# The Determinant of a Matrix

Department of Mathematics

Salt Lake Community College

## Determinant of a $2 \times 2$ Matrix

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$$|\boldsymbol{A}| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

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## Example 1

$$\begin{vmatrix} 3 & 8 \\ 5 & -1 \end{vmatrix} = 3 \cdot (-1) + 8 \cdot 5 = 37$$

## Minors of a Matrix

For every element $a_{ij}$ of a $n \times n$ matrix $\boldsymbol{A}$, the **minor** $\boldsymbol{M_{ij}}$ is an $(n-1) \times (n-1)$ matrix obtained by deleting the $i$th row and the $j$th column of $\boldsymbol{A}$.

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## Cofactors of a Matrix

For every element $a_{ij}$ of a $n \times n$ matrix $\boldsymbol{A}$, the **cofactor** of $a_{ij}$ is the scalar

$$C_{ij} = (-1)^{(i+j)} \left| \boldsymbol{M_{ij}} \right|$$

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I recommend expanding across the first row.

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Compute the determinant:

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- If $\boldsymbol{A}$ is an diagonal, upper triangular, or lower triangular matrix, the determinant is the product of the diagonal elements:

$$|\boldsymbol{A}| = \prod_{i=1}^{m} a_{ii}$$

## Cramer's Rule

Consider the matrix equation:

$$\boldsymbol{A}\vec{\boldsymbol{x}} = \vec{\boldsymbol{b}} \quad \text{where} \quad |\boldsymbol{A}| \neq 0$$

The matrix $\boldsymbol{A_j}$ is obtained by replacing the $j$th column of $\boldsymbol{A}$ with $\vec{\boldsymbol{b}}$.

The $j$th solution is:

$$x_j = \frac{|\boldsymbol{A_j}|}{|\boldsymbol{A}|}$$

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Consider the system

$$\begin{array}{rcrcl} x & + & 2y & = & 5 \\ 2x & + & 3y & = & 8 \end{array}$$

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Let us solve this system using Cramer's Rule.
This means we need to calculate the following determinats.

$$|\boldsymbol{A}| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \qquad |\boldsymbol{A_x}| = \begin{vmatrix} 5 & 2 \\ 8 & 3 \end{vmatrix} \qquad |\boldsymbol{A_y}| = \begin{vmatrix} 1 & 5 \\ 2 & 8 \end{vmatrix}$$

