Scatterplots, Correlation, and Regression

Colby Community College

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A **scatterplot** is a plot of paired (x, y) quantitative data with a horizontal x-axis and a vertical y-axis. The horizontal axis is used for the first variable (x), and the vertical axis for the second variable (y).

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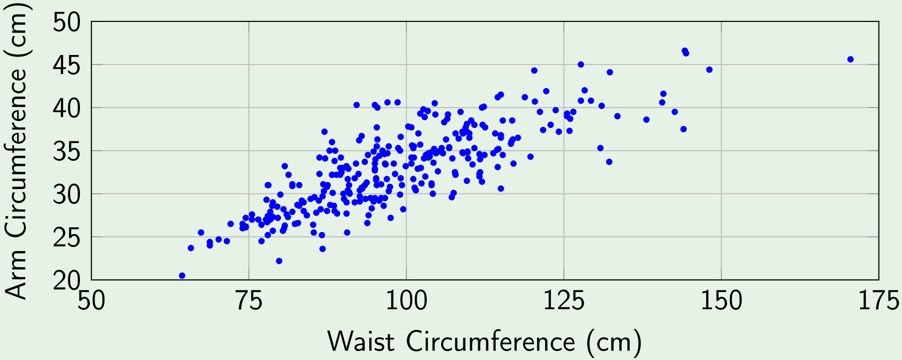
Warning

The presence of a correlation between two variables is not evidence that one of the variables causes the other.

Correlation does not imply causality!

Example 1

Data Set 1 "Body Data" in Appendix B includes waist circumference and arm circumference (cm) of randomly selected adult subjects.



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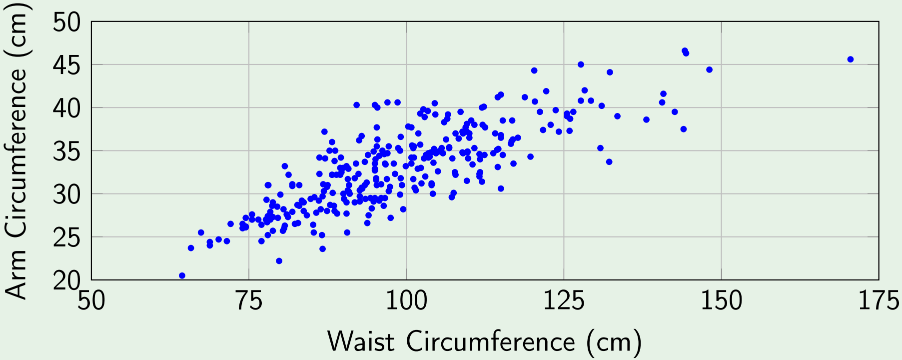
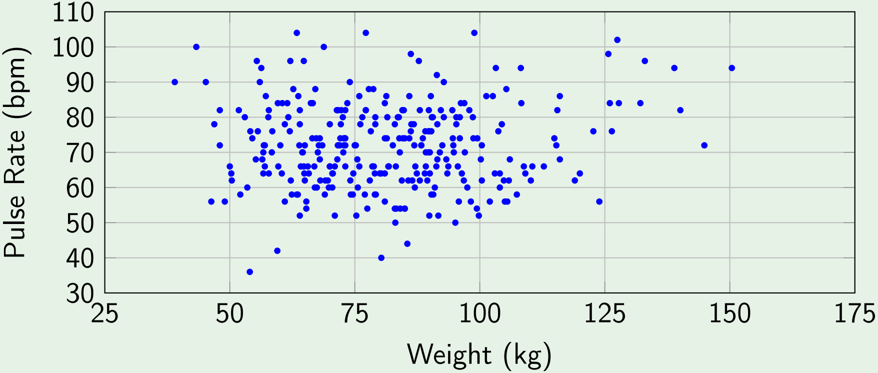


The points show a pattern of increasing values from left to right. This pattern suggests that there is a relationship between waist circumferences and arm circumferences.

Example 2

Data Set 1 "Body Data" in Appendix B includes weights (kg) and pulse rates (bpm) of randomly selected adult subjects.



