One-sample means with the t-distribution

Colby Community College

Central Limit Theorem for the Sample Mean

When we collect a sufficiently large sample of n independent observations from a population with mean μ and standard deviation σ, the sampling distribution of $\bar{x}$ will be nearly normal with:

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Note

It's rare to need to estimate the population mean μ, but somehow know the population standard deviation σ. In most cases σ will need to be estimated.

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- $n \geq 30$: If the sample size n is at least 30 and there are no *particularly extreme outliers*, then we typically assume the sampling distribution of $\bar{x}$ is nearly normal, even if the underlying population is not.

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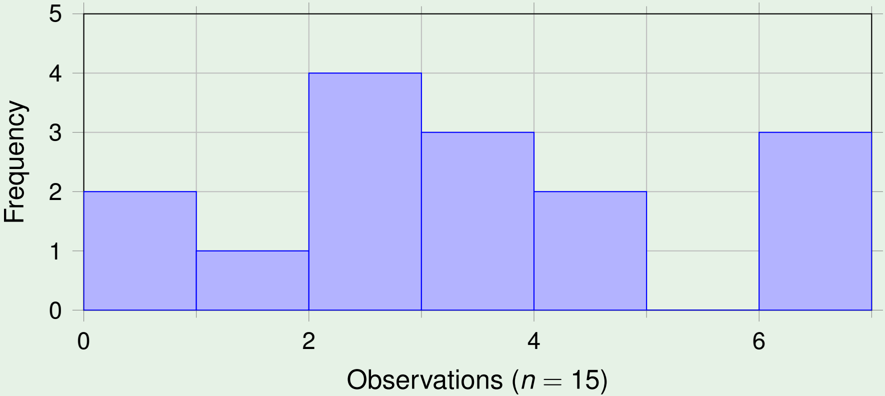
Note

In a first course in statistics, you aren't expected to develop perfect judgment on the normality condition.

Example 1



Is the normality condition met?

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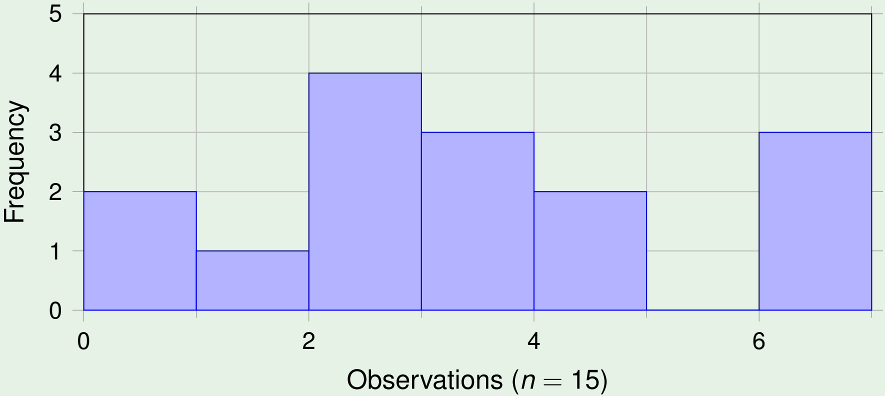
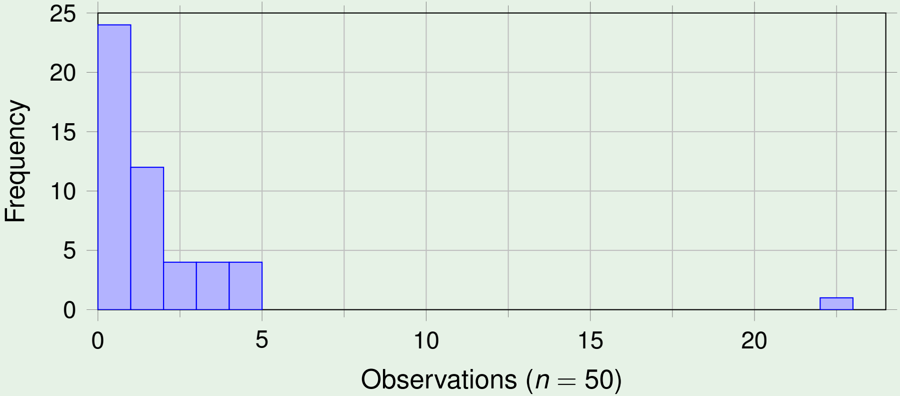


Is the normality condition met?