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We can now find $x$

$$x = \frac{|\boldsymbol{A_x}|}{|\boldsymbol{A}|}$$

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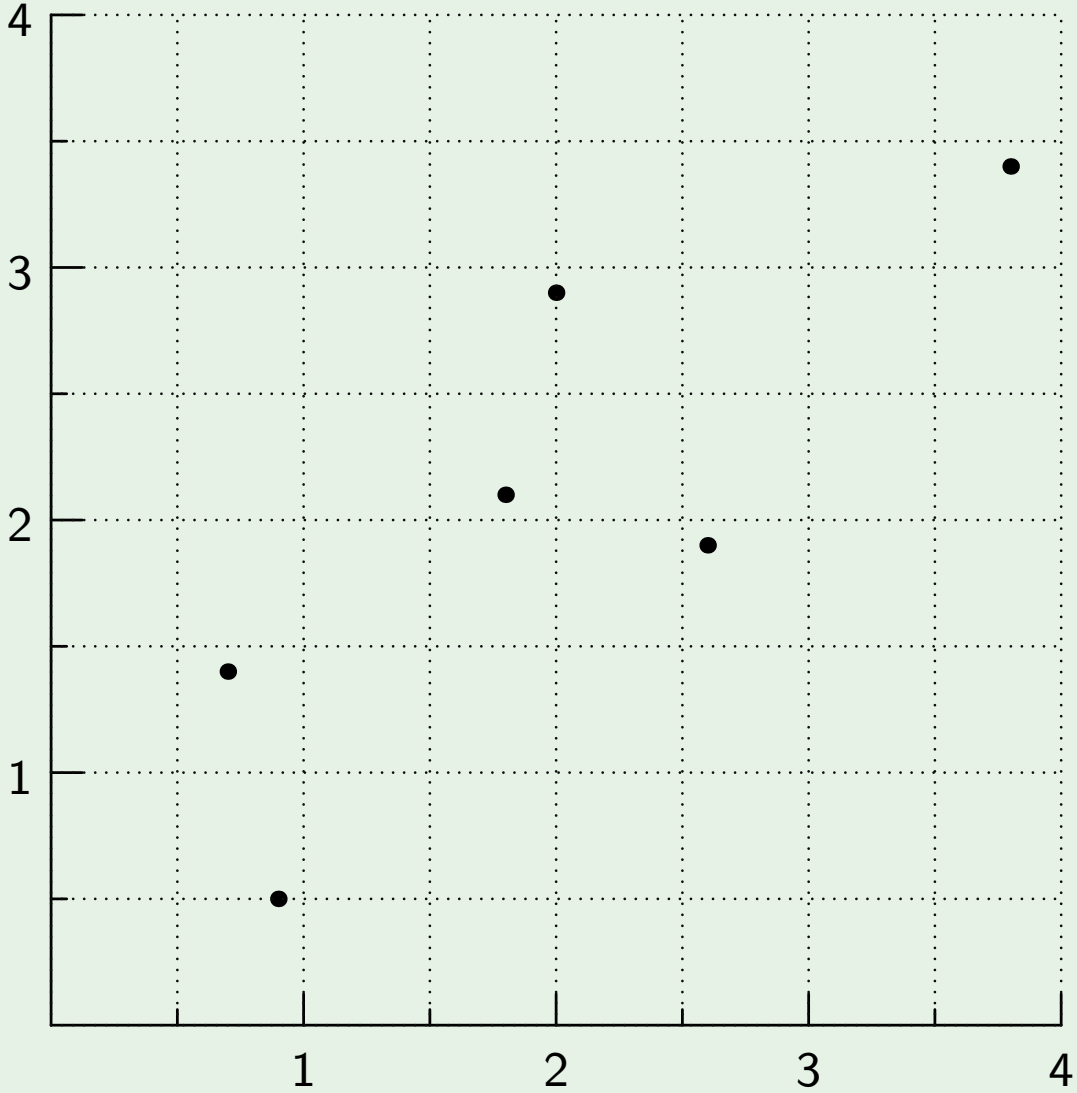
## Example 5

The **line of best fit** is the line that gets "closest" to every point.



