Conditional Probability

Colby Community College

Example 1

The `photo_classify` data set represents a machine learning algorithm classifying a sample of 1822 photos as either about fashion or not.

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		truth		
		fashion	*not*	Total
mach_learn	*pred_fashion*	197	22	219
	pred_not	112	1491	1603
	Total	309	1513	1822

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The `photo_classify` data set represents a machine learning algorithm classifying a sample of 1822 photos as either about fashion or not.

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		fashion	*not*	Total
mach_learn	*pred_fashion*	197	22	219
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If a photo is actually about fashion, what is the chance the algorithm will correctly identify the photo as being about fashion?

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The `photo_classify` data set represents a machine learning algorithm classifying a sample of 1822 photos as either about fashion or not.

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		fashion	*not*	Total
mach_learn	*pred_fashion*	197	22	219
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If a photo is actually about fashion, what is the chance the algorithm will correctly identify the photo as being about fashion?

Of the 309 fashion photos, the algorithm correctly classifies 197 of them.

$$P\left(\texttt{mach\_learn} \text{ is } \textit{pred\_fashion} \text{ given } \texttt{truth} \text{ is } \textit{fashion}\right) = \frac{197}{309} = 0.638$$

Example 2

Using the same data set as in Example 1.

		truth		
		fashion	*not*	Total
mach_learn	*pred_fashion*	197	22	219
	pred_not	112	1491	1603
	Total	309	1513	1822

If the algorithm predicts the photo as being about fashion, what is the probability is actually is?

Example 2

Using the same data set as in Example 1.

		truth		
		fashion	*not*	Total
mach_learn	*pred_fashion*	197	22	219
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	Total	309	1513	1822

If the algorithm predicts the photo as being about fashion, what is the probability is actually is?

Of the 1603 photos predicted to be about fashion, 112 we actually about fashion.

$$P\left(\texttt{truth} \text{ is } \textit{fashion} \text{ given } \texttt{mach\_learn} \text{ is } \textit{pred\_fashion}\right) = \frac{112}{1603} = 0.070$$

Note

It can be helpful to draw Venn Diagrams of these contingency tables using rectangles.

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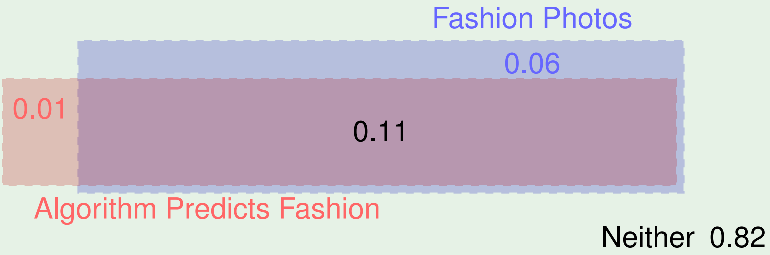
Example 3

The Venn Diagram for Example 1 is:



Definition

A **marginal probability** is a probability based on a single variable without regard to other variables.

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Example 4

$$P\left(\texttt{mach\_learn}\text{ is }\textit{pred\_fashion}\right) = \frac{219}{1822} = 0.12$$

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Example 4

$$P\left(\texttt{mach\_learn} \text{ is } \textit{pred\_fashion}\right) = \frac{219}{1822} = 0.12$$

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A probability of outcomes for two or more variables is called a **joint probability**.

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$$P\left(\texttt{mach\_learn} \text{ is } \mathit{pred\_fashion}\right) = \frac{219}{1822} = 0.12$$

Definition

A probability of outcomes for two or more variables is called a **joint probability**.

Example 5

$$P\left(\texttt{mach\_learn} \text{ is } \mathit{pred\_fashion} \text{ and } \texttt{truth} \text{ is } \mathit{fashion}\right) = \frac{197}{1822} = 0.11$$

Note

Sometimes a comma is substituted for "and" in a joint probability.

$$P\left(\texttt{mach\_learn}\text{ is }\textit{\texttt{pred\_fashion}}, \texttt{truth}\text{ is }\textit{\texttt{fashion}}\right)$$

means the same thing as

$$P\left(\texttt{mach\_learn}\text{ is }\textit{\texttt{pred\_fashion}}\text{ and }\texttt{truth}\text{ is }\textit{\texttt{fashion}}\right)$$

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A **table proportions** is a table that summarizes joint probabilities. The proportions are computed by dividing each count by table's total.

