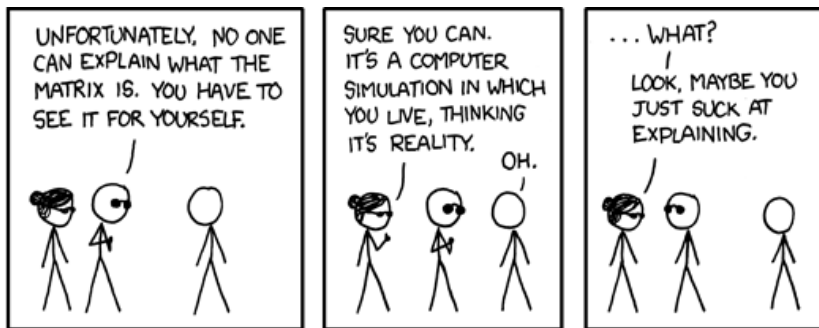


# Matrices: Sum and Products

Adam Wilson

Salt Lake Community College



Source: <http://xkcd.com/566/>

# Basic Terminology

## Matrix

A **matrix** is a rectangular array of **elements** or **entries** (numbers or functions) arranged in **rows** (horizontal) and **columns** (vertical).

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

The **order** of  $A$  is  $m \times n$ . If  $m = n$ , we call the matrix **square**.

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## Equal Matrices

Two matrices of the same order are **equal** if their corresponding entries are equal. If matrices  $A = [a_{ij}]$  and  $B = [b_{ij}]$  are both  $m \times n$ , then

$$A = B \Leftrightarrow a_{ij} = b_{ij}, \quad 1 \leq i \leq m, 1 \leq j \leq n$$

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- The  $n \times n$  **identity matrix**, denoted  $\mathbf{I}_n$  is:

$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

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## Matrix Addition

Two matrices of the same order are added (or subtracted) by adding (or subtracting) corresponding entries and recording the results in a matrix of the same size. Using matrix notation, if  $A = [a_{ij}]$  and  $B = [b_{ij}]$  are both  $m \times n$ .

$$A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$$

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## Multiplication by a Scalar

To find the product of a matrix and a scalar (a complex number), multiply each entry of the matrix by that number. This is called **multiplication by a scalar**. Using matrix notation, if  $A = [a_{ij}]$ , then

$$c \cdot A = [c \cdot a_{ij}] = [a_{ij} \cdot c] = A \cdot c$$



# Basic Properties

## Properties of Matrix Addition and Scalar Multiplication

Suppose  $A$ ,  $B$ , and  $C$  are  $m \times n$  matrices and  $c$  and  $k$  are scalars. Then the following properties hold:

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# Vectors

(are just tiny matrices)

A vector  $\vec{v} = \langle v_1, \dots, v_n \rangle$  can be represented by either by a  $1 \times n$  row matrix, or a  $n \times 1$  column matrix.



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## Vector addition and Scalar Multiplication

Let

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad \vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

be vectors in  $\mathbb{R}^n$  and  $c$  be any scalar. Then, we have:

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix} \quad \text{and} \quad c \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} c \cdot x_1 \\ \vdots \\ c \cdot x_n \end{bmatrix}$$

# Vector Properties

## Properties of Vector Addition and Multiplication

For vectors  $\vec{u}$ ,  $\vec{v}$ , and  $\vec{w}$  in  $\mathbb{R}^n$  and scalars  $c$  and  $k$ .

- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$  (Commutativity)
- $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$  (Associativity)
- $c(k\vec{v}) = (ck)\vec{v}$  (Associativity)
- $\vec{u} + \vec{0} = \vec{u}$  (Zero Element)
- $\vec{u} + (-\vec{u}) = \vec{0}$  (Inverse Element)
- $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$  (Distributivity)
- $(c + k)\vec{u} = c\vec{u} + k\vec{u}$  (Distributivity)

# Dot Product

(also called the Scalar Product)

