Defining Probability

Colby Community College

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What is the chance of rolling a 1 on this die?

If the dice is fair, each side has the same chance of being rolled. So a 1 has a one-in-six chance, equivalently $\frac{1}{6}$.

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Example 2

What is the probability of rolling a 1 or 2 on a die?

There are two outcomes, a 1 or a 2, and six faces on a die.

$$P(\text{roll 1 or 2}) = \frac{2}{6} = \frac{1}{3}$$

Note

A standard deck of 52 playing cards consists of four **suits** in two colors: Hearts ♥, Spades ♠, Diamonds ♦, and Clubs ♣

Each suit contains 13 cards, each of a different **rank**:
2 through 10, Jack, Queen, King, and Ace

The Jack, Queen, and King cards are called **face cards.**

The Jack, Queen, King, and Ace cards are called **honour cards.**

The cards numbered 2 to 10 are called **numerals.**

♣2	♣3	♣4	♣5	♣6	♣7	♣8	♣9	♣10	♣J	♣Q	♣K	♣A
♦2	♦3	♦4	♦5	♦6	♦7	♦8	♦9	♦10	♦J	♦Q	♦K	♦A
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There are four aces in a deck of 52 cards. Which gives the probability

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Every side of the die is listed, so

$$P\left(\text{roll 1 or 2 or 3 or 4 or 5 or 6}\right) = \frac{6}{6} = 1 = 100\%$$

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An outcome with a probability of 1 is called **certain.**

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Probabilities are always between 0 and 1.

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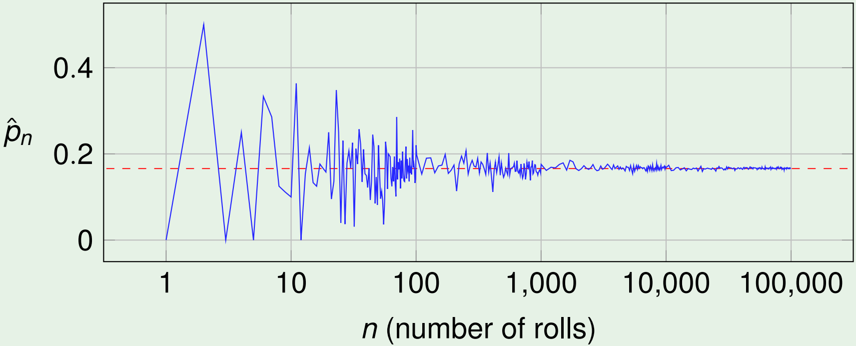
Let $\hat{p}_n$ be the proportion of number of 1's rolled after n rolls.



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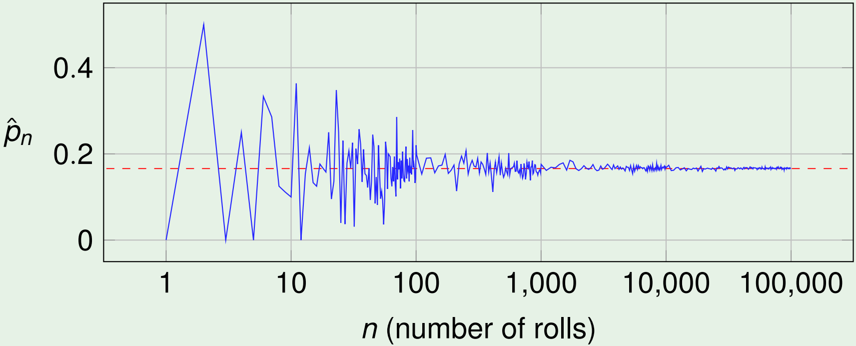

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It is not a coincidence that $\hat{p}_n$ get closer to p as n increases.

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As more observations are collected, the proportion $\hat{p}_n$ of occurrences with a particular outcome converges to the probability p of that outcome.

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- If we know nothing about the likelihood of different possible outcomes, we should not assume that they are equally likely.
  - You should not think that the probability of passing the next exam is $\frac{1}{2}$, or 0.5. The actual probability depends on factors such as the amount of preparation and the difficulty of the exam.

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Example 9

Are the outcomes "draw an ace" and "draw a diamond" disjoint?

No, the ♦A is both an ace and a diamond.

