Applications of Normal Distributions

Colby Community College

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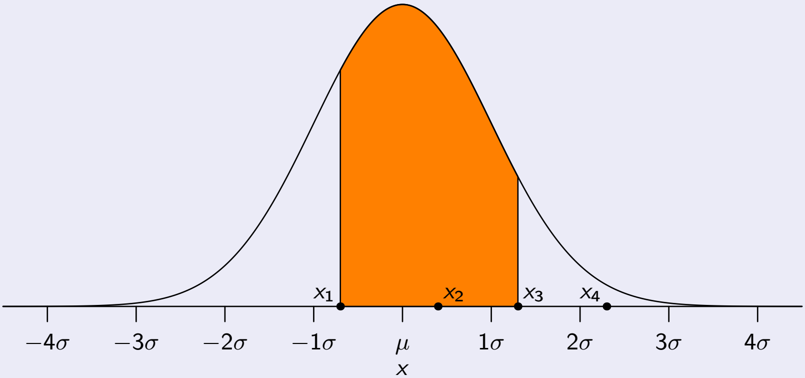
Note

If your calculator or software lets you enter values for μ and σ, you do not need to perform this conversion yourself.

Procedure for Finding Areas with a Nonstandard Normal Distribution

1. Sketch a normal curve, label the mean and any specific x values, and then shade the region representing the desimediumyellow probability.



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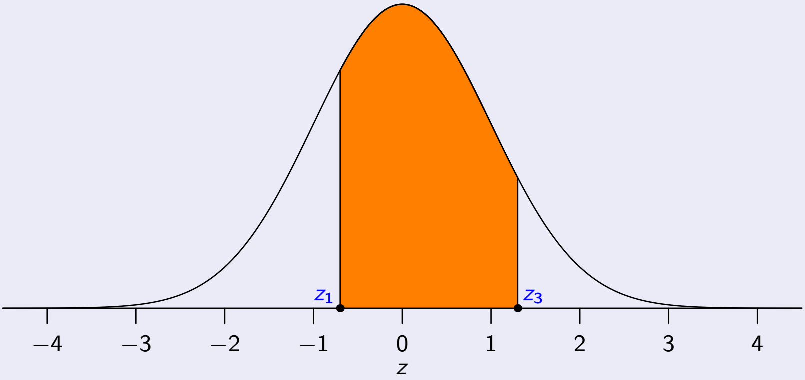

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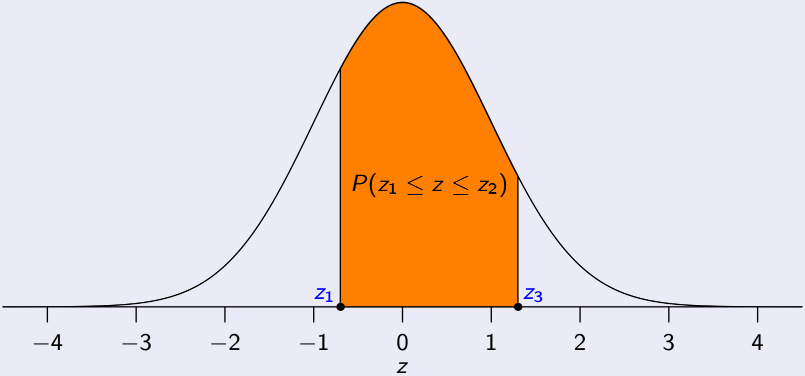
3. Use technology to find the area of the shaded region.



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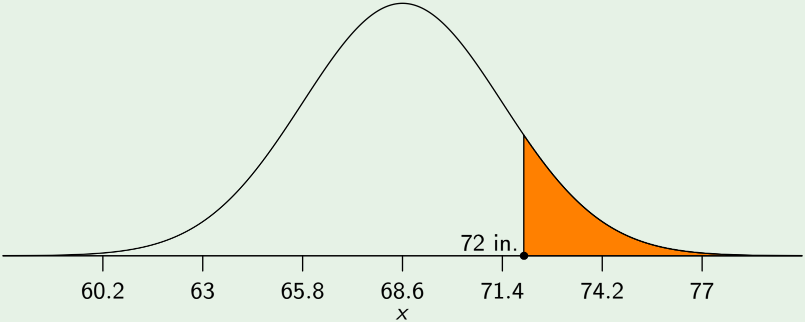

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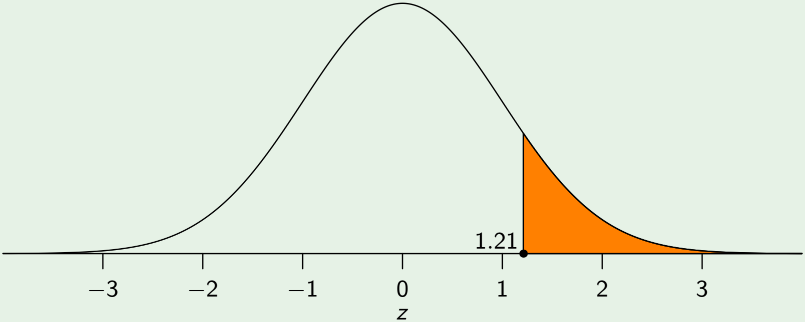

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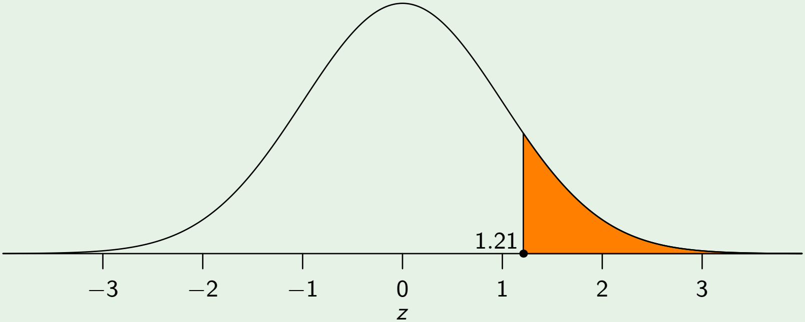

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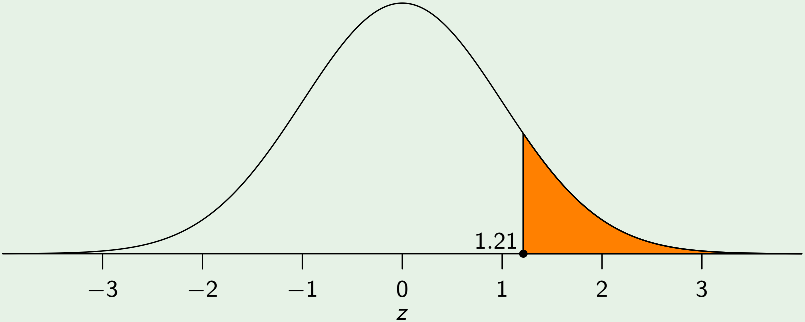

We then need to compute

$$P(z \geq 1.21) = 0.1123 \quad \text{(rounded)}$$

So, we see that 11.23% of men are taller than the showerhead.

Example 2

The U.S. Air Force requires that pilots have heights between 64 and 77 in. From Data Set 1 "Body Data" we see that heights of women have a mean of 63.7 in. and a standard deviation of 2.9 in. What percentage of women meet that requirement?

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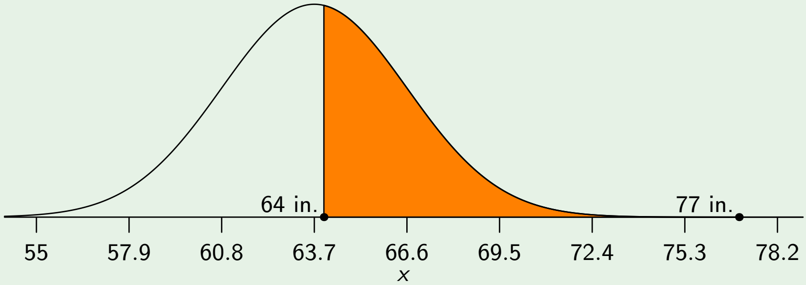

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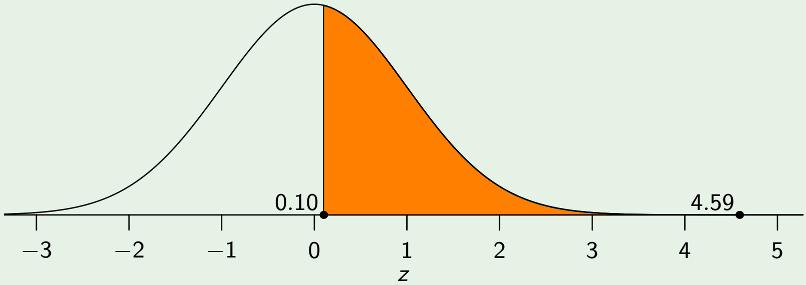

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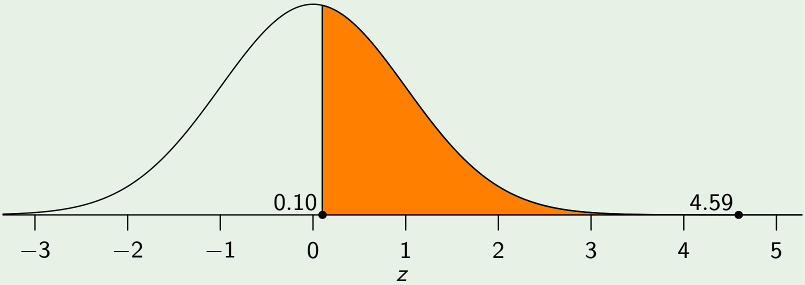

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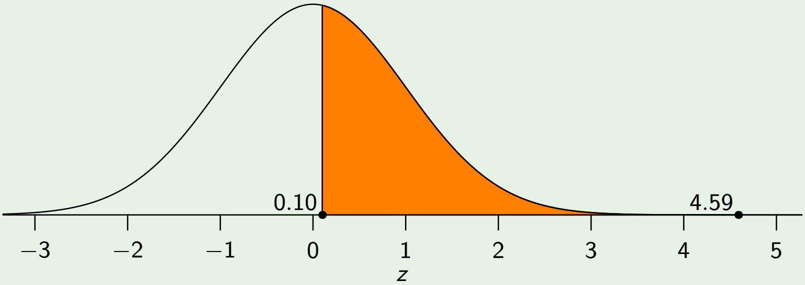

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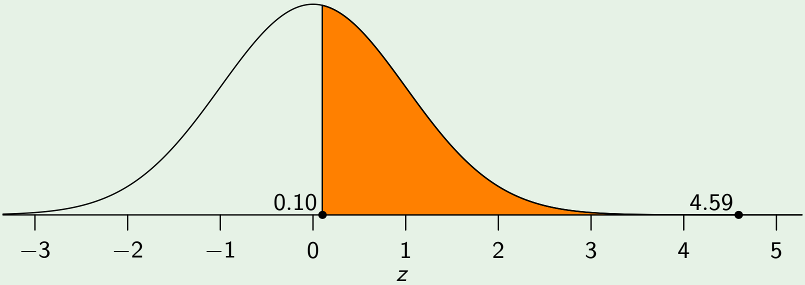

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$$P\left(0.10 \leq z \leq 4.59\right) = 0.4601 \quad \text{(rounded)}$$

So, we see that about 46% of women meet the U.S. Air Force requirements.

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4. Use your sketch to verify that the solution makes sense.

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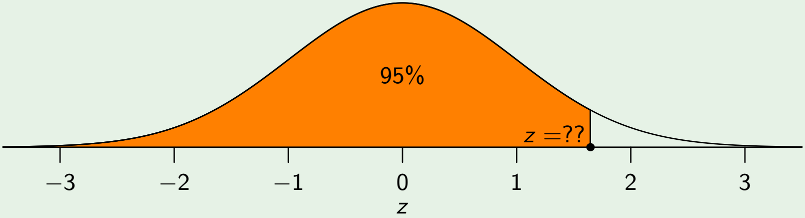

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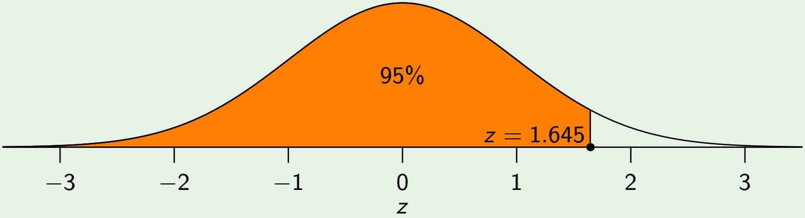

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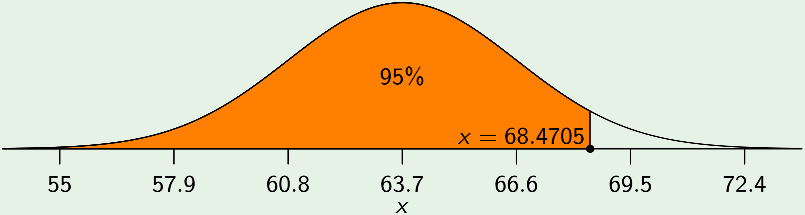

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$$x = \mu + z \cdot \sigma = 63.7 + 1.645 \cdot 2.9 = 68.4705$$

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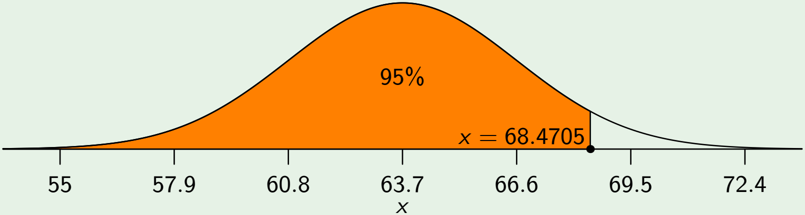

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A requirement of a height less than or equal to 68.5 in. would allow 95% of women to be eligible.

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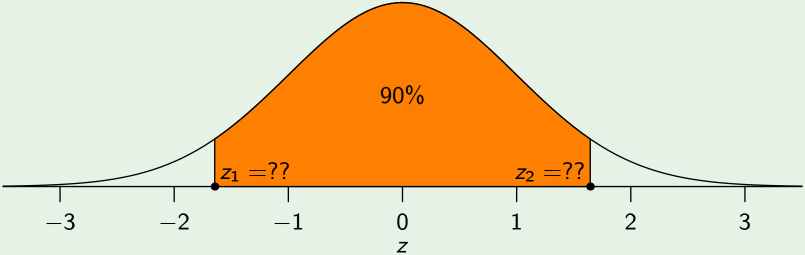

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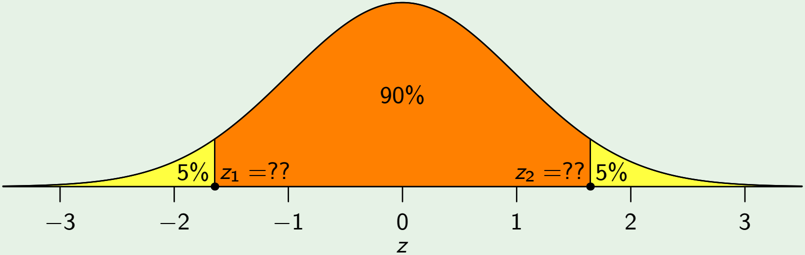

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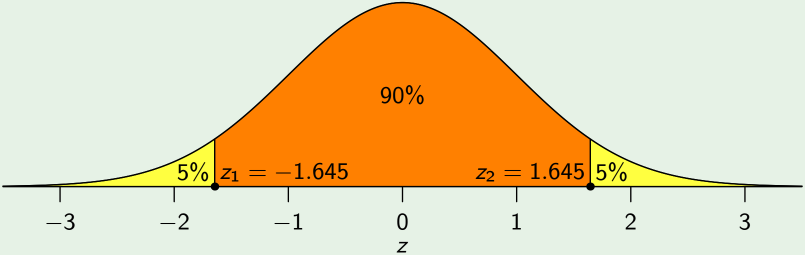

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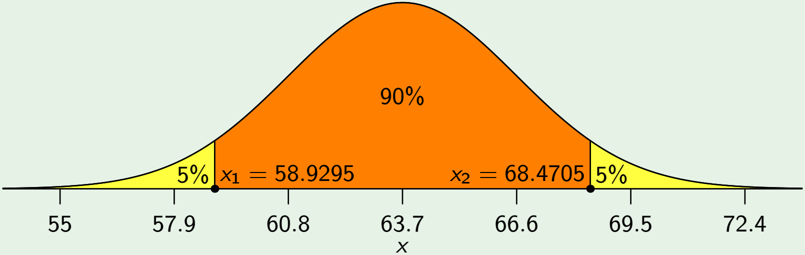

Using either technology or a table, we find that $z_1 = -1.645$ and $z_2 = 1.645$.

