Central Limit Theorem

Colby Community College

Definition

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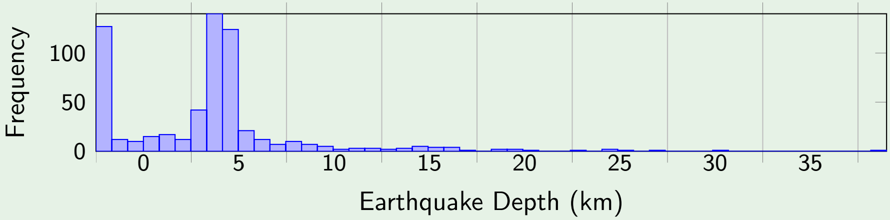
There are some rare cases where the requirement $n > 30$ isn't quite enough.

Example 1

If we look at depths of the 600 earthquakes in Data Set 21 we see that this distribution is not normal.



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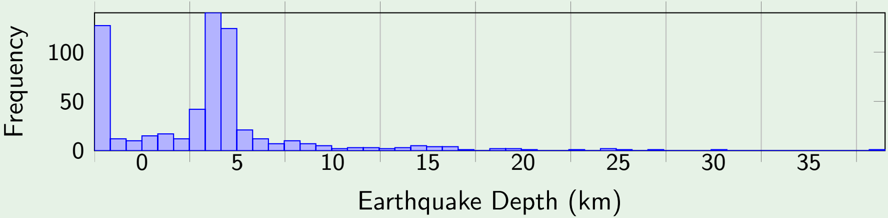
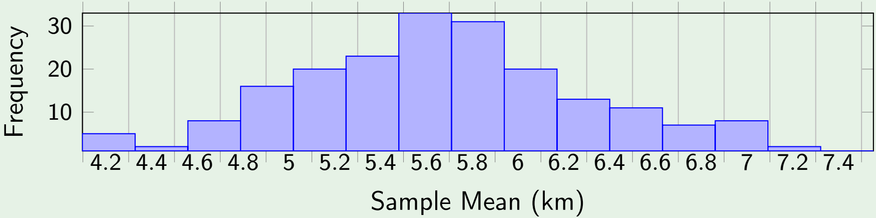


Looking at 200 sample means, each includes 50 randomly selected earthquake depths, we see that the distribution is approximately normal.



Universal Truth

The Central Limit Theorem describes a rule of nature that works throughout the universe.

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Notation

If all possible simple random samples of size n are selected from a population with mean μ and standard deviation σ, the mean of all sample means is denoted by:

Mean of all values of $\bar{x}$: $\mu_{\bar{x}} = \mu$

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Note

$\sigma_{\bar{x}}$ is called the **standard error of the mean**.

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Let us find the probability that 1 randomly selected adult male has weight greater than 148 lb.

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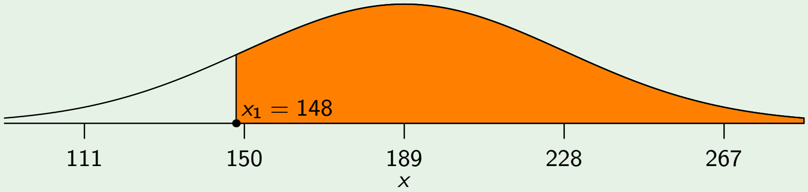

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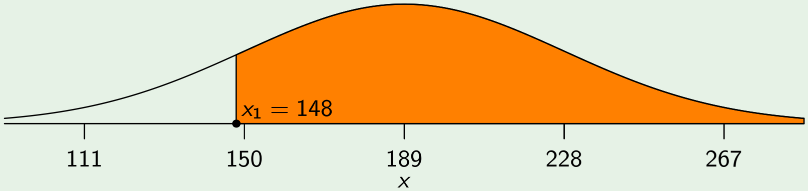

Using technology we find that 0.8531 or 85.31% of males have weights greater than 148 lb.

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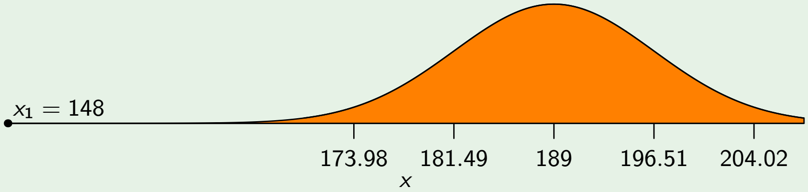

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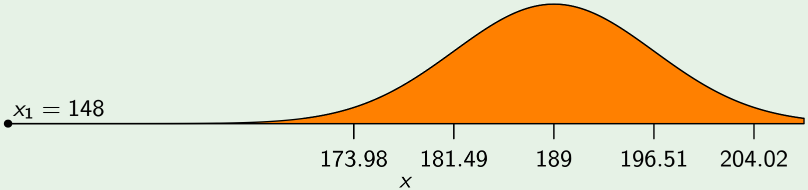

Using technology we find that 0.9999 or 99.99% of the time 27 randomly selected males have a mean weight greater than 148 lb.

Rare Event Rule for Inferential Statistics

If, under a given assumption the probability of a particular observed event is very small and the observed event occurs *significantly less than* or *significantly greater than* what we typically expect with that assumption, we conclude that the assumption is probably not correct.

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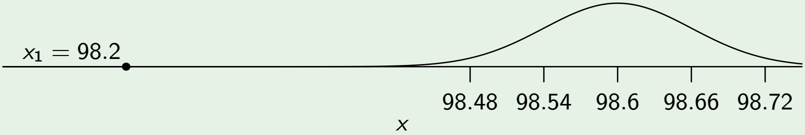

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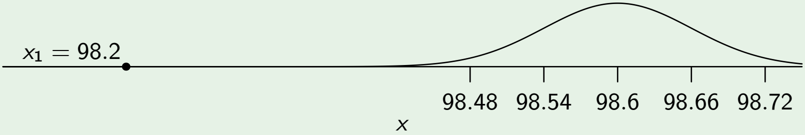

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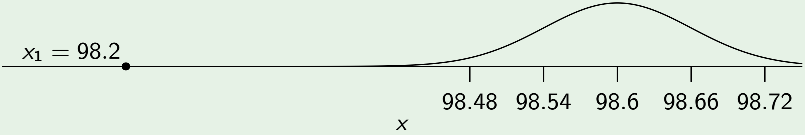

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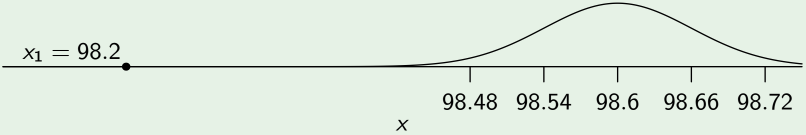

Using technology we see that the probability is 0.0000000000155.

If 98.6°F is the mean of our body temperature, there is a very small probability of getting a sample mean of 98.2°F.

It is then reasonable to conclude that the population mean is lower than 98.6°F. (It's really closer to 98.2°F.)

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Correction for a Finite Population

When sampling without replacement and the sample size n is greater than 5% of the finite population size N (that is $n > 0.05N$), adjust the standard deviation of sample means $\sigma_{\bar{x}}$ by multiplying it by this **finite population correction factor**:

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Note

You *do not* use the finite population correction factor when:

- You sample with replacement.
- The population if infinite.
- Sample size does not exceed 5% of the population size.