

## Class 16 Instrumental Variables

Dr Wei Miao

UCL School of Management

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## Section 1

# Instrumental Variable

# Causal Inference from OLS

- The necessary condition for OLS to reveal causal effect is: all confounding variables (i.e., variables that are correlated with  $X$  and affect outcome variable) are controlled in the regression.
- Otherwise, the OLS estimator will be biased and we can only obtain the **total effect (correlation)** rather than the **direct effect (causal effect)**.
- From secondary data, we will **never** be able to control all confounding factors, which means we can never obtain causation from OLS regressions.
- So, is there still a way for us to obtain causal inference from secondary data?

# What is an Instrumental Variable

## Instrumental Variable

An instrumental variable is a variable  $z$  that satisfies two requirements:

- ①  $z$  is uncorrelated with  $\epsilon$ ; that is,  $cov(z, \epsilon) = 0$
- ②  $z$  is correlated with  $x$ ; that is,  $cov(z, x) \neq 0$

- Point 1 is called **exogeneity requirement**: the instrumental variable should be beyond an individual's control, such that the instrumental variable will not be correlated with any individual's choices/omitted variables.
  - The spirit is similar to RCT's randomization
- Point 2 is called **relevance requirement**: though beyond an individual's control, the instrumental variable should still affect the individual's  $X$ , causing some exogenous variations in  $X$ .

## Graphical Illustration of IV



# A Classic Example of Instrumental Variable

## Return of Military Service to Lifetime Income<sup>1</sup>

$$Income = \beta_0 + \beta_1 MilitaryService + \epsilon$$

- OLS suffers from endogeneity problems, for example
  - individual ability correlates with military service and affects income
  - individual health status correlates with military service and affects income
- A draft lottery was used to determine if a soldier with a certain birthday goes to the war.
- The date of birth ( $z$ ) is an **instrumental variable**
  - is correlated with military service:  $cov(z, x) \neq 0$
  - but does not directly affect income:  $cov(z, \epsilon) = 0$

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<sup>1</sup>Angrist, Joshua D., Stacey H. Chen, and Jae Song. "Long-term consequences of Vietnam-era conscription: New estimates using social security data." *American Economic Review* 101, no. 3 (2011): 334-38.

- Exogeneity requires that  $z$  should only affect  $Y$  through  $X$ , but not directly affect  $Y$ .
- The instrumental variable should be beyond an individual's control. Because omitted variable bias is often caused by individual's own selection, instrumental variables are thus not correlated with omitted variable bias.

## IV Requirement II: Relevance

- The instrumental variable must be sufficiently correlated with  $x$ .
- If the correlation between  $z$  and  $x$  is too small, we have a **weak IV** problem.
- For more mathematical details of the weak IV issue, refer to this [resource](#).



## More Examples of IVs

Can you come up with IV candidates for the following causation questions?<sup>2</sup>

- COVID-19 cases  $\Rightarrow$  Uber Driver Supply
- Number of restaurants on UberEat  $\Rightarrow$  Number of orders on UberEat
- Retail price  $\Rightarrow$  Sales

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<sup>2</sup>See html version for answers.

## Section 2

# Two-Stage Least Square

# Solving Endogeneity Using IV

- Given an endogenous OLS regression,

$$y_i = X_i\beta + \varepsilon_i, \quad \text{cov}(X_i, \varepsilon_i) \neq 0$$

- Find instrumental variables  $Z_i$  that do not (directly) influence  $y_i$ , but are correlated with  $X_i$

# Two-Stage Least Squares: Stage 1

- ① Run a regression with  $X \sim Z$ . The predicted  $X$  is predicted by  $Z$ , which should be uncorrelated with the error term  $\epsilon$ .
- $\hat{X}$  (predicted  $X$  from  $Z$ ) is exogenous, because  $Z$  is exogenous
  - All endogenous parts are now left in the error term in the first-stage regression  $\epsilon_i$

$$X_i = Z_i\eta + \epsilon_i$$

## Two-Stage Least Squares: Stage 2

- ② Run a regression with  $Y \sim \hat{X}$ : now  $\hat{X}$  is uncorrelated with the error term and thus we can get causal inference from the second stage regression.

$$y_i = \hat{X}\beta + \varepsilon_i, \quad \text{cov}(\hat{X}_i, \varepsilon_i) = 0$$

## Section 3

### **Application: Causal Effects of COVID-19**

# Causal Impact of COVID-19

- The COVID-19 pandemic has brought unprecedented disruptions to many industries, and platform businesses, especially sharing economy platforms, are among the most disrupted ones.
- A common data science interview question: how would you evaluate causal impact of COVID-19 on the company's business and profits?
  - Can we collect data on the *COVID cases* and *KPI measures*, and run an OLS regression to get the causal effect?  $KPI \sim NumCovid$
  - What would hinder us from causal inference from the above OLS regressions?

# Causal Impact of COVID-19 on UberEat Delivery Drivers' Labor Supply

- In this case workshop, we will see an application of instrumental variable in evaluating the causal impact of COVID-19 on UberEat delivery drivers' labor supply decision.
- Let's take out the Quarto document.



# Beyond the Impact of COVID-19 on Labor Supply

- You can follow this case study and propose similar topics for your term 3 dissertation project, depending on the company you work with.
  - The causal impact of COVID-19 on Uber/Bolt drivers' labor supply
  - The causal impact of COVID-19 on customer demand for offline shopping
  - etc.
- For similar causal inference interview questions/data science tasks, when RCTs are difficult to implement, instrumental variable method can be a very powerful solution.

## After-Class Readings

- (optional) [Econometrics with R: Instrumental Variables Regression](#)