Sampling From A Small Population

Colby Community College

Note

While we usually sample from a much larger population, there are times where our sample size is large enough or the population small enough that we sample more that 10% of a population without replacement.

Example 1

Teachers sometimes select a student at random to answer a question. We assume each student has an equal chance of being selected and there are 15 students in the class.

What is the chance you will be picked for the next question?

Probability is $\frac{1}{5} \approx 0.067$.

Example 2

If the teacher asks 3 questions, what is the probability that you will not be selected? (Assume that she only picks a student once.)

For any single question, if you are not picked, then she picked one of the other students.

Using the General Multiplication rule we get:

$$
\begin{aligned}
&P(\text{not picked in 3 questions}) \\
&\quad = P(\texttt{Q1} \text{ is } \texttt{not picked} \text{ and } \texttt{Q2} \text{ is } \texttt{not picked} \text{ and } \texttt{Q3} \text{ is } \texttt{not picked}) \\
&\quad = P(\texttt{Q3} \text{ is } \texttt{not picked} \mid \texttt{Q1} \text{ is } \texttt{not picked} \text{ and } \texttt{Q2} \text{ is } \texttt{not picked}) \\
&\qquad \times P(\texttt{Q1} \text{ is } \texttt{not picked} \text{ and } \texttt{Q2} \text{ is } \texttt{not picked}) \\
&\quad = P(\texttt{Q3} \text{ is } \texttt{not picked} \mid \texttt{Q1} \text{ is } \texttt{not picked} \text{ and } \texttt{Q2} \text{ is } \texttt{not picked}) \\
&\qquad \times P(\texttt{Q2} \text{ is } \texttt{not picked} \mid \texttt{Q1} \text{ is } \texttt{not picked}) \times P(\texttt{Q1} \text{ is } \texttt{not picked}) \\
&\quad = \frac{12}{13} \cdot \frac{13}{14} \cdot \frac{14}{15} = 0.80
\end{aligned}
$$

So, there is a 80% chance you won't be picked.

Example 3

If the teacher asks 3 questions without regard to who she has already selected, what is the probability that you will not be selected?

If she is willing to selected the same student twice, then the questions become independent events and the calculations become easier.

Using the Multiplication Rule for Independent Events we get:

$$\begin{aligned} &P(\text{not picked in 3 questions}) \\ &\quad = P(\texttt{Q1} \text{ is } \textit{not picked} \text{ and } \texttt{Q2} \text{ is } \textit{not picked} \text{ and } \texttt{Q3} \text{ is } \textit{not picked}) \\ &\quad = P(\texttt{Q1} \text{ is } \textit{not picked}) \cdot P(\texttt{Q2} \text{ is } \textit{not picked}) \cdot P(\texttt{Q3} \text{ is } \textit{not picked}) \\ &\quad = \frac{14}{15} \cdot \frac{14}{15} \cdot \frac{14}{15} = 0.813 \end{aligned}$$

So, there is a 81.3% chance you won't be picked.

Note

Notice that this is different than the 80% chance of not being picked when she was picking without replacement.

Example 4

Your department is holding a raffle. They sell 30 tickets and offer seven prizes. They place the tickets in a hat and draw one for each prize, without replacing the winning tickets.

What is your chance of winning a prize if you buy one ticket?

This sampling is without replacement, so the events are not independent and we have to use the General Multiplication Rule.

$$
\begin{aligned}
P(\text{win at least one prize}) &= 1 - P(\text{win no prizes}) \\
&= 1 - \frac{29}{30} \cdot \frac{28}{29} \cdot \frac{27}{28} \cdot \frac{26}{27} \cdot \frac{25}{26} \cdot \frac{24}{25} \cdot \frac{23}{24} \\
&= 1 - \frac{29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23}{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24} \\
&= 1 - \frac{\cancel{29} \cdot \cancel{28} \cdot \cancel{27} \cdot \cancel{26} \cdot \cancel{25} \cdot \cancel{24} \cdot 23}{30 \cdot \cancel{29} \cdot \cancel{28} \cdot \cancel{27} \cdot \cancel{26} \cdot \cancel{25} \cdot \cancel{24}} \\
&= 1 - \frac{23}{30} = \frac{7}{30} \approx 0.233
\end{aligned}
$$

Example 5

Your department is holding a raffle. They sell 30 tickets and offer seven prizes. They place the tickets in a hat and draw one for each prize, with replacing the winning tickets.

What is your chance of winning a prize if you buy one ticket?

This sampling is with replacement, so the events are independent and we have to use the Multiplication Rule for Independent Events.

$$\begin{aligned} P(\text{win at least one prize}) &= 1 - P(\text{win no prizes}) \\ &= 1 - \frac{29}{30} \cdot \frac{29}{30} \cdot \frac{29}{30} \cdot \frac{29}{30} \cdot \frac{29}{30} \cdot \frac{29}{30} \cdot \frac{29}{30} \\ &= 1 - \left(\frac{29}{30}\right)^7 \approx 0.211 \end{aligned}$$

Note

The chances of winning a prize when sampling without replacement almost 10% larger than when sampling with replacement.