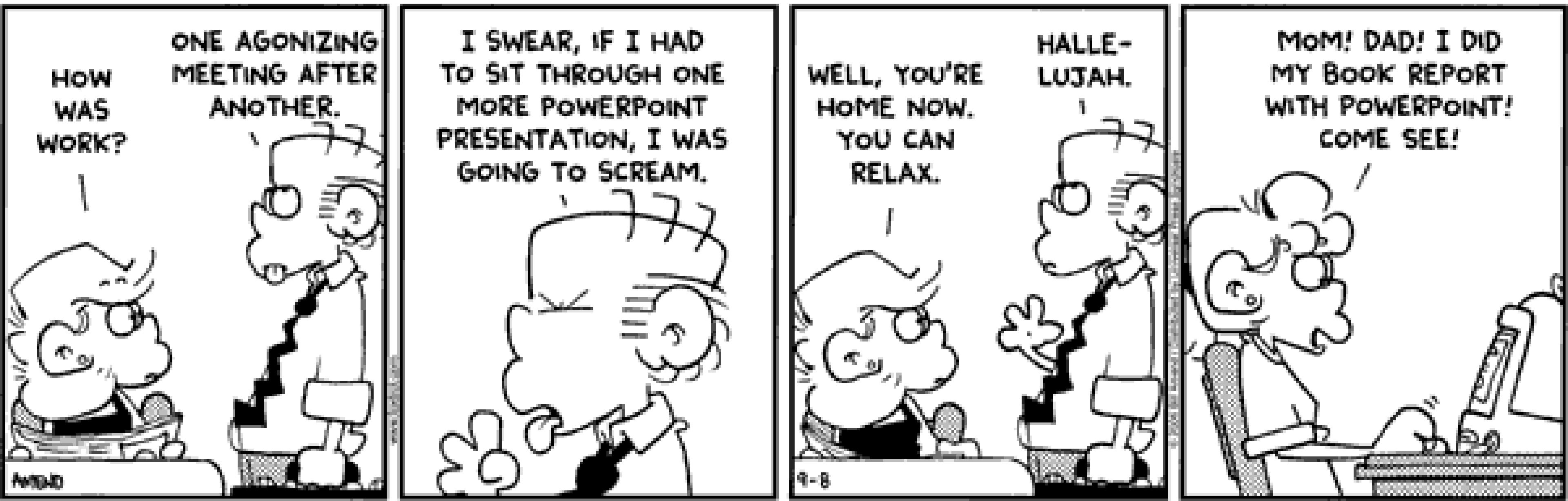
Lexical Analysis

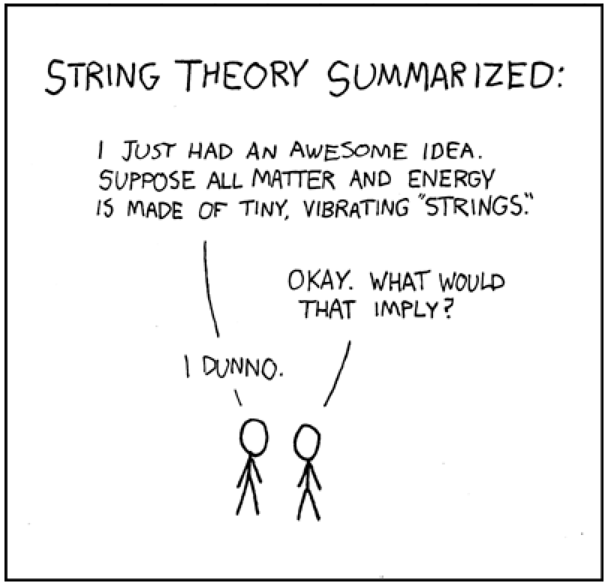
Finite Automata

(Part 2 of 2)


HOW WAS WORK?
ONE AGONIZING MEETING AFTER ANOTHER.
I SWEAR, IF I HAD TO SIT THROUGH ONE MORE POWERPOINT PRESENTATION, I WAS GOING TO SCREAM.
WELL, YOU'RE HOME NOW. YOU CAN RELAX.
HALLE-LUJAH.
MOM! DAD! I DID MY BOOK REPORT WITH POWERPOINT! COME SEE!
9-8

Cunning Plan

- Regular expressions provide a concise notation for **string patterns**
- Use in lexical analysis requires extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (next)
  - Require only single pass over the input
  - Few operations per character (table lookup)


STRING THEORY SUMMARIZED:
I JUST HAD AN AWESOME IDEA. SUPPOSE ALL MATTER AND ENERGY IS MADE OF TINY, VIBRATING "STRINGS."
OKAY. WHAT WOULD THAT IMPLY?
I DUNNO.

One-Slide Summary

- **Finite automata** are formal models of computation that can accept regular languages corresponding to regular expressions.
- **Nondeterministic** finite automata (NFA) feature epsilon transitions and multiple outgoing edges for the same input symbol.
- Regular expressions can be **converted** to NFAs.
- Tools will **generate** DFA-based lexer code for you from regular expressions.

Finite Automata

- Regular expressions = specification
- Finite automata = implementation

- A finite automaton consists of
  - An input alphabet **Σ**
  - A set of states **S**
  - A start state **n**
  - A set of accepting states **F $\subseteq$ S**
  - A set of transitions **state $\rightarrow^{\text{input}}$ state**

Finite Automata

- Transition

$$s_1 \rightarrow^a s_2$$

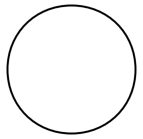
- Is read

  In state s_1 on input "a" go to state s_2

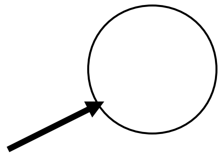
- If end of input
  - If in accepting state $\Rightarrow$ accept
  - Otherwise $\Rightarrow$ reject
- If still input, no transitions possible $\Rightarrow$ reject

Finite Automata State Graphs

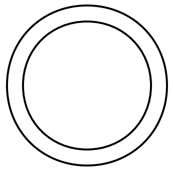
- A state



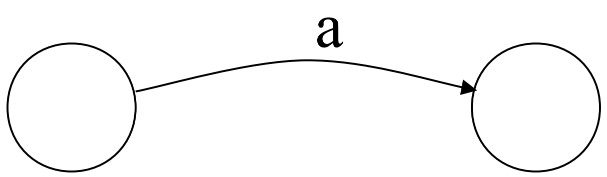
- The start state



- An accepting state



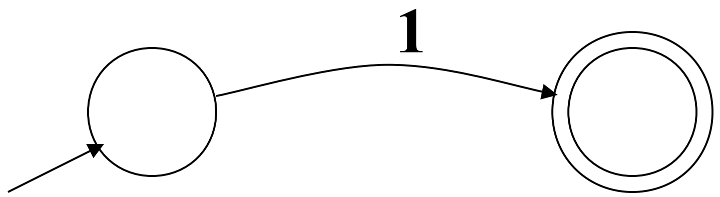
- A transition



You can hand-write RS1.
Wait, what's RS1?

