Normal Distribution

Colby Community College

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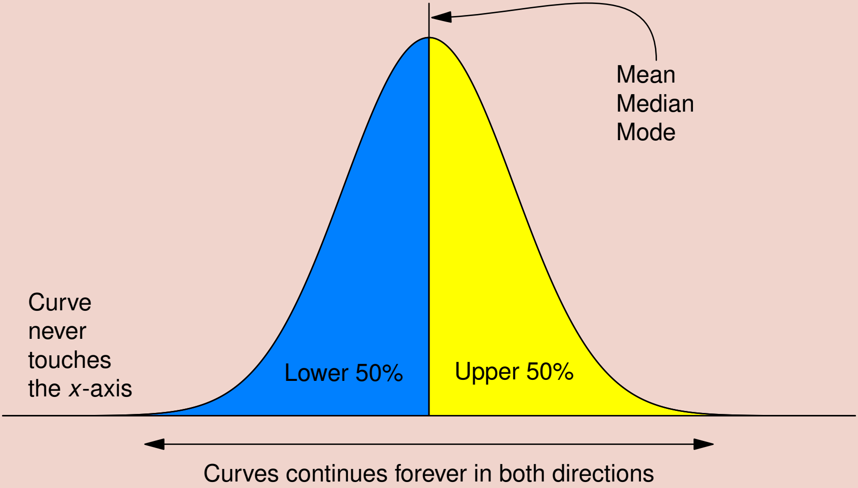
This goes one for a minute or so, and the popping gradually tappers off.

Most of the popping happens in that brief, noisy moment.

This demonstrates a typical pattern that is part of many phenomena.

Definition

A **normal distribution** is a perfectly symmetric, bell-shaped distribution. It is also referred to as a **normal curve** or a **bell curve**.



Note

The normal distribution with mean μ and standard distribution σ is denoted $N(\mu, \sigma)$, where μ and σ are called the parameters.

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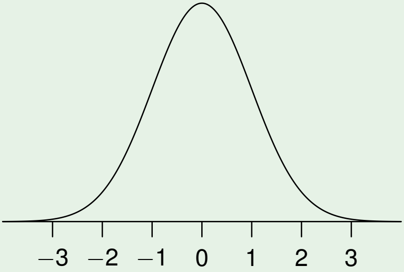
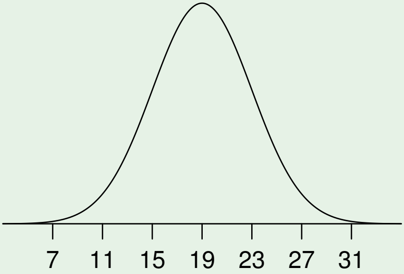
Example 2

Both are normal distributions, but with different center and spread.



(a) $N(\mu = 0, \sigma = 1)$



(b) $N(\mu = 19, \sigma = 4)$

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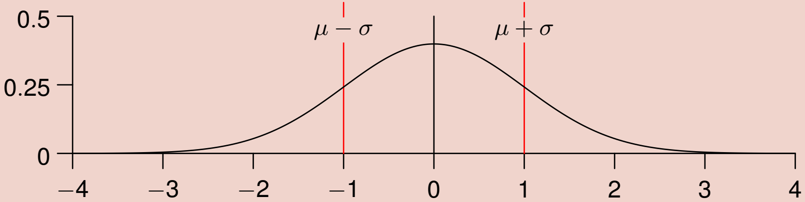
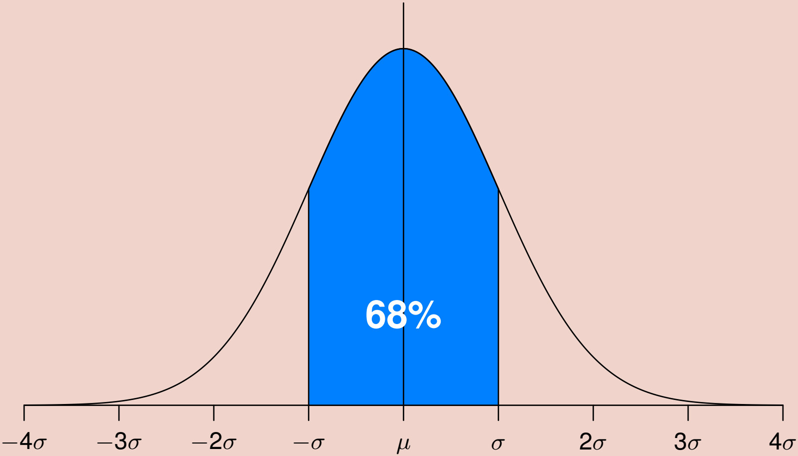
The special case $N(\mu = 0, \sigma = 1)$ is called the **standard normal distribution.** The total area under the curve is exactly equal to 1.



