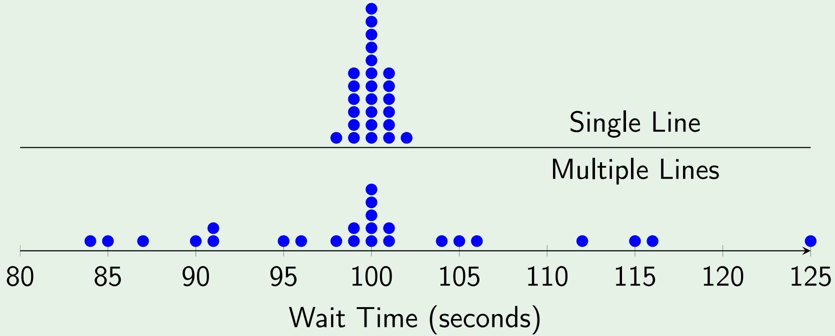
Measures of Variation

Colby Community College

Example 1

Consider the dotplot of waiting times at a bank.



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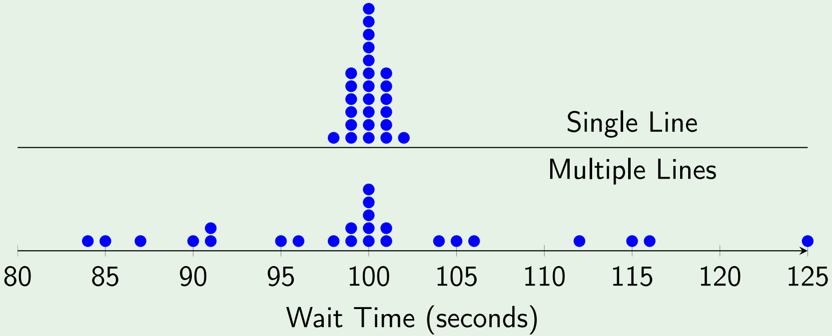

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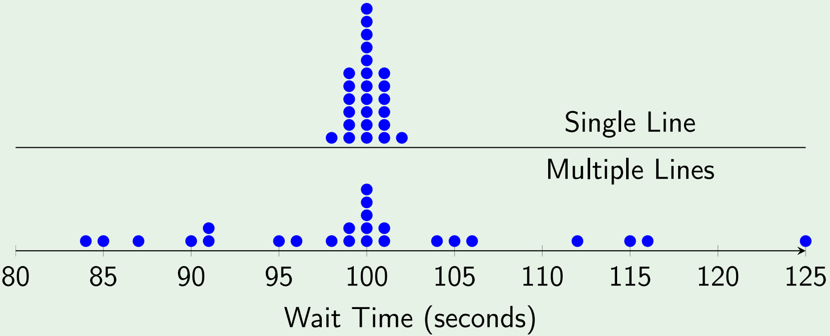

Both of these data sets have the same mean, but there are clearly different.

The bank didn't switch to multiple lines because it made them more efficient, nor because customer wait times were reduced, but because customers prefer waiting times with less variation.

Definition

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Properties

- The range is very sensitive to extreme values.
- Because the range only uses two values it does not reflect the true variation among all of the data values.

Example 2

Data set 32 "Airport Data Speeds" in Appendix B includes measures of data speeds of smartphones from four different carriers. The table contains five data speeds, in megabits per second (Mbps), from the data set.

Verizon	38.5	55.6	22.4	14.1	23.1

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Note

Just like measures of center, we want to round to one more decimal place that our data contains.

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A shortcut version that tends to be used by software is

$$s = \sqrt{\frac{n(\sum x^2) - (\sum x)^2}{n(n - 1)}}$$

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- The units of the standard deviation are the same units as the original data values.
- The sample standard deviation is a **biased estimator** of the population standard deviation. This means that the values of s for not center around the value of σ.

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Step 6: Find the square root of the result of **Step 5**.

The result of **Step 6** is s, the standard deviation.

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$$\sum x = 153.7$$
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This is the same result we found in Example 3.

Range Rule of Thumb for Identifying Significant Values

A crude, but simple tool for interpreting the standard deviation.

Significantly low values are $\mu - 2\sigma$ or lower.

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Range Estimate of the Standard Deviation

A rough estimate of the standard deviation is

$$s \approx \frac{\text{range}}{4}$$

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Using the full data set of Verizon data speeds (Data Set 32 Appendix B)

$$\bar{x} = 17.60 \text{ Mbps}$$
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Note

Based on this these results, we expect that typical airport Verizon data speeds are between -14.44 Mbps and 49.64 Mbps.

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Note

When using a calculator or computer, make sure you use proper button for either the sample or population standard deviation.

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Biased and Unbiased Estimators

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- The sample variance s^2 is an **unbiased estimator** of the population variance σ^2, which means that values of s^2 tend to center around the value of σ^2 instead of overestimating or underestimating.