Disclaimer:
Please take the following things into account when using this exam for your preparation:

- This exam was used in an earlier version of the unit with somewhat different content, priorities, and modalities.
- In particular, this exam was designed for a duration of 3 hours as opposed to your final exam, which will only be 2 hours and 10 minutes.
- Questions that are completely irrelevant to this semester's version of the unit have been removed. However, the remaining questions might still contain alternative wording to the one introduced in the lecture.

Good luck with your preparation!

For Questions 1-6 circle only one letter for each question corresponding to the correct response.

Question 1 [1 mark]

What is printed by the following code?

```
def anon(n):
    if n %2 == 1:
        return 0
    else:
        return 1 + anon(n//2)

print(anon(36))
```

(a) 5
(b) 4
(c) 2
(d) 1

Question 2 [1 mark]

Suppose you have the following function:

```
def secret(aList):
    val = 0
    for i in range(0, len(aList)//2):
        val += aList[i]*aList[len(aList)-i-1]
    return val
```

What is printed by the following code?

```
myList = [4, 3, 2, 1, 5]
print(secret(myList))
```

(a) 7
(b) 23
(c) 25
(d) 27

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Question 3 [1 mark]

The base case for Merge sort is a list of size?

(a) 0
(b) 1
(c) Both 0 and 1
(d) None of the above.

Question 4 [1 mark]

A function g(n) is said to be O(f(n)) if there exists constants k and L such that.

(a) $g(n) > k*f(n)$ for all $n < L$
(b) $g(n) < k*f(n)$ for all $n > L$
(c) $g(n) < k*f(n)$ for all $n < L$
(d) $g(n) > k*f(n)$ for all $n > L$

Question 5 [1 mark]

How many solutions are there for the 3 Queens problem?

(a) 0
(b) 1
(c) 2
(d) None of these.

Question 6 [1 mark]

An O(nlog(n)) algorithm always runs faster than an $O(n^2)$ algorithm. True or False? Why?

(a) False. For small n, constant factors may dominate the running time.
(b) True. O(nlog(n)) complexity is better than $O(n^2)$.
(c) True, but only if the input given to both algorithms is the same.
(d) False. $O(n^2)$ complexity is better than O(nlog(n)).

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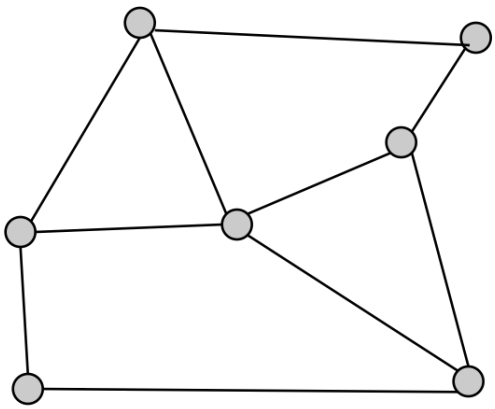
Question 7 [2 + 2 + 2 = 6 marks]

(a) [REMOVED]

(b) Give a definition of a Hamiltonian cycle in a connected graph G.

(c) How many cliques of each size less than 5 are there in the following graph?



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Question 8 [1 + 1 + 1 + 1 = 4 marks]

This question is about *time complexity*. For each of the given Python functions, state the time complexity in big O notation and provide a brief explanation.

(a)

```
def total_func(n):
    total = 0
    for k in range(n):
        for j in range(n-k, 0, -1):
            total += k*j
    return total
```

(b)

```
def fraction_func(n):
    fraction = 1
    for k in range(100):
        for j in range(k):
            fraction = k + j + 1/fraction
    return fraction
```

(c)

```
def another_total_func(n):
    total = 0
    for k in range(n):
        total += k
    for k in range(10*n):
        total += num
    return total
```

(d)

```
def mid_func(n):
    low = 0
    high = n
    while low <= high:
        mid = (low + high)//2
        low = mid + 1
    return mid
```

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Question 9 [5 marks]

Write a Python function, **unique**, which takes as input a sorted list of strings, and returns **True** if the all the items in the list are unique. Otherwise the function should return **False**.

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Question 10 [1 + 3 + 2 = 6 marks]

The number comparisons for a version of Quick sort, **C(n)**, is defined by the following relations.

$$C(0) = 1 \text{ and } C(n) = C(n//2) + n + 1,$$

(a) Give the value of **C(3)**.

(b) Write a recursive Python function which has as input a non-negative integer, **n**, and returns **C(n)**.

(c) [REMOVED]

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Question 11 [5 + 5 = 10 marks]

Consider the 4 Queen problem. Suppose we use the representation of a list of numbers where the kth number represents the row which the kth Queen, $\mathbf{Q_k}$, is in, e.g., [2, 1, 3, 0] would represent the following board.

Row 0				Q_3
Row 1		Q_1		
Row 2	Q_0			
Row 3			Q_2	

(a) Write a Python function, **lastQueenOk**, which has as input a list of numbers representing the positions of the Queens on 4x4 board. This function should return **True** if no Queen is attacking the Queen represented by the last number in the list, otherwise it should return **False**.

(b) Using the function, **lastQueenOk**, write a Python function**, isSolution**, which has as input a list of numbers representing the positions of the Queens on 4x4 board. This function should return **True** if no Queen is attacking any other Queen, otherwise it should return **False**.

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Question 12 [5 marks]

Suppose you have a collection of **n** items. All the items have the same weight, **w**, and you can choose at most **one** of each item. Write a Python function which is given as input, the capacity of the knapsack, **capacity**, a list (sorted in ascending order) of values, **values**, of each item, and the weight, **w**, and returns the maximum value that the knapsack can hold.

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Question 13 [5 marks]

Write a Python function, **duplicate,** which takes two lists sorted in ascending order) as input and returns a list of items that appear in both lists.

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Question 14 [6 marks]

Insert the following numbers, in the order they appear, into a Heap. You are allowed to choose whether the Heap is a min-Heap or a max-Heap.

10 6 11 -6 13 2

Show the Heap after each number has been inserted. The answer should consist of **6 Heaps.**

Question 15 [2 marks]

Insert the following numbers, in the order they appear, into Binary Search Tree.

10 6 11 -6 13 2

Show the Binary Search Tree after all the numbers have been inserted.

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Question 16 [2 + 6 = 8 marks]

Consider the problem of finding all the permutations of N different items.

a) Describe how a partial solution can be represented as list of numbers.

b) Show how you could use backtracking to solve this problem, when N = 3.

In particular, show a search tree and indicate on your diagram for each position the corresponding partial solution (represented as a list of numbers).

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Question 17 [4 + 3 + 3 = 10 marks]

(a) Explain the difference between the P and NP classes of problems; What would be the consequences of discovering for certain that these are different?

(b) Give **3** examples of NP problems given in lectures and state their certificates.

(c) [REMOVED]

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Question 18 [1 + 1 = 2 Marks]

(a) Give an example of a sorting method that has linear time complexity is the best case.

(b) Give an example of a sorting method that uses divide and conquer and has quadratic time complexity in the worst case.

Question 19 [5 Marks]

Write a Python function, **isVertexCover,** that checks whether a list of vertices, **vertexList,** is a vertex cover in a given graph.

The graph is represented as an adjacency matrix, **graphTable,** where **graphTable[j][k] = 1** if vertex **j** is adjacent to vertex **k** and **len(graphTable)** returns the number of vertices in the graph. The function, **isVertexCover**, takes **vertexList** and **graphTable** as input and returns **True** if the vertices in **vertexList** form a vertex cover of the graph represented by **graphTable**, and returns **False** otherwise.

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END OF EXAM