Fitting a Line, Residuals, and Correlation

Colby Community College

Definition

Linear Regression is the statistical method for fitting a line to data where the relationship between two variables, x and y, can be modeled by a straight line with some error:

$$y = \beta_0 + \beta_1 x + \epsilon$$

Note

We don't use $y = mx + b$ because the format $y = b_0 + b_1 x$ can easily be expanded in include more variables:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \cdots$$

This is used when performing a multiple regression.

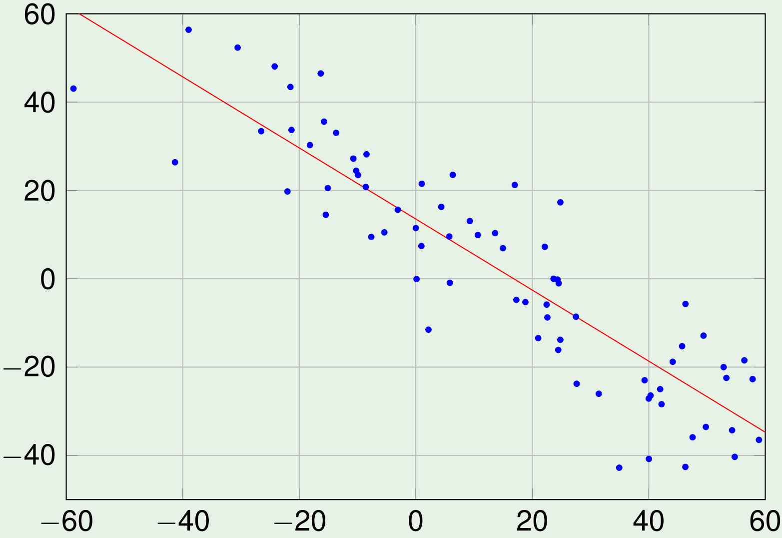
Definition

We call x the **explanatory variable**or **predictor variable**.

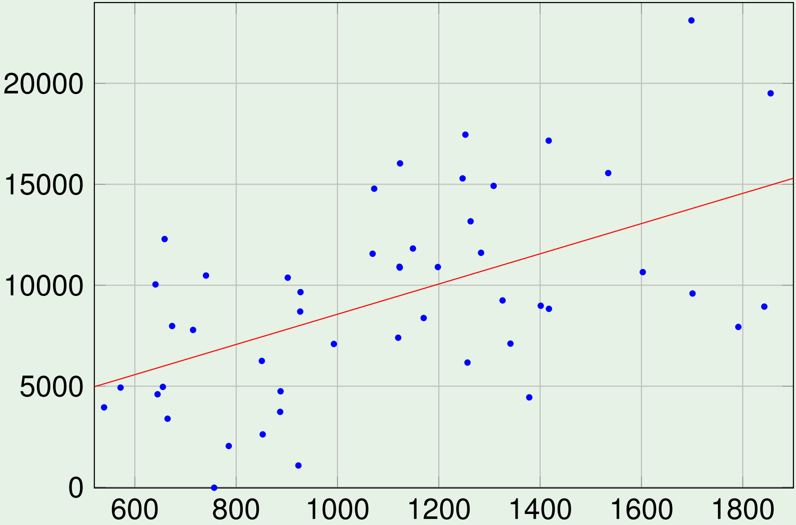
Definition

We call y the **response variable**.

