Conditional Probability

Colby Community College

Example 1

The `photo_classify` data set represents a machine learning algorithm classifying a sample of 1822 photos as either about fashion or not.

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The `photo_classify` data set represents a machine learning algorithm classifying a sample of 1822 photos as either about fashion or not.

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		fashion	*not*	Total
mach_learn	*pred_fashion*	197	22	219
	pred_not	112	1491	1603
	Total	309	1513	1822

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If a photo is actually about fashion, what is the chance the algorithm will correctly identify the photo as being about fashion?

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		fashion	*not*	Total
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If a photo is actually about fashion, what is the chance the algorithm will correctly identify the photo as being about fashion?

Of the 309 fashion photos, the algorithm correctly classifies 197 of them.

$$P\left(\texttt{mach\_learn}\text{ is }\textit{pred\_fashion}\text{ given }\texttt{truth}\text{ is }\textit{fashion}\right) = \frac{197}{309} = 0.638$$

Example 2

Using the same data set as in Example 1.

		truth		
		fashion	*not*	Total
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If the algorithm predicts the photo as being about fashion, what is the probability is actually is?

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Using the same data set as in Example 1.

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mach_learn	*pred_fashion*	197	22	219
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If the algorithm predicts the photo as being about fashion, what is the probability is actually is?

Of the 1603 photos predicted to be about fashion, 112 we actually about fashion.

$$P\left(\texttt{truth}\text{ is }\textit{fashion}\text{ given }\texttt{mach\_learn}\text{ is }\textit{pred\_fashion}\right) = \frac{197}{219} = 0.900$$

Note

It can be helpful to draw Venn Diagrams of these contingency tables using rectangles.

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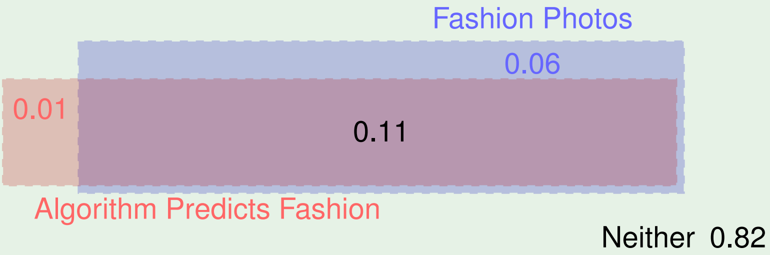
Example 3

The Venn Diagram for Example 1 is:



Definition

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Example 4

$$P\left(\texttt{mach\_learn}\text{ is } \textit{pred\_fashion}\right) = \frac{219}{1822} = 0.12$$

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$$P\left(\texttt{mach\_learn} \text{ is } \mathit{pred\_fashion}\right) = \frac{219}{1822} = 0.12$$

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A probability of outcomes for two or more variables is called a **joint probability**.

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A probability of outcomes for two or more variables is called a **joint probability**.

Example 5

$$P\left(\texttt{mach\_learn} \text{ is } \textit{pred\_fashion} \text{ and } \texttt{truth} \text{ is } \textit{fashion}\right) = \frac{197}{1822} = 0.11$$

Note

Sometimes a comma is substituted for “and” in a joint probability.

$$P(\texttt{mach\_learn} \text{ is } \mathtt{\mathit{pred\_fashion}}, \texttt{truth} \text{ is } \mathtt{\mathit{fashion}})$$

means the same thing as

$$P(\texttt{mach\_learn} \text{ is } \mathtt{\mathit{pred\_fashion}} \text{ and } \texttt{truth} \text{ is } \mathtt{\mathit{fashion}})$$

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Example 6

The table proportions for `photo_classify` are:

	`truth`: *`fashion`*	`truth`: *`not`*	Total
`mach_learn`: *`pred_fashion`*	$\frac{197}{1822}$		
`mach_learn`: *`pred_not`*			
Total			

$\downarrow\downarrow\downarrow$

	`truth`: *`fashion`*	`truth`: *`not`*	Total
`mach_learn`: *`pred_fashion`*	0.1081		
`mach_learn`: *`pred_not`*			
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`mach_learn`: *pred_fashion*	$\frac{197}{1822}$	$\frac{22}{1822}$	
`mach_learn`: *pred_not*			
Total			

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	`truth`: *fashion*	`truth`: *not*	Total
`mach_learn`: *pred_fashion*	0.1081	0.0121	
`mach_learn`: *pred_not*			
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Total			

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	`truth`: *fashion*	`truth`: *not*	Total
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Total			