Definition

The **empirical rule**, or **68-95-99.7 rule**, states approximately how much of the area is contained when stepping one, two, or three standard deviations from the mean.

One standard deviation from the mean.



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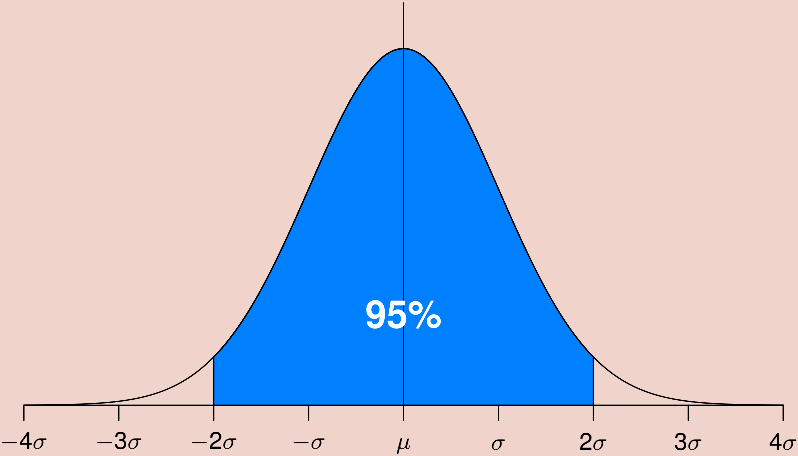
Two standard deviations from the mean.



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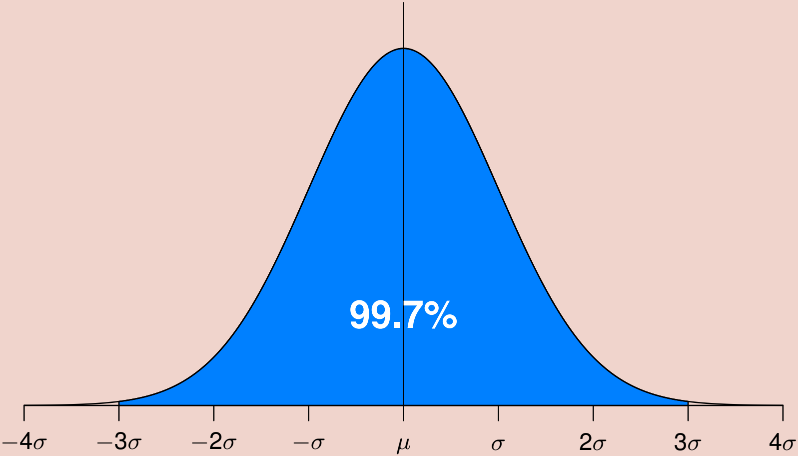
Three standard deviations from the mean.



Definition

A ***z*-score** is a measure of the number of standard deviations a particular data point is away from the mean.

