Counting

Colby Community College

Multiplication Counting Rule

For a sequence of events in which the first event can occur n_1 ways, the second event can occur n_2 ways, the third event can occur n_3 ways, and so on. The total number of outcomes $n_1 \cdot n_2 \cdot n_3 \cdot \cdots$.

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Example 1

When making random guesses for an unknown four-digit passcode, each digit can be $0, 1, \ldots, 9$. What is the total number of different possible passcodes?

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Example 1

When making random guesses for an unknown four-digit passcode, each digit can be $0, 1, \ldots, 9$. What is the total number of different possible passcodes?

There are 10 possible choices for each digit, so the total number of passcodes is $10 \cdot 10 \cdot 10 \cdot 10 = 10,000$.

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- How many different travel iterations are there?
  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.
- What is the probability that the presidents are visited in order from younger to oldest? There is only one: $1/120 = .0083$.

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$$_{48}P_r = \frac{48!}{(48-6)!} = 8,835,488,640$$

Example 5

In a horse race, a trifecta bet is won by correctly selecting, in the correct order, the horses that finish first, second, and third. The 104th running Kentucky Derby has a field of 19 horses.

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There are $n = 19$ horses available, and we must select $r = 3$ of them without replacement. The number of permutations is

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We are assuming that all horses are equally likely to win the Kentucky Derby. In practice, this is not true. Some horses are faster than others.

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Example 7

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$$\frac{({}_4C_2)\,({}_{48}C_3)}{{}_{52}C_5} = \frac{6 \cdot 17,296}{2,598,960} = \frac{103,776}{2,598,960} \approx 4\%$$

Example 8

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The lowest ranked straight is A,2,3,4,5 and the highest ranked straight is 10,J,Q,K,A. Thus, for the any of the ten ranks A through 10, we can build a straight. Each card can be from any of the four suits.
This means the probability is:

$$P\left(\text{straight}\right) = \frac{10\left({}_4C_1\right)\left({}_4C_1\right)\left({}_4C_1\right)\left({}_4C_1\right)\left({}_4C_1\right)}{{}_{52}C_5} = \frac{10,240}{2,598,960} \approx 3.94\%$$

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Example 9

What is the probability that a poker hand will contain a straight, but not a straight flush?
We calculated in Example 7 the number of straights possible. Now, just need to subtract out the number of straight flushes.

$$P\,(\text{straight}) = \frac{10,240 - 40}{{}_{52}C_5} = \frac{10,200}{2,598,960} \approx 3.92\%$$