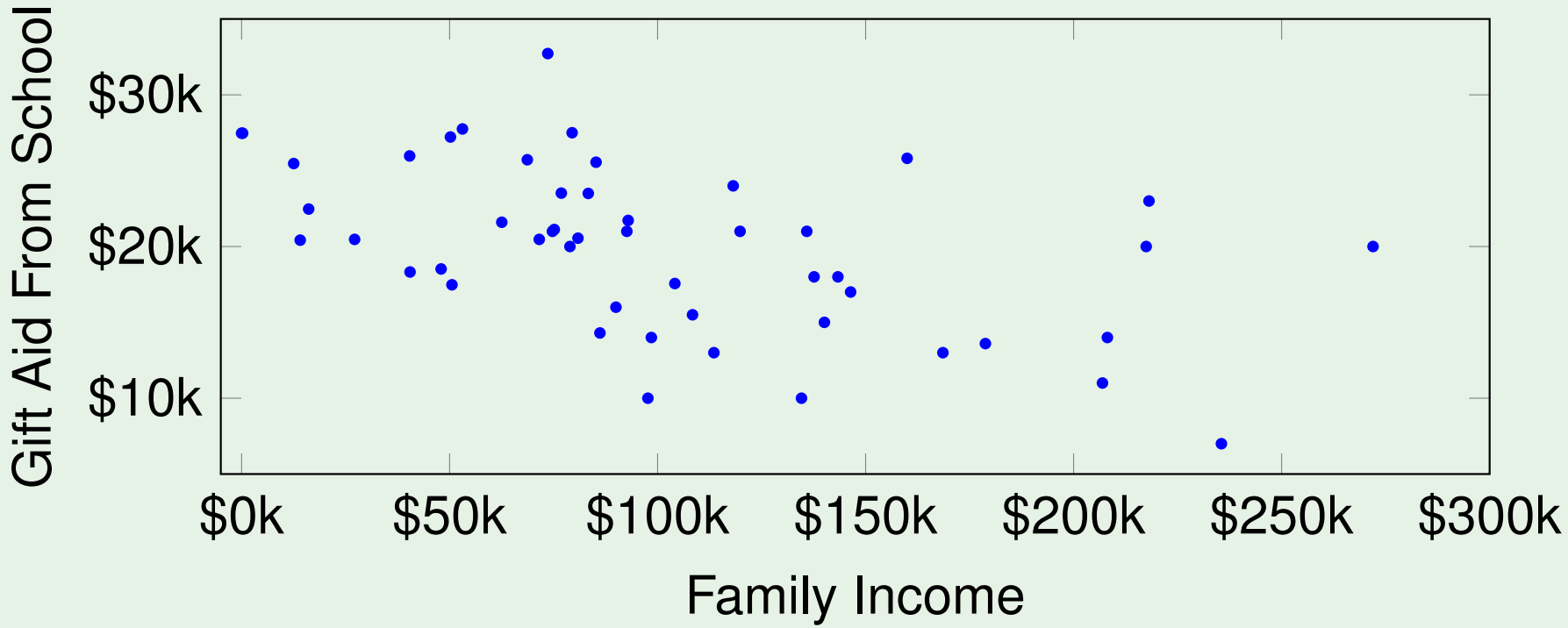
Least Squares Regression

Colby Community College

Example 1

Gift aid is financial aid that does not need to be paid back.

A sample of 50 random freshmen at Elmhurst College is shown, comparing the student's family income against gift aid received.



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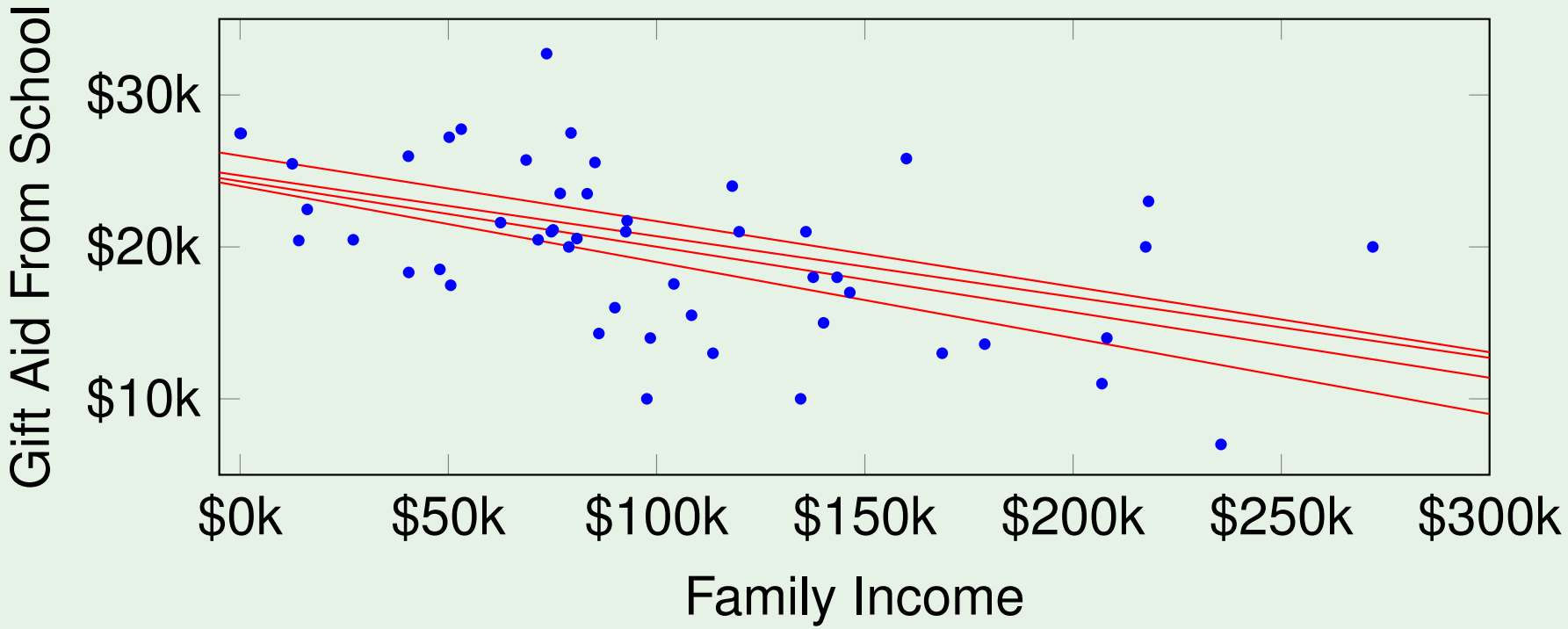

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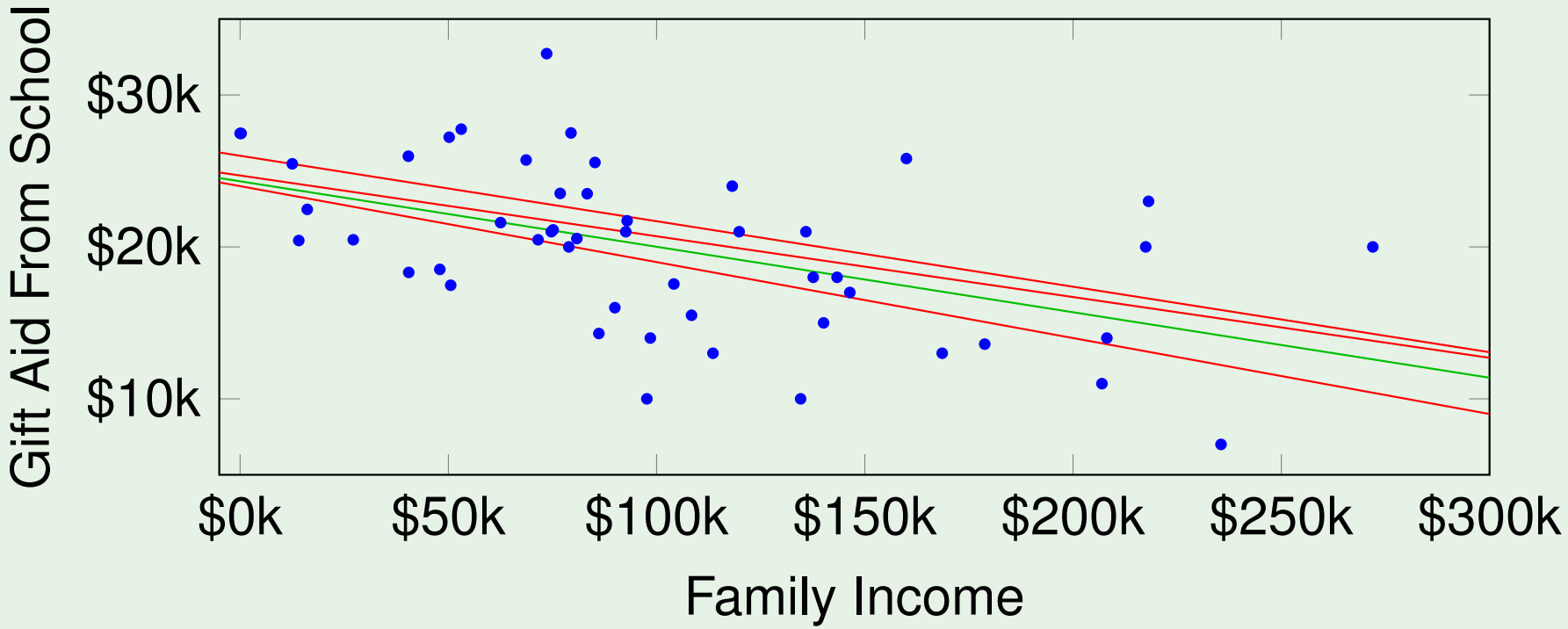

Which of the lines best fits the data?

Without an objective definition of measure of "best", the answer will vary from person to person.

What does “best” mean?

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Note

In many applications, a residual twice as large as another is more than twice as bad. Squaring the residuals helps account for this discrepancy.

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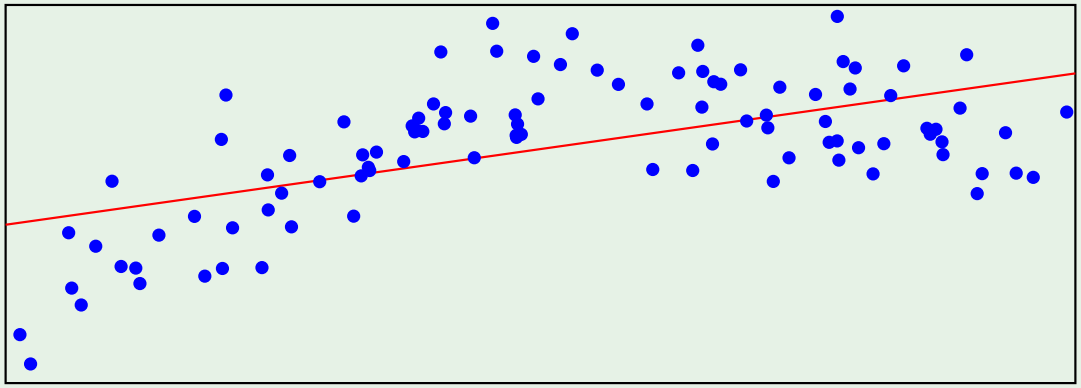
Independent Observations: Be careful about applying regression to **time series** data, which are sequential observations in time such as a stock price each day.

Example 2

Scatter plot where linearity fails:



Residual plot: