

# Updates pT Training: KNN

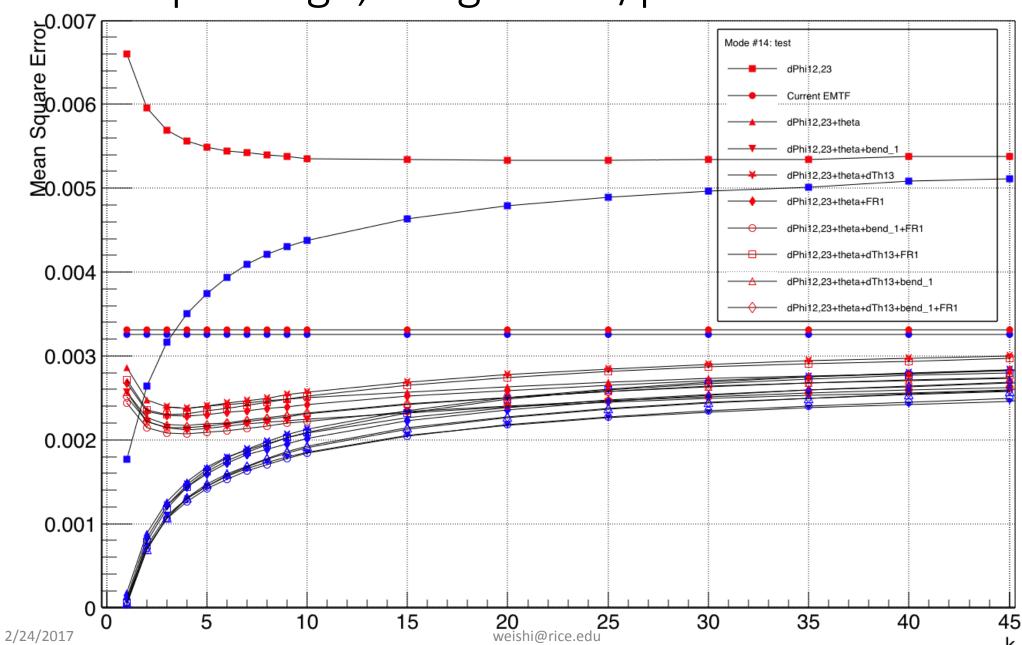
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EMTF Working Meeting

#### Mode 14: Station 1-2-3

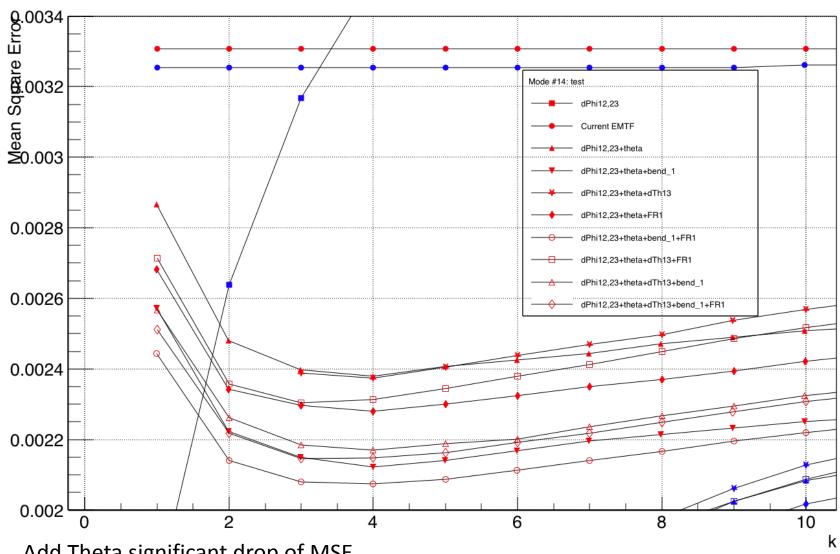
#### Compare to last time:

- Much higher statistics(10M events, mode 14 has 787858 train and 788544 test), 42 \* last time: 18746 train)
- 2016 LUT(dPhi12, 23; theta; CLCT1; dTheta13; FR 1)(mainly tune value k)
- Low/high pT divide up
- Weight 1/pT(unweight not plotted yet)
- Tune KNN scale frac
- Add more input combinations of dPhis?
- Doing classification instead of regression?

