Determinants and Cramer's Rule

Department of Mathematics

Salt Lake Community College

Determinant of a 2×2 Matrix

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Example 2

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Example 2

$$\begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix} = 3 \cdot (1) - (-2) \cdot 6 = 15$$

Minors of a Matrix

For very element a_{ij} of a $n \times n$ matrix $\boldsymbol{A}$, the **minor** $\boldsymbol{M_{ij}}$ is an $(n-1) \times (n-1)$ matrix obtained by deleting the ith row and the jth column of $\boldsymbol{A}$.

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Cofactors of a Matrix

For every element a_{ij} of a $n \times n$ matrix $\boldsymbol{A}$, the **cofactor** of a_{ij} is the scalar

$$C_{ij} = (-1)^{(i+j)} \left| \boldsymbol{M_{ij}} \right|$$

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Expansion by the *i*th row:

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Expansion by the jth column:

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I recommend you always expand across the first row.

Example 4

Compute the determinant

$$\begin{vmatrix} 3 & 1 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 2 \end{vmatrix}$$

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Expanding across the first row gives:

$$\begin{vmatrix} \boxed{3} & 1 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 2 \end{vmatrix} = \boxed{+}\ \boxed{(+3)} \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}$$

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Example 4

Compute the determinant

$$\begin{vmatrix} 3 & 1 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 2 \end{vmatrix}$$

Expanding across the first row gives:

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$$= 3(1 \cdot 2 - 3 \cdot 1) - (2 \cdot 2 - 3 \cdot 0) - (2 \cdot 1 - 1 \cdot 0)$$

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&= 3(1 \cdot 2 - 3 \cdot 1) - (2 \cdot 2 - 3 \cdot 0) - (2 \cdot 1 - 1 \cdot 0) \\
&= -3 - 4 - 2
\end{aligned}$$

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Expanding across the first row gives:

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$$= 3(6 \cdot 3 - 2 \cdot (-2)) + 0(4 \cdot 3 - 2 \cdot 8) - (4 \cdot (-2) - 6 \cdot 8)$$

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&= 3(6 \cdot 3 - 2 \cdot (-2)) + 0(4 \cdot 3 - 2 \cdot 8) - (4 \cdot (-2) - 6 \cdot 8) \\
&= 66 + 0 + 56 \\
&= 122
\end{aligned}$$

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Invertibility Criterion

If $\boldsymbol{A}$ is a square matrix, then A has an inverse if and only if $|\boldsymbol{A}| \neq 0$.

Cramer's Rule

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$$\begin{array}{ccccc} ax & + & by & = & s \\ cx & + & dy & = & t \end{array}$$

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We then have the following three determinants:

$$|\boldsymbol{D}| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \qquad |\boldsymbol{D_x}| = \begin{vmatrix} s & b \\ t & d \end{vmatrix} \qquad |\boldsymbol{D_y}| = \begin{vmatrix} a & s \\ c & t \end{vmatrix}$$

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The solutions is:

$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{\begin{vmatrix} s & b \\ t & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad y = \frac{|\boldsymbol{D_y}|}{|\boldsymbol{D}|} = \frac{\begin{vmatrix} a & s \\ c & t \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

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Note

This method can be extended to any size system of equations.

Example 6

Consider the system

$$\begin{array}{rcrcr} x & + & 2y & = & 5 \\ 2x & + & 3y & = & 8 \end{array}$$

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Let us solve this system using Cramer's Rule.
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$$|\boldsymbol{D}| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \qquad |\boldsymbol{D_x}| = \begin{vmatrix} 5 & 2 \\ 8 & 3 \end{vmatrix} \qquad |\boldsymbol{D_y}| = \begin{vmatrix} 1 & 5 \\ 2 & 8 \end{vmatrix}$$

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We can now find x

$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|}$$

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$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{-1}{-1}$$

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$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{-1}{-1} = 1$$

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We can now find x and y

$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{-1}{-1} = 1 \qquad y = \frac{|\boldsymbol{D_y}|}{|\boldsymbol{D}|}$$

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$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{-1}{-1} = 1 \qquad y = \frac{|\boldsymbol{D_y}|}{|\boldsymbol{D}|} = \frac{-2}{-1} = 2$$

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Consider the system

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Let us solve this system using Cramer's Rule.
This means we need to calculate the following determinants.

$$|\boldsymbol{D}| = \begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix} = 15 \qquad |\boldsymbol{D_x}| = \begin{vmatrix} 4 & -2 \\ 13 & 1 \end{vmatrix} \qquad |\boldsymbol{D_y}| = \begin{vmatrix} 3 & 4 \\ 6 & 13 \end{vmatrix}$$

Example 7

Consider the system

$$\begin{array}{rcrcr} 3x & - & 2y & = & 4 \\ 6x & + & y & = & 13 \end{array}$$

Let us solve this system using Cramer's Rule.
This means we need to calculate the following determinants.

$$|\boldsymbol{D}| = \begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix} = 15 \quad |\boldsymbol{D_x}| = \begin{vmatrix} 4 & -2 \\ 13 & 1 \end{vmatrix} = 30 \quad |\boldsymbol{D_y}| = \begin{vmatrix} 3 & 4 \\ 6 & 13 \end{vmatrix}$$

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We can now find x

$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|}$$

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$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{30}{15}$$

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We can now find x

$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{30}{15} = 2$$

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We can now find x and y

$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{30}{15} = 2 \qquad y = \frac{|\boldsymbol{D_y}|}{|\boldsymbol{D}|}$$

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$$x = \frac{|\boldsymbol{D_x}|}{|\boldsymbol{D}|} = \frac{30}{15} = 2 \qquad y = \frac{|\boldsymbol{D_y}|}{|\boldsymbol{D}|} = \frac{15}{15} = 1$$