

Describing Data

Department of Mathematics

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(Slides by Adam Wilson)

Definition

A **frequency table** is a table with two columns. One column lists the categories, and the other the frequencies with which the items in the categories occur. (i.e. how many items fit into each category.)

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Example 1

An insurance company determines vehicle insurance premiums based on known risk factors. If a person is considered a higher risk, their premiums will be higher. One potential factor is the color of your car. The insurance company believes that people with some color cars are more likely to get in accidents.

To research this, they examined police reports for recent total-loss collisions. The data is summarized in the frequency table.

Color	Frequency
Blue	25
Green	52
Red	41
White	36
Black	39
Grey	23

Definition

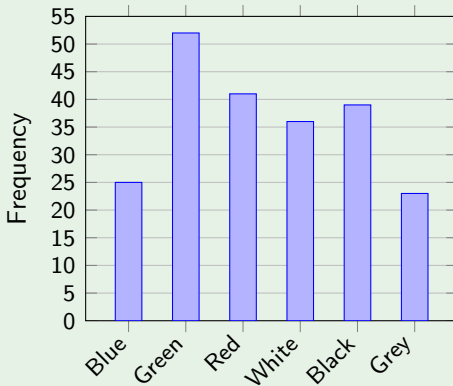
A **bar graph** is a graph that displays a bar for each category with the length of each bar indicating the frequency of that category.

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Example 2

Using the car data from Example 1, we have the following bar graph.



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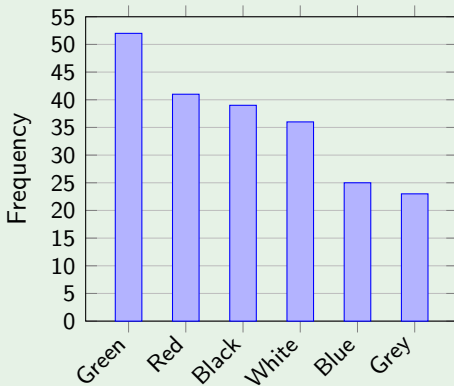
A **Pareto chart** is a bar graph ordered from highest to lowest frequency.

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Example 3

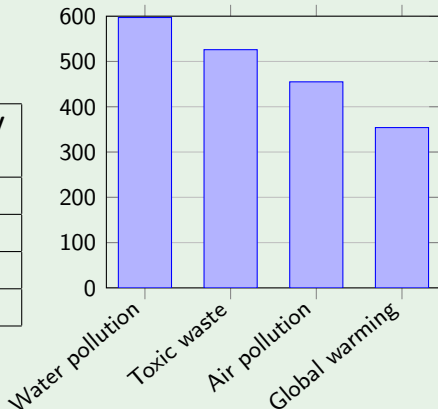
Using the car data from Example 1, we have the following Pareto chart.



Example 4

In a survey¹, adults were asked whether they personally worried about a variety of environmental concerns. The numbers (out of 1012 surveyed) who indicated that they worried “a great deal” about some selected concerns are summarized below.

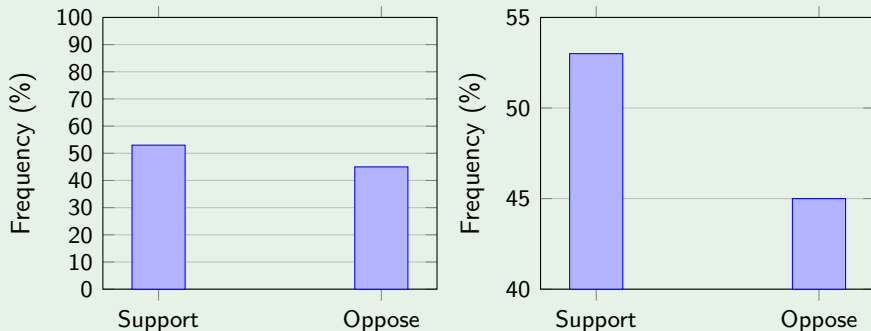
Environmental Issue	Frequency
Water pollution	597
Toxic waste	526
Air pollution	455
Global warming	354



¹Gallup Poll. March 5-8, 2009. <http://www.pollingreport.com/enviro.htm>

Example 5

Compare the two graphs below showing support for same-sex marriage rights from a poll taken in May 2013².



