	72.79(0.3)	46.42(0.1)	70.51(5.0)	74.73(0.8)	**82.57(0.0)**
	F-score	61.12(1.3)	72.80(2.2)	75.03(0.3)	73.20(1.3)	79.97(0.2)	65.44(2.4)	76.65(0.2)	61.62(0.0)	70.02(0.0)	***81.12(1.3)***	**85.02(0.0)**
BBCsport	ACC	***89.71(0.0)***	69.34(0.4)	79.62(0.0)	77.96(0.0)	80.63(0.0)	85.12(12.3)	48.78(3.4)	80.98(2.5)	72.05(0.1)	79.74(3.5)	**98.16(0.0)**
	NMI	***79.20(0.0)***	56.39(0.1)	77.91(0.0)	73.30(0.1)	75.68(0.0)	74.48(13.6)	27.21(1.6)	84.93(0.0)	68.39(8.0)	76.43(2.5)	**93.27(0.0)**
	ARI	***83.20(0.0)***	70.48(0.5)	65.54(0.0)	68.48(2.0)	72.20(0.0)	77.04(16.5)	31.45(1.4)	81.34(0.2)	60.04(1.0)	71.30(1.1)	**95.03(0.0)**
	F-score	***87.32(0.0)***	73.54(0.8)	72.44(0.0)	82.04(0.0)	79.41(0.0)	82.88(11.4)	39.34(5.6)	86.03(0.0)	68.13(0.0)	80.13(0.0)	**96.22(0.0)**
COIL20	ACC	77.85(2.2)	73.21(1.3)	***86.21(1.1)***	72.43(0.0)	79.11(0.1)	77.97(0.1)	70.83(0.3)	70.56(1.2)	85.98(0.0)	76.54(1.2)	**90.00(0.0)**
	NMI	84.63(0.2)	79.54(0.8)	***96.10(0.4)***	84.46(0.0)	94.10(0.2)	85.92(0.1)	81.49(0.2)	79.97(0.0)	94.56(3.2)	81.34(0.1)	**97.17(0.0)**
	ARI	73.21(0.5)	66.32(1.5)	***83.56(0.6)***	71.44(0.0)	78.20(0.0)	67.52(0.2)	59.41(0.6)	62.89(0.3)	82.32(0.2)	74.56(0.5)	**84.34(0.0)**
	F-score	74.50(0.5)	69.40(0.4)	***84.43(1.3)***	73.02(0.0)	80.21(0.0)	71.61(0.2)	63.95(0.5)	64.98(0.0)	81.65(0.0)	77.38(0.0)	**86.80(0.0)**
Caltech101-7	ACC	51.74(0.1)	68.73(0.3)	62.70(0.0)	59.65(0.0)	67.52(2.2)	***86.33(0.1)***	83.46(0.1)	46.47(0.0)	81.33(0.0)	78.54(0.0)	**90.50(0.0)**
	NMI	31.73(0.7)	49.43(11.5)	54.67(0.0)	46.24(0.0)	61.99(1.9)	***72.97(0.4)***	58.50(0.4)	18.52(0.0)	69.79(3.4)	70.33(2.8)	**81.79(0.0)**
	ARI	38.63(0.6)	53.21(0.0)	41.47(0.0)	51.23(2.0)	41.17(0.4)	41.17(0.4)	38.04(0.5)	13.00(0.0)	***54.33(7.3)***	51.32(0.0)	**55.05(0.0)**
	F-score	42.11(0.8)	61.53(6.9)	61.88(0.0)	57.51(0.0)	68.29(0.0)	***91.28(0.2)***	78.28(2.2)	44.70(0.0)	77.35(0.3)	75.53(1.2)	**95.89(0.0)**
HW	ACC	86.61(0.3)	87.32(0.0)	96.05(0.0)	88.90(2.5)	88.20(2.5)	89.35(0.3)	92.66(0.0)	35.36(0.0)	94.32(0.0)	***96.41(1.1)***	**96.59(0.0)**
	NMI	79.20(0.1)	83.72(6.4)	92.22(0.0)	83.61(0.0)	90.41(0.1)	83.70(0.2)	86.01(0.1)	57.36(0.0)	89.90(0.0)	***92.42(0.2)***	**92.59(0.0)**
	ARI	75.04(0.2)	71.30(0.0)	82.45(0.0)	79.67(0.0)	80.12(0.0)	78.93(0.3)	86.13(0.1)	22.49(0.0)	83.44(1.3)	***91.12(0.0)***	**91.65(0.0)**
	F-score	77.53(0.2)	76.32(10.3)	84.32(0.0)	82.15(0.0)	86.53(0.0)	82.02(0.2)	84.69(0.1)	55.64(0.0)	82.13(0.0)	***90.03(0.0)***	**92.48(0.0)**