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$$z = \frac{x - \mu}{\sigma} \quad \Rightarrow \quad z\sigma = x - \mu \quad \Rightarrow \quad x = z\sigma + \mu = (-1.3)(8) + 70 = 59.6$$

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Moreover, we know that roughly 95% of the scores fall within two standard deviations of the mean.

Which means that $95\% - 68\% = 27\%$ of the scores are more than one standard deviation from the mean, but less than two.

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Since the curve is symmetric, we know that 13.5% of the students scored between 89 and 96, as well as 13.5% between 68 and 75

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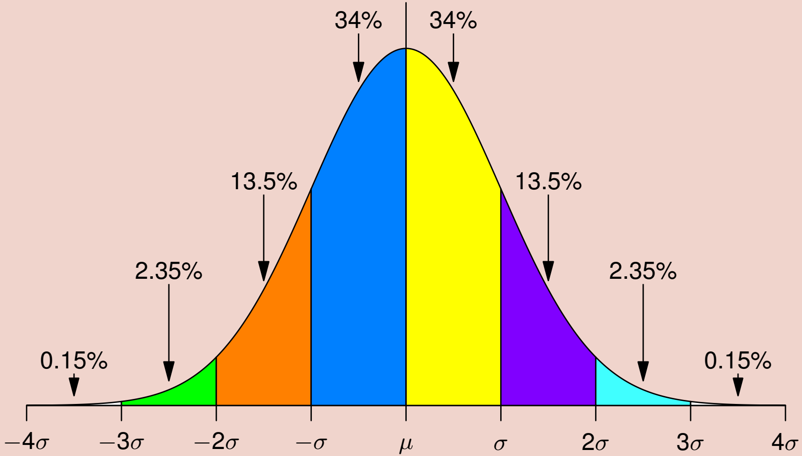
For each standard deviation.



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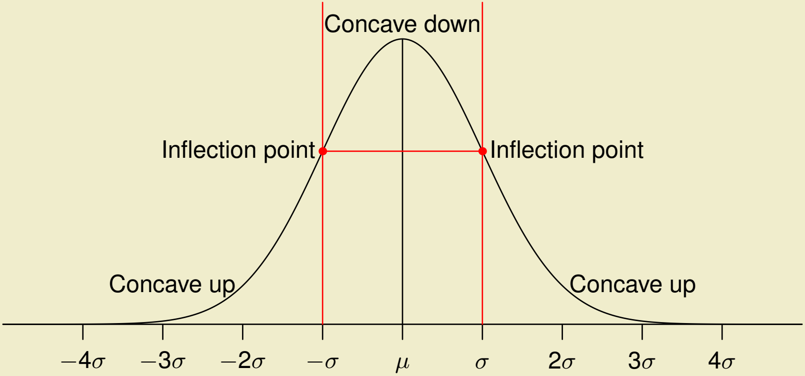
A normal density curve always has two inflection points, each one standard deviation from the mean.



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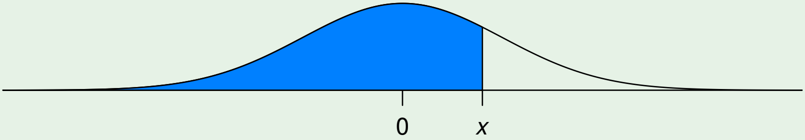

The area of the shaded region is the probability that a z score is less than or equal to x, $P(z \leq x)$.

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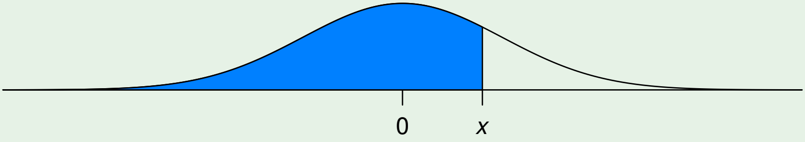

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Note

Most statistical software, programming languages, spreadsheets programs, and calculators are able to calculate the area for you.