Since there are less than 30 observations, we need to look for *clear* outliers. While there is a gap on the right, the gap is small and 20% of the observations fall in rightmost bar. We can't really call these clear outliers, so the normality condition is reasonably met.

Example 2



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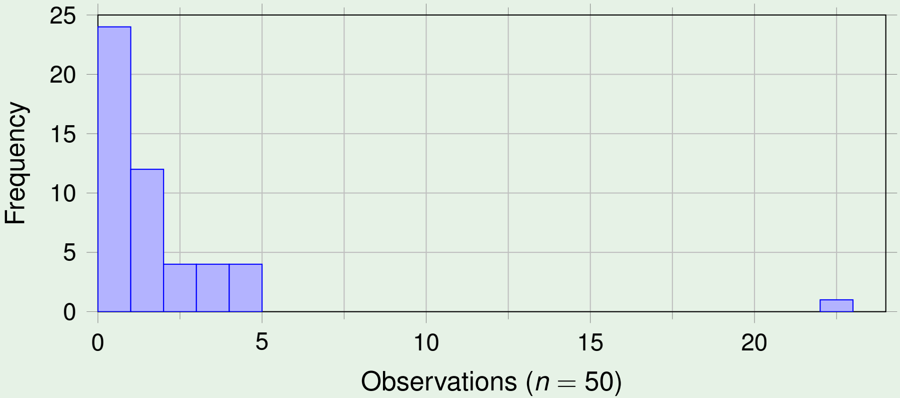

Is the normality condition met?

The sample size is greater than 30, so we need to look for an extreme outlier. The gap is more than four times the width of the cluster on the left side, so this is clearly an extreme outlier and the normality condition is not met.

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If a population has a normal distribution, then the distribution of

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A Student t distribution is commonly called a ***t* distribution**.

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When modeling $\bar{x}$ using the t-distribution, use:

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We can freely assign values to the first 9 scores, but the 10th score would need to be:

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Hence 9 degrees of freedom.

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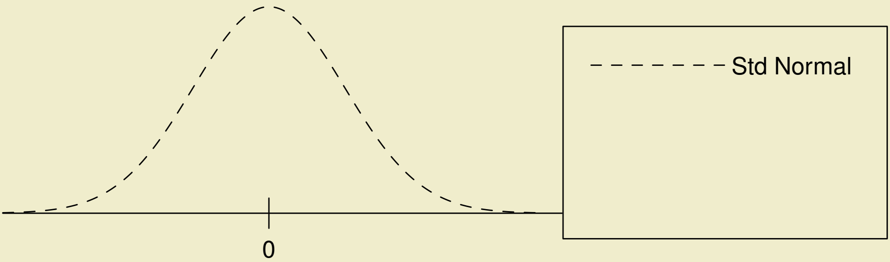
The Student *t* distribution changes for different degrees of freedom.



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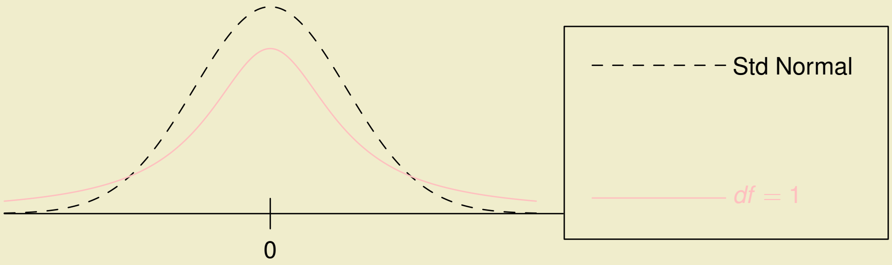

