Confidence Intervals for a Proportion

Colby Community College

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Based on this result, what is the best point estimate of the proportion of all adults who have a Facebook page.

The sample proportion, 0.43, is the best point estimate of the population proportion.

Note

We have no indication of how *good* of an estimate 0.43 is, just that it is the best of the available options.

Definition

A **confidence interval** is a range of values around the point estimate used to estimate the true value of a population parameter.

$$[\text{point estimate} - \text{some value}, \text{point estimate} + \text{some value}]$$

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Round the confidence interval limits to three significant digits.

The Process Success Rate

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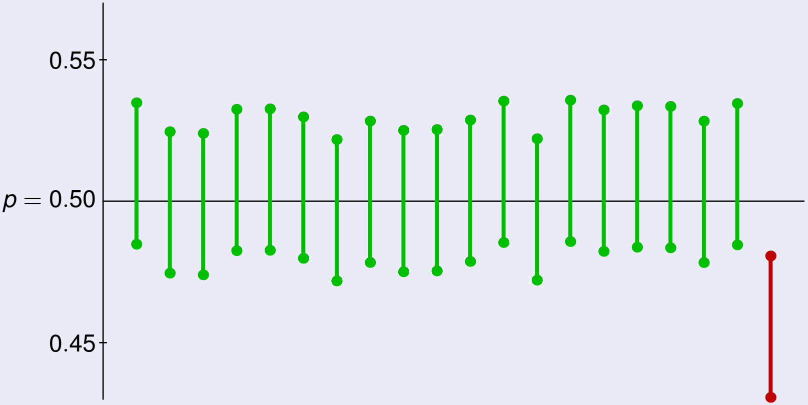
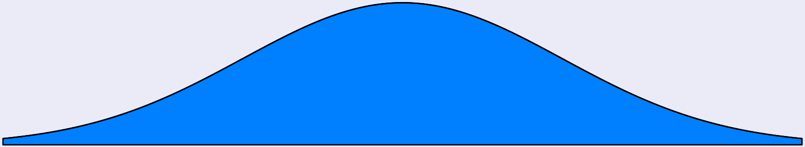
If the true population proportion is $p = 0.5$, then we expect around 19 of 20 confidence intervals to contain the true value of p.



A Few Observations



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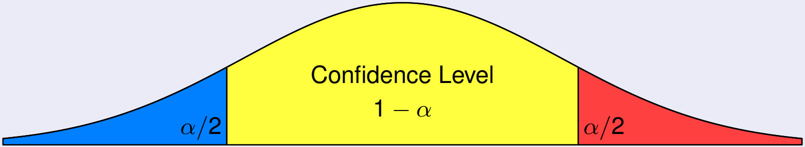

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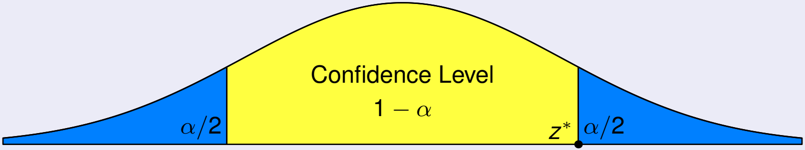

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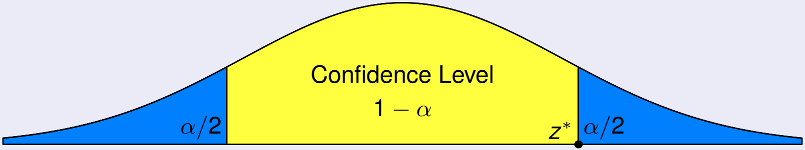

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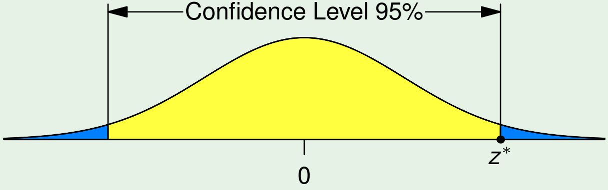
Definition

The value z^* is called a **critical value**.

Example 2

Let us find the critical value corresponding to a 95% confidence level.



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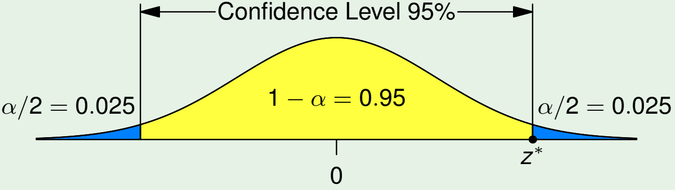
A 95% confidence interval gives $\alpha = 0.05$ and $\alpha/2 = 0.025$.



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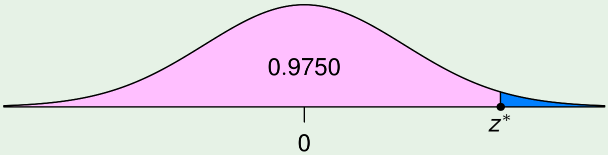
To find the z value using the inverse normal distribution, we need to know the area to the left of the right tail, $0.025 + 0.95 = 0.9750$.



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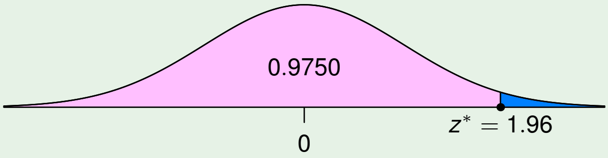

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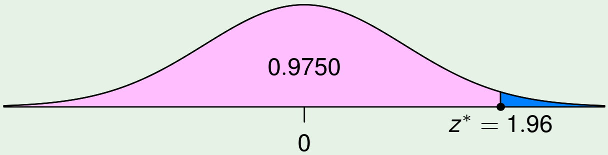

Common Confidence Levels

Confidence Level	α	Critical Value
90%	0.10	1.645
95%	0.05	1.960
99%	0.01	2.575

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Note

The margin of error for $\hat{p}$ is:

$$z^* \cdot SE_{\hat{p}} = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

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Round the confidence interval limits to four digits.

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5. We are 90% confident that between 87.1% and 90.4% of American adults supported the expansion of solar power in 2018.

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5. We are 95% confident that between 79.7% and 84.3% of NYC adults supported the mandatory Ebola quarantine in 2014.

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Incorrect: "There is a 90% chance that the true value of p will fall between 0.8705 and 0.9035."

Reason: The population proportion p is a fixed value.

Interpreting a Confidence Interval

For the confidence interval $(0.8705, 0.9035)$ there is one correct interpretation and many creatively incorrect interpretations.

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Reason: The values 0.8705 and 0.9035 result from one sample, they are not parameters describing the behavior of all samples.

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Caution

Never think that poll results are unreliable if the sample size if a small percentage of the population size.

Finding $\hat{p}$ from a Confidence Interval

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The article "High-Dose Nicotine Patch Therapy", by Dale, Hurt, et al. (*Journal of the American Medical Association*, Vol 274, No. 17) includes the statement:

> Of the 71 subjects, 70% were abstinent from smoking at 8 weeks (95% confidence interval [CI], 58% to 81%).

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Rounding

If the computed sample size n is not a whole number, round the value of n up to the next larger whole number.

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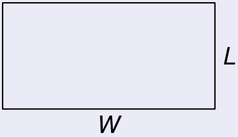
To see why, consider the rectangle:



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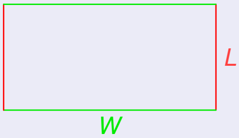

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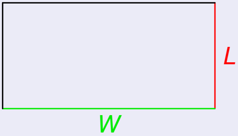

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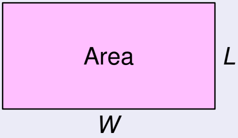

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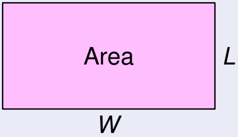

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Since this is a parabola that opens down, we know that the vertex, $(0.5, 0.5)$, is the maximum value.

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