Sampling Distributions and Estimators

Colby Community College

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Each of these $50,000$ newbie surveyors reported the percentage that they found, with results such as 68%, 72%, 70%, etc. . .

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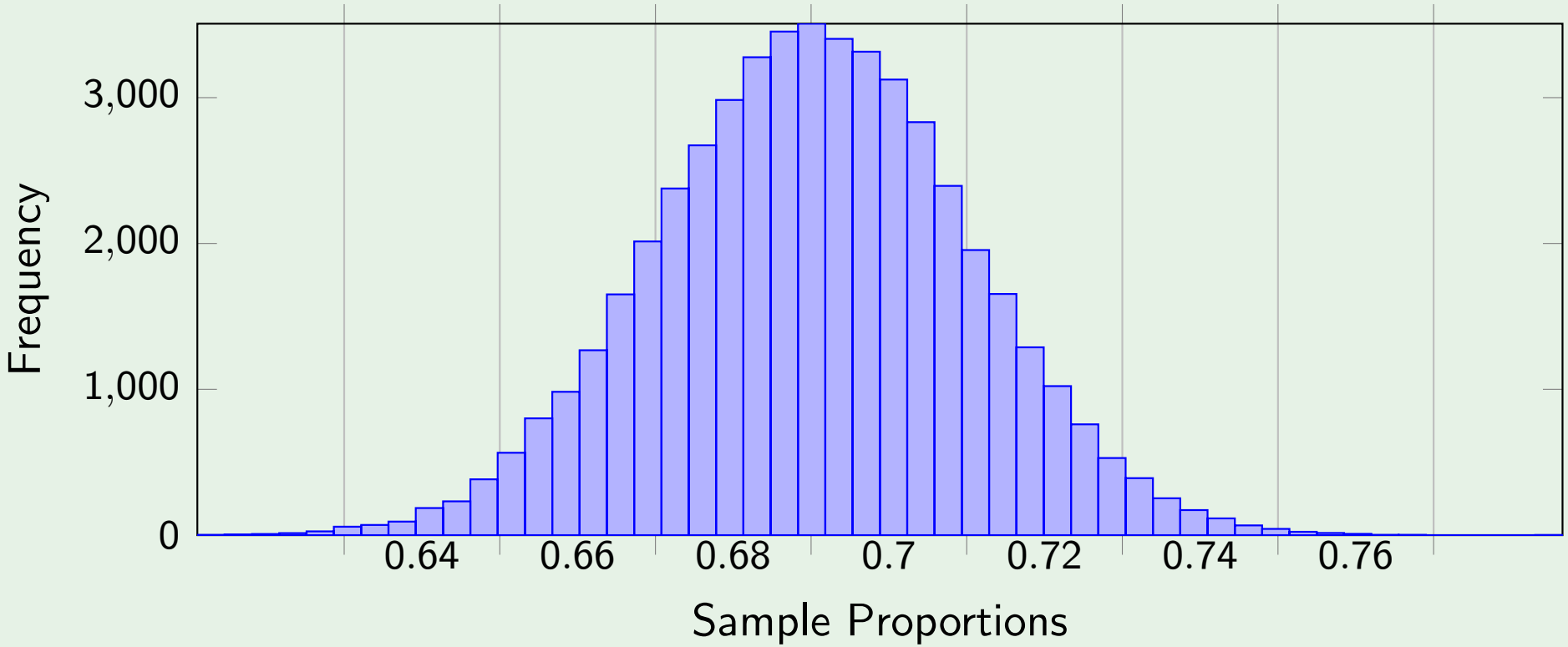
Lets assume we gather all $50,000$ of the percentages and convert them to proportions. We can then construct the histogram:



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When samples of the same size are taken from the same population:

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Definition

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Notation

$$\begin{aligned} p &= \text{population proportion} \\ \hat{p} &= \text{sample proportion} \end{aligned}$$

Behavior of Sample Proportions

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2. Sample proportions target the value of the population proportion in the sense that the mean of all the sample proportions $\hat{p}$ is equal to the population proportion p. The expected value of the sample proportion is equal to the population proportion.

