Geometric Distribution

Colby Community College

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The events `success` and `failure` are complements.

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Note

`success` and `failure` are not moral descriptions. We could have just as easily labeled the universal donors as `failure`.

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Bernoulli Random Variable

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The probability that the first universal donor is the second person.

$$P\left(1^{\text{st}} \text{ no and } 2^{\text{nd}} \text{ yes}\right) = (1 - 0.06)(0.06) = (0.94)(0.06) = 0.0564$$

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The probability that the first universal donor is the n^{th} person.

$$\begin{aligned} P\left(1^{\text{st}} \text{ through } (n-1)^{\text{th}} \text{ no and } n^{\text{th}} \text{ yes}\right) &= (0.94)\cdots(0.94)(0.06) \\ &= (0.94)^{n-1}\cdot 0.06 \end{aligned}$$

Geometric Distribution

If the probability of a success in one trial is p and the probability of failure is $1 - p$, then the probability of finding the first success in the n^{th} trial is given by

$$(1 - p)^{n-1} \cdot p$$

The mean, variance and standard deviation of this wait time are

$$\mu = \frac{1}{p} \quad \sigma^2 = \frac{1-p}{p^2} \quad \sigma = \sqrt{\frac{1-p}{p^2}}$$

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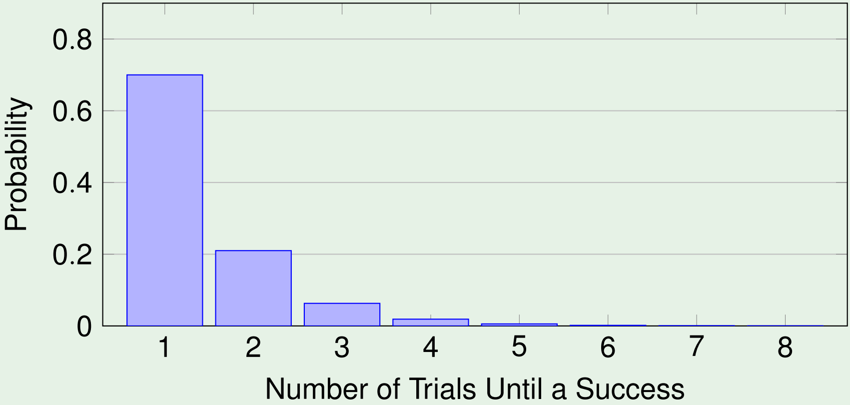
Note

The trials need to be both independent and identical to use the geometric distribution.

Example 5

The geometric distribution for $p = 0.7$



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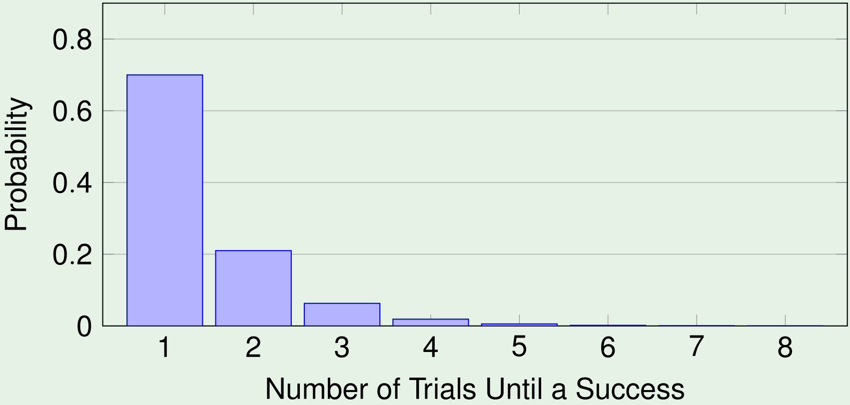

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We call this the geometric distribution because the probabilities decrease exponentially fast as n increases.

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The expected value, μ, tells us, on average, how many trials it takes to get a success. The higher p, the fewer it takes.

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The probability of getting a "heads" on a fair coin is $p = 0.5$, so the expected value is:

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On average, it should only take two flips to get a "heads".

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