# Conditional Probability

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## Example 1

The `photo_classify` data set represents a machine learning algorithm classifying a sample of 1822 photos as either about fashion or not.

| | | truth | | |
|---|---|---|---|---|
| | | *fashion* | *not* | Total |
| mach_learn | *pred_fashion* | 197 | 22 | 219 |
| | *pred_not* | 112 | 1491 | 1603 |
| | Total | 309 | 1513 | 1822 |

*If a photo is actually about fashion, what is the chance the algorithm will correctly identify the photo as being about fashion?*

Of the 309 fashion photos, the algorithm correctly classifies 197 of them.

$$P\left(\texttt{mach\_learn}\text{ is }\textit{pred\_fashion}\text{ given }\texttt{truth}\text{ is }\textit{fashion}\right) = \frac{197}{309} = 0.638$$

## Example 2

Using the same data set as in Example 1.

| | | truth | | |
|---|---|---|---|---|
| | | *fashion* | *not* | Total |
| mach_learn | *pred_fashion* | 197 | 22 | 219 |
| | *pred_not* | 112 | 1491 | 1603 |
| | Total | 309 | 1513 | 1822 |

*If the algorithm predicts the photo as being about fashion, what is the probability is actually is?*

Of the 1603 photos predicted to be about fashion, 112 we actually about fashion.

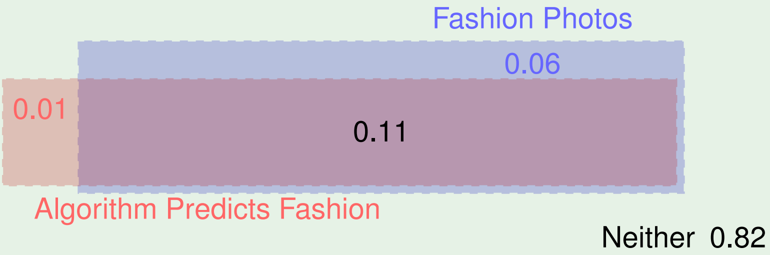
$$P\left(\texttt{truth}\text{ is }\mathit{fashion}\text{ given }\texttt{mach\_learn}\text{ is }\mathit{pred\_fashion}\right) = \frac{112}{1603} = 0.070$$

### Note

It can be helpful to draw Venn Diagrams of these contingency tables using rectangles.

### Example 3

The Venn Diagram for Example 1 is:



## Definition

A **marginal probability** is a probability based on a single variable without regard to other variables.

## Example 4

$$P\left(\texttt{mach\_learn} \text{ is } \mathit{pred\_fashion}\right) = \frac{219}{1822} = 0.12$$

## Definition

A probability of outcomes for two or more variables is called a **joint probability**.

## Example 5

$$P\left(\texttt{mach\_learn} \text{ is } \mathit{pred\_fashion} \text{ and } \texttt{truth} \text{ is } \mathit{fashion}\right) = \frac{197}{1822} = 0.11$$

### Note

Sometimes a comma is substituted for "and" in a joint probability.

$$P\left(\texttt{mach\_learn} \text{ is } \mathtt{\mathit{pred\_fashion}}, \texttt{truth} \text{ is } \mathtt{\mathit{fashion}}\right)$$

means the same thing as

$$P\left(\texttt{mach\_learn} \text{ is } \mathtt{\mathit{pred\_fashion}} \text{ and } \texttt{truth} \text{ is } \mathtt{\mathit{fashion}}\right)$$

## Definition

A **table proportions** is a table that summarizes joint probabilities. The proportions are computed by dividing each count by table's total.

## Example 6

The table proportions for `photo_classify` are:

| | `truth`: *fashion* | `truth`: *not* | Total |
|---|---|---|---|
| `mach_learn`: *pred_fashion* | $\frac{197}{1822}$ | $\frac{22}{1822}$ | $\frac{219}{1822}$ |
| `mach_learn`: *pred_not* | $\frac{112}{1822}$ | $\frac{1491}{1822}$ | $\frac{1603}{1822}$ |
| Total | $\frac{309}{1822}$ | $\frac{1513}{1822}$ | $\frac{1822}{1822}$ |

$\downarrow\downarrow\downarrow$

| | `truth`: *fashion* | `truth`: *not* | Total |
|---|---|---|---|
| `mach_learn`: *pred_fashion* | 0.1081 | 0.0121 | 0.1202 |
| `mach_learn`: *pred_not* | 0.0615 | 0.8183 | 0.8798 |
| Total | 0.1696 | 0.8304 | 1.0 |

## Example 7

The table proportions from Example 6 make a probability distribution.

| Joint Outcome | Probability |
|---|---|
| `mach_learn` is *pred_fashion* and `truth` is *fashion* | 0.1081 |
| `mach_learn` is *pred_fashion* and `truth` is *not* | 0.0121 |
| `mach_learn` is *pred_not* and `truth` is *fashion* | 0.0615 |
| `mach_learn` is *pred_not* and `truth` is *not* | 0.8182 |

## Note

Joint probabilities can be used to calculate marginal probabilities in simple cases.

## Example 8

$$
\begin{aligned}
P\left(\texttt{truth} \text{ is } \textit{fashion}\right) &= P\left(\texttt{mac\_learn} \text{ is } \textit{pred\_fashion} \text{ and } \texttt{truth} \text{ is } \textit{fashion}\right) \\
&\quad + P\left(\texttt{mac\_learn} \text{ is } \textit{pred\_not} \text{ and } \texttt{truth} \text{ is } \textit{fashion}\right) \\
&= 0.1081 + 0.0615 = 0.1696
\end{aligned}
$$