Defining Probability

Colby Community College

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The 1 that was rolled.

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If the dice is fair, each side has the same chance of being rolled. So a 1 has a one-in-six chance, equivalently $\frac{1}{6}$.

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Example 2

What is the probability of rolling a 1 or 2 on a die?

There are two outcomes, a 1 or a 2, and six faces on a die.

$$P(\text{roll 1 or 2}) = \frac{2}{6} = \frac{1}{3}$$

Note

A standard deck of 52 playing cards consists of four **suits** in two colors: Hearts ♥, Spades ♠, Diamonds ♦, and Clubs ♣

Each suit contains 13 cards, each of a different **rank**:
2 through 10, Jack, Queen, King, and Ace

The Jack, Queen, and King cards are called **face cards.**

The Jack, Queen, King, and Ace cards are called **honour cards.**

The cards numbered 2 to 10 are called **numerals.**

♣2	♣3	♣4	♣5	♣6	♣7	♣8	♣9	♣10	♣J	♣Q	♣K	♣A
♦2	♦3	♦4	♦5	♦6	♦7	♦8	♦9	♦10	♦J	♦Q	♦K	♦A
♥2	♥3	♥4	♥5	♥6	♥7	♥8	♥9	♥10	♥J	♥Q	♥K	♥A
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Every side of the die is listed, so

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An outcome with a probability of 1 is called **certain.**

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An outcome with a probability of 0 is called **impossible**.

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Probabilities are always between 0 and 1.

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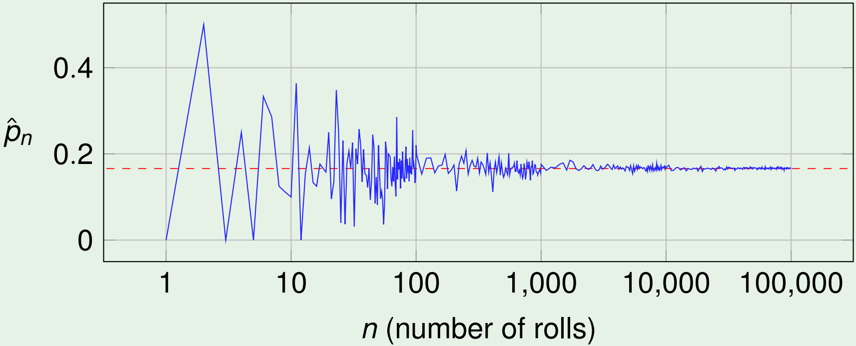
Let $\hat{p}_n$ be the proportion of number of 1's rolled after n rolls.



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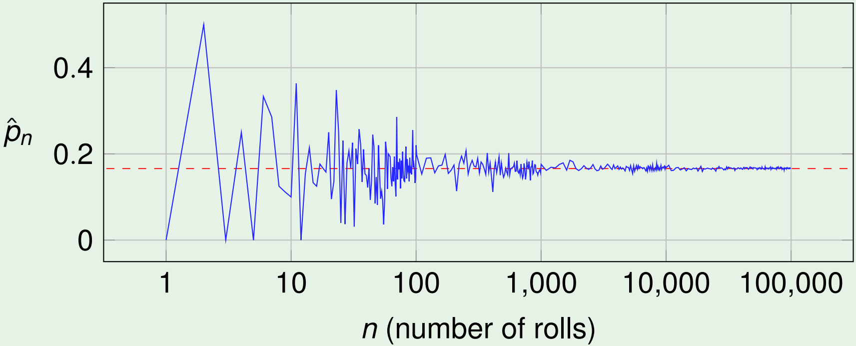

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It is not a coincidence that $\hat{p}_n$ get closer to p as n increases.

Law of Large Numbers

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- If we know nothing about the likelihood of different possible outcomes, we should not assume that they are equally likely.
  - You should not think that the probability of passing the next exam is $\frac{1}{2}$, or 0.5. The actual probability depends on factors such as the amount of preparation and the difficulty of the exam.

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Are the outcomes "draw an ace" and "draw a diamond" disjoint?

No, the ♦A is both an ace and a diamond.

Addition Rule of Disjoint Outcomes

If A_1 and A_2 represent two disjoint outcomes, then the probability that one of them occurs is given by:

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2)$$

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$$\begin{aligned} P(\text{roll a 1 or roll a 2}) &= P(\text{roll a 1}) + P(\text{roll a 2}) \\ &= \frac{1}{6} + \frac{1}{6} = \frac{1+1}{6} = \frac{2}{6} = \frac{1}{3} \end{aligned}$$

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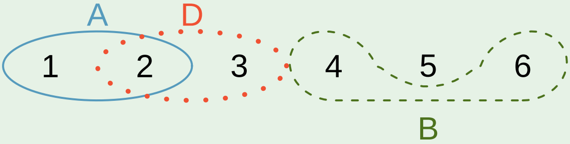

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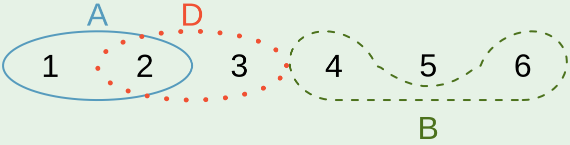

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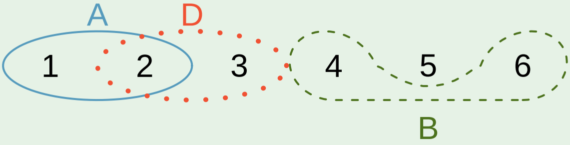

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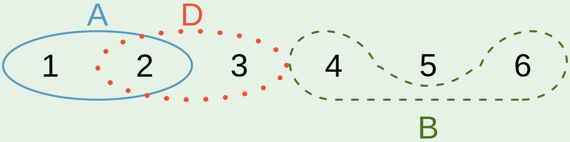

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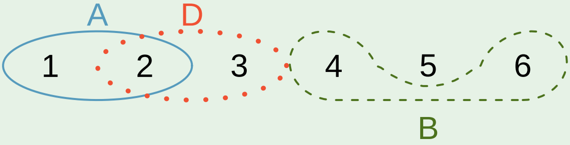

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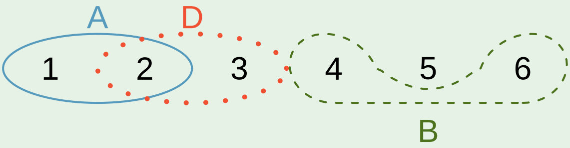

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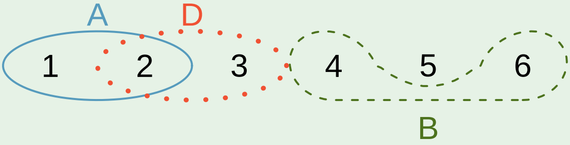

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Suppose we flip a fair coin and roll a fair die.

The list of all possible outcomes is:

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$$P\,(\text{H or 6}) = P\,(\text{H}) + P\,(6) - P\,(\text{H and 6}) = \frac{1}{2} + \frac{1}{6} - \frac{1}{12}$$

Example 14

Suppose we flip a fair coin and roll a fair die.

The list of all possible outcomes is:

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

We want to calculate the probability of getting a head or a six.

The relevant outcomes are: H1, H2, H3, H4, H5, H6, T6.

Meaning that $P\,(\text{H or 6}) = \frac{7}{12}$.

Notice that $\frac{6}{12} = \frac{1}{2}$ of the outcomes have heads and $\frac{2}{12} = \frac{1}{6}$ have a six.

But, $\frac{1}{2} + \frac{1}{6} = \frac{8}{12}$, is wrong because we have double counted H6.

The correct probability is:

$$P\,(\text{H or 6}) = P\,(\text{H}) + P\,(6) - P\,(\text{H and 6}) = \frac{1}{2} + \frac{1}{6} - \frac{1}{12} = \frac{7}{12}$$

Example 15

Let us consider the events “draw a diamond” and “draw a face card”.

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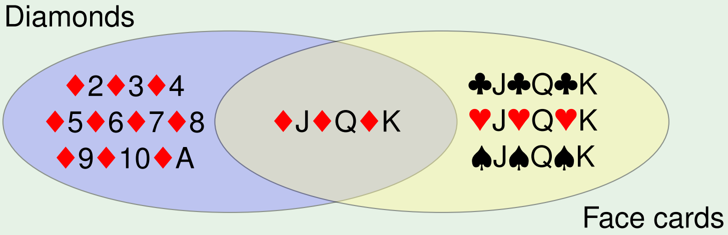
These outcomes are not disjoint, since three cards are both:



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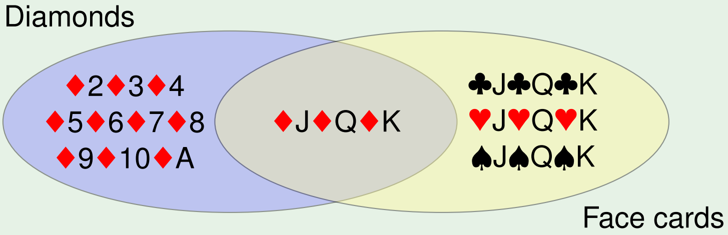

The Addition Rule for Disjoint Outcomes would count ♦J♦Q♦K twice!

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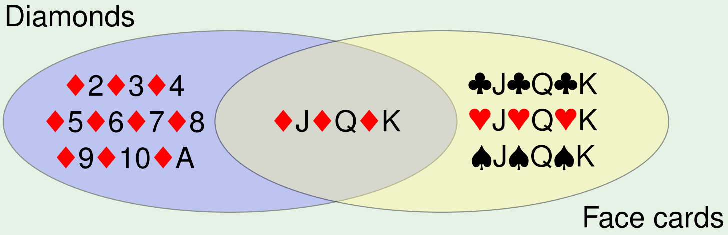

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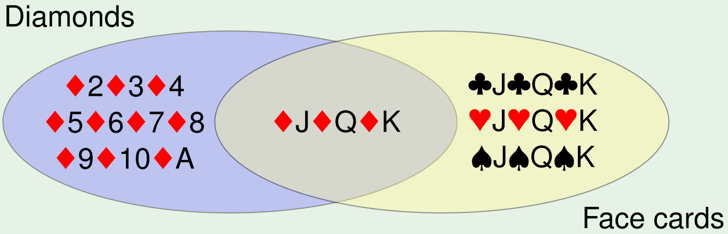

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$$
\begin{aligned}
P(\blacklozenge \text{ and face}) &= P(\blacklozenge) + P(\text{face}) - P(\blacklozenge \text{ and face}) \\
&= \frac{13}{52}
\end{aligned}
$$

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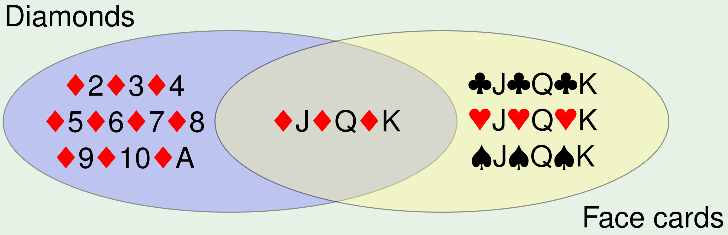

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$$P(\text{♦ and face}) = P(\text{♦}) + P(\text{face}) - P(\text{♦ and face})$$
$$= \frac{13}{52} + \frac{12}{52}$$

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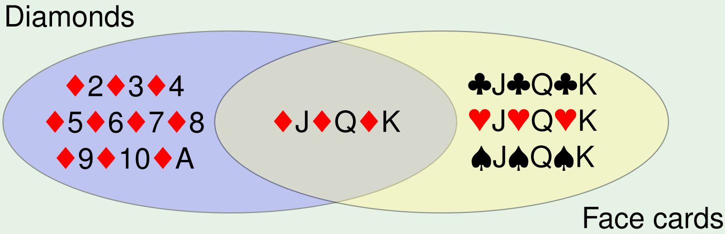

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$$P(\diamondsuit \text{ and face}) = P(\diamondsuit) + P(\text{face}) - P(\diamondsuit \text{ and face})$$
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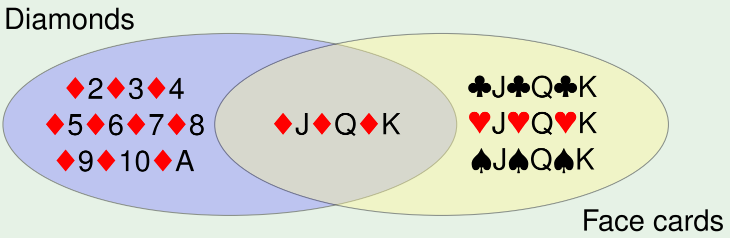

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$$P(\diamondsuit \text{ and face}) = P(\diamondsuit) + P(\text{face}) - P(\diamondsuit \text{ and face})$$
$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$
$$= \frac{22}{52} = \frac{11}{26}$$

General Addition Rule

If A and B are any two events, disjoint or not, then the probability that at least one of them will occur is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

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Note

In statistics, when we write "or" what we mean is "and/or", unless we explicitly say otherwise.

In other words, "A or B" occurring means A, B, or both A and B occur.

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In statistics, when we write "or" what we mean is "and/or", unless we explicitly say otherwise.

In other words, "A or B" occurring means A, B, or both A and B occur.

Note

If A and B are disjoint this means $P(A \text{ and } B) = 0$, and so we get the Addition Rule for Disjoint Outcomes:

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= P(A) + P(B) - 0 \\ &= P(A) + P(B) \end{aligned}$$

Example 16

Suppose we draw one card from a standard deck. Let us find the probability that we get a Queen or a King.

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There are 4 Queens and 4 Kings in the deck, hence eight outcomes out of 52 possible outcomes. So, the probability is

$$P\left(\text{Q or K}\right) = \frac{8}{52}$$

Since there are no cards that are both Kings and Queens, we have

$$P\left(\text{Q or K}\right) = P\left(\text{Q}\right) + P\left(K\right) - P\left(\text{Q and K}\right) = \frac{4}{52} + \frac{4}{52} - \frac{0}{52} = \frac{8}{52}$$

Definition

A **probability distribution** is a table of all disjoint outcomes and their associated probabilities.

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Example 17

The probability distribution for the sum of two dies:

Dice sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

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Rules for Probability Distributions

All probability distributions must satisfy the following rules:

1. The outcomes listed must be disjoint.
2. Each probability must be between 0 and 1.
3. The probabilities must sum to 1.

Example 18

The follow table contain three possible distributions for household income in the United States.

Income Range	\$0-\$25k	\$25k-\$50k	\$50k-\$100k	\$100k+
a	0.18	0.39	0.33	0.16
b	0.38	-0.27	0.52	0.37
c	0.28	0.27	0.29	0.16

Only one of the three is actually a probability distribution. Which one?

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- a sums to 2.5.
- b contains a negative number.
- c is the real probability distribution.

Note

A probability distribution can be represented with a bar plot, where each outcome is represented by a bar, the height of the bar being the probability of the outcome.

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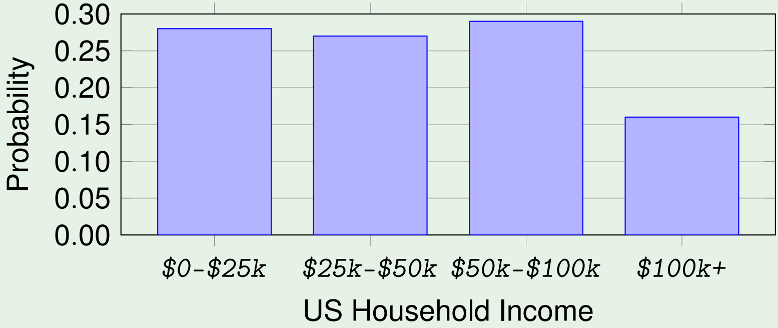
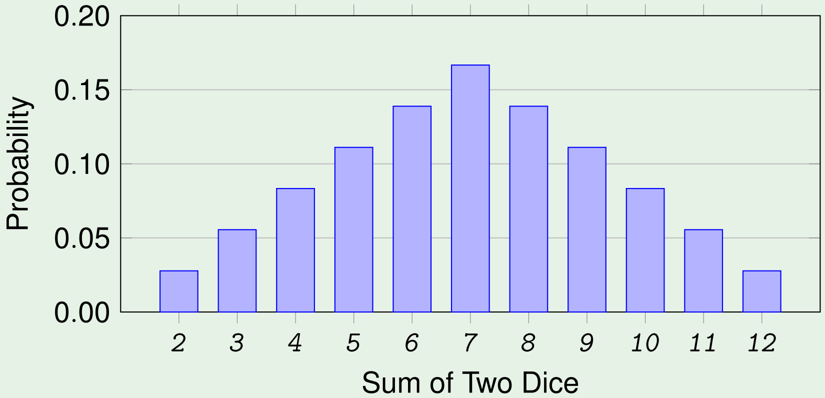
Example 19

Here is the bar plot for the probability distribution in Example 18:



Example 20

Here is the bar plot for the dice sum distribution in Example 17



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The sample space of rolling a single die is:

$$\{\text{roll a } 1, \text{roll a } 2, \text{roll a } 3, \text{roll a } 4, \text{roll a } 5, \text{roll a } 6\}$$

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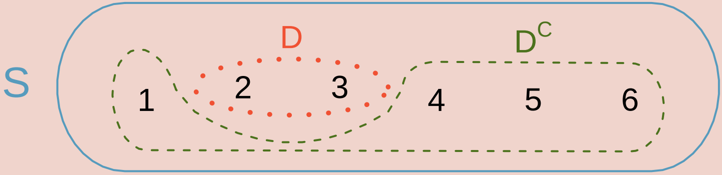
Definition

The **complement** of an event D is all the outcomes in the sample space that are not in D. Denoted D^C.



Note

A and A^C are disjoint events.

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$$P\left(A \text{ or } A^C\right) = 1$$

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We can also use the Addition Rule $\left(P\left(A \text{ and } A^C\right) = 0\right)$:

$$P\left(A \text{ or } A^C\right) = P(A) + P\left(A^C\right)$$

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Note

Finally, $P\left(A^C\right) = 1 - P(A)$ and $P(A) = 1 - P\left(A^C\right)$.

Example 22

Consider rolling two dice and summing the numbers.

What is the complement of the event “the total is less than 12?”

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&= 1 - \left(\frac{1}{36} + \frac{2}{36}\right) \\
&= 1 - \frac{3}{36}
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1. A fair coin is tossed two times. The two events are:
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Note

If two events are not independent, they are called **dependent**.

Multiplication Rule For Independent Events

If events A and B are independent, then the probability of both A and B occurring is

$$P\left(A \text{ and } B\right) = P\left(A\right) \cdot P\left(B\right)$$

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The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

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The probability of getting tails on the coin and three on the die is

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The probability of getting tails on the coin and three on the die is

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Since a coin and dice are independent, we could have used the Multiplication Rule:

$$P(\text{T and 3}) = P(\text{T}) \cdot P(3) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

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- *If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight?* $P(\text{safe flight}) = 1 - P(\text{system fails}) = 1 - 0.002 = 0.998$
- *Given the Airbus 310 has three independent hydraulic systems, what is the probability that the flight control system would work for an entire flight?*

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Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- *If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight?* $P\,(\text{safe flight}) = 1 - P\,(\text{system fails}) = 1 - 0.002 = 0.998$
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Note

A good rule of thumb is that the sample size is small if it is less than 10% of the total population size.

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Because the three adults are randomly selected without replacement, the three events are dependent. This means the exact probability is rather cumbersome to calculate:

$$\begin{aligned} P\,(\text{all use drugs}) &= P\left(1^{\text{st}} \text{ uses drugs and } 2^{\text{nd}} \text{ uses drugs and } 3^{\text{rd}} \text{ uses drugs}\right) \\ &= \left(\frac{33,337,369}{333,373,690}\right) \cdot \left(\frac{33,337,368}{333,373,689}\right) \cdot \left(\frac{33,337,367}{333,373,688}\right) \\ &= 0.000999999919 \end{aligned}$$

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Since 3 adults is a very small part of the total population, we can simplify the calculations considerably:

$$\begin{aligned} P\,(\text{all use drugs}) &= P\left(1^{\text{st}} \text{ uses drugs and } 2^{\text{nd}} \text{ uses drugs and } 3^{\text{rd}} \text{ uses drugs}\right) \\ &= 0.1 \cdot 0.1 \cdot 0.1 = 0.00100 \end{aligned}$$

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Since the equation holds, the two events must be independent.