Table 1: Clustering results (in %) including mean and std (in the brackets) of evaluated methods on the used benchmark data sets. The best results are emphasized in red and bold, the second-best results are denoted in bold and italics.

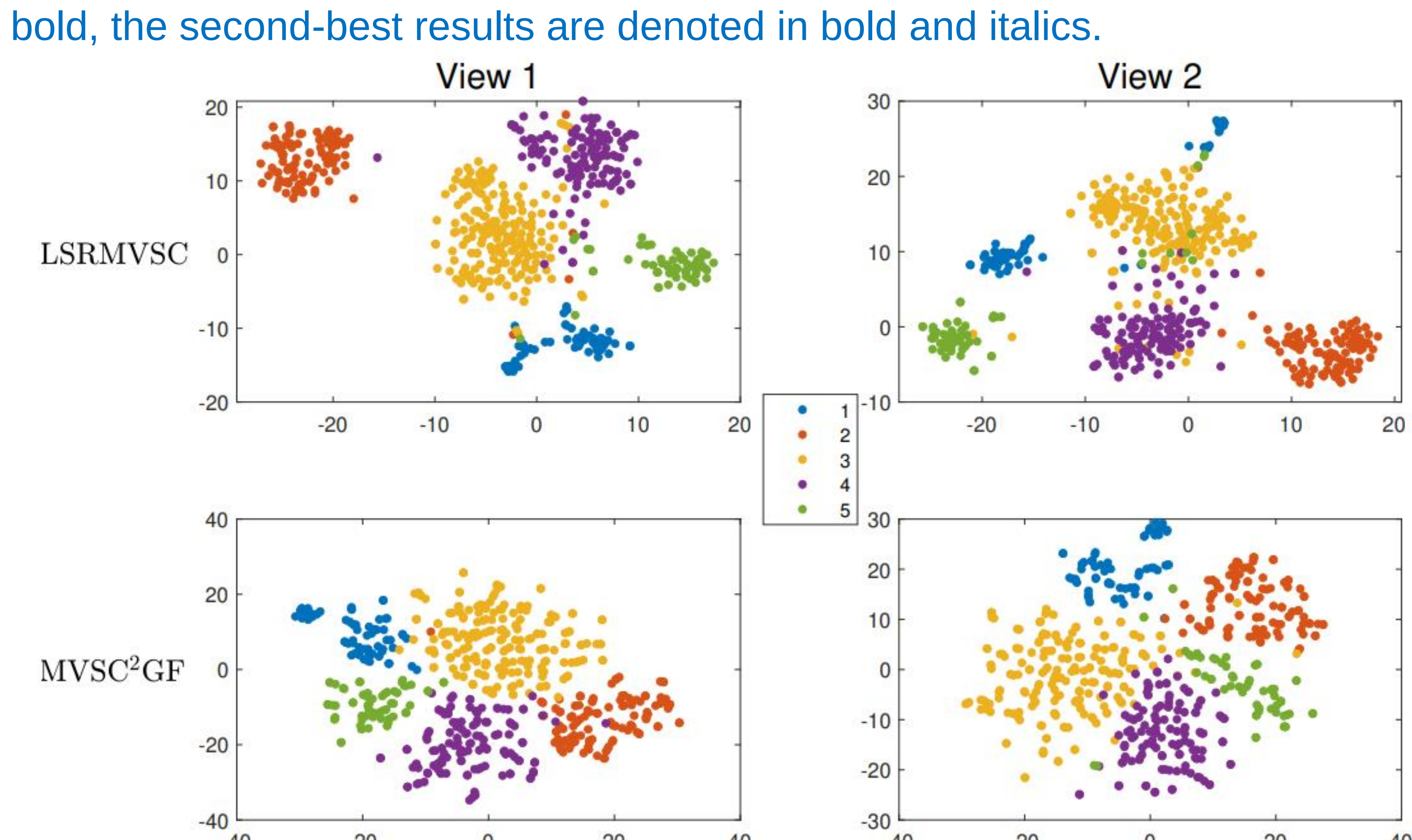

Figure 2: The filtered features in each view obtained by LSRMVSC (the first row) and MVSC²GF (the second row). The filtered features from the first and second views are listed in the first and second columns respectively.

