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Math142
Homework 4
Due: Monday, Octoboer 7, 2019

A: Problems on Reviewing of Rigid Motions in R^3.

- a) Show that the set of rigid motions $E(3)$ forms a group. (Later, we will see that $E(3)$ is in fact a Lie group.)

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B: Problems from Lectures

- a) Show that of all simple closed curves in the plane with given length l, a circle bounds the largest area.

  See The isoperimetric inequality on Do Carmo page 33. ∎

C: Other Problems

- a) Problem 2 on page 29, Section 1-6, Baby Do Carmo.

  The osculating plane is the unique plane containing $\alpha(s), \alpha(s) + \alpha'(s), \alpha(s) + \alpha''(s)$. Let P_{h_1,h_2} be the plane containing $\alpha(s), \alpha(s+h_1), \alpha(s+h_2)$. It is given that $\alpha(s) \in P_{h_1,h_2}$.
  Now we show $\alpha(s) + \alpha'(s) \in P_{h_1,h_2}$. All affine combinations of those points are contained in P_{h_1,h_2} so $\alpha(s) + \alpha'(s) = \alpha(s) + \frac{1}{h_1}(\alpha(s+h_1) - \alpha(s)) \in P_{h_1,h_2}$. Now we show $\alpha(s) + \alpha''(s) \in P_{h_1,h_2}$.

  $$\begin{aligned}\alpha(s) + \alpha''(s) &= \alpha(s) + \frac{1}{h_2}(\alpha'(s+h_2) - \alpha'(s)) \\ &= \alpha(s) + \frac{1}{h_2}(\frac{\alpha(s+h_2) - \alpha(s+h_1)}{h_2 - h_1} - \alpha'(s)) \\ &\in P_{h_1,h_2}\end{aligned}$$

  (Since this is an affine combination of poitns in the plane)

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- b) Problem 1 on page 47, Section 1-7, Baby Do Carmo.

  No. That would violate the isoperimetric inequality.

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- c) Problem 2 on page 47, Section 1-7, Baby Do Carmo.

  Suppose that we have a curve E of length l from A to B that is part of a larger circle D with length g. We know from the isoperimetric inequality that this circle is the closed cuve of length g that bounds the largest possible area. If there was a curve C of length l from A to B that together with $\overline{AB}$ bounds a larger area than E with $\overline{AB}$ that would contradict the isoperimetric theorem because that would imply that replaceing E with C in the circle D would create a shape with length g that bounds more area than the circle D. ∎

- d) Problem 3 on page 65, Section 2-2, Baby Do Carmo.
- e) Problem 5 on page 65, Section 2-2, Baby Do Carmo.
- f) Problem 10 on page 66, Section 2-2, Baby Do Carmo.
- g) Problem 16 on page 67, Section 2-2, Baby Do Carmo.

D: Extra Credit Problems

- Give a different solution to B a).