A Simple Example

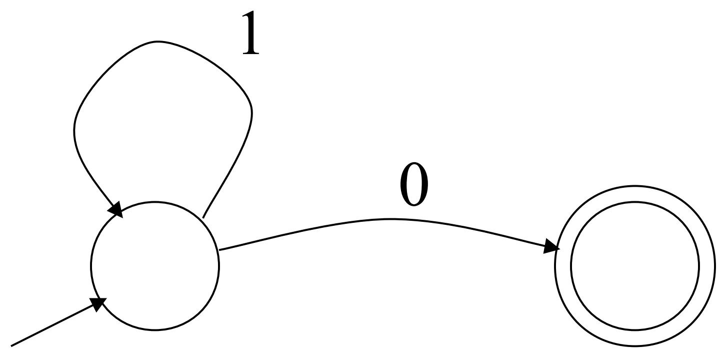
- A finite automaton that accepts only "1"



- A finite automaton accepts a string if we can follow transitions labeled with the characters in the string from the start to some accepting state

Another Simple Example

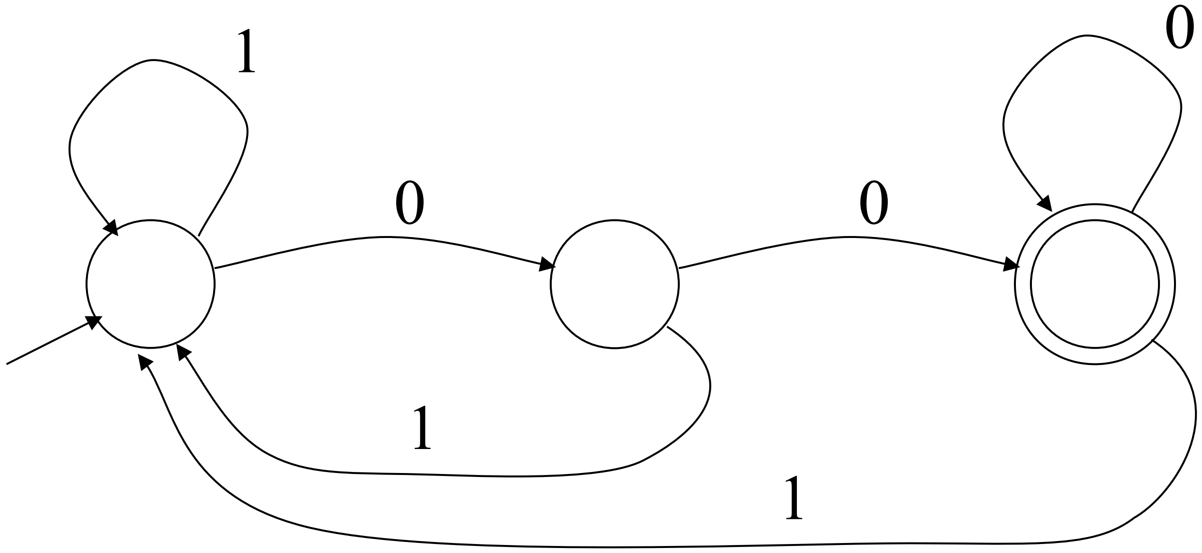
- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet $\Sigma = \{0,1\}$



- Check that "**1110**" is accepted but "**110…**" is not

And Another Example

- Alphabet Σ = {0,1}
- What language does this recognize?



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how to hook up a hose to a kitchen sink

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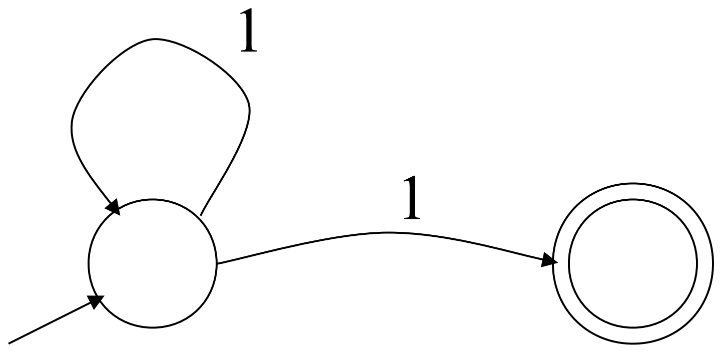
Did you mean: how to hook up a ***horse*** to a kitchen sink

And A Fourth Example

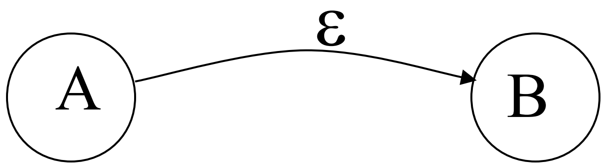
- Alphabet still $\Sigma = \{ 0, 1 \}$



- The operation of the automaton is not completely defined by the input
  - On input "11" the automaton could be in either state

Epsilon Moves

- Another kind of transition: ε-moves



$$\begin{tikzcd} A \arrow[r, bend left, "\varepsilon"] & B \end{tikzcd}$$

- Machine can move from state A to state B *without reading input*

Deterministic and Nondeterministic Automata

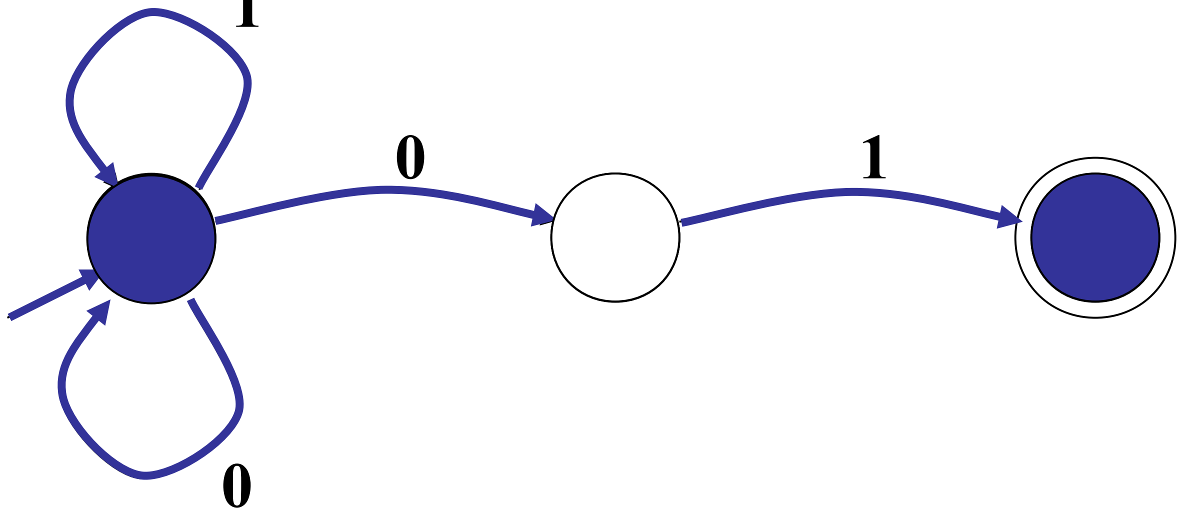
- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
- Finite automata have finite memory
  - Need only to encode the current state

Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



- Input: **1 0 1**
- Rule: NFA accepts if it <u>can</u> get in a final state

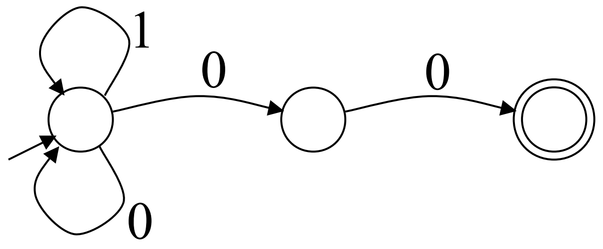
NFA vs. DFA (1)

