Addition Rule and Multiplication Rule

Colby Community College

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If events A and B are independent, then the probability of both A and B occurring is

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Suppose we flip a fair coin and roll a fair die.

The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

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The probability of getting tails on the coin and three on the die is

$$P(\text{T3}) = \frac{1}{12}$$

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We could have also calculated

$$P(\text{H and } 3) = P(\text{H}) \cdot P(3) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

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The sample space is $\{\text{H1}, \text{H2}, \text{H3}, \text{H4}, \text{H5}, \text{H6}, \text{T1}, \text{T2}, \text{T3}, \text{T4}, \text{T5}, \text{T6}\}$.

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But, $\frac{1}{2} + \frac{1}{6} = \frac{8}{12}$, is wrong because we have double counted H6. Thus, we need to subtract $P(\text{H6}) = \frac{1}{12}$.

$$P(\text{H or } 6) = P(\text{H}) + P(6) - P(\text{H and } 6) = \frac{1}{2} + \frac{1}{6} - \frac{1}{12} = \frac{7}{12}$$

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Note

If two events are **disjoint**, then $P(A \text{ or } B) = P(A) + P(B)$.

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Two cards are red kings, so $P(\text{Red and K}) = \dfrac{2}{52}$.

Thus,

$$P(\text{Red or K}) = P(\text{Red}) + P(\text{K}) - P(\text{Red and K}) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{7}{13}$$

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The probability the event B occurs, given that event A has happened, is represented as $P(B \mid A)$. This is called a **conditional probability**.

Read as "the probability of B given A."

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Car color	Speeding ticket	No speeding ticket	Total
Red	15	135	150
Not red	45	470	515
Total	60	605	665

Find the probability someone has gotten a speeding ticket *given* they drive a red car.

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$$P(\text{red} \mid \text{ticket}) = \frac{15}{60} = \frac{1}{4} = 25\%$$

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Note

In general $P\left(B \mid A\right) \neq P\left(A \mid B\right)$.

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The probability that the first card is a heart is $P(1^{\text{st}} \heartsuit) = \dfrac{13}{52}$.

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If you pull two cards out of a deck, find the probability that both are hearts.

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The probability that the second card is a heart, given that the first card was a heart, is $P\left(2^{\text{nd}} \heartsuit \mid 1^{\text{st}} \heartsuit\right) = \dfrac{12}{51}$.

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The probability that the second card is a heart, given that the first card was a heart, is $P\left(2^{\text{nd}} \heartsuit \mid 1^{\text{st}} \heartsuit\right) = \dfrac{12}{51}$.

So, the probability that both are spades is

$$P(\text{both } \heartsuit) = P\left(1^{\text{st}} \heartsuit\right) \cdot P\left(2^{\text{nd}} \heartsuit \;\middle|\; 1^{\text{st}} \heartsuit\right) = \frac{13}{52} \cdot \frac{12}{52} = \frac{156}{2652} \approx 5.9\%$$

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Event A Drawing the Ace of Diamonds then a black card.

$$\begin{aligned} P\left(\text{A}\blacklozenge \text{ and Black}\right) &= P\left(\text{A}\blacklozenge\right) \cdot P\left(\text{Black} \mid \text{A}\blacklozenge\right) \\ &= \frac{1}{52} \cdot \frac{26}{51} = \frac{1}{102} \end{aligned}$$

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Event B Drawing a black card then the Ace of Diamonds.

$$\begin{aligned} P\left(\text{Black and A}\blacklozenge\right) &= P\left(\text{Black}\right) \cdot P\left(\text{A}\blacklozenge \mid \text{Black}\right) \\ &= \frac{26}{52} \cdot \frac{1}{51} = \frac{1}{102} \end{aligned}$$

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These events are independent and mutually exclusive, so

$$P\left(A \text{ or } B\right) = P\left(A\right) + P\left(B\right) = \frac{1}{102} + \frac{1}{102} = \frac{2}{102} \approx 1.96\%$$

Sampling

Sampling methods are critically important, and the following relationships hold:

- Sampling *with replacement*: Selections are *independent* events.
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Treating Dependent Events as Independent

When sampling without replacement and the sample size is no more than 5% of the size of the population, treat the selections as being independent (even though they are actually dependent).

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Assume that three adults are randomly selected without replacement from the $247,436,830$ adults in the United States. If we assume that 10% of adults use drugs, what is the probability that the three selected adults all use drugs?

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Because the three adults are randomly selected without replacement, the three events are dependent. This means the exact probability would be rather cumbersome.

$$\begin{aligned}P(\text{all use drugs}) &= P(\text{first use drugs and second use drugs and third use drugs})\\ &= \left(\frac{24,743,683}{247,436,830}\right)\cdot\left(\frac{24,743,682}{247,436,829}\right)\cdot\left(\frac{24,743,681}{247,436,828}\right)\\ &= 0.0009999998909\end{aligned}$$

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Assume that three adults are randomly selected without replacement from the 247, 436, 830 adults in the United States. If we assume that 10% of adults use drugs, what is the probability that the three selected adults all use drugs?

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Since 5 adults is less that 5% of the total population, we can simplify the calculations considerably.

$$\begin{aligned} P\,(\text{all use drugs}) &= P\,(\text{first use drugs and second use drugs and third use drugs}) \\ &= 0.1\cdot 0.1\cdot 0.1 = 0.00100 \end{aligned}$$

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When two different people are randomly selected from those at your school, we can assume that birthdays occur on the days of the week with equal frequencies.

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The probability that the any person is born on a Monday is 1/7. Since birth dates are independent, the probability that both are born on Monday is

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Caution

In any probability calculation, it is very important to carefully identify the event being considered.