Correlation

Colby Community College

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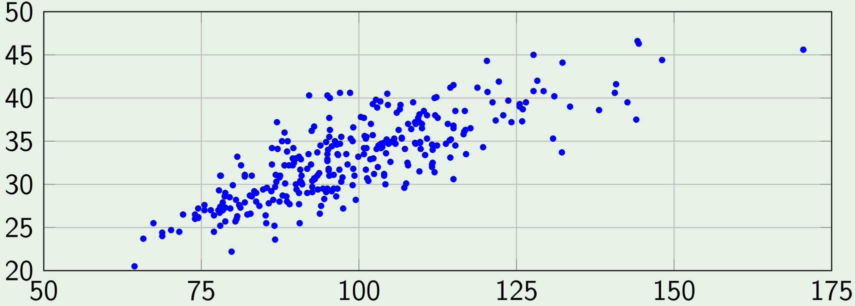
Note

Because conclusions based on visual examinations of scatterplots are largely subjective, we need more objective measures. We use the linear correlation coefficient r, which is a number that measures the strength of the linear association between the two variables.

Example 1



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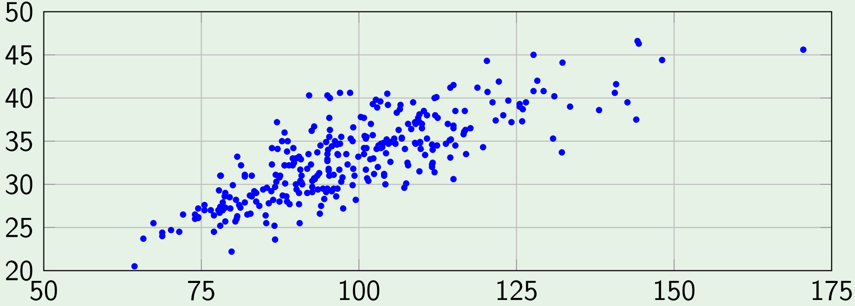
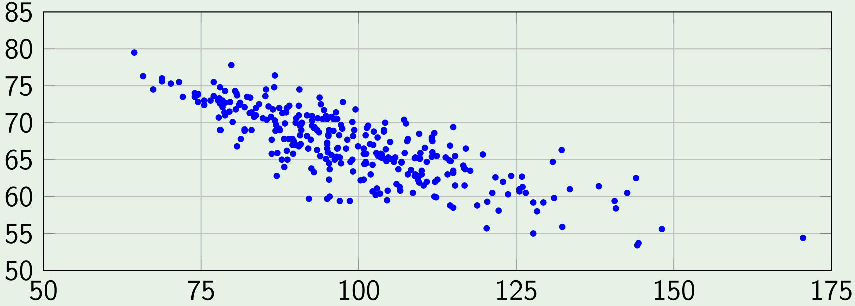


Distinct straight-line, or linear, pattern. We say that there is a positive linear correlation ($r = 0.80241$) between x and y, since as the x values increase, the corresponding y values also increase.

Example 2



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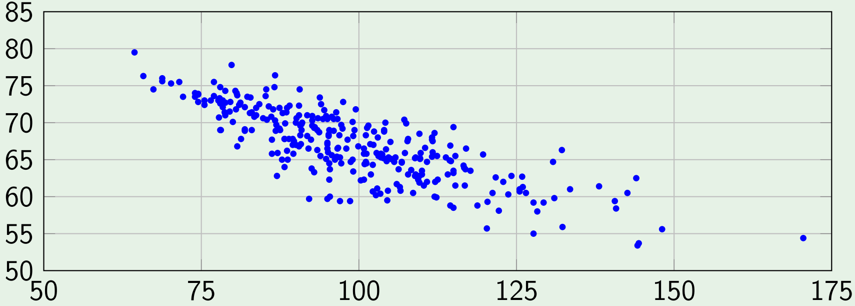

Distinct straight-line, or linear, pattern. We say that there is a positive linear correlation ($r = -0.80241$) between x and y, since as the x values increase, the corresponding y values also increase.

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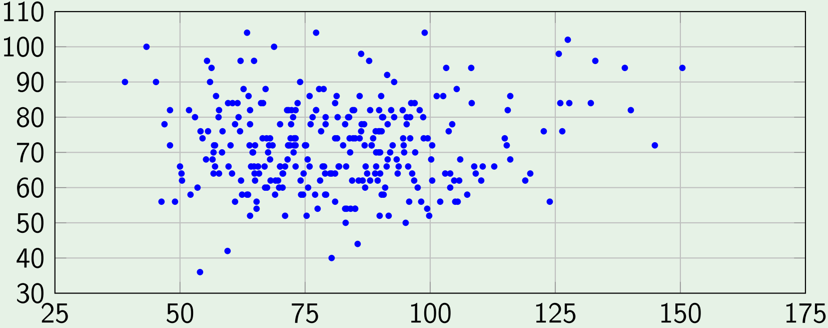
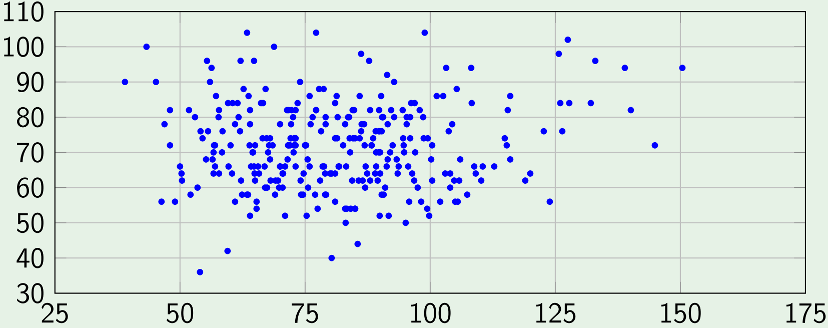


Example 3



The points do not show any obvious pattern ($r = 0.08161$), and this lack of a pattern suggests that there is no relationship between the variables.

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1. The sample of paired data is a simple random of quantitative data.
2. Visual examination of the scatterplot must confirm that the points approximate a straight-line pattern.
3. Because the results can be strongly affected by outliers, any outliers mist be removed if they are known to be errors.

Caution

A linear correlation coefficient r can always be calculated, wether or not it applies.

Formula

The formula for r is:

$$r = \frac{\sum(z_x z_y)}{n-1}$$

where z_x and z_y are the z-scores for the sample values x and y, respectively.

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Alternative Formula

A formula for r that is better for hand calculations is:

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2}\sqrt{n(\sum y^2) - (\sum y)^2}}$$

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- r measures the strength of a linear relationship. It is not designed to measure the strength of a relationship that is nor linear.
- r is very sensitive to outliers in the sense that a single outlier could dramatically affect its value.

Is There a Linear Correlation?

Technology will generate a P-value along with r. If we have a significance level α, then

P-value $\leq \alpha$: Supports the claim of a linear correlation.

P-value $> \alpha$: Does not support the claim of a linear correlation.

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Example 4

For Data Set 2 "Foot and Height," when we use technology to calculate the linear correlation between the foot length and age of 40 randomly selected prople we get:

$$r = 0.3591 \quad \text{and} \quad P\text{-value} = 0.02287$$

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Because $0.02287 \leq 0.05$ we have evidence of a linear correlation.

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this implies that about 87.1% of the variation in ages cannot be explained by rates of chocolate consumption.

Do Not Assume That Correlation Implies Causality!

For several years, the data suggested a linear correlations between the stork population in Copenhagen and the number of human births.

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Do Not Ignore The Possibility of a Nonlinear Relationship

If there is no linear correlation, there might be some correlation that is not linear.

Formal Hypothesis Test

If conducting a formal hypothesis test to determine whether there is a significant linear correlation between two variables, use:

$$H_0: \; p = 0 \text{ (There is no linear correlation.)}$$
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The test statistic (with $n-2$ degrees of freedom) is

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

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Technology give $r = 0.801$ and $n = 23$ which means the test statistic is

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Because the P-value is less than the significance level, we reject H_0.

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Claim of Negative Correlation

When testing a claim of a negative linear correlation use

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Claim of Positive Correlation

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