Measures of Relative Standing and Boxplots

Colby Community College

Definition

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- A data value is *significantly high* if $z \geq 2$.
- If a data value is less than the mean, its z-score will be negative.

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The weights of a sample of 400 newborn baby weights has mean $\bar{x} = 3152.0$ g and standard deviation $s = 693.4$ g. What is the z-score of a 4000 g baby?

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Rounding

Round z-scores to two decimal places.

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Round percentiles to the nearest whole number.

Example 3

The table lists the 50 Verizon airport data speeds, in Mbps, from Data Set 32 in Appendix B.

38.5	55.6	22.4	14.1	23.1	24.5	6.5	21.5	25.7	14.7
77.8	71.3	43.0	20.2	15.5	13.7	11.1	13.5	10.2	21.1
15.1	14.2	4.5	7.9	9.9	10.3	6.2	17.5	22.2	13.1
18.2	28.5	15.8	15.0	11.1	11.8	16.0	10.9	1.8	34.6
4.6	12.0	11.6	3.6	1.9	7.7	0.8	4.5	1.4	3.2

What percentile is the data value 11.8 Mbps in?

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Note

This can be interpreted loosely as 40% of Verizon data speeds are slower than 11.8 Mbps and 60% of Verizon data speeds are faster than 11.8 Mbps.

Converting a Percentile to a Data Value

Notation:

- n is the total number of values in the data set.
- k is the percentile being used.
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   - If L is a whole number, the value of the kth percentile is midway between the Lth value and the next value in the sorted data. Add the Lth value and $(L+1)$th value, then divide by 2.
   - If L is not a whole number, round L up to the nearest whole number. $P_k$is the Lth data value.

Example 4

The table lists the 50 Verizon airport data speeds, in Mbps, from Data Set 32 in Appendix B.

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What is the value in the 25th percentile, P_{25}?

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11.8	12.0	13.1	13.5	13.7	14.1	14.2	14.7	15.0	15.1
15.5	15.8	16.0	17.5	18.2	20.2	21.1	21.5	22.2	22.4
23.1	24.5	25.7	28.5	34.6	38.5	43.0	55.6	71.3	77.8

What is the value in the 25th percentile, P_{25}?
First, sort the data.

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What is the value in the 25th percentile, P_{25}?

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$$L = \frac{k}{100} \cdot n$$

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What is the value in the 25th percentile, P_{25}?
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Since $L = 12.5$ is not a whole number, we round up to 13.
So, P_{25} is the 13th data value, 7.9 Mbps.

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Note

Use the same procedure for calculating percentiles to calculate quartiles.