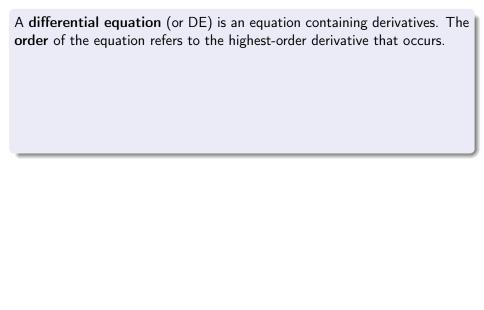
Solution and Direction Fields: Qualitative Analysis

Department of Mathematics

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In this chapter we will focus on DEs that can be written as:

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 or $y' = f(t, y)$

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Analytic Definition of a Solution

Analytically, y(t) is a **solution** of a differential equation if substituting y(t) for y reduced the equation to an identity:

$$y'(t) = f(t, y(t))$$

on an appropriate domain for t.

Verify that y(t) is a solution to the DE.

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Substituting into the DE gives:

$$y'(t) = \frac{d}{dt}e^{2t}$$

$$= 2e^{2t}$$

$$= 2y(t)$$

$$= f(t, y(t))$$

Similarly, we could show that

$$y(t) = 2e^{2t}$$
 and $y(t) = \frac{-3}{2}e^{2t}$

are also solutions. In fact, any constant multiple of e^{2t} is a solution.

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$$y$$

$$y = 2e^{2t}$$

$$y = e^{2t}$$

$$y = \frac{3}{2}e^{2t}$$