Addition Rule of Disjoint Outcomes

If A_1 and A_2 represent two disjoint outcomes, then the probability that one of them occurs is given by:

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2)$$

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$$\begin{aligned} P(\text{roll a 1 or roll a 2}) &= P(\text{roll a 1}) + P(\text{roll a 2}) \\ &= \frac{1}{6} + \frac{1}{6} = \frac{1+1}{6} = \frac{2}{6} = \frac{1}{3} \end{aligned}$$

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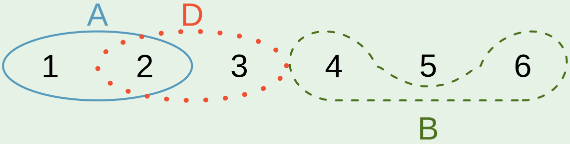

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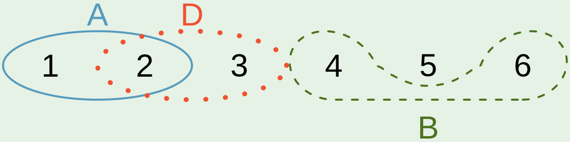

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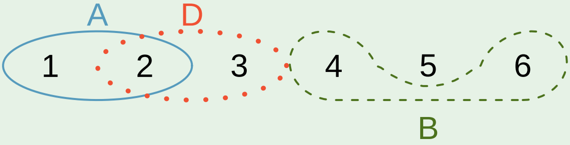

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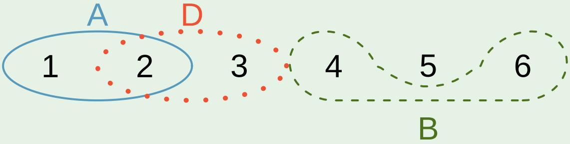

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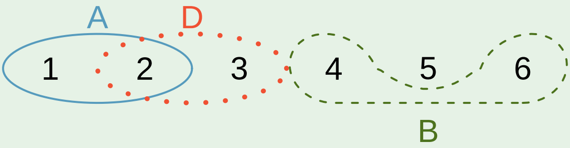

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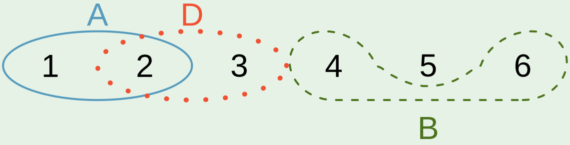

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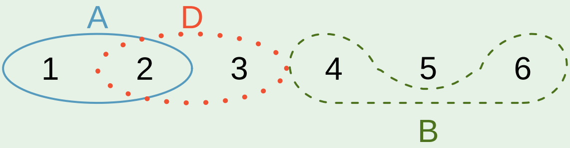

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Suppose we flip a fair coin and roll a fair die.

The list of all possible outcomes is:

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$$P(\text{H or 6}) = P(\text{H}) + P(6) - P(\text{H and 6}) = \frac{1}{2} + \frac{1}{6} - \frac{1}{12}$$

Example 14

Suppose we flip a fair coin and roll a fair die.

The list of all possible outcomes is:

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

We want to calculate the probability of getting a head or a six.

The relevant outcomes are: H1, H2, H3, H4, H5, H6, T6.

Meaning that $P\,(\text{H or } 6) = \frac{7}{12}$.

Notice that $\frac{6}{12} = \frac{1}{2}$ of the outcomes have heads and $\frac{2}{12} = \frac{1}{6}$ have a six.

But, $\frac{1}{2} + \frac{1}{6} = \frac{8}{12}$, is wrong because we have double counted H6.

The correct probability is:

$$P\,(\text{H or } 6) = P\,(\text{H}) + P\,(6) - P\,(\text{H and } 6) = \frac{1}{2} + \frac{1}{6} - \frac{1}{12} = \frac{7}{12}$$

Example 15

Let us consider the events “draw a diamond” and “draw a face card”.

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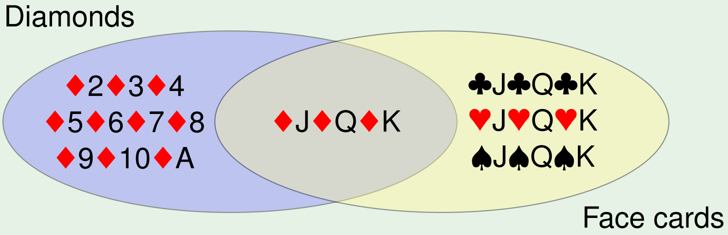
These outcomes are not disjoint, since three cards are both:



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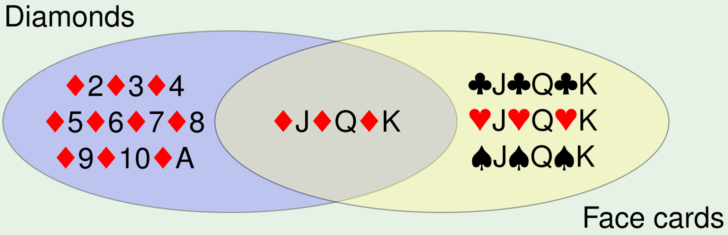