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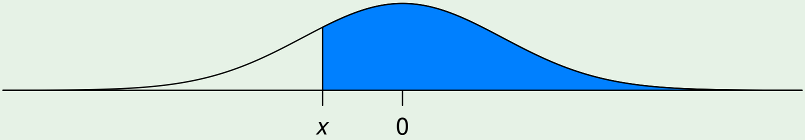

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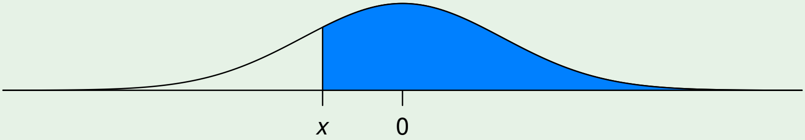

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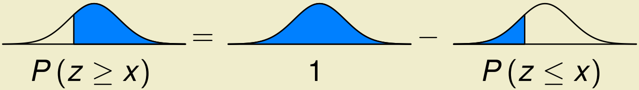
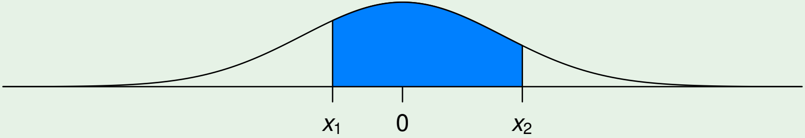


Example 8



The area of the shaded region is the probability that a z score lies between x_1 and x_2, $P(x_1 \leq z \leq x_2)$.

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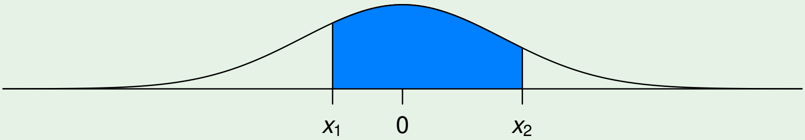

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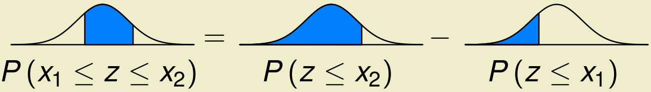

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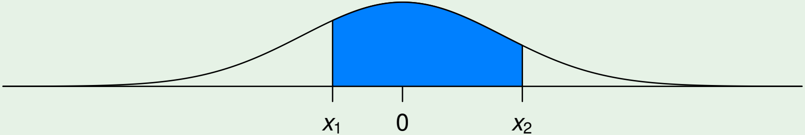

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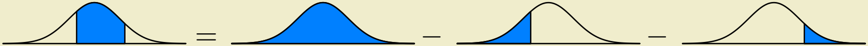
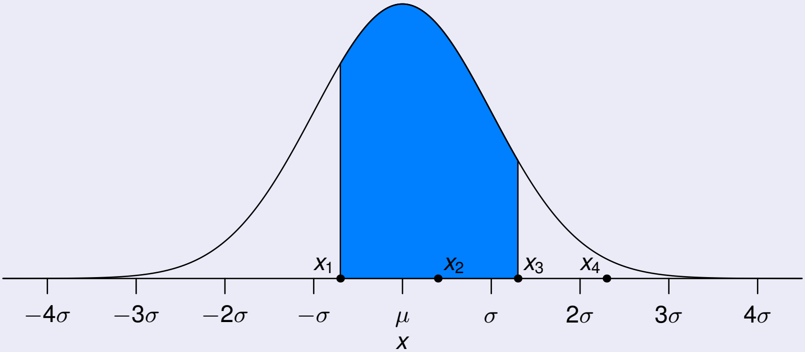


$$P(x_1 \leq z \leq x_2) = 1 - P(z \leq x_1) - P(z \geq x_2)$$

Procedure for Finding Areas with a Nonstandard Normal Distribution

1. Sketch a normal curve, label the mean and any specific *x* values, and then shade the region representing the desired probability.
2.
3.