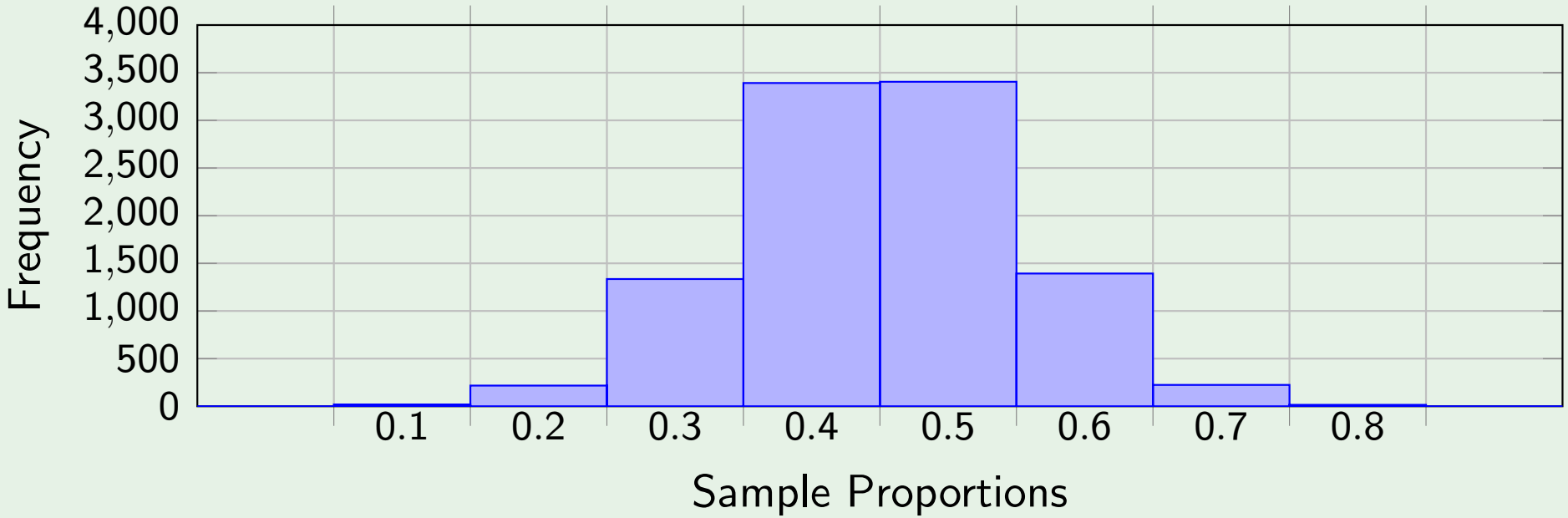
Example 2

Consider repeating the following process:

- Roll a six-sided die 5 times.
- Find the proportion of odd numbers.

What can we say about the behavior of all sample proportions that are generated as this process continues indefinitely?

Repeating this process $10,000$ times gives the following distribution of sample proportions.



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Sample	$\bar{x}$	**Probability**
4,4	4.0	1/9
4,5	4.5	1/9
4,9	6.5	1/9
5,4	4.5	1/9
5,5	5.0	1/9
5,9	7.0	1/9
9,4	6.5	1/9
9,5	7.0	1/9
9,9	9.0	1/9

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Condensing the sample means gives:

$\bar{x}$	Probability	$\bar{x}$	Probability
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The mean of the population $\{4, 5, 9\}$ is $\mu = 0.6$. Using either version of the table, we find that the mean of the sample values is 6.0.

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- Select 5 values from the population, then find the mean $\bar{x}$ of the results.

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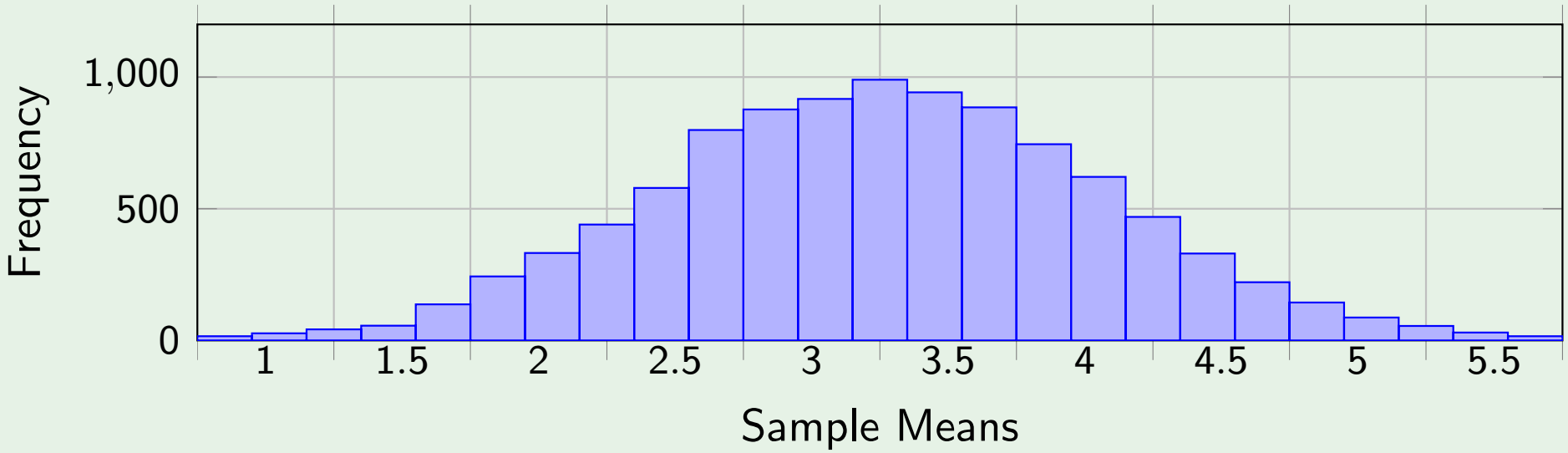

