The Standard Normal Distribution

Colby Community College

Example 1

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So the probability of crashing on land is

$$\frac{54,255,000}{196,970,000} = 0.275$$

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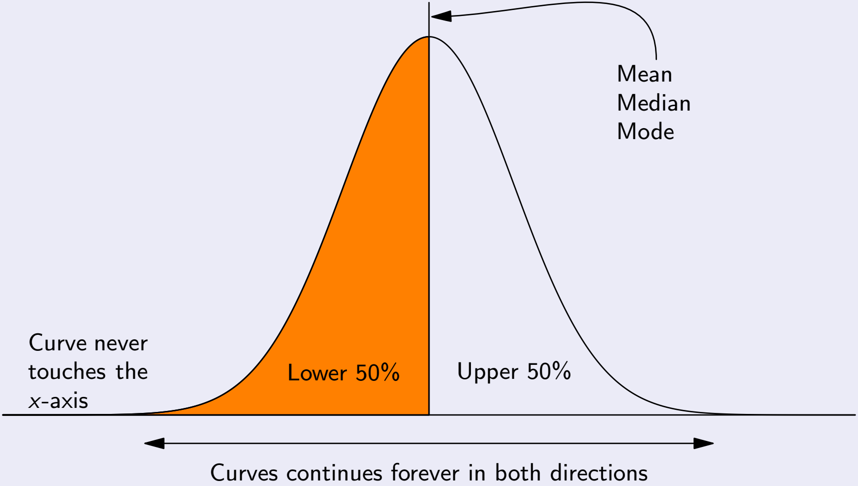
Consider making popcorn. You put some oil and corn kernels in a pan and start heating.

For the first few minutes nothing happens, then a few kernels start to pop. A little while later more and more start to pop. This goes one for a minute or so, and the popping gradually tappers off.

Most of the popping happens in that brief, noisy moment. This demonstrates a typical pattern that is part of many phenomena.

Definition

A **normal distribution** is a perfectly symmetric, bell-shaped distribution. It is also referred to as a **normal curve** or a **bell curve**.



Note

It is extremely important you master the procedures in this section. We will be using normal distributions often throughout the remainder of the course.

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Equation

The equation for the normal curve is

$$y = \frac{e^{-\frac{1}{2}z^2}}{\sigma\sqrt{2\pi}} \qquad \text{where} \qquad z = \frac{x-\mu}{\sigma}$$

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Note

We will not be working directly from the equation in this course.

The equation is only included for the rare chance you encounter technology that doesn't already have the normal distribution included.

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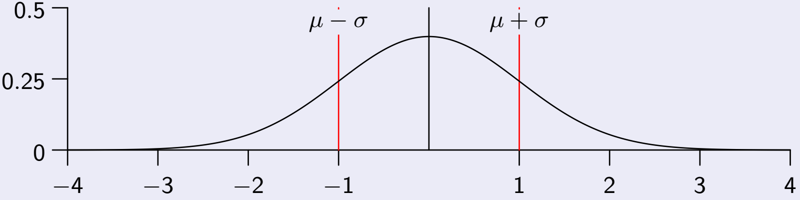
Definition

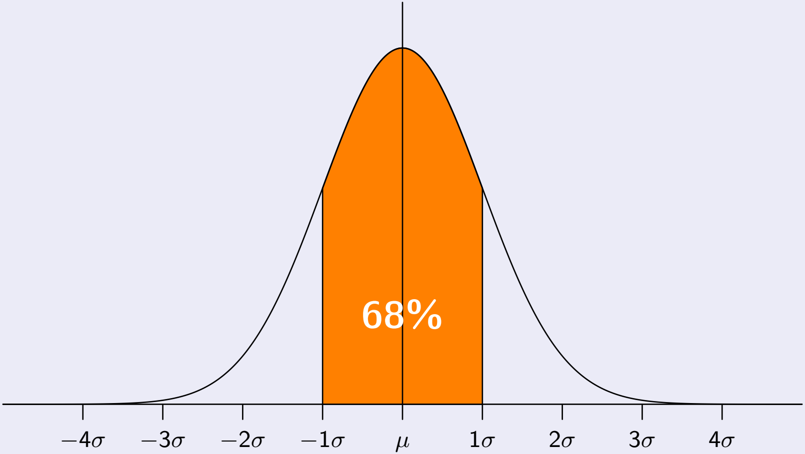
When $\mu = 0$ and $\sigma = 1$, the curve is called the **standard normal distribution**. The total area under its density curve is equal to 1.



