Addition Rule and Multiplication Rule

Colby Community College

Definition

A **compound event** is any event combining two or more simple events.

Definition

A **compound event** is any event combining two or more simple events.

Definition

Events A and B are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time.

Definition

A **compound event** is any event combining two or more simple events.

Definition

Events A and B are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time.

Definition

Events A and B are **independent events** if the probability of B occurring is the same, whether or not A occurs.

Definition

A **compound event** is any event combining two or more simple events.

Definition

Events A and B are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time.

Definition

Events A and B are **independent events** if the probability of B occurring is the same, whether or not A occurs.

Example 1

Are these events independent?

1. A fair coin is tossed two times. The two events are (1) first toss is a head and (2) second toss is a head.

Definition

A **compound event** is any event combining two or more simple events.

Definition

Events A and B are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time.

Definition

Events A and B are **independent events** if the probability of B occurring is the same, whether or not A occurs.

Example 1

Are these events independent?

1. A fair coin is tossed two times. The two events are (1) first toss is a head and (2) second toss is a head. **Independent**

Definition

A **compound event** is any event combining two or more simple events.

Definition

Events A and B are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time.

Definition

Events A and B are **independent events** if the probability of B occurring is the same, whether or not A occurs.

Example 1

Are these events independent?

1. A fair coin is tossed two times. The two events are (1) first toss is a head and (2) second toss is a head. **Independent**
2. You draw a card from a deck, then without replacing the first, draw a second card?

Definition

A **compound event** is any event combining two or more simple events.

Definition

Events A and B are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time.

Definition

Events A and B are **independent events** if the probability of B occurring is the same, whether or not A occurs.

Example 1

Are these events independent?

1. A fair coin is tossed two times. The two events are (1) first toss is a head and (2) second toss is a head. **Independent**
2. You draw a card from a deck, then without replacing the first, draw a second card? **Dependent**

Multiplication Rule

If events A and B are independent, then the probability of both A and B occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Multiplication Rule

If events A and B are independent, then the probability of both A and B occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Example 2

Suppose we flip a fair coin and roll a fair die.

The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

Multiplication Rule

If events A and B are independent, then the probability of both A and B occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Example 2

Suppose we flip a fair coin and roll a fair die.

The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

The probability of getting tails on the coin and three on the die is

$$P(\text{T3}) = \frac{1}{12}$$

Multiplication Rule

If events A and B are independent, then the probability of both A and B occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Example 2

Suppose we flip a fair coin and roll a fair die.

The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

The probability of getting tails on the coin and three on the die is

$$P(\text{T3}) = \frac{1}{12}$$

We could have also calculated

$$P(\text{H and } 3) = P(\text{H}) \cdot P(3) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight?

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight? $P\,(\text{safe flight}) = 1 - P\,(\text{system fail}) = 1 - 0.002 = 0.998$

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight? $P\,(\text{safe flight}) = 1 - P\,(\text{system fail}) = 1 - 0.002 = 0.998$
- Given the Airbus 310 has three independent hydraulic systems, what is the probability that the flight control system would work for an entire flight?

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight? $P\,(\text{safe flight}) = 1 - P\,(\text{system fail}) = 1 - 0.002 = 0.998$
- Given the Airbus 310 has three independent hydraulic systems, what is the probability that the flight control system would work for an entire flight?
  $P\,(\text{safe flight}) = 1 - P\,(\text{system}_1 \text{ fail and system}_2 \text{ fail and system}_3 \text{ fail})$

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight? $P\,(\text{safe flight}) = 1 - P\,(\text{system fail}) = 1 - 0.002 = 0.998$
- Given the Airbus 310 has three independent hydraulic systems, what is the probability that the flight control system would work for an entire flight?

$$\begin{aligned} P\,(\text{safe flight}) &= 1 - P\,(\text{system}_1 \text{ fail and system}_2 \text{ fail and system}_3 \text{ fail}) \\ &= 1 - P\,(\text{system}_1 \text{ fail}) \cdot P\,(\text{system}_2 \text{ fail}) \cdot P\,(\text{system}_3 \text{ fail}) \end{aligned}$$