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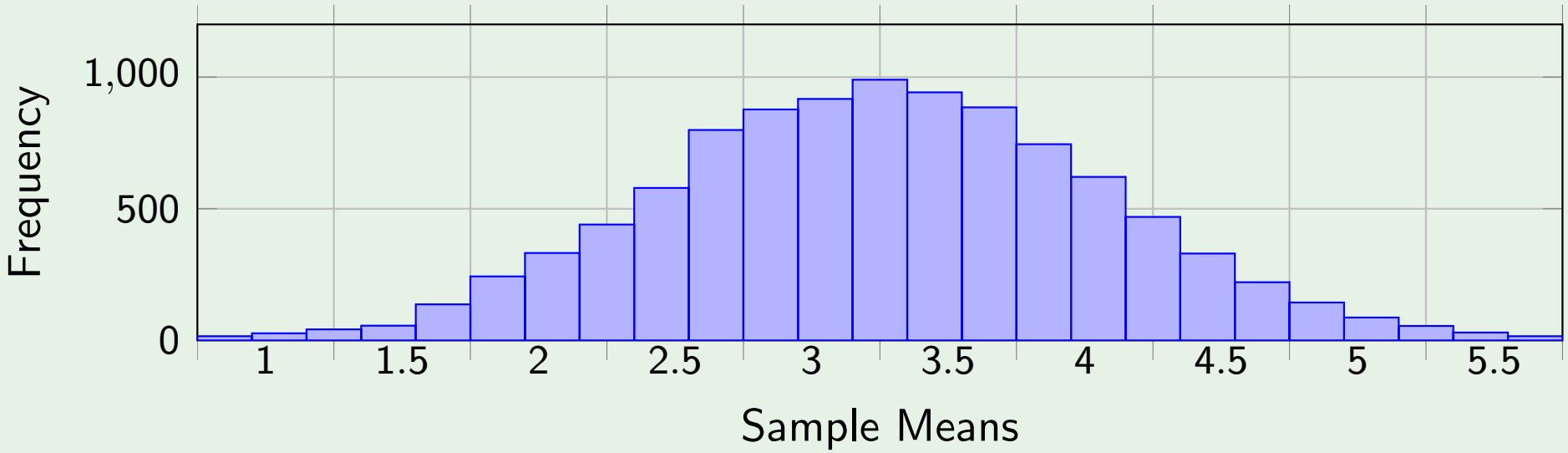

If the process continues, the mean of the sample means will be 3.5.

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Reminder

When working with population you divide by the population size N:

$$\text{Population standard deviation:} \quad \sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}}$$

$$\text{Population Variance:} \quad \sigma^2 = \frac{\sum (x-\mu)^2}{N}$$

Behavior of Sample Variances

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2. The sample variances target the value of the population variance in the sense that mean of the sample variances is the population variance. The expected value of the sample variance is equal to the population variance.