- NFAs and DFAs recognize the *same* set of languages (regular languages)
  - They have the same expressive power
- DFAs are easier to implement
  - There are no choices to consider

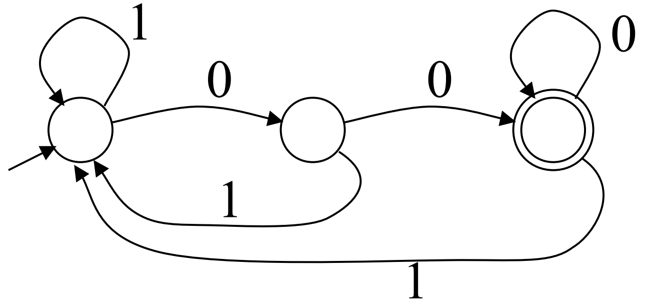
NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA



DFA



- DFA can be *exponentially* larger than NFA

Natural Languages

- This North Germanic language is generally mutually intelligible with Norwegian and Danish, and descends from Old Norse of the Viking Era to a modern speaking population of about 10 million people. The language contains two genders, nouns that are rarely inflected, and a typical subject-verb-object ordering. Its home country is one of the largest music exporters of the modern world, often targeting English-speaking audiences. Bands such as Ace of Base, ABBA and Roxette are examples, with over 420m combined album sales.

Unnatural Languages

- This stack-based structured computer programming language appeared in the 1970's and went on to influence PostScript and RPL. It is typeless and was often used in bootloaders and embedded apps. Example:

```
25 10 * 50 +
```

- Simple C Program:

```
int floor5(int v) { return (v < 6) ? 5 : (v – 1); }
```

- Same program in *this* Language:

```
: FLOOR5 ( n -- n' ) DUP 6 < IF DROP 5 ELSE 1 – THEN ;
```

Chemistry

- Which of these chemical reactions would be the hardest to carry out in a school chemistry class?
  - Nitrating cellulose to produce guncotton
  - Reacting thermite with iron oxide (2500 °C)
  - Dissolving bauxite in cryolite to make aluminum
  - Cross-linking polyvinyl alcohol with sodium borate

Chemistry

- The Hall–Héroult process (1886) extracts aluminum from the ore bauxite. Aluminum is the most abundant metallic element on Earth but not in its elemental state.
- Before this process aluminum was *more expensive than gold or platinum*:
  - "Bars of aluminum were exhibited alongside the French crown jewels at the Exposition Universelle of 1855, and Emperor Napoleon III of France was said to have reserved his few sets of aluminum dinner plates and eating utensils for his most honored guests."

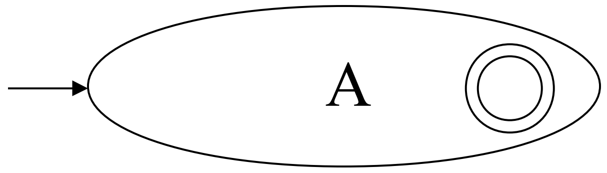
Regular Expressions to Finite Automata

- High-level sketch

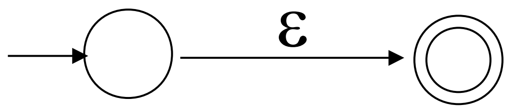
$$\begin{tikzcd}
 & \text{NFA} \arrow[dr] & \\
\text{Regular expressions} \arrow[ur] & & \text{DFA} \arrow[d] \\
\text{Lexical Specification} \arrow[u] & & \text{Table-driven Implementation of DFA}
\end{tikzcd}$$

Regular Expressions to NFA (1)

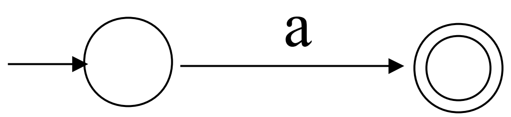
- For each kind of regexp, define an NFA
  - Notation: NFA for regexp A



- For ε

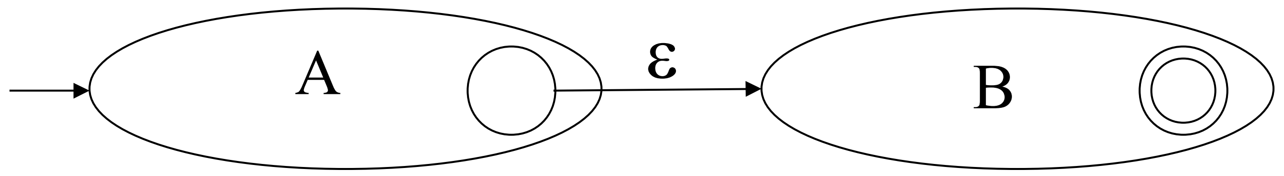


- For input a

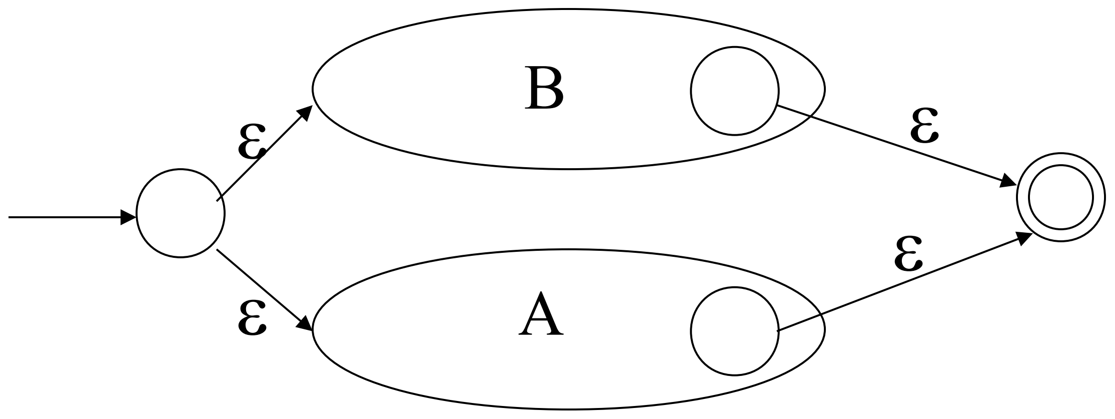


Regular Expressions to NFA (2)

- For AB

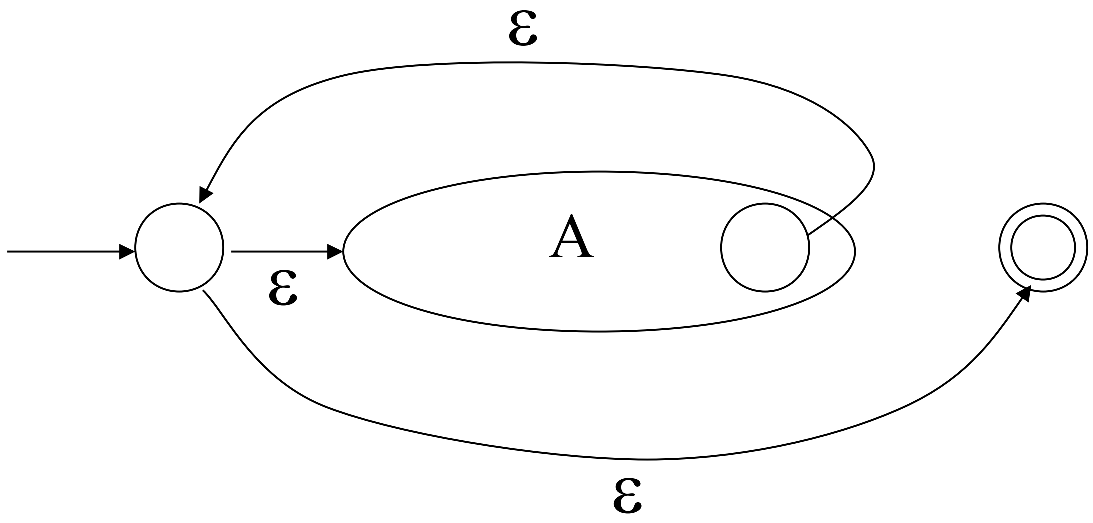


- For A | B



Regular Expressions to NFA (3)

- For A*



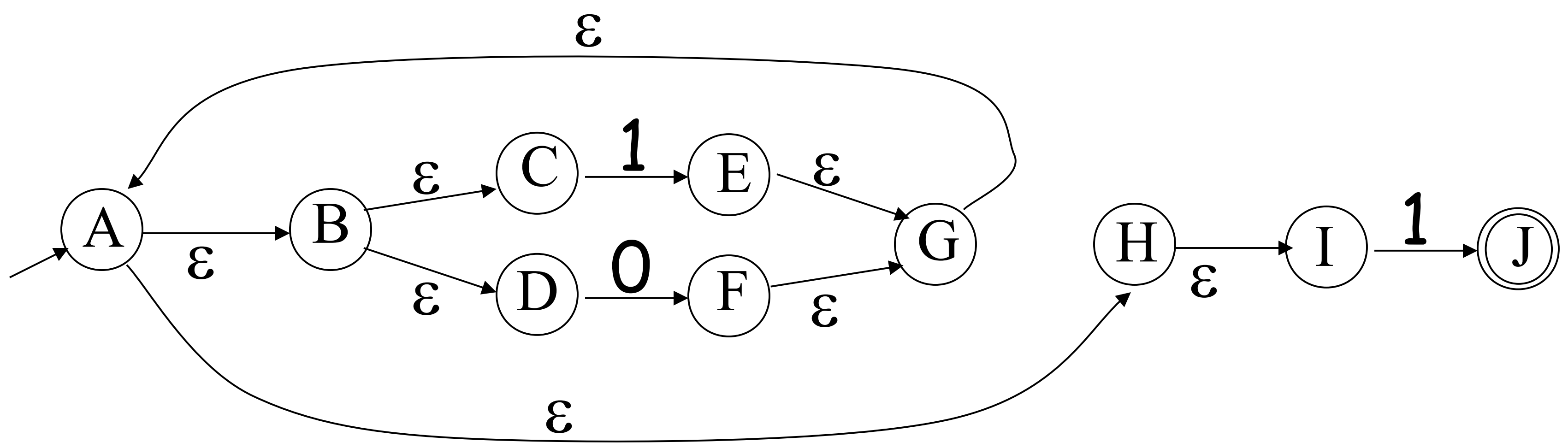
DO YOU BELIEVE OUR DESTINIES ARE CONTROLLED BY THE STARS?
NO, I THINK WE CAN DO WHATEVER WE WANT WITH OUR LIVES.
NOT TO HEAR MOM AND DAD TELL IT.
WATERSON

Example of RegExp -> NFA Conversion

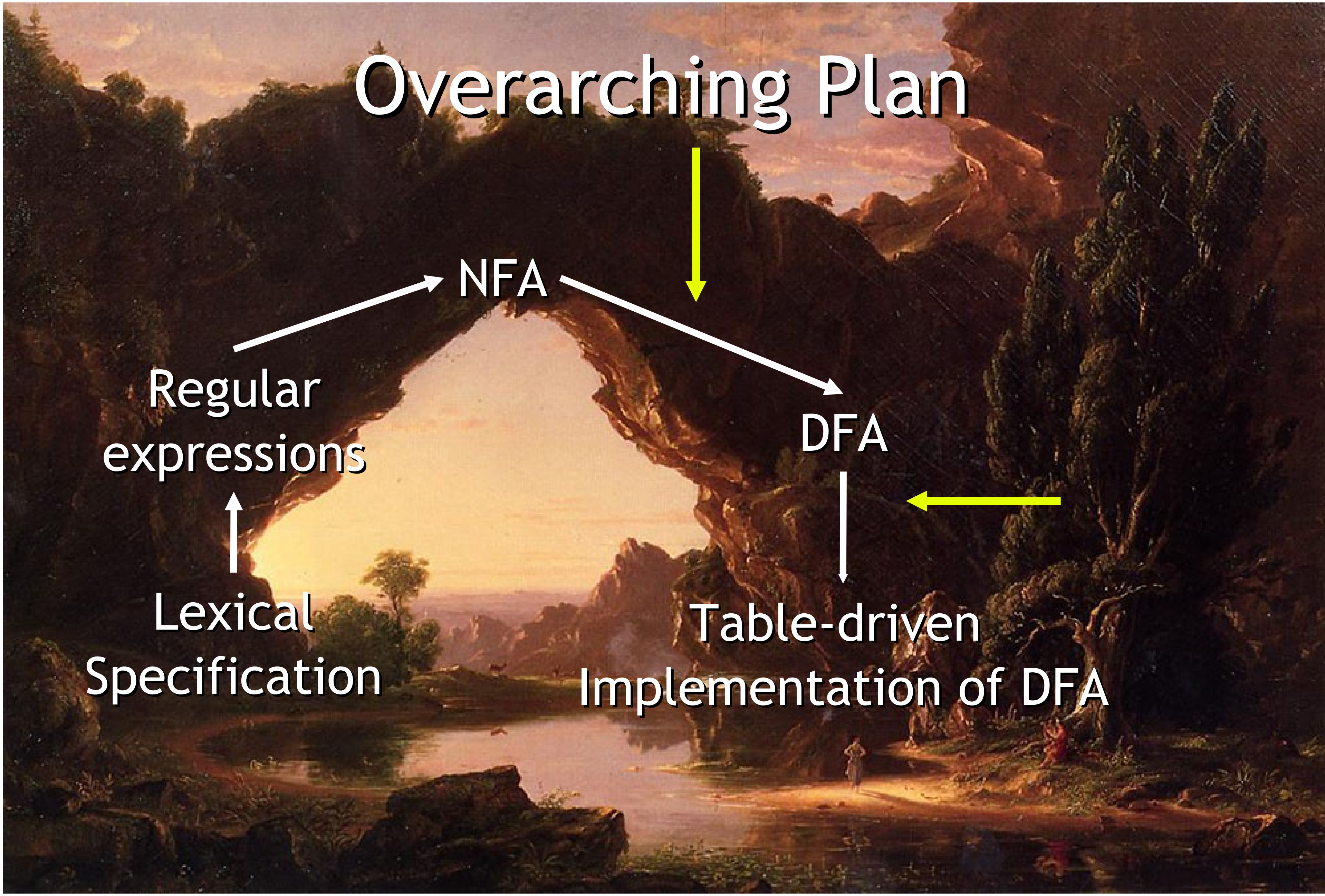
- Consider the regular expression

  (1 | 0)* 1

- The NFA is



Overarching Plan



$$\begin{tikzcd}
 & \text{NFA} \arrow[dr] & \\
\text{Regular expressions} \arrow[ur] & & \text{DFA} \arrow[d] \\
\text{Lexical Specification} \arrow[u] & & \text{Table-driven Implementation of DFA}
\end{tikzcd}$$