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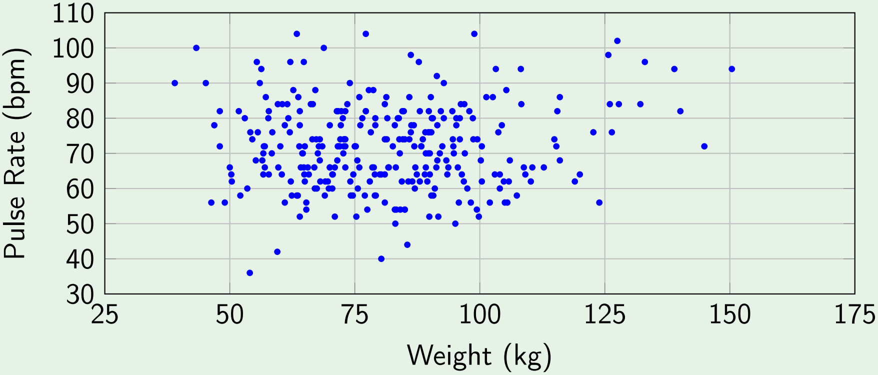
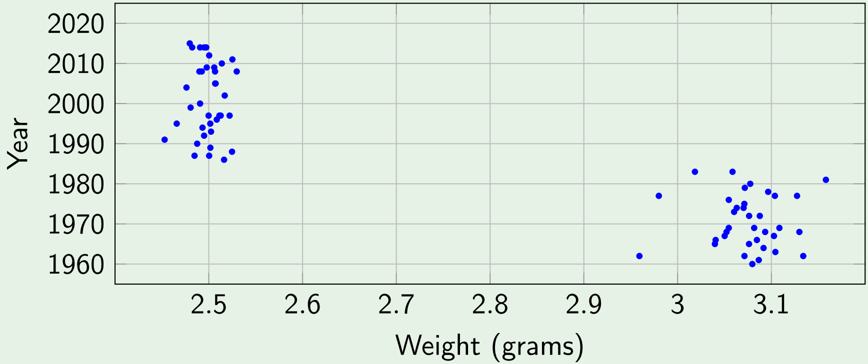


The points do not show any obvious pattern, and this lack of a pattern suggests that there is no relationship between weights and pulse rates.

Example 3

Consider the scatterplot that depicts data consisting of the weight (grams) and year of production for 72 pennies.



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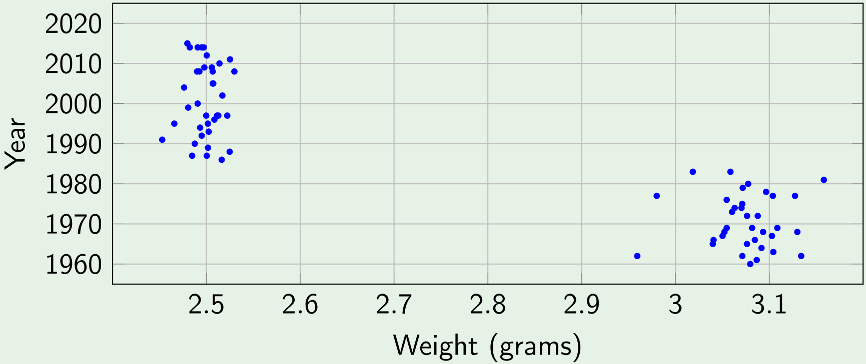

While it may look like there is a relationship, looking at the individual clusters we see that there is not a relationship between the weight of a penny and year is was produced.

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Note

In chapter 10 we will talk in detail about how correlation is calculated. For now, we will use software to calculate correlation and focus on how to interpret the results.

Example 4

The table contains the shoe size and height of five random people. (Data Set 2 Appendix B)

Shoe Print Length (cm)	29.7	29.7	31.4	31.8	27.6
Height (cm)	175.3	177.8	185.4	175.3	172.7

Example 4

The table contains the shoe size and height of five random people. (Data Set 2 Appendix B)

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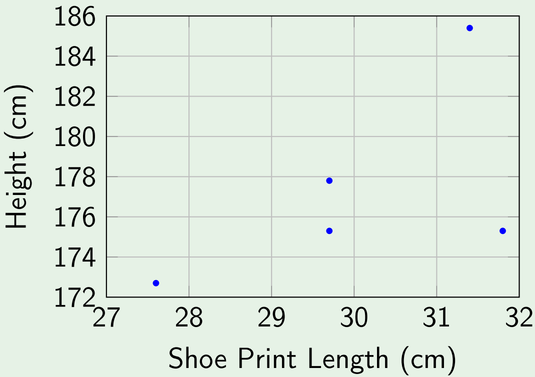

