

## Class 13 OLS Regression Advanced

Dr Wei Miao

UCL School of Management

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## Section 1

# Categorical Variables

# Categorical variables

- So far, the independent variables we have used are Income and Kidhome, which are **continuous variables**.
- Some variables are intrinsically not countable; we need to treat them as **categorical variables**
  - e.g., gender, education group, city.

# Handling Categorical Variables in R using factor()

- In R, we need to use a function `factor()` to explicitly inform R that this variable is a categorical variable, such that statistical models will treat them differently from continuous variables.
  - e.g., we can use `factor(Education)` to indicate that, Education is a categorical variable.

```
1 data_full <- data_full %>%
2   mutate(Education_factor = factor(Education))
```

- We can use `levels()` to check how many categories there are in the factor variable.
  - e.g., Education has 5 different levels.

```
1 # check levels of a factor
2 levels(data_full$Education_factor)
```

```
[1] "2n Cycle" "Basic" "Graduation" "Master" "PhD"
```

## Handling Categorical Variables using factor()

- `factor()` will check all levels of the categorical variables, and then choose the default level based on alphabetic order.
- If needed, we can revise the baseline group to another group using `relevel()` function.

```

1 # Create a new factor variable, with Basic as the baseline.
2 data_full <- data_full %>%
3   mutate(Education_factor_2 = relevel(Education_factor,
4                                       ref = "Basic") )
5
6 levels(data_full$Education_factor_2)

```

```
[1] "Basic"      "2n Cycle"   "Graduation" "Master"     "PhD"
```

# Running Regression with Factor Variables

```

1  pacman::p_load(fixest,modelsummary)
2  feols_categorical <- feols(data = data_full,
3    fml = total_spending ~ Income + Kidhome + Education_factor_2)
4  modelsummary(feols_categorical,
5    stars = T,
6    gof_map = c('nobs','r.squared'))

```

	(1)
(Intercept)	-180.297** (56.305)
Income	0.020*** (0.000)
Kidhome	-227.761*** (16.961)
Education_factor_2n Cycle	-164.044** (60.448)
Education_factor_2Graduation	-119.695* (56.176)
Education_factor_2Master	-143.015* (58.443)
Education_factor_2PhD	-153.190** (57.751)
Num.Obs.	2000
R2	0.662

+ p < 0.1, \* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

# One-Hot Encoding of factor()

- In the raw data, Education is label-encoded with 5 levels.

	ID	Education
1	5524	Graduation
2	2174	Graduation
3	4141	Graduation
4	6182	Graduation
5	5324	PhD
6	7446	Master
7	965	Graduation
8	6177	PhD
9	4855	PhD
10	5899	PhD

- After factorizing education with “*Basic*” as the baseline group, internally, we have 4 binary indicators as follows. Because we have the intercept, “*Basic*” is omitted as the baseline group. Other groups represent the comparison relative to the baseline group.

	ID	Edu_2n Cycle	Edu_Graduation	Edu_Master	Edu_PhD
1:	5524	0	1	0	0
2:	2174	0	1	0	0
3:	4141	0	1	0	0
4:	6182	0	1	0	0
5:	5324	0	0	0	1
6:	7446	0	0	1	0
7:	965	0	1	0	0
8:	6177	0	0	0	1
9:	4855	0	0	0	1
10:	5899	0	0	0	1

# Interpretation of Coefficients for Categorical Variables

- In general, R uses **one-hot encoding** to encode factor variables with **K** levels into **K-1** binary variables.
  - As we have the intercept term, we can only have **K-1** binary variables.
- The interpretation of coefficients for factor variables: Ceteris paribus, compared with the **[baseline group]**, the **[outcome variable]** of **[group X]** is higher/lower by **[coefficient]**, and the coefficient is statistically **[significant/insignificant]**.
  - Ceteris paribus, compared with the basic education group, the total spending of PhD group is lower by 153.190 dollars. The coefficient is statistically significant at the 1% level.
- Now please rerun the regression using `Education_factor` and interpret the coefficients. What's your finding?
  - Conclusion: factor variables can only measure the relative difference in outcome variable across different groups rather than the absolute levels.



