

## Class 12 OLS Regression Basics

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November 8, 2023

## Section 1

# Background of Regression

# Conditional Mean in Causal Inference

- In causal inference, we often care about the expected mean of the outcome variable ( $Y$ ) conditional on treatment variables ( $X$ ).
- For example, in an RCT,  $Y$  is the outcome variable (e.g., purchase rate),  $X$  is whether or not customers receive the treatment (e.g., BMW ads), then from the **basic identity of causal inference**, we have

$$ATE = E[Y|X = 1] - E[Y|X = 0]$$

- Question: how can we model the expected mean of outcome variable conditional on  $X$ ,  $E[Y|X = x]$ ?

# Linear Regression Models

- If we assume a **linear**, **additive** function for  $E[Y|X = x]$ , we have a simple linear regression model, as follows,

$$Y_i = \beta_0 + x_1\beta_1 + x_2\beta_2 + \dots + x_k\beta_k + \epsilon_i$$

- $y_i$ : Outcome variable/dependent variable/regressand/response variable/LHS variable
- $\beta$ : Regression coefficients/estimates/parameters;  $\beta_0$ : intercept
- $x_k$ : control variable/independent variable/regressor/explanatory variable/RHS variable
  - Lower case such as  $x_1$  usually indicates a single variable while upper case such as  $X_{ik}$  indicates several variables
- $\epsilon_i$ : error term/disturbance, which has the expected mean of 0, i.e.,  $E[\epsilon|X] = 0$
- If we take the expectation of  $Y$ , we have:

$$E[Y|X] = \beta_0 + x_1\beta_1 + x_2\beta_2 + \dots + x_k\beta_k$$

# Why the Name “Regression”?

- The term “regression” was coined by Francis Galton to describe a biological phenomenon: The heights of descendants of tall ancestors tend to regress down towards a normal average.
- The term “regression” was later extended by Udny Yule and Karl Pearson to a more general statistical context (Pearson, 1903).
- In supervised learning models, “regression” can have different meanings:<sup>1</sup>
  - The regression-class models (OLS, Lasso, Ridge, etc.)
  - Regression task
- To establish causal inference, **OLS regression model** is all we need.

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<sup>1</sup>ML models are developed by computer science; causal inference models are developed by economists.

## Section 2

# Estimation

# How to Run Regression in R

- In R, there are tons of packages that can run OLS regression.
- In this module, we will be using the `fixest` package, because it's able to estimate high-dimensional fixed effects.

```
1  pacman::p_load(modelsummary,fixest)
2
3  OLS_result <- feols(
4    fml = total_spending ~ Income, # Y ~ X
5    data = data_full, # dataset from Tesco
6  )
```

## Report Regression Results

```
1 modelsummary(OLS_result,  
2   stars = TRUE # export statistical significance  
3   )
```

| (1)         |                         |
|-------------|-------------------------|
| (Intercept) | −552.235***<br>(20.722) |
| Income      | 0.021***<br>(0.000)     |
| Num.Obs.    | 2000                    |
| R2          | 0.630                   |
| R2 Adj.     | 0.630                   |
| AIC         | 29 130.1                |
| BIC         | 29 141.3                |
| RMSE        | 351.63                  |
| Std.Errors  | IID                     |

+ p < 0.1, \* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001



## Parameter Estimation: Univariate Regression Case

- Let's take a **univariate regression**<sup>2</sup> as an example

$$y = a + bx_1 + \epsilon$$

- For each guess of  $a$  and  $b$ , we can compute the error for customer  $i$ ,

$$e_i = y_i - a - bx_{1i}$$

- We can compute the **sum of squared residuals (SSR)** across all customers

$$SSR = \sum_{i=1}^n (y_i - a - bx_{1i})^2$$

- Objective of estimation:** Search for the unique set of  $a$  and  $b$  that can minimize the SSR.
- This estimation method that minimizes SSR is called **Ordinary Least Square (OLS)**.

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<sup>2</sup>Regressions with a single regressor is called univariate regressions.

## Visualization: Estimation of Univariate Regression

- If in the Tesco dataset, if we regress **total spending** (Y) on **income** (X)



| Model                  | Color  | Sum of Squared Error       |
|------------------------|--------|----------------------------|
| $Y = -552 + 0.06 * X$  | Purple | $1.6176047 \times 10^{13}$ |
| $Y = 0 + 0.004 * X$    | Red    | $5.093683 \times 10^{11}$  |
| $Y = -552 + 0.021 * X$ | Green  | $2.0205681 \times 10^9$    |

# Multivariate Regression

- The OLS estimation also applies to multivariate regression with multiple regressors.

$$y_i = b_0 + b_1x_1 + \dots + b_kx_k + \epsilon_i$$

- **Objective of estimation:** Search for the set of  $b$  that can minimize the sum of squared residuals.

$$SSR = \sum_{i=1}^n (y_i - b_0 - b_1x_1 - \dots - b_kx_k)^2$$

## Section 3

### Interpretation

## Coefficients Interpretation

- Now on your Quarto document, let's run a new regression, where the DV is *total\_spending*, and X includes *Income* and *Kidhome*.

| (1)         |                         |
|-------------|-------------------------|
| (Intercept) | -316.878***<br>(26.972) |
| Income      | 0.019***<br>(0.000)     |
| Kidhome     | -210.613***<br>(16.282) |
| Num.Obs.    | 2000                    |
| R2          | 0.658                   |
| R2 Adj.     | 0.658                   |
| AIC         | 28 971.2                |
| BIC         | 28 988.0                |
| RMSE        | 337.77                  |
| Std.Errors  | IID                     |

+ p < 0.1, \* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

- Controlling for Kidhome** / everything else being equal / *ceteris paribus*, one unit increase in Income increases total spending by 0.019 pounds.

# Standard Errors and P-values

- Due to randomness of the error term, all coefficients estimates follow a  $t$  distribution.
- Therefore, we need **p-values** to check whether the coefficients are statistically different from 0.
- Income/Kidhome is statistically significant at the 1% level.

# R-squared

- R-squared ( $R^2$ ) is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable or variables in a regression model.
- Interpretation: 65.8% of the variation in Spending can be explained by Income and Kidhome.
- As the number of variables increases, the  $R^2$  will naturally increase.
- In causal inference tasks,  $R^2$  does not mean much.