Definition

The **empirical rule**, or **68-95-99.7 rule**, states approximately how much of the area is contained when stepping one, two, or three standard deviations from the mean.

One standard deviation from the mean.



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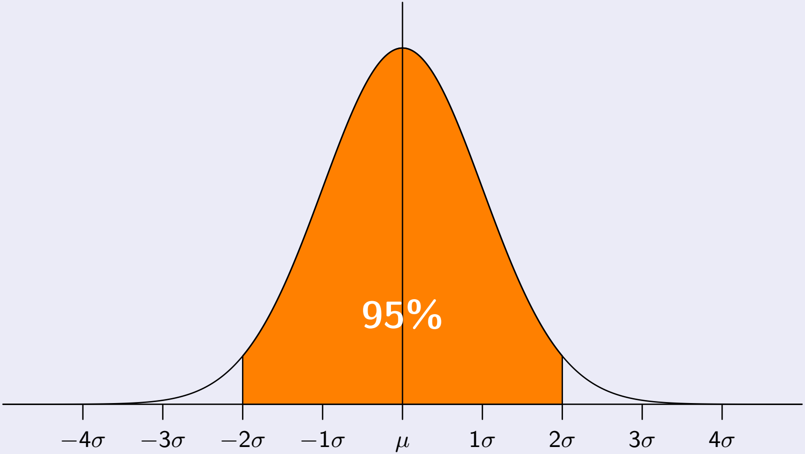
Two standard deviations from the mean.



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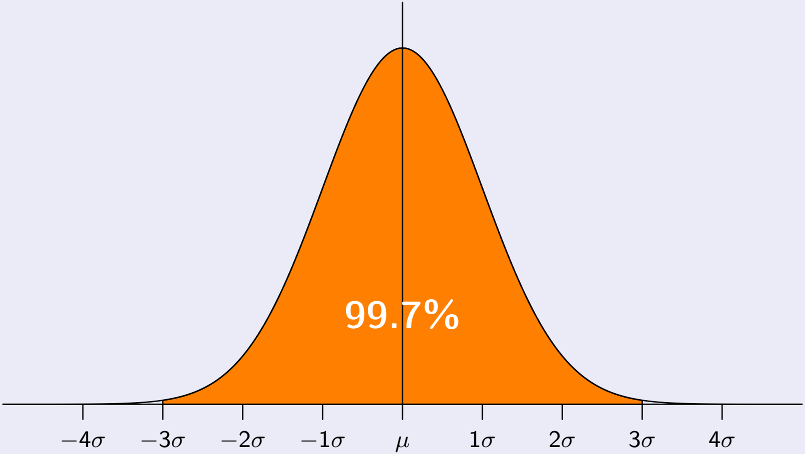
Three standard deviations from the mean.



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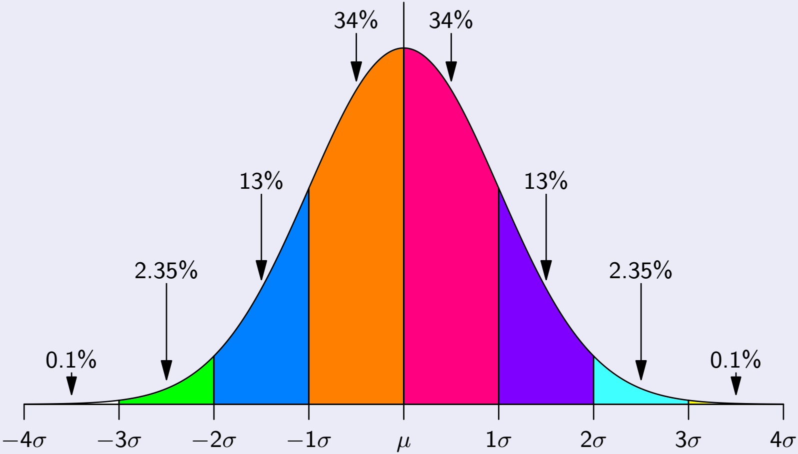
For each standard deviation from the mean.



Definition

A **z-score** is a measure of the number of standard deviations a particular data point is away from the mean. It is calculated with:

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$$z = \frac{x - \mu}{\sigma} \quad \Rightarrow \quad z\sigma = x - \mu \quad \Rightarrow \quad x = z\sigma + \mu = (-1.3)(8) + 70 = 59.6$$

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Note

We know from the empirical rule that roughly 68% of the scores fall within one standard deviation of the mean. This means that 68% of the students scored between 75 and 89.

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Moreover, we know that roughly 95% of the scores fall within two standard deviations of the mean. Which means that $95\% - 68\% = 32\%$ of the scores are more than one standard deviation from the mean, but less than two.