# Whole pT range, weighted 1/pT

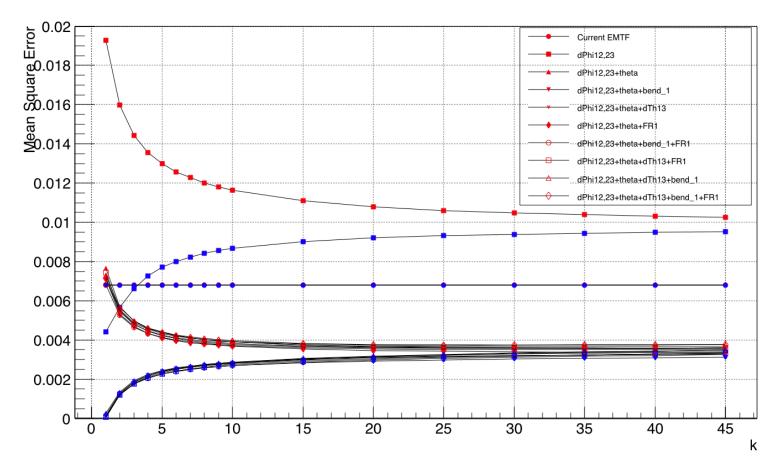


#### Zoom in



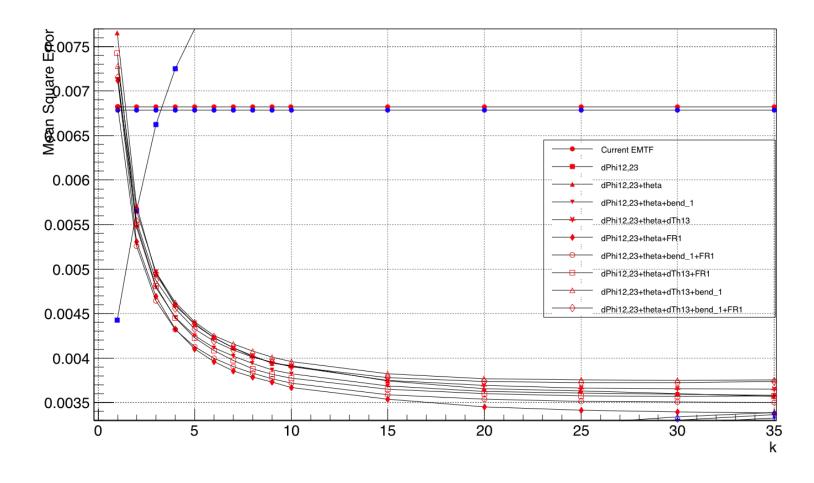
- Add Theta significant drop of MSE
- dPhis+theta+bend\_1+FR1 give best performance

### pT 1-8GeV

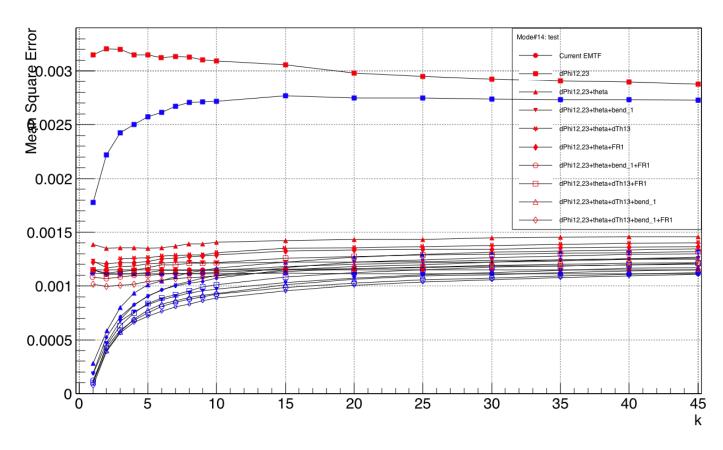


- Over fit starts when k<10, main contribution to overtrain in whole pT range plot
- Large k performance unchanged, still not enough statistics?