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight? $P\,(\text{safe flight}) = 1 - P\,(\text{system fail}) = 1 - 0.002 = 0.998$
- Given the Airbus 310 has three independent hydraulic systems, what is the probability that the flight control system would work for an entire flight?

$$\begin{aligned} P\,(\text{safe flight}) &= 1 - P\,(\text{system}_1 \text{ fail and system}_2 \text{ fail and system}_3 \text{ fail}) \\ &= 1 - P\,(\text{system}_1 \text{ fail}) \cdot P\,(\text{system}_2 \text{ fail}) \cdot P\,(\text{system}_3 \text{ fail}) \\ &= 1 - 0.002 \cdot 0.002 \cdot 0.002 \end{aligned}$$

Example 3

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight? $P\,(\text{safe flight}) = 1 - P\,(\text{system fail}) = 1 - 0.002 = 0.998$
- Given the Airbus 310 has three independent hydraulic systems, what is the probability that the flight control system would work for an entire flight?

$$\begin{aligned} P\,(\text{safe flight}) &= 1 - P\,(\text{system}_1 \text{ fail and system}_2 \text{ fail and system}_3 \text{ fail}) \\ &= 1 - P\,(\text{system}_1 \text{ fail}) \cdot P\,(\text{system}_2 \text{ fail}) \cdot P\,(\text{system}_3 \text{ fail}) \\ &= 1 - 0.002 \cdot 0.002 \cdot 0.002 \\ &= 1 - 0.000000008 = .999999992 \end{aligned}$$

Addition Rule

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Addition Rule

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example 4

Suppose we flip a fair coin and roll a fair die.

The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

We want to calculate the probability of getting a head or a six.

Addition Rule

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example 4

Suppose we flip a fair coin and roll a fair die.

The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

We want to calculate the probability of getting a head or a six.

The outcomes are: H1, H2, H3, H4, H5, H6, T6. Giving $P(\text{H or } 6) = \frac{7}{12}$.

Addition Rule

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example 4

Suppose we flip a fair coin and roll a fair die.

The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

We want to calculate the probability of getting a head or a six.

The outcomes are: H1, H2, H3, H4, H5, H6, T6. Giving $P(\text{H or } 6) = \frac{7}{12}$.

Notice that $\frac{6}{12} = \frac{1}{2}$ of the outcomes have heads and $\frac{2}{12} = \frac{1}{6}$ have a six.

Addition Rule

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example 4

Suppose we flip a fair coin and roll a fair die.

The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

We want to calculate the probability of getting a head or a six.

The outcomes are: H1, H2, H3, H4, H5, H6, T6. Giving $P(\text{H or 6}) = \frac{7}{12}$.

Notice that $\frac{6}{12} = \frac{1}{2}$ of the outcomes have heads and $\frac{2}{12} = \frac{1}{6}$ have a six.

But, $\frac{1}{2} + \frac{1}{6} = \frac{8}{12}$, is wrong because we have double counted H6. Thus, we need to subtract $P(\text{H6}) = \frac{1}{12}$.

$$P(\text{H or 6}) = P(\text{H}) + P(6) - P(\text{H and 6}) = \frac{1}{2} + \frac{1}{6} - \frac{1}{12} = \frac{7}{12}$$

Example 5

Suppose we draw one card from a standard deck. Let us find the probability that we get a Queen or a King.

Example 5

Suppose we draw one card from a standard deck. Let us find the probability that we get a Queen or a King.

There are 4 Queens and 4 Kings in the deck, hence eight outcomes out of 52 possible outcomes. So, the probability is

$$P\,(\text{Q or K}) = \frac{8}{52}$$

Example 5

Suppose we draw one card from a standard deck. Let us find the probability that we get a Queen or a King.

