

Hypothesis Testing For A Proportion

Colby Community College

Example 1

The following question comes from a book written by Hans Rosling, Anna Rosling Rönnlund, and Ola Rosling called *Factfulness*.

How many of the world's 1 year old children today have been vaccinated against some disease?

(a) 20%

(b) 50%

(c) 80%

What is your answer (or guess)?

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Note

If we take a multiple choice test, then we might like to distinguish between the two possibilities:

- People never learn these particular topics and their responses are simply equivalent to random guessing.
- People have knowledge that helps them do better than random guessing, or perhaps, they have false knowledge that leads them to actually do worse than random guessing.

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The **null hypothesis** (H_0) represents a skeptical perspective or a claim to be tested.

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The **alternative hypotheses** (H_A) represents an alternative claim under consideration and is often represented by a range of possible parameter values.

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We may reject or fail to reject the alternative hypothesis, but we typically never accept the null hypothesis as true.

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The value we are comparing the parameter to is called the **null value**.

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No. While we may not believe the null hypotheses, we need strong evidence before we reject the null hypothesis and conclude something more interesting.

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If we don't believe the null hypothesis, should we simply reject it?

No. While we may not believe the null hypotheses, we need strong evidence before we reject the null hypothesis and conclude something more interesting.

Even if we don't believe the proportion is *exactly* 33.3%, that doesn't tell us anything useful about if people do better or worse than random guessing.

Note

We will be using the `rosling_responses` data set to evaluate the hypothesis test evaluating whether college-educated adults who get the question about infant vaccination correct is different from 33.3%.

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We will be using the `rosling_responses` data set to evaluate the hypothesis test evaluating whether college-educated adults who get the question about infant vaccination correct is different from 33.3%. This data set summarizes the answers of 50 college-educated adults. Of these 50 adults, 24% of respondents got the question correct.

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The conditions are met, so let's construct a 95% confidence interval.

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The confidence interval is (0.122, 0.358), which means we are 95% confident that the proportion of college-educated adults to correctly answer the question is between 12.2% and 35.8%.

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The confidence interval is (0.122, 0.358), which means we are 95% confident that the proportion of college-educated adults to correctly answer the question is between 12.2% and 35.8%.

Since the null hypothesis of $p = 0.333$ falls in this range, we cannot say the null hypotheses is implausible. We fail to reject the null hypothesis.

Just because we conclude that it's plausible that $p = 0.333$ does not mean we actually accept the null hypothesis.

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The confidence interval is (0.103, 0.195), which means we are 95% confident that the proportion of college-educated adults that answered the question correctly is between 10.3% and 19.5%. Since $p = 0.333$ is implausible, we reject the null hypothesis.

Note

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This means you always need to write what confidence level you used.

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A **Type 1 Error** is rejecting H_0 , when H_0 is actually true.

A **Type 2 Error** is failing to reject H_0 , when H_A is actually true.

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		do not reject H_0	reject H_0 in favor of H_A
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In a US court, the defendant is either innocent (H_0) or guilty (H_A).

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The defendant is innocent, but wrongly convicted.

What does a Type 2 Error represent in this context?

The defendant really did commit the crime, but was not found guilty.

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We typically use a summary statistic of the data, in this section the sample proportion, to help compute the p-value and evaluate the hypotheses.

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$$H_A : p \neq 0.5$$

Note

In this case, the null value would be $p_0 = 0.5$.

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It is natural to wonder if 37% represents a real difference from the null hypothesis of 50%. That is, if we assume the null hypothesis were true, how likely is a sample with $\hat{p} = 0.37$?

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So, if the null hypothesis is true, the sampling distribution would be approximately normal, with:

$$\mu_{\hat{p}} = p$$

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Pew Research's sample show that 37% of American adults support the increased usage of coal.

It is natural to wonder if 37% represents a real difference from the null hypothesis of 50%. That is, if we assume the null hypothesis were true, how likely is a sample with $\hat{p} = 0.37$?

Independence. The poll was based on a random sample.

Success-Failure. Based on the poll's sample size $n = 1000$:

$$np \stackrel{H_0}{=} 1000 \cdot 0.5 = 500 \geq 10$$

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Definition

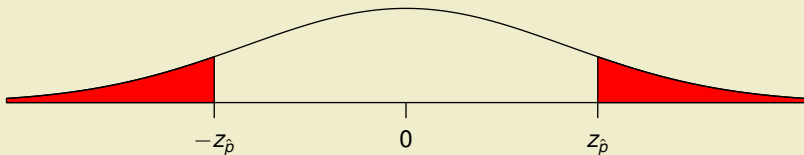
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Note

The p-value represents the probability of the observed \hat{p} , or a \hat{p} that is more extreme, if the null hypothesis is true.



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With $\mu = 0.5$ and $\sigma = 0.016$, the z -value for $\hat{p} = 0.37$ is:

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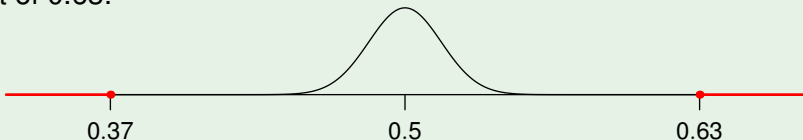
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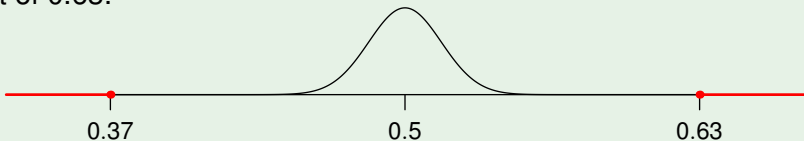
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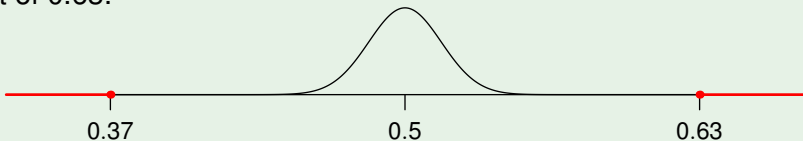
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Inferential Statistics

If, under an given assumption, the probability of a particular observed event is very small, we conclude that the assumption is probably nor correct.

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Note

Make sure to provide a conclusion in the context of the problem. Most people don't understand what rejecting or failing to reject H_0 means.

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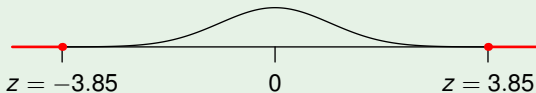
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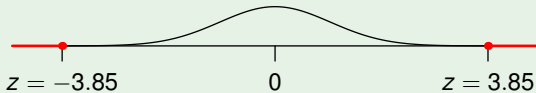
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The p-value is 0.0002, which is smaller than $\alpha = 0.05$, so we reject H_0 .

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Note

Type 2 Errors can be reduced by collecting more data, without affecting Type I Errors. But this often adds time and expense to a study. There is typically a cost-benefit analysis to be considered.

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Since neither Type 1 nor Type 2 errors should be dangerous or, relatively, much more expensive, 5% would be a reasonable choice.

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Since safety is involved, the car manufacturer should be eager to switch to the slightly more expensive supplier. A slightly larger significance level, such as $\alpha = 0.1$, might be appropriate.

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What significance level should be used in such a hypothesis test?

Since it doesn't seem to be a major problem if the part stays broken, we require very strong evidence before we pay to replace the part. A small significance, such as $\alpha = 0.01$, might be appropriate.

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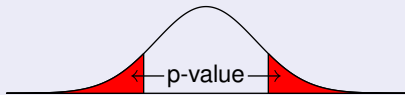
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Note

When using a one-sided hypothesis test, there is a risk of overlooking data supporting the opposite conclusion.

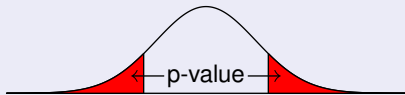
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The critical region is in the two extreme regions under the curve.



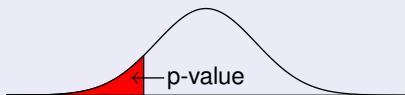
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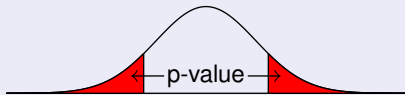
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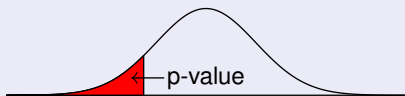
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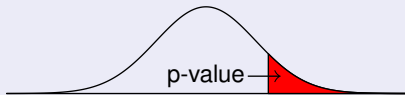
Left-sided Test ($H_A : <$)

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Right-sided Test ($H_A : >$)

The critical region is in the extreme right region under the curve.



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When considering one, ask yourself:

What would I, or others, conclude if the data happens to go clearly in the opposite direction than my alternative hypothesis?