## Dot Product

The **dot product** of a row vector  $\vec{x}$  and a column vector  $\vec{y}$  of equal length  $n$  is the result of adding the products of the corresponding entries as follows:

$$\begin{aligned}\vec{x} \cdot \vec{y} &= [x_1 \quad \cdots \quad x_n] \cdot \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \\ &= x_1 \cdot y_1 + x_2 \cdot y_2 + \cdots + x_n \cdot y_n \\ &= \sum_{k=1}^n x_k \cdot y_k\end{aligned}$$

# Dot Product

## Some Properties

### Orthogonality

Two vectors  $\vec{x}$  and  $\vec{y}$  in  $\mathbb{R}^n$  are called **orthogonal** when:

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### Magnitude

For any vector  $\vec{v}$  in  $\mathbb{R}^n$  the **length**, or **magnitude**, of  $\vec{v}$  is a nonnegative scalar, denoted by  $||\vec{v}||$  and defined to be

$$||\vec{v}|| = \sqrt{\vec{v} \cdot \vec{v}}$$

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### Unit Vectors

Vectors of length one are called **unit vectors**.

# Matrix Multiplication

## Matrix Product

The **matrix product** of a  $m \times r$  matrix  $A$  and a  $r \times n$  matrix  $B$  is denoted

$$C = A \cdot B = AB$$

where the  $ij$ th entry of  $C$  is the dot product of the  $i$ th row vector of  $A$  and the  $j$ th column vector of  $B$ :

$$\begin{aligned} c_{ij} &= [a_{i1} \quad a_{i2} \quad \cdots \quad a_{ir}] \cdot \begin{bmatrix} b_{1j} \\ \vdots \\ b_{rj} \end{bmatrix} \\ &= \sum_{k=1}^r a_{ik} b_{kj} \end{aligned}$$

The matrix  $C$  has order  $m \times n$ .

# Matrix Multiplication

## The Good Notation

### Example

Perform  $AB$  where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -4 \\ -1 & 0 \end{bmatrix}$$



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$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix}$	$\begin{bmatrix} \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \end{bmatrix}$

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	$\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$
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# Matrix Multiplication

## Properties

### Properties of Matrix Multiplication

- $(AB)C = A(BC)$  (Associativity)
- $A(B + C) = AB + AC$  (Distributivity)
- $(B + C)A = BA + CA$  (Distributivity)

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### Properties of Identity Matrices

For a  $m \times n$  matrix  $A$ :

- $A \cdot I_n = A$  and  $I_m \cdot A = A$
- $A \cdot \mathbf{0}_n = \mathbf{0}_{mn}$  and  $\mathbf{0}_m \cdot A = \mathbf{0}_{mn}$



# Matrix Transpose

## Transpose

For a matrix  $A = [a_{ij}]$  the **transpose** of the  $m \times n$  matrix  $A$  is defined as the  $n \times m$  matrix:

$$A^T = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T = \begin{bmatrix} & \\ & \end{bmatrix}$$

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# Matrices with Function Entries

Matrices can have functions as entries, not just real numbers.

## Example

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}(t) & a_{m1}(t) & \cdots & a_{mn}(t) \end{bmatrix}$$



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We say that a matrix is **continuous**, **piecewise continuous**, or **differentiable** if every element in the matrix has that property.

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For a differentiable matrix  $A$ , the derivative of  $A$  is defined as:

$$A'(t) = \frac{dA}{dt} = \begin{bmatrix} a'_{11}(t) & a'_{12}(t) & \cdots & a'_{1n}(t) \\ a'_{21}(t) & a'_{22}(t) & \cdots & a'_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a'_{m1}(t) & a'_{m1}(t) & \cdots & a'_{mn}(t) \end{bmatrix}$$

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## Matrix Differentiation Rules

For differentiable matrices  $A(t)$  and  $B(t)$  and scalar constant  $c$ .

- $(A(t) + B(t))' = A'(t) + B'(t)$
- $(cA(t))' = cA'(t)$
- $(A(t) \cdot B(t))' = A(t) \cdot B'(t) + A'(t) \cdot B(t)$

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## Examples

### Example

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \quad g'(t) = \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix}$$

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$$A(t) = \begin{bmatrix} e^t & t^2 \\ \sin t & 2t \end{bmatrix} \quad A'(t) = \begin{bmatrix} & \\ & \end{bmatrix}$$



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