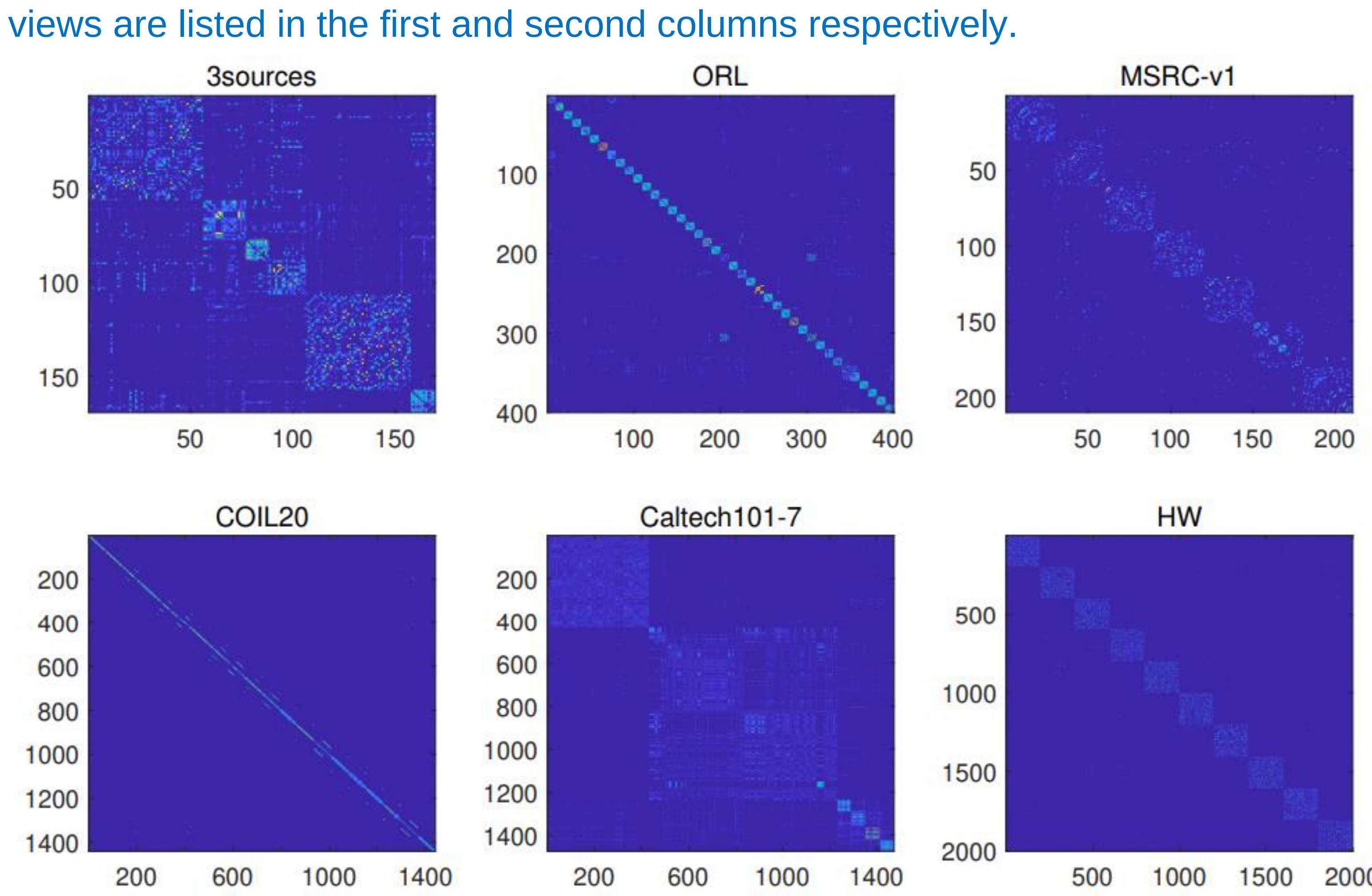

Figure 3: The consensus reconstruction coefficient matrices of MVSC²GF obtained on the rest six data sets.