Thomas Cole – Evening in Arcady (1843)

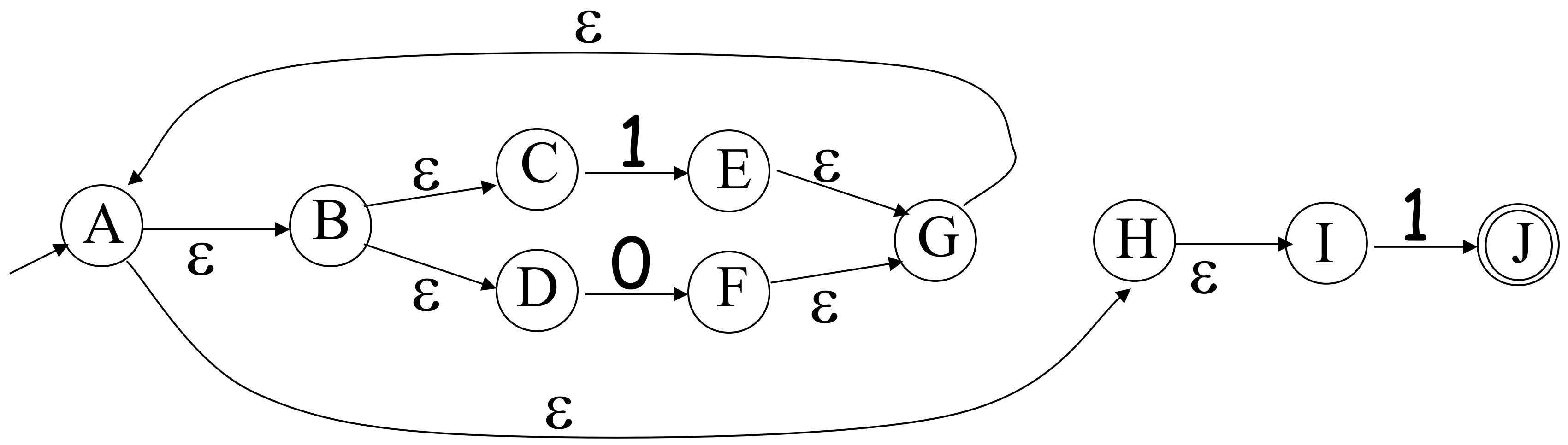
NFA to DFA: The Trick

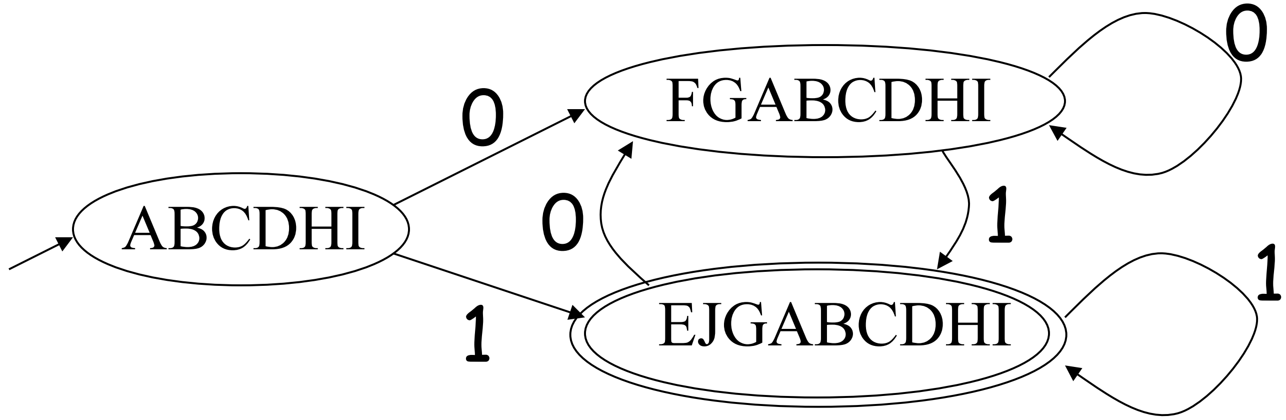
- Simulate the NFA
- Each state of DFA

  = a non-empty *subset of states* of the NFA
- Start state

  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \rightarrow^{a} S'$ to DFA iff
  - S' is the set of NFA states reachable from the states in S after seeing the input a
    - considering ε-moves as well

NFA → DFA Example





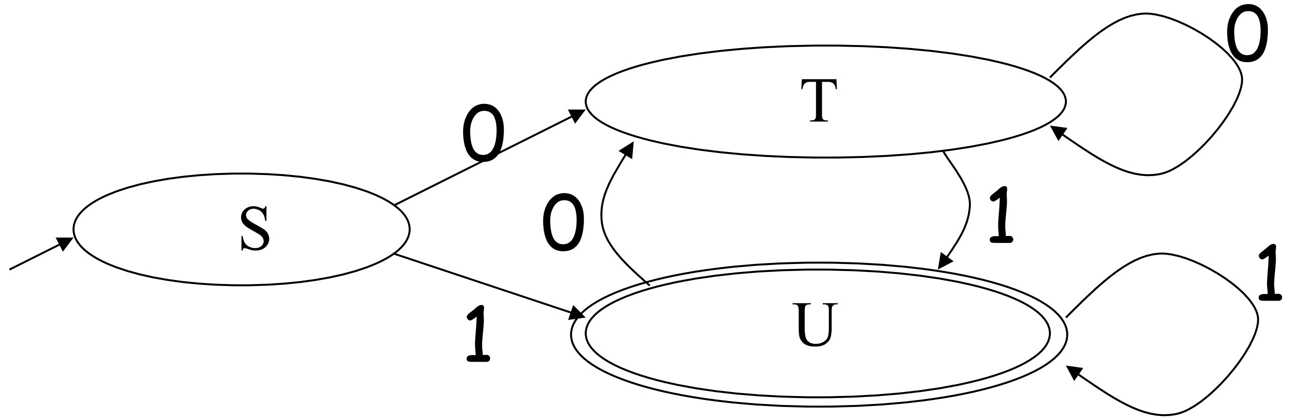
NFA $\rightarrow$ DFA: Remark

- An NFA may be in many states at any time
- How many different states?
- If there are N states, the NFA must be in some subset of those N states
- How many non-empty subsets are there?
  - $2^N - 1$ = finitely many

Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is “states”
  - Other dimension is “input symbols”
  - For every transition $S_i \to^a S_k$ define T[i,a] = k
- DFA “execution”
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

Table Implementation of a DFA



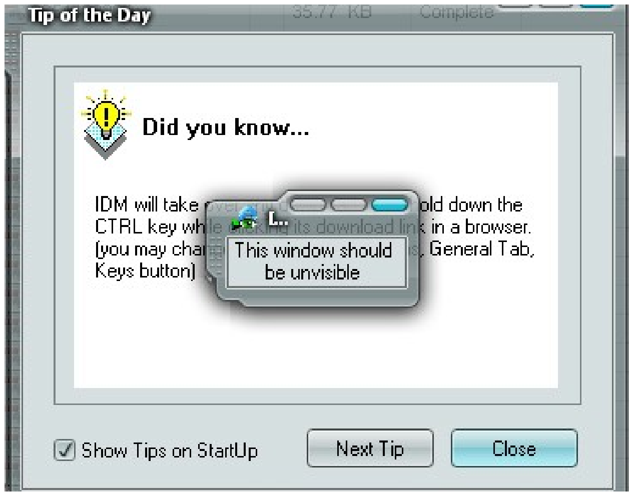
	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- NFA $\rightarrow$ DFA conversion is at the heart of tools (such as flex or or ply or ocamllex)
- But, DFAs can be huge
- In practice, lexer tools trade off speed for space in the choice of NFA and DFA representations

