Difference of Two Proportions

Colby Community College

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But, what we really want to know is, if blood thinners have an effect of heart attack survival rates in the general population?

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When these conditions are satisfied, the standard error of $\hat{p}_1 - \hat{p}_2$ is

$$SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

where p_1 and p_2 represent the population proportions, and n_1 and n_2 represent the sample sizes.

Confidence Intervals for $\hat{p}_1 - \hat{p}_2$

When the independence and success-failure conditions are met, we can build confidence interval in the same general manner and before:

$$\begin{array}{ccc} \text{point estimate} & \pm & z^* \cdot SE \\ & \Downarrow & \\ \hat{p}_1 - \hat{p}_2 & \pm & z^* \sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}} \end{array}$$

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We can summarize the results from the experiment in Example 1:

	Survived	Died	Total
Control	11	29	50
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The treatment group had 11 survivals and 29 deaths, and the control group had 14 survivals and 26 deaths. All are more than 10, so yes.

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Since 0% is in the confidence interval, we don't have enough evidence to say if blood thinners had any impact.

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We'll consider heart attack outcomes in these patients:

	heart attack	no event	Total
fish oil	145	12788	12933
placebo	200	12738	12938