Random Variables

Colby Community College

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We expect about 25 students will buy none, 55 will buy just the textbook, and 25 will buy both. A total of $1 \cdot 55 + 2 \cdot 25 = 105$ books.

How much revenue should be expected to be made?

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We expect about 25 students will buy none, 55 will buy just the textbook, and 25 will buy both. A total of $1 \cdot 55 + 2 \cdot 25 = 105$ books.

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$\$11,785 \div 100$ students $= \$117.85$ per student.

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The probability distribution is:

i	1	2	Total
x_i	0	1	–
$P(X = x_i)$	0.50	0.50	1.00

Example 3

If we let X be the amount a student spends in Example 1, then the probability distribution is:

i	1	2	3	Total
x_i	\$0	\$137	\$170	–
$P(X = x_i)$	0.20	0.55	0.25	1.00

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If we let X be the amount a student spends in Example 1, then the probability distribution is:

i	1	2	3	Total
x_i	\$0	\$137	\$170	–
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The random variable in Example 3 is a discrete random variable.

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Let's consider tossing two coins, with the following random variable:

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Example 5

Hiring managers were asked to identify the biggest mistakes that job applicants make during an interview.

Consider the following table:

i	x_i	$P(X = x_i)$
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2	Being late	0.44
3	Lack of Eye Contact	0.33
4	Checking phone or texting	0.30
	Total	1.57

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So, we see that X is not a random variable.

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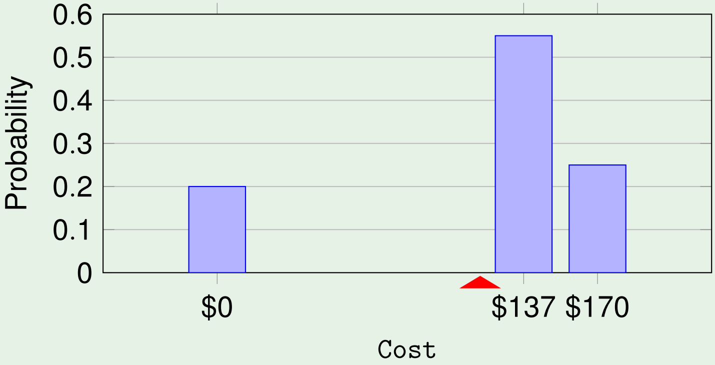
Example 6

In Example 1 the average revenue, \$117.85 per student, is the expected value for the bookstore's revenue.



Expected Value Of A Discrete Random Variable

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$$\begin{aligned} E(X) &= x_1 \cdot P(X = x_1) + x_2 \cdot P(X = x_2) + \cdots + x_k \cdot P(X = x_k) \\ &= \sum_{i=1}^{k} x_i \cdot P(X = x_i) \end{aligned}$$

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Note

The Greek letter μ is sometimes used in place of $E(X)$.

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If X is the net winnings, then the probability distribution is:

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On *average*, the player will lose 5.3 cents per bet.

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Note

It makes sense that an insurance policy would have a negative expected value, otherwise the insurance company couldn't stay in business.

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A company estimates that 0.7% of their products will fail, with a replacement cost of \$350, after the original warranty period but within 2 years of the purchase.

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The probabilities and values for the two outcomes are:

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The company makes, on average, \$45.55 for each extended warranty.

Note

If you ran the university bookstore in Example 1, then not only would you want to know your expected revenue, but also how much variability there is in your revenue.

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General Variance Formula

If X takes outcomes $x_1, \ldots, x_k$ with probabilities $P(X = x_1), \ldots,$ $P(X = x_k)$ and expected value $\mu = E(x)$, then the variance of X, denoted by $Var(X)$ or the symbol σ^2, is:

$$\begin{aligned}\sigma^2 &= (x_1 - \mu)^2 \cdot P(X = x_1) + \cdots + (x_2 - \mu)^2 \cdot P(X = x_2) \\ &= \sum_{j=1}^{k} (x_j - \mu)^2 \cdot P\left(X = x_j\right)\end{aligned}$$

The standard deviation of X, denoted σ, is the square root of the variance. i.e. $\sigma = \sqrt{\sigma^2}$

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x_i	\$0.00	\$137.00	\$170.00	–
$P(x = x_i)$	0.20	0.55	0.25	–
$x_i \cdot P(X = x_i)$	0.00	75.35	42.50	117.85

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Let us find the expected value of the bookstore in Example 1.

It is useful to construct a table to hold the computations:

i	1	2	3	Total	
x_i	\$0.00	\$137.00	\$170.00	–	
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The variance of X is $\sigma^2 = 3659.33$ and so the standard deviation is $\sigma = \sqrt{3659.33} = \$60.49$.

So, on average, we can expect a variability of revenue of around \$60.49 per student.

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The bookstore also offers a chemistry textbook for \$159 with a supplement for \$41. From past expereince, they know about 25% of chemistry students just buy the textbook while 60% buy both.

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We will use:

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Note

By breaking the week into the individual days we can better understand the source of each randomness and is useful for modeling W.

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$$\begin{aligned} E(W) &= E(X_1 + X_2 + X_3 + X_4 + X_5) \\ &= E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5) \\ &= 18 + 18 + 18 + 18 + 18 \\ &= 90 \text{ minutes} \end{aligned}$$

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Would you be surprised if John's weekly commute wasn't exactly 90 minutes long?

There is always some variability with probabilities, so we can reasonably expect his commute to be a bit different from 90 minutes.

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Elena is selling a TV on eBay and plans to use the money to buy a toaster oven.

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$$E(X - Y) = E(X) - E(Y) = 175 - 23 = 152$$

So, she should expect to make about \$152.

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where a and b are some fixed and known numbers.

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Expected Value of Linear Combinations of Random Variables

If X and Y are random variables, then

$$E(aX + bY) = a \cdot E(X) + b \cdot E(Y)$$

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Would it be surprising to learn Leonard actually had a loss this month?

While stocks tend to rise over time, they are often volatile in the short term.

Variability of Linear Combinations of Random Variables

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$$Var(aX + bY) = a^2 \cdot Var(X) + b^2 \cdot Var(y)$$

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As usual, you can get the standard deviation by taking the square root of the variance.

Example 18

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What is the uncertainty in his total weekly commute?

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It depends on traffic patterns and what mode of transport John uses.

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Suppose that the standard deviation of TV auctions is \$25 and toaster oven auctions is \$8.

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Recall:

- X is the profit from the TV, so $Var(X) = 25^2 = 625$, and

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The standard deviation for Elena's net gain is about \$26.25.