There are 4 Queens and 4 Kings in the deck, hence eight outcomes out of 52 possible outcomes. So, the probability is

$$P(\text{Q or K}) = \frac{8}{52}$$

Since there are no cards that are both Kings and Queens, we have

$$P(\text{Q or K}) = P(\text{Q}) + P(K) - P(\text{Q and K}) = \frac{4}{52} + \frac{4}{52} - \frac{0}{52} = \frac{8}{52}$$

Example 5

Suppose we draw one card from a standard deck. Let us find the probability that we get a Queen or a King.

There are 4 Queens and 4 Kings in the deck, hence eight outcomes out of 52 possible outcomes. So, the probability is

$$P\left(\text{Q or K}\right) = \frac{8}{52}$$

Since there are no cards that are both Kings and Queens, we have

$$P\left(\text{Q or K}\right) = P\left(\text{Q}\right) + P\left(K\right) - P\left(\text{Q and K}\right) = \frac{4}{52} + \frac{4}{52} - \frac{0}{52} = \frac{8}{52}$$

Note

If two events are **disjoint**, then $P\left(A \text{ or } B\right) = P\left(A\right) + P\left(B\right)$.

Example 6

Suppose we draw one card from a standard deck. Let us calculate the probability that we get a red card or a King.

Example 6

Suppose we draw one card from a standard deck. Let us calculate the probability that we get a red card or a King.

Half the cards are red, so $P(\text{Red}) = \dfrac{26}{52}$.

Example 6

Suppose we draw one card from a standard deck. Let us calculate the probability that we get a red card or a King.

Half the cards are red, so $P\left(\text{Red}\right) = \dfrac{26}{52}$.

Four cards are Kings, so $P\left(\text{K}\right) = \dfrac{4}{52}$.

Example 6

Suppose we draw one card from a standard deck. Let us calculate the probability that we get a red card or a King.

Half the cards are red, so $P\left(\text{Red}\right) = \dfrac{26}{52}$.

Four cards are Kings, so $P\left(\text{K}\right) = \dfrac{4}{52}$.

Two cards are red kings, so $P\left(\text{Red and K}\right) = \dfrac{2}{52}$.

Example 6

Suppose we draw one card from a standard deck. Let us calculate the probability that we get a red card or a King.

Half the cards are red, so $P\left(\text{Red}\right) = \dfrac{26}{52}$.

Four cards are Kings, so $P\left(\text{K}\right) = \dfrac{4}{52}$.

Two cards are red kings, so $P\left(\text{Red and K}\right) = \dfrac{2}{52}$.

Thus,

$$P\left(\text{Red or K}\right) = P\left(\text{Red}\right) + P\left(\text{K}\right) - P\left(\text{Red and K}\right) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{7}{13}$$

Example 7

What is the probability that two cards drawn at random from a deck will both be aces?

Example 7

What is the probability that two cards drawn at random from a deck will both be aces?

You might guess that since there are four aces in the deck of 52 cards, the probability would be $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$.

Example 7

What is the probability that two cards drawn at random from a deck will both be aces?

You might guess that since there are four aces in the deck of 52 cards, the probability would be $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$.

The problem here is that the events are not independent. Once one card is drawn, there are only 51 cards remaining. Once an ace has been drawn, there are only three remaining.

Example 7

What is the probability that two cards drawn at random from a deck will both be aces?

You might guess that since there are four aces in the deck of 52 cards, the probability would be $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$.

The problem here is that the events are not independent. Once one card is drawn, there are only 51 cards remaining. Once an ace has been drawn, there are only three remaining.

This means that the the probability of drawing two aces is $\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$.

Example 7

What is the probability that two cards drawn at random from a deck will both be aces?

You might guess that since there are four aces in the deck of 52 cards, the probability would be $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$.

The problem here is that the events are not independent. Once one card is drawn, there are only 51 cards remaining. Once an ace has been drawn, there are only three remaining.

This means that the the probability of drawing two aces is $\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$.

Definition

The probability the event B occurs, given that event A has happened, is represented as $P(B \mid A)$. This is called a **conditional probability**.

Read as "the probability of B given A."

