# One-sample means with the *t*-distribution

Colby Community College

## Central Limit Theorem for the Sample Mean

When we collect a sufficiently large sample of $n$ independent observations from a population with mean $\mu$ and standard deviation $\sigma$, the sampling distribution of $\bar{x}$ will be nearly normal with:

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## Note

It's rare to need to estimate the population mean $\mu$, but somehow know the population standard deviation $\sigma$. In most cases $\sigma$ will need to be estimated.

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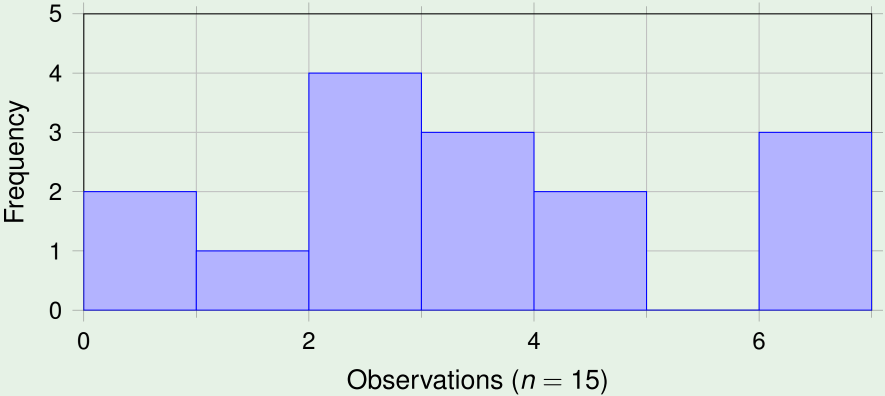
## Note

In a first course in statistics, you aren't expected to develop perfect judgment on the normality condition.

# Example 1

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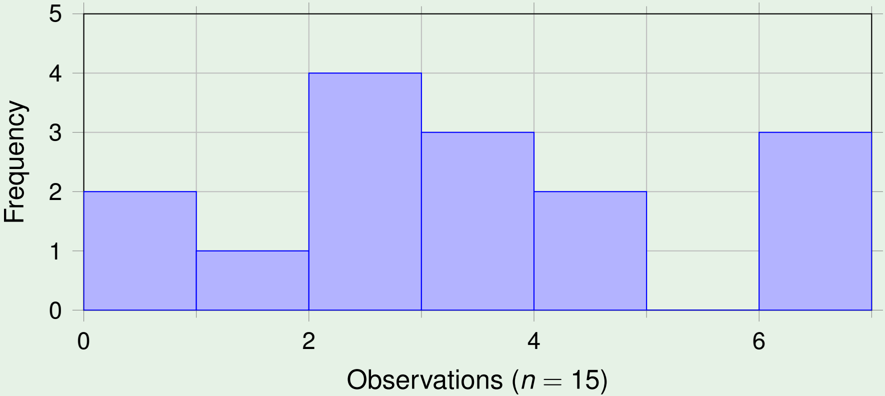


*Is the normality condition met?*

Since there are less than 30 observations, we need to look for *clear* outliers. While there is a gap on the right, the gap is small and 20% of the observations fall in rightmost bar. We can't really call these clear outliers, so the normality condition is reasonably met.

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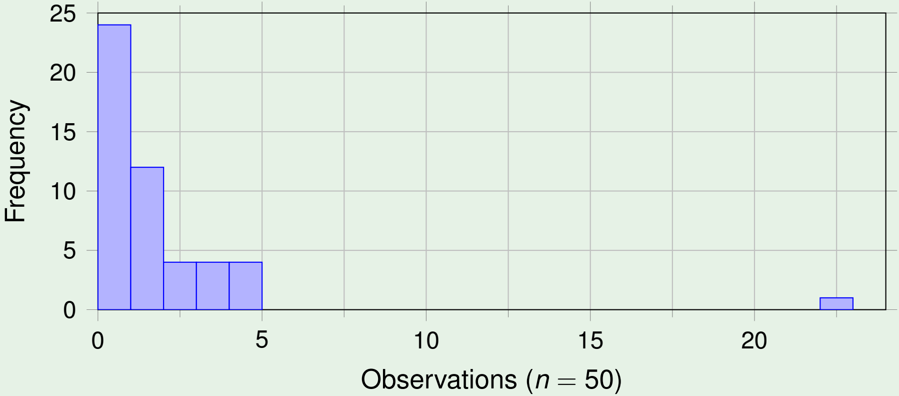


