Regression

Colby Community College

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Definition

Given a collection of paired sample data, the **regression line** (or **line of best fit**) is the straight line that "best" fits the scatter plot of the data. (We will discuss that "best" means later.)

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We call y the **response variable**, or **dependent variable**.

Note

We don't use $y = mx + b$ because the format $y = b_0 + b_1 x$ can easily be expanded in include more variables:

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + \cdots$$

This is used when performing a multiple regression.

Requirements

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2. Visual examination of the scatterplot shows that the points approximate a straight-line pattern.
3. Outliers can have a strong effect on the regression equation, so remove any outliers if they are known errors.

Slope

The slope of the regression line is

$$b_1 = r \cdot \frac{s_y}{s_x}$$

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y-intercept

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$$b_0 = \bar{y} - b_1\bar{x}$$

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Note

Technology will calculate both of these values for you.

Making Predictions

When making predictions, keep the following in mind:

Bad Model: If the regression equation does not appear to be useful for making predictions, don't use the regression equation.

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- Predicting too far beyond the scope of the available sample data is called **extrapolation** and can easily result in bad predictions.

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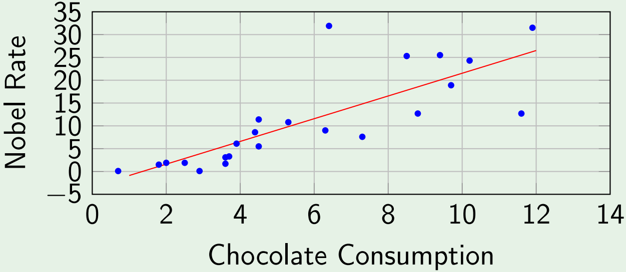

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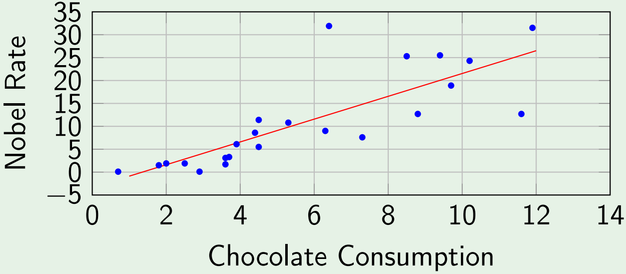

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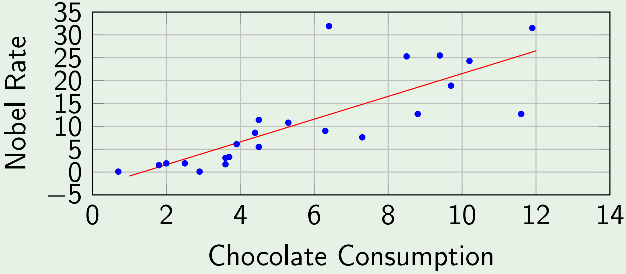

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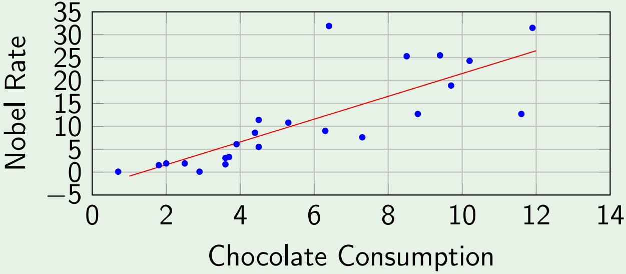

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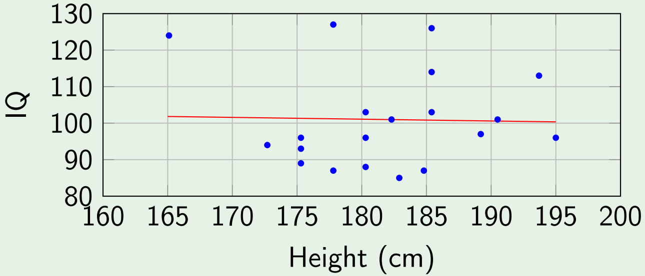
So, we expect 21.5 Nobel Laureates per 10 million people.

Example 2

The scatterplot shows sample data recording subject height and IQ score.



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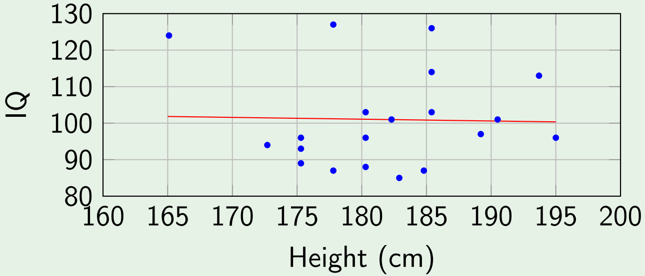

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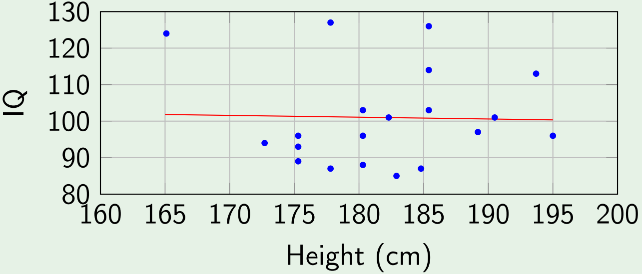

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This means the regression line is a bad model and should not be used to make predictions.