### 1-8GeV zoom in

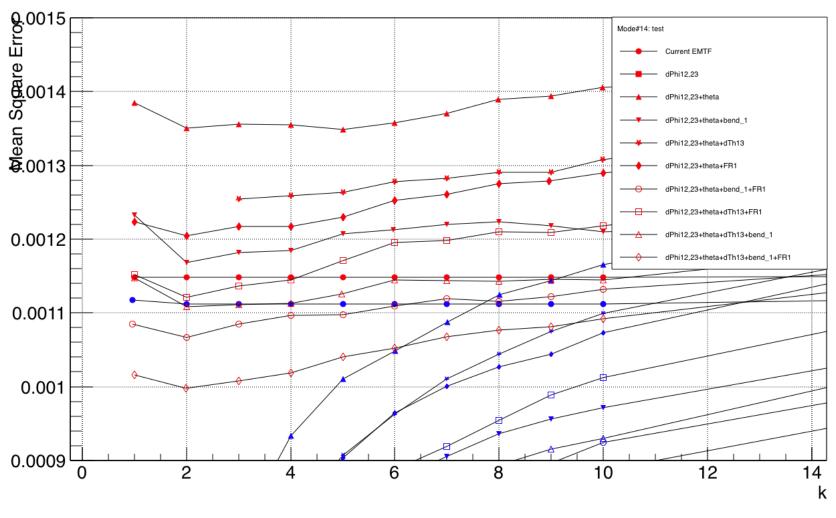


### 8-30GeV

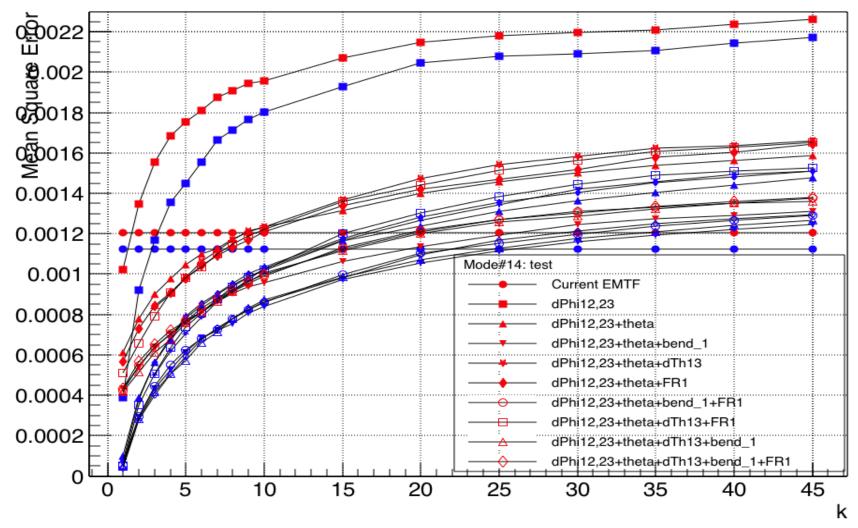


Minima at k~2

### 8-30GeV zoom in

