Estimating a Population Proportion

Colby Community College

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Recall

An unbiased estimator is a statistic that targets the value of the corresponding population parameter in the sense that the sampling distribution of the statistic has a mean that is equal to the corresponding population parameter.

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The sample proportion is 0.43, so the best estimate of p is 0.43.

Note

We have no indication of how *good* of an estimate 0.43 is, just that it is the best of the available options.

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Rounding

Round the confidence interval limits for p to three significant digits.

Interpreting a Confidence Interval

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Incorrect: "There is a 95% chance that the true value of p will fall between 0.405 and 0.455."

Reason: The population proportion p is a fixed value, it is not a random variable.

Incorrect: "95% of sample proportions will fall between 0.405 and 0.455."

Reason: The values 0.405 and 0.455 result from one sample, they are not parameters describing the behavior of all samples.

The Process Success Rate

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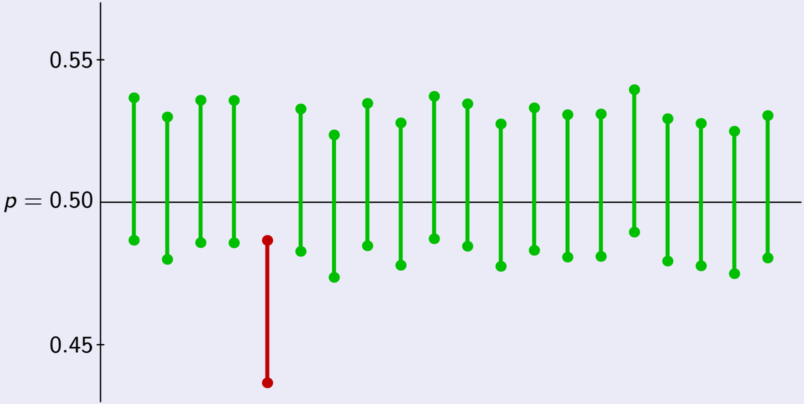
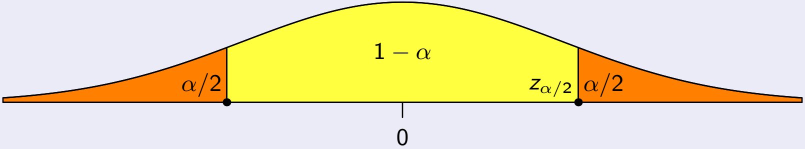
If the true population proportion is $p = 0.5$, then we expect about 19 out of 20 confidence intervals to contain the true value of p.



A Few Observations



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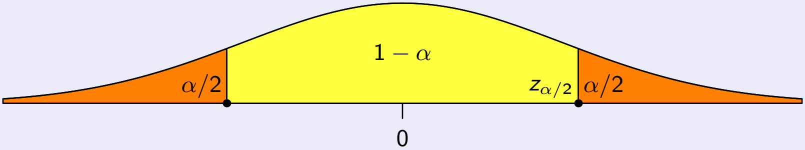

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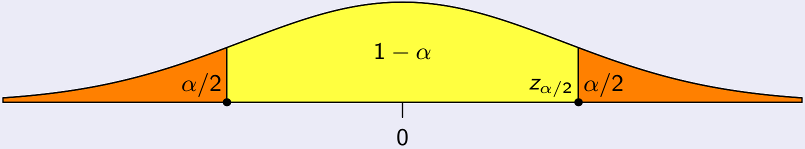

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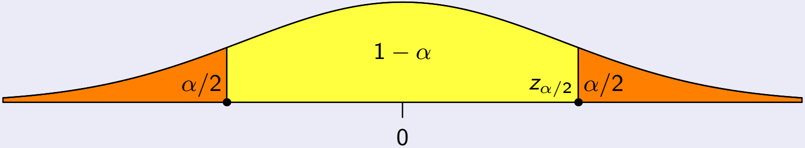

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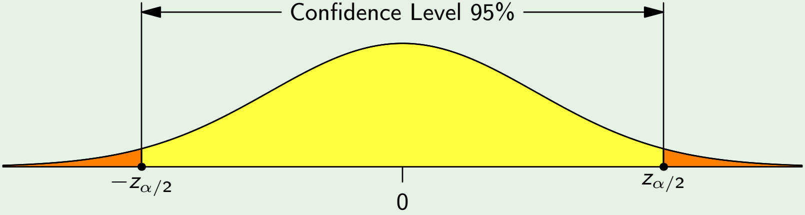
Definition

A **critical value** is the number on the borderline separating sample statistics that are significantly high or low from those that are not significant.

Example 2

Let us find the critical value $z_{\alpha/2}$ corresponding to a 95% confidence level.



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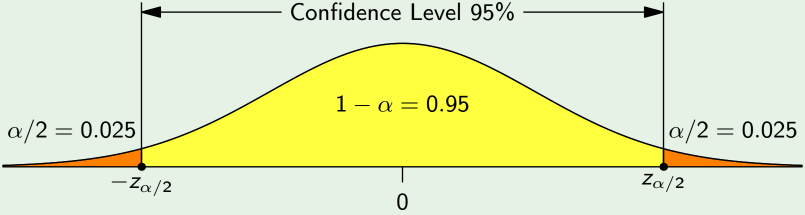
Let us find the critical value $z_{\alpha/2}$ corresponding to a 95% confidence level.

A 95% confidence interval gives $\alpha = 0.05$ and $\alpha/2 = 0.025$.



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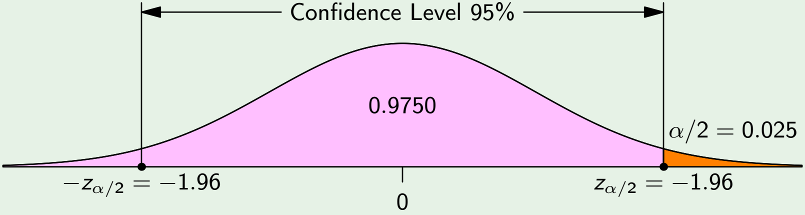

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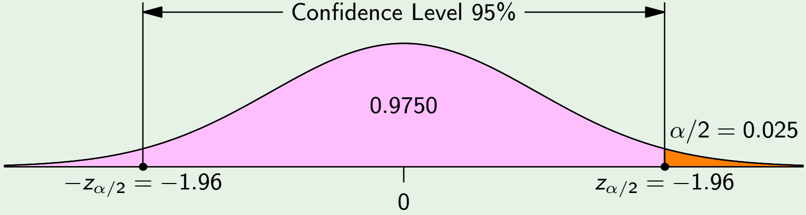

Common Confidence Levels

Confidence Level	α	Critical Value
90%	0.10	1.645
95%	0.05	1.960
99%	0.01	2.575

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There is a probability of $1 - \alpha$ that $\hat{p} - E < p < \hat{p} + E$.

Note

The margin of error E is also called the **maximum error of the estimate** and can be found by multiplying the critical value and the estimated standard deviation of sample proportions.

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

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Note

Statistics software, such as Statdisk, can calculate the confidence interval for you.

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Note

Looking at the confidence interval we can conclude that less than half of adults have a Facebook page.

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Caution

Never think that poll results are unreliable if the sample size if a small percentage of the population size.

Finding $\hat{p}$ from a Confidence Interval

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If you know the confidence interval limits, we can calculate the margin of error:

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We have $z_{\alpha/2} = 1.96$ and $E = 0.03$, but what about $\hat{p}$?

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Rounding

If the computed sample size n is not a whole number, round the value of n up to the next larger whole number.

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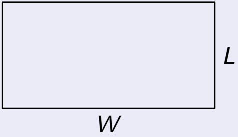
To see why, consider the rectangle:



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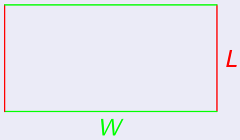

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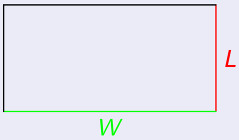

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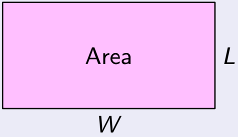

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Since this is a parabola that opens down, we know that the vertex $(0.5, 0.5)$ is the maximum value.

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There are other types of confidence intervals:

- The Wilson score.
- The Clopper-Pearson Method.