We then need to convert to the x values.

$$x_1 = 63.7 - 1.645 \cdot 2.9 = 58.9295$$

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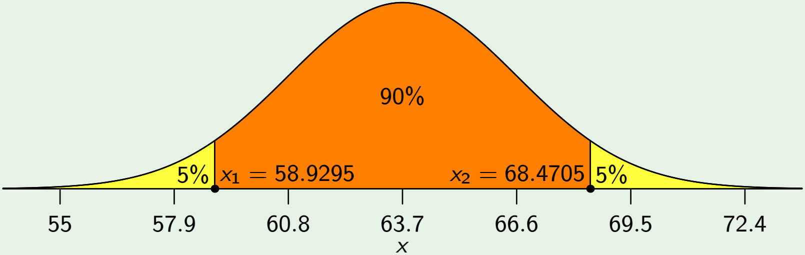

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A requirement of a heights between 58.9 in. and 68.5 in. would allow 90% of women to be eligible.

Recall

Significantly high: The value x is significantly high if

$$P\,(x \text{ or greater}) \leq 0.05$$

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Note

The value of 0.05 is not absolutely rigid, and other values such as 0.01 may make more sense for a given situation.

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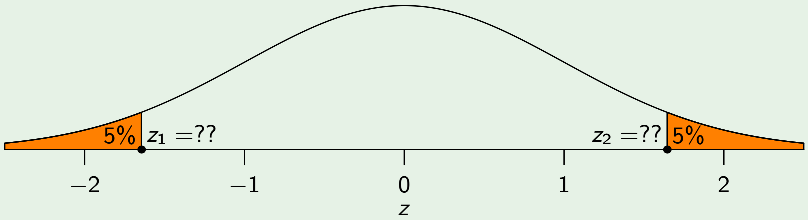

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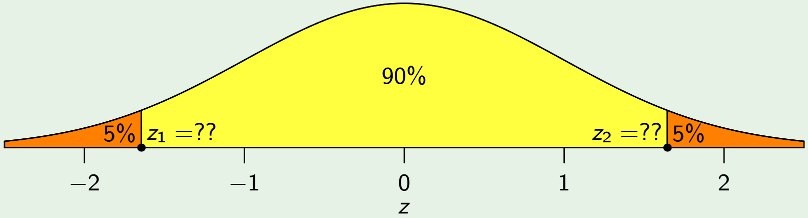

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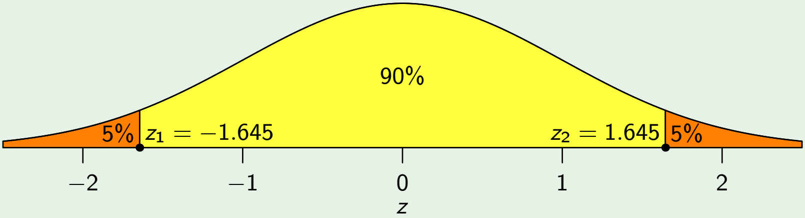

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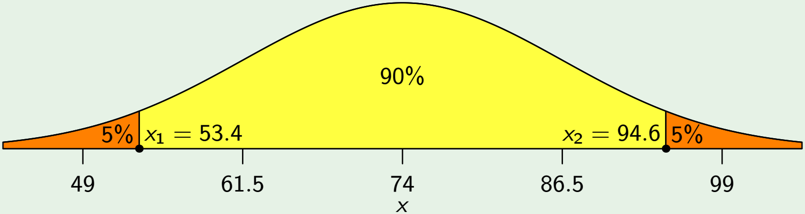

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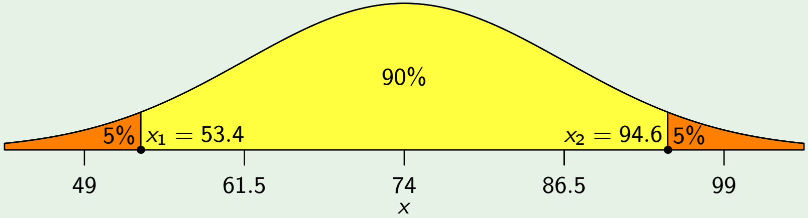

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The pulse rates of women that are significant:

- Significantly low: 53.4 bpm or lower.
- Significantly high: 94.6 bpm or higher.