Dynamical Systems: Modeling

Department of Mathematics

Salt Lake Community College

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The study of multidimensional systems will be aided by the study of **linear algebra** in chapters 3 and 5.

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But, if all we care about is the temperature of the coffee, we can use a limited model called Newton's Law of Cooling, which incorporates the surrounding temperature to give an accurate description of the coffee's temperature.

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Note

We will only be studying ODE's in this class.

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- $\dfrac{\partial^2 y}{\partial x^2} + \dfrac{\partial^2 z}{\partial t^2} = xyz$ is a second-order PDE with independent variables x and t and dependent variables y and z.

Constants of Proportionality

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- The rate of change of y is **directly proportional** to y^2 and **inversely proportional** to $\sqrt{t}$:

$$\frac{dy}{dt} = k\frac{y^2}{\sqrt{t}}$$

Example 3

Exponential Growth The population P is growing at a rate proportional to the population at any time t:

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Example 4

Exponential Decay Let A be the amount of radioactive material in a sample at any time t. The amount A is decreasing at a rate proportional to the amount at any time t:

$$\frac{dA}{dt} = kA, \quad k < 0$$

Example 5

Newton's Law of Cooling or Heating the rate of change of temperature T of an object is proportional to the difference between the temperature M of the surroundings and the temperature of the object:

$$\frac{dT}{dt} = k(M - T), \quad k > 0$$