Point Estimates and Sampling Variability

Colby Community College

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In general, there are two sources of error: sampling error and bias.

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Much of statistics is focused on understanding and quantifying sampling error.

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We try to minimize bias by using thoughtful data collection procedures.

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While this method seems silly, a compute can do these steps in a short amount of time.

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0.867	-0.013	0.876	-0.004	0.883	0.003
0.889	0.009	0.887	0.007	0.874	-0.006
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The **sampling distribution** is the distribution of sample proportions.

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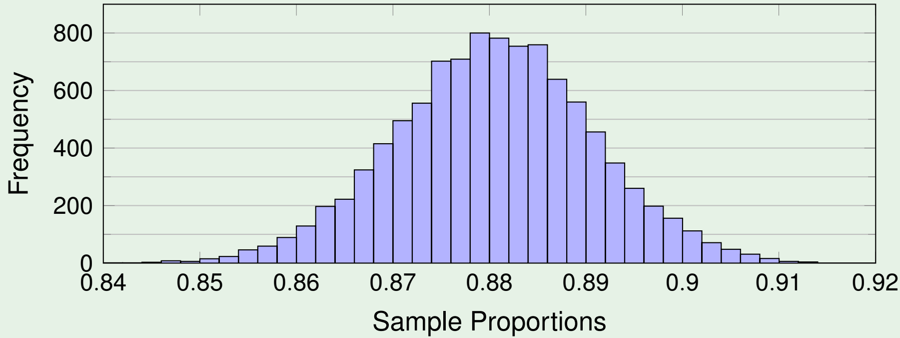
Running the simulation 10,000 times gives the histogram:



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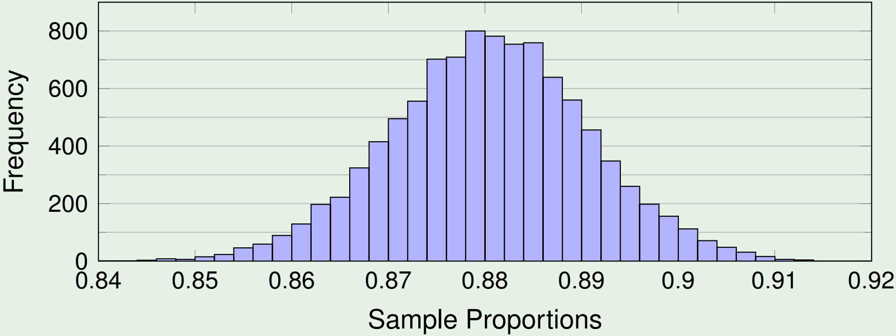

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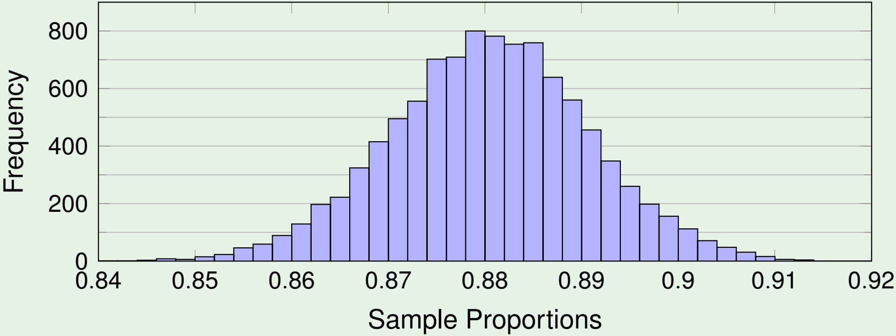

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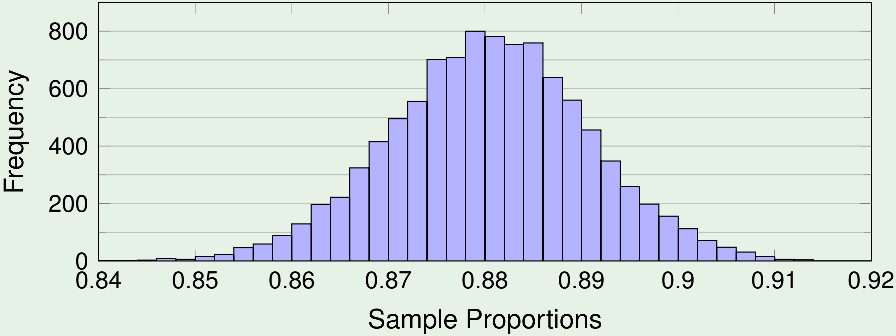