The difference in the vertical scale on the first graph suggests a different story than the true differences in percentages; the second graph makes it look like more than twice as many people support marriage rights as oppose it.

²Gallup Poll. May 2-7, 2013, from <http://www.pollingreport.com/civil.htm>

Definition

A **histogram** a bar graph, where the horizontal axis is a number line.

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Definition

Class intervals are groupings of the data. In general, we define class intervals so that:

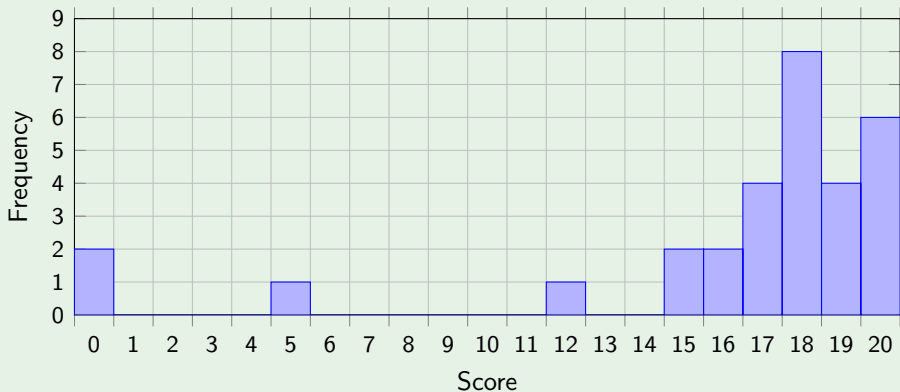
- Each interval is equal size. For example, if the first class contains values from 120-129, then the second class should include 130-139.
- We have somewhere between 5 and 20 classes, typically, depending upon the number of data we are working with.

Example 6

A teacher records scores on a 20-point quiz for the 30 students in his class.

19	20	18	18	17	18	19	17	20	18	20	16	20	15	17
12	18	19	18	19	17	20	18	16	15	18	20	5	0	0

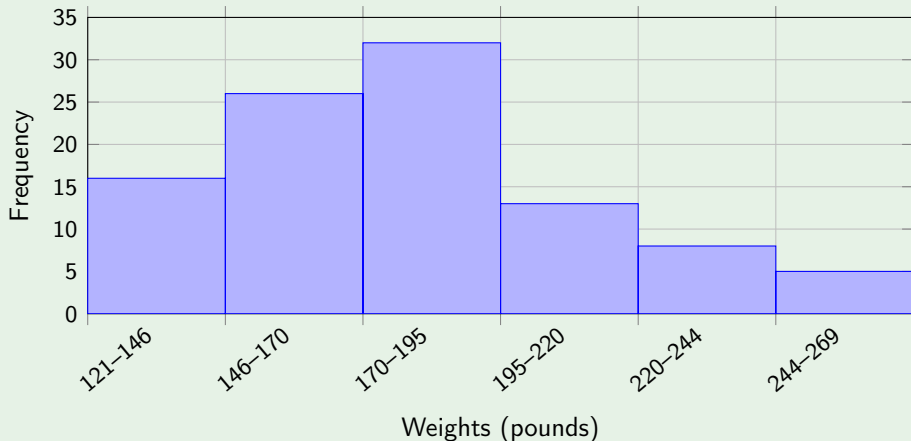
These scores could be summarized in the following histogram.



Example 7

Suppose that we have collected weights from 100 male subjects as part of a nutrition study. For our weight data, we have values ranging from a low of 121 pounds to a high of 263 pounds, giving a total span of $263 - 121 = 142$.