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	`truth`: *fashion*	`truth`: *not*	Total
`mach_learn`: *pred_fashion*	0.1081	0.0121	0.1202
`mach_learn`: *pred_not*	0.0615	0.8183	
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Total	$\frac{309}{1822}$		

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	truth: *fashion*	truth: *not*	Total
mach_learn: *pred_fashion*	0.1081	0.0121	0.1202
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Total	0.1696		

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Total	$\frac{309}{1822}$	$\frac{1513}{1822}$	$\frac{1822}{1822}$

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	`truth`: *`fashion`*	`truth`: *`not`*	Total
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Total	0.1696	0.8304	1.0

Example 7

The table proportions from Example 6 make a probability distribution.

Joint Outcome	Probability
`mach_learn` is *pred_fashion* and `truth` is *fashion*	0.1081
`mach_learn` is *pred_fashion* and `truth` is *not*	0.0121
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Note

Joint probabilities can be used to calculate marginal probabilities in simple cases.

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Example 8

$$\begin{aligned} P\left(\texttt{truth}\text{ is }\textit{fashion}\right) &= P\left(\texttt{mac\_learn}\text{ is }\textit{pred\_fashion}\text{ and }\texttt{truth}\text{ is }\textit{fashion}\right) \\ &\quad + P\left(\texttt{mac\_learn}\text{ is }\textit{pred\_not}\text{ and }\texttt{truth}\text{ is }\textit{fashion}\right) \end{aligned}$$

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Example 9

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Definition

There are two parts to a conditional probability, the **outcome of interest** and the **condition.**

$$P\left(\text{outcome of interest given condition}\right)$$

is the same as

$$P\left(\text{outcome of interest} \mid \text{condition}\right)$$

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Example 10

$$P\left(\texttt{truth}\text{ is }\mathtt{\mathit{fashion}} \mid \texttt{mach\_learn}\text{ is }\mathtt{\mathit{pred\_fashion}}\right) = \frac{197}{219} = 0.900$$

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$$\begin{aligned}
&P\left(\texttt{truth}\text{ is }\textit{fashion} \mid \texttt{mach_learn}\text{ is }\textit{pred_fashion}\right)\\
&\quad = \frac{P\left(\texttt{truth}\text{ is }\textit{fashion}\text{ and }\texttt{mach_learn}\text{ is }\textit{pred_fashion}\right)}{P\left(\texttt{mach_learn}\text{ is }\textit{pred_fashion}\right)}\\
&\quad = \frac{0.0615}{0.1696}\\
&\quad = 0.3626
\end{aligned}$$

Example 12

The `smallpox` data set provides a sample of 6,224 individuals from the year 1721 who were exposed to smallpox in Boston.

		inoculated			inoculated		
		yes	*no*	Total	*yes*	*no*	Total
result	*lived*	238	5136	5374	0.0382	0.8252	0.8634
	died	6	844	850	0.0010	0.1356	0.1366
	Total	244	5980	6224	0.0392	0.9608	1.0000

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$$=\frac{0.1356}{0.9608}$$

$$=0.1411\approx 14.11\%$$

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$$P\left(\texttt{result} \text{ is } \textit{died} \;\middle|\; \texttt{inoculated} \text{ is } \textit{yes}\right)$$

$$= \frac{P\left(\texttt{result} \text{ is } \textit{died} \text{ and } \texttt{inoculated} \text{ is } \textit{yes}\right)}{P\left(\texttt{inoculated} \text{ is } \textit{yes}\right)}$$

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$$= \frac{0.0010}{0.0392}$$

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The `smallpox` data set provides a sample of 6,224 individuals from the year 1721 who were exposed to smallpox in Boston.

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People die for many reasons, wealth determines level of medical care available, etc. . .

General Multiplication Rule

If A and B represent two outcomes or events, then

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Let's use

- $P(\texttt{inoculated} \text{ is } \textit{yes}) = 0.0392$
- $P(\texttt{result} \text{ is } \textit{lived} \mid \texttt{inoculated} \text{ is } \textit{yes}) = 0.9754$

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&\qquad = 0.0382 \approx 3.82\%
\end{aligned}
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Sum of Conditional Probabilities

Let $A_1, A_2, \ldots, A_k$ represent all the disjoint outcomes for a variable. Then if B is an event, possibly for another variable, we have:

$$P(A_1 \mid B) + P(A_2 \mid B) + \cdots + P(A_k \mid B) = 1$$

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Example 17

If 97.54% of the inoculated people lived, what is the proportion of people that must have died?

There are only two outcomes: `lived` and `died`. Which means that $100\% - 97.54\% = 2.46\%$ people who were inoculated died.

Example 18

Let X and Y represent the outcomes of rolling two dice.

Let us compute the following:

$$P(Y = 1 \mid X = 1)$$

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Note

We have shown that if two events are independent, then knowing the outcome of one should provide no information about the other.

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Ron is watching a roulette table in a casino and notices that the last five outcomes were `black`. He figures that the chances of getting `black` six times in a row is very small and puts his paycheck on red.

Why is this a really bad idea?

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Note

Posting the last several outcomes of a betting game is a real practice casinos use to trick people into believing the odds are in their favor. It's known as the **gambler's fallacy**.

Definition

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They are most useful when two or more processes occur in a sequence and each process is conditioned on its predecessor.

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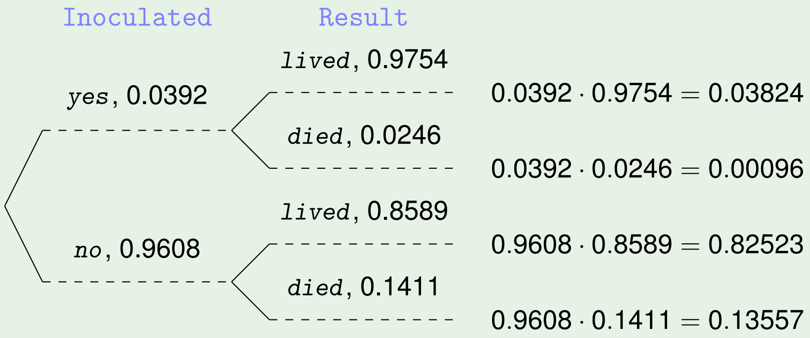
Here is the tree diagram for `smallpox` dataset.



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Suppose that 13% of students earned an A on the midterm. Of those students that earned an A, 47% received an A on the final and 11% of the student who earned a lower grade than an A on the midterm received an A on the final.

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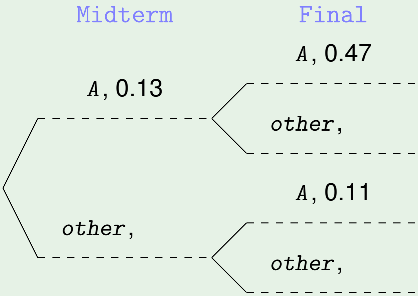

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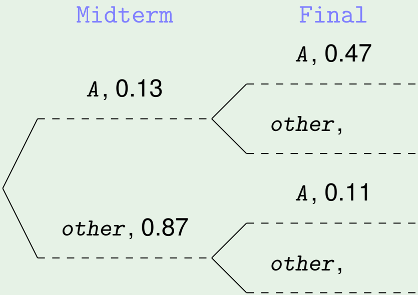

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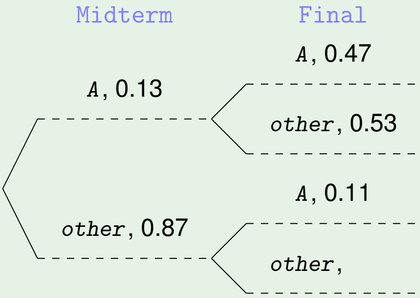

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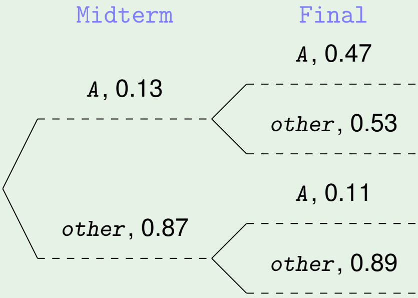

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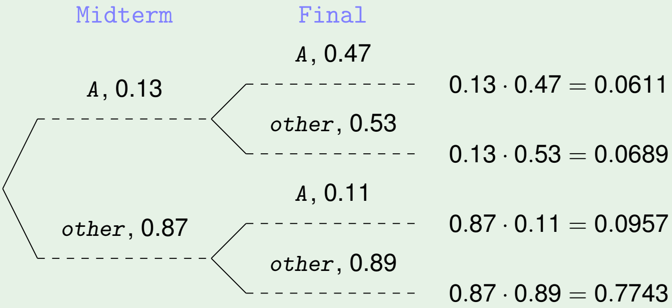

