Basics of Hypothesis Testing

Colby Community College

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Note

the "property of a population" is often the value of a population parameter.

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Using technology we have

$$P\,(545 \text{ or more consumers}) \approx 0.005386$$

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Notation

The null hypotheses is denoted by $\boldsymbol{H_0}$.

The alternative hypotheses is denoted $\boldsymbol{H_1}$ or $\boldsymbol{H_a}$ or $\boldsymbol{H_A}$.

Example 2

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Example 4

Returning to Example 1, for

"the majority of consumers are not comfortable with drone deliveries."

we have the hypotheses:

$$H_0: \; p = 0.5$$
$$H_A: \; p > 0.5$$

Note

If you are conducting a study and want to use a hypothesis test to *support* your claim, your claim must be worded such that it becomes the alternative hypothesis and can be expressed using only the symbols $>$, $<$, or $\neq$.

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Caution

You *never* support a claim that a parameter is equal to a specified value.

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$$\text{Significance level } \alpha = P\left(\text{rejecting } H_0 \text{ when } H_0 \text{ is true}\right)$$

Note

The significance level α is the same α we talked about in Chapter 7, when discussing confidence intervals.

Definition

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Test Statistic for Proportion p

Sampling Distribution: Normal (z)

Requirements: $np \geq 5$ and $nq \geq 5$

Test Statistic: $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}}$

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Which means we are working form the assumption that $p = 0.5$ and so $q = 1 - p = 0.5$.

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$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.540 - 0.5}{\sqrt{\frac{0.5 \cdot 0.5}{1009}}} = 2.54$$

Test Statistic for Mean μ

Sampling Distribution: Student t

Requirements: Both of the following:

- σ not known.
- Normally distributed or $n > 30$.

Test Statistic: $t = \dfrac{\bar{x} - \mu}{\dfrac{s}{\sqrt{n}}}$

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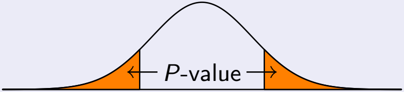
Caution

Be careful not to confuse the notation.

***P*-value**	The probability of a test statistic at least as extreme as the one obtained.
$\boldsymbol{p}$	The population proportion.
$\boldsymbol{\hat{p}}$	The sample proportion.

Two-tailed Test ($H_A : \neq$)

The critical region is in the two extreme regions under the curve.



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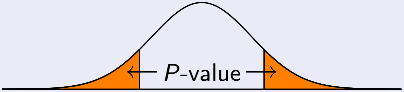
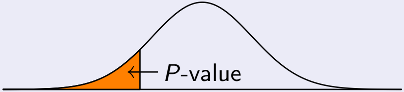


Left-tailed Test ($H_A : <$)

The critical region is in the extreme left region under the curve.



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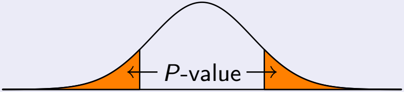

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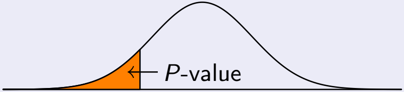
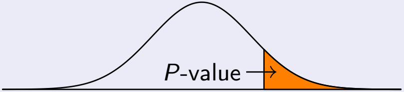


Right-tailed Test ($H_A: >$)

The critical region is in the extreme right region under the curve.



Decision Criteria

- If P-value $\leq \alpha$, reject H_0.
- If P-value $\geq \alpha$, fail to reject H_0.

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The alternative hypothesis is $H_A: \ p > 0.5$, so this is a right-tailed test.

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If we are working with a significance level $\alpha = 0.05$, so we reject the null hypothesis.

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Note

Technology will compute P-values for you.

Restate the Decision Using Nontechnical Terms

After you have decided to reject or not reject the null hypothesis, you need to restate the decision in terms that a layperson can understand.

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Example 7

In Example 1 we restate the decision to reject the null hypothesis as:

"There is sufficient evidence to support the claim that the majority of consumers are uncomfortable with drone deliveries."

Helpful Wording

Original claim does not include equality and you reject H_0:

> "There is sufficient evidence to support the claim that . . . (claim)."

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Caution

We say "fail to reject the null hypothesis" instead of "accept the null hypothesis."

Procedure for Hypothesis Tests Flow Chart

Page 360 in your textbook contains a summary of all the steps.

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Note

A confidence interval estimate of a population parameter contains the likely values of that parameter.

We should therefore reject a claim that the population parameter has a value that is not included in the confidence interval.

Definition

A **type I error** is the mistake of rejecting the null hypothesis when it is actually true.

The symbol α is used to represent the probability of a type I error.

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A **type II error** is the mistake of failing to reject the null hypothesis when it is actually false.

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Describing Type I and Type II Errors

When wording a statement representing a type I / II error, be sure that the conclusion addresses the original claim, which may or may not be H_0.

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$$H_A: \ p > 0.5$$

what is a statement that describes a type I error?

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In reality $p = 0.5$, but sample evidence leads us to conclude the $p > 0.5$. That is, we conclude that the medical procedure is effective when it reality it has no effect.

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What is a statement that describes a type II error?

In reality $p > 0.5$, but we fail to support that conclusion. That is, we conclude that the medical procedure has no effect, when it really is effective in increasing the likelihood of a baby girl.