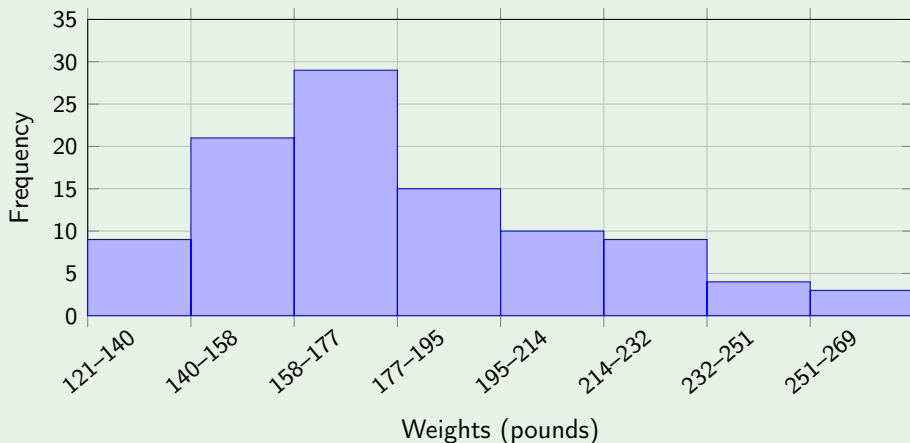
We could create 6 intervals with a width of around 20.



Example 7

Suppose that we have collected weights from 100 male subjects as part of a nutrition study. For our weight data, we have values ranging from a low of 121 pounds to a high of 263 pounds, giving a total span of $263 - 121 = 142$.

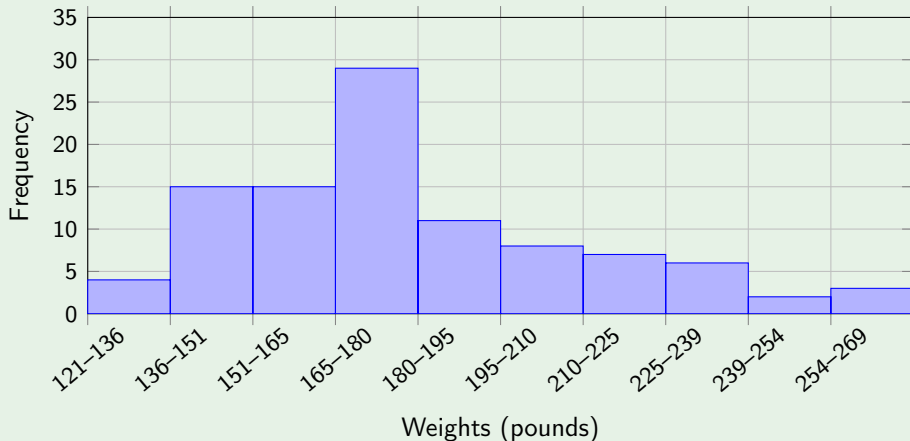
We could create 8 intervals with a width of around 18.



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Suppose that we have collected weights from 100 male subjects as part of a nutrition study. For our weight data, we have values ranging from a low of 121 pounds to a high of 263 pounds, giving a total span of $263 - 121 = 142$.

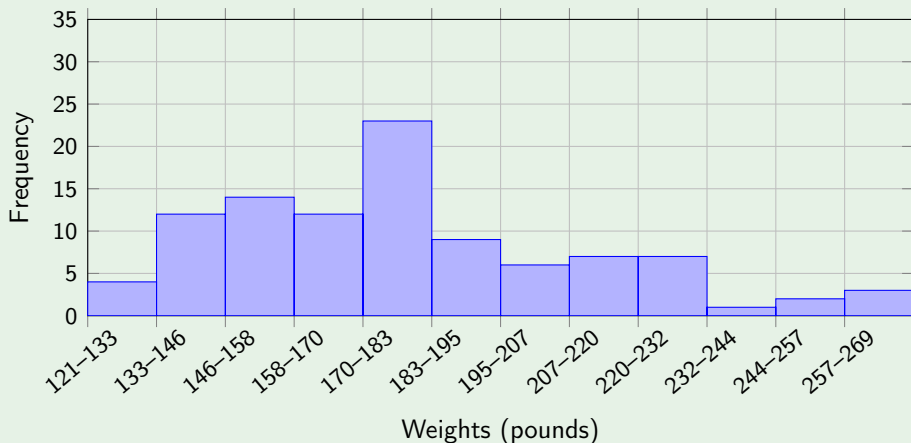
We could create 10 intervals with a width of around 14.