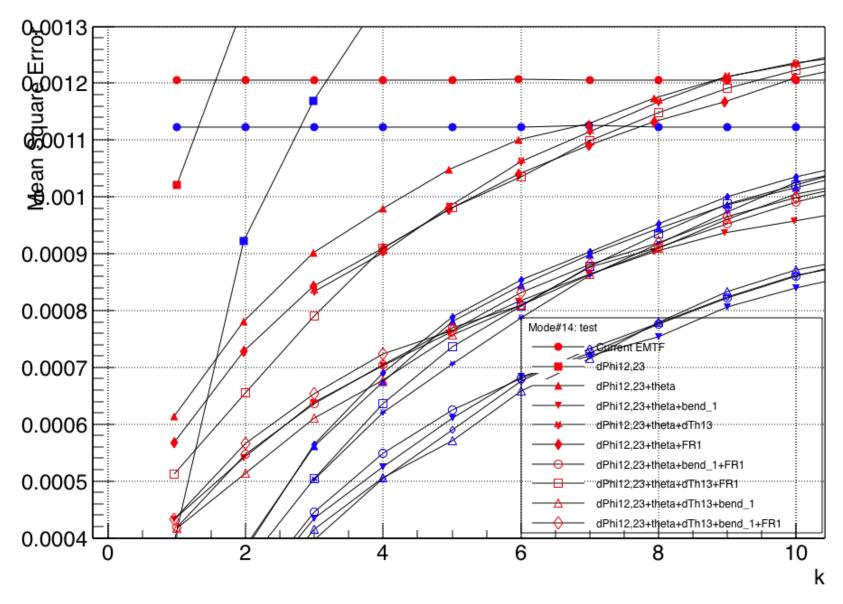


### 30-120GeV

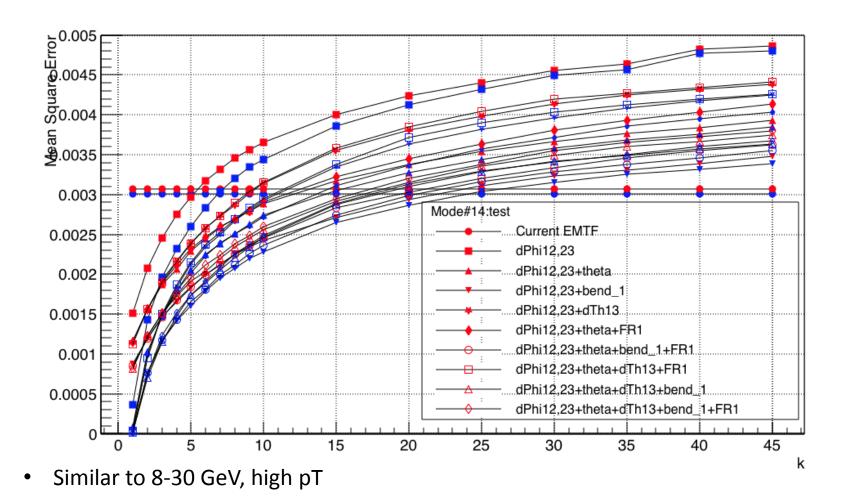


MSE keeps decreasing even when k=1, too sparse input variable space?