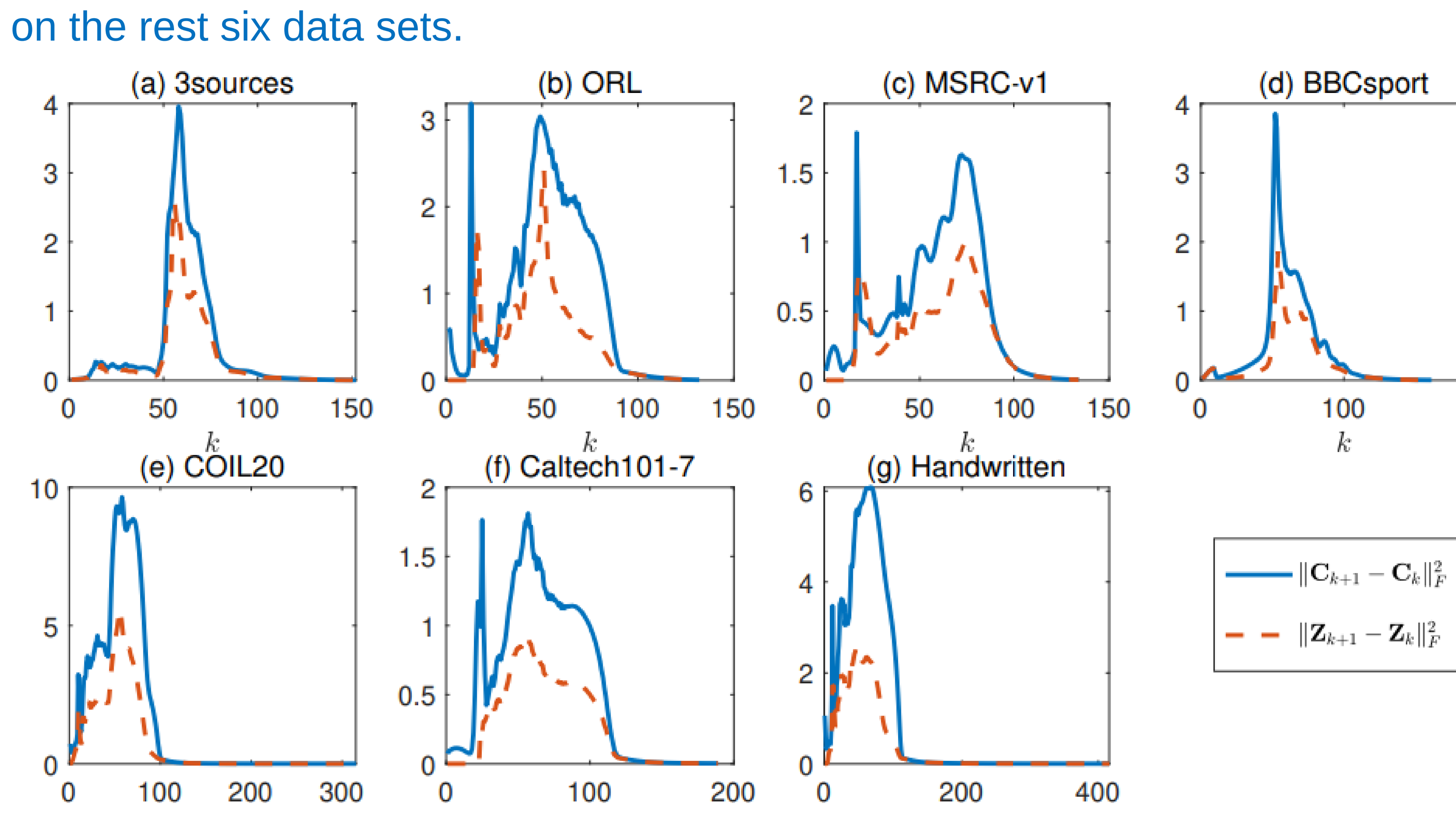

Figure 4: The convergence of MVSC²GF