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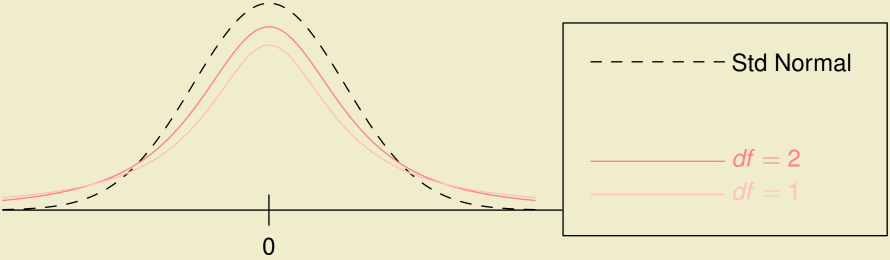

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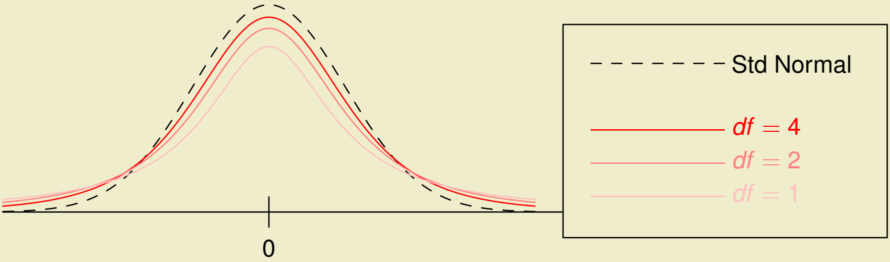

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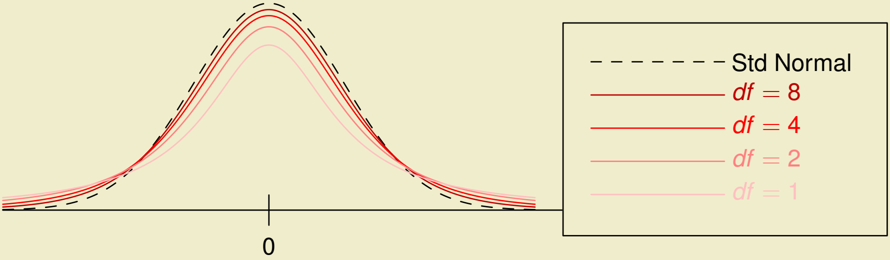

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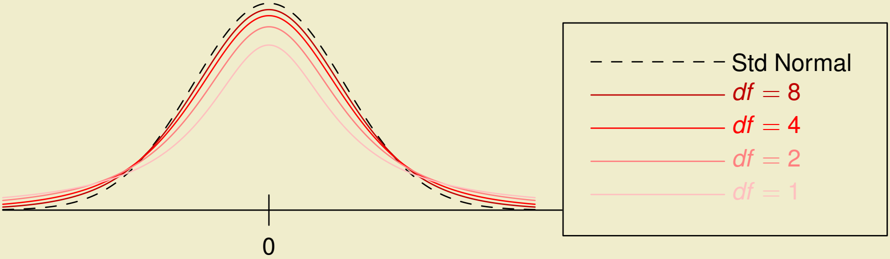

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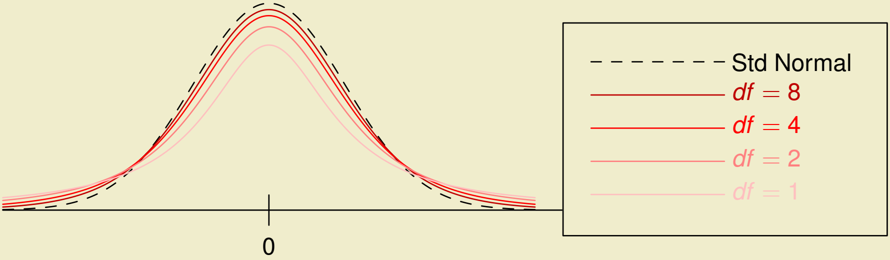

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Note

As the sample size gets larger, the Student t distribution gets closer to the standard normal distribution.