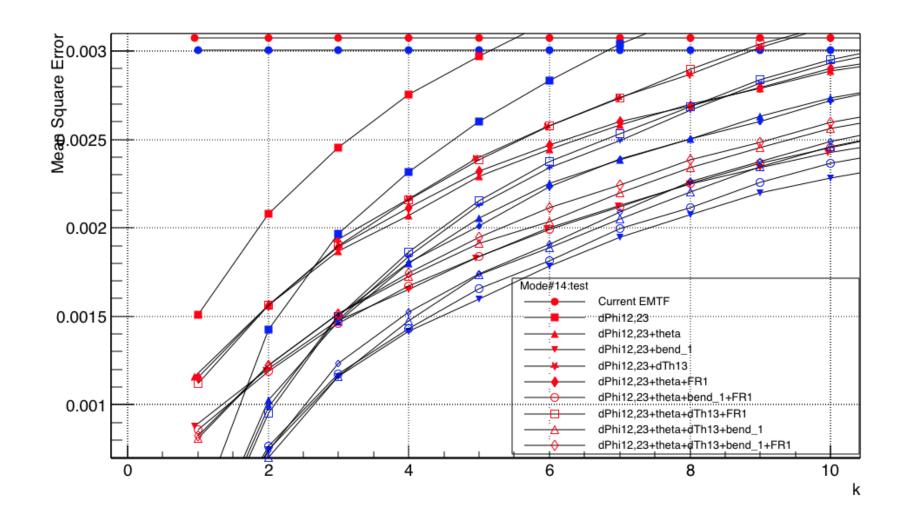
### 30-120GeV zoom in



### 120-1000GeV

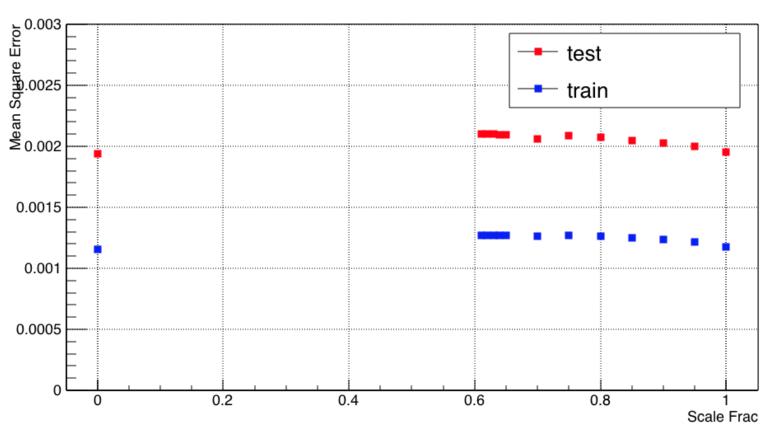


### 120-1000GeV



### Scale the variables

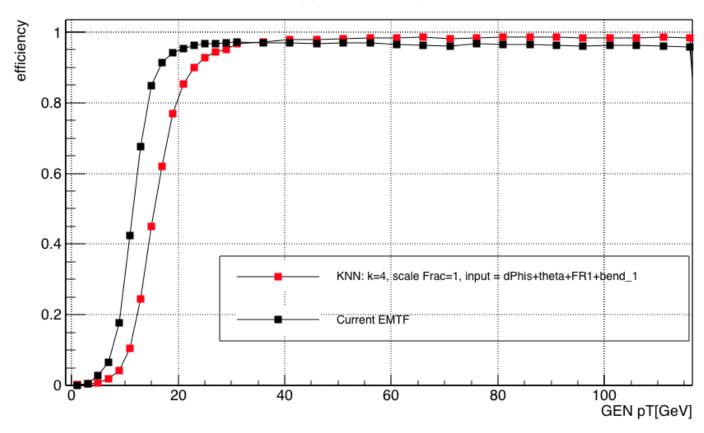
- Input variables have different distribution
- e.g. theta has larger distribution than dphis
- Need to standardize variables when calculating distance



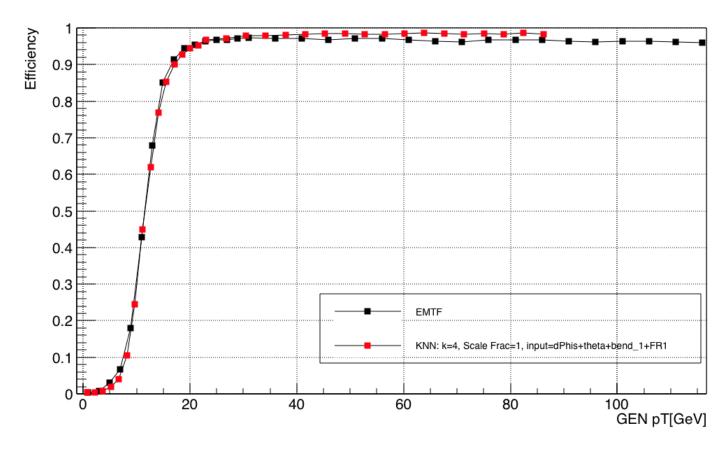
- Scale Frac = 0 means turning off scale
- Scale Frac = 1 gives best performance

