

Defining Probability

Colby Community College

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What is the chance of rolling a 1 on this die?

If the dice is fair, each side has the same chance of being rolled. So a 1 has a one-in-six chance, equivalently $\frac{1}{6}$.

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Example 2

What is the probability of rolling a 1 or 2 on a die?

There are two outcomes, a 1 or a 2, and six faces on a die.

$$P(\text{roll 1 or 2}) = \frac{2}{6} = \frac{1}{3}$$

Note

A standard deck of 52 playing cards consists of four **suits** in two colors: Hearts ♥, Spades ♠, Diamonds ♦, and Clubs ♣

Each suit contains 13 cards, each of a different **rank**:
2 through 10, Jack, Queen, King, and Ace

The Jack, Queen, and King cards are called **face cards**.

The Jack, Queen, King, and Ace cards are called **honour cards**.

The cards numbered 2 to 10 are called **numerals**.

♣2	♣3	♣4	♣5	♣6	♣7	♣8	♣9	♣10	♣J	♣Q	♣K	♣A
♦2	♦3	♦4	♦5	♦6	♦7	♦8	♦9	♦10	♦J	♦Q	♦K	♦A
♥2	♥3	♥4	♥5	♥6	♥7	♥8	♥9	♥10	♥J	♥Q	♥K	♥A
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There are four aces in a deck of 52 cards. Which gives the probability

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Every side of the die is listed, so

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An outcome with a probability of 1 is called **certain**.

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An outcome with a probability of 0 is called **impossible**.

Note

Probabilities are always between 0 and 1.

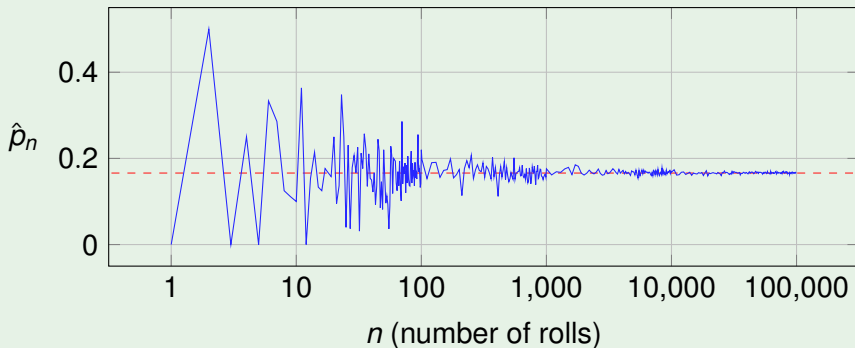
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The probability of rolling a 1 on a die is $p = 1/6 \approx 0.167$, but if we roll six dice, we may get no 1's or multiple 1's.

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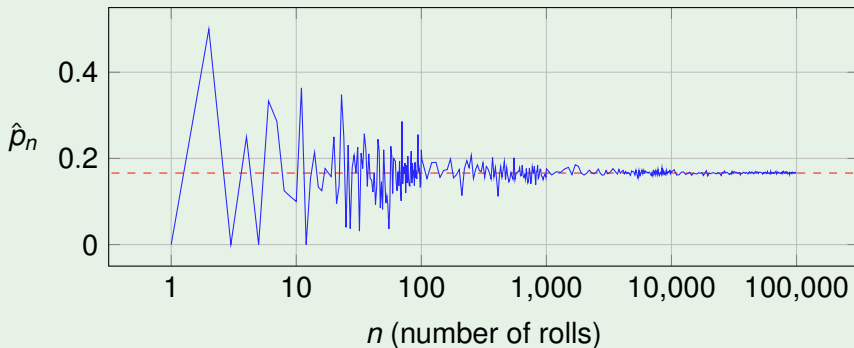
Let \hat{p}_n be the proportion of number of 1's rolled after n rolls.



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Note

It is not a coincidence that \hat{p}_n get closer to p as n increases.

Law of Large Numbers

As more observations are collected, the proportion \hat{p}_n of occurrences with a particular outcome converges to the probability p of that outcome.

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- If we know nothing about the likelihood of different possible outcomes, we should not assume that they are equally likely.
 - You should not think that the probability of passing the next exam is $\frac{1}{2}$, or 0.5. The actual probability depends on factors such as the amount of preparation and the difficulty of the exam.

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Are the outcomes “draw an ace” and “draw a diamond” disjoint?

No, the ♦A is both an ace and a diamond.

Addition Rule of Disjoint Outcomes

If A_1 and A_2 represent two disjoint outcomes, then the probability that one of them occurs is given by:

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2)$$

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$$\begin{aligned} P(\text{roll a 1 or roll a 2}) &= P(\text{roll a 1}) + P(\text{roll a 2}) \\ &= \frac{1}{6} + \frac{1}{6} = \frac{1+1}{6} = \frac{2}{6} = \frac{1}{3} \end{aligned}$$

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If two events have no elements in common, they are disjoint.