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Example 6

The table proportions for `photo_classify` are:

	`truth`: *fashion*	`truth`: *not*	Total
`mach_learn`: *pred_fashion*	$\frac{197}{1822}$		
`mach_learn`: *pred_not*			
Total			

$\downarrow\downarrow\downarrow$

	`truth`: *fashion*	`truth`: *not*	Total
`mach_learn`: *pred_fashion*	0.1081		
`mach_learn`: *pred_not*			
Total			

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mach_learn: *pred_fashion*	$\frac{197}{1822}$	$\frac{22}{1822}$	
mach_learn: *pred_not*			
Total			

$$\downarrow\downarrow\downarrow$$

	truth: *fashion*	truth: *not*	Total
mach_learn: *pred_fashion*	0.1081	0.0121	
mach_learn: *pred_not*			
Total			

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`mach_learn`: *pred_fashion*	$\frac{197}{1822}$	$\frac{22}{1822}$	$\frac{219}{1822}$
`mach_learn`: *pred_not*			
Total			

$\downarrow\downarrow\downarrow$

	`truth`: *fashion*	`truth`: *not*	Total
`mach_learn`: *pred_fashion*	0.1081	0.0121	0.1202
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`mach_learn`: *`pred_fashion`*	$\frac{197}{1822}$	$\frac{22}{1822}$	$\frac{219}{1822}$
`mach_learn`: *`pred_not`*	$\frac{112}{1822}$		
Total			

$\downarrow\downarrow\downarrow$

	`truth`: *`fashion`*	`truth`: *`not`*	Total
`mach_learn`: *`pred_fashion`*	0.1081	0.0121	0.1202
`mach_learn`: *`pred_not`*	0.0615		
Total			

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`mach_learn`: *`pred_fashion`*	$\frac{197}{1822}$	$\frac{22}{1822}$	$\frac{219}{1822}$
`mach_learn`: *`pred_not`*	$\frac{112}{1822}$	$\frac{1491}{1822}$	
Total			

$$\downarrow\downarrow\downarrow$$

	`truth`: *`fashion`*	`truth`: *`not`*	Total
`mach_learn`: *`pred_fashion`*	0.1081	0.0121	0.1202
`mach_learn`: *`pred_not`*	0.0615	0.8183	
Total			

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	truth: *fashion*	truth: *not*	Total
mach_learn: *pred_fashion*	$\frac{197}{1822}$	$\frac{22}{1822}$	$\frac{219}{1822}$
mach_learn: *pred_not*	$\frac{112}{1822}$	$\frac{1491}{1822}$	$\frac{1603}{1822}$
Total			

$\downarrow\downarrow\downarrow$

	truth: *fashion*	truth: *not*	Total
mach_learn: *pred_fashion*	0.1081	0.0121	0.1202
mach_learn: *pred_not*	0.0615	0.8183	0.8798
Total			

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`mach_learn`: *pred_fashion*	$\frac{197}{1822}$	$\frac{22}{1822}$	$\frac{219}{1822}$
`mach_learn`: *pred_not*	$\frac{112}{1822}$	$\frac{1491}{1822}$	$\frac{1603}{1822}$
Total	$\frac{309}{1822}$		

$\downarrow\downarrow\downarrow$

	`truth`: *fashion*	`truth`: *not*	Total
`mach_learn`: *pred_fashion*	0.1081	0.0121	0.1202
`mach_learn`: *pred_not*	0.0615	0.8183	0.8798
Total	0.1696		

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	`truth`: *`fashion`*	`truth`: *`not`*	Total
`mach_learn`: *`pred_fashion`*	$\frac{197}{1822}$	$\frac{22}{1822}$	$\frac{219}{1822}$
`mach_learn`: *`pred_not`*	$\frac{112}{1822}$	$\frac{1491}{1822}$	$\frac{1603}{1822}$
Total	$\frac{309}{1822}$	$\frac{1513}{1822}$	

$\downarrow\downarrow\downarrow$

	`truth`: *`fashion`*	`truth`: *`not`*	Total
`mach_learn`: *`pred_fashion`*	0.1081	0.0121	0.1202
`mach_learn`: *`pred_not`*	0.0615	0.8183	0.8798
Total	0.1696	0.8304	

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	`truth`: *`fashion`*	`truth`: *`not`*	Total
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`mach_learn`: *`pred_not`*	$\frac{112}{1822}$	$\frac{1491}{1822}$	$\frac{1603}{1822}$
Total	$\frac{309}{1822}$	$\frac{1513}{1822}$	$\frac{1822}{1822}$

$\downarrow\downarrow\downarrow$

	`truth`: *`fashion`*	`truth`: *`not`*	Total
`mach_learn`: *`pred_fashion`*	0.1081	0.0121	0.1202
`mach_learn`: *`pred_not`*	0.0615	0.8183	0.8798
Total	0.1696	0.8304	1.0

Example 7

The table proportions from Example 6 make a probability distribution.

Joint Outcome	Probability
`mach_learn` is *pred_fashion* and `truth` is *fashion*	0.1081
`mach_learn` is *pred_fashion* and `truth` is *not*	0.0121
`mach_learn` is *pred_not* and `truth` is *fashion*	0.0615
`mach_learn` is *pred_not* and `truth` is *not*	0.8182

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`mach_learn` is *pred_not* and `truth` is *fashion*	0.0615
`mach_learn` is *pred_not* and `truth` is *not*	0.8182

Note

Joint probabilities can be used to calculate marginal probabilities in simple cases.

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`mach_learn` is *pred_not* and `truth` is *fashion*	0.0615
`mach_learn` is *pred_not* and `truth` is *not*	0.8182

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Joint probabilities can be used to calculate marginal probabilities in simple cases.

Example 8

$$P(\texttt{truth}\text{ is }\textit{fashion}) = P(\texttt{mac\_learn}\text{ is }\textit{pred\_fashion}\text{ and }\texttt{truth}\text{ is }\textit{fashion}) + P(\texttt{mac\_learn}\text{ is }\textit{pred\_not}\text{ and }\texttt{truth}\text{ is }\textit{fashion})$$

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`mach_learn` is *pred_fashion* and `truth` is *fashion*	0.1081
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`mach_learn` is *pred_not* and `truth` is *fashion*	0.0615
`mach_learn` is *pred_not* and `truth` is *not*	0.8182

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$$\begin{aligned} P\left(\texttt{truth}\text{ is }\textit{fashion}\right) &= P\left(\texttt{mac\_learn}\text{ is }\textit{pred\_fashion}\text{ and }\texttt{truth}\text{ is }\textit{fashion}\right) \\ &\quad + P\left(\texttt{mac\_learn}\text{ is }\textit{pred\_not}\text{ and }\texttt{truth}\text{ is }\textit{fashion}\right) \\ &= 0.1081 + 0.0615 \end{aligned}$$

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`mach_learn` is *pred_not* and `truth` is *fashion*	0.0615
`mach_learn` is *pred_not* and `truth` is *not*	0.8182

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Joint probabilities can be used to calculate marginal probabilities in simple cases.

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$$\begin{aligned} P(\texttt{truth} \text{ is } \textit{fashion}) &= P(\texttt{mac\_learn} \text{ is } \textit{pred\_fashion} \text{ and } \texttt{truth} \text{ is } \textit{fashion}) \\ &\quad + P(\texttt{mac\_learn} \text{ is } \textit{pred\_not} \text{ and } \texttt{truth} \text{ is } \textit{fashion}) \\ &= 0.1081 + 0.0615 = 0.1696 \end{aligned}$$