# Efficiency on test sample(Not on zerobias!)



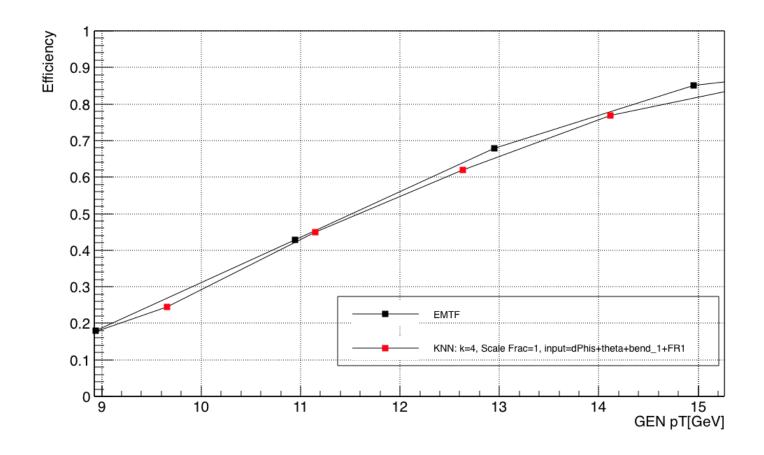


#### Cross at 50%



- Similar performance on test set
- Not accurate, hard to tell by eye; Need to evaluate using zerobias data, rate reduction plot?

### Zoom in



### Conclusions

- Significant drop when adding theta
- Almost for all sets of input variables, k=4, Scale Frac=1, gives best performace
- dPhi(12,23),theta,bend\_1,FR1 are best input variables
- Overfit at small k contributed from low pT 1-30GeV
- high momentum seems need more data(weight 1/pT)
- Flat curve at large k indicates statistics still not high enough(?)
- Non-parametric method/depend on local data structure/no mapping
- Large dataset/cross validation dataset

#### **BACK UP**

#### Steps

- Find optimized input/target/k/scalfrac for KNN for other modes
- Select the better method used for other modes: change inputs
- Include RPC hits and repeat 1-3 step
- Remove/truncate to fit in 29 bits
- Repeat for other modes
- Train charge assignment(classification)

#### Mode 14

- train events: 787858(blue)
- test events: 788544 (red)

Overall EMTF performance[marker 20]

- EMTF SD of train: 0.00325463
- EMTF SD of test: 0.00330735
- 1-8GeV:215837, 215442
- EMTF SD of train: 0.00678562
- EMTF SD of test: 0.0068219

- 8-30GeV:161893, 163132
- EMTF SD of train: 0.00111222
- EMTF SD of test: 0.00114859
- 30-120GeV:166127, 165505
- EMTF SD of train: 0.00112246
- EMTF SD of test: 0.00120542
- 120-1000GeV:244001, 244465
- EMTF SD of train: 0.00300435
- EMTF SD of test: 0.00307362

### dPhis weighted marker 21

| K(overall) | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| train      | 0.00176646 | 0.00263829 | 0.00316725 | 0.00350307 | 0.00374539 | 0.00393243 | 0.00409194 | 0.00421173 | 0.00430717 |
| test       | 0.00660051 | 0.00595772 | 0.00569287 | 0.00556525 | 0.00548916 | 0.00544611 | 0.00542184 | 0.00540143 | 0.00537927 |
| K          | 10         | 15         | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train      | 0.00437654 | 0.004639   | 0.00479074 | 0.00489073 | 0.00496563 | 0.00500657 | 0.00508776 | 0.00511004 |            |
| test       | 0.00535497 | 0.00533783 | 0.00532939 | 0.00532983 | 0.00534399 | 0.0053438  | 0.00538274 | 0.00537939 |            |