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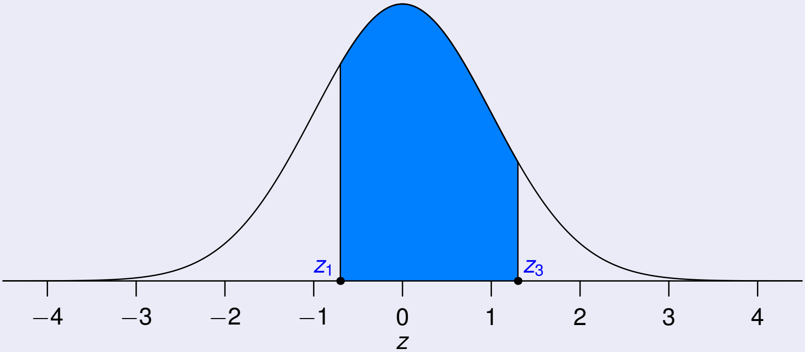

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1. Sketch a normal curve, label the mean and any specific x values, and then shade the region representing the desired probability.
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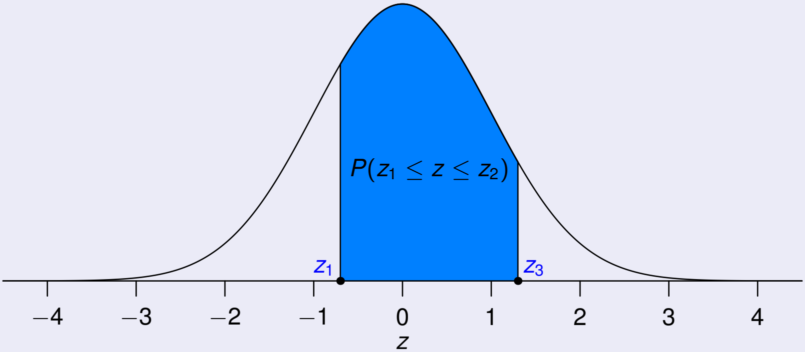
3. Use technology to find the area of the shaded region.



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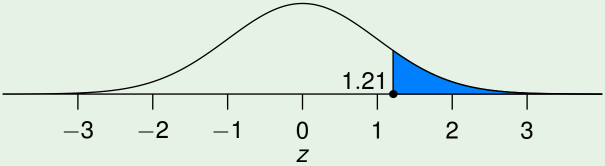
Next, we sketch a picture and shade the area we wish to find:



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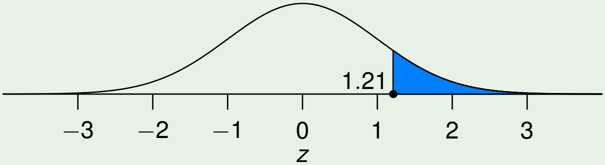

We can then use technology to compute:

$$P\left(z \geq 1.21\right) \approx 0.1123 \quad \text{(rounded)}$$

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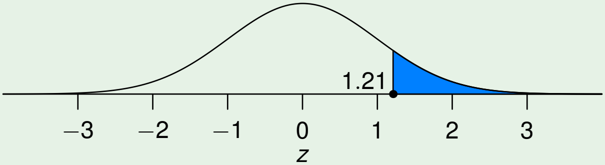

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So, about 11.23% of men are taller than the shower head.

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Recall that the percentile of a data value is the percentage of data less than the data value.

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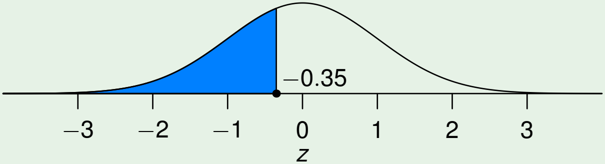

This means $P\left(z \leq -0.35\right)$ is the percentile of Edwards SAT score.