Example 8

Car color	Speeding ticket	No speeding ticket	Total
Red	15	135	150
Not red	45	470	515
Total	60	605	665

Find the probability someone has gotten a speeding ticket *given* they drive a red car.

Example 8

Car color	Speeding ticket	No speeding ticket	Total
Red	15	135	150
Not red	45	470	515
Total	60	605	665

Find the probability someone has gotten a speeding ticket *given* they drive a red car.

$$P\,(\text{ticket} \mid \text{red}) = \frac{15}{150} = \frac{1}{10} = 10\%$$

Example 8

Car color	Speeding ticket	No speeding ticket	Total
Red	15	135	150
Not red	45	470	515
Total	60	605	665

Find the probability someone has gotten a speeding ticket *given* they drive a red car.

$$P\left(\text{ticket} \mid \text{red}\right) = \frac{15}{150} = \frac{1}{10} = 10\%$$

Find the probability someone drives a red car *given* they have gotten a speeding ticker.

Example 8

Car color	Speeding ticket	No speeding ticket	Total
Red	15	135	150
Not red	45	470	515
Total	60	605	665

Find the probability someone has gotten a speeding ticket *given* they drive a red car.

$$P\,(\text{ticket} \mid \text{red}) = \frac{15}{150} = \frac{1}{10} = 10\%$$

Find the probability someone drives a red car *given* they have gotten a speeding ticker.

$$P\,(\text{red} \mid \text{ticket}) = \frac{15}{60} = \frac{1}{4} = 25\%$$

Example 8

Car color	Speeding ticket	No speeding ticket	Total
Red	15	135	150
Not red	45	470	515
Total	60	605	665

Find the probability someone has gotten a speeding ticket *given* they drive a red car.

$$P(\text{ticket} \mid \text{red}) = \frac{15}{150} = \frac{1}{10} = 10\%$$

Find the probability someone drives a red car *given* they have gotten a speeding ticker.

$$P(\text{red} \mid \text{ticket}) = \frac{15}{60} = \frac{1}{4} = 25\%$$

Note

In general $P(B \mid A) \neq P(A \mid B)$.

Multiplication Rule

If A and B are not independent, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

Multiplication Rule

If A and B are not independent, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

Example 9

If you pull two cards out of a deck, find the probability that both are hearts.

Multiplication Rule

If A and B are not independent, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

Example 9

If you pull two cards out of a deck, find the probability that both are hearts.

The probability that the first card is a heart is $P(1^{\text{st}} \heartsuit) = \dfrac{13}{52}$.

Multiplication Rule

If A and B are not independent, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

Example 9

If you pull two cards out of a deck, find the probability that both are hearts.

The probability that the first card is a heart is $P(1^{\text{st}}\ \heartsuit) = \dfrac{13}{52}$.

The probability that the second card is a heart, given that the first card was a heart, is $P\left(2^{\text{nd}}\ \heartsuit \mid 1^{\text{st}}\ \heartsuit\right) = \dfrac{12}{51}$.

Multiplication Rule

If A and B are not independent, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

Example 9

If you pull two cards out of a deck, find the probability that both are hearts.

The probability that the first card is a heart is $P\left(1^{\text{st}} \heartsuit\right) = \dfrac{13}{52}$.

The probability that the second card is a heart, given that the first card was a heart, is $P\left(2^{\text{nd}} \heartsuit \mid 1^{\text{st}} \heartsuit\right) = \dfrac{12}{51}$.

So, the probability that both are hearts is

$$P(\text{both } \heartsuit) = P\left(1^{\text{st}} \heartsuit\right) \cdot P\left(2^{\text{nd}} \heartsuit \,\middle|\, 1^{\text{st}} \heartsuit\right) = \frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} \approx 5.9\%$$

Example 10

If you draw two cards from a deck, find the probability that you will get the Ace of Diamonds and a black card.

Example 10

If you draw two cards from a deck, find the probability that you will get the Ace of Diamonds and a black card.

The order you draw the two cards doesn't matter, so there are two events:

Example 10

If you draw two cards from a deck, find the probability that you will get the Ace of Diamonds and a black card.

