

Picard's Theorem: Theoretical Analysis

Department of Mathematics

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Why Study Theory?

When considering a mathematical model two important questions are often considered:

- Does a solution actually exist? (Existence Theorems.)
- Is that solution unique? (Uniqueness Theorems.)

The theorems we will study can answer both of these questions, without actually needing to find any solutions.

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Note

Questions of existence and uniqueness are not limited to the study of differential equations. All branches of mathematics ask these questions, as well as many other fields such as philosophy and physics.

Example 1

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Further, given that

$$f'(x) = 5x^4 + 1$$

we know, from Rolle's theorem, that there can't exist more than one root in $[0, 1]$. Thus, Rolle gives us uniqueness.