Solution and Direction Fields: Qualitative Analysis

Department of Mathematics

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Analytic Definition of a Solution

Analytically, $y(t)$ is a **solution** of a differential equation if substituting $y(t)$ for y reduced the equation to an identity:

$$y'(t) = f(t, y(t))$$

on an appropriate domain for t.

Example 1

Verify that $y(t)$ is a solution to the DE.

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Similarly, we could show that

$$y(t) = 2e^{2t} \quad \text{and} \quad y(t) = \frac{-3}{2} e^{2t}$$

are also solutions. In fact, any constant multiple of e^{2t} is a solution.

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