Hypothesis Testing For A Proportion

Colby Community College

Example 1

The following question comes from a book written by Hans Rosling, Anna Rosling Rönnlund, and Ola Rosling called *Factfulness*.

How many of the world's 1 year old children today have been vaccinated against some disease?

(a) 20% (b) 50% (c) 80%

What is your answer (or guess)?

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Note

If we take a multiple choice test, then we might like to distinguish between the two possibilities:

- People never learn these particular topics and their responses are simply equivalent to random guessing.
- People have knowledge that helps them do better than random guessing, or perhaps, they have false knowledge that leads them to actually do worse than random guessing.

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The **alternative hypotheses** (H_A) represents an alternative claim under consideration and is often represented by a range of possible parameter values.

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We may reject or fail to reject the alternative hypothesis, but we typically never accept the null hypothesis as true.

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The value we are comparing the parameter to is called the **null value.**

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No. While we may not believe the null hypotheses, we need strong evidence before we reject the null hypothesis and conclude something more interesting.

Even if we don't believe the proportion is *exactly* 33.3%, that doesn't tell us anything useful about if people do better or worse than random guessing.

Note

We will be using the `rosling_responses` data set to evaluate the hypothesis test evaluating whether college-educated adults who get the question about infant vaccination correct is different from 33.3%.

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This data set summarizes the answers of 50 college-educated adults. Of these 50 adults, 24% of respondents got the question correct.

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The conditions are met, so let's construct a 95% confidence interval.

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Since the null hypothesis of $p = 0.333$ falls in this range, we cannot say the null hypotheses is implausible. We fail to reject the null hypothesis.

Just because we conclude that it's plausible that $p = 0.333$ does not mean we actually accept the null hypothesis.

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The confidence interval is $(0.103, 0.195)$, which means we are 95% confident that the proportion of college-educated adults that answered the question correctly is between 10.3% and 19.5% Since $p = 0.333$ is implausible, we reject the null hypothesis.

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This means you always need to write what confidence level you used.