Statdisk Output

```
Sample Size, n:   5
Degrees of Freedom: 3

Correlation Results:
Correlation Coeff, r:  0.59127
Critical r:            ±0.87834
P-Value (two-tailed):  0.29369

Regression Results:
Y= b0 + b1x:
Y Intercept, b0:      125.40733
Slope, b1:            1.72745

Total Variation:       95.02
Explained Variation:   33.21891
Unexplained Variation: 61.80109
Standard Error:        4.53876
Coeff of Det, R^2:     0.3496
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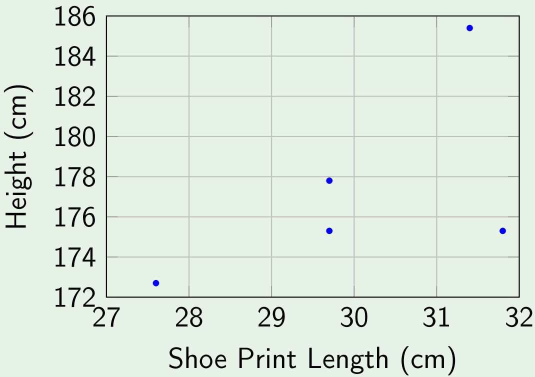

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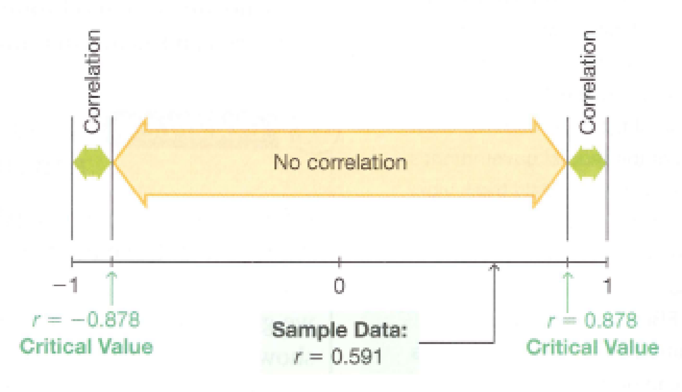
Is $r = 0.59127$ "close" to 1? Or is r "close" to 0?

Interpreting r

The critical value tells us the separation between "close to -1" or "close to 1" and "close to 0."



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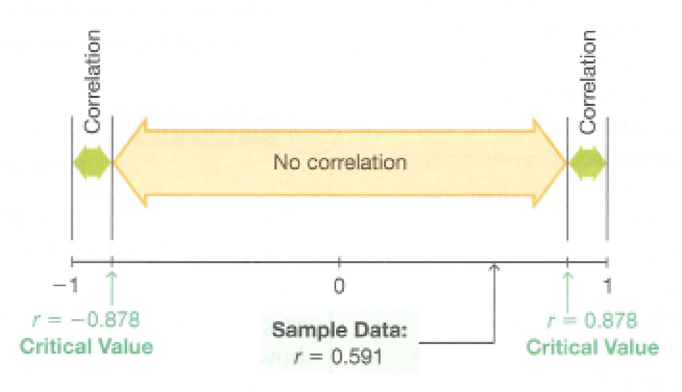

We see that there isn't sufficient evidence in Example 4 for correlation between shoe print length and height.

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(We will talk more about P-values in chapter 8.)

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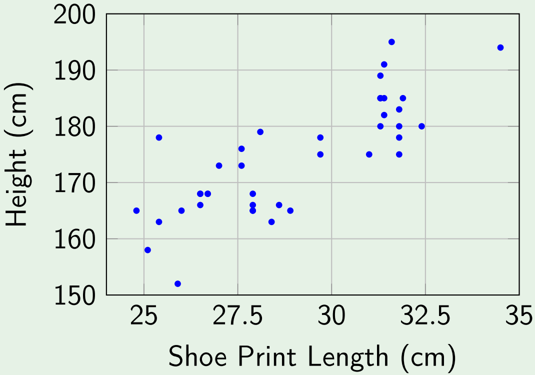
(We will talk more about P-values in chapter 8.)

Note

Only a small P-value, such as 0.05 or less, suggests that the sample results are not likely to occur by chance when there is no linear correlation. The smaller the P-value the stronger the evidence that there is a linear correlation between the two variables.

Example 5

