Prediction Intervals and Variation

Colby Community College

Definition

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Formula

Given a fixed and known value x_0, the prediction interval for an individual y value is

$$\hat{y} - E < y < \hat{y} + E$$

where

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\sum x^2) - (\sum x)^2}} \text{ and } s_e = \sqrt{\frac{\sum (y - \hat{y})^2}{n-2}}$$

and $t_{\alpha/2}$ has $n - 2$ degrees of freedom.

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Note

We could narrow down the interval by using a much larger set of data.

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For the following definitions, we assume that we have a collection of paired data containing the sample point (x, y), that $\hat{y}$ is the predicted value of y (obtained by using the regression equation), and that the mean of the sample y values is $\bar{y}$.

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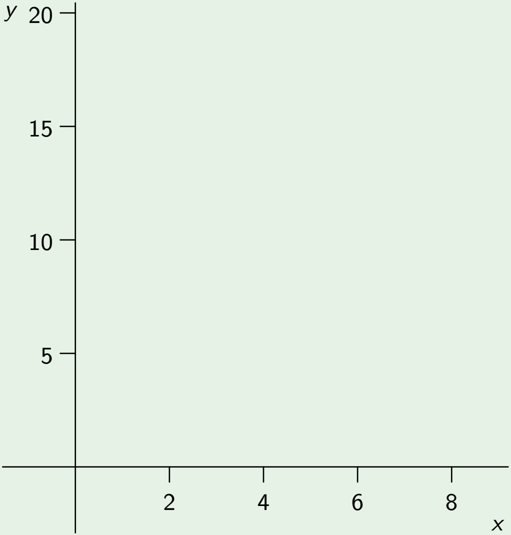
The **unexplained deviation** is the vertical distance $(y - \hat{y})$, which is the vertical distance between the point (x, y) and the regression line.

Example 2



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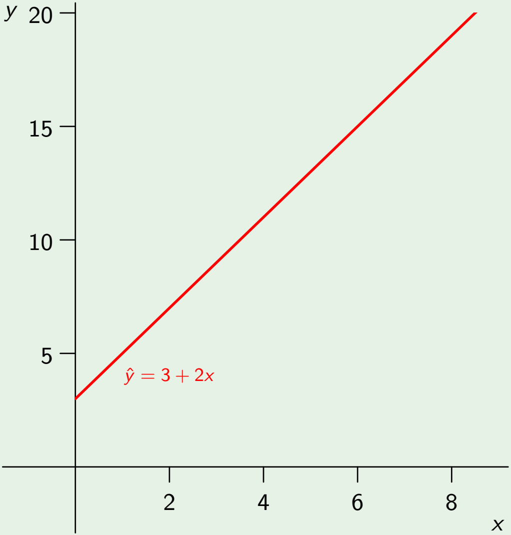

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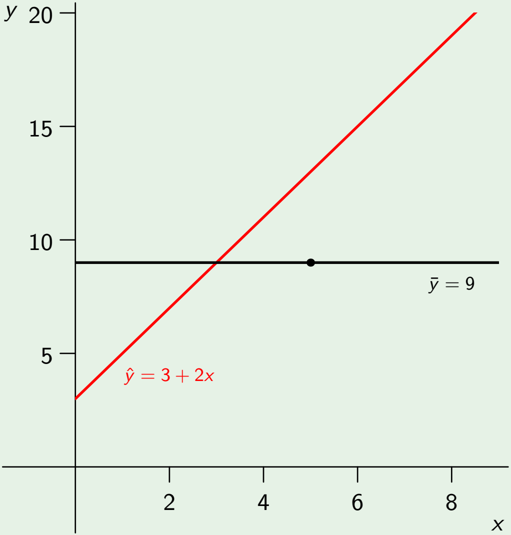

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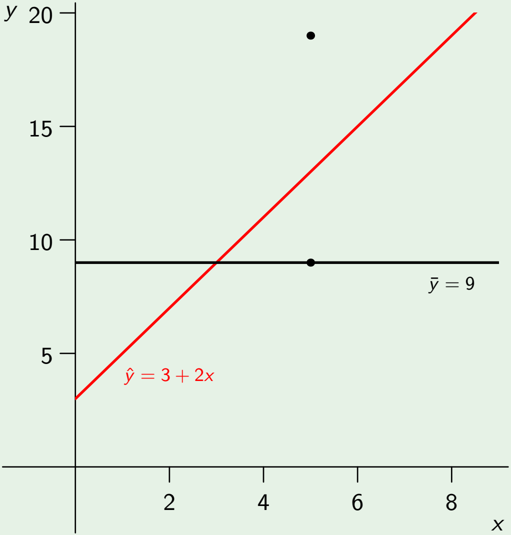

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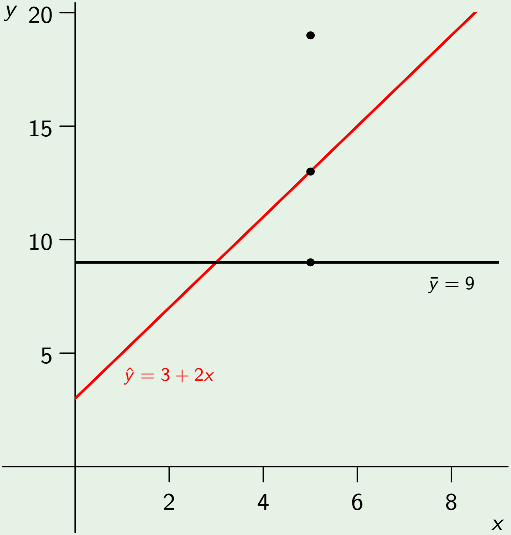

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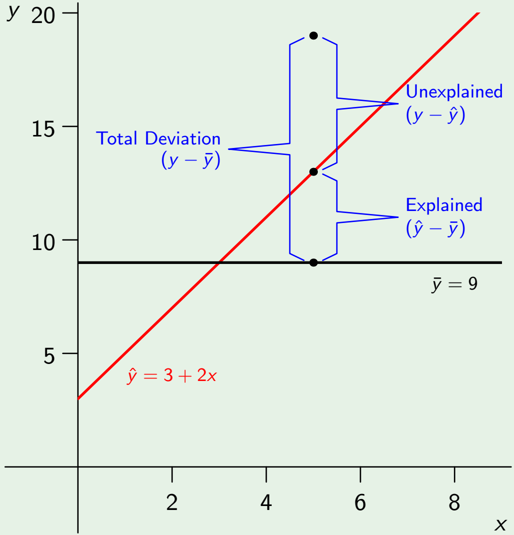

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We can either compute r^2 using the variation or we can square the linear correlation coefficient r.

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Note

The other 35.8% might be explained by some other factors. But it is pretty silly to seriously think that chocolate consumption in a country is going to directly effect the country's rate of Nobel Laureates.