Class 13 OLS Regression Advanced

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Section 1

Categorical Variables

Categorical variables

- So far, the independent variables we have used are `Income` and `Kidhome`, which are **continuous variables**.
- Some variables are intrinsically not countable; we need to treat them as **categorical variables**
  - e.g., gender, education group, city.

Handling Categorical Variables using `factor()`

- In R, we need to use a function `factor()` to inform R that this variable is a categorical variable, such that statistical models will treat them differently from continuous variables.
  - Refer to this link for more examples in datacamp.
- We can use `factor(Education)` to indicate that, `Education` is a categorical variable.

```
data_full <- data_full %>%
  mutate(Education_factor = factor(Education))
```

- We can use `levels()` to check how many categories are there in the factor variable.

```
# check levels of a factor
levels(data_full$Education_factor)
```

```
[1] "2n Cycle"   "Basic"      "Graduation" "Master"     "PhD"
```

Handling Categorical Variables using `factor()`

- We can also change the baseline group to another group using `relevel()`.

```
data_full <- data_full %>%
  mutate(Education_factor_2 = relevel(Education_factor,
                                      ref = "Basic") )

levels(data_full$Education_factor_2)
```

```
[1] "Basic"      "2n Cycle"   "Graduation" "Master"     "PhD"
```

Running Regression with Factor Variables

```
pacman::p_load(fixest,modelsummary)
feols_categorical <- feols(data = data_full,
  fml = total_spending ~ Income + Kidhome + Education_factor_2)
modelsummary(feols_categorical,
             stars = T)
```

	(1)
(Intercept)	−180.297**
	(56.305)
Income	0.020***
	(0.000)
Kidhome	−227.761***
	(16.961)
Education_factor_22n Cycle	−164.044**
	(60.448)
Education_factor_2Graduation	−119.695*
	(56.176)
Education_factor_2Master	−143.015*
	(58.443)
Education_factor_2PhD	−153.190**
	(57.751)
Num.Obs.	2000
R2	0.662
R2 Adj.	0.661
AIC	29128.2
BIC	29167.4
RMSE	350.59
Std.Errors	IID

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Interpretation of Coefficients for Categorical Variables

- Internally, R uses **one-hot encoding** to encode factor variables with **K** levels into **K-1** binary variables.
  - Because we have the intercept, we can only have K-1 binary variables.
  - The intercept stands for the effects of the baseline group.
  - In the regression result table, `Basic` group is suppressed if we use `Education_factor_2`, because this group is chosen as the **baseline group**.
- The interpretation template of coefficients for factor variables: Ceteris paribus, compared with the [baseline group], the [outcome variable] of [group XXX] is higher/lower by [coefficient], and the coefficient is statistically [significant/insignificant].
  - Ceteris paribus, compared with the basic education group, the total spending of PhD group is lower by 153.190 dollars. The coefficient is statistically significant at the 1% level.

After-class exercise: change the baseline group to Master, rerun the regression, and interpret the coefficients.

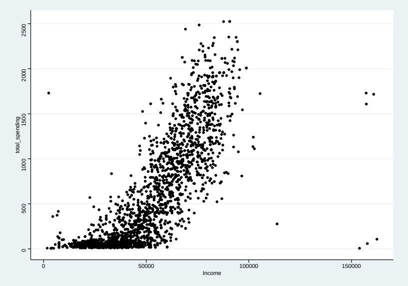
Section 2

Non-linear Effects

Quadratic Terms

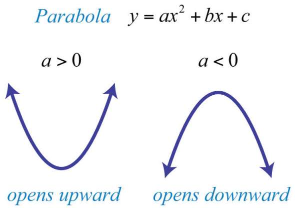
- If we believe the relationship between the outcome variable and explanatory variable is a quadratic function, we can include **an additional quadratic term** in the regression to model such non-linear relationship.

$$Spending = \beta_0 + \beta_1 Income + \beta_2 Income^2 + \epsilon$$



Quadratic Terms

- If after estimation, the coefficient for $Income^2$, β_2, is negative, then we have an down open parabola.



- That is, as income increases, total spending first increases and then decreases, i.e., a non-linear effect.

Quadratic Terms in Linear Regression

- Let's run two regressions, with and without the quadratic term.

```
data_full <- data_full %>%
  mutate(Income_quadartic = Income^2 )

# model 1: without quadratic term
feols_noquadratic <- feols(data = data_full,
  fml = total_spending ~ Income )

# model 2: with quadratic term
feols_quadratic <- feols(data = data_full,
  fml = total_spending ~ Income  + Income_quadartic )
```

Quadratic Terms in Linear Regression

```r
modelsummary(list(feols_noquadratic,feols_quadratic),
             stars = T)
```

	(1)	(2)
(Intercept)	−556.823***	−627.040***
	(21.654)	(36.522)
Income	0.022***	0.025***
	(0.000)	(0.001)
Income_quadartic		0.000*
		(0.000)
Num.Obs.	2000	2000
R2	0.629	0.630
R2 Adj.	0.629	0.630
AIC	29 306.1	29 302.4
BIC	29 317.3	29 319.2
RMSE	367.45	366.92
Std.Errors	IID	IID

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Quadratic Terms: Compute the Vertex

- We can compute the vertex point where total spending is maximized by income

```
# extract the coeffcient vector
feols_coefficient <- feols_quadratic$coefficients
feols_coefficient
```

```
   (Intercept)           Income Income_quadartic
 -6.270403e+02     2.533276e-02    -2.663682e-08
```

```
# Use b / (-2a) to get the vertex
- feols_coefficient[2]/
  (2 * feols_coefficient[3])
```

```
  Income
475521.5
```

Section 3

Linear Probability Model

Linear Probability Model

- In Predictive Analytics, we learned how to use decision tree and random forest to make predictions. In fact, linear regression can also be used as another supervised learning model.
- On the one hand, regression predicts the expectation of response Y conditional on X; that is
$$E[Y|X] = X\beta$$
- On the other hand, for a binary outcome variable, if the probability of outcome occurring is p, then we can write the expectation of Y is
$$E[Y|X] = 1 * p + 0 * (1 - p) = p$$
- As a result, we have the following equation
$$\text{Probability}[Y = 1|X] = E[Y|X] = X\beta$$
- Interpretation of LPM: Everything else equal, a unit change in x will change the probability of the outcome occurring by β units.

Pros and Cons of LPM

- The procedures of training LPM is similar to training a decision tree `rpart()`/random forest `ranger()`: we use linear regression function `feols()` to train the LPM on the **training data** and make predictions on the **test data**.
- Advantages
  - Easy and fast to run
  - High interpretability: coefficients have clear economic meanings
- Disadvantages
  - Predicted probabilities of occurring may fall out of the [0,1] range
  - Accuracy tends to be low