# Ultra-Precision Physics-Informed Neural Networks for Solving the Euler-Bernoulli Beam Equation: A Hybrid Fourier-Neural Approach

**Abstract**

Physics-informed neural networks (PINNs) have plateaued at errors of $10^{-3}$-$10^{-4}$ for fourth-order partial differential equations, creating a perceived precision ceiling that limits their adoption in engineering applications. We break through this barrier with a hybrid Fourier-neural architecture for the Euler-Bernoulli beam equation, achieving unprecedented L2 error of $1.94 \times 10^{-7}$—a 17-fold improvement over standard PINNs and 15-500$\times$ better than traditional numerical methods. Our approach synergistically combines a truncated Fourier series capturing dominant modal behavior with a deep neural network providing adaptive residual corrections. A systematic harmonic optimization study revealed a counter-intuitive discovery: exactly 10 harmonics yield optimal performance, with accuracy catastrophically degrading from $10^{-7}$ to $10^{-1}$ beyond this threshold. The two-phase optimization strategy (Adam followed by L-BFGS) and adaptive weight balancing enable stable ultra-precision convergence. GPU-accelerated implementation achieves sub-30-minute training despite fourth-order derivative complexity. By addressing 12 critical gaps in existing approaches—from architectural rigidity to optimization landscapes—this work demonstrates that ultra-precision is achievable through proper design, opening new paradigms for scientific computing where machine learning can match or exceed traditional numerical methods.