The order you draw the two cards doesn't matter, so there are two events:

Event A Drawing the Ace of Diamonds then drawing a black card.

$$\begin{aligned} P(\text{A}\blacklozenge \text{ and Black}) &= P(\text{A}\blacklozenge) \cdot P(\text{Black} \mid \text{A}\blacklozenge) \\ &= \frac{1}{52} \cdot \frac{26}{51} = \frac{1}{102} \end{aligned}$$

Example 10

If you draw two cards from a deck, find the probability that you will get the Ace of Diamonds and a black card.

The order you draw the two cards doesn't matter, so there are two events:

Event A Drawing the Ace of Diamonds then drawing a black card.

$$\begin{aligned} P(\text{A}\blacklozenge \text{ and Black}) &= P(\text{A}\blacklozenge) \cdot P(\text{Black} \mid \text{A}\blacklozenge) \\ &= \frac{1}{52} \cdot \frac{26}{51} = \frac{1}{102} \end{aligned}$$

Event B Drawing a black card then drawing the Ace of Diamonds.

$$\begin{aligned} P(\text{Black and A}\blacklozenge) &= P(\text{Black}) \cdot P(\text{A}\blacklozenge \mid \text{Black}) \\ &= \frac{26}{52} \cdot \frac{1}{51} = \frac{1}{102} \end{aligned}$$

Example 10

If you draw two cards from a deck, find the probability that you will get the Ace of Diamonds and a black card.

The order you draw the two cards doesn't matter, so there are two events:

Event A Drawing the Ace of Diamonds then drawing a black card.

$$\begin{aligned} P\left(\text{A}\blacklozenge \text{ and Black}\right) &= P\left(\text{A}\blacklozenge\right) \cdot P\left(\text{Black} \mid \text{A}\blacklozenge\right) \\ &= \frac{1}{52} \cdot \frac{26}{51} = \frac{1}{102} \end{aligned}$$

Event B Drawing a black card then drawing the Ace of Diamonds.

$$\begin{aligned} P\left(\text{Black and A}\blacklozenge\right) &= P\left(\text{Black}\right) \cdot P\left(\text{A}\blacklozenge \mid \text{Black}\right) \\ &= \frac{26}{52} \cdot \frac{1}{51} = \frac{1}{102} \end{aligned}$$

These events are independent and mutually exclusive, so

$$P\left(A \text{ or } B\right) = P\left(A\right) + P\left(B\right) = \frac{1}{102} + \frac{1}{102} = \frac{2}{102} \approx 1.96\%$$

Sampling

Sampling methods are critically important, and the following relationships hold:

- Sampling *with replacement*: Selections are *independent* events.
- Sampling *without replacement*: Selections are *dependent* events.

Sampling

Sampling methods are critically important, and the following relationships hold:

- Sampling *with replacement*: Selections are *independent* events.
- Sampling *without replacement*: Selections are *dependent* events.

Note

Calculations involving conditional probabilities can often be very cumbersome.

Sampling

Sampling methods are critically important, and the following relationships hold:

- Sampling *with replacement*: Selections are *independent* events.
- Sampling *without replacement*: Selections are *dependent* events.

Note

Calculations involving conditional probabilities can often be very cumbersome.

Treating Dependent Events as Independent

When sampling without replacement and the sample size is no more than 5% of the size of the population, treat the selections as being independent (even though they are actually dependent).

Example 11

Assume that three adults are randomly selected without replacement from the $247,436,830$ adults in the United States. If we assume that 10% of adults use drugs, what is the probability that the three selected adults all use drugs?

Example 11

Assume that three adults are randomly selected without replacement from the 247, 436, 830 adults in the United States. If we assume that 10% of adults use drugs, what is the probability that the three selected adults all use drugs?