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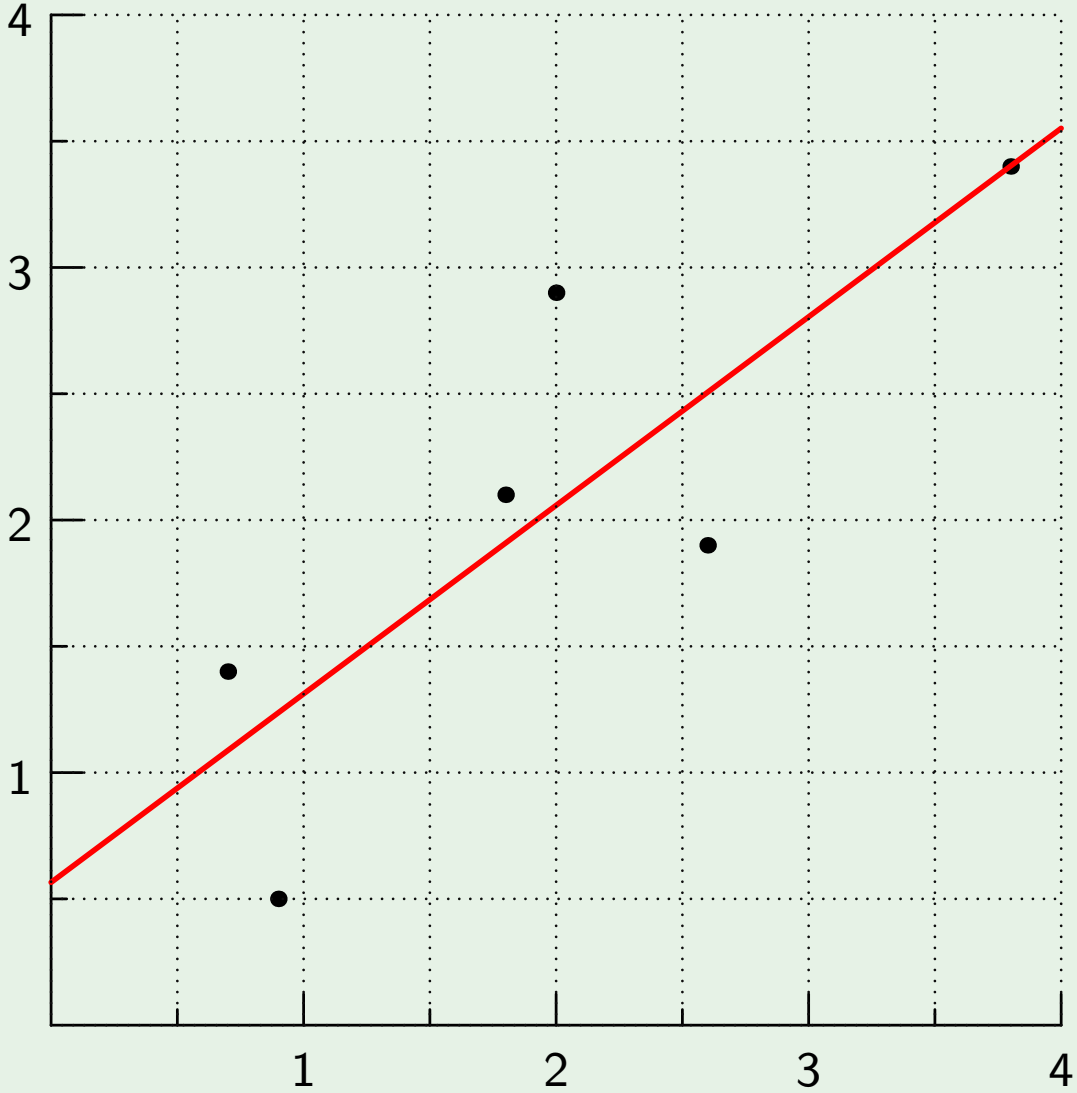

## Least Squares Approximation

A general strategy for finding the line $y = mx + b$ that best describes a data set is to find $b$ and $m$ that minimizes the sums of the squares of the vertical distances between the data points and the line, given by $F(b, m)$

$$F(b, m) = \sum_{i=1}^{n} (y_i - (b + mx_i))^2$$

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To find such a $b$ and $m$, we need to solve the system:

$$\frac{\partial F}{\partial b} = 0 \quad \text{and} \quad \frac{\partial F}{\partial m} = 0$$

## Least Squares Method

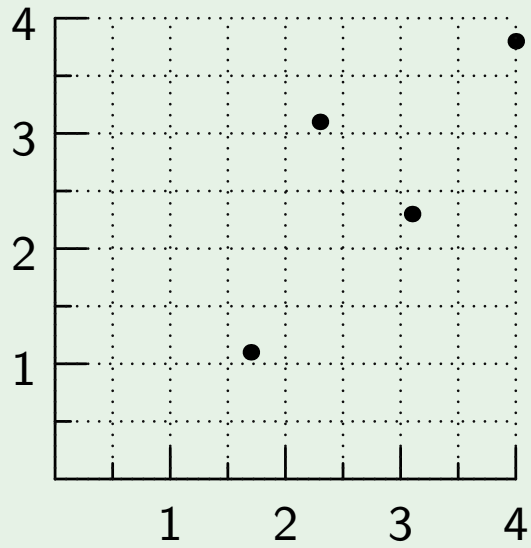
The best-fit straight line for $n$ data points $(x_i, y_i)$, $i = 1, 2, \ldots, n$, has y-intercept $b$ and slope $m$ as determined by the system

$$\begin{bmatrix} \sum_{i=1}^{n} 1 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix}$$