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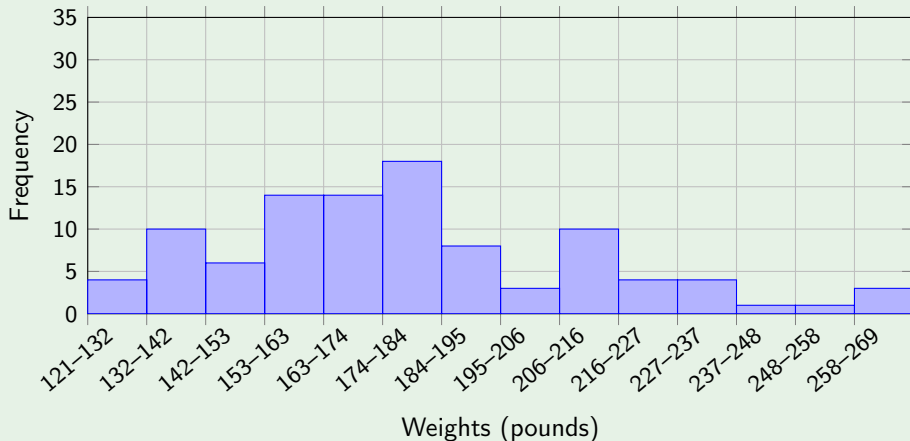
We could create 10 intervals with a width of around 12.



Example 7

Suppose that we have collected weights from 100 male subjects as part of a nutrition study. For our weight data, we have values ranging from a low of 121 pounds to a high of 263 pounds, giving a total span of $263 - 121 = 142$.

We could create 14 intervals with a width of around 10.



Definition

The **mean** of a set of data is the sum of the data values divided by the number of values.

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It is not uncommon to see the word “average” used instead of “mean”.

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Example 8

Marci's exam scores for her last math class were: 79, 86, 82, 94.
What is the mean of these values?

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It is not uncommon to see the word “average” used instead of “mean”.

Example 8

Marci's exam scores for her last math class were: 79, 86, 82, 94.
What is the mean of these values?

$$\frac{79 + 86 + 82 + 94}{4} = \frac{341}{4} = 82.25 \approx 85.3$$

Typically we round means to one more decimal than the original data.

Example 9

The one hundred families in a particular neighborhood are asked their annual household income, to the nearest \$5 thousand dollars.

Income	Frequency	Income	Frequency
15	6	35	19
20	8	40	20
25	11	45	12
30	17	50	7

What is the mean of this data?

Example 9

The one hundred families in a particular neighborhood are asked their annual household income, to the nearest \$5 thousand dollars.

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15	6	35	19
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What is the mean of this data?

$$\begin{aligned}& \overbrace{15 + \cdots + 15}^{6 \text{ terms}} + \overbrace{20 + \cdots + 20}^{8 \text{ terms}} + \overbrace{25 + \cdots + 25}^{11 \text{ terms}} + \cdots + \overbrace{50 + \cdots + 50}^{7 \text{ terms}} \\&= \frac{15 \cdot 6 + 20 \cdot 8 + 25 \cdot 11 + 30 \cdot 17 + 35 \cdot 19 + 40 \cdot 20 + 45 \cdot 12 + 50 \cdot 7}{100} \\&= \frac{3390}{100} = 33.9\end{aligned}$$

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The **median** of a set of data is the value in the middle when the data is in numerical order.

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Note

To find the median, begin by listing the data in order from smallest to largest, or largest to smallest.

If the number of data values, N , is odd, then the median is the middle data value. This value can be found by rounding $\frac{N}{2}$ up to the next whole number.

If the number of data values is even, there is no one middle value, so we find the mean of the two middle values (values $\frac{N}{2}$ and $\frac{N}{2} + 1$)

Example 10

Find the media of these quiz scores: 8 6 4 1 7 1 1

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List the data in order: 1 1 1 4 6 7 8
 ↑

Since we have an odd number of data, we see the median is 4.

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Find the median of these quiz scores: 5 9 8 6 4 8 2 5 7 7

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 ↑

Since we have an odd number of data, we see the median is 4.

Example 11

Find the median of these quiz scores: 5 9 8 6 4 8 2 5 7 7

List the data in order: 2 4 5 5 6 7 7 8 8 9
 ↑

Since we have an even number of data, there is no middle item. In this case, we say the median is the mean of the two middle numbers, 6 and 7, which is 6.5.

Example 12

The one hundred families in a particular neighborhood are asked their annual household income, to the nearest \$5 thousand dollars.