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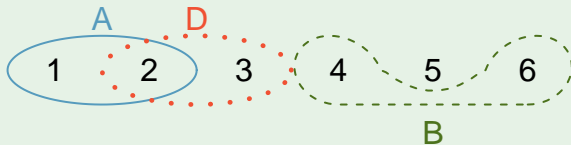
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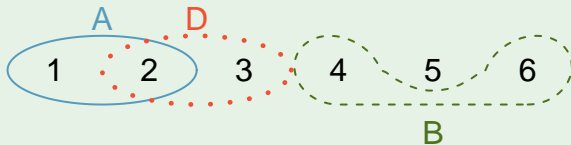
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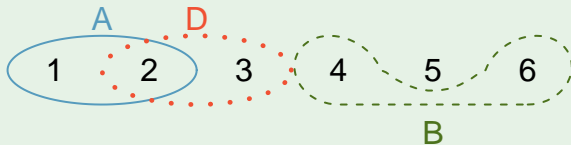
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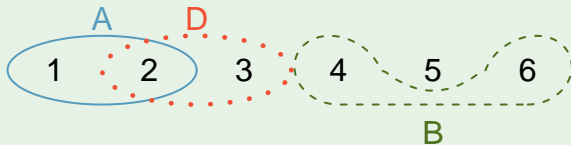
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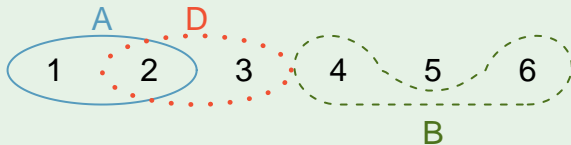
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Are A and D disjoint? No, 2 is in both.

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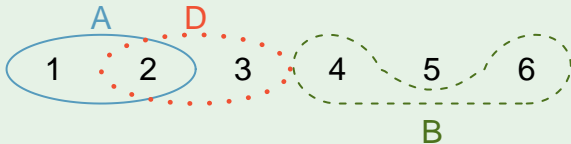
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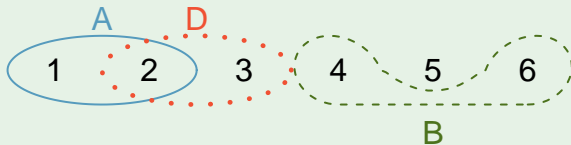
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The list of all possible outcomes is:

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

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The relevant outcomes are: H1, H2, H3, H4, H5, H6, T6.

Meaning that $P(\text{H or 6}) = \frac{7}{12}$.

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The correct probability is:

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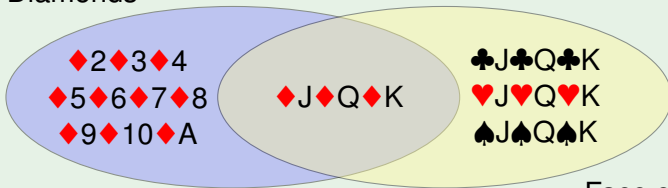
Example 15

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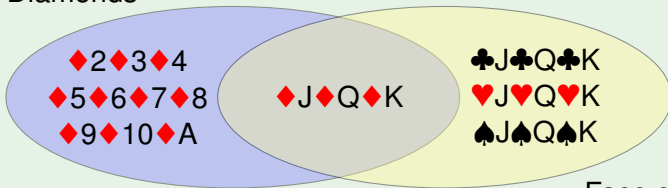


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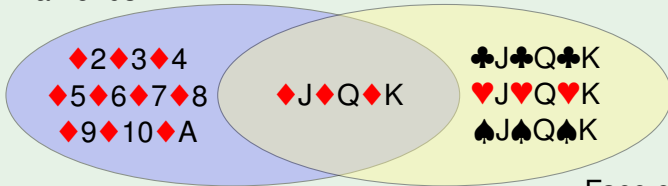
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The Addition Rule for Disjoint Outcomes would count $\diamondsuit J$, $\diamondsuit Q$, $\diamondsuit K$ twice!

