Estimating a Population Mean

Colby Community College

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1. The sample is a simple random sample.
2. The population is normally distributed or $n > 30$.
    - The method we will use is robust against departure from normality.
    - If the distribution is approximately normal, a sample size of 15 to 30 may be acceptable.

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If a population has a normal distribution, then the distribution of

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A Student t distribution is commonly called a ***t* distribution**.

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We can freely assign values to the first 9 scores, but the 10th score would need to be:

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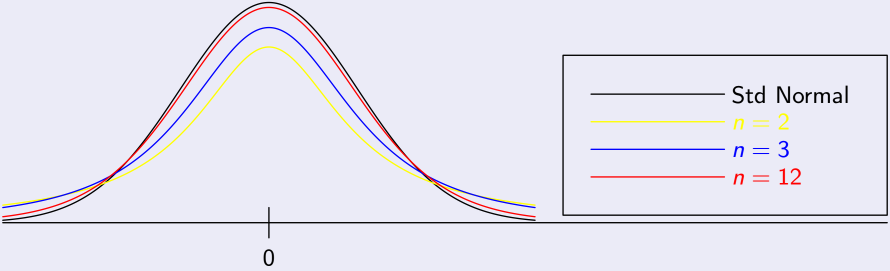
Hence 9 degrees of freedom.

Note

The Student t distribution is different for different sample sizes.



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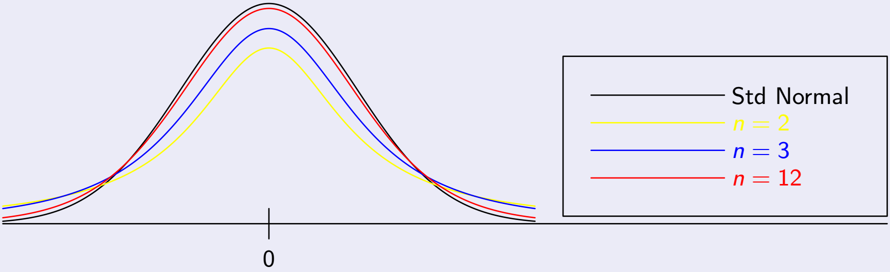

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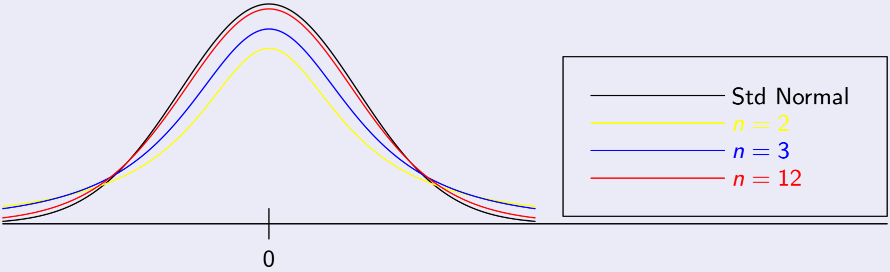

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As the sample size n gets larger, the Student t distribution gets closer to the standard normal distribution.

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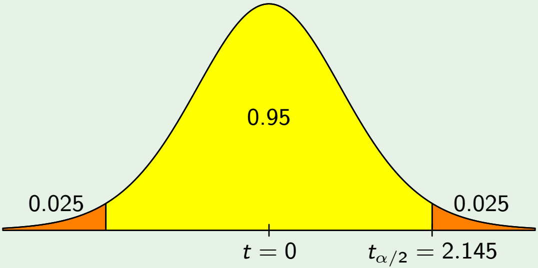
We can then use technology to find the critical value.



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4. Using the value of the calculated margin of error E and the value of the sample mean $\bar{x}$, find the values of the confidence interval limits $\bar{x} - E$ and $\bar{x} + E$.
5. Round the resulting confidence interval limits:
   - With an original data set, round to three significant digits.
   - When using summary statistics, round to the same number of decimal places.

Example 3

The weights (in hectograms, hg) of randomly selected girls at birth are

33	28	33	37	31	32	31	28	34	28	33	26	30	31	28

(Based on data from the National Center for Health Statistics.)

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We are 95% confident that the interval from 29.2 hg to 32.5hg actually does contain the true value of μ.

Finding $\bar{x}$ from a Confidence Interval

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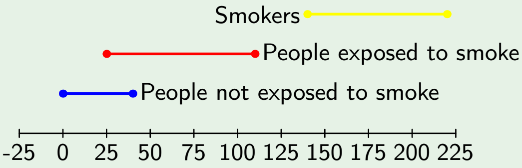
Example 4

We can compare the confidence intervals of the mean cotinine level in each of three samples (Data Set 12).



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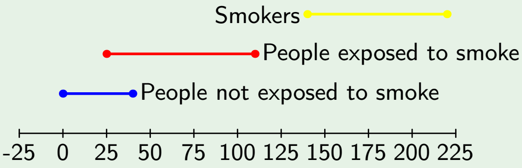

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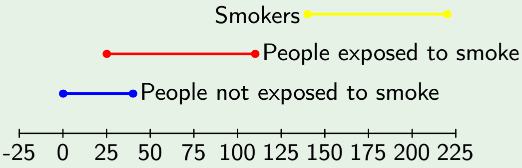

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The two non-smoking groups have overlapping confidence intervals, so it possible that they have the same mean cotinine level.

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The required sample size is

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

(Recall that $z_{\alpha/2}$ is the critical value for the standard normal distribution.)

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Rounding

If the computed sample size n is not a whole number, round the value of n up to the next larger whole number.

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Caution

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To be 95% confident that out interval contains the true population standard deviation, we would need a sample size of at least 97.

Note

If we somehow know the population standard deviation but do not know the population mean, then we calculate the confidence interval using the methods in section 7.1.

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Choosing the Appropriate Distribution

Conditions	**Method**
σ not known and normal population or σ not known and $n > 30$	Student t distribution
σ known and normal population or σ known and $n > 30$	Normal distribution
Population is not normal and $n \leq 30$	Use other methods.