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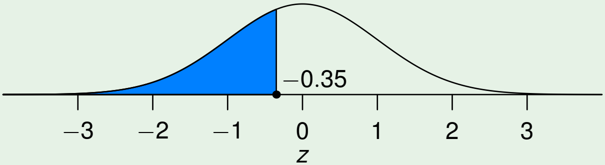

This means $P(z \leq -0.35)$ is the percentile of Edwards SAT score.

We can then use technology to compute:

$$P(z \leq -0.35) \approx 0.3632 \quad \text{(rounded)}$$

Example 10

Cumulative SAT scores are approximately normal with a mean of 1100 and standard distribution of 200. Edward earned a 1030 on his SAT.

The z-value of his score is:

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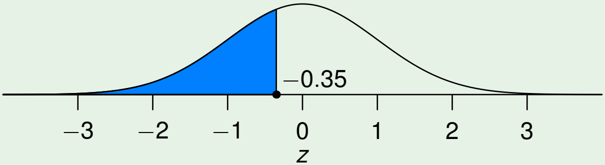
Recall that the percentile of a data value is the percentage of data less than the data value.



This means $P(z \leq -0.35)$ is the percentile of Edwards SAT score.

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So, Edward is in the 36$^{\text{th}}$ percentile.

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The U.S. Air Force requires that pilots have heights between 64 and 77 in. The heights of men are normally distributed with a mean of 68.6 in. and a standard deviation of 2.8 in.

Let's find the percentage of men meet that requirement.

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We start by finding the *z*-values of the height requirements.

$$z_1 = \frac{x - \mu}{\sigma} = \frac{64 - 68.6}{2.8} = -1.64$$

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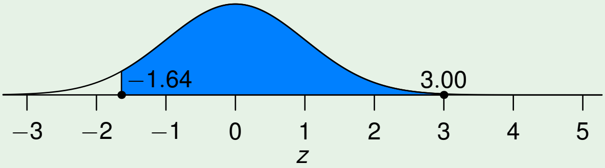

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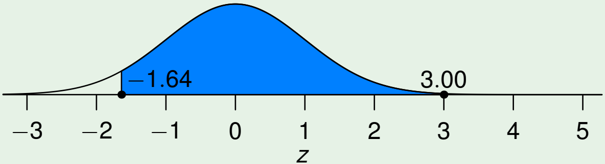

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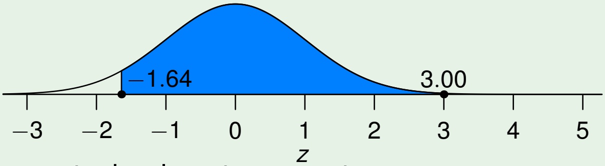

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So, we see that about 94.84% of men meet the requirements.

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Let's find the percentage of women meet that requirement.

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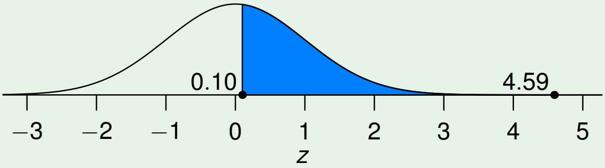

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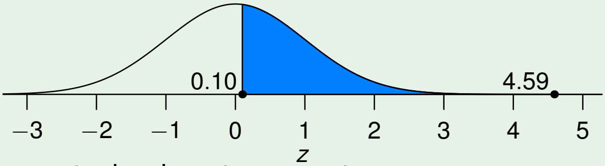

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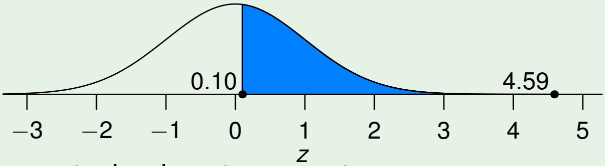

We can then use technology to compute:

$$P\,(0.10 \le z \le 4.59) \approx 0.4601 \quad \text{(rounded)}$$

So, we see that only about 46% of women meet the requirements.

