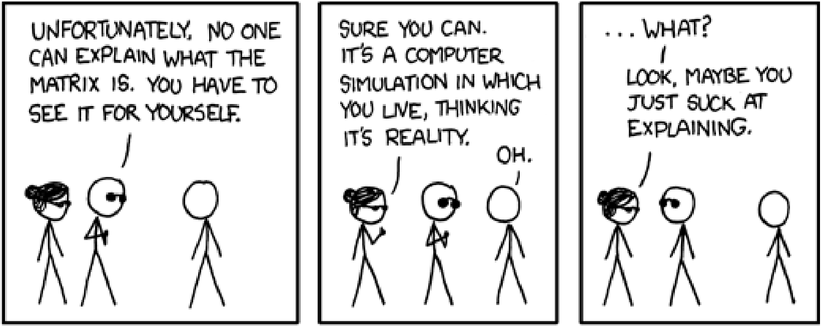
Matrices: Sum and Products

Department of Mathematics

Salt Lake Community College



Matrix

A **matrix** is a rectangular array of **elements** or **entries** (numbers or functions) arranged in **rows** (horizontal) and **columns** (vertical).

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

The **order** of A is $m \times n$. If $m = n$, we call the matrix **square**.

Matrix

A **matrix** is a rectangular array of **elements** or **entries** (numbers or functions) arranged in **rows** (horizontal) and **columns** (vertical).

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

The **order** of A is $m \times n$. If $m = n$, we call the matrix **square**.

Equal Matrices

Two matrices of the same order are **equal** if their corresponding entries are equal. If matrices $A = [a_{ij}]$ and $B = [a_{ij}]$ are both $m \times n$, then

$$A = B \Leftrightarrow a_{ij} = b_{ij}, \quad 1 \leq i \leq m,\ 1 \leq j \leq n$$

Special Matrices

- The $m \times n$ **zero matrix**, denoted $\mathbf{0}_{mn}$, has all its entries equal to zero.

Special Matrices

- The $m \times n$ **zero matrix**, denoted $\mathbf{0}_{mn}$, has all its entries equal to zero.
- A **diagonal matrix** is:

$$D = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{mn} \end{bmatrix}$$

Special Matrices

- The $m \times n$ **zero matrix**, denoted $\mathbf{0}_{mn}$, has all its entries equal to zero.
- A **diagonal matrix** is:

$$D = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{mn} \end{bmatrix}$$

- The $n \times n$ **identity matrix**, denoted $\mathbf{I}_n$ is:

$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Matrix Addition

Two matrices of the same order are added (or subtracted) by adding (or subtracting) corresponding entries and recording the results in a matrix of the same size. Using matrix notation, if $A = [a_{ij}]$ and $B = [b_{ij}]$ are both $m \times n$.

$$A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$$
$$A - B = [a_{ij}] - [b_{ij}] = [a_{ij} - b_{ij}]$$

Matrix Addition

Two matrices of the same order are added (or subtracted) by adding (or subtracting) corresponding entries and recording the results in a matrix of the same size. Using matrix notation, if $A = [a_{ij}]$ and $B = [b_{ij}]$ are both $m \times n$.

$$A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$$
$$A - B = [a_{ij}] - [b_{ij}] = [a_{ij} - b_{ij}]$$

Multiplication by a Scalar

To find the product of a matrix and a scalar (a complex number), multiply each entry of the matrix by that number. This is called **multiplication by a scalar**. Using matrix notation, if $A = [a_{ij}]$, then

$$c \cdot A = [c \cdot a_{ij}] = [a_{ij} \cdot c] = A \cdot c$$

Properties of Matrix Addition and Scalar Multiplication

Suppose A, B, and C are $m \times n$ matrices and c and k are scalars. Then the following properties hold:

- $A + B = B + A$ (Commutativity)

Properties of Matrix Addition and Scalar Multiplication

Suppose A, B, and C are $m \times n$ matrices and c and k are scalars. Then the following properties hold:

- $A + B = B + A$ (Commutativity)
- $A + (B + C) = (A + B) + C$ (Associativity)

Properties of Matrix Addition and Scalar Multiplication

Suppose A, B, and C are $m \times n$ matrices and c and k are scalars. Then the following properties hold:

- $A + B = B + A$ (Commutativity)
- $A + (B + C) = (A + B) + C$ (Associativity)
- $c(kA) = (ck)A$ (Associativity)

Properties of Matrix Addition and Scalar Multiplication

Suppose A, B, and C are $m \times n$ matrices and c and k are scalars. Then the following properties hold:

- $A + B = B + A$ (Commutativity)
- $A + (B + C) = (A + B) + C$ (Associativity)
- $c(kA) = (ck)A$ (Associativity)
- $A + \mathbf{0} = A$ (Zero Element)

Properties of Matrix Addition and Scalar Multiplication

Suppose A, B, and C are $m \times n$ matrices and c and k are scalars. Then the following properties hold:

- $A + B = B + A$ (Commutativity)
- $A + (B + C) = (A + B) + C$ (Associativity)
- $c(kA) = (ck)A$ (Associativity)
- $A + \mathbf{0} = A$ (Zero Element)
- $A + (-A) = \mathbf{0}$ (Inverse Element)

Properties of Matrix Addition and Scalar Multiplication

Suppose A, B, and C are $m \times n$ matrices and c and k are scalars. Then the following properties hold:

- $A + B = B + A$ (Commutativity)
- $A + (B + C) = (A + B) + C$ (Associativity)
- $c(kA) = (ck)A$ (Associativity)
- $A + \mathbf{0} = A$ (Zero Element)
- $A + (-A) = \mathbf{0}$ (Inverse Element)
- $c(A + B) = cA + cB$ (Distributivity)

Properties of Matrix Addition and Scalar Multiplication

Suppose A, B, and C are $m \times n$ matrices and c and k are scalars. Then the following properties hold:

- $A + B = B + A$ (Commutativity)
- $A + (B + C) = (A + B) + C$ (Associativity)
- $c(kA) = (ck)A$ (Associativity)
- $A + \mathbf{0} = A$ (Zero Element)
- $A + (-A) = \mathbf{0}$ (Inverse Element)
- $c(A + B) = cA + cB$ (Distributivity)
- $(c + k)A = cA + kA$ (Distributivity)

Vectors (are just tiny matrices)

A vector $\vec{\boldsymbol{v}} =< v_1, \ldots, v_n >$ can be represented by either by a $1 \times n$ row matrix, or a $n \times 1$ column matrix.

Vectors (are just tiny matrices)

A vector $\vec{\boldsymbol{v}} = < v_1, \dots, v_n >$ can be represented by either by a $1 \times n$ row matrix, or a $n \times 1$ column matrix.

Vector addition and Scalar Multiplication

Let

$$\vec{\boldsymbol{x}} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad \vec{\boldsymbol{y}} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

be vectors in $\mathbb{R}^n$ and c be any scalar. Then, we have:

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix} \quad \text{and} \quad c \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} c \cdot x_1 \\ \vdots \\ c \cdot x_n \end{bmatrix}$$

Properties of Vector Addition and Multiplication

For vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ in $\mathbb{R}^n$ and scalars c and k.

- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (Commutativity)
- $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$ (Associativity)
- $c(k\vec{v}) = (ck)\vec{v}$ (Associativity)
- $\vec{u} + \vec{0} = \vec{u}$ (Zero Element)
- $\vec{u} + (-\vec{u}) = \vec{0}$ (Inverse Element)
- $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$ (Distributivity)
- $(c + k)\vec{u} = c\vec{u} + k\vec{u}$ (Distributivity)

Dot Product (also called the Scalar Product)

The **dot product** of a row vector $\vec{\boldsymbol{x}}$ and a column vector $\vec{\boldsymbol{y}}$ of equal length n is the result of adding the products of the corresponding entries as follows:

$$\begin{aligned}\vec{\boldsymbol{x}} \bullet \vec{\boldsymbol{y}} &= \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix} \bullet \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \\ &= x_1 \cdot y_1 + x_2 \cdot y_2 + \cdots + x_n \cdot y_n \\ &= \sum_{k=1}^{n} x_k \cdot y_k\end{aligned}$$