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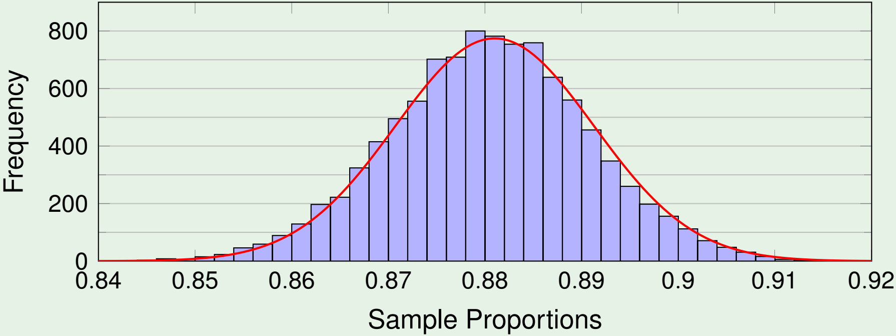

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Shape : This distribution is approximately normal.

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In real-world applications, we never actually observe the sampling distribution, yet it is useful to always think of a point estimate as coming from a hypothetical distribution.

Central Limit Theorem

When observations are independent and the sample size, n, is sufficiently large, the sample proportions $\hat{p}$ will tend to follow a normal distribution with the following mean and standard deviation:

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The Central Limit Theorem is incredibly important, and provides a foundation for much of statistics.

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This is very close to the observed standard error, 0.0102.

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Note

If a sample is larger than 10% of the population, the methods we discuss tend to overestimate the sampling error slightly. In these cases more advanced methods are needed.

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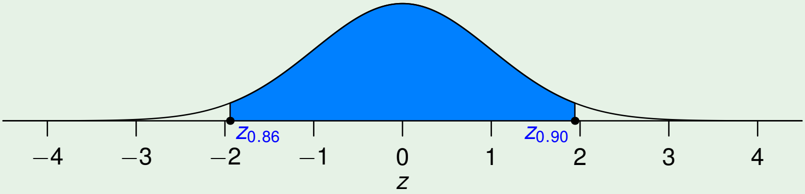

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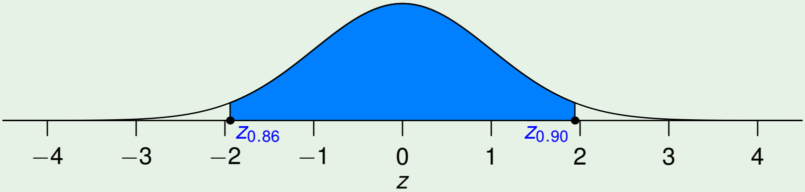

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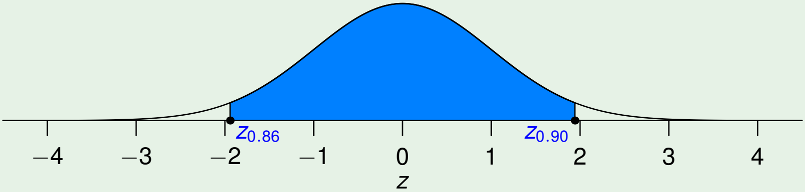

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We expect $\hat{p}$ to be within 0.02 of 0.88 about 94.78% of the time.

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Independence Pew Research is a well known non-profit think tank, so we can believe that the poll is a simple random sample, and hence the observations are independent.

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Success-Failure Conditions Since we don't actually know p, the next best thing we have is $\hat{p}$.

$$n\hat{p} =$$

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We do not actually know the population proportion unless we conduct a full census of the entire population.

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Because $n\hat{p}$ and $n(1-\hat{p})$ are both well above 10, we can conclude that $\hat{p}$ is a reasonable substitute for p.

Substitution Approximation

When np and $n(1-p)$ are much larger than 10, we can use $\hat{p}$ in place of p and the Centeral Limit Theorem becomes:

$$\mu_{\hat{p}} = p \approx \hat{p}$$
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These values are the same to three decimal places, so using the substitution approximation won't make a major difference.

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- When np and $n(1-p)$ are both very large, the distributions discreteness is hardly evident, and the distribution looks much more like a normal distribution.