

The Determinant of a Matrix

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Determinant of a 2×2 Matrix

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$$\begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix}$$

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Example

$$\begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix} = 3 \cdot (1) - (-2) \cdot 6 = 15$$

Minors of a Matrix

For every element a_{ij} of a $n \times n$ matrix \mathbf{A} , the **minor** M_{ij} is an $(n - 1) \times (n - 1)$ matrix obtained by deleting the i th row and the j th column of \mathbf{A} .

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Example

$$\mathbf{A} = \begin{bmatrix} 5 & 4 & -3 \\ 2 & -8 & 1 \\ 9 & 3 & 0 \end{bmatrix} \quad M_{12} =$$

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Cofactors of a Matrix

For every element a_{ij} of a $n \times n$ matrix \mathbf{A} , the **cofactor** of a_{ij} is the scalar

$$C_{ij} = (-1)^{(i+j)} |M_{ij}|$$

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For a $n \times n$ matrix **A**, choose any row or column.

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Expansion by the j th column:

$$|\mathbf{A}| = \sum_{i=1}^n a_{ij} C_{ij} = \sum_{i=1}^n a_{ij} (-1)^{(i+j)} |\mathbf{M}_{ij}|$$

I recommend you always expand across the first row.

Example

Compute the determinant

$$\begin{vmatrix} 3 & 1 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 2 \end{vmatrix}$$

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Expanding across the first row gives:

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$$= 3(1 \cdot 2 - 3 \cdot 1) - (2 \cdot 2 - 3 \cdot 0) - (2 \cdot 1 - 1 \cdot 0)$$

Example

Compute the determinant

$$\begin{vmatrix} 3 & 1 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 2 \end{vmatrix}$$

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Example

Compute the determinant

$$\begin{vmatrix} 3 & 0 & -1 \\ 4 & 6 & 2 \\ 8 & -2 & 3 \end{vmatrix}$$

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$$= 3(6 \cdot 3 - 2 \cdot (-2)) + 0(4 \cdot 3 - 2 \cdot 8) - (4 \cdot (-2) - 6 \cdot 8)$$

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$$= 3(6 \cdot 3 - 2 \cdot (-2)) + 0(4 \cdot 3 - 2 \cdot 8) - (4 \cdot (-2) - 6 \cdot 8)$$
$$= 66 + 0 + 56$$

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- If any row (or any column) of \mathbf{A} contains all zeros, then $|\mathbf{A}| = 0$

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- If the entries of any row (or any column) of $|\mathbf{A}|$ are multiplied by a nonzero number k and the result is added to another row, the value of $|\mathbf{A}|$ remains unchanged.

Cramer's Rule

Consider the system:

$$\begin{array}{rclcl} ax & + & by & = & s \\ cx & + & dy & = & t \end{array}$$

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We then have the following three determinants:

$$|D| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad |D_x| = \begin{vmatrix} s & b \\ t & d \end{vmatrix} \quad |D_y| = \begin{vmatrix} a & s \\ c & t \end{vmatrix}$$

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The solutions is:

$$x = \frac{|D_x|}{|D|} = \frac{\begin{vmatrix} s & b \\ t & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad y = \frac{|D_y|}{|D|} = \frac{\begin{vmatrix} a & s \\ c & t \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

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Note

This method can be extended to any size system of equations.

Example

Consider the system

$$\begin{array}{rclcl} x & + & 2y & = & 5 \\ 2x & + & 3y & = & 8 \end{array}$$

Example

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$$\begin{array}{rcrcrcrcrcrl} x & + & 2y & = & 5 \\ 2x & + & 3y & = & 8 \end{array}$$

Let us solve this system using Cramer's Rule.

This means we need to calculate the following determinants.

$$|D| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$$

$$|D_x| = \begin{vmatrix} 5 & 2 \\ 8 & 3 \end{vmatrix}$$

$$|D_y| = \begin{vmatrix} 1 & 5 \\ 2 & 8 \end{vmatrix}$$

Example

Consider the system

$$\begin{array}{rcrcrcrcrcrcl} x & + & 2y & = & 5 \\ 2x & + & 3y & = & 8 \end{array}$$

Let us solve this system using Cramer's Rule.

This means we need to calculate the following determinants.

$$|D| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \qquad |D_x| = \begin{vmatrix} 5 & 2 \\ 8 & 3 \end{vmatrix} \qquad |D_y| = \begin{vmatrix} 1 & 5 \\ 2 & 8 \end{vmatrix}$$

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Example

Consider the system

$$\begin{array}{rcrcrcrcrcrcl} x & + & 2y & = & 5 \\ 2x & + & 3y & = & 8 \end{array}$$

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We can now find x

$$x = \frac{|D_x|}{|D|}$$

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We can now find x

$$x = \frac{|D_x|}{|D|} = \frac{-1}{-1}$$

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$$x = \frac{|D_x|}{|D|} = \frac{-1}{-1} = 1$$

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We can now find x and y

$$x = \frac{|D_x|}{|D|} = \frac{-1}{-1} = 1 \quad y = \frac{|D_y|}{|D|}$$

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$$x = \frac{|D_x|}{|D|} = \frac{-1}{-1} = 1 \quad y = \frac{|D_y|}{|D|} = \frac{-2}{-1}$$

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$$x = \frac{|D_x|}{|D|} = \frac{-1}{-1} = 1 \quad y = \frac{|D_y|}{|D|} = \frac{-2}{-1} = 2$$

Example

Consider the system

$$3x - 2y = 4$$

$$6x + y = 13$$

Example

Consider the system

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Let us solve this system using Cramer's Rule.

This means we need to calculate the following determinants.

$$|D| = \begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix}$$

$$|D_x| = \begin{vmatrix} 4 & -2 \\ 13 & 1 \end{vmatrix}$$

$$|D_y| = \begin{vmatrix} 3 & 4 \\ 6 & 13 \end{vmatrix}$$

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Consider the system

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Let us solve this system using Cramer's Rule.

This means we need to calculate the following determinants.

$$|D| = \begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix} = 15 \quad |D_x| = \begin{vmatrix} 4 & -2 \\ 13 & 1 \end{vmatrix} \quad |D_y| = \begin{vmatrix} 3 & 4 \\ 6 & 13 \end{vmatrix}$$

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Consider the system

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Let us solve this system using Cramer's Rule.

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$$|D| = \begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix} = 15 \quad |D_x| = \begin{vmatrix} 4 & -2 \\ 13 & 1 \end{vmatrix} = 30 \quad |D_y| = \begin{vmatrix} 3 & 4 \\ 6 & 13 \end{vmatrix}$$

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We can now find x

$$x = \frac{|D_x|}{|D|}$$

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$$|D| = \begin{vmatrix} 3 & -2 \\ 6 & 1 \end{vmatrix} = 15 \quad |D_x| = \begin{vmatrix} 4 & -2 \\ 13 & 1 \end{vmatrix} = 30 \quad |D_y| = \begin{vmatrix} 3 & 4 \\ 6 & 13 \end{vmatrix} = 15$$

We can now find x

$$x = \frac{|D_x|}{|D|} = \frac{30}{15}$$

Example

Consider the system

$$\begin{array}{rclcrcl} 3x & - & 2y & = & 4 \\ 6x & + & y & = & 13 \end{array}$$

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We can now find x

$$x = \frac{|D_x|}{|D|} = \frac{30}{15} = 2$$

Example

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We can now find x and y

$$x = \frac{|D_x|}{|D|} = \frac{30}{15} = 2 \quad y = \frac{|D_y|}{|D|}$$

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$$x = \frac{|D_x|}{|D|} = \frac{30}{15} = 2 \quad y = \frac{|D_y|}{|D|} = \frac{15}{15} = 1$$