## Example 6

Consider the data comparing the high school and college GPA for four students.

| $i$ | $x_i$ | $y_i$ |
|---|---|---|
| 1 | 1.7 | 1.1 |
| 2 | 2.3 | 3.1 |
| 3 | 3.1 | 2.3 |
| 4 | 4.0 | 3.8 |



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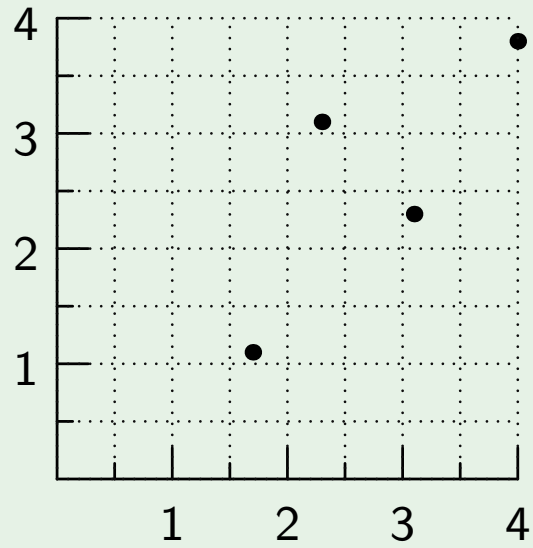


The Least Squares Method system for this dataset is:

$$\begin{bmatrix} 4 & 11.1 \\ 11.1 & 33.79 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 10.3 \\ 31.33 \end{bmatrix}$$

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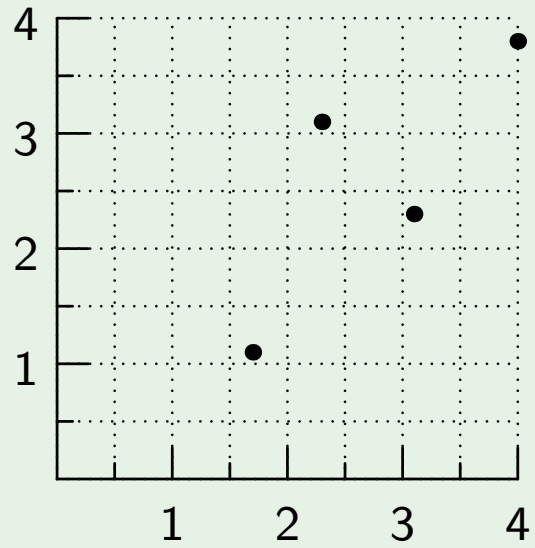


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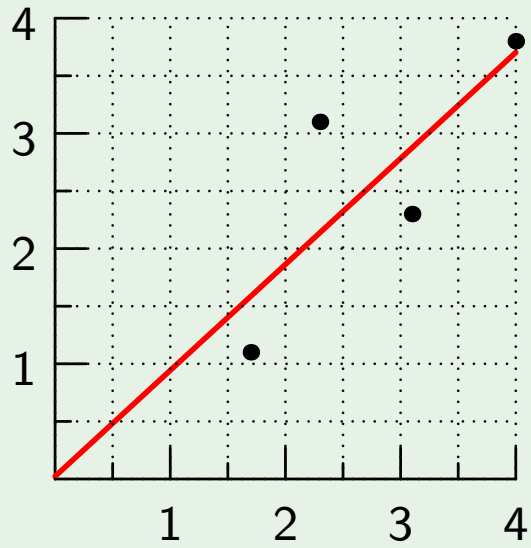


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So, the line of best fit is $y = 0.92x + 0.023$.

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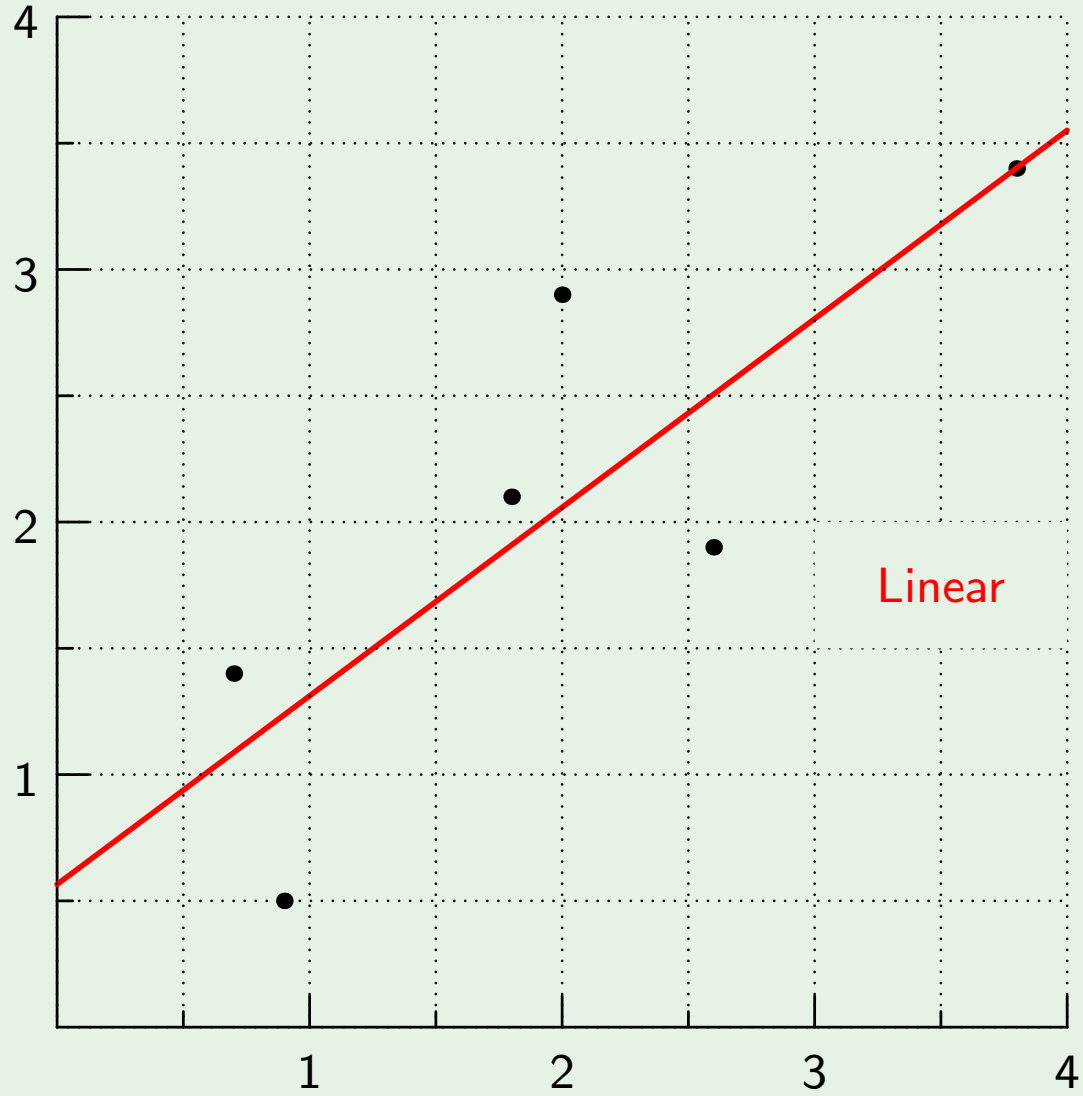
Let us consider some nonlinear approximations.



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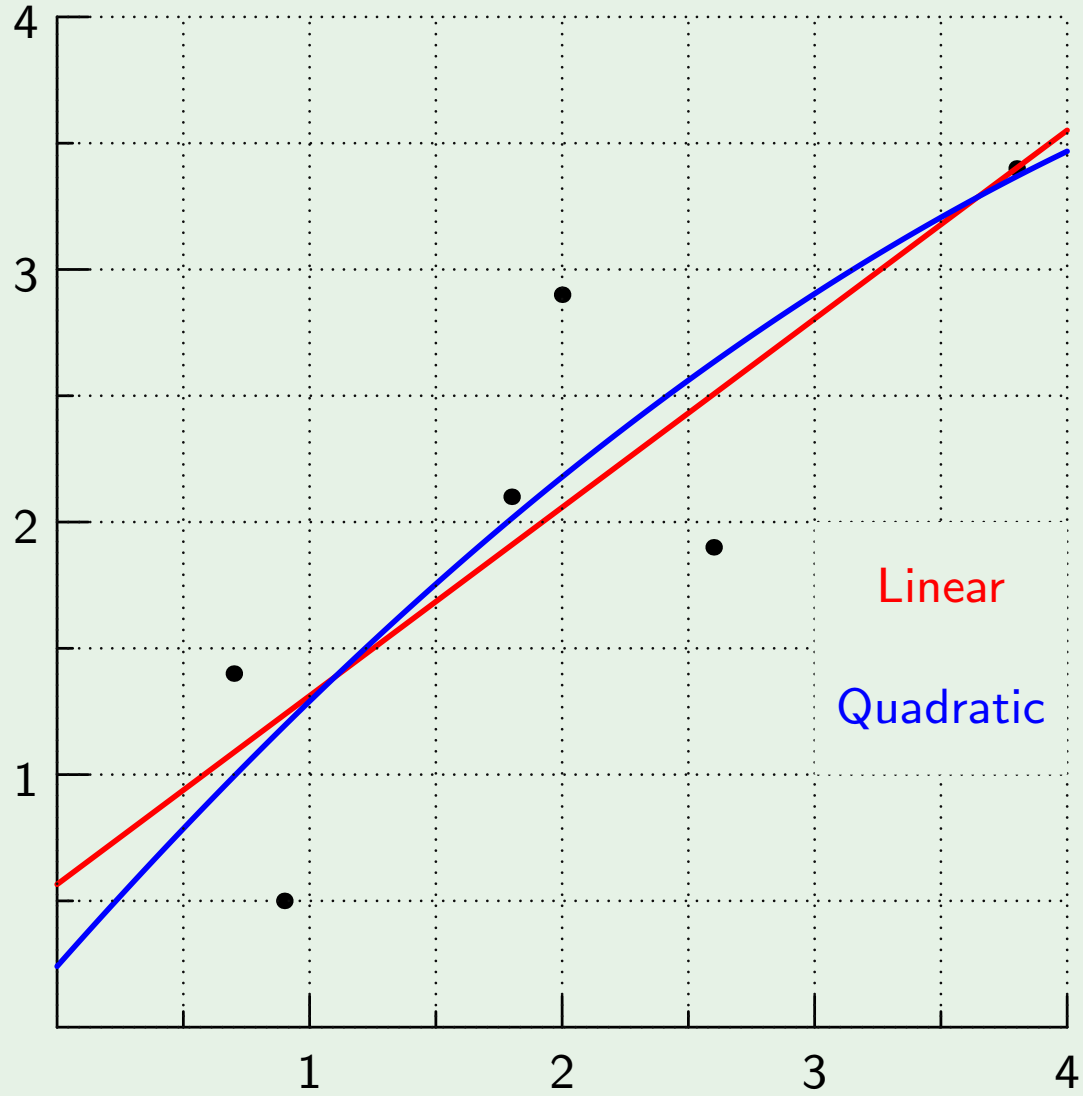

