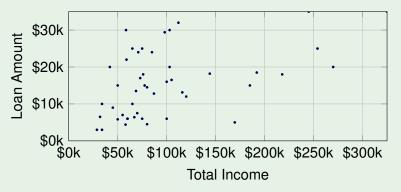
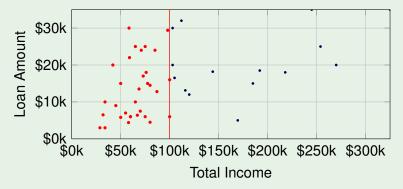
# **Examining Numerical Data**

Colby Community College

Let us consider a scatterplot of borrowers total income and the loan amount from the loan50 data set.

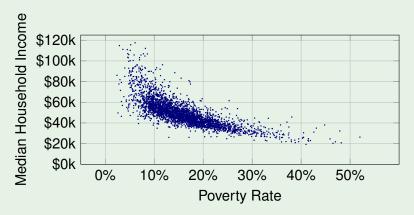


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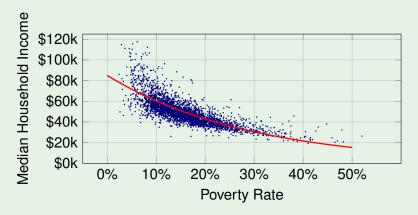


We can see that the many of borrowers earn \$100,000 a year or less.

Let us consider a scatterplot of borrowers total income and the loan amount from the loan50 data set.



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It is clear there is a **nonlinear** association between the median household income and the poverty rate.

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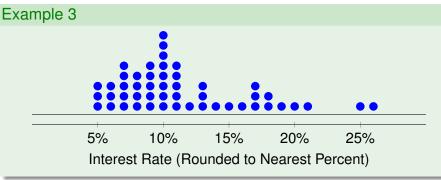
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Dot plots work best with integer data. It is common to round decimals before building a dot plot.

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#### Note

Parameter and population both start with a "P." Statistic and sample both start with a "S."

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 Sample means drawn from the same population tend to vary less than other measures of center.

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# Properties of the Mean

- Sample means drawn from the same population tend to vary less than other measures of center.
- A disadvantage of the mean is that just one extreme value can change the value of the mean substantially.

Sample statistics are usually represented by English letters, such as  $\bar{x}$ , while population parameters are usually represented by Greek letters, such as  $\mu$ .

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 $\bar{x} = \frac{\sum x}{n}$  is the mean of a set of sample values.

 $\mu = \frac{\sum X}{N}$  is the mean of all values in a population.

Suppose we measure the of data speeds of smartphones from the four major carriers. The table contains five data speeds, in megabits per second (Mbps), from this data set.

Carrier	Verizon	Verizon	Verizon	Verizon	Verizon
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### Note

Round statistics and parameters to one more decimal place than found in the data.

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It is common to mark the mean on a dot plot.

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# Example 5

The mean of interest\_rate is: (Do not round the data values.)

$$\bar{x} = \frac{\left(\begin{array}{c} 5.31\% + 5.31\% + 5.32\% + 6.08\% + 6.08\% + 6.08\% + 6.71\% + 6.71\% + 7.34\% \\ + 7.35\% + 7.35\% + 7.96\% + 7.96\% + 7.96\% + 7.97\% + 9.43\% + 9.43\% + 9.44\% \\ + 9.44\% + 9.44\% + 9.92\% + 9.92\% + 9.92\% + 9.92\% + 9.93\% + 9.93\% + 10.42\% \\ + 10.42\% + 10.9\% + 10.91\% + 10.91\% + 11.98\% + 12.62\% \\ + 12.62\% + 12.62\% + 14.08\% + 15.04\% + 16.02\% + 17.09\% + 17.09\% + 17.09\% \\ + 18.06\% + 18.45\% + 19.42\% + 20\% + 21.45\% + 24.85\% + 26.3\% \\ \hline 50 \\ \end{array} = 11.567\%$$

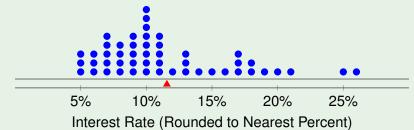
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We saw in Example 5 that the average loan interest rate was 11.567%.

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### **Definition**

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#### Note

We will discuss tools in Chapter 5 and beyond to determine how well a point estimate estimates a parameter.

We would like to determine if a new drug is more effective at treating asthma attacks than the standard drug. A trial of 1500 adults is setup, giving the following data.