Using the full data set on shoe print length and height. (Data Set 2)



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Sample Size, n:   40
Degrees of Freedom: 38

Correlation Results:
Correlation Coeff, r:  0.81295
Critical r:            ±0.31201
P-Value (two-tailed):  0

Regression Results:
Y= b0 + b1x:
Y Intercept, b0:      80.93041
Slope, b1:            3.21856

Total Variation:       3958.755
Explained Variation:   2616.27965
Unexplained Variation: 1342.47535
Standard Error:        5.94376
Coeff of Det, R^2:     0.66088
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Example 5

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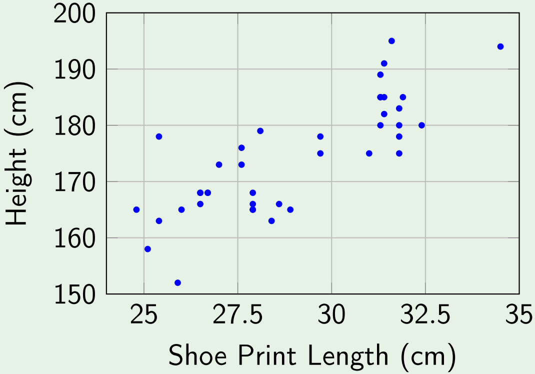

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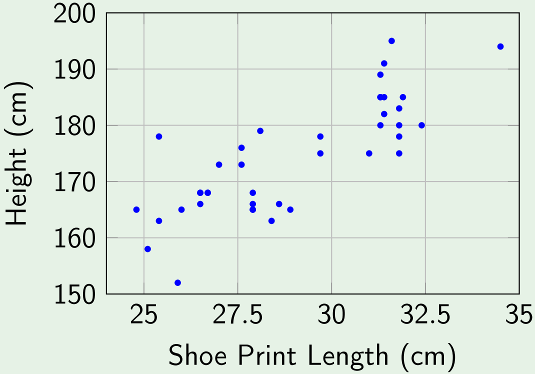

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But, with the full 40 data pairs, we get a P-value of 0. This is strong evidence of a correlation between shoe print length and height.

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Note

A regression line is used to make predictions about a population using the sample data.

Example 6

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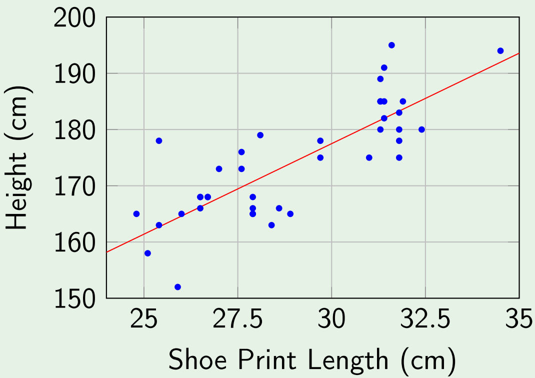

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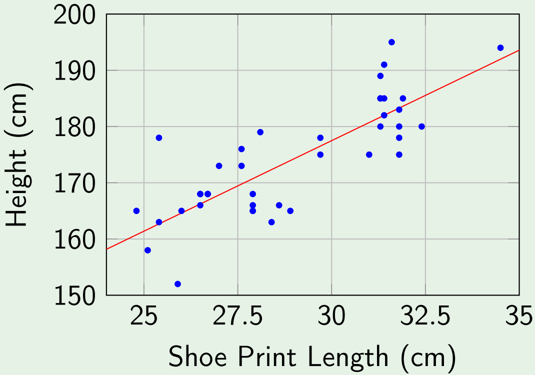

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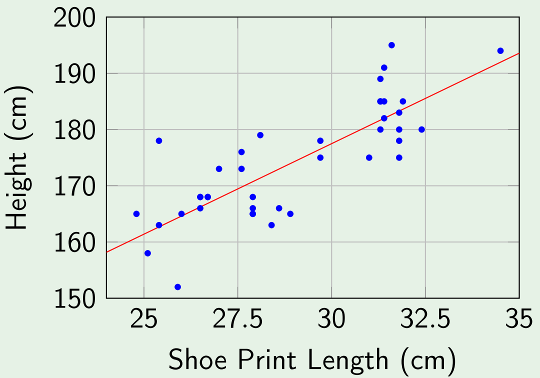

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We can expect a person with a shoe length of 30 cm to be $80.9 + 3.2(30) = 176.9$ cm tall.