Orthogonality

Two vectors $\vec{\boldsymbol{x}}$ and $\vec{\boldsymbol{y}}$ in $\mathbb{R}^n$ are called **orthogonal** when:

$$\vec{\boldsymbol{x}} \cdot \vec{\boldsymbol{y}} = 0$$

Orthogonality

Two vectors $\vec{\boldsymbol{x}}$ and $\vec{\boldsymbol{y}}$ in $\mathbb{R}^n$ are called **orthogonal** when:

$$\vec{\boldsymbol{x}} \cdot \vec{\boldsymbol{y}} = 0$$

Magnitude

For any vector $\vec{\boldsymbol{v}}$ in $\mathbb{R}^n$ the **length**, or **magnitude**, of $\vec{\boldsymbol{v}}$ is a nonnegative scalar, denoted by $\|\vec{\boldsymbol{v}}\|$ and defined to be

$$\|\vec{\boldsymbol{v}}\| = \sqrt{\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}}$$

Orthogonality

Two vectors $\vec{\boldsymbol{x}}$ and $\vec{\boldsymbol{y}}$ in $\mathbb{R}^n$ are called **orthogonal** when:

$$\vec{\boldsymbol{x}} \cdot \vec{\boldsymbol{y}} = 0$$

Magnitude

For any vector $\vec{\boldsymbol{v}}$ in $\mathbb{R}^n$ the **length**, or **magnitude**, of $\vec{\boldsymbol{v}}$ is a nonnegative scalar, denoted by $\|\vec{\boldsymbol{v}}\|$ and defined to be

$$\|\vec{\boldsymbol{v}}\| = \sqrt{\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}}$$

Unit Vectors

Vectors of unit length are called **unit vectors**.

Matrix Product

The **matrix product** of a $m \times r$ matrix A and a $r \times n$ matrix B is denoted

$$C = A \cdot B = AB$$

where the ijth entry of C is the dot product of the ith row vector of A and the jth column vector of B:

$$c_{ij} = \begin{bmatrix} a_{i1} & a_{2j} & \cdots & a_{ir} \end{bmatrix} \bullet \begin{bmatrix} b_{1j} \\ \vdots \\ b_{rj} \end{bmatrix} = \sum_{k=1}^{r} a_{ik} b_{kj}$$

The matrix C has order $m \times n$.

Example 1

Perform AB where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -4 \\ -1 & 0 \end{bmatrix}$$

Example 1

Perform AB where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -4 \\ -1 & 0 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{bmatrix} 3 & 1 \\ 2 & -4 \\ -1 & 0 \end{bmatrix} \\ \hline \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \end{array}$$

Example 1

Perform AB where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -4 \\ -1 & 0 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{bmatrix} \boxed{\begin{matrix} 3 \\ 2 \\ -1 \end{matrix}} & \begin{matrix} 1 \\ -4 \\ 0 \end{matrix} \end{bmatrix} \\ \hline \begin{bmatrix} \boxed{1 \quad -1 \quad 3} \\ 0 \quad\quad 4 \quad\quad 2 \end{bmatrix} & \begin{bmatrix} \boxed{-2} & \\ & \end{bmatrix} \end{array}$$

Example 1

Perform AB where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -4 \\ -1 & 0 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{bmatrix} 3 & \boxed{1} \\ 2 & \boxed{-4} \\ -1 & \boxed{0} \end{bmatrix} \\ \hline \begin{bmatrix} \boxed{1 \quad -1 \quad 3} \\ 0 \quad\;\; 4 \quad\; 2 \end{bmatrix} & \begin{bmatrix} -2 & \boxed{5} \\ & \end{bmatrix} \end{array}$$

Example 1

Perform AB where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -4 \\ -1 & 0 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{bmatrix} \boxed{\begin{matrix} 3 \\ 2 \\ -1 \end{matrix}} & \begin{matrix} 1 \\ -4 \\ 0 \end{matrix} \end{bmatrix} \\ \hline \begin{bmatrix} 1 & -1 & 3 \\ \boxed{0 \quad 4 \quad 2} \end{bmatrix} & \begin{bmatrix} -2 & 5 \\ \boxed{6} & \end{bmatrix} \end{array}$$

Example 1

Perform AB where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -4 \\ -1 & 0 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{bmatrix} 3 & \boxed{\begin{matrix} 1 \\ -4 \\ 0 \end{matrix}} \\ 2 & \\ -1 & \end{bmatrix} \\ \hline \begin{bmatrix} 1 & -1 & 3 \\ \boxed{0} & \boxed{4} & \boxed{2} \end{bmatrix} & \begin{bmatrix} -2 & 5 \\ 6 & \boxed{-16} \end{bmatrix} \end{array}$$

Properties of Matrix Multiplication

- $(AB)C = A(BC)$ (Associativity)
- $A(B + C) = AB + AC$ (Distributivity)
- $(B + C)A = BA + CA$ (Distributivity)

Properties of Matrix Multiplication

- $(AB)C = A(BC)$ (Associativity)
- $A(B + C) = AB + AC$ (Distributivity)
- $(B + C)A = BA + CA$ (Distributivity)

Note

Commutativity is not present in the above list.

