Random Variables

Colby Community College

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$\$11,785 \div 100$ students $= \$117.85$ per student.

Definition

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The probability distribution is:

i	1	2	Total
x_i	0	1	–
$P(X = x_i)$	0.50	0.50	1.00

Example 3

If we let X be the amount a student spends in Example 1, then the probability distribution is:

i	1	2	3	Total
x_i	\$0	\$137	\$170	–
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The random variable in Example 3 is a discrete random variable.

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Hiring managers were asked to identify the biggest mistakes that job applicants make during an interview.

Consider the following table:

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	Total	1.57

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So, we see that X is not a random variable.

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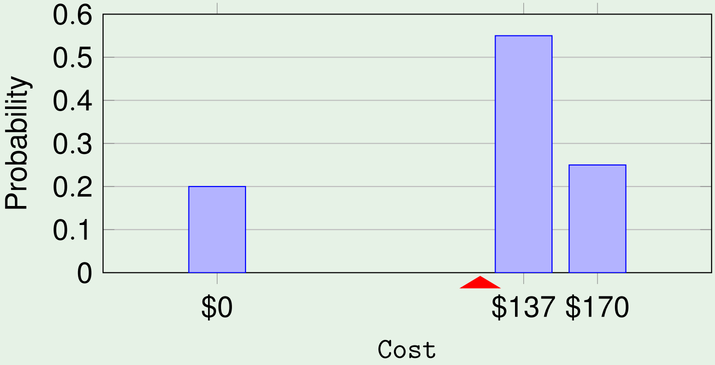
Example 6

In Example 1 the average revenue, \$117.85 per student, is the expected value for the bookstore's revenue.



Expected Value Of A Discrete Random Variable

If X takes outcomes $x_1, \ldots, x_k$ with probabilities $P(X = x_1), \ldots,$ $P(X = x_k)$, the expected value of X is:

$$\begin{aligned} E(X) &= x_1 \cdot P(X = x_1) + x_2 \cdot P(X = x_2) + \cdots + x_k \cdot P(X = x_k) \\ &= \sum_{i=1}^{k} x_i \cdot P(X = x_i) \end{aligned}$$

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Note

The Greek letter μ is sometimes used in place of $E(X)$.

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If X is the net winnings, then the probability distribution is:

i	1	2	Total
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On *average*, the player will lose 5.3 cents per bet.

Note

Pick back up here. The following slides need to be editied

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Note

It makes sense that a insurance policy would have a negative expected value, otherwise the insurance company couldn't stay in business.

The benefit for the consumer is the security that the policy provides.

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A company estimates that 0.7% of their products will fail after the original warranty period but within 2 years of the purchase, with a replacement cost of \$350. If they offer a 2 year extended warranty for \$48, what is the company's expected value?

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The company makes, on average, $45.55 on each extended warranty.

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Note

The expected value for the consumer may be different. The consumer is likely to pay the more to repair or replace the item out of warranty. (The company pays manufacturing cost, consumer has to pay retail