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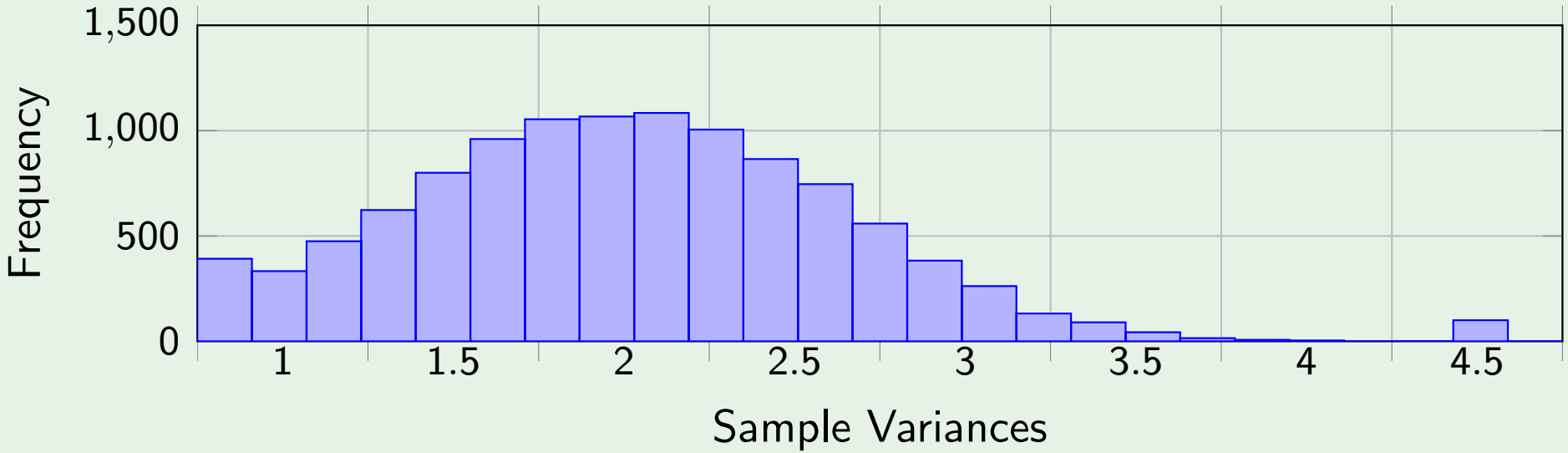

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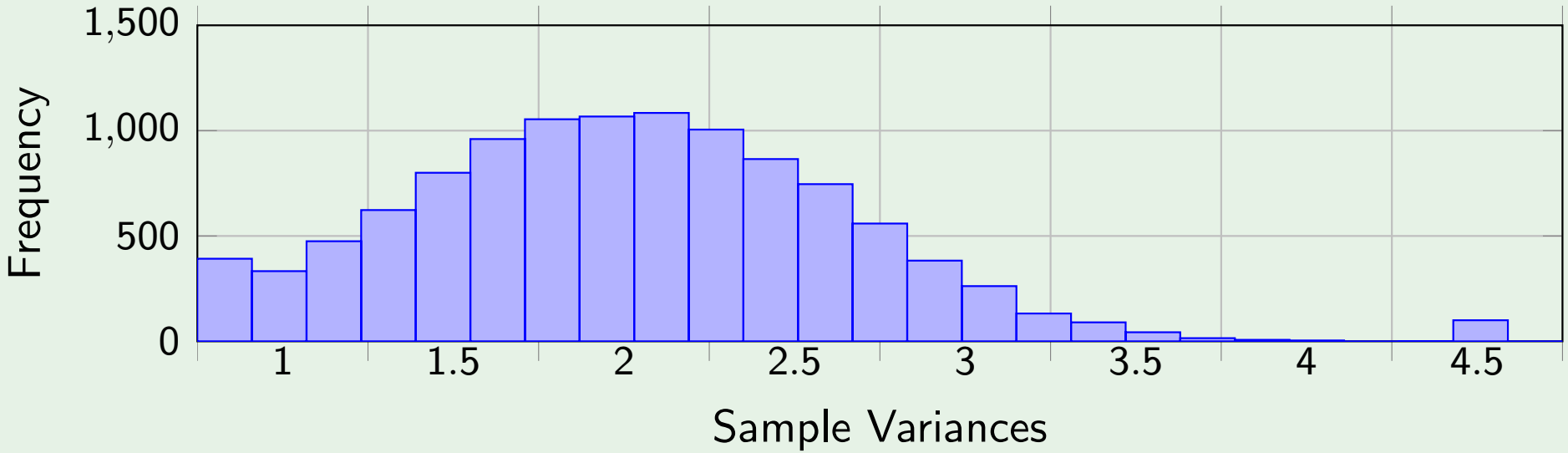

The histogram shows that the distribution of the sample variances is a skewed distribution, not a normal distribution.

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An **biased estimator** is a statistic that is not an unbiased estimator.

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Biased Estimators

These statistics do not target the value of the corresponding population parameters.

- Median
- Range
- Standard deviation s

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1. When selecting a relatively small sample from a large population, it makes no significant difference whether we sample with replacement or without replacement.
2. Sampling with replacement results in independent events that are unaffected by previous outcomes, and independent events are easier to analyze and result in simpler calculation and formulas.