

## Class 12 OLS Regression Basics

Dr Wei Miao

UCL School of Management

November 8, 2023

## Section 1

# Basics of Linear Regression

# Linear Regression Models

- A simple linear regression is a model as follows.

$$Y_i = \beta_0 + x_1\beta_1 + x_2\beta_2 + \dots + x_k\beta_k + \epsilon_i$$

- $y_i$ : Outcome variable/dependent variable/regressand/response variable/LHS variable
- $\beta$ : Regression coefficients/estimates/parameters;  $\beta_0$ : intercept
- $x_k$ : Control variable/independent variable/regressor/explanatory variable/RHS variable
  - Lower case such as  $x_1$  usually indicates a single variable while upper case such as  $X_{ik}$  indicates a set of several variables
- $\epsilon_i$ : Error term, which captures the deviation of  $Y$  from the prediction
  - Expected mean should be 0, i.e.,  $E[\epsilon|X] = 0$
  - If we take the expectation of  $Y$ , we should have

$$E[Y|X] = \beta_0 + x_1\beta_1 + x_2\beta_2 + \dots + x_k\beta_k$$

# Why the Name “Regression”?

- The term “regression” was coined by Francis Galton to describe a biological phenomenon: The heights of descendants of tall ancestors tend to regress down towards a normal average.
- The term “regression” was later extended by statisticians Udny Yule and Karl Pearson to a more general statistical context (Pearson, 1903).
- In supervised learning models, “regression” has a different meaning: when outcome is continuous, the task is called regression task.<sup>1</sup>

---

<sup>1</sup>ML models are developed by computer science; causal inference models are developed by economists.

## Section 2

### Estimation of Coefficients

# How to Run Regression in R

- In R, there are tons of packages that can run OLS regression.
- In this module, we will be using the `fixest` package, because it's able to estimate high-dimensional fixed effects.

```
1  pacman::p_load(modelsummary,fixest)
2
3  OLS_result <- feols(
4    fml = total_spending ~ Income, # Y ~ X
5    data = data_full, # dataset from Tesco
6  )
```

# Report Regression Results

```

1  modelsummary(OLS_result,
2      stars = TRUE # export statistical significance
3  )

```

(1)	
(Intercept)	−552.235*** (20.722)
Income	0.021*** (0.000)
Num.Obs.	2000
R2	0.630
R2 Adj.	0.630
AIC	29 130.1
BIC	29 141.3
RMSE	351.63
Std.Errors	IID

+  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

# Parameter Estimation: Univariate Regression Case

- Let's take a **univariate regression**<sup>2</sup> as an example

$$y = a + bx_1 + \epsilon$$

- For each guess of  $a$  and  $b$ , we can compute the error for customer  $i$ ,

$$e_i = y_i - a - bx_{1i}$$

- We can compute the **sum of squared residuals (SSR)** across all customers

$$SSR = \sum_{i=1}^n (y_i - a - bx_{1i})^2$$

- Objective of estimation:** Search for the unique set of  $a$  and  $b$  that can minimize the SSR.
- This estimation method that minimizes SSR is called **Ordinary Least Square (OLS)**.

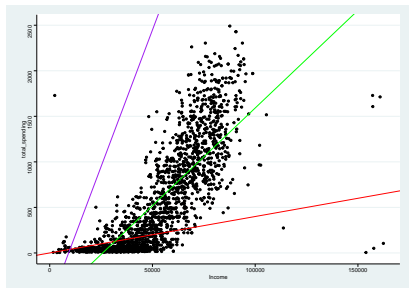
---

<sup>2</sup>Regressions with a single regressor is called univariate regressions.



# Visualization: Estimation of Univariate Regression

- If in the Tesco dataset, if we regress **total spending** (Y) on **income** (X)



Model	Color	Sum of Squared Error
$Y = -552 + 0.06 * X$	Purple	$1.6176047 \times 10^{13}$
$Y = 0 + 0.004 * X$	Red	$5.093683 \times 10^{11}$
$Y = -552 + 0.021 * X$	Green	$2.0205681 \times 10^9$

# Multivariate Regression

- The OLS estimation also applies to multivariate regression with multiple regressors.

$$y_i = b_0 + b_1x_1 + \dots + b_kx_k + \epsilon_i$$

- **Objective of estimation:** Search for the **unique** set of  $b$  that can minimize the **sum of squared residuals**.

$$SSR = \sum_{i=1}^n (y_i - b_0 - b_1x_1 - \dots - b_kx_k)^2$$

## Section 3

# Interpretation of Coefficients

## Coefficients Interpretation

- Now on your Quarto document, let's run a new regression, where the DV is *total\_spending*, and X includes *Income* and *Kidhome*.

(1)	
(Intercept)	-316.878*** (26.972)
Income	0.019*** (0.000)
Kidhome	-210.613*** (16.282)
Num.Obs.	2000
R2	0.658
R2 Adj.	0.658
AIC	28 971.2
BIC	28 988.0
RMSE	337.77
Std.Errors	IID
+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001	

- Controlling for Kidhome**, one unit increase in *Income* increases *totalspending* by £0.019.

# Standard Errors and P-Values

- Because the regression is estimated on a random sample of the population, so if we rerun the regression on different samples, we would get a different set of regression coefficients each time.
- In theory, the regression coefficients estimates follows a **t-distribution**: the mean is the true  $\beta$ . The **standard error** of the estimates is the estimated standard deviation of the error.
- We can test whether the coefficients are statistically different from 0 using **hypothesis testing**.
  - Null hypothesis: the true regression coefficient  $\beta$  is 0
- Income/Kidhome is statistically significant at the 1% level.

# R-Squared

- R-squared ( $R^2$ ) is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by all included variables in a regression.
- Interpretation: 65.8% of the variation in `totalspending` can be explained by `Income` and `Kidhome`.
- As the number of variables increases, the  $R^2$  will naturally increase, so sometimes we may need to penalize the number of variables using the so-called **adjusted R-squared**.

## ! Important

R-Squared is only important for supervised learning prediction tasks, because it measures the predictive power of the X. However, In causal inference tasks,  $R^2$  does not matter much.