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Number of patients	500	1000
Total asthma attacks	200	300

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The average number of asthma attacks per patient is higher with the new drug, so it's not more effective.

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#### Note

The mean gives a standardized a metric into something easier to interpret and compare.

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- 3 Then divide by the total number of people in the country.

Using this method we would find the average income per person in the US is \$30,861. If we had used the simple mean of per\_capita\_income the result would have been \$26,093, which is much lower.

A **weighted mean** is a mean where some values contribute more than others.

$$\bar{x} = \frac{\sum w_{x} \cdot x}{\sum w_{x}}$$

The values  $w_x$  are called the **weights**.

Value	% of Grade	Weight
Your average attendance score	10%	10

Value	% of Grade	Weight
Your average attendance score	10%	10
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Your final exam score	20%	20

Your final grade in this class is a weighted mean of the following four values:

Value	% of Grade	Weight
Your average attendance score	10%	10
Your average assignment score	30%	30
Your average exam score	40%	40
Your final exam score	20%	20

So, your final grade is calculated using the formula:

$$Grade = \frac{10 \cdot \overline{attendance} + 30 \cdot \overline{assignments} + 40 \cdot \overline{exams} + 20 \cdot \overline{final}}{10 + 30 + 40 + 20}$$

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### Note

We could also use the decimal versions of the percentages as the weights, instead of the whole numbers.

The **median** of a data set is the middle value when the original data values are arranged in order of smallest to largest.

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### **Notation**

The median of a sample is denoted  $\tilde{x}$ .

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# **Properties**

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### **Notation**

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#### **Procedure**

- 1 Sort the values.
- If the number of data values is odd, the median is the number located in the exact middle of the sorted list.
  - If the number of data values is even, the median is found by computing the mean of the two middle numbers in the sorted list.

Let find the median data speed using the table from Example 4.

Carrier	Verizon	Verizon	Verizon	Verizon	Verizon
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We have 5 data values so the median is  $\tilde{x} = 23.1$  Mbps.

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We have 5 data values so the median is  $\tilde{x} = 23.1$  Mbps.

#### Note

This different than the mean 30.74 Mbps.

Let find the median data speed using the table from Example 4, but with an extreme value added in.

Carrier	Verizon	Verizon	Verizon	Verizon	Verizon	Verizon
Mbps	38.5	55.6	22.4	14.1	23.1	192.6

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First sort the data values.

We have 6 data values so 
$$\tilde{x} = \frac{23.1 + 38.5}{2} = 30.80$$
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$$\tilde{x} = \frac{23.1 + 38.5}{2} = 30.80$$
 Mbps.

#### Note

This is very different from the mean of this table.

$$\bar{x} = \frac{14.1 + 22.4 + 23.1 + 38.5 + 55.6 + 192.6}{6} = 173.15 \text{ Mbps}$$

A **histogram** is a graph consisting of bars of equal width drawn adjacent to each other. Each bar represents a "bin" of data values and the height of each bar is how many data values are in the "bin".

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### Note

Histograms are similar to dot plots, except that each bar represents a range of data values.

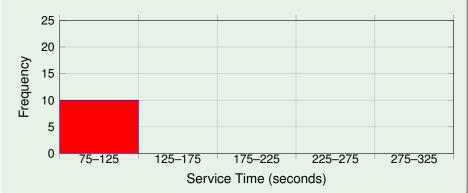
The table contains drive-through service times, in seconds.

107	139	197	209	281	254	163	150	127	308	206
169	83	127	133	140	143	130	144	91	113	153
252	200	117	167	148	184	123	153	155	154	100
101	138	186	196	146	90	144	119	135	151	197



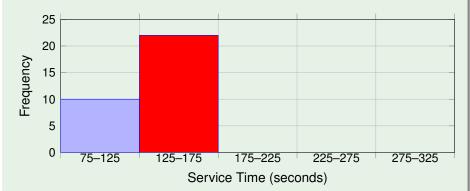
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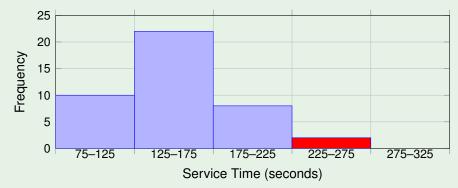
The table contains drive-through service times, in seconds.

107	139	197	209	281	254	163	150	127	308	206
169	83	127	133	140	143	130	144	91	113	153
252	200	117	167	148	184	123	153	155	154	100
101	138	186	196	146	90	144	119	135	151	197



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