Binomial Distribution

Colby Community College

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So, the probability exactly one has heard of Twitter is

$$0.002869 + 0.002869 + 0.002869 + 0.002869 = 0.11475 = 11.475\%$$

Definition

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If all the scenarios are independent of each other, then we can calculate the final probability as:

$$[\# \text{ of scenarios}] \cdot P\,(\text{single scenario})$$

Definition

The **factorial**, for any positive integer n, is

$$\begin{aligned}
0! &= 1 \\
1! &= 1 \\
2! &= 2 \cdot 1 = 2 \\
3! &= 3 \cdot 2 \cdot 1 = 6 \\
4! &= 4 \cdot 3 \cdot 2 \cdot 1 = 24 \\
&\vdots \\
n! &= n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 4 \cdot 3 \cdot 2 \cdot 1
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Note

Factorials can be calculated iteratively. i.e.

$$(n+1)! = n! \cdot (n+1)$$

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$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Read "n choose k."

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Binomial Distribution

Suppose the probability of a single trial being a success is p. Then the probability of observing exactly k successes in n independent trials is given by

$$\binom{n}{k} p^k (1-p)^{n-k} = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

The mean, variance, and standard deviation of the number of observed successes are

$$\mu = np, \qquad \sigma^2 = np(1-p), \qquad \sigma = \sqrt{np(1-p)}$$

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- The trials are independent.
- The number of trials, n, is fixed.
- Each trial outcome can be classified as either a *success* or *failure*.
- The probability of a success, p, is the same for each trial.

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From Example 1 we have $p = 0.85$, $n = 4$, and $k = 1$.

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$$\begin{aligned} P\,(\text{exactly one has heard of twitter}) &= \binom{n}{k} p^k (1-p)^{n-k} \\ &= \binom{4}{1} (0.85)^1 (1-0.85)^{4-1} \end{aligned}$$

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&= \frac{4!}{1!(4-1)!} (0.85)^1 (0.15)^3
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&= 4 \cdot (0.85)^1 (0.15)^3 \\
&= 0.11475
\end{aligned}$$

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&= \frac{8!}{5!(8-5)!} (0.7)^5 (0.3)^3 = \frac{8!}{5!(3)!} (0.7)^5 (0.3)^3 \\
&= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} (0.7)^5 (0.3)^3 \\
&= \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot 3 \cdot 2 \cdot 1} (0.7)^5 (0.3)^3
\end{aligned}$$

Example 4

Assume that 70% of customers won't exceed their car insurance deductible. Let's the probability that 5 of 8 randomly selected customers won't exceed their premium.

Start by identifying

$$p = 0.7, \qquad q = 1 - p = 0.3, \qquad n = 8, \qquad k = 5$$

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&= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} (0.7)^5 (0.3)^3 \\
&= \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot 3 \cdot 2 \cdot 1} (0.7)^5 (0.3)^3 \\
&= 56 \cdot (0.7)^5 (0.3)^3
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&= 56 \cdot (0.7)^5 (0.3)^3 \\
&= 0.254122
\end{aligned}$$

Example 5

Assume the probability that a smoker will develop a severe lung condition in their life time is 0.3.

If you have four friends who smoke, are the conditions for the binomial model satisfied?

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Example 6

Suppose instead four people are randomly selected.

Is the binomial model appropriate to find the probability that none of them will develop a severe lung condition?

We are assuming that the four are randomly selected, yes.

$$\binom{n}{k} p^k (1-p)^{n-k} = \binom{4}{0} (0.3)^0 (1-0.3)^{4-0} = \frac{4!}{0!(4-0)!} (0.3)^0 (0.7)^4$$
$$= 1 \cdot 1 \cdot (0.7)^4 = 0.2401$$

Example 7

Let consider finding the probability than no more than one of the four people will develop a severe lung condition.

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$$P\,(\text{none}) + P\,(\text{exactly one})$$
$$= \binom{4}{0}(0.3)^0(1-0.3)^{4-0} + \binom{4}{1}(0.3)^1(1-0.3)^{4-1}$$

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&\qquad = 0.2401 + 0.4116
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&\quad = 0.2401 + 0.4116 \\
&\quad = 0.6517 = 65.17\%
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Out of seven randomly selected smokers, how many would we expect to develop a severe lung condition?

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The mean of the binomial model is

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Out of seven randomly selected smokers, how many would we expect to develop a severe lung condition?

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$$\mu = np = 7 \cdot 0.3$$

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Out of seven randomly selected smokers, how many would we expect to develop a severe lung condition?

The mean of the binomial model is

$$\mu = np = 7 \cdot 0.3 = 2.1$$

On average, we would expect 2.1 of 7 randomly chosen smokers to develop a severe lung condition.