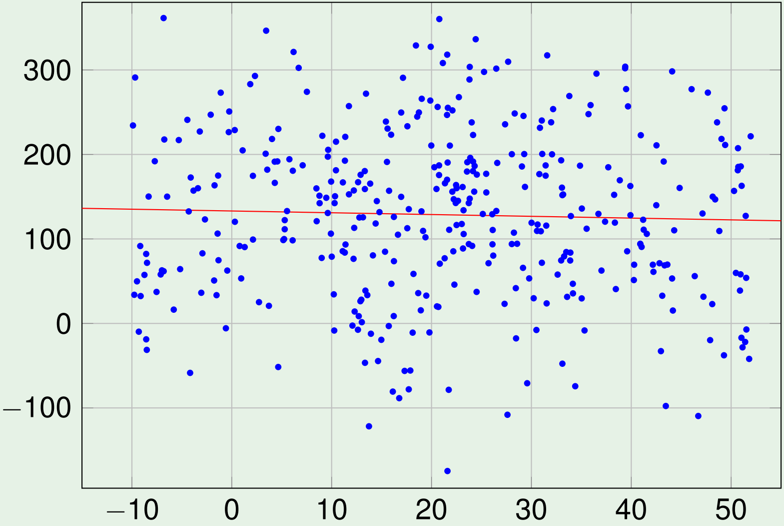
Example 1



Example 2

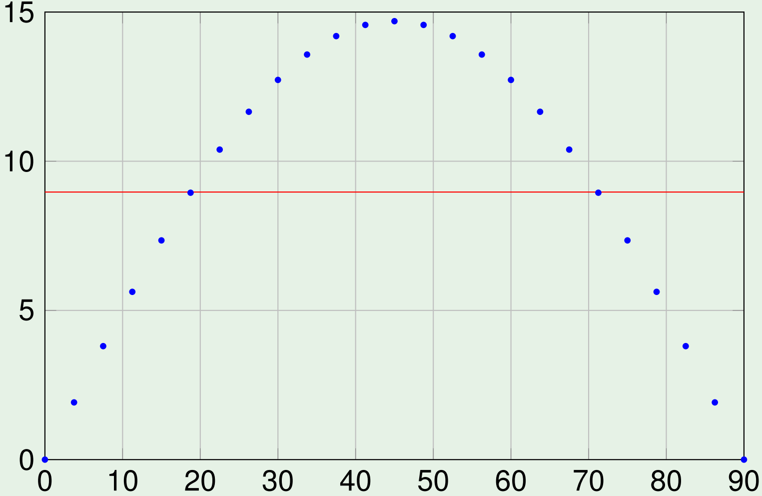


Example 3



Even though this looks like just a cloud, the linear model may be useful.

Example 4



Because, there is a clear non-linear pattern, the linear model is a poor choice for this data.

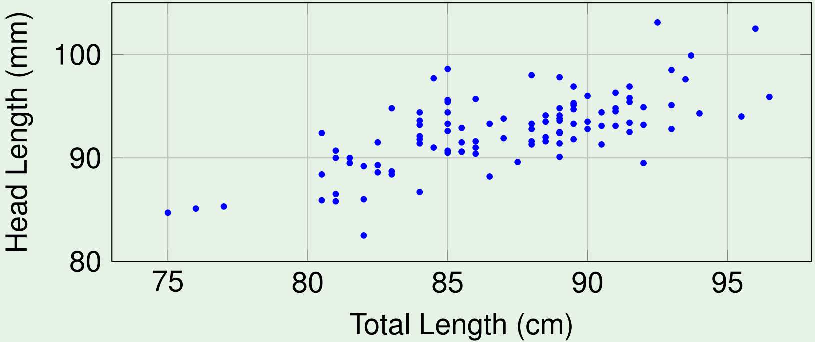
Example 5

Brushtail possums are a marsupial that lives in Australia.

Researchers captured 104 of these animal and took body measurements before releasing the animals back into the wild.

We will consider two measurements:

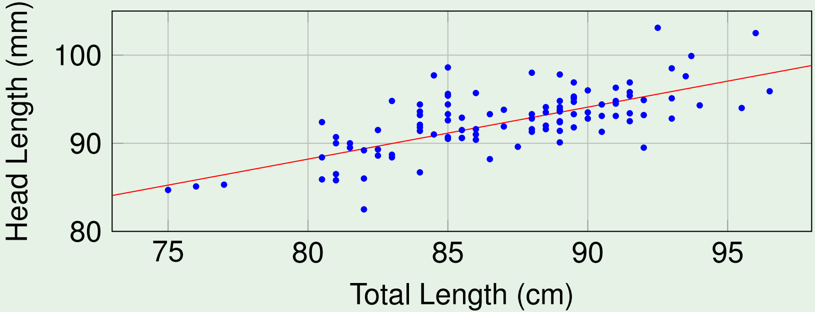
- The length of each possum from head to tail.
- The length of each possum's head.



Example 5 (Continued)

We could fit the linear relationship by eye, giving the equation:

$$\hat{y} = 41 + 0.59x$$



This allows us to make estimates of the possum population.

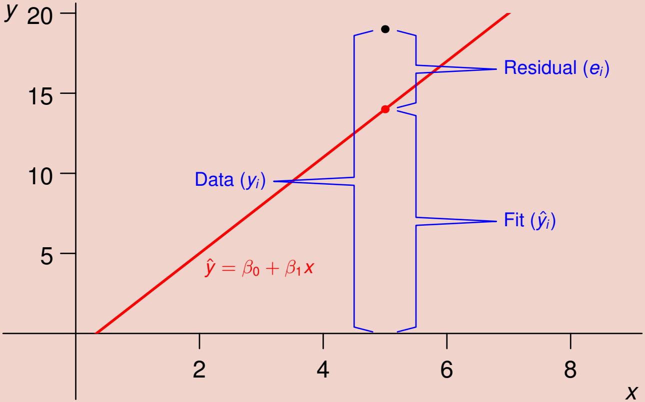
$$\hat{y} = 41 + 0.59(80) = 88.2$$

We expect that a possum with a total length of 80cm would have a head length of about 88.2mm.