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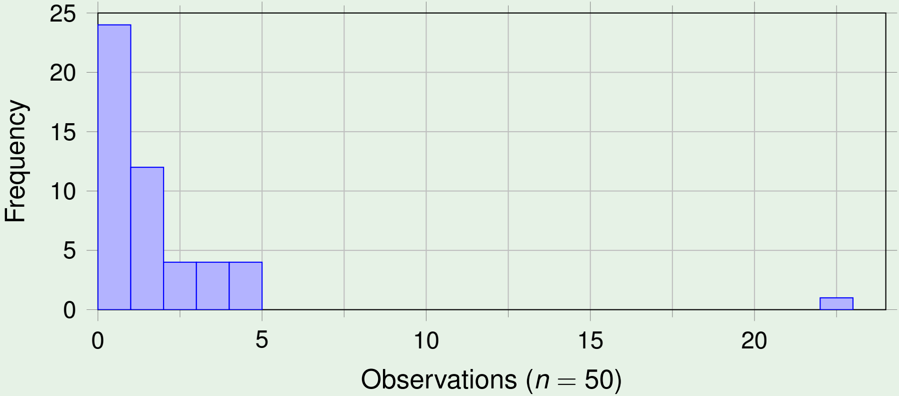


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The sample size is greater than 30, so we need to look for an extreme outlier. The gap is more than four times the width of the cluster on the left side, so this is clearly an extreme outlier and the normality condition is not met.

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## Definition

If a population has a normal distribution, then the distribution of

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A Student $t$ distribution is commonly called a ***t* distribution**.

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We can freely assign values to the first 9 scores, but the 10th score would need to be:

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Hence 9 degrees of freedom.

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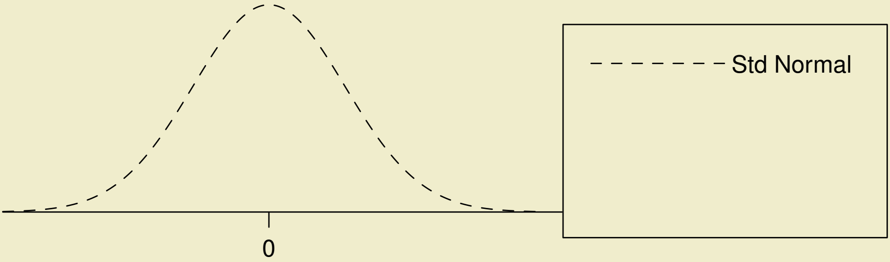
The Student *t* distribution changes for different degrees of freedom.



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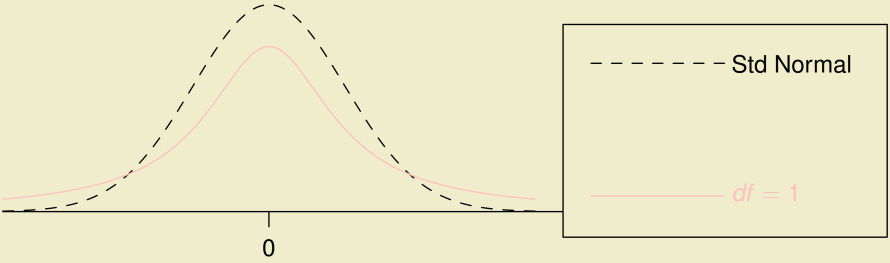

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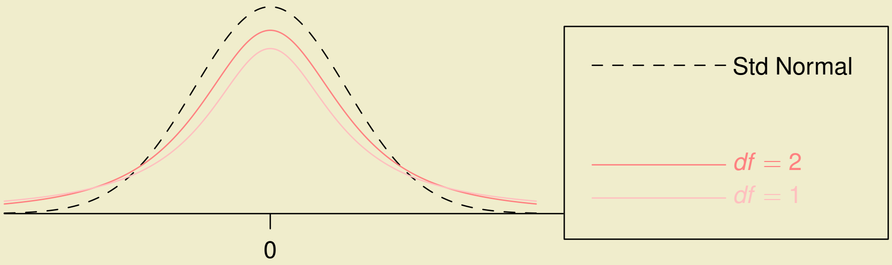

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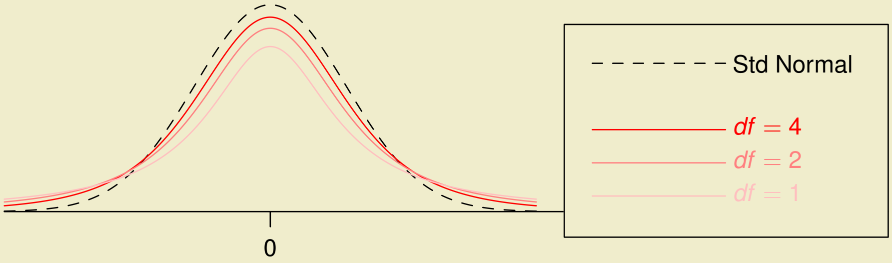

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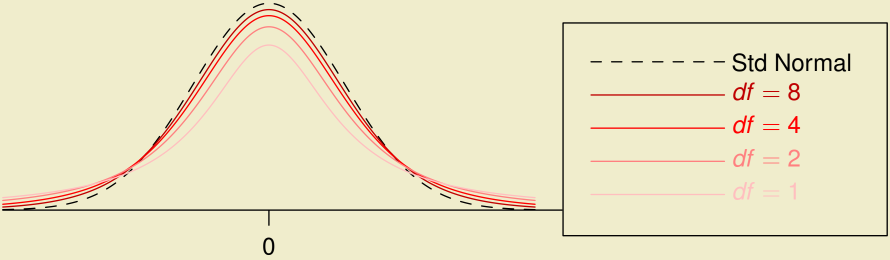

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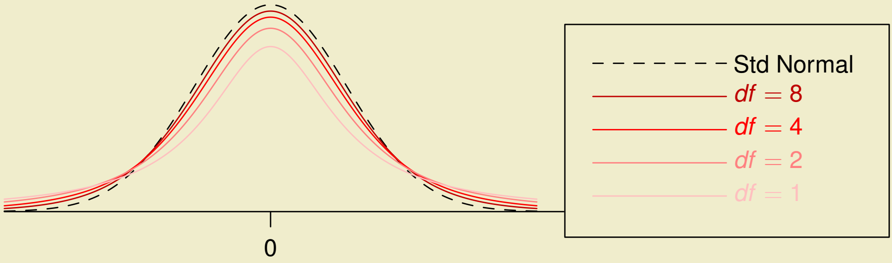


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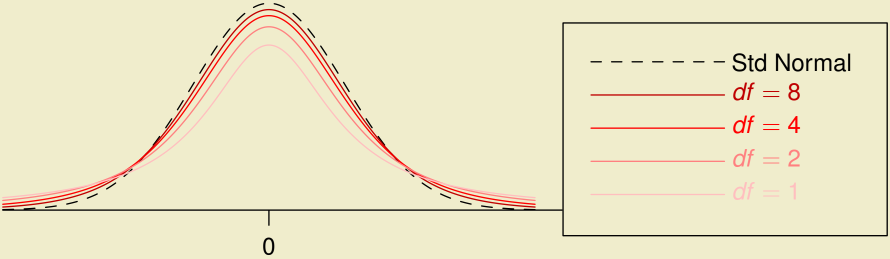


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## Note

As the sample size gets larger, the Student $t$ distribution gets closer to the standard normal distribution.