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4. Use your sketch to verify that the solution makes sense.

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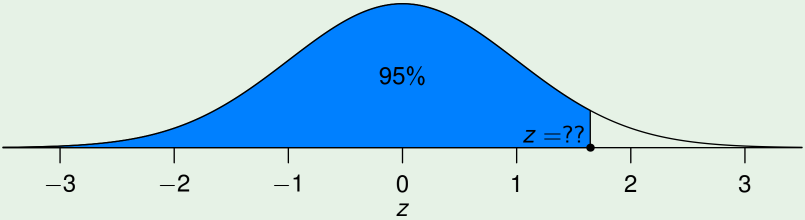

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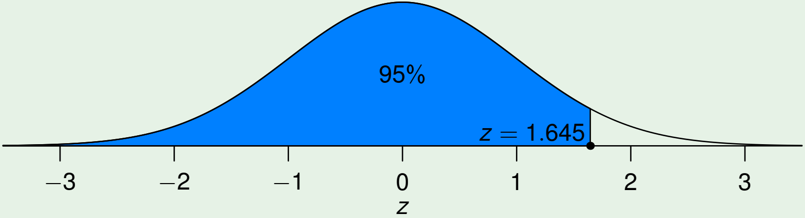

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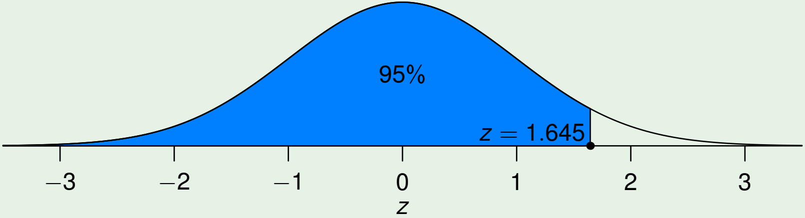

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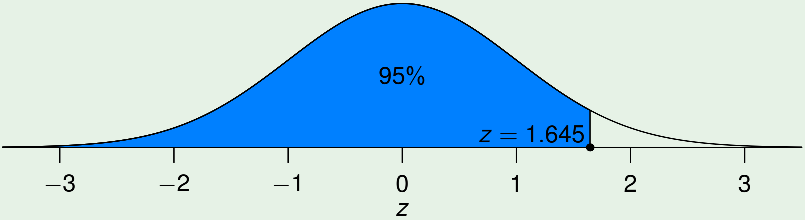

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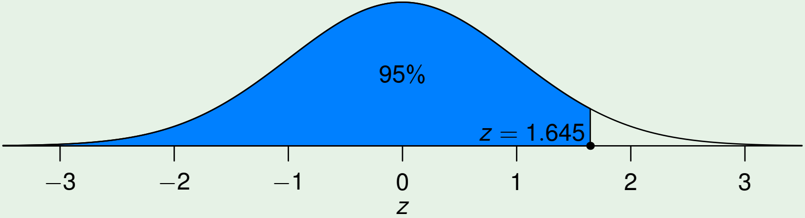

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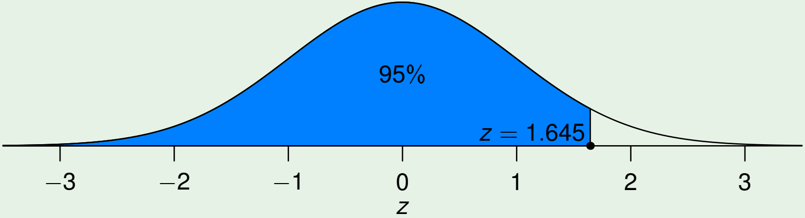

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A requirement of a height less than or equal to 68.5 in. would allow 95% of women to be eligible.