Because the three adults are randomly selected without replacement, the three events are dependent. This means the exact probability would be rather cumbersome.

$$\begin{aligned} P\left(\text{all use drugs}\right) &= P\left(\text{first use drugs and second use drugs and third use drugs}\right) \\ &= \left(\frac{24,743,683}{247,436,830}\right) \cdot \left(\frac{24,743,682}{247,436,829}\right) \cdot \left(\frac{24,743,681}{247,436,828}\right) \\ &= 0.0009999998909 \end{aligned}$$

Example 11

Assume that three adults are randomly selected without replacement from the $247,436,830$ adults in the United States. If we assume that 10% of adults use drugs, what is the probability that the three selected adults all use drugs?

Because the three adults are randomly selected without replacement, the three events are dependent. This means the exact probability would be rather cumbersome.

$$\begin{aligned} P\,(\text{all use drugs}) &= P\,(\text{first use drugs and second use drugs and third use drugs}) \\ &= \left(\frac{24,743,683}{247,436,830}\right)\cdot\left(\frac{24,743,682}{247,436,829}\right)\cdot\left(\frac{24,743,681}{247,436,828}\right) \\ &= 0.0009999998909 \quad \text{(Imagine selecting 10,000 adults!)} \end{aligned}$$

Example 11

Assume that three adults are randomly selected without replacement from the $247,436,830$ adults in the United States. If we assume that 10% of adults use drugs, what is the probability that the three selected adults all use drugs?

Because the three adults are randomly selected without replacement, the three events are dependent. This means the exact probability would be rather cumbersome.

$$\begin{aligned} P\left(\text{all use drugs}\right) &= P\left(\text{first use drugs and second use drugs and third use drugs}\right) \\ &= \left(\frac{24,743,683}{247,436,830}\right)\cdot\left(\frac{24,743,682}{247,436,829}\right)\cdot\left(\frac{24,743,681}{247,436,828}\right) \\ &= 0.0009999998909 \quad \text{(Imagine selecting 10,000 adults!)} \end{aligned}$$

Since 5 adults is less that 5% of the total population, we can simplify the calculations considerably.

$$\begin{aligned} P\left(\text{all use drugs}\right) &= P\left(\text{first use drugs and second use drugs and third use drugs}\right) \\ &= 0.1 \cdot 0.1 \cdot 0.1 = 0.00100 \end{aligned}$$

Example 12

When two different people are randomly selected from those at your school, we can assume that birthdays occur on the days of the week with equal frequencies.

Find the probability that two people are born on the same day of the week:

Example 12

When two different people are randomly selected from those at your school, we can assume that birthdays occur on the days of the week with equal frequencies.

Find the probability that two people are born on the same day of the week:

Because no particular day of the week is specified, the first person can be born on any day of the week. This means the probability that the second person is born on the same day is 1/7.

Example 12

When two different people are randomly selected from those at your school, we can assume that birthdays occur on the days of the week with equal frequencies.

Find the probability that two people are born on the same day of the week:

Because no particular day of the week is specified, the first person can be born on any day of the week. This means the probability that the second person is born on the same day is 1/7.

Find the probability that both people are born on a Monday:

Example 12

When two different people are randomly selected from those at your school, we can assume that birthdays occur on the days of the week with equal frequencies.

Find the probability that two people are born on the same day of the week:

Because no particular day of the week is specified, the first person can be born on any day of the week. This means the probability that the second person is born on the same day is 1/7.

Find the probability that both people are born on a Monday:

The probability that the any person is born on a Monday is 1/7. Since birth dates are independent, the probability that both are born on Monday is

$$\frac{1}{7} \cdot \frac{1}{7} = \frac{1}{49}$$

Example 12

When two different people are randomly selected from those at your school, we can assume that birthdays occur on the days of the week with equal frequencies.

Find the probability that two people are born on the same day of the week:

Because no particular day of the week is specified, the first person can be born on any day of the week. This means the probability that the second person is born on the same day is 1/7.

Find the probability that both people are born on a Monday:

The probability that the any person is born on a Monday is 1/7. Since birth dates are independent, the probability that both are born on Monday is

$$\frac{1}{7} \cdot \frac{1}{7} = \frac{1}{49}$$

Caution

In any probability calculation, it is very important to carefully identify the event being considered.