| K(1-8GeV) | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|-----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| train     | 0.0044069  | 0.00564745 | 0.00662196 | 0.00725022 | 0.00768767 | 0.00799273 | 0.00821802 | 0.00840296 | 0.00855085 |
| test      | 0.0192802  | 0.0159831  | 0.0144378  | 0.0135641  | 0.0129895  | 0.0125784  | 0.0122711  | 0.0120097  | 0.0118032  |
| K         | 10         | 15         | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train     | 0.00866189 | 0.0090073  | 0.00919383 | 0.00930826 | 0.0093862  | 0.00944135 | 0.00947819 | 0.00951042 |            |
| test      | 0.0116233  | 0.0110895  | 0.0107902  | 0.0106024  | 0.0104928  | 0.0103955  | 0.010321   | 0.010256   |            |

| K(8-30GeV)     | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|----------------|-------------|-------------|------------|------------|------------|------------|------------|------------|------------|
| train          | 0.00177577  | 0.0022171   | 0.00242412 | 0.00249948 | 0.002576   | 0.00261698 | 0.00266968 | 0.00270696 | 0.00271329 |
| test           | 0.00314753  | 0.00320737  | 0.00320031 | 0.00315078 | 0.00314676 | 0.00312423 | 0.00313284 | 0.00312907 | 0.00310248 |
| K              | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train          | 0.00271906  | 0.00276767  | 0.00274619 | 0.00274698 | 0.00273733 | 0.00273397 | 0.0027329  | 0.00272946 |            |
| test           | 0.00309081  | 0.00305881  | 0.00298001 | 0.00294883 | 0.00292446 | 0.00290772 | 0.00289705 | 0.00287683 |            |
| K(30-120GeV)   | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
| train          | 0.000384502 | 0.000920427 | 0.0011663  | 0.00135327 | 0.0014471  | 0.00155499 | 0.00166553 | 0.0017135  | 0.00176665 |
| test           | 0.00102133  | 0.00134885  | 0.00155429 | 0.0016842  | 0.00175421 | 0.00181135 | 0.00187518 | 0.00190838 | 0.00194351 |
| K              | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train          | 0.00180276  | 0.0019294   | 0.0020449  | 0.00207842 | 0.00208877 | 0.00210901 | 0.00214203 | 0.00217054 |            |
| test           | 0.00195574  | 0.00206901  | 0.00214635 | 0.00217922 | 0.0021962  | 0.00221023 | 0.00223516 | 0.00226244 |            |
| K(120—1000GeV) | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
| train          | 0.000365503 | 0.00142551  | 0.00196672 | 0.00231801 | 0.00259883 | 0.00283227 | 0.0030378  | 0.00320359 | 0.00334057 |
| test           | 0.00150747  | 0.00207809  | 0.00245131 | 0.00275476 | 0.00297092 | 0.00317072 | 0.00331427 | 0.00345886 | 0.00356337 |
| K              | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train          | 0.0034379   | 0.00386133  | 0.00412191 | 0.0043202  | 0.00449246 | 0.00456431 | 0.00477212 | 0.00479842 |            |
| test           | 0.003643    | 0.00400286  | 0.00423953 | 0.00440501 | 0.00455207 | 0.00463889 | 0.00482045 | 0.00486192 |            |

### dPhis+theta weighted marker 22

| K(overall) | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|------------|-------------|-------------|------------|------------|------------|------------|------------|------------|------------|
| train      | 0.000175655 | 0.000890127 | 0.00126378 | 0.00150348 | 0.00167575 | 0.00179383 | 0.00188252 | 0.00195635 | 0.00202567 |
| test       | 0.00286361  | 0.00248023  | 0.00239592 | 0.00237648 | 0.0023978  | 0.00241887 | 0.00244029 | 0.00246701 | 0.00248143 |
| K          | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train      | 0.00208292  | 0.00226585  | 0.00238524 | 0.00247187 | 0.00254425 | 0.00259589 | 0.0026331  | 0.00267777 | 0.00271159 |
|            |             |             |            |            |            |            | 0.0020001  |            |            |

