

Class 12 OLS Regression Basics

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Section 1

Background of Regression

Conditional Mean in Causal Inference

- In causal inference, we often care about the expected mean of the outcome variable (Y) conditional on treatment variables (X).
- For example, in an RCT, Y is the outcome variable (e.g., purchase rate), X is whether or not customers receive the treatment (e.g., BMW ads), then from the **basic identity of causal inference**, we have

$$ATE = E[Y|X = 1] - E[Y|X = 0]$$

- Question: how can we model the expected mean of outcome variable conditional on X , $E[Y|X = x]$?

Linear Regression Models

- If we assume a **linear**, **additive** function for $E[Y|X = x]$, we have a simple linear regression model, as follows,

$$Y_i = \beta_0 + x_1\beta_1 + x_2\beta_2 + \dots + x_k\beta_k + \epsilon_i$$

- y_i : Outcome variable/dependent variable/regressand/response variable/LHS variable
- β : Regression coefficients/estimates/parameters; β_0 : intercept
- x_k : control variable/independent variable/regressor/explanatory variable/RHS variable
 - Lower case such as x_1 usually indicates a single variable while upper case such as X_{ik} indicates several variables
- ϵ_i : error term/disturbance, which has the expected mean of 0, i.e., $E[\epsilon|X] = 0$
- If we take the expectation of Y , we have:

$$E[Y|X] = \beta_0 + x_1\beta_1 + x_2\beta_2 + \dots + x_k\beta_k$$

Why the Name “Regression”?

- The term “regression” was coined by Francis Galton to describe a biological phenomenon: The heights of descendants of tall ancestors tend to regress down towards a normal average.
- The term “regression” was later extended by Udny Yule and Karl Pearson to a more general statistical context (Pearson, 1903).
- In supervised learning models, “regression” can have different meanings:¹
 - The regression-class models (OLS, Lasso, Ridge, etc.)
 - Regression task
- To establish causal inference, **OLS regression model** is all we need.

¹ML models are developed by computer science; causal inference models are developed by economists.

Section 2

Estimation

How to Run Regression in R

- In R, there are tons of packages that can run OLS regression.
- In this module, we will be using the `fixest` package, because it's able to estimate high-dimensional fixed effects.

```
1  pacman::p_load(modelsummary,fixest)
2
3  OLS_result <- feols(
4    fml = total_spending ~ Income, # Y ~ X
5    data = data_full, # dataset from Tesco
6  )
```

Report Regression Results

```
1 modelsummary(OLS_result,  
2   stars = TRUE # export statistical significance  
3   )
```

| (1) | |
|-------------|-------------------------|
| (Intercept) | −552.235*** (20.722) |
| Income | 0.021*** (0.000) |
| Num.Obs. | 2000 |
| R2 | 0.630 |
| R2 Adj. | 0.630 |
| AIC | 29 130.1 |
| BIC | 29 141.3 |
| RMSE | 351.63 |
| Std.Errors | IID |

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

Parameter Estimation: Univariate Regression Case

- Let's take a **univariate regression**² as an example

$$y = a + bx_1 + \epsilon$$

- For each guess of a and b , we can compute the error for customer i ,

$$e_i = y_i - a - bx_{1i}$$

- We can compute the **sum of squared residuals (SSR)** across all customers

$$SSR = \sum_{i=1}^n (y_i - a - bx_{1i})^2$$

- Objective of estimation:** Search for the unique set of a and b that can minimize the SSR.
- This estimation method that minimizes SSR is called **Ordinary Least Square (OLS)**.

²Regressions with a single regressor is called univariate regressions.

Visualization: Estimation of Univariate Regression

- If in the Tesco dataset, if we regress **total spending** (Y) on **income** (X)



| Model | Color | Sum of Squared Error |
|------------------------|--------|----------------------------|
| $Y = -552 + 0.06 * X$ | Purple | 1.6176047×10^{13} |
| $Y = 0 + 0.004 * X$ | Red | 5.093683×10^{11} |
| $Y = -552 + 0.021 * X$ | Green | 2.0205681×10^9 |

Multivariate Regression

- The OLS estimation also applies to multivariate regression with multiple regressors.

$$y_i = b_0 + b_1x_1 + \dots + b_kx_k + \epsilon_i$$

- **Objective of estimation:** Search for the set of b that can minimize the sum of squared residuals.

$$SSR = \sum_{i=1}^n (y_i - b_0 - b_1x_1 - \dots - b_kx_k)^2$$

Section 3

Interpretation

Coefficients Interpretation

- Now on your Quarto document, let's run a new regression, where the DV is *total_spending*, and X includes *Income* and *Kidhome*.

| (1) | |
|-------------|-------------------------|
| (Intercept) | -316.878*** (26.972) |
| Income | 0.019*** (0.000) |
| Kidhome | -210.613*** (16.282) |
| Num.Obs. | 2000 |
| R2 | 0.658 |
| R2 Adj. | 0.658 |
| AIC | 28 971.2 |
| BIC | 28 988.0 |
| RMSE | 337.77 |
| Std.Errors | IID |

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

- Controlling for Kidhome** / everything else being equal / *ceteris paribus*, one unit increase in Income increases total spending by 0.019 pounds.

Standard Errors and P-values

- Due to randomness of the error term, all coefficients estimates follow a t distribution.
- Therefore, we need **p-values** to check whether the coefficients are statistically different from 0.
- Income/Kidhome is statistically significant at the 1% level.

R-squared

- R-squared (R^2) is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable or variables in a regression model.
- Interpretation: 65.8% of the variation in Spending can be explained by Income and Kidhome.
- As the number of variables increases, the R^2 will naturally increase.
- In causal inference tasks, R^2 does not mean much.