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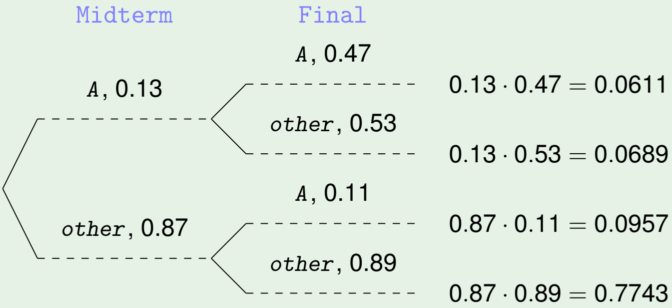

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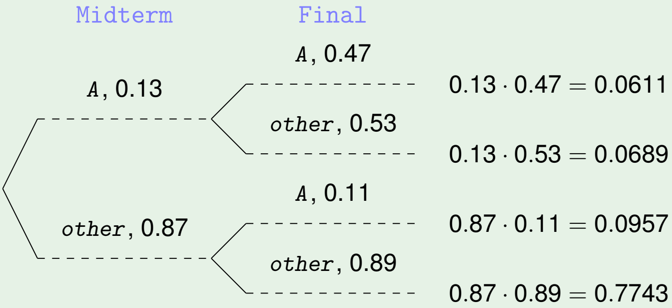

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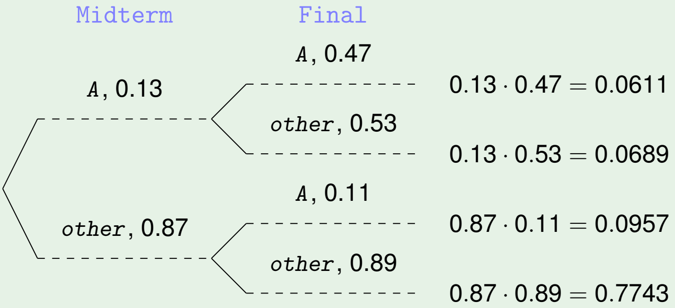

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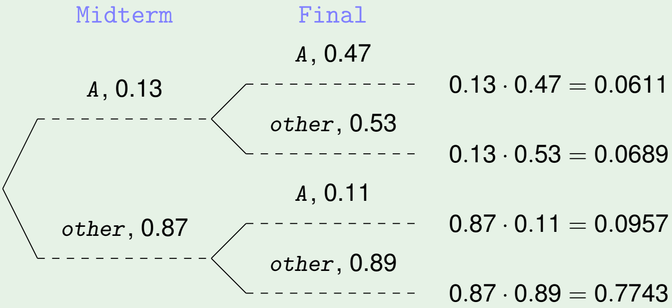

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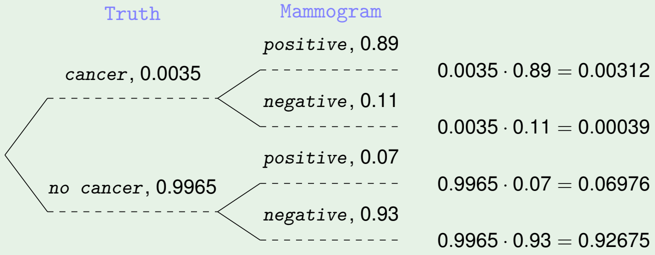

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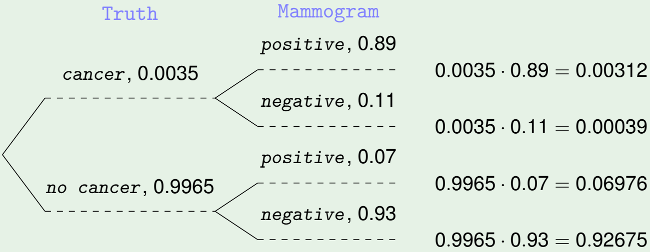

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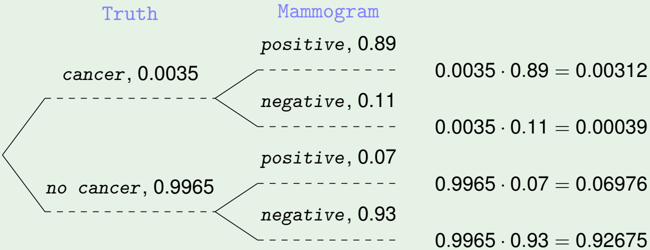

