

ATM/MPO 663 Homework 6

Due March 19 (Tuesday after Spring Break)

1. Vorticity instability (in chapter 3 of Markowski and Richardson). I want you to understand shear instability conceptually, whether mathematics or pictures underpins that for you. The assignment is: **write in your own words an accurate, complete, clear caption for these figures explaining the essential mechanisms of shear instability.**

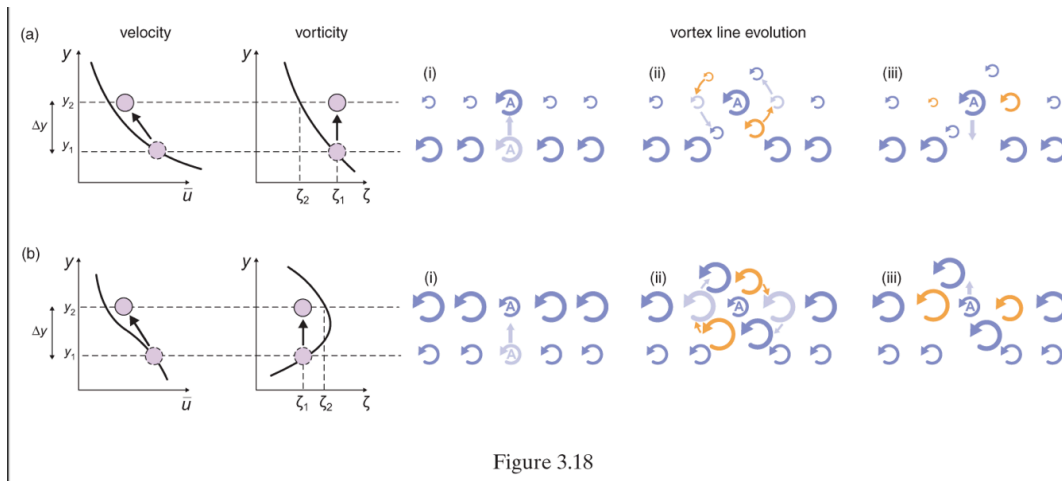


Figure 3.18

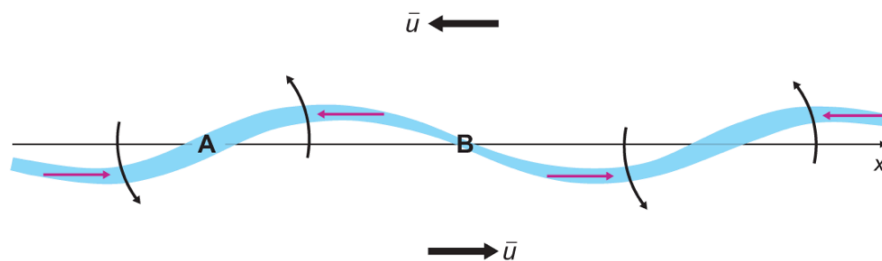


Figure 3.19

2. Do the same for these diagrams about Kelvin-Helmholtz instability, where the shear effect must work against static stability.

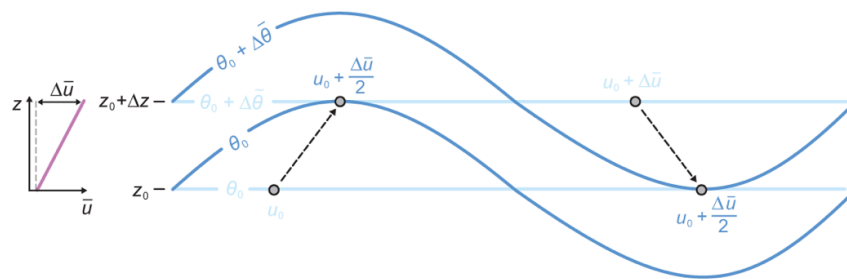


Figure 3.21

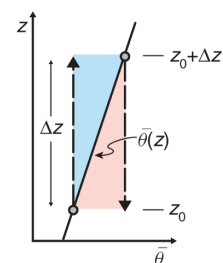


Figure 3.22

3. Do the same for these diagrams about gravity waves

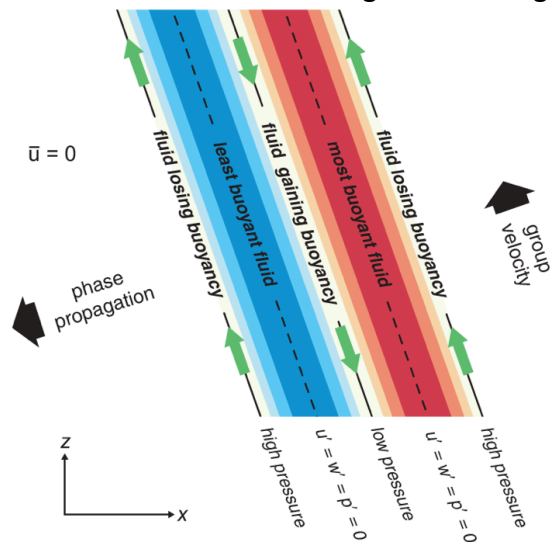


Figure 6.6

energy and phase propagation of a wave packet

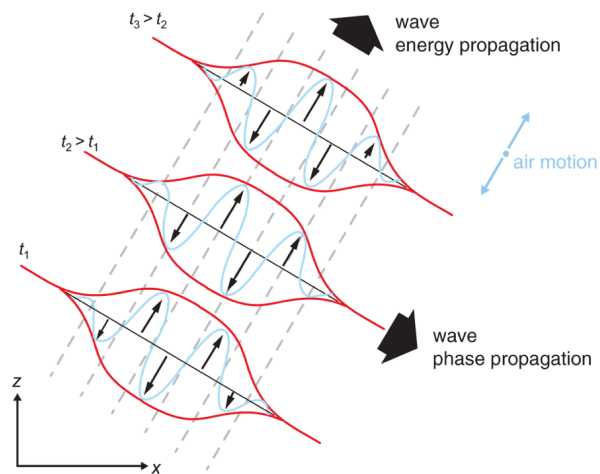


Figure 6.5

4. From the inviscid Boussinesq equations, derive the internal wave dispersion relation (as in class), but now permitting a sheared mean background wind $U(z)$ only).

$$u_t = fv - \pi_x$$

u momentum equation

$$v_t = -fu - \pi_y$$

v momentum equation

$$w_t = b - \pi_z$$

w momentum equation (=0 for hydrostatic)

$$u_x + v_y + w_z = 0$$

continuity equation

$$b_t = -N^2 w + q$$

thermodynamic equation w/ heating

$$(q = g/\theta \, D\theta/Dt)$$

Steps:

i. Neglect v and all y derivatives and f (work in the nonrotating x - z plane).

ii. Eliminate π by forming a vorticity equation

iii. Use the b equation to eliminate the b term in ii, neglecting q .

iv. Use mass continuity to eliminate u .

v. Assume solutions proportional to $\exp[i(kx + mz + \sigma t)]$. This means you can replace time derivatives with a factor $i\sigma$, z derivatives with im , and x derivatives with ik .

Solve for σ . This is the *dispersion relation*. Without $U(z)$, it is $\sigma = N \, k \, (k^2 + m^2)^{1/2}$.

5. Show me that you understand each part of the 2017 homework (attached; a solution set is on Github area).