Definition

Residuals are the leftover variation in the data after accounting for the model fit:

$$\text{Data} = \text{Fit} + \text{Residual}$$

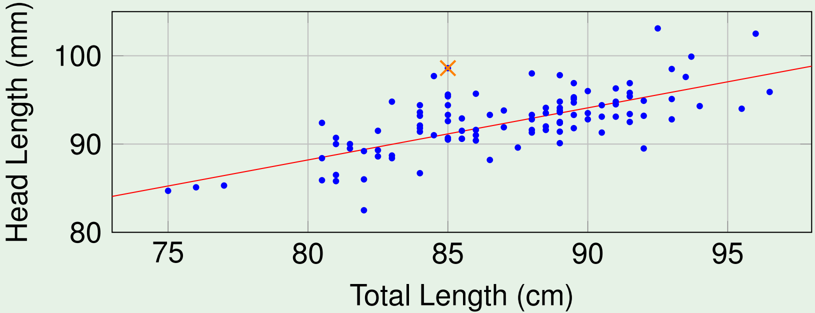


The residuals are calculated as:

$$e_i = y_i - \hat{y}_i$$

Example 5 (Continued)

Let's calculate the residual for the observation $(85.0, 96.6)$.



We first need to find $\hat{y}$:

$$\hat{y}_{\times} = 41 + 0.59(85) = 91.15$$

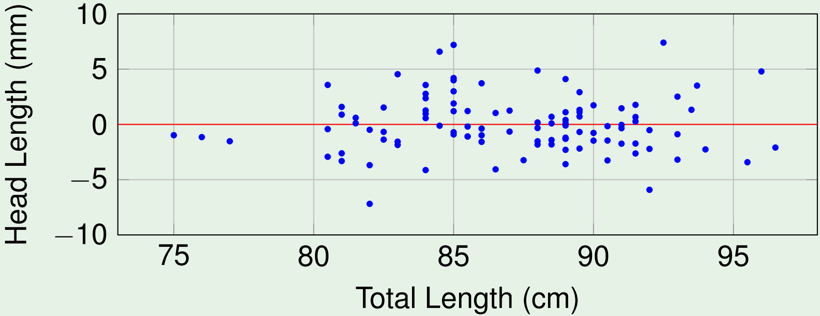
Next, the residual:

$$e_{\times} = y_{\times} - \hat{y}_{\times} = 96.6 - 91.15 = 7.45$$

Definition

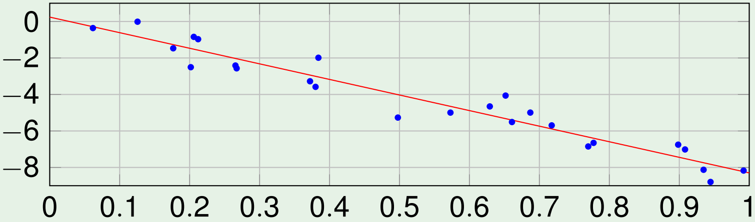
If the residual for each point is calculated, the corresponding graph is called a **residual plot**.

Example 5 (Continued)

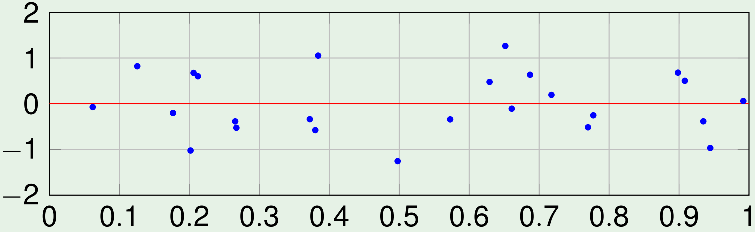


Example 6

Scatter plot with linear regression:

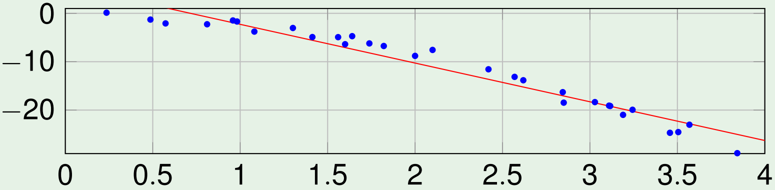


Residual plot:

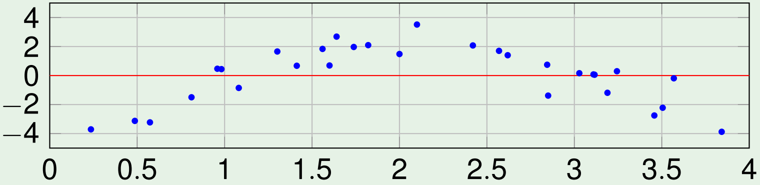


Example 7

Scatter plot with linear regression:



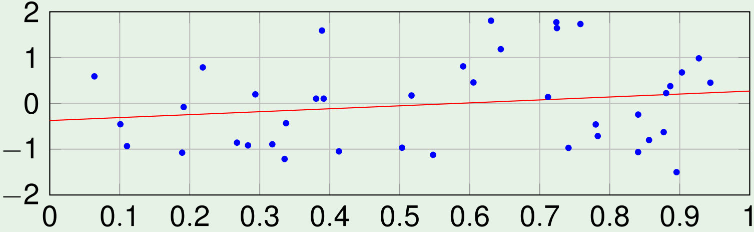
Residual plot:



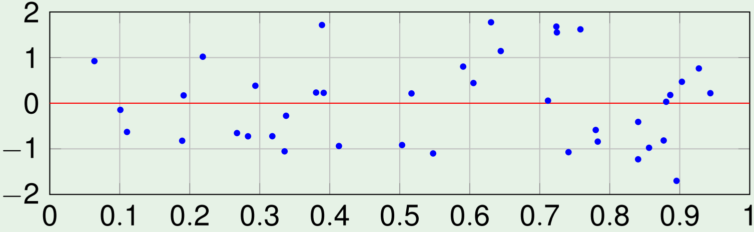
Since there is a clear curve in the residual plot, we should not use a linear model. A more advanced method is needed.

Example 8

Scatter plot with linear regression:



Residual plot:



Definition

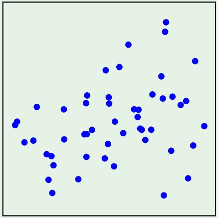
Correlation, which is always between -1 and 1, describes the strength of the linear relationship between two values. We denote the correlation by R.

Note

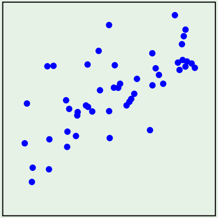
While technology is often used, the formula for correlation is:

$$R = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s_x} \cdot \frac{y_i - \bar{y}}{s_y}\right)$$

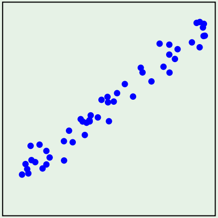
Example 9



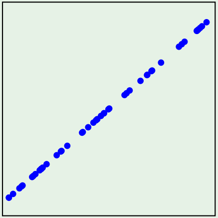
$R = 0.33$



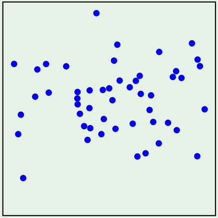
$R = 0.69$



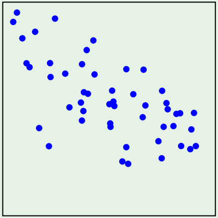
$R = 0.98$



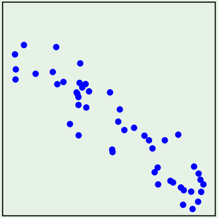
$R = 1.00$



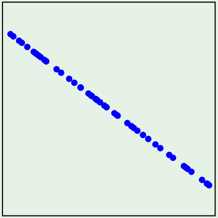
$R = 0.08$



$R = -0.64$

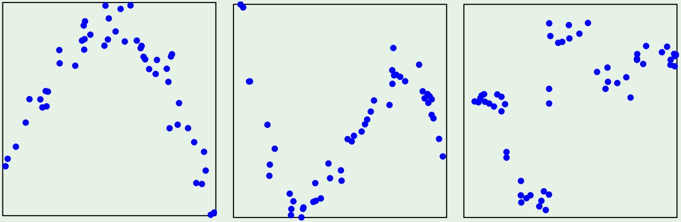


$R = -0.92$



$R = -1.00$

Example 10



$R = -0.23$ $R = 0.31$ $R = 0.50$

Since each of these scatter plots has a clear non-linear pattern, a linear model is not appropriate and correlation shouldn't have been calculated.

Note

Given a table of x and y values, a computer will happily compute correlation. It is your job to determine if a linear model makes sense.