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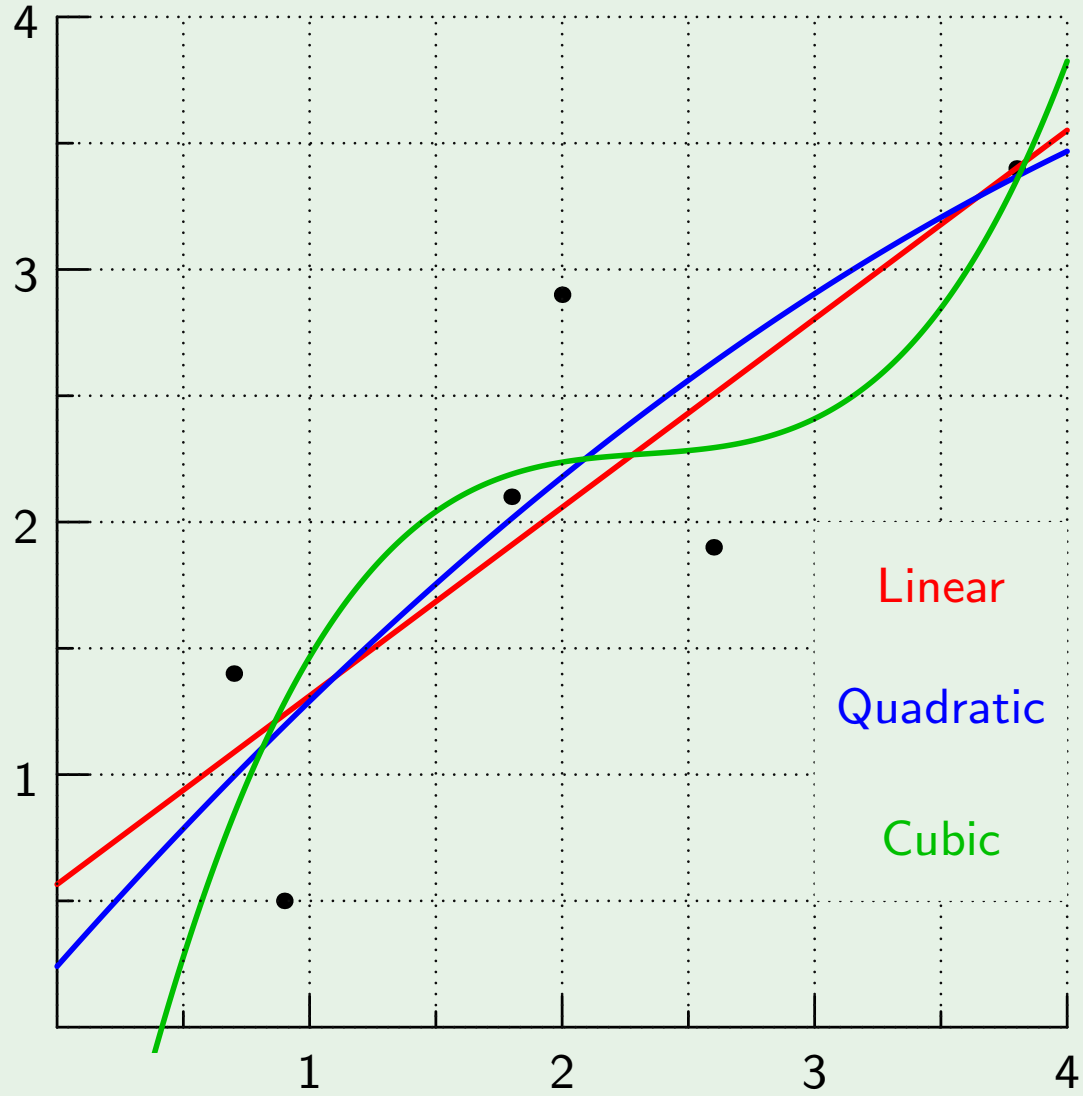

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