# Application of Categorical Variables in Marketing

- Analyze the treatment effects in A/B/N testing, where  $Treatment_i$  is a categorical variable that specifies the treatment group customer  $i$  is in:

$$Outcome_i = \beta_0 + \delta Treatment_i + \epsilon$$

- Analyze the brand premiums or country-of-origin effects:

$$Sales_i = \beta_0 + \beta_1 Brand_i + \beta_2 Country_i + X\beta + \epsilon$$

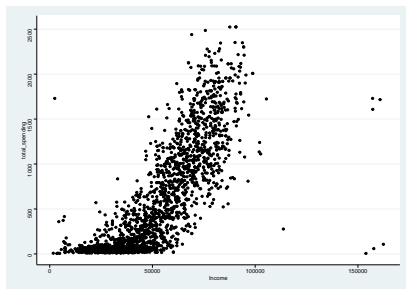
## Section 2

### Non-linear Effects

# Quadratic Terms

- If we believe the relationship between the outcome variable and explanatory variable is a quadratic function, we can include **an additional quadratic term** in the regression to model such non-linear relationship.

$$totalspending = \beta_0 + \beta_1 Income + \beta_2 Income^2 + \epsilon$$

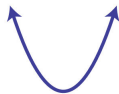


# Quadratic Terms

- If the coefficient for  $Income^2$  is negative, then we have an downward open parabola. That is, as income increases, total spending first increases and then decreases, i.e., a non-linear, non-monotonic effect.
  - As income first increases, customers increase their spending with Tesco due to the **income effect**; however, as customers get even richer, they may switch to more premium brands such as Waitrose, so their spending may decrease due to the **substitution effect**.

*Parabola*  $y = ax^2 + bx + c$

$a > 0$



*opens upward*

$a < 0$



*opens downward*

# Quadratic Terms in Linear Regression

- Let's run two regressions in the Quarto document, with and without the quadratic term.

```
1 # model 1: without quadratic term
2 feols_noquadratic <- feols(data = data_full,
3   fml = total_spending ~ Income )
4
5 # model 2: with quadratic term
6 feols_quadratic <- feols(data = data_full%>%
7   mutate(Income_squared = Income^2 ),
8   fml = total_spending ~ Income + Income_squared )
```

# Quadratic Terms in Linear Regression

```

1  modelsummary(list(feols_noquadratic,
2                    feols_quadratic),
3                    stars = T,
4                    fmt = fmt_sprintf("%.2e"),
5                    gof_map = c('nobs', 'r.squared'))

```

	(1)	(2)
(Intercept)	$-5.57 \times 10^2***$ ( $2.17 \times 10^1$ )	$-6.27 \times 10^2***$ ( $3.65 \times 10^1$ )
Income	$2.24 \times 10^{-2}***$ ( $3.84 \times 10^{-4}$ )	$2.53 \times 10^{-2}***$ ( $1.30 \times 10^{-3}$ )
Income_squared		$-2.66 \times 10^{-8}*$ ( $1.12 \times 10^{-8}$ )
Num.Obs.	2000	2000
R2	0.629	0.630

+  $p < 0.1$ , \*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

# Quadratic Terms: Compute the Vertex

- We can compute the vertex point where total spending is maximized by income

```
1 # extract the coefficient vector using $ sign
2 feols_coefficient <- feols_quadratic$coefficients
3 feols_coefficient
```

(Intercept)	Income	Income_squared
-6.270403e+02	2.533276e-02	-2.663682e-08

```
1 # Use b / (-2a) to get the vertex
2 - feols_coefficient[2]/
3   (2 * feols_coefficient[3])
```

Income  
475521.5

## Section 3

# Linear Probability Model



# Linear Probability Model

- In Predictive Analytics, we learned how to use decision tree and random forest to make predictions for binary outcome variables.
- In fact, linear regression can also be used as another supervised learning model to predict binary outcomes. When the outcome variable is a binary variable, the linear regression model is also called linear probability model.

- On the one hand, regression predicts the expectation of response  $Y$  conditional on  $X$ ; that is

$$E[Y] = E[X\beta + \epsilon] = X\beta$$

- On the other hand, for a binary outcome variable, if the probability of outcome occurring is  $p$ , then we can write the expectation of  $Y$  is

$$E[Y] = 1 * p + 0 * (1 - p) = p$$

- As a result, we have the following equation

$$p = X\beta$$

- Interpretation of LPM coefficients: Everything else equal, a unit change in  $x$  will change the **probability of the outcome occurring** by  $\beta$ .

# Pros and Cons of LPM

- We use linear regression function `feols()` to train the LPM on the **training data** and make predictions using `predict(LPM, data_test)` to make predictions on the **test data**.
- Advantages
  - Fast to run, even with a large number of fixed effects and features
  - High interpretability: coefficients have clear economic meanings
- Disadvantages
  - Predicted probabilities of occurring may fall out of the  $[0,1]$  range
  - Accuracy tends to be low