Considering Categorical Data

Colby Community College

The following table summarizes two categorical variables from the full loans data set.

| | | h | | | |
|----------|------------|------|----------|------|-------|
| | | rent | mortgage | own | Total |
| app_type | individual | 3496 | 3839 | 1170 | 8505 |
| | joint | 362 | 950 | 183 | 1495 |
| | Total | 3858 | 4789 | 1353 | 10000 |

Definition

A table that summarizes data for two categorical variables in this way is called a **contingency table**.

Definition

The **row totals** provide the total counts across each row.

The **column totals** provide the total counts down each column.

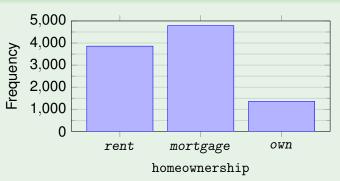
You can also create a table that considers only a single variable.

Example 2

| homeownership | Count |
|---------------|-------|
| rent | 3858 |
| mortgage | 4789 |
| own | 1353 |
| Total | 10000 |

A **bar plot** plots a bar for each variable outcome, the height is the frequency of the outcome.

Example 3



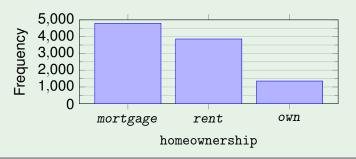
Note

A histogram has no gaps between the bars, where as bar plot does.

A bar plot that has been sorted so the largest bar is on the left and the smallest bar on the right is called **Pareto chart**.

Example 4

Here is the Pareto chart for the bar plot in Example 3.



Note

Named after Vilfredo Pareto, a noted Italian economist.

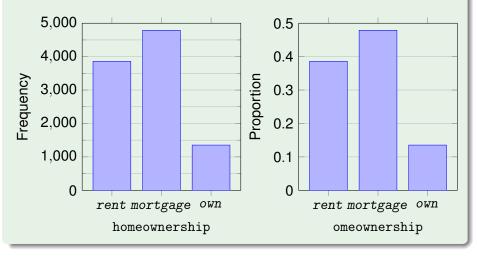
Instead of using frequencies, we could instead use proportions.

Example 5

To find the proportion, divide each frequency by the total count.

| homeownership | Frequency | Proportion |
|---------------|-----------|-------------------------------|
| rent | 3858 | $\frac{3858}{10000} = 0.3858$ |
| mortgage | 4789 | $\frac{4789}{10000} = 0.4789$ |
| own | 1353 | $\frac{1353}{10000} = 0.1353$ |
| Total | 10000 | 1.0000 |

Here are both the frequency and proportion for homeownership.



Here, we use the **row proportions** for the contingency table from Example 1. Where we divide each count by their row total.

| | | homeownership | | | | | |
|----------|------------|---------------|----------------------------------|--------|--------|--|--|
| | | rent | mortgage | own | Total | | |
| | individual | 3496 | 3839 | 1170 | 8505 | | |
| app_type | joint | 362 | 950 | 183 | 1495 | | |
| | Total | 3858 | 4789 | 1353 | 10000 | | |
| | | | $\downarrow\downarrow\downarrow$ | | | | |
| | | h | homeownership | | | | |
| | | rent | mortgage | own | Total | | |
| app_type | individual | 0.4111 | 0.4514 | 0.1376 | 1.0000 | | |
| | joint | 0.2421 | 0.6355 | 0.1224 | 1.0000 | | |
| | Total | 0.3858 | 0.4789 | 0.1353 | 1.0000 | | |

What does the number 0.4111 represent?

That 41.11% of those that applied as individuals are renters.

Here, we use the **column proportions** for the contingency table from Example 1. Where we divide each count by their column total.

| | | h | homeownership | | | | |
|----------|------------|--------|----------------------------------|--------|--------|--|--|
| | | rent | mortgage | own | Total | | |
| + | individual | 3496 | 3839 | 1170 | 8505 | | |
| app_type | joint | 362 | 950 | 183 | 1495 | | |
| | Total | 3858 | 4789 | 1353 | 10000 | | |
| | | | $\downarrow\downarrow\downarrow$ | | | | |
| | | h | homeownership | | | | |
| | | rent | mortgage | own | Total | | |
| app_type | individual | 0.9062 | 0.8016 | 0.8647 | 0.8505 | | |
| | joint | 0.0946 | 0.1984 | 0.1353 | 0.1495 | | |
| | Total | 1.0000 | 1.0000 | 1.0000 | 1.0000 | | |

What does the number 0.9062 represent?

That 90.62% of renters applied as individuals.

Let us look at the column proportions from Example 8 more closely.

| | | h | | | |
|----------|------------|--------|----------|--------|--------|
| | | rent | mortgage | own | Total |
| app_type | individual | 0.9062 | 0.8016 | 0.8647 | 0.8505 |
| | joint | 0.0946 | 0.1984 | 0.1353 | 0.1495 |
| | Total | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

Notice that 90.62% of renters applied as individuals, which is higher than for those with mortgages (80.16%) or those who own (86.47%). Because these rates vary between the three levels of homeowndership (rent, mortgage, own), there is evidence that the app_type and homeownsership variables are associated.

Note

If we had considered row proportions instead, we would look down columns to see if the fraction of loans where the borrower rents, has a mortgage, or owns varied across the <code>individual</code> to <code>joint</code> types.

Data scientists use statistics to filter spam from incoming email messages. By noting specific characteristics of an email, an email may be classified as spam or not spam.

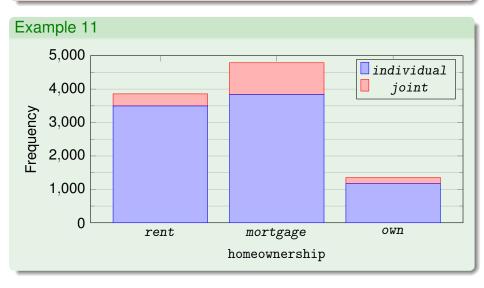
One such characteristic is the email format, which indicates whether or not an email has any HTML content, such as bolded text.

| | text | HTML | Total | | text | HTML | Total |
|----------|------|------|-------|----------|-------|-------|-------|
| spam | 209 | 158 | 367 | spam | 0.175 | 0.058 | 0.094 |
| not spam | 986 | 2568 | 2554 | not spam | 0.825 | 0.942 | 0.651 |
| Total | 1195 | 2726 | 3921 | Total | 1.000 | 1.000 | 1.000 |

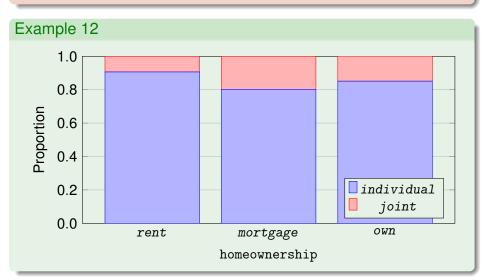
If we generate column proportions, we can see that a higher fraction of plain text emails are spam (17.5%) than compared to HTML emails (5.8%).

But, this information is not enough on its own to classify an email and spam or not spam, since more than 80% of plain text emails are not spam.

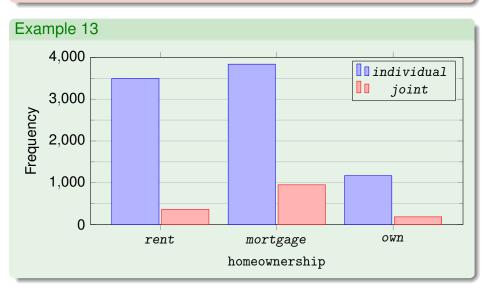
A **stacked bar plot** is a graphical display of contingency tables.



A stacked bar plot generated from column proportions is called a **standardized stacked bar plot**.



A **side-by-side bar plot** is graphical display of contingency tables.



The stacked bar plot is most useful when it's reasonable to assign one variable as the explanatory variable and the other variable as the response.

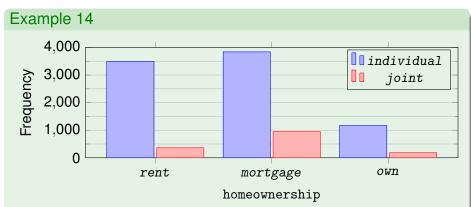
Note

Side-by-side bar plots are more agnostic in their display about which variable, if any, represents the explanatory variable and which the response variable.

Note

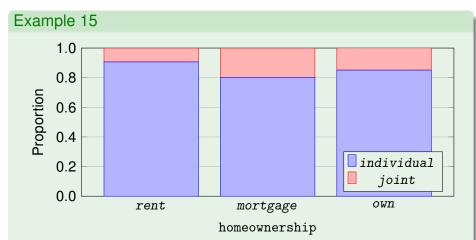
The standardized stacked bar plot is useful when the primary variable in the stacked bar plot is relatively imbalanced.

When a group of bars have very different sizes, relative to the other groups, it is difficult to discern if there is an association between the variables.



The bars for the own group are both much shorter than the other bars, so you can't tell from this plot if there is an association.

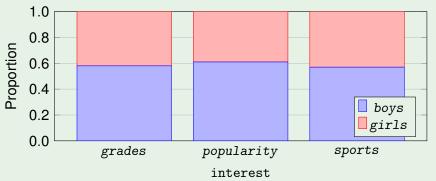
A standardized stacked bar plot is useful for checking for association.



Since the proportions vary for each outcome of homeownership, there is an association.

Suppose grade school students were surveyed and asked which of the following they though was the most important: grades, popularity, sports.

The standardized stacked bar plot represents the responses.



Since the proportions do not significantly vary for each outcome, the variables gender and interest are independent.

To compare numerical and categorical variable, you can group the numbers based on the possible outcomes of the categorical variable, then draw a plot for each group.

Example 17

If we wish to compare the median income to whether a county gained population, we could build the table.

Median Income for U.S. Counties, in \$1000s

| Population Gain | | | | | | No P | opulatio | n Gain |
|-----------------|------|------|------|------|------|------|----------|--------|
| 38.2 | 43.6 | 42.2 | 61.5 | 51.1 | 45.7 | 48.3 | 60.3 | 50.7 |
| 38.2 | 43.6 | 44.6 | 51.8 | 40.6 | 63.3 | 48.3 | 60.3 | 50.7 |
| 51.1 | 34.1 | 80.8 | 46.3 | 75.2 | 40.6 | 39.3 | 40.4 | 40.3 |
| 51.9 | 34.7 | 61.0 | 51.4 | 53.8 | 57.6 | 57.0 | 47.2 | 45.9 |
| 53.1 | 54.6 | 63.0 | 49.1 | 46.6 | 46.5 | 42.3 | 41.5 | 46.1 |
| 74.2 | 63.0 | 63.2 | 47.6 | 50.4 | 49.0 | 44.9 | 51.7 | 46.4 |
| : | | | : | | | : | : | : |
| | | | | | | | | • |

A **frequency polygon** or **hollow histogram** allows us to compare numerical and categorical variables. It uses line segments connected to points located where the top of the bar would be.

Example 18

The plot shows the wait times for both McDonald's and Dunkin' Donuts.



For small data sets (20 values or fewer), use a table instead of a graph.

Note

A graph of data should draw focus to the true nature of the data, not on other elements, such as eye-catching but distracting design features.

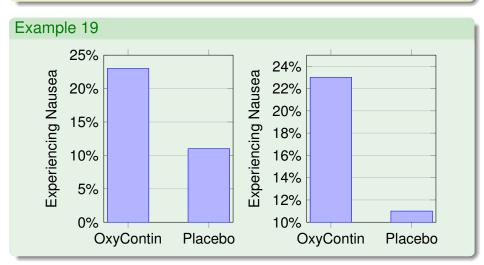
Note

Do not distort data. Construct a graph to reveal the true nature of the data.

Note

Almost all of the ink in a graph should be used for data, not for other design elements.

Always examine a graph carefully to see whether a vertical axis begins at some point other than zero so that differences are exaggerated.



When examining data depicted with a pictograph, beware when area or volume is used to depict amounts that are actually one-dimensional.

Example 20

The pictographs show data from the CDC.

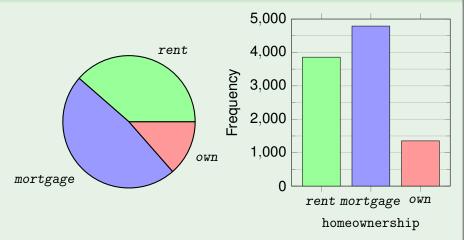


- (a) 1970: 36% of U.S. adults smoked.
- (b) 2013: 18% of U.S. adults smoked.

The larger cigarette is about twice as long as the smaller, which means it has four times the area of the smaller cigarette. While the data shows that 36% is only double 18%.

A pie chart is a worse version of the bar plot.





The pie chart makes it harder to tell the relative sizes of each group, where the bar plot makes it much easier and gives the actual frequencies.