PA2: Lexical Analysis

- Correctness is job #1.
  - And job #2 and #3!
- Tips on building large systems:
  - Keep it simple
  - Design systems that can be tested
  - Don’t optimize prematurely
  - It is easier to modify a working system than to get a system working



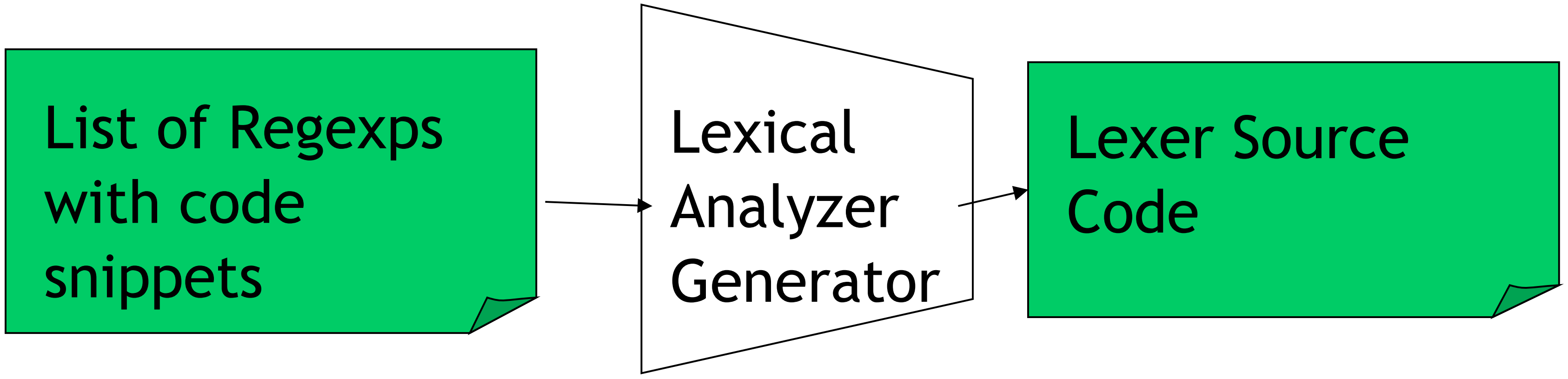
Did you know...

IDM will take ... old down the CTRL key wh... its download link in a browser. (you may cha... General Tab, Keys button)

This window should be unvisible

Lexical Analyzer Generator

- Tools like *ply* and *flex* and *ocamllex* will build lexers for you!
- You must use such a tool for PA2


List of Regexps with code snippets
Lexical Analyzer Generator
Lexer Source Code

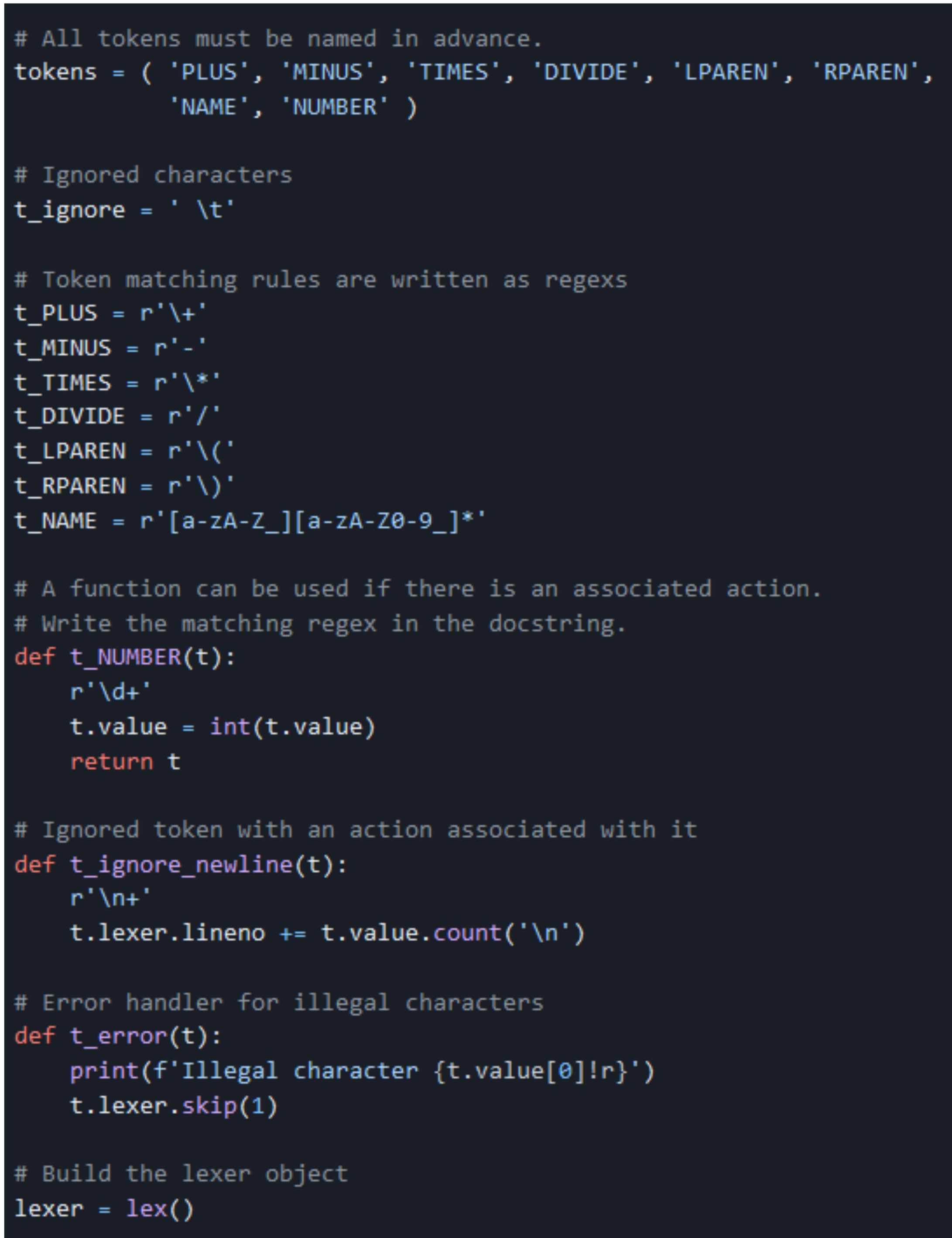
- I'll explain Python (OCaml in bonus slides)
  - See PA2 documentation.