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We have 100 items, this means the median will be the mean of the 50th and 51st data values. So, we need to start counting up from the bottom:

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There are 6 data values of 15. \Rightarrow Values 1 to 6 are 15.

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- The next 11 data values are 25. \Rightarrow Values 15 to $(14+11)=25$ are 25.

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- The next 8 data values are 20. \Rightarrow Values 7 to $(6+8)=14$ are 20.
- The next 11 data values are 25. \Rightarrow Values 15 to $(14+11)=25$ are 25.
- The next 17 data values are 30. \Rightarrow Values 26 to $(25+17)=42$ are 30.

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- The next 11 data values are 25. \Rightarrow Values 15 to $(14+11)=25$ are 25.
- The next 17 data values are 30. \Rightarrow Values 26 to $(25+17)=42$ are 30.
- The next 19 data values are 35. \Rightarrow Values 43 to $(42+19)=61$ are 35.

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- The next 17 data values are 30. \Rightarrow Values 26 to $(25+17)=42$ are 30.
- The next 19 data values are 35. \Rightarrow Values 43 to $(42+19)=61$ are 35.

So, the median is $\frac{35+35}{2} = 35$, \$35 thousand dollars.

Definition

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Example 13

In Example 1 we collected the data:

Color	Frequency	Color	Frequency
Blue	3	White	3
Green	5	Black	2
Red	4	Grey	3

For this data, Green is the mode.

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Example 13

In Example 1 we collected the data:

Color	Frequency	Color	Frequency
Blue	3	White	3
Green	5	Black	2
Red	4	Grey	3

For this data, Green is the mode.

Note

It is possible for a data set to have more than one mode.

Example 14

Consider these three sets of quiz scores:

Section A	5	5	5	5	5	5	5	5	5	5
Section B	0	0	0	0	0	10	10	10	10	10
Section C	4	4	4	5	5	5	5	6	6	6

Example 14

Consider these three sets of quiz scores:

Section A	5	5	5	5	5	5	5	5	5	5
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All three sets of data have a mean of 5 and a median of 5, yet each section has quite different scores.

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The **range** is the difference between the maximum and minimum values.

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Example 15

Using the quiz scores from Example 14.

For Section A, the range is $5 - 5 = 0$.

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Example 15

Using the quiz scores from Example 14.

For Section A, the range is $5 - 5 = 0$.

For Section B, the range is $10 - 0 = 10$.

Example 14

Consider these three sets of quiz scores:

Section A	5	5	5	5	5	5	5	5	5	5
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Definition

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Example 15

Using the quiz scores from Example 14.

For Section A, the range is $5 - 5 = 0$.

For Section B, the range is $10 - 0 = 10$.

For Section C, the range is $6 - 4 = 2$.

Definition

The **standard deviation** is a measure of variation based on measuring how far each data value deviates, or is different, from the mean.

The standard deviation

- is always positive.

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The standard deviation

- is always positive.
- will be zero if all the data values are equal.
- will get larger the more the data spreads out.
- has the same units as the original data.
- can be highly influenced by outliers.

Computing Standard Deviations

To compute a standard deviation of a data set with size n .

- ① Find the deviation of each data from the mean.

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To compute a standard deviation of a data set with size n .

- ① Find the deviation of each data from the mean.
- ② Square each deviation.

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Computing Standard Deviations

To compute a standard deviation of a data set with size n .

- 1 Find the deviation of each data from the mean.
- 2 Square each deviation.
- 3 Add the squared deviations.
- 4
 - If the data represents the whole population, divide the sum by n .

Computing Standard Deviations

To compute a standard deviation of a data set with size n .

- ① Find the deviation of each data from the mean.
- ② Square each deviation.
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- ④
 - If the data represents the whole population, divide the sum by n .
 - If the data is from a sample, divide the sum by $(n - 1)$.

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- ① Find the deviation of each data from the mean.
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- ⑤ Compute the square root of the result.

Computing Standard Deviations

To compute a standard deviation of a data set with size n .

- 1 Find the deviation of each data from the mean.
- 2 Square each deviation.
- 3 Add the squared deviations.
- 4
 - If the data represents the whole population, divide the sum by n .
 - If the data is from a sample, divide the sum by $(n - 1)$.
- 5 Compute the square root of the result.