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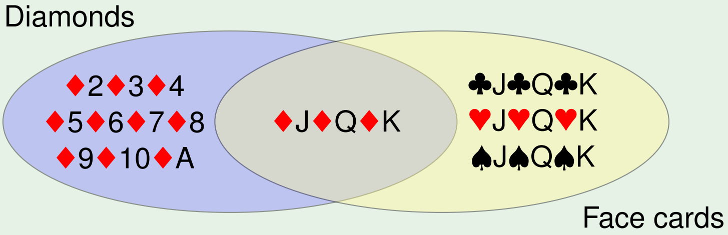

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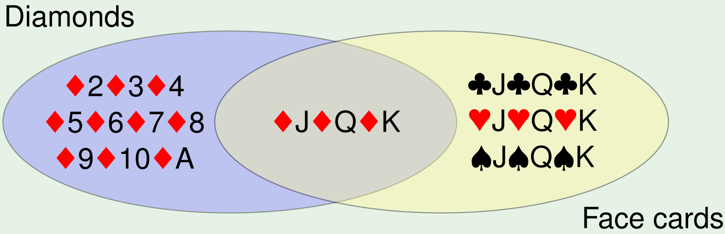

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$$= \frac{13}{52}$$

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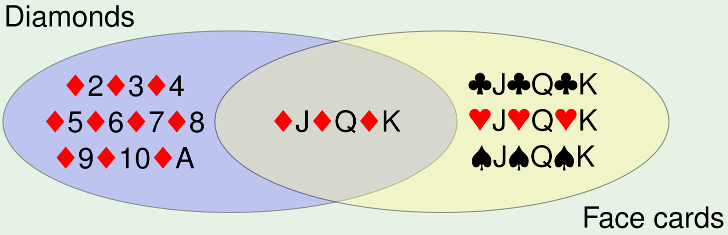

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$$P(\diamondsuit \text{ and face}) = P(\diamondsuit) + P(\text{face}) - P(\diamondsuit \text{ and face})$$
$$= \frac{13}{52} + \frac{12}{52}$$

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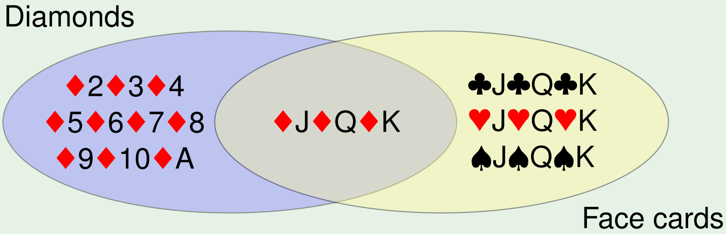

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$$P(\blacklozenge \text{ and face}) = P(\blacklozenge) + P(\text{face}) - P(\blacklozenge \text{ and face})$$
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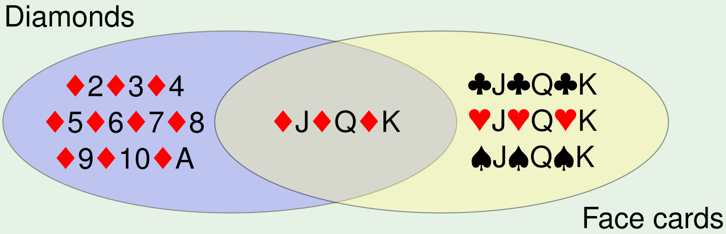

The Addition Rule for Disjoint Outcomes would count ♦J♦Q♦K twice!

$$\begin{aligned} P(\blacklozenge \text{ and face}) &= P(\blacklozenge) + P(\text{face}) - P(\blacklozenge \text{ and face}) \\ &= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} \\ &= \frac{22}{52} = \frac{11}{26} \end{aligned}$$

General Addition Rule

If A and B are any two events, disjoint or not, then the probability that at least one of them will occur is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

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In statistics, when we write “or” what we mean is “and/or”, unless we explicitly say otherwise.

In other words, “A or B” occurring means A, B, or both A and B occur.

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In statistics, when we write "or" what we mean is "and/or", unless we explicitly say otherwise.

In other words, "A or B" occurring means A, B, or both A and B occur.

Note

If A and B are disjoint this means $P(A \text{ and } B) = 0$, and so we get the Addition Rule for Disjoint Outcomes:

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= P(A) + P(B) - 0 \\ &= P(A) + P(B) \end{aligned}$$

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Suppose we draw one card from a standard deck. Let us find the probability that we get a Queen or a King.

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There are 4 Queens and 4 Kings in the deck, hence eight outcomes out of 52 possible outcomes. So, the probability is

$$P(\text{Q or K}) = \frac{8}{52}$$

Since there are no cards that are both Kings and Queens, we have

$$P(\text{Q or K}) = P(\text{Q}) + P(K) - P(\text{Q and K}) = \frac{4}{52} + \frac{4}{52} - \frac{0}{52} = \frac{8}{52}$$