

Sampling From A Small Population

Colby Community College

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Example 1

Teachers sometimes select a student at random to answer a question. We assume each student has an equal chance of being selected and there are 15 students in the class.

What is the chance you will be picked for the next question?

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Teachers sometimes select a student at random to answer a question. We assume each student has an equal chance of being selected and there are 15 students in the class.

What is the chance you will be picked for the next question?

Probability is $\frac{1}{15} \approx 0.067$.

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If the teacher asks 3 questions, what is the probability that you will not be selected? (Assume that she only picks a student once.)

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Using the General Multiplication rule we get:

$$\begin{aligned} &P(\text{not picked in 3 questions}) \\ &= P(\text{Q1 is not picked and Q2 is not picked and Q3 is not picked}) \end{aligned}$$

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$$\begin{aligned} &P(\text{not picked in 3 questions}) \\ &= P(\text{Q1 is not picked and Q2 is not picked and Q3 is not picked}) \\ &= P(\text{Q3 is not picked} \mid \text{Q1 is not picked and Q2 is not picked}) \\ &\quad \times P(\text{Q1 is not picked and Q2 is not picked}) \end{aligned}$$

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$$= \frac{12}{13} \cdot \frac{13}{14} \cdot \frac{14}{15}$$

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$$= \frac{12}{13} \cdot \frac{13}{14} \cdot \frac{14}{15} = 0.80$$

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So, there is a 80% chance you won't be picked.

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So, there is a 81.3% chance you won't be picked.

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So, there is a 81.3% chance you won't be picked.

Note

Notice that this is different than the 80% chance of not being picked when she was picking without replacement.

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Your department is holding a raffle. They sell 30 tickets and offer seven prizes. They place the tickets in a hat and draw one for each prize, without replacing the winning tickets.

What is your chance of winning a prize if you buy one ticket?

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Note

The chances of winning a prize when sampling without replacement almost 10% larger than when sampling with replacement.