In general, matrix multiplication does not commute:

- $AB \neq BA$ (Generally Noncommutative)

Properties of Matrix Multiplication

- $(AB)C = A(BC)$ (Associativity)
- $A(B + C) = AB + AC$ (Distributivity)
- $(B + C)A = BA + CA$ (Distributivity)

Note

Commutativity is not present in the above list.

In general, matrix multiplication does not commute:

- $AB \neq BA$ (Generally Noncommutative)

Properties of Identity Matrices

For a $m \times n$ matrix A:

- $A \cdot I_n = A$ and $I_m \cdot A = A$
- $A \cdot \mathbf{0}_n = \mathbf{0}_{mn}$ and $\mathbf{0}_m \cdot A = \mathbf{0}_{mn}$

Transpose

For a matrix $A = [a_{ij}]$ the **transpose** of the $m \times n$ matrix A is defined as the $n \times m$ matrix:

$$A^{\mathsf{T}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Transpose

For a matrix $A = [a_{ij}]$ the **transpose** of the $m \times n$ matrix A is defined as the $n \times m$ matrix:

$$A^\intercal = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}^\intercal = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Example 2

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^\intercal = \begin{bmatrix} 1 & \\ 3 & \end{bmatrix}$$

Transpose

For a matrix $A = [a_{ij}]$ the **transpose** of the $m \times n$ matrix A is defined as the $n \times m$ matrix:

$$A^{\mathsf{T}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Example 2

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Transpose

For a matrix $A = [a_{ij}]$ the **transpose** of the $m \times n$ matrix A is defined as the $n \times m$ matrix:

$$A^{\mathsf{T}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Example 2

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Example 3

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Transpose

For a matrix $A = [a_{ij}]$ the **transpose** of the $m \times n$ matrix A is defined as the $n \times m$ matrix:

$$A^T = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Example 2

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Example 3

$$\begin{bmatrix} \boxed{1 \quad 2 \quad -1} \\ 3 \quad 0 \quad\;\; 5 \end{bmatrix}^T = \begin{bmatrix} \boxed{\begin{matrix} 1 \\ 2 \\ -1 \end{matrix}} & \end{bmatrix}$$

Transpose

For a matrix $A = [a_{ij}]$ the **transpose** of the $m \times n$ matrix A is defined as the $n \times m$ matrix:

$$A^{\mathsf{T}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Example 2

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Example 3

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 5 \end{bmatrix}$$

Matrices with Function Entries

Matrices can have functions as entries, not just real numbers.

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}(t) & a_{m1}(t) & \cdots & a_{mn}(t) \end{bmatrix}$$

Matrices with Function Entries

Matrices can have functions as entries, not just real numbers.

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}(t) & a_{m1}(t) & \cdots & a_{mn}(t) \end{bmatrix}$$

Example 4

$$A(t) = \begin{bmatrix} t^2 & \sin(2t) & 5t-1 \\ t^3 & \frac{1}{3t} & \ln(t+1) \end{bmatrix}$$

Where,

$$A(1) =$$

Matrices with Function Entries

Matrices can have functions as entries, not just real numbers.

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}(t) & a_{m1}(t) & \cdots & a_{mn}(t) \end{bmatrix}$$

Example 4

$$A(t) = \begin{bmatrix} t^2 & \sin(2t) & 5t-1 \\ t^3 & \frac{1}{3t} & \ln(t+1) \end{bmatrix}$$

Where,

$$A(1) = \begin{bmatrix} (1)^2 & \sin(2(1)) & 5(1)-1 \\ (1)^3 & \frac{1}{3(1)} & \ln((1)+1) \end{bmatrix}$$

Matrices with Function Entries

Matrices can have functions as entries, not just real numbers.

