Confidence Intervals for a Proportion

Colby Community College

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Note

We have no indication of how *good* of an estimate 0.43 is, just that it is the best of the available options.

Definition

A **confidence interval** is a range of values around the point estimate used to estimate the true value of a population parameter.

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Round the confidence interval limits to three significant digits.

The Process Success Rate

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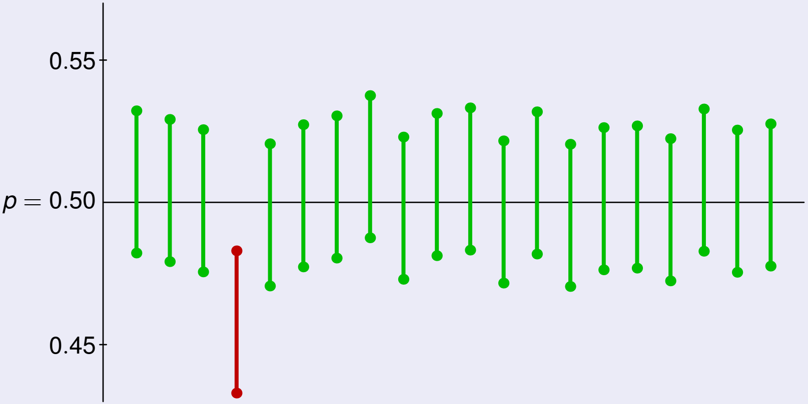
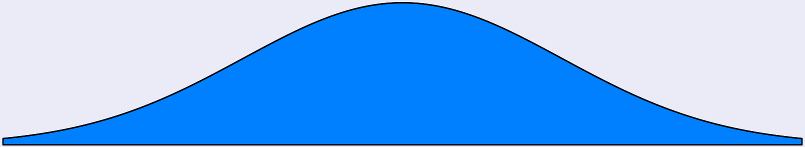
If the true population proportion is $p = 0.5$, then we expect around 19 of 20 confidence intervals to contain the true value of p.



A Few Observations



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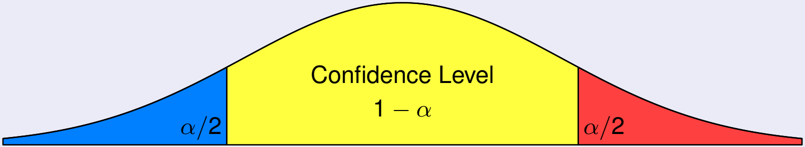

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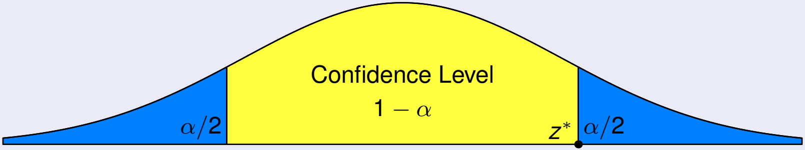

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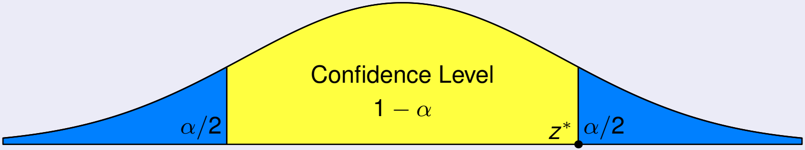

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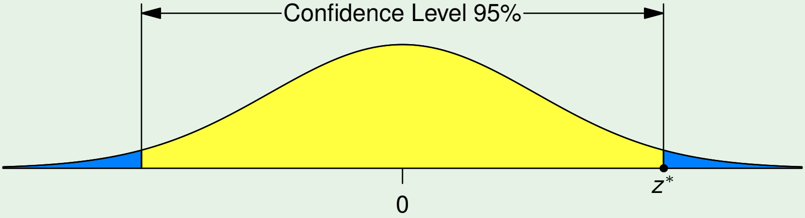
Definition

The value z^* is called a **critical value**.

Example 2

Let us find the critical value z^* corresponding to a 95% confidence level.



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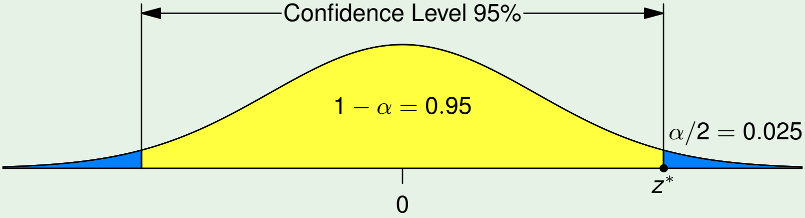
A 95% confidence interval gives $\alpha = 0.05$ and $\alpha/2 = 0.025$.



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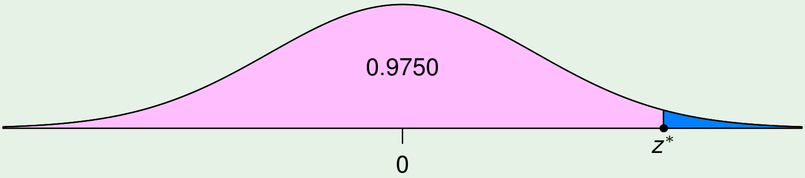
To find the z value using the inverse normal distribution, we need to know the cumulative area to the left of the right tail, $0.025 + 0.95 = 0.9750$.



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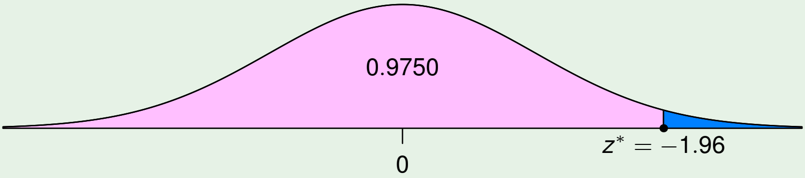
Using technology we get

$$z^* = 1.96$$



Common Confidence Levels

Confidence Level	α	Critical Value
90%	0.10	1.645
95%	0.05	1.960
99%	0.01	2.575