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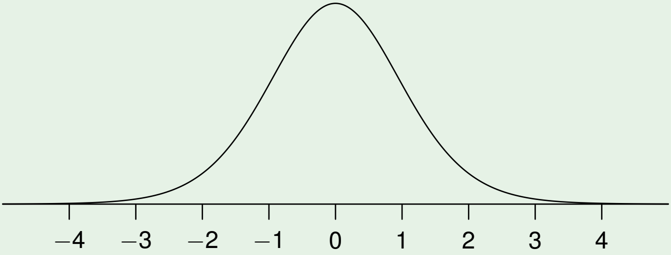
The $t$-distribution with 13 degrees of freedom is shown.



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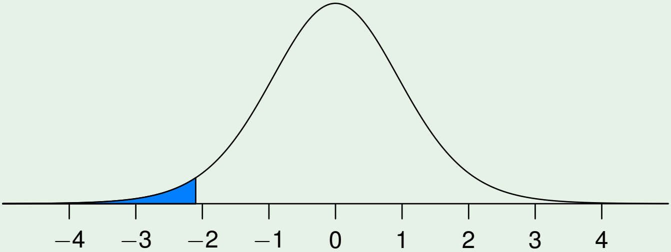
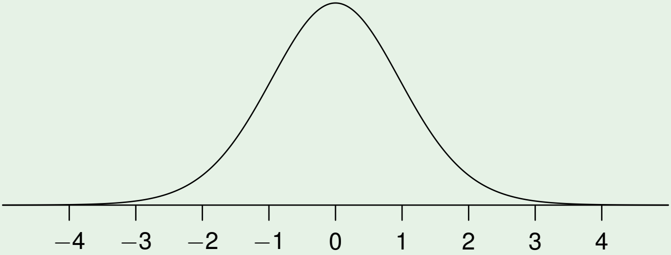



The area to the left of $t = -2.1$ is:

$$P\left(t \leq 2.1\right) \approx 0.0279$$

## Example 5

The $t$-distribution with 20 degrees of freedom is shown.



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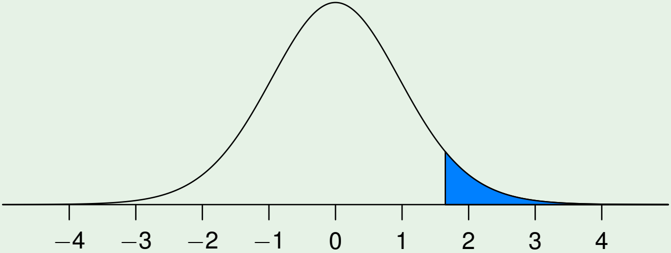


The area to the right of $t = 1.65$ is:

$$P\,(t \geq 1.65) \approx 0.0573$$

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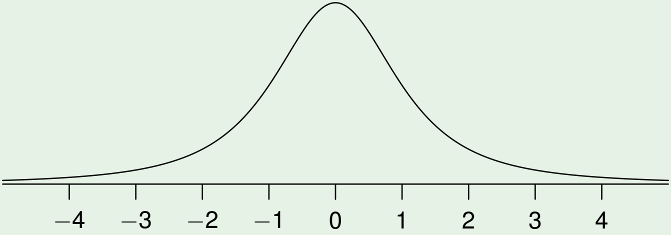
The $t$-distribution with 2 degrees of freedom is shown.



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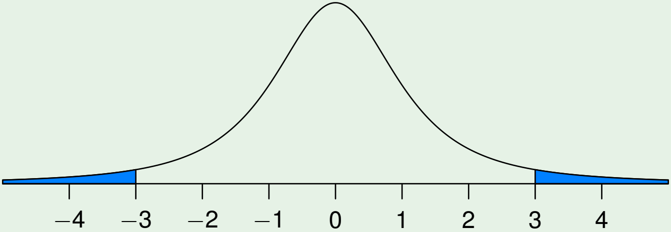


The area more than three units from the mean is:

$$P\,(t \leq -3 \text{ or } t \geq 3) \approx 0.0955$$

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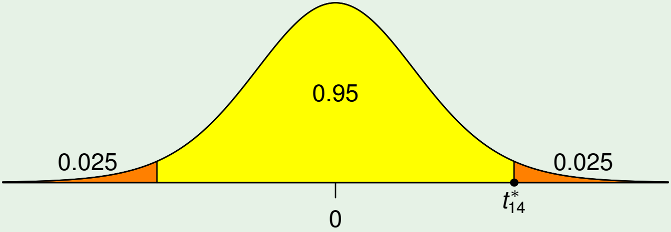

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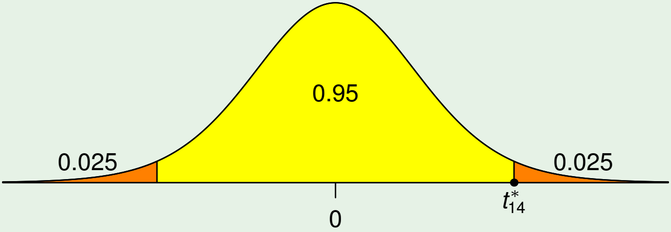


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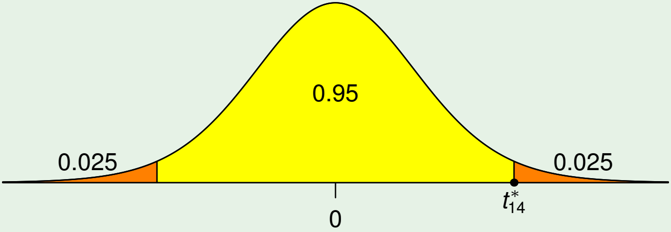


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## Note

The critical value $t^*_{df}$ must be found every time, since the $t$-distribution changes for different sample sizes.

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**Conclude:** Interpret the confidence interval in the context of the problem.

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We are 95% confident that the interval from 29.2 hg to 32.5 hg actually does contain the true value of $\mu$.

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**Conclude:** Evaluate the hypothesis test by comparing the $p$-value to $\alpha$, and provide a conclusion in the context of the problem.

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$H_0$ :The average time was the same in 2007 and 2017

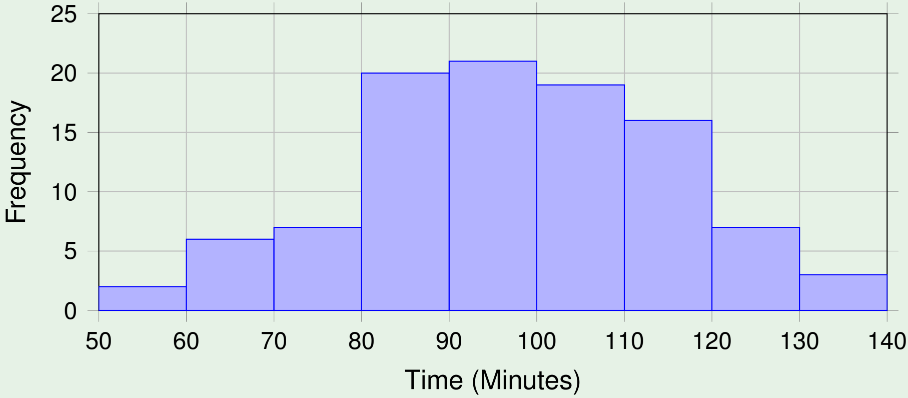
$\mu = 93.29$

$H_A$ :The average time was different in 2017 compared to 2007

$\mu \neq 93.29$

## Example 9 (Continued)

The histogram shows the times for 100 of the runners in the 2017 race.

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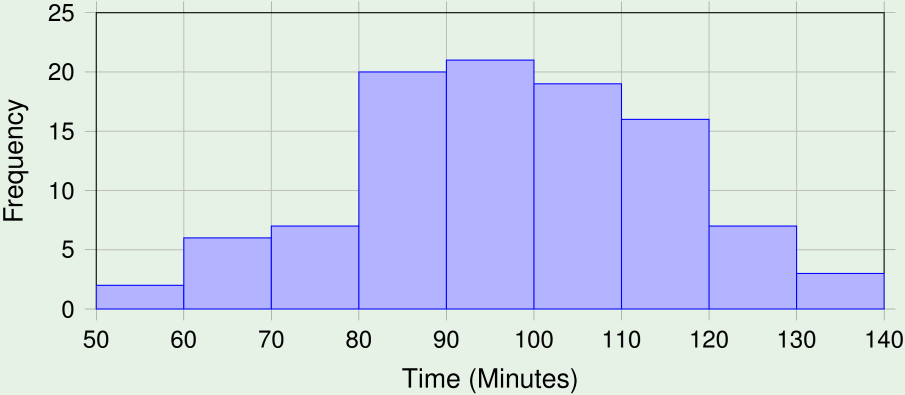


*Is the normality condition satisfied?*

We have more than 30 observations and there are no outliers, so yes.

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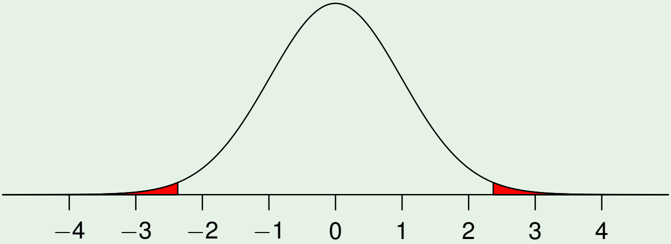
$$t = \frac{\bar{x} - \text{null value}}{SE} = \frac{97.32 - 93.29}{1.70} = 2.37$$

## Example 9 (Continued)

So, we wish to find the area of the tails:



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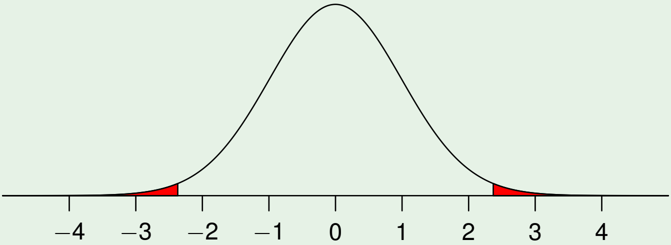


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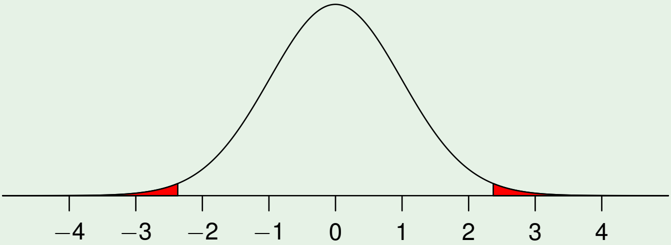


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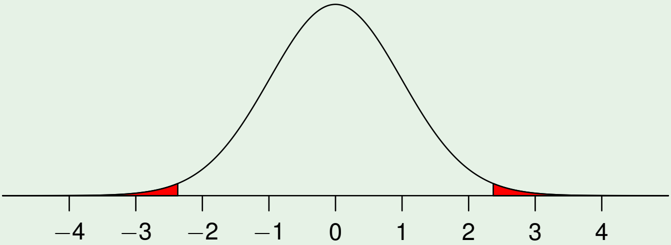


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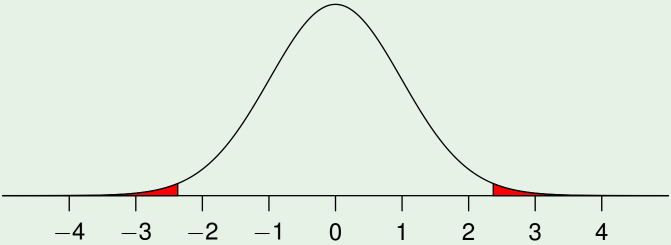


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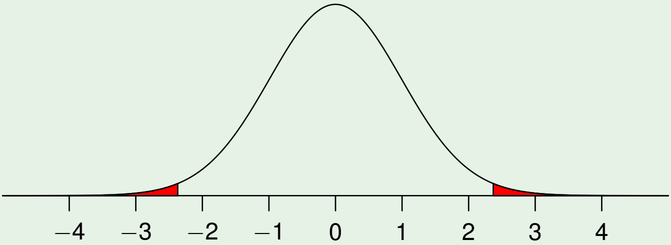


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Since the average in 2017 (97.32 minutes) is larger than the average in 2007 (93.29 minutes), it is likely that racers in 2017 were slower.