Normal Distribution

Colby Community College

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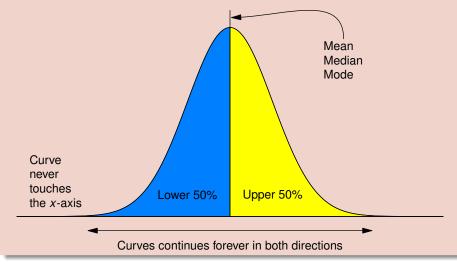
A little while later more and more start to pop.

This goes one for a minute or so, and the popping gradually tappers off.

Most of the popping happens in that brief, noisy moment.

This demonstrates a typical pattern that is part of many phenomena.

A **normal distribution** is a perfectly symmetric, bell-shaped distribution. It is also referred to as a **normal curve** or a **bell curve**.



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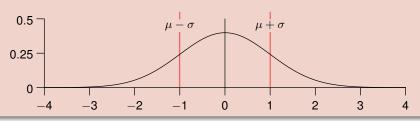
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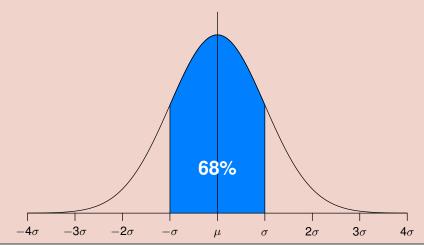
Definition

When $\mu=0$ and $\sigma=1$, the curve is called the **standard normal** distribution. The total area under the curve is exactly equal to 1.



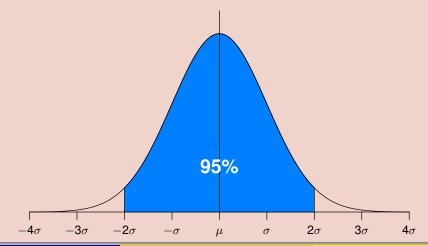
The **empirical rule**, or **68-95-99.7 rule**, states approximately how much of the area is contained when stepping one, two, or three standard deviations from the mean.

One standard deviation from the mean.



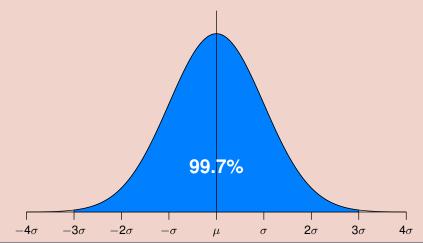
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Two standard deviations from the mean.



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Three standard deviations from the mean.



A **z-score** is a measure of the number of standard deviations a particular data point is away from the mean.

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On a college entrance exam, the mean was 70, and the standard deviation was 8. Rose scored a 85, what is her *z*-score?

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$$\mathbf{Z} = \frac{\mathbf{X} - \boldsymbol{\mu}}{\sigma} \quad \Rightarrow \quad \mathbf{Z} \sigma = \mathbf{X} - \boldsymbol{\mu} \quad \Rightarrow \quad \mathbf{X} = \mathbf{Z} \sigma + \boldsymbol{\mu}$$

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$$z = \frac{x - \mu}{\sigma} \implies z\sigma = x - \mu \implies x = z\sigma + \mu = (-1.3)(8) + 70 = 59.6$$

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Moreover, we know that roughly 95% of the scores fall within two standard deviations of the mean.

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Which means that 95% - 68% = 27% of the scores are more than one standard deviation from the mean, but less than two.

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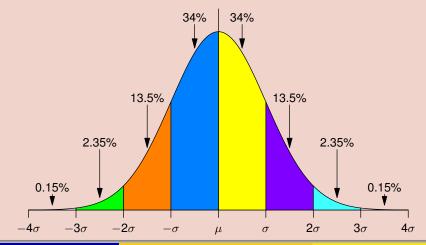
Moreover, we know that roughly 95% of the scores fall within two standard deviations of the mean.

Which means that 95% - 68% = 27% of the scores are more than one standard deviation from the mean, but less than two.

Since the curve is symmetric, we know that 13.5% of the students scored between 89 and 96, as well as 13.5% between 68 and 75

The **empirical rule**, or **68-95-99.7 rule**, states approximately how much of the area is contained when stepping one, two, or three standard deviations from the mean.

For each standard deviation.

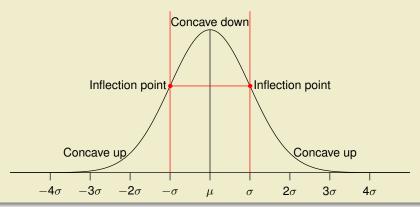


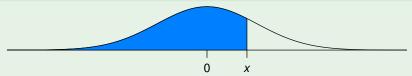
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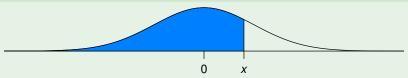
Note

A normal density curve always has two inflection points, each one standard deviation from the mean.



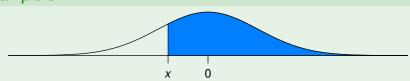


The area of the shaded region is the probability that a z score is less than or equal to x, $P(z \le x)$.



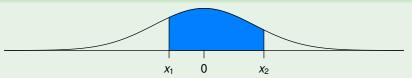
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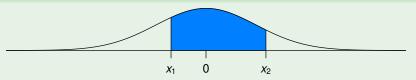


The area of the shaded region is the probability that a z score is greater than or equal to x, $P(z \ge x)$.

Can also be calculated as $1 - P(z \le x)$.

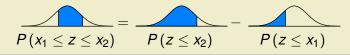


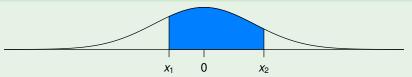
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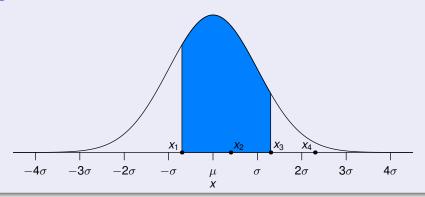


Procedure for Finding Areas with a Nonstandard Normal Distribution

1 Sketch a normal curve, label the mean and any specific *x* values, and then shade the region representing the desired probability.

2

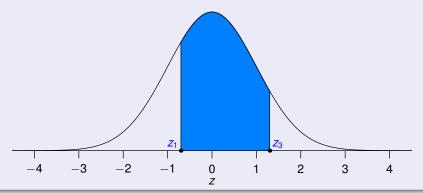
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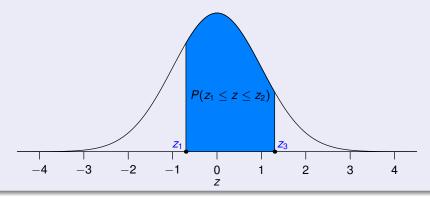
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Procedure for Finding Areas with a Nonstandard Normal Distribution

- 1 Sketch a normal curve, label the mean and any specific *x* values, and then shade the region representing the desired probability.
- 2 For each relevant *x* value that is a boundary for the shaded region, convert that value to the equivalent *z* score.
- 3 Use technology to find the area of the shaded region.



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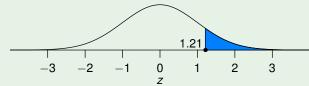
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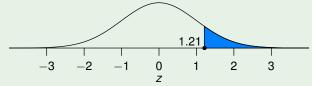
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We can then use technology to compute:

$$P(z \ge 1.21) \approx 0.1123$$
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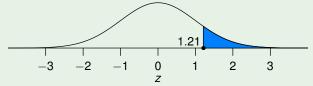
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So, about 11.23% of men are taller than the shower head.

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Let's find the percentage of men meet that requirement.

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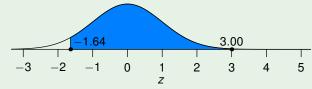
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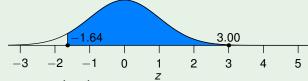
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So, we see that about 94.84% of men meet the requirements.

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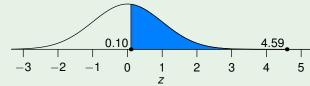
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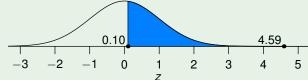
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Next, we sketch a picture and shade the area we wish to find:



We can then use technology to compute:

$$P(0.10 \le z \le 4.59) \approx 0.4601$$
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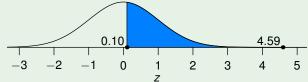
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So, we see that only about 46% of women meet the requirements.

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Procedure

Sketch the normal distribution curve, write the given probability or percentage in the appropriate region of the graph, and identify the x values being sought.

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- Don't confuse z scores and areas.
- Choose the correct side of the graph.
- A z score must be negative whenever it is located in the left half of the normal distribution.
- Areas are always between 0 and 1, and are never negative.

Procedure

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- 4 Use your sketch to verify that the solution makes sense.