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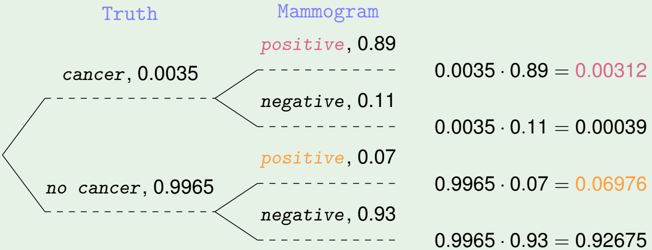

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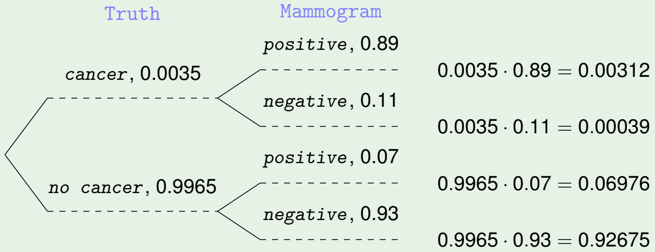

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Note

There are times where we are given:

$$P\left(\text{statement about } \texttt{variable 1|} \mid \text{statement about } \texttt{variable 2}\right)$$

but we would rather know the inverted conditional probability:

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Bayes' Theorem

Consider the following conditional probability for `variable 1` and `variable 2`:

$$P\left(\text{outcome } A_1 \text{ of } \texttt{variable 1} \,\middle|\, \text{outcome } B \text{ of } \texttt{variable 2}\right)$$

Bayes' Theorem states that this conditional probability is the same as:

$$\frac{P(B \mid A_1)\, P(A_1)}{P(B \mid A_1)\, P(A_1) + P(B \mid A_2)\, P(A_2) + \cdots + P(B \mid A_k)\, P(A_k)}$$

where $A_2, A_3, \ldots, A_k$ represent all other possible outcomes of `variable 1`.

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In Example 23, we computed $P\,(\text{has BC} \mid \text{test positive})$ using a tree diagram.

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\begin{aligned}
&P\left(\text{has BC} \mid \text{test pos}\right) \\
&\quad = \frac{P\left(\text{test pos} \mid \text{has BC}\right) P\left(\text{has BC}\right)}{P\left(\text{test pos} \mid \text{has BC}\right) P\left(\text{has BC}\right) + P\left(\text{test pos} \mid \text{no BC}\right) P\left(\text{no BC}\right)} \\
&\quad = \frac{0.89 \cdot 0.0035}{0.89 \cdot 0.0035 + 0.07 \cdot 0.9965}
\end{aligned}
$$

Example 24

In Example 23, we computed $P\left(\text{has BC} \mid \text{test positive}\right)$ using a tree diagram.

We could have also used Bayes' Theorem:

$$
\begin{aligned}
&P\left(\text{has BC} \mid \text{test pos}\right)\\
&\quad = \frac{P\left(\text{test pos} \mid \text{has BC}\right) P\left(\text{has BC}\right)}{P\left(\text{test pos} \mid \text{has BC}\right) P\left(\text{has BC}\right) + P\left(\text{test pos} \mid \text{no BC}\right) P\left(\text{no BC}\right)}\\
&\quad = \frac{0.89 \cdot 0.0035}{0.89 \cdot 0.0035 + 0.07 \cdot 0.9965} = \frac{0.03115}{0.07287}
\end{aligned}
$$

Example 24

In Example 23, we computed $P\,(\text{has BC} \mid \text{test positive})$ using a tree diagram.

We could have also used Bayes' Theorem:

$$
\begin{aligned}
& P\,(\text{has BC} \mid \text{test pos}) \\
& \quad = \frac{P\,(\text{test pos} \mid \text{has BC})\,P\,(\text{has BC})}{P\,(\text{test pos} \mid \text{has BC})\,P\,(\text{has BC}) + P\,(\text{test pos} \mid \text{no BC})\,P\,(\text{no BC})} \\
& \quad = \frac{0.89 \cdot 0.0035}{0.89 \cdot 0.0035 + 0.07 \cdot 0.9965} = \frac{0.03115}{0.07287} = 0.0428 \approx 4.28\%
\end{aligned}
$$

Example 24

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&\quad = \frac{0.89 \cdot 0.0035}{0.89 \cdot 0.0035 + 0.07 \cdot 0.9965} = \frac{0.03115}{0.07287} = 0.0428 \approx 4.28\%
\end{aligned}
$$

Note

This strategy of updating beliefs using Bayes' Theorem is the foundation of an entire branch of statistics called **Bayesian statistics**.