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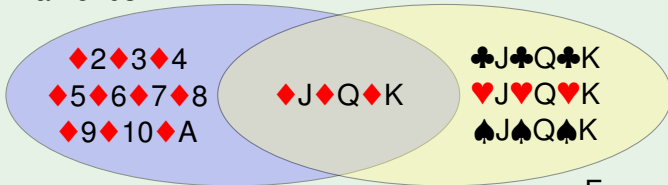
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$$P(\diamondsuit \text{ and face}) = P(\diamondsuit) + P(\text{face}) - P(\diamondsuit \text{ and face})$$

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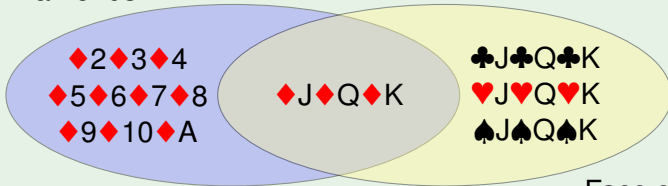
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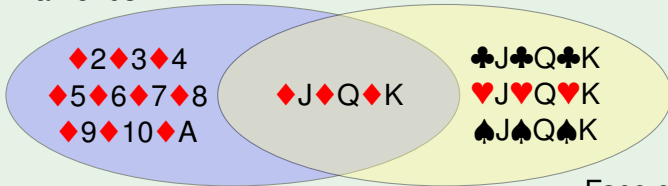
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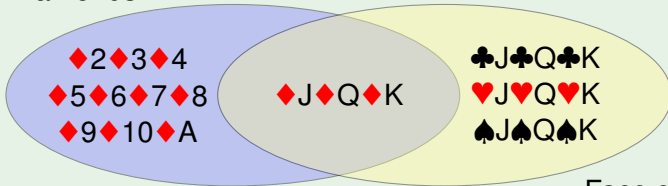
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General Addition Rule

If A and B are any two events, disjoint or not, then the probability that at least one of them will occur is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

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Note

In statistics, when we write “or” what we mean is “and/or”, unless we explicitly say otherwise.

In other words, “ A or B ” occurring means A , B , or both A and B occur.

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In other words, “ A or B ” occurring means A , B , or both A and B occur.

Note

If A and B are disjoint this means $P(A \text{ and } B) = 0$, and so we get the Addition Rule for Disjoint Outcomes:

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= P(A) + P(B) - 0 \\ &= P(A) + P(B) \end{aligned}$$

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Suppose we draw one card from a standard deck. Let us find the probability that we get a Queen or a King.

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There are 4 Queens and 4 Kings in the deck, hence eight outcomes out of 52 possible outcomes. So, the probability is

$$P(Q \text{ or } K) = \frac{8}{52}$$

Since there are no cards that are both Kings and Queens, we have

$$P(Q \text{ or } K) = P(Q) + P(K) - P(Q \text{ and } K) = \frac{4}{52} + \frac{4}{52} - \frac{0}{52} = \frac{8}{52}$$

Definition

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Example 17

The probability distribution for the sum of two dies:

Dice sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

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Rules for Probability Distributions

All probability distributions must satisfy the following rules:

- 1 The outcomes listed must be disjoint.
- 2 Each probability must be between 0 and 1.
- 3 The probabilities must sum to 1.

Example 18

The follow table contain three possible distributions for household income in the United States.

Income Range	\$0-\$25k	\$25k-\$50k	\$50k-\$100k	\$100k+
(a)	0.18	0.39	0.33	0.16
(b)	0.38	-0.27	0.52	0.37
(c)	0.28	0.27	0.29	0.16

Only one of the three is actually a probability distribution. Which one?

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Only one of the three is actually a probability distribution. Which one?

- (a) sums to 2.5.
- (b) contains a negative number.
- (c) is the real probability distribution.

Note

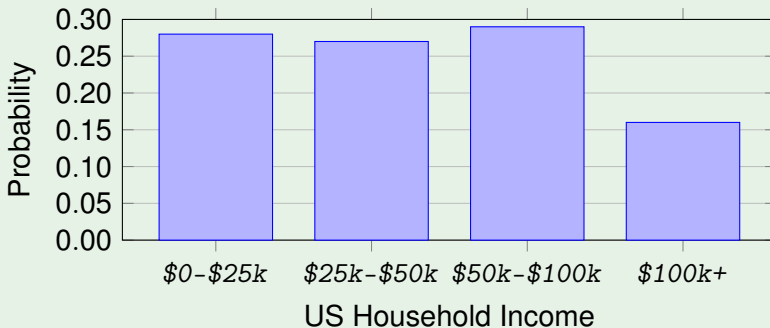
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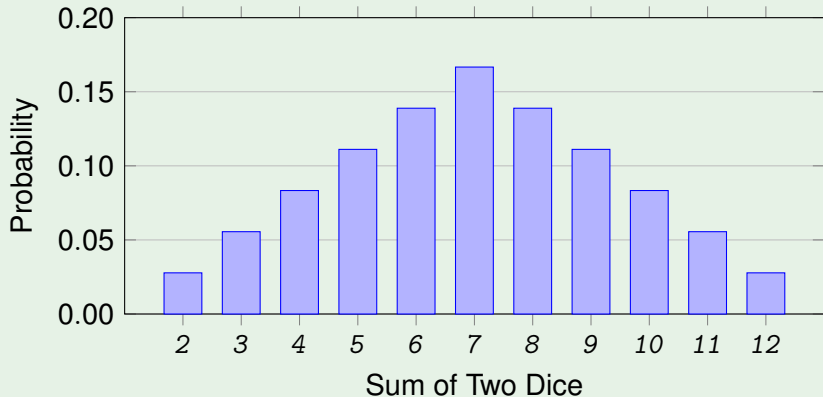
Example 19

Here is the bar plot for the probability distribution in Example 18:



Example 20

Here is the bar plot for the dice sum distribution in Example 17



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$\{\text{roll a 1, roll a 2, roll a 3, roll a 4, roll a 5, roll a 6}\}$

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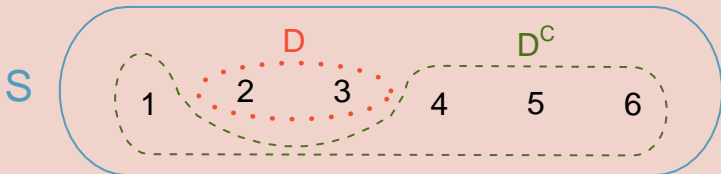
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Definition

The **complement** of an event D is all the outcomes in the sample space that are not in D . Denoted D^C .



Note

A and A^C are disjoint events.

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Note

Finally, $P(A^C) = 1 - P(A)$ and $P(A) = 1 - P(A^C)$.

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Consider rolling two dice and summing the numbers.

What is the complement of the event “the total is less than 12?”

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Note

If two events are not independent, they are called **dependent**.

Multiplication Rule For Independent Events

If events A and B are independent, then the probability of both A and B occurring is

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The sample space is $\{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$.

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Since a coin and dice are independent, we could have used the Multiplication Rule:

$$P(T \text{ and } 3) = P(T) \cdot P(3) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

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$$\begin{aligned} P(\text{safe flight}) &= 1 - P(\text{system}_1 \text{ fails and system}_2 \text{ fails and system}_3 \text{ fails}) \\ &= 1 - P(\text{system}_1 \text{ fails}) \cdot P(\text{system}_2 \text{ fails}) \cdot P(\text{system}_3 \text{ fails}) \end{aligned}$$

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$$\begin{aligned} P(\text{safe flight}) &= 1 - P(\text{system}_1 \text{ fails and system}_2 \text{ fails and system}_3 \text{ fails}) \\ &= 1 - P(\text{system}_1 \text{ fails}) \cdot P(\text{system}_2 \text{ fails}) \cdot P(\text{system}_3 \text{ fails}) \\ &= 1 - 0.002 \cdot 0.002 \cdot 0.002 \end{aligned}$$

Example 26

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of **redundancy**, whereby critical components are duplicated so that if one fails, the other will still work.

The Airbus 310 airliner has three independent hydraulic systems, so if one system fails, full flight control is maintained. Let us assume the probability of a hydraulic system failing is 0.002.

- *If the Airbus 310 had only a single hydraulic system, what is the probability that the flight control system would work for an entire flight?* $P(\text{safe flight}) = 1 - P(\text{system fails}) = 1 - 0.002 = 0.998$
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Note

We can also use the Multiplication Rule For Independent Events to check if two events are independent.

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Example 27

If we shuffle a deck of playing cards and draw one card, let us check that the events “draw a heart” and “draw an ace” are independent.

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We know that

$$P(\heartsuit A) = \frac{1}{52}$$

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If we shuffle a deck of playing cards and draw one card, let us check that the events “draw a heart” and “draw an ace” are independent.

We know that

$$P(\heartsuit A) = \frac{1}{52}$$
$$P(\heartsuit) = \frac{13}{52} = \frac{1}{4}$$

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We can also use the Multiplication Rule For Independent Events to check if two events are independent.

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$$P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$$

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So,

$$P(\heartsuit) \cdot P(\text{Ace})$$

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$$P(\heartsuit) \cdot P(\text{Ace}) = \frac{1}{4} \cdot \frac{1}{13}$$

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So,

$$P(\heartsuit) \cdot P(\text{Ace}) = \frac{1}{4} \cdot \frac{1}{13} = \frac{1}{4 \cdot 13}$$

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$$P(\heartsuit) \cdot P(\text{Ace}) = \frac{1}{4} \cdot \frac{1}{13} = \frac{1}{4 \cdot 13} = \frac{1}{52} = P(\heartsuit \text{ and Ace})$$

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Since the equation holds, the two events must be independent.