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}(t) & a_{m1}(t) & \cdots & a_{mn}(t) \end{bmatrix}$$

Example 4

$$A(t) = \begin{bmatrix} t^2 & \sin(2t) & 5t-1 \\ t^3 & \frac{1}{3t} & \ln(t+1) \end{bmatrix}$$

Where,

$$A(1) = \begin{bmatrix} (1)^2 & \sin(2(1)) & 5(1)-1 \\ (1)^3 & \frac{1}{3(1)} & \ln((1)+1) \end{bmatrix} = \begin{bmatrix} 1 & \sin(2) & 4 \\ 1 & \frac{1}{3} & \ln(2) \end{bmatrix}$$

We say that a matrix is **continuous**, **piecewise continuous**, or **differentiable** if every element in the matrix has that property.

We say that a matrix is **continuous**, **piecewise continuous**, or **differentiable** if every element in the matrix has that property.

Derivative of a Matrix

For a differentiable matrix A, the derivative of A is defined as:

$$A'(t) = \frac{dA}{dt} = \begin{bmatrix} a'_{11}(t) & a'_{12}(t) & \cdots & a'_{1n}(t) \\ a'_{21}(t) & a'_{22}(t) & \cdots & a'_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a'_{m1}(t) & a'_{m1}(t) & \cdots & a'_{mn}(t) \end{bmatrix}$$

We say that a matrix is **continuous**, **piecewise continuous**, or **differentiable** if every element in the matrix has that property.

Derivative of a Matrix

For a differentiable matrix A, the derivative of A is defined as:

$$A'(t) = \frac{dA}{dt} = \begin{bmatrix} a'_{11}(t) & a'_{12}(t) & \cdots & a'_{1n}(t) \\ a'_{21}(t) & a'_{22}(t) & \cdots & a'_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a'_{m1}(t) & a'_{m1}(t) & \cdots & a'_{mn}(t) \end{bmatrix}$$

Matrix Differentiation Rules

For differentiable matrices $A(t)$ and $B(t)$ and scalar constant c.

- $(A(t) + B(t))' = A'(t) + B'(t)$
- $(cA(t))\prime = cA'(t)$
- $(A(t) \cdot B(t))' = A(t) \cdot B'(t) + A'(t) \cdot B(t)$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \dfrac{1}{t} \\ \\ \\ \end{bmatrix}$$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \frac{1}{t} \\ -3t^2 \\ \end{bmatrix}$$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \dfrac{1}{t} \\ -3t^2 \\ -2\sin 2t \end{bmatrix}$$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \dfrac{1}{t} \\ -3t^2 \\ -2\sin 2t \end{bmatrix}$$

Example 6

$$A(t) = \begin{bmatrix} e^t & t^2 \\ \sin t & 2t \end{bmatrix} \qquad A'(t) = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \dfrac{1}{t} \\ -3t^2 \\ -2\sin 2t \end{bmatrix}$$

Example 6

$$A(t) = \begin{bmatrix} e^t & t^2 \\ \sin t & 2t \end{bmatrix} \qquad A'(t) = \begin{bmatrix} e^t & \\ & \end{bmatrix}$$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \dfrac{1}{t} \\ -3t^2 \\ -2\sin 2t \end{bmatrix}$$

Example 6

$$A(t) = \begin{bmatrix} e^t & t^2 \\ \sin t & 2t \end{bmatrix} \qquad A'(t) = \begin{bmatrix} e^t & 2t \\ & \end{bmatrix}$$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \dfrac{1}{t} \\ -3t^2 \\ -2\sin 2t \end{bmatrix}$$

Example 6

$$A(t) = \begin{bmatrix} e^t & t^2 \\ \sin t & 2t \end{bmatrix} \qquad A'(t) = \begin{bmatrix} e^t & 2t \\ \cos t & \end{bmatrix}$$

Example 5

$$g(t) = \begin{bmatrix} \ln t \\ -t^3 \\ \cos 2t \end{bmatrix} \qquad g'(t) = \begin{bmatrix} \dfrac{1}{t} \\ -3t^2 \\ -2\sin 2t \end{bmatrix}$$

Example 6

$$A(t) = \begin{bmatrix} e^t & t^2 \\ \sin t & 2t \end{bmatrix} \qquad A'(t) = \begin{bmatrix} e^t & 2t \\ \cos t & 2 \end{bmatrix}$$