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Moreover, we know that roughly 95% of the scores fall within two standard deviations of the mean. Which means that $95\% - 68\% = 32\%$ of the scores are more than one standard deviation from the mean, but less than two.

Since the curve is symmetric, we know that 16% of the students scored between 89 and 96, as well as 16% between 68 and 75

Definition

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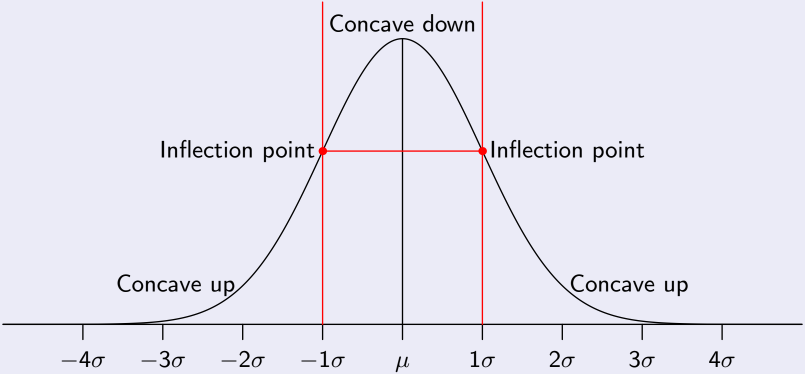
Note

A normal density curve always has two inflection points, each one standard deviation from the mean.



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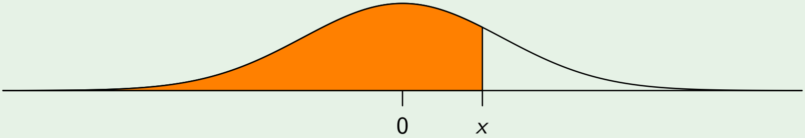

The area of the shaded region is the probability that a z score is less than or equal to x, $P(z \leq x)$.

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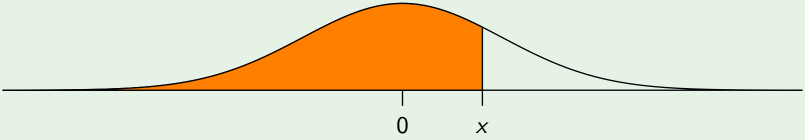

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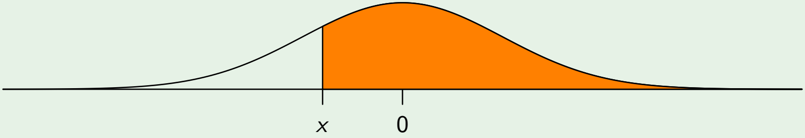
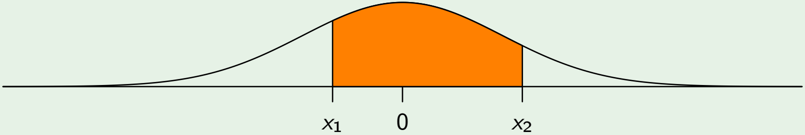


The area of the shaded region is the probability that a z score is greater than or equal to x, $P(z \geq x)$. Can also be calculated as $1 - P(z \leq x)$.

Example 8



The area of the shaded region is the probability that a z score lies between x_1 and x_2, $P\left(x_1 \leq z \leq x_2\right)$.

Can also be calculated as $P\left(x \leq x_2\right) - P\left(z \leq x_1\right)$.

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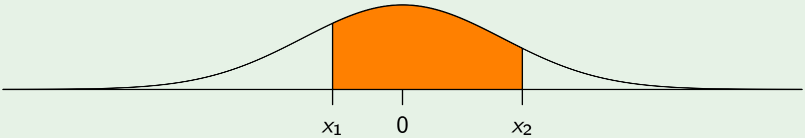

The area of the shaded region is the probability that a z score lies between x_1 and x_2, $P(x_1 \leq z \leq x_2)$.

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Note

It is common for technology to only calculate $P(z \leq n)$.

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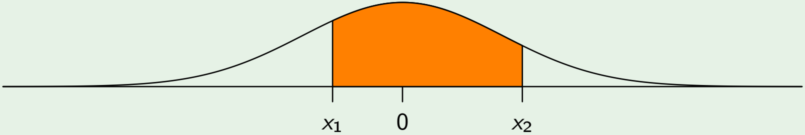

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Note

It is always a very good idea to sketch a graph, shading the area you want to find. You can then determine how to find the desired area by working with cumulative areas from the left.