Note

Standard deviations are only computed by hand for very small data sets.

Example 16

Let us find the standard deviation for the quiz scores:

0 5 5 5 5 5 5 5 5 10

Example 16

Let us find the standard deviation for the quiz scores:

0 5 5 5 5 5 5 5 5 10

The mean is 5 and the deviations are:

Data Value	Deviation	Deviation squared
0		
5		
5		
5		
5		
5		
5		
5		
5		
5		
10		

Example 16

Let us find the standard deviation for the quiz scores:

0 5 5 5 5 5 5 5 5 10

The mean is 5 and the deviations are:

Data Value	Deviation	Deviation squared
0	$0 - 5 = -5$	
5	$5 - 5 = 0$	
5	$5 - 5 = 0$	
5	$5 - 5 = 0$	
5	$5 - 5 = 0$	
5	$5 - 5 = 0$	
5	$5 - 5 = 0$	
5	$5 - 5 = 0$	
5	$5 - 5 = 0$	
5	$5 - 5 = 0$	
10	$10 - 5 = 5$	

Example 16

Let us find the standard deviation for the quiz scores:

0 5 5 5 5 5 5 5 5 10

The mean is 5 and the deviations are:

Data Value	Deviation	Deviation squared
0	$0 - 5 = -5$	$(-5)^2 = 25$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
10	$10 - 5 = 5$	$(5)^2 = 25$

Example 16

Let us find the standard deviation for the quiz scores:

0 5 5 5 5 5 5 5 5 10

The mean is 5 and the deviations are:

Data Value	Deviation	Deviation squared
0	$0 - 5 = -5$	$(-5)^2 = 25$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
5	$5 - 5 = 0$	$(0)^2 = 0$
10	$10 - 5 = 5$	$(5)^2 = 25$

The standard deviation is $\sqrt{\frac{50}{10}} = \sqrt{5} \approx 2.2$.

Example 17

The standard deviations for the quiz scores from Example 14 and Example 16 are:

Data Set											Std. Dev.
Section A:	5	5	5	5	5	5	5	5	5	5	0
Section B:	0	0	0	0	0	10	10	10	10	10	5
Section C:	4	4	4	5	5	5	5	6	6	6	0.8
Section D:	0	5	5	5	5	5	5	5	5	10	2.2

Example 18

The price of a jar of peanut butter at 5 stores were: \$3.29, \$3.59, \$3.75, \$3.79, and \$3.99. Let us find the standard deviation of the prices.

Example 18

The price of a jar of peanut butter at 5 stores were: \$3.29, \$3.59, \$3.75, \$3.79, and \$3.99. Let us find the standard deviation of the prices.

The mean is 3.682 and the deviations are:

Data Value	Deviation	Deviation squared
3.29		
3.59		
3.75		
3.79		
3.99		

Example 18

The price of a jar of peanut butter at 5 stores were: \$3.29, \$3.59, \$3.75, \$3.79, and \$3.99. Let us find the standard deviation of the prices.

The mean is 3.682 and the deviations are:

Data Value	Deviation	Deviation squared
3.29	$3.29 - 3.682 = -0.391$	
3.59	$3.59 - 3.682 = -0.092$	
3.75	$3.75 - 3.682 = 0.068$	
3.79	$3.79 - 3.682 = 0.108$	
3.99	$3.99 - 3.682 = 0.308$	

Example 18

The price of a jar of peanut butter at 5 stores were: \$3.29, \$3.59, \$3.75, \$3.79, and \$3.99. Let us find the standard deviation of the prices.

The mean is 3.682 and the deviations are:

Data Value	Deviation	Deviation squared
3.29	$3.29 - 3.682 = -0.391$	$(-0.391)^2 = 0.153664$
3.59	$3.59 - 3.682 = -0.092$	$(-0.092)^2 = 0.008464$
3.75	$3.75 - 3.682 = 0.068$	$(0.068)^2 = 0.011664$
3.79	$3.79 - 3.682 = 0.108$	$(0.108)^2 = 0.004624$
3.99	$3.99 - 3.682 = 0.308$	$(0.308)^2 = 0.094864$

Example 18

The price of a jar of peanut butter at 5 stores were: \$3.29, \$3.59, \$3.75, \$3.79, and \$3.99. Let us find the standard deviation of the prices.

The mean is 3.682 and the deviations are:

Data Value	Deviation	Deviation squared
3.29	$3.29 - 3.682 = -0.391$	$(-0.391)^2 = 0.153664$
3.59	$3.59 - 3.682 = -0.092$	$(-0.092)^2 = 0.008464$
3.75	$3.75 - 3.682 = 0.068$	$(0.068)^2 = 0.011664$
3.79	$3.79 - 3.682 = 0.108$	$(0.108)^2 = 0.004624$
3.99	$3.99 - 3.682 = 0.308$	$(0.308)^2 = 0.094864$

Note that we are dealing with a sample. The standard deviation is:

$$\begin{aligned} & \sqrt{\frac{0.153664 + 0.008464 + 0.011664 + 0.004624 + 0.094864}{4}} \\ &= \sqrt{\frac{0.27328}{4}} = \sqrt{0.06832} \approx \$0.261 \end{aligned}$$

Definition

Quartiles are values that divide the data into quarters.

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The first quartile, Q_1 , is the value so that 25% of the data values are below it.

The third quartile, Q_3 , is the value so that 75% of the data values are below it.

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The first quartile, Q_1 , is the value so that 25% of the data values are below it.

The third quartile, Q_3 , is the value so that 75% of the data values are below it.

This divides the data into quarters:

- 25% of the data is between the minimum and Q_1 .
- 25% of the data is between Q_1 and the median.
- 25% of the data is between the media and Q_3 .
- 25% of the data is between Q_3 and the maximum.

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Five-Number Summary

A common summary of a data set takes the form:

Minimum, Q_1 , Median, Q_3 , Maximum

How to Calculate Q_1

- 1 Order the data from smallest to largest.

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- ① Order the data from smallest to largest.
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 - If L is a decimal value:
 - Round up to the next whole number, $L+$.
 - Q_1 is the data value at position $L+$.

How to Calculate Q_1

- ① Order the data from smallest to largest.
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 - If L is a decimal value:
 - Round up to the next whole number, $L+$.
 - Q_1 is the data value at position $L+$.
 - If L is a whole number:
 - Q_1 is the mean of the data values in position L and $L + 1$.

How to Calculate Q_1

- ① Order the data from smallest to largest.
- ② Compute the locator: $L = 0.25n$
- ③
 - If L is a decimal value:
 - Round up to the next whole number, $L+$.
 - Q_1 is the data value at position $L+$.
 - If L is a whole number:
 - Q_1 is the mean of the data values in position L and $L + 1$.

How to Calculate Q_3

Use the same procedure as for Q_1 , but with locator: $L = 0.75n$.

Example 19

Suppose we have measured the height (in inches) of 9 women. The data, sorted from smallest to largest is:

59 60 62 64 66 67 69 70 72

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59 60 62 64 66 67 69 70 72

Finding Q_1 : The locator is $L = 0.25 \cdot 9 = 2.25$, so we need to round up, which gives $L+ = 3$. Thus, $Q_1 = 62$.

Example 19

Suppose we have measured the height (in inches) of 9 women. The data, sorted from smallest to largest is:

59 60 62 64 66 67 69 70 72

Finding Q_1 : The locator is $L = 0.25 \cdot 9 = 2.25$, so we need to round up, which gives $L+ = 3$. Thus, $Q_1 = 62$.

Finding Q_2 : This is just the median, which is 66.

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59 60 62 64 66 67 69 70 72

Finding Q_1 : The locator is $L = 0.25 \cdot 9 = 2.25$, so we need to round up, which gives $L+ = 3$. Thus, $Q_1 = 62$.

Finding Q_2 : This is just the median, which is 66.

Finding Q_3 : The locator is $L = 0.75 \cdot 9 = 6.75$, so we need to round up, which gives $L+ = 7$. Thus, $Q_3 = 69$.

Example 19

Suppose we have measured the height (in inches) of 9 women. The data, sorted from smallest to largest is:

59 60 62 64 66 67 69 70 72

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Finding Q_2 : This is just the median, which is 66.

Finding Q_3 : The locator is $L = 0.75 \cdot 9 = 6.75$, so we need to round up, which gives $L+ = 7$. Thus, $Q_3 = 69$.

The five-number summary for this data set is: 59, 62, 66, 69, 72

Example 20

Suppose, as in Example 19, we have measured the height (in inches) of 8 women. The data, sorted from smallest to largest is:

59 60 62 64 66 67 69 70

Example 20

Suppose, as in Example 19, we have measured the height (in inches) of 8 women. The data, sorted from smallest to largest is:

59 60 62 64 66 67 69 70

Finding Q_1 : The locator is $L = 0.25 \cdot 8 = 2$, so Q_1 will be the median of the second and third data values. Thus, $Q_1 = \frac{60 + 62}{2} = 61$.

Example 20

Suppose, as in Example 19, we have measured the height (in inches) of 8 women. The data, sorted from smallest to largest is:

59 60 62 64 66 67 69 70

Finding Q_1 : The locator is $L = 0.25 \cdot 8 = 2$, so Q_1 will be the median of the second and third data values. Thus, $Q_1 = \frac{60 + 62}{2} = 61$.

Finding Q_2 : This is just the median, which is 65.

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Suppose, as in Example 19, we have measured the height (in inches) of 8 women. The data, sorted from smallest to largest is:

59 60 62 64 66 67 69 70

Finding Q_1 : The locator is $L = 0.25 \cdot 8 = 2$, so Q_1 will be the median of the second and third data values. Thus, $Q_1 = \frac{60 + 62}{2} = 61$.

Finding Q_2 : This is just the median, which is 65.

Finding Q_3 : The locator is $L = 0.75 \cdot 8 = 6$, so Q_3 will be the median of the sixth and seventh data values. Thus, $Q_3 = \frac{67 + 69}{2} = 68$.

Example 20

Suppose, as in Example 19, we have measured the height (in inches) of 8 women. The data, sorted from smallest to largest is:

59 60 62 64 66 67 69 70

Finding Q_1 : The locator is $L = 0.25 \cdot 8 = 2$, so Q_1 will be the median of the second and third data values. Thus, $Q_1 = \frac{60 + 62}{2} = 61$.

Finding Q_2 : This is just the median, which is 65.

Finding Q_3 : The locator is $L = 0.75 \cdot 8 = 6$, so Q_3 will be the median of the sixth and seventh data values. Thus, $Q_3 = \frac{67 + 69}{2} = 68$.

The five-number summary for this data set is: 59, 61, 65, 68, 70

Definition

A **box plot** is a graphical representation of a five-number summary.



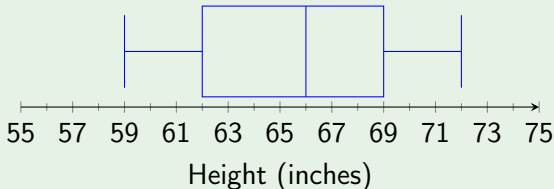
Definition

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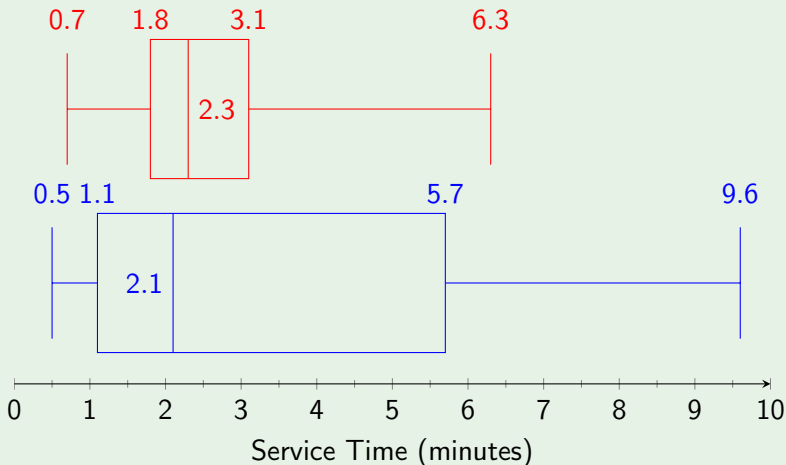
Example 21

Example 19 has the five-number summary: 59, 62, 66, 69, 72



Example 22

The box plot shows the service times for two fast-food restaurants.



What comparisons between the two restaurants can we make?
If you are in a hurry, which restaurant should you go to?