| K(1-8GeV) | 1           | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9            |
|-----------|-------------|------------|------------|------------|------------|------------|------------|------------|--------------|
| train     | 0.000265564 | 0.00135399 | 0.00192063 | 0.00224078 | 0.00244116 | 0.00256898 | 0.00266991 | 0.00274679 | 0.00280904   |
| test      | 0.00764548  | 0.00568607 | 0.00493178 | 0.00457527 | 0.00435141 | 0.00420696 | 0.00409707 | 0.00401509 | 0.00393585   |
| K         | 10          | 15         | 20         | 25         | 30         | 35         | 40         | 45         | 50(not used) |
| train     | 0.00285759  | 0.00302045 | 0.00310326 | 0.00316005 | 0.00320347 | 0.00324047 | 0.00326432 | 0.00329008 | 0.0033141    |
| test      | 0.00388252  | 0.00371944 | 0.0036491  | 0.00360799 | 0.00358896 | 0.0035599  | 0.00355234 | 0.00354343 | 0.00354362   |

| K(8-30GeV)     | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| train          | 0.000274707 | 0.000578462 | 0.00079865  | 0.000928425 | 0.00100565  | 0.00104388  | 0.00108337  | 0.00111979  | 0.00113931  |
| test           | 0.00138541  | 0.00135015  | 0.00135582  | 0.00135485  | 0.00134893  | 0.00135819  | 0.00137005  | 0.00138784  | 0.0013938   |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00116074  | 0.00121879  | 0.00126228  | 0.0012905   | 0.0013092   | 0.00132194  | 0.00132774  | 0.00133875  | 0.00134939  |
| test           | 0.00140122  | 0.00142112  | 0.00143084  | 0.00143514  | 0.00144861  | 0.00145503  | 0.00145747  | 0.0014567   | 0.00146603  |
| K(30-120GeV)   | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 9.88163e-05 | 0.000386895 | 0.000561537 | 0.000674731 | 0.000780775 | 0.000844393 | 0.000898348 | 0.000945289 | 0.000986059 |
| test           | 0.000611453 | 0.000777158 | 0.000899182 | 0.000975824 | 0.00104728  | 0.00109806  | 0.00112913  | 0.0011714   | 0.00120756  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.0010162   | 0.00116374  | 0.00123706  | 0.00130896  | 0.00136497  | 0.00140467  | 0.00143884  | 0.0014755   | 0.00149806  |
| test           | 0.00123401  | 0.00131643  | 0.00139887  | 0.00145648  | 0.00150097  | 0.00153995  | 0.0015616   | 0.00158847  | 0.00160449  |
| K(120—1000GeV) | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 8.27185e-05 | 0.00102922  | 0.00146949  | 0.0017971   | 0.00205262  | 0.00225216  | 0.0023863   | 0.00250057  | 0.00262862  |
| test           | 0.00116058  | 0.00156208  | 0.0018685   | 0.00206872  | 0.00229036  | 0.00244506  | 0.00258204  | 0.0027      | 0.00278789  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00273579  | 0.00304343  | 0.00327691  | 0.00343871  | 0.00358348  | 0.00368201  | 0.00375393  | 0.00384313  | 0.00390867  |
| test           | 0.00288712  | 0.00316911  | 0.00337927  | 0.00354014  | 0.00366184  | 0.00376743  | 0.00383383  | 0.00392785  | 0.00399696  |

### dPhis+theta+bend\_1 weighted marker 23

| K(overall) | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|------------|-------------|-------------|------------|------------|------------|------------|------------|------------|------------|
| train      | 0.000109535 | 0.000737917 | 0.00110068 | 0.00130193 | 0.00144304 | 0.00156527 | 0.00165723 | 0.00173176 | 0.0017999  |
| test       | 0.00256406  | 0.00221768  | 0.0021426  | 0.00212016 | 0.00213592 | 0.0021635  | