Homework 2

6601 Artificial Intelligence

September 20, 2013

1 Gaussian Mixture Models

In this assignment we will perform image segmentation using Gaussian Mixture Models. For simplicity we will use gray scale images only. A gray scale image is a 2D matrix with each value representing the gray scale value. Our goal is to segment an image into different regions of same color / gray value. Therefore, we will fit a Gaussian Mixture using the EM algorithm. Our data set is the set of all pixels (gray values) in the image. As discussed in lecture, a Gaussian Mixture Model is defined by:

$$f(x) = \sum_{i=1}^{k} w_i N(x|\mu_i, \sigma_i^2) \tag{1}$$

With x being a gray scale value and $f(x)$ being the joint probability for that value. The w_i is the weight on component i, μ_i and σ_i^2 are mean and variance of the i ths gaussian. If we want the component a pixel x belongs to, we can use the maximum aposteriori probability:

$$component(x) = argmax_i w_i N(x|\mu_i, \sigma_i^2) \tag{2}$$

So for the segmentation we will replace every pixel x in the input image with the mean of the maximum component μ_{max} using the above equation.

An example for a mixture of size 3 is shown in image 1

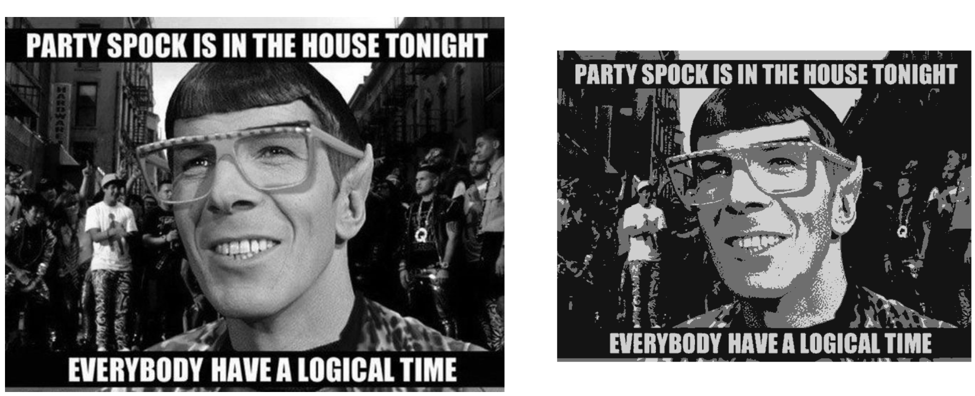

Figure 1: Input and Output for a Mixture of Size 3

For more clearity please read the attached chapter from "Pattern Recognition and Machine Learning" from Bishop named "em.pdf".

2 Implement: Gaussian Mixture Models and EM 30%

Using the attached reading implement a Gaussian Mixture Model for gray scale image segmentation. It should support computing the **Joint Probability** for a given pixel, the **Maximum Aposteriori** probability and estimating the model using **EM**. Furthermore, you have to be able to **Segment** an image by first estimating the model and then replacing all pixels in the image by the mean of the maximum aposteriori component. As an initialization for EM, pick random pixels for each component as the mean and set the variance to 1. Repeat this multiple times and keep the best result. Use the formula (9.14) in the "em.pdf" document as a score. **Segment the attached image of party spock using 3, 5 and 8 components with the best model from your random initializations**. Your results should look similar to the image above. Explain your design choices for the implementation in a pdf document.

HINT: if your implementation is numerically unstable please read the appendix.

HINT2: in the book's algorithm description mean and variance are vectors and matrices. However you can see each vector or matrix as one number in the equations since we only consider one dimensional data.

3 Experiment: Another initialization 40%

Develop a better initialization algorithm than the random one and implement it. Run an experiment in which you run the above procedure (Section 2) on the image 100 times and report the average score. Then run your procedure on the image 100 times and report the average score as well. Also create a histogram of all scores from the 100 runs for each condition for a total of two histograms.

HINT: Maybe another clustering algorithm might help :D

4 Research: Baysian Information Criterion and Mode Selection 30%

Read about the Baysian Information Criterion. One text you can read is:

"http://arxiv.org/PS_cache/astro-ph/pdf/0701/0701113v2.pdf"

The parameters are the likelihood L, the number of parameters K and the number of data points N. **a) How many parameters does a Gaussian Mixture Model have as implemented with 3 components? b) For a log likelihood of** $L = -200$ **and** $N = 50$ **and a model with 3 components what is the BIC? c) For a log likelihood of** $L = -200$ **and** $N = 50$ **and a model with 100 components what is the BIC? d) For a log likelihood of** $L = -2000$ **and** $N = 50$ **and a model with 3 components what is the BIC? e) Explain how the BIC behaves with changing k in relation**

to the likelihood.? f) Why is the BIC score useful for finding k?. Explain all your answer in full detail and show all calculations completely.

5 Late Submission: Implementation of BIC

Implement the Baysian Information Criterion. For the above image find the appropriate K by iterating from $k = 1...8$. For each k estimate the model using multiple iterations of initialization compute the BIC and choose the best model. Report the best k, attach the segmented image for that k and make a hisogram showing the BIC score for each k.

6 Hand in

All your code in source format as well as instructions on how to run it. Please use png format for all images requested above. A pdf file with all explanations and calculations.

7 Appendix 1: Log - Probabilities

You might observe that your probabilities will get really small and you get a lot of 0s, NaN or Inf values. You may want to use log probability calculations. A Gaussian in log form is:

$$logN(x|\mu, \sigma^2) = -0.5log(2\pi\sigma^2) - \frac{(x-\mu)^2}{2\sigma^2} \tag{3}$$

The other operations such as sums and products are defined in the document "addition.png" and pages 2 and 3 from the document "hmm_scaling_revised.pdf"