In [174…
```
import pandas as pd
import numpy as np
import regex as re
from matplotlib import pyplot as plt
from docplex.mp.model import Model
import importlib

import load_data as ld
import analytics as an
import models as md

# Reloads all local modules
def reload_all():
    for _ in [ld, an, md]:
        importlib.reload(_)
```

Load in Map Data

Small Data

In [2]:
```
specs_small = ld.get_specs('smallexample')
s_c, s_t, s_b = ld.load_data('smallexample', specs_small)
```

```
Number of beams: 5
Vertical pixel resolution: 8
Horizontal pixel resolution: 8
Maximum Dose Allowed Over Critical Area: 2
Minimum Dose Required Over Tumor Area: 10

Map Loaded: Critical
Map Loaded: Tumor
Map Loaded: Beams
```

Actual Data

In [141…
```
specs_actual = ld.get_specs('actualexample')
a_c, a_t, a_b = ld.load_data('actualexample', specs_actual)
```

```
Number of beams: 126
Vertical pixel resolution: 60
Horizontal pixel resolution: 80
Maximum Dose Allowed Over Critical Area: 2
Minimum Dose Required Over Tumor Area: 10

Map Loaded: Critical
Map Loaded: Tumor
Map Loaded: Beams
```

Model 1

min $\sum_i \sum_j \sum_k$ x(i) b(ijk) (t(jk))

st

$\sum_i$ x(i) b(ijk) <= 2 for every critical cell (jk)

$\sum_i$ x(i) b(ijk) >= 10 for every tumor cell (jk)

x(i) >= 0 for every beam (i)

In [4]:
```
if False:
    sol = md.build_model_1(specs_actual, a_c, a_t, a_b)
    an.plot_beams(sol, a_b, a_c, a_t, print_vars = True)
else:
    sol = md.build_model_1(specs_small, s_c, s_t, s_b)
    an.plot_beams(sol, s_b, s_c, s_t, print_vars = True)
```

```
Intensity variables added.
CRITICAL ERROR FOR 0 1
CRITICAL ERROR FOR 0 2
CRITICAL ERROR FOR 1 2
CRITICAL ERROR FOR 2 0
Intensity constraints added.
Object Function Constructed.
-5.200x1-4.200x2-3.600x3+1.200x4-3.100x5
Model Exported.
Model: m1
 - number of variables: 5
   - binary=0, integer=0, continuous=5
 - number of constraints: 25
   - linear=25
 - parameters: defaults
 - objective: minimize
 - problem type is: LP
ERROR: NO SOLUTION
---------------------------------------------------------------------------
AttributeError                            Traceback (most recent call last)
<ipython-input-4-6a4c505c8710> in <module>
      4 else:
      5     sol = md.build_model_1(specs_small, s_c, s_t, s_b)
----> 6     an.plot_beams(sol, s_b, s_c, s_t, print_vars = True)

~\Desktop\Homework\Radiation-Therapy-Optimization\analytics.py in plot_beams(sol, b, c, t, cmap_choice, print_vars)
     28     """Plot the path of the beams in python"""
     29
---> 30     m = calc_m(sol, b, print_vars)
     31
```

```
     32     from matplotlib.colors import colorConverter

~\Desktop\Homework\Radiation-Therapy-Optimization\analytics.py in calc_m(sol, b, print_vars)
      7
      8     sol_l = []
----> 9     for var, value in sol.iter_var_values():
     10         if str(var)[0] == 'x':
     11             if print_vars == True:

AttributeError: 'NoneType' object has no attribute 'iter_var_values'
```

Model 2

min $\sum_i \sum_j \sum_k$ x(i) b(ijk) (c(jk))

st

$\sum_i$ x(i) b(ijk) - s(jk) <= 2 for every critical cell (jk)

$\sum_i$ x(i) b(ijk) + s(jk) >= 10 for every tumor cell (jk)

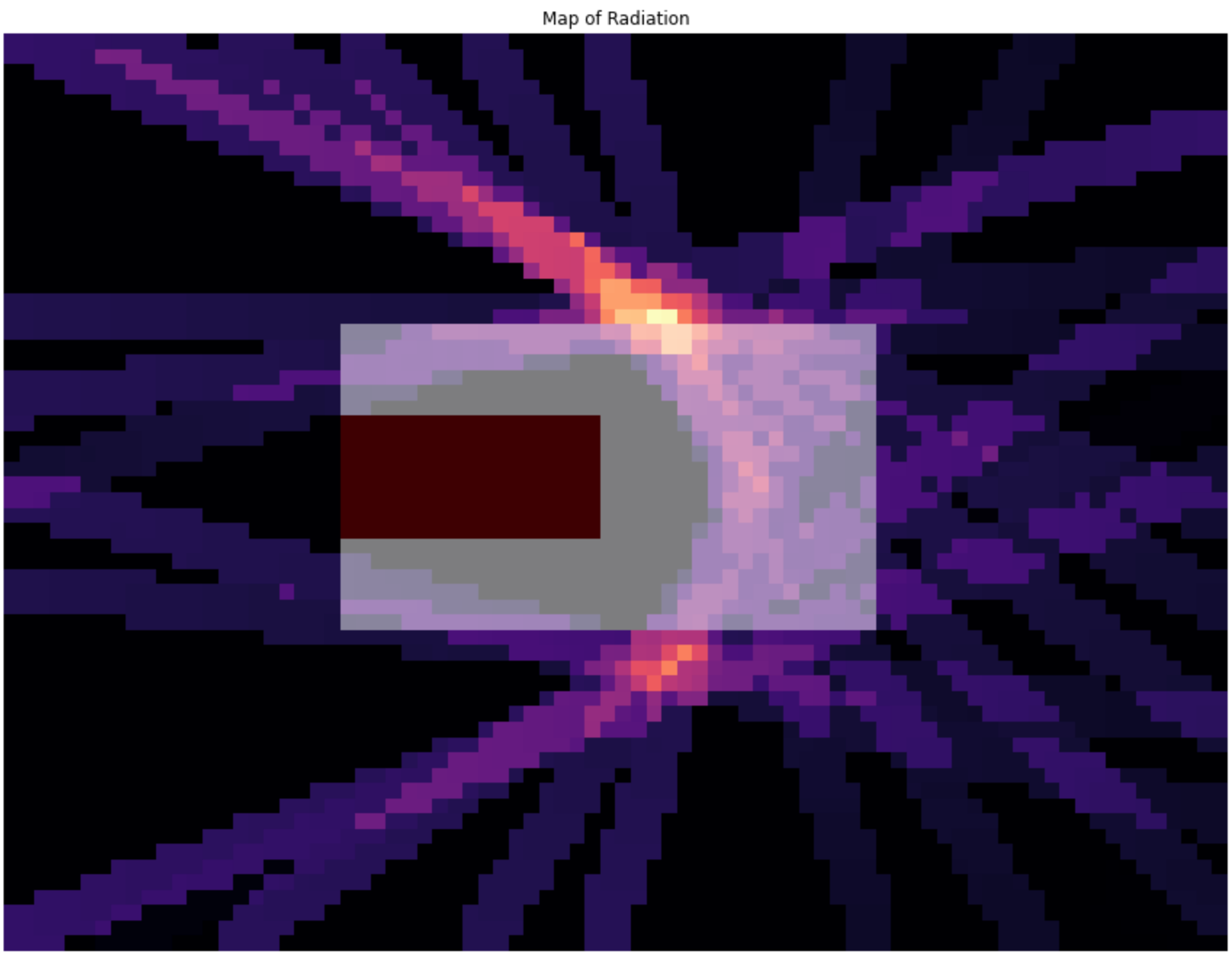
x(i) >= 0 for every beam (i)

0 <= s(jk) <= 10 for every slack/surplus variable

In [5]:
```
if True:
    sol = md.build_model_2(specs_actual, a_c, a_t, a_b)
    an.plot_beams(sol, a_b, a_c, a_t, print_vars = True)
    an.report_effectiveness(sol, a_b, a_c, a_t, plot=False)
else:
    sol = md.build_model_2(specs_small, s_c, s_t, s_b)
    an.plot_beams(sol, s_b, s_c, s_t, print_vars = True)
    an.report_effectiveness(sol, s_b, s_c, s_t, plot=False)
```

```
Intensity variables added.
Intensity constraints added.
Object Function Constructed.
Model Exported.
Model: m2
 - number of variables: 9726
   - binary=0, integer=0, continuous=9726
 - number of constraints: 20026
   - linear=20026
 - parameters: defaults
 - objective: minimize
 - problem type is: LP
Version identifier: 20.1.0.0 | 2020-11-10 | 9bedb6d68
CPXPARAM_Read_DataCheck                          1
Parallel mode: deterministic, using up to 4 threads for concurrent optimization:
```

```
 * Starting dual Simplex on 1 thread...
 * Starting Barrier on 3 threads...
Tried aggregator 1 time.
LP Presolve eliminated 20026 rows and 9726 columns.
All rows and columns eliminated.
Presolve time = 0.03 sec. (5.55 ticks)

Dual simplex solved model.

Model Solved.
x3 15.625
x5 15.625
x9 14.492753623188408
x11 1.93661971830986
x15 13.513513513513512
x17 13.513513513513512
x21 13.513513513513512
x27 12.5
x29 12.345679012345679
x33 12.82051282051282
x35 13.157894736842106
x66 0.3523608174770967
x77 15.625
x87 11.904761904761905
x93 12.82051282051282
x98 12.987012987012987
x104 11.627906976744187
MODEL REPORT

No units of radiation were delivered to any critical cells.

8625.8 units of radiation were delivered to tumor cells.
22.8 units were delivered to each cell, on average.
368 cells were found to have acceptable levels of radiation, out of 564.
65.25% of cells were found to have acceptable levels of radiation.
```



Model 2.1

min $\sum_i \sum_j \sum_k$ x(i) b(ijk) (c(jk)) + s1(jk) t(jk) + s2(jk) c(jk)

st

$\sum_i$ x(i) b(ijk) - s1(jk) <= 2 for every critical cell (jk)

$\sum_i$ x(i) b(ijk) + s2(jk) >= 10 for every tumor cell (jk)

0 <= x(i) <= 100 for every beam (i)

0 <= s(jk) <= 10 for every slack/surplus variable (jk)

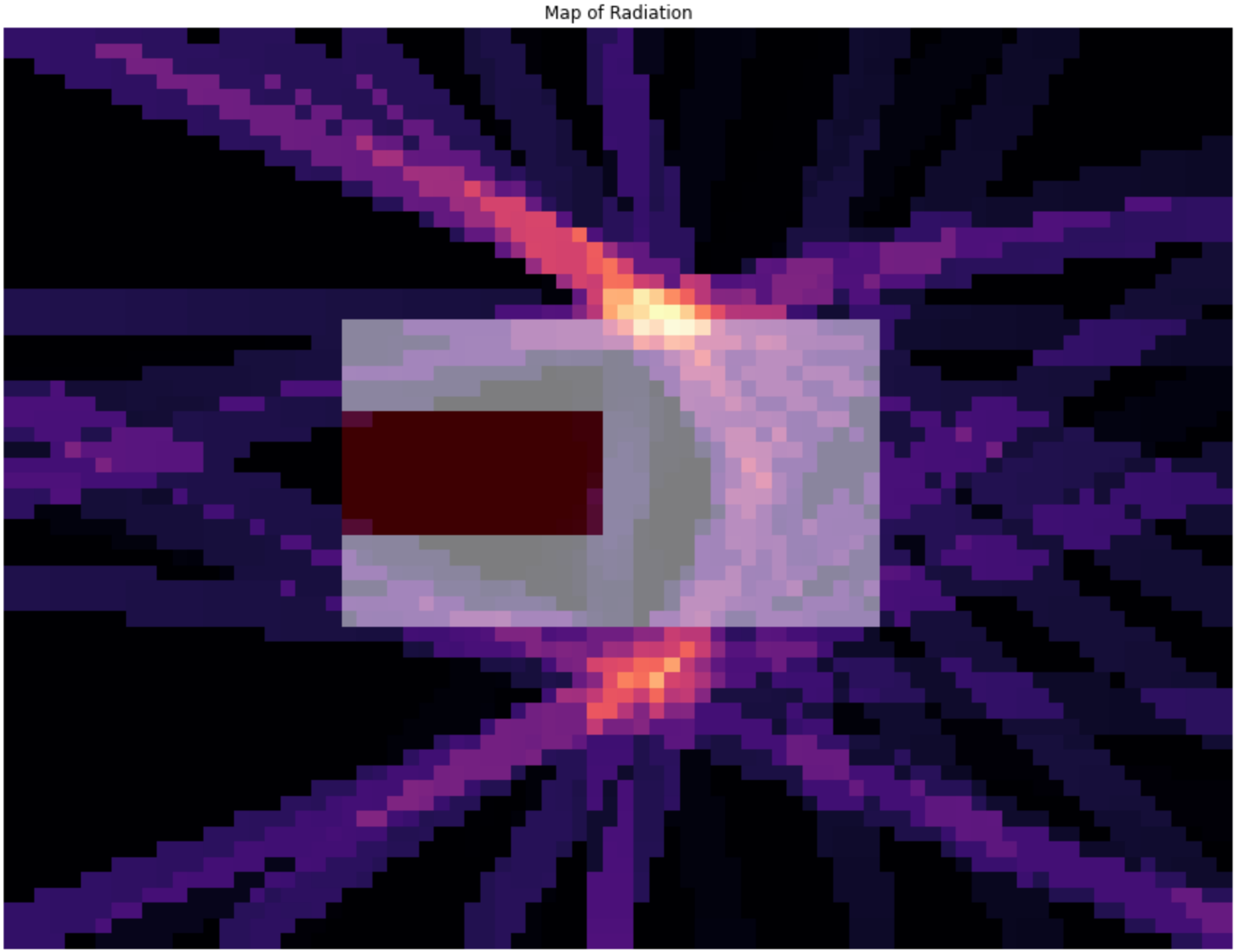
In [6]:
```
if True:
    sol = md.build_model_2_1(specs_actual, a_c, a_t, a_b)
    an.plot_beams(sol, a_b, a_c, a_t, print_vars = True)
    an.report_effectiveness(sol, a_b, a_c, a_t, plot=False)
else:
    sol = md.build_model_2_1(specs_small, s_c, s_t, s_b)
    an.plot_beams(sol, s_b, s_c, s_t, print_vars = True)
    an.report_effectiveness(sol, s_b, s_c, s_t, plot=False)
```

```
Intensity variables added.
Intensity constraints added.
Object Function Constructed.
Model Exported.
Model: m1
 - number of variables: 9726
   - binary=0, integer=0, continuous=9726
 - number of constraints: 20026
   - linear=20026
 - parameters: defaults
 - objective: minimize
 - problem type is: LP
Version identifier: 20.1.0.0 | 2020-11-10 | 9bedb6d68
CPXPARAM_Read_DataCheck                          1
Parallel mode: deterministic, using up to 4 threads for concurrent optimization:
 * Starting dual Simplex on 1 thread...
 * Starting Barrier on 3 threads...
Tried aggregator 1 time.
LP Presolve eliminated 19704 rows and 9331 columns.
Reduced LP has 322 rows, 395 columns, and 4393 nonzeros.
Presolve time = 0.06 sec. (7.12 ticks)

Iteration log . . .
Iteration:     1   Dual objective     =            10.000000
Iteration:    62   Dual objective     =           550.355840
Iteration:   124   Dual objective     =           881.711126
Iteration:   186   Dual objective     =          1078.154005
Iteration:   248   Dual objective     =          1128.821369

Dual simplex solved model.

Model Solved.
x3 15.625
x5 15.625
x9 14.492753623188408
```

x11 4.637457502263864
x15 13.513513513513512
x17 13.513513513513512
x21 13.513513513513512
x27 9.720877062887908
x29 12.345679012345679
x33 12.82051282051282
x34 7.97180298237686
x35 13.157894736842106
x37 2.6655620826429707
x40 2.388190992619371
x43 0.5918793763348669
x49 2.2433267290022836
x66 12.778087586210042
x83 14.994361348528017
x87 11.904761904761905
x92 12.195121951219512
x93 0.4748338081671406
x98 2.143557582147733
x99 12.345679012345679
x104 11.627906976744187
MODEL REPORT

93.8 units of radiation were delivered to critical cells.
5.5 units were delivered to each cell, on average.
8 cells were found to have acceptable levels of radiation, out of 564.
5.88% of cells were found to have acceptable levels of radiation.

9612.7 units of radiation were delivered to tumor cells.
18.9 units were delivered to each cell, on average.
418 cells were found to have acceptable levels of radiation, out of 564.
74.11% of cells were found to have acceptable levels of radiation.



Model 3

min $\sum_i \sum_j \sum_k$ x(i) b(ijk) (c(jk) + p(critical) c_neighbor(jk)) + s1(jk) t(jk) + s2(jk) c(jk)

st

$\sum_i$ x(i) b(ijk) - s1(jk) <= 2 for every critical cell (jk)

$\sum_i$ x(i) b(ijk) + s2(jk) >= 10 for every tumor cell (jk)

0 <= x(i) <= 100 for every beam (i)

0 <= s(jk) <= 2 for every slack variable (jk)

0 <= s(jk) <= 10 for every surplus variable (jk)

In [178…

```
reload_all()
if True:
    sol = md.build_model_3(specs_actual, a_c, a_t, a_b)
    an.plot_beams(sol, a_b, a_c, a_t, print_vars = True, cmap_choice = 'magma')
    an.report_effectiveness(sol, a_b, a_c, a_t, plot=False)
else:
    sol = build_model_3(specs_small, s_c, s_t, s_b)
    an.plot_beams(sol, s_b, s_c, s_t, print_vars = True)
    an.report_effectiveness(sol, s_b, s_c, s_t, plot=False)
```

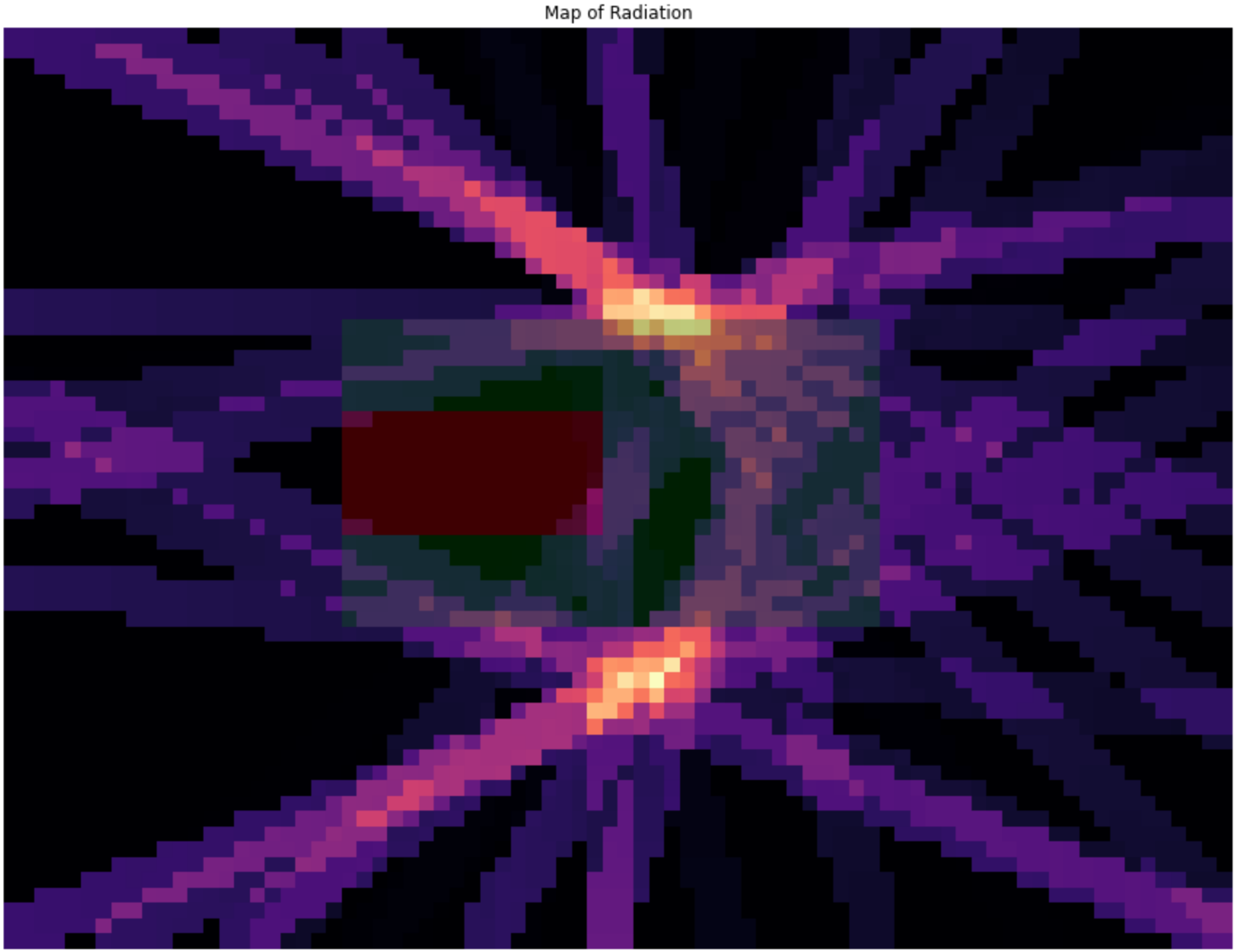
```
Intensity variables added.
Intensity constraints added.
Object Function Constructed.
Model Exported.
Model: m1
 - number of variables: 9726
   - binary=0, integer=0, continuous=9726
 - number of constraints: 20026
   - linear=20026
 - parameters: defaults
 - objective: minimize
 - problem type is: LP
Version identifier: 20.1.0.0 | 2020-11-10 | 9bedb6d68
CPXPARAM_Read_DataCheck                          1
Parallel mode: deterministic, using up to 4 threads for concurrent optimization:
 * Starting dual Simplex on 1 thread...
 * Starting Barrier on 3 threads...
Tried aggregator 1 time.
LP Presolve eliminated 19695 rows and 9321 columns.
Reduced LP has 331 rows, 405 columns, and 4494 nonzeros.
Presolve time = 0.05 sec. (7.16 ticks)

Iteration log . . .
Iteration:     1   Dual objective     =            10.000000
Iteration:    71   Dual objective     =           639.638573
Iteration:   137   Dual objective     =          1100.107912

Dual simplex solved model.

Model Solved.
x3 15.625
```

x5 15.625
x9 14.492753623188408
x10 0.39617920816496266
x11 13.23969367749179
x15 12.802493819087317
x17 13.513513513513512
x21 12.78449332049425
x23 0.6660184479435245
x27 1.0460206082224968
x28 0.2582340885105248
x29 12.345679012345679
x33 12.08434727050849
x34 11.031162575675468
x35 13.157894736842106
x37 1.4035980705089934
x40 3.254551710038818
x43 1.140669068501193
x49 11.288743764464163
x66 15.248547335600907
x76 0.3450655624568668
x83 14.794402751719826
x87 11.904761904761905
x92 11.904761904761905
x93 0.625390869293307
x98 0.2834467120181401
x99 12.195121951219512
x104 11.627906976744187

MODEL REPORT

150.0 units of radiation were delivered to critical cells.
6.3 units were delivered to each cell, on average.
119 cells were found to have acceptable levels of radiation, out of 136.
87.5% of cells were found to have acceptable levels of radiation.

9953.2 units of radiation were delivered to tumor cells.
18.9 units were delivered to each cell, on average.
485 cells were found to have acceptable levels of radiation, out of 564.
85.99% of cells were found to have acceptable levels of radiation.



Model 4: Surgical Removal

Model

min $\sum_i \sum_j \sum_k$ x(i) b(ijk) (c(jk) + α c_n(jk)) + s1(jk) (t(jk) - β tr(jk)) + s2(jk) (c(jk) + α c_n(jk))

st

$\sum_i$ x(i) b(ijk) - s1(jk) <= 2 $\forall$ (j,k) if c(j,k) = 1 $\in$ (J, K)

$\sum_i$ x(i) b(ijk) tr(jk) + s2(jk) >= 10 $\forall$ (j,k) if t(j,k) = 1 $\in$ (J, K)

0 <= x(i) <= 100 $\forall$ i $\in$ (I)

0 <= s(jk) <= 2 $\forall$ (j,k) if c(j,k) = 1 $\in$ (J, K)

0 <= s(jk) <= 10 $\forall$ (j,k) if t(j,k) = 1 $\in$ (J, K)

New Variables

α = P(Critical Region Not Identified Properly)

β = P(Tumor Regrew After Removal)

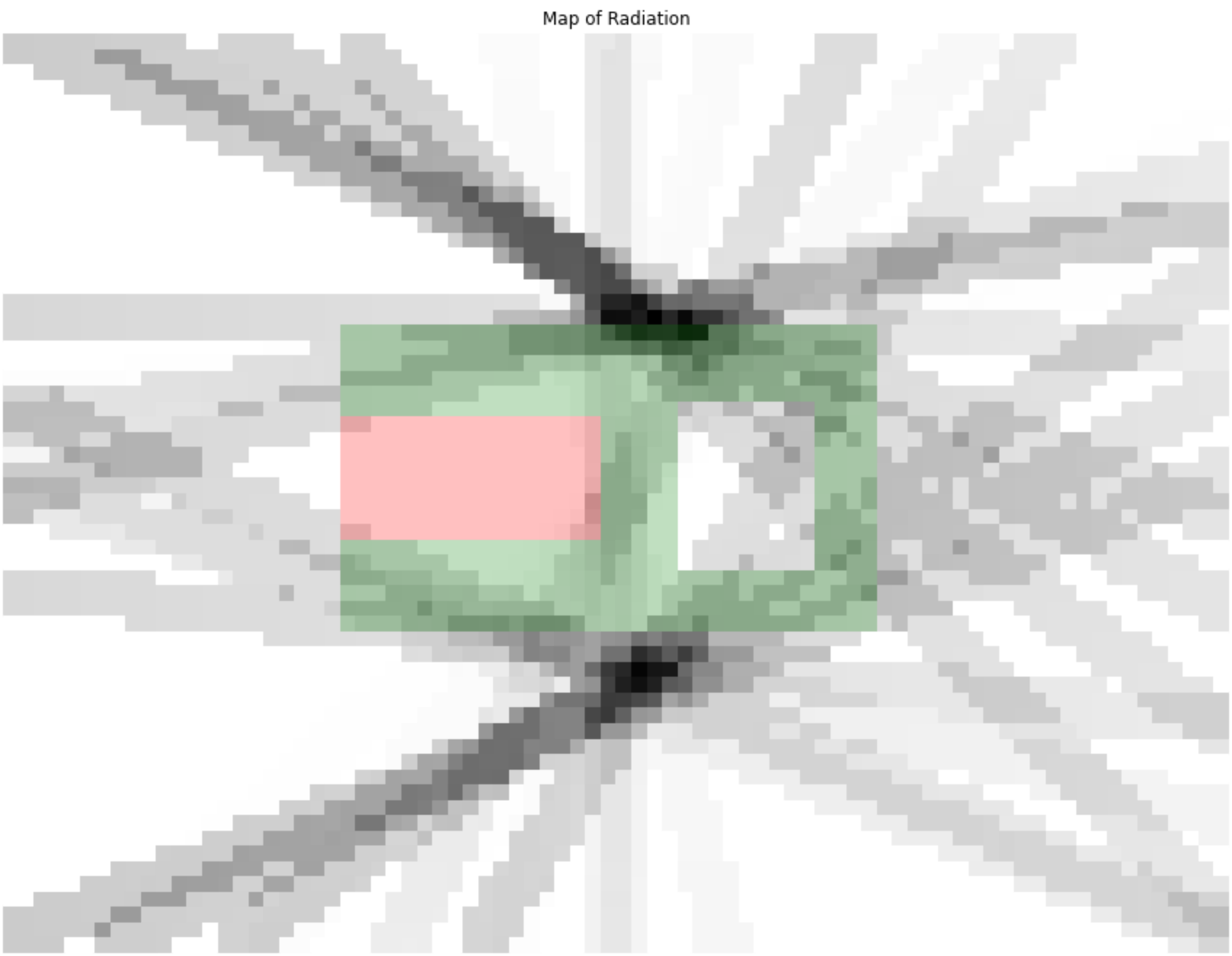
tr(jk) = Tumor Removed from Cell (jk)

In [181…

```
reload_all()
if True:
    sol, t_removed = md.build_model_4(specs_actual, a_c, a_t, a_b)
    an.plot_beams(sol, a_b, a_c, t_removed, print_vars = True, cmap_choice = 'binary')
    an.report_effectiveness(sol, a_b, a_c, t_removed, plot=False)
else:
    sol = md.build_model_4(specs_small, s_c, s_t, s_b)
    an.plot_beams(sol, s_b, s_c, t_removed, print_vars = True)
    an.report_effectiveness(sol, s_b, s_c, t_removed, plot=False)
```

```
Intensity variables added.
Intensity constraints added.
Object Function Constructed.
Model Exported.
Model: m1
 - number of variables: 9726
   - binary=0, integer=0, continuous=9726
 - number of constraints: 15127
   - linear=15127
 - parameters: defaults
 - objective: minimize
 - problem type is: LP
Version identifier: 20.1.0.0 | 2020-11-10 | 9bedb6d68
CPXPARAM_Read_DataCheck                          1
Parallel mode: deterministic, using up to 4 threads for concurrent optimization:
 * Starting dual Simplex on 1 thread...
 * Starting Barrier on 3 threads...
Tried aggregator 1 time.
LP Presolve eliminated 14809 rows and 9334 columns.
```

```
Reduced LP has 318 rows, 392 columns, and 4328 nonzeros.
Presolve time = 0.03 sec. (6.01 ticks)

Iteration log . . .
Iteration:     1   Dual objective     =            10.000000
Iteration:    85   Dual objective     =           770.005784
Iteration:   147   Dual objective     =          1064.564531

Dual simplex solved model.

Model Solved.
x3 15.625
x5 15.625
x9 14.492753623188408
x10 0.4358136782146525
x11 14.285714285714286
x15 13.20339947790928
x17 13.513513513513512
x21 13.195548489666136
x23 0.29048656499636927
x28 0.30239095321973153
x29 12.345679012345679
x31 6.58537353877178
x34 4.0032139143586365
x37 1.5237266885792256
x40 2.8169014084507045
x43 10.260094802346735
x49 2.1301916948003385
x66 6.26584079381505
x76 0.15729569498595464
x83 14.794402751719826
x87 11.904761904761905
x92 11.904761904761905
x93 0.625390869293307
x98 7.0561946238335915
x99 12.195121951219512
x104 11.627906976744187

MODEL REPORT

151.9 units of radiation were delivered to critical cells.
6.3 units were delivered to each cell, on average.
121 cells were found to have acceptable levels of radiation, out of 136.
88.97% of cells were found to have acceptable levels of radiation.

7460.9 units of radiation were delivered to tumor cells.
17.2 units were delivered to each cell, on average.
389 cells were found to have acceptable levels of radiation, out of 465.
83.66% of cells were found to have acceptable levels of radiation.
```



Model 5: Magnetic Fields

Model

min $\sum_m \sum_i \sum_j \sum_k$ x(mi) b(mijk) (c(jk) + α c_n(jk)) + s1(jk) (t(jk) - β tr(jk)) + s2(jk) (c(jk) + α c_n(jk))

st

$\sum_m \sum_i$ x(mi) b(mijk) - s1(jk) <= 2 $\forall$ (j,k) if c(j,k) = 1 $\in$ (J, K)

$\sum_m \sum_i$ x(mi) b(mijk) + s2(jk) >= 10 $\forall$ (j,k) if t(j,k)-tr(j,k) = 1 $\in$ (J, K)

0 <= x(mi) <= 100 $\forall$ (m,i) $\in$ (M, I)

0 <= s(jk) <= 2 $\forall$ (j,k) if c(j,k) = 1 $\in$ (J, K)

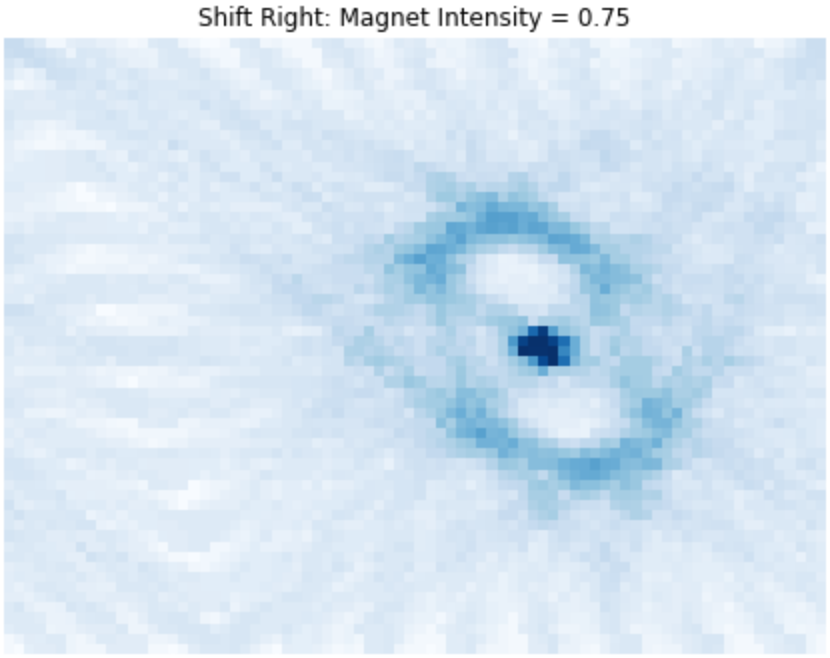
0 <= s(jk) <= 10 $\forall$ (j,k) if t(j,k) = 1 $\in$ (J, K).

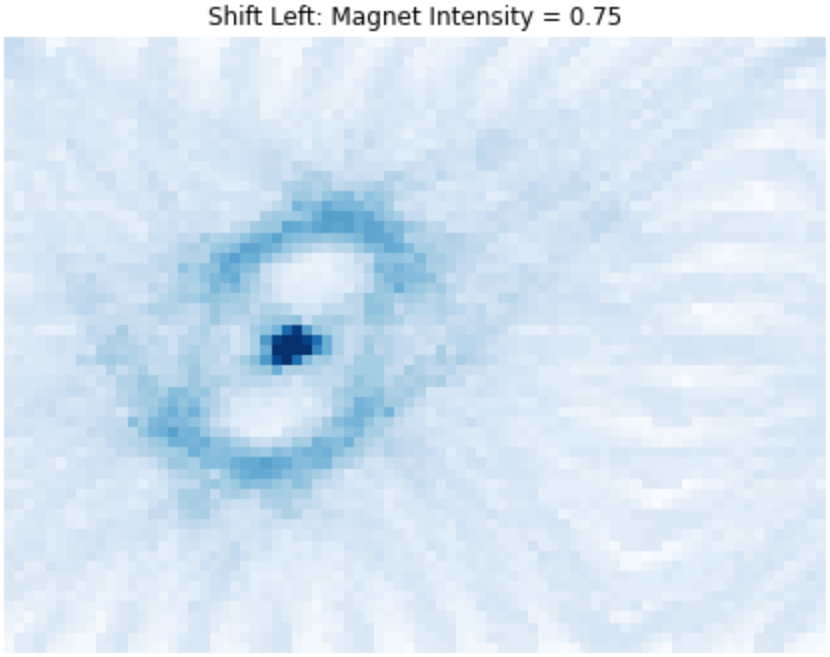
New Variables

m = Choice of magnetic field $\in$ M = {'None', 'Left', 'Right'}

In [177…

```
an.plot_magnetic_shifts(a_b)
```





In [180…

```
reload_all()
if True:
    sol, t_removed = md.build_model_5(specs_actual, a_c, a_t, a_b)
    an.plot_beams(sol, a_b, a_c, t_removed, print_vars = True, magnetic=True)
    an.report_effectiveness(sol, a_b, a_c, t_removed, plot=False, magnetic=True)
else:
    sol = md.build_model_5(specs_small, s_c, s_t, s_b)
```

```
    an.plot_beams(sol, s_b, s_c, t_removed, print_vars = True)
    an.report_effectiveness(sol, s_b, s_c, t_removed, plot=False)
```

```
Intensity variables added.
Intensity constraints added.
Object Function Constructed.
Model Exported.
Model: m1
 - number of variables: 9978
   - binary=0, integer=0, continuous=9978
 - number of constraints: 20179
   - linear=20179
 - parameters: defaults
 - objective: minimize
 - problem type is: LP
Version identifier: 20.1.0.0 | 2020-11-10 | 9bedb6d68
CPXPARAM_Read_DataCheck                          1
Parallel mode: deterministic, using up to 4 threads for concurrent optimization:
 * Starting dual Simplex on 1 thread...
 * Starting Barrier on 3 threads...
Tried aggregator 1 time.
LP Presolve eliminated 19965 rows and 9587 columns.
Reduced LP has 214 rows, 391 columns, and 7934 nonzeros.
Presolve time = 0.03 sec. (8.67 ticks)

Iteration log . . .
Iteration:     1   Dual objective     =            10.000000
Iteration:    77   Dual objective     =           480.231748

Dual simplex solved model.

Model Solved.
x3 15.625
x5 15.625
x9 14.492753623188408
x11 14.285714285714286
x15 12.990040657987992
x17 5.693896264972257
x21 12.5
x29 8.746261238224246
x66 10.105269170418492
x77 0.2648305084745779
x83 15.367685633575466
x87 6.6054180980812465
x92 0.08159934720522166
x98 4.156032624626077
x131 9.421628666646846
x137 2.4052849885479368e-05
x138 0.2421307506053251
x182 0.003228410008070698
x191 8.147936445559852
x216 0.1417434443656962
x222 0.001707752341755353
x270 13.157894736842106
```

x272 0.33738191632928505
x276 5.023635936538845
x325 7.228915662650603
x331 4.819277108433735
x335 16.949152542372882
x361 11.764705882352942
x372 10.7171999766859
x378 7.017543859649123

MODEL REPORT

27.9 units of radiation were delivered to critical cells.
4.0 units were delivered to each cell, on average.
129 cells were found to have acceptable levels of radiation, out of 136.
94.85% of cells were found to have acceptable levels of radiation.

9087.4 units of radiation were delivered to tumor cells.
20.2 units were delivered to each cell, on average.
432 cells were found to have acceptable levels of radiation, out of 465.
92.9% of cells were found to have acceptable levels of radiation.

Map of Radiation