Counting

Colby Community College

Definition

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What is the probability that a poker hand will contain exactly two jacks?

$$\frac{(_4C_2)\,(_{48}C_3)}{_{52}C_5} = \frac{6 \cdot 17,296}{2,598,960} = \frac{103,776}{2,598,960} \approx 4\%$$

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The lowest ranked straight is A,2,3,4,5 and the highest ranked straight is 10,J,Q,K,A. Thus, for the any of the ten ranks A through 10, we can build a straight. Each card can be from any of the four suits.
This means the probability is:

$$P\left(\text{straight}\right) = \frac{10\left({}_4C_1\right)\left({}_4C_1\right)\left({}_4C_1\right)\left({}_4C_1\right)\left({}_4C_1\right)}{{}_{52}C_5} = \frac{10,240}{2,598,960} \approx 3.94\%$$

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Example 4

What is the probability that a poker hand will contain a straight, but not a straight flush?
We calculated in Example 2 the number of straights possible. Now, just need to subtract out the number of straight flushes.

$$P\,(\text{straight}) = \frac{10,240 - 40}{{}_{52}C_5} = \frac{10,200}{2,598,960} \approx 3.92\%$$

Example 5

In the casino game Roulette, a wheel with 38 spaces (18 red, 18 black, and 2 green) is spun. In one possible bet, the players bet \$1 on a single number. If that number is spun on the wheel, then they receive \$36. Otherwise, they lose their \$1.

On average, how much money should a player expect to win or lose if they play this game repeatedly?

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Any number you bet on will have the following probabilities:

Outcome	Probability
\$35 (win)	1/38
-\$1 (lose)	37/38

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So, **on average**, we will have a net change of

$$\$35 \cdot \frac{1}{38} + -\$1 \cdot \frac{37}{38} = \$0.9211 - \$0.9737 \approx -\$0.053$$

That is, **on average**, we will lose 5.3 cents per space we bet on.