PLY

- Example lexer definition for a calculator:

  (34 + y) * 5

- `“hi”` is a string, and `r”hi”` is a regexp.
- In `r'\*'` the `\` starts an *escape sequence*. It means “really literally `*`, not zero-or-more”.



```
# All tokens must be named in advance.
tokens = ( 'PLUS', 'MINUS', 'TIMES', 'DIVIDE', 'LPAREN', 'RPAREN',
           'NAME', 'NUMBER' )

# Ignored characters
t_ignore = ' \t'

# Token matching rules are written as regexs
t_PLUS = r'\+'
t_MINUS = r'-'
t_TIMES = r'\*'
t_DIVIDE = r'/'
t_LPAREN = r'\('
t_RPAREN = r'\)'
t_NAME = r'[a-zA-Z_][a-zA-Z0-9_]*'

# A function can be used if there is an associated action.
# Write the matching regex in the docstring.
def t_NUMBER(t):
    r'\d+'
    t.value = int(t.value)
    return t

# Ignored token with an action associated with it
def t_ignore_newline(t):
    r'\n+'
    t.lexer.lineno += t.value.count('\n')

# Error handler for illegal characters
def t_error(t):
    print(f'Illegal character {t.value[0]!r}')
    t.lexer.skip(1)

# Build the lexer object
lexer = lex()
```

PLY

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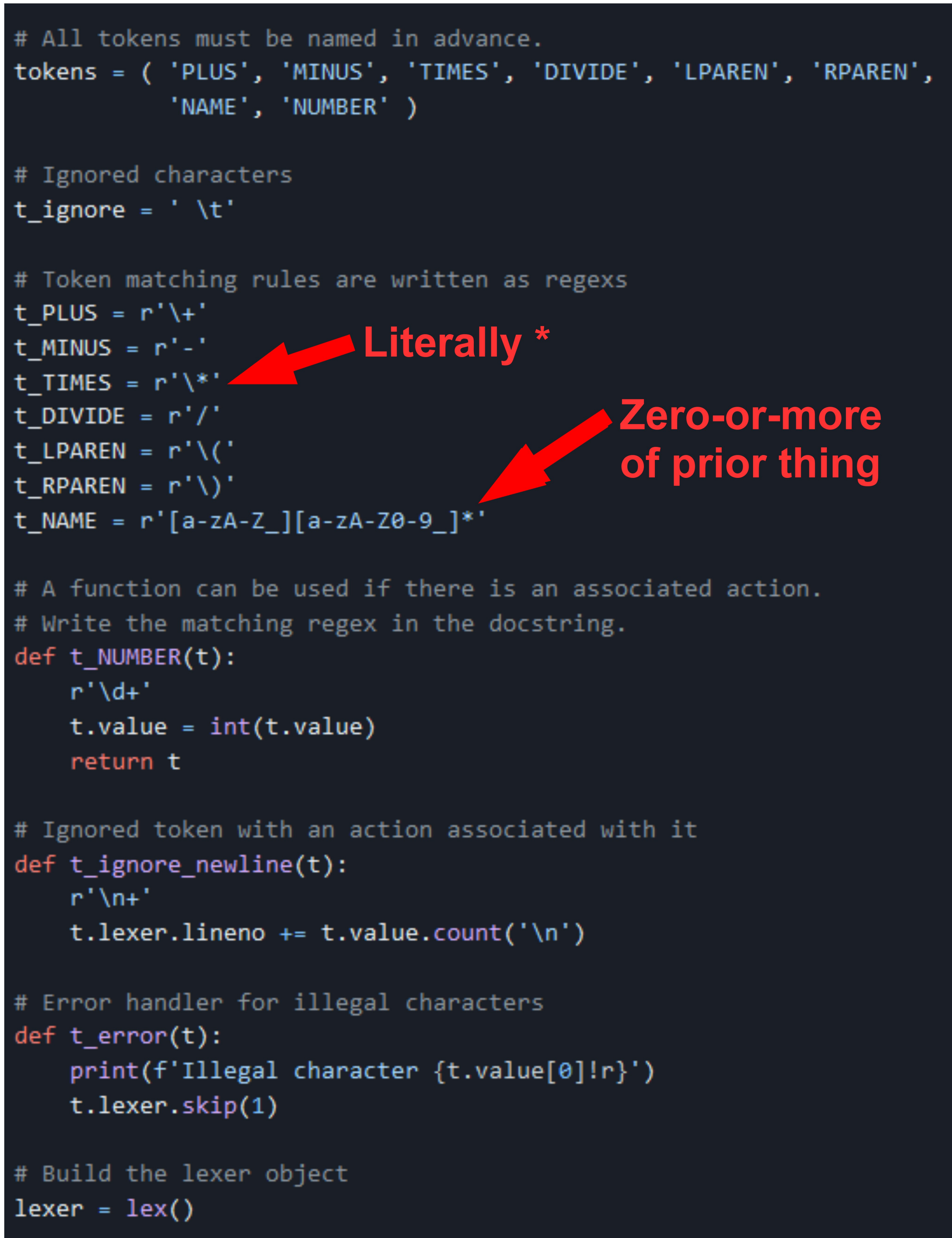

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How Big Is PA2?

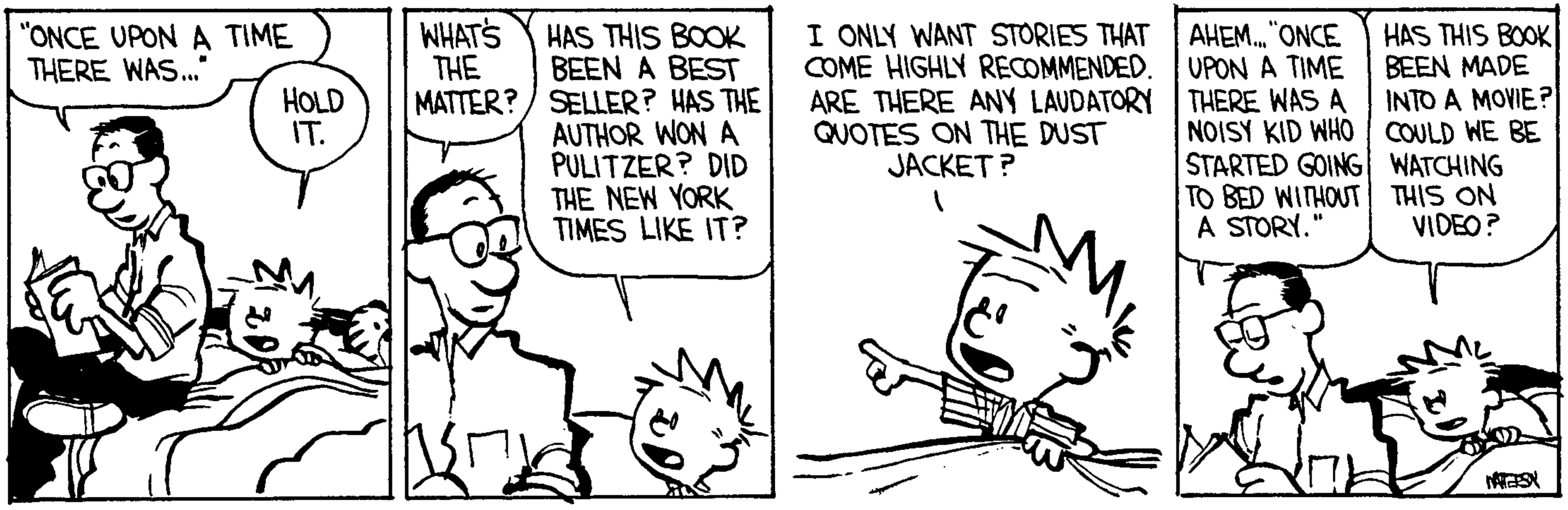
- The reference "lexer.mll" file is 88 lines
  - Perhaps another 20 lines to keep track of input line numbers
  - Perhaps another 20 lines to open the file and get a list of tokens
  - Then 65 lines to serialize the output
  - I'm sure it's possible to be smaller!
- Conclusion:
  - This isn't a code slog, it's about careful forethought and precision.

Warning!

- You may be tempted to use C for PA2 because the "flex" docs are the best.
- However, you can use any language (in any combination) for PA2-PA5.

Test Yourself! Exam Practice.

- Are practical parsers and scanners based on deterministic or non-deterministic automata?
- How can regular expressions be used to specify nested constructs?
- How is a two-dimensional *transition table* used in table-driven scanning?


"ONCE UPON A TIME THERE WAS..."
HOLD IT.
WHAT'S THE MATTER?
HAS THIS BOOK BEEN A BEST SELLER? HAS THE AUTHOR WON A PULITZER? DID THE NEW YORK TIMES LIKE IT?
I ONLY WANT STORIES THAT COME HIGHLY RECOMMENDED. ARE THERE ANY LAUDATORY QUOTES ON THE DUST JACKET?
AHEM... "ONCE UPON A TIME THERE WAS A NOISY KID WHO STARTED GOING TO BED WITHOUT A STORY."
HAS THIS BOOK BEEN MADE INTO A MOVIE? COULD WE BE WATCHING THIS ON VIDEO?
WATTERSON

Practice RegExp -> NFA

- Consider the regular expression

  (11 | 0)*

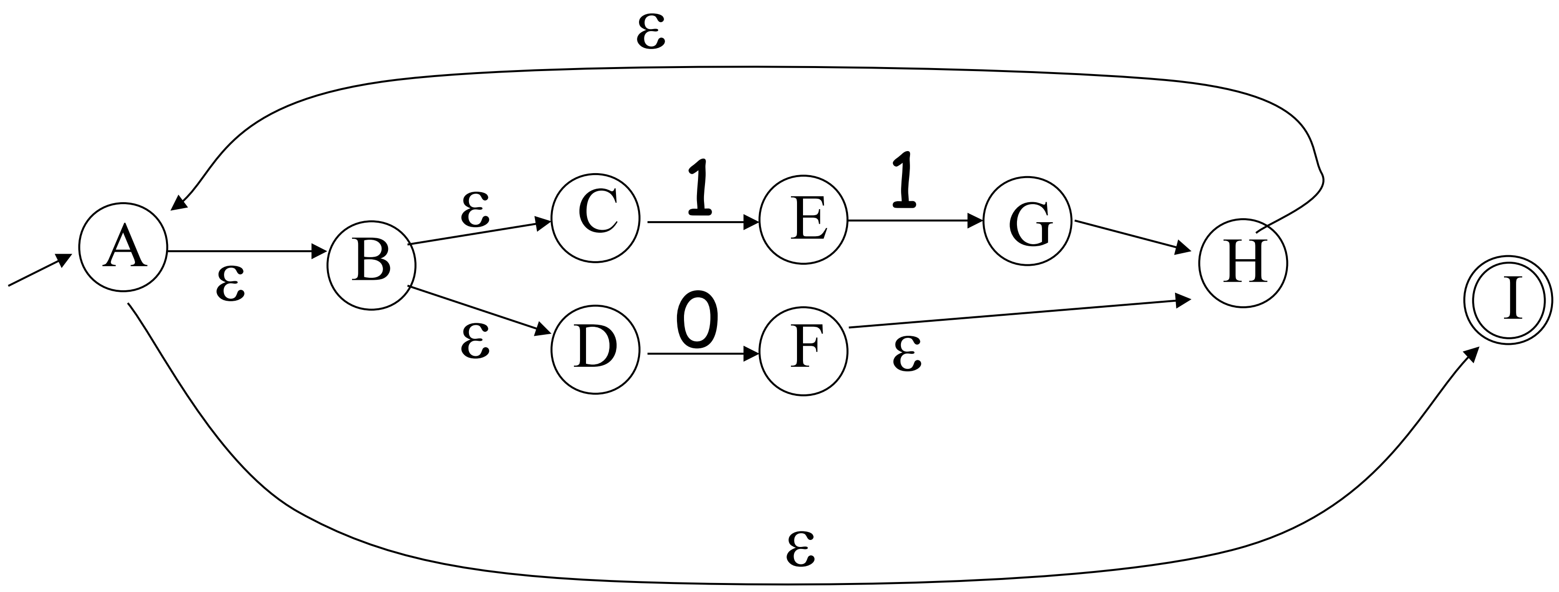
- What is an NFA that accepts that language?
  - Time permitting, let's do it now ...

Practice Answer (1/2)

- Consider the regular expression

(11 | 0)*

- One NFA is

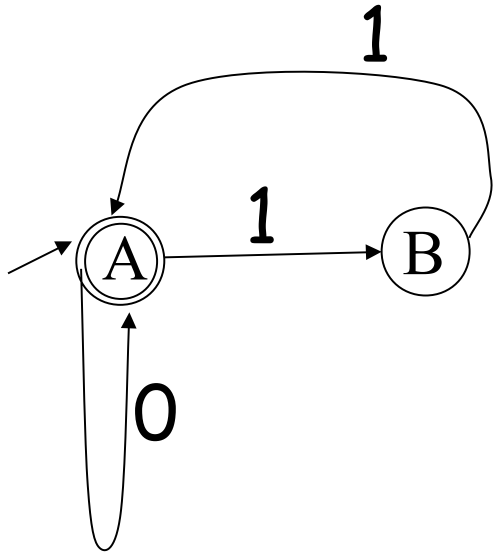


Practice Answer (2/2)

- Consider the regular expression

  (11 | 0)*

- A smaller also-full-credit NFA is:



Homework

- Textbook Reading, CD Reading – 2.4

Ocamllex “lexer.mll” file

```
{
   (* raw preamble code
        type declarations, utility functions, etc. *)
}
let re_name_i = re_i
rule normal_tokens = parse
   re_1          { token_1 }
|  re_2          { token_2 }
and special_tokens = parse
|  re_n          { token_n }
```

Example “lexer.mll”

```
{
   type token = Tok_Integer of int       (* 123 *)
        | Tok_Divide                     (*  /  *)
}
let digit = [‘0’ - ‘9’]
rule initial = parse
   ‘/’            { Tok_Divide }
| digit digit* { let token_string = Lexing.lexeme lexbuf in
                 let token_val = int_of_string token_string in
                 Tok_Integer(token_val) }
| _             { Printf.printf “Error!\n”; exit 1 }
```

Adding Winged Comments

```
{
    type token = Tok_Integer of int      (* 123 *)
           | Tok_Divide                  (*  /  *)
}
let digit = ['0' - '9']
rule initial = parse
    "//"            { eol_comment }
|   '/'             { Tok_Divide }
| digit digit*      { let token_string = Lexing.lexeme lexbuf in
                      let token_val = int_of_string token_string in
                      Tok_Integer(token_val) }
| _                 { Printf.printf "Error!\n"; exit 1 }

and eol_comment = parse
  '\n'     { initial lexbuf }
| _        { eol_comment lexbuf }
```

Using Lexical Analyzer Generators

```
$ ocamllex lexer.mll
45 states, 1083 transitions, table size 4602 bytes

(* your main.ml file … *)
let file_input = open_in “file.cl” in
let lexbuf = Lexing.from_channel file_input in
let token = Lexer.initial lexbuf in
match token with
| Tok_Divide -> printf “Divide Token!\n”
| Tok_Integer(x) -> printf “Integer Token = %d\n” x
```