Algorithmic Trading (COMP0051) Coursework2

Abstract

This project investigates two leveraged trading strategies on the SPTL bond ETF over the period from January 1, 2023 to December 31, 2023, using the Effective Federal Funds Rate (EFFR) as the risk-free rate. The coursework is separated into three parts:
1. Time Series Preparation: Compute daily returns and excess returns.
2. Trading Strategies: Develop a constant-leverage Buy & Hold strategy and a moving average crossover strategy under the appropriate margin constraints.
3. Performance Indicators: Evaluate the strategies using Sharpe and Calmar Ratios and present in a comparable table.

Results indicate that the moving average crossover strategy outperforms the leveraged Buy & Hold approach under the tested market conditions.

1 Time Series Prep

1.1 Download SPTL bond ETF and EFFR

To start with, we describe the steps taken to prepare the data required for the analysis:

Data Acquisition: We obtained the daily adjusted close prices for the SPTL ETF from Yahoo Finance library, covering the period from January 1, 2023 to December 31, 2023. The Effective Federal Funds Rate (EFFR) data was downloaded from the NewYork fed website.

Data Alignment: Both datasets were reindexed to a common business day frequency. Missing values were filled using forward-fill to ensure that every business day has an associated SPTL price and EFFR value.

Conversion of EFFR: We define the daily excess return, r_t^e, of the SPTL ETF relative to the annual risk-free rate r_t^f as follows:

$$r_t^e = \frac{\Delta p_t}{p_t} - r_t^f,$$

where

- p_t: the price of the asset (SPTL) at time t,
- $\Delta p_t = p_{t+1} - p_t$: the change in price from day t to $t+1$,
- $\frac{\Delta p_t}{p_t}$: the daily percentage change in the asset's price,
- r_t^f: the daily risk-free rate, given by

  $$r_t^f = \text{EFFR}(t) \cdot dc,$$

  and $dc \approx \frac{1}{252}$ reflects the typical number of trading days in a year.

By subtracting r_t^f from the asset's daily return, we measure how much the asset outperforms (or underperforms) the risk-free rate on each trading day.

1.2 Plot

Figure 1 contains 3 subplots which examines the SPTL performance in 2023:

1. SPTL daily returns range from -2% to 2%, with high volatility due to rising interest rates.
2. The daily risk-free rate, derived from the EFFR, rises from 0.00017 (4.28% annualized) to 0.00020 (5.04% annualized) in steps, reflecting the Fed's 2023 rate hikes.
3. Excess returns (SPTL returns minus the risk-free rate) mirror raw returns (-2% to 2%) due to the small risk-free rate.

2 Trading Strategies

This section implements and evaluates two leveraged trading strategies on the SPTL ETF over 2023: a constant-leverage Buy & Hold strategy and a Moving Average Crossover strategy. Both strategies operate under an initial capital $V_0 = \$100{,}000$ and a leverage factor $L = 10$, adhering to margin constraints. We compute the total Profit and Loss (PnL) series by considering trading PnL and capital account PnL, and visualize the daily and cumulative PnL components to assess each strategy's performance.

Figure 1: Time Series Analysis of SPTL Returns, Daily Risk-Free Rate, and Excess Returns for 2023.

2.1 Buy & Hold

The Buy & Hold strategy is a simple investment approach where an investor purchases an asset and holds it over an extended period, regardless of short-term price fluctuations.

Instead of buying a fixed number of shares and leaving the position unchanged, our approach continuously rebalances the position so that the dollar value held, θ_t, always remains at the maximum allowed by the current account value $V_{\text{total},t}$. In other words, we maintain:

$$|\theta_t| \leq V_t^{total} \times L$$

is strictly satisfied on every trading day.

This constant-leverage or "daily rebalanced" Buy & Hold strategy amplifies the asset's returns (and losses) in proportion to the account value. It serves as a benchmark to compare against more dynamic strategies, such as our second strategy: Moving Average Crossover (section 2.2)

2.2 Moving Average Crossover

The Moving Average Crossover strategy is a popular technical trading method that uses two moving averages of different lengths to generate trading signals. In our implementation, we calculate:

- **Short-term Moving Average ($\mathbf{MA_{short}}$)**: This is the average of the SPTL closing prices over a short period (e.g. 20 days).
- **Long-term Moving Average ($\mathbf{MA_{long}}$)**: This is the average over a longer period (e.g. 50 days).

The strategy works as follows:

- When the short-term moving average crosses above the long-term moving average, it indicates a potential upward trend. In this case, we go fully long by setting the dollar position to the maximum allowed, i.e.,

$$\theta_t = V_t^{total} \times L.$$

- Conversely, when the short-term moving average crosses below the long-term moving average, it suggests a potential downward trend. The strategy then goes fully short, setting

$$\theta_t = -V_t^{total} \times L.$$

This approach ensures that the position always adheres to the leverage constraint, i.e.,

$$|\theta_t| \leq V_t^{total} \times L,$$

and allows the strategy to dynamically adjust to market conditions by switching between long and short positions based on the relative values of the two moving averages.

2.3 Total PnL Series Calculation

In the final part of task 2, we create a total PnL series for the trading strategies by considering two components:

- **Trading PnL** (ΔV_t): This is the profit or loss arising from the changes in the asset's price. It is computed using the formula

$$\Delta V_t = \left(\frac{\Delta p_t}{p_t} - r_t^f \right) \theta_t,$$

where:

  - $\frac{\Delta p_t}{p_t}$ is the daily percentage change in the asset price (with $\Delta p_t = p_{t+1} - p_t$),
  - $r_{f,t}$ is the daily risk-free rate, and
  - θ_t is the dollar value of the SPTL position held at time t.

- **Capital Account PnL** (ΔV_t^{cap}): This component represents the interest earned on any unused capital that is placed in a money-market account. Unused capital is given by the difference between the total account value $V_{\text{total},t}$ and the margin used, where the margin M_t is defined as

$$M_t = \frac{|\theta_t|}{L}.$$

The daily capital PnL is computed as:

$$\Delta V_t^{cap} = \left(V_t^{total} - M_t \right) r_t^f.$$

The total daily PnL, which reflects the overall change in the account value, is then the sum of these two components:

$$\Delta V_t^{total} = \Delta V_t + \Delta V_t^{cap}.$$

Thus, the update to the total account value from day t to day $t+1$ is given by:

$$V_{t+1}^{total} - V_t^{total} = \Delta V_t^{total}.$$

2.4 Visualisation for strategies

The plots in Figure 3 and 4 illustrate the daily and cumulative Profit and Loss (PnL) components for the MA Crossover and Buy & Hold strategies over 2023, providing insights into their performance under a 10x leverage factor.

For the **Buy & Hold strategy**, the top subplot displays the daily PnL components. The trading PnL (ΔV_t) also fluctuates between -\$20,000 and +\$20,000, driven by the SPTL price movements amplified by 10x leverage. The capital account PnL (ΔV_t^{cap}) is consistently near zero, as the strategy is fully leveraged with no unused capital. The total daily PnL ($\Delta V_t^{\text{total}}$) mirrors the trading PnL, showing similar volatility.

The bottom subplot reveals the cumulative PnL components, with the cumulative total PnL declining steadily over the year, dropping to approximately -\$75,000 by December 2023.

For the **MA Crossover strategy**, the top subplot shows the daily PnL components: ΔV_t (trading PnL), ΔV_t^{cap} (capital account PnL), and $\Delta V_t^{\text{total}}$ (total PnL). The trading PnL (ΔV_t) fluctuates significantly, ranging from approximately -\$20,000 to +\$20,000, reflecting the strategy's dynamic switching between long and short positions based on moving average signals. The capital account PnL (ΔV_t^{cap}) remains near zero until the 50-day moving average window is reached, after which it contributes small positive amounts (around \$100) due to interest on unused capital. The total daily PnL ($\Delta V_t^{\text{total}}$) closely follows the trading PnL due to the minimal contribution of ΔV_t^{cap}.

The bottom subplot shows the cumulative PnL components, with the cumulative total PnL rising steadily after the initial 50-day period, reaching approximately \$150,000 by the end of 2023. This indicates that the strategy successfully capitalized on short-term trends, achieving a positive return despite market volatility.

This approach allows us to clearly distinguish between the profit or loss from the trading activity (adjusted for financing costs) and the income generated from the unutilized capital, giving a complete picture of the leveraged strategy's performance.

3 Performance Indicator

To assess the effectiveness of our leveraged trading strategies, we evaluate their performance using two key metrics: the Sharpe Ratio (SR) and the Calmar Ratio (CR). We first compute the total return for each strategy as:

$$r_t = \frac{\Delta V_t^{\text{total}}}{V_{\text{total},t}},$$

where $\Delta V_t^{\text{total}}$ is the total change in account value on day t (which includes both trading PnL and capital account PnL) and $V_{\text{total},t}$ is the account value at the beginning of day t. The performance metrics are defined as follows:

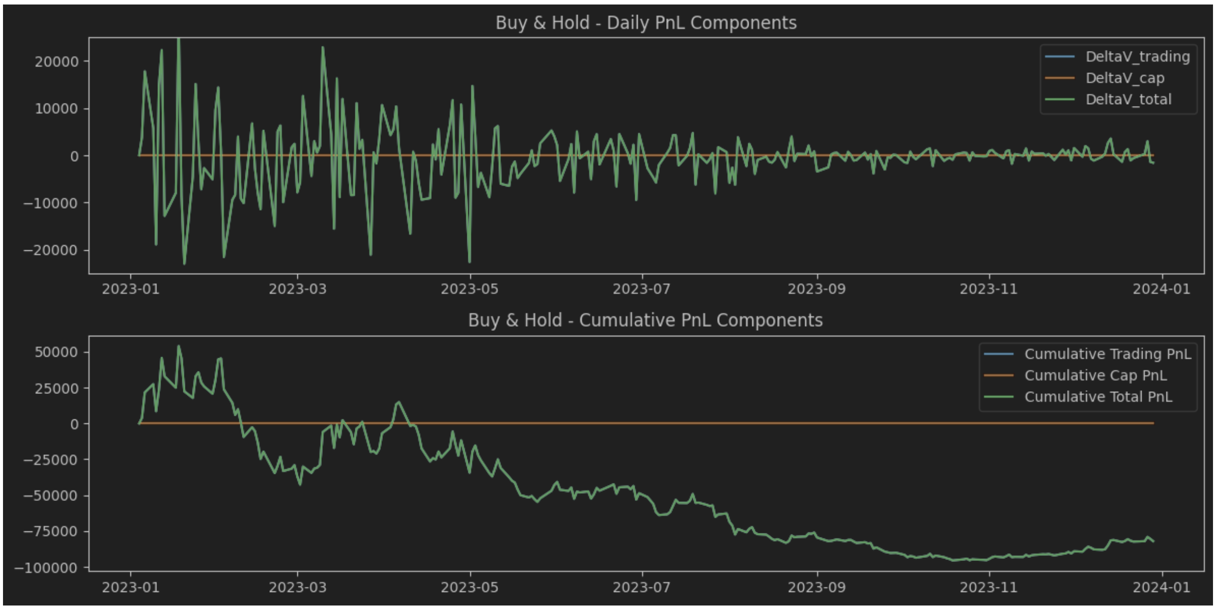

Figure 2: Buy & Hold strategy: Daily and cumulative PnL components.

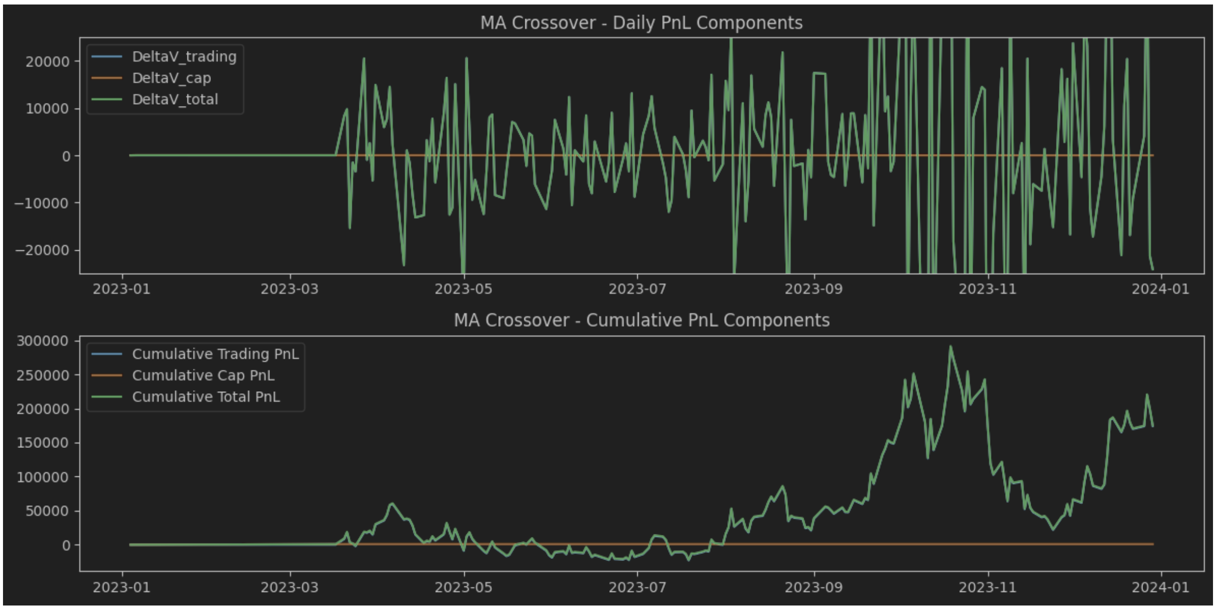

Figure 3: MA Crossover strategy: Daily and cumulative PnL components.

- **Sharpe Ratio (SR)**: This is computed using the updated function `annualized_sharpe_ratio`, which now accounts for the risk-free rate to calculate excess returns. The formula is defined as follows:

$$\text{SR} = \frac{\text{Mean}(r_t^e)}{\text{Std}(r_t^e)} \cdot \sqrt{\text{freq}},$$

  where $r_t^e = r_t - r_t^f$ represents the excess return, with r_t as the daily return of the strategy ($r_t = \frac{\Delta V_t^{\text{total}}}{V_{\text{total},t}}$) and r_t^f as the daily risk-free rate (derived from the EFFR, $r_t^f = \frac{\text{EFFR}(t)}{252}$). The $\text{Mean}(r_t^e)$ and $\text{Std}(r_t^e)$ of the excess returns are computed over the period, and the result is annualized by multiplying by $\sqrt{\text{freq}}$, where $\text{freq} = 252$ (the number of trading days in a year). If $\text{Std}(r_t^e) = 0$, the function returns `np.nan` to avoid division by zero. This adjustment ensures a more accurate measure of risk-adjusted returns by considering the opportunity cost of the risk-free rate.

- **Maximum Drawdown (MDD)**: The Maximum Drawdown measures the largest drop from a peak to a trough in the equity curve. It is defined as:

$$\text{MDD} = \frac{\text{MinThroughValue} - \text{MaxPeakValue}}{\text{MaxPeakValue}},$$

where:

- MaxPeakValue is the highest value of the equity curve (or account value) observed up to a certain time, and
- MinThroughValue is the lowest value reached after that peak.

The MDD is typically expressed as a negative value, representing the percentage loss from the peak.

- **Calmar Ratio (CR)**:

$$\text{CR} = \frac{\mu_{\text{annualized}}}{|\text{MDD}|},$$

where $\mu_{\text{annualized}}$ is the annualized return of the strategy. A higher Calmar Ratio indicates better performance relative to the worst-case drawdown.

3.1 Summary of Performance Metrics

The performance of the Buy & Hold and MA Crossover strategies is evaluated using Sharpe and Calmar ratios, as shown in Table 1.

Strategy	Sharpe Ratio	Calmar Ratio
Buy & Hold	-0.226868	-0.850427
MA Crossover	1.391962	2.615341

Table 1: Sharpe and Calmar ratios for the Buy & Hold and MA Crossover strategies in 2023.

The Buy & Hold strategy yields a negative Sharpe ratio of -0.226868, indicating poor risk-adjusted returns due to a $75,000 loss driven by the decline in SPTL prices under 10x leverage. Its Calmar ratio of -0.850427 further reflects this underperformance, with a significant maximum drawdown. In contrast, the MA Crossover strategy achieves a positive Sharpe ratio of 1.391962, demonstrating strong risk-adjusted returns with profit by adapting to market trends. Its Calmar ratio of 2.615341 indicates robust performance relative to its maximum drawdown.

Key Insight: The MA Crossover strategy significantly outperforms the Buy & Hold strategy, offering better risk-adjusted returns and resilience in a volatile market. However, these metrics do not account for the risk-free rate, which may slightly adjust the Sharpe ratios if included.