Difference of Two Proportions

Colby Community College

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But, what we really want to know is, if blood thinners have an effect of heart attack survival rates in the general population?

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Success-Failure: The success-failure condition holds for both groups, where we check successes and failures in each group separately.

When these conditions are satisfied, the standard error of $\hat{p}_1 - \hat{p}_2$ is

$$SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

where p_1 and p_2 represent the population proportions, and n_1 and n_2 represent the sample sizes.

Confidence Intervals for $\hat{p}_1 - \hat{p}_2$

When the independence and success-failure conditions are met, we can build confidence interval in the same general manner and before:

$$\begin{array}{ccc} \text{point estimate} & \pm & z^* \cdot SE \\ & \Downarrow & \\ \hat{p}_1 - \hat{p}_2 & \pm & z^* \sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}} \end{array}$$

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We can summarize the results from the experiment in Example 1:

	Survived	Died	Total
Control	11	29	50
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This is a randomized experiment, so yes.

Are the success-failure conditions satisfied?

The treatment group had 11 survivals and 29 deaths, and the control group had 14 survivals and 26 deaths. All are more than 10, so yes.

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What can be conclude about whether blood thinners help or harm?

Since 0% is in the confidence interval, we don't have enough evidence to say if blood thinners had any impact.

Example 3

A 5-year experiment was conducted to evaluate the effectiveness of fish oils on reducing cardiovascular events, where each subject was randomized into one of two groups.

We'll consider heart attack outcomes in these patients:

	heart attack	no event	Total
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Since the entire interval is negative, we have strong evidence that fish oil supplements reduce heart attacks.

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	Death from breast cancer?	
	Yes	No
Mammogram	500	44,425
Control	505	44,405

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Definition

When the null hypothesis is that $p_1 - p_2 = 0$, we use a special proportion called the **pooled proportion** to check the success-failure condition and compute the standard error:

$$\hat{p}_{\text{pooled}} = \frac{\text{number of “successes”}}{\text{number of cases}} = \frac{\hat{p}_1 n_1 + \hat{p}_2 n_2}{n_1 + n_2}$$

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Remember that we check the success-failure condition under the assumption that the null hypothesis is true. We must also check for each group:

$$\begin{aligned}
\hat{p}_{\text{pooled}} \cdot n_{\text{mgm}} &= \\
(1 - \hat{p}_{\text{pooled}}) n_{\text{mgm}} &= \\
\hat{p}_{\text{pooled}} \cdot n_{\text{ctrl}} &= \\
(1 - \hat{p}_{\text{pooled}}) \cdot n_{\text{ctrl}} &=
\end{aligned}$$

Example 4 (Continued)

Lets check if it's reasonable to model the difference in proportion using a normal distribution in this study?

Because the patients are randomized, they can be treated as independent.

Remember that we check the success-failure condition under the assumption that the null hypothesis is true. We must also check for each group:

$$\begin{aligned}
\hat{p}_{\text{pooled}} \cdot n_{\text{mgm}} &= 0.0112 \cdot 44,925 \\
(1 - \hat{p}_{\text{pooled}}) n_{\text{mgm}} &= \\
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Example 4 (Continued)

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Example 4 (Continued)

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(1 - \hat{p}_{\text{pooled}}) \cdot n_{\text{ctrl}} &= 0.9888 \cdot 44,910 = 44,407
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Example 4 (Continued)

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$$\begin{aligned}
\hat{p}_{\text{pooled}} \cdot n_{\text{mgm}} &= 0.0112 \cdot 44,925 = 503 \geq 10 \\
(1 - \hat{p}_{\text{pooled}})n_{\text{mgm}} &= 0.9888 \cdot 44,925 = 44,422 \geq 10 \\
\hat{p}_{\text{pooled}} \cdot n_{\text{ctrl}} &= 0.0112 \cdot 44,910 = 503 \geq 10 \\
(1 - \hat{p}_{\text{pooled}}) \cdot n_{\text{ctrl}} &= 0.9888 \cdot 44,910 = 44,407 \geq 10
\end{aligned}$$

Since each is at least 10, we can safely model the difference in proportions using a normal distribution.

Example 4 (Continued)

The point estimate for the difference in breast cancer rates is:

$$p_{\text{mgm}} - p_{\text{ctrl}} =$$

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p_{\text{mgm}} - p_{\text{ctrl}} &= \frac{500}{500 + 44,425} + \frac{505}{505 + 44,405} \\
&= 0.01113 - 0.01125
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$$= 0.01113 - 0.01125 = -0.00012$$

The standard error is:

$$SE = \sqrt{\frac{\hat{p}_{\text{pooled}}(1 - \hat{p}_{\text{pooled}})}{n_{\text{mgm}}} + \frac{\hat{p}_{\text{pooled}}(1 - \hat{p}_{\text{pooled}})}{n_{\text{ctrl}}}}$$

Example 4 (Continued)

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$$= \sqrt{\frac{0.0112(1 - 0.0112)}{500 + 44,425} + \frac{0.0112(1 - 0.0112)}{505 + 44,405}} = 0.00070$$

Example 4 (Continued)

Next, we calculate the z-score:

$$z = \frac{\text{point estimate - null value}}{\text{SE}}$$

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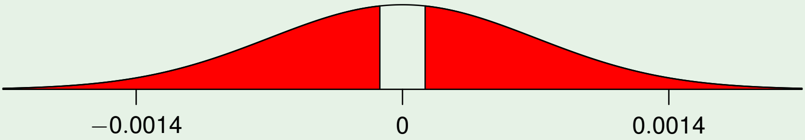
$$z = \frac{\text{point estimate - null value}}{\text{SE}} = \frac{-0.00012 - 0}{0.00070} = -0.17$$

Example 4 (Continued)

Next, we calculate the *z*-score:

$$z = \frac{\text{point estimate - null value}}{\text{SE}}$$
$$= \frac{-0.00012 - 0}{0.00070} = -0.17$$

The *p*-value is the area in both tails:



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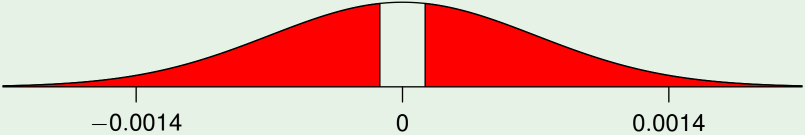

The left tail has area 0.4325 and so the total area is 0.8560. We do not reject the null hypothesis.

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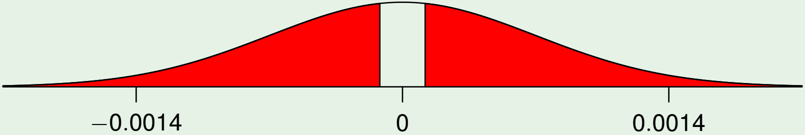

The left tail has area 0.4325 and so the total area is 0.8560. We do not reject the null hypothesis.

That is, the difference between breast cancer death rates is reasonably explained by random chance, and we do not observe benefits or harm from mammograms relative to a regular breast exam.

Example 4 (Continued)

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- If mammograms are helpful or harmful, the data suggests the effect isn't very large.

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- We do not accept null hypothesis, which means we don't have sufficient evidence to conclude that mammograms reduce or increase breast cancer deaths.
- If mammograms are helpful or harmful, the data suggests the effect isn't very large.
- Are mammograms more or less expensive than a non-mammogram breast exam?

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- We do not accept null hypothesis, which means we don't have sufficient evidence to conclude that mammograms reduce or increase breast cancer deaths.
- If mammograms are helpful or harmful, the data suggests the effect isn't very large.
- Are mammograms more or less expensive than a non-mammogram breast exam?
  - If one option is much more expensive than the other and doesn't offer clear benefits, then we should lean towards the less expensive option.

Example 4 (Continued)

- The study's authors also found that mammograms led to overdiagosis of breast cancer:
    - This means that some breast cancers were found, or thought to be found, but that these cancers would not cause symptoms during the patients' lifetimes.

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  - That is, something else would kill the patient before breast cancer symptoms appeared.
  - This means some patients may have been treated for breast cancer unnecessarily, and this treatment is another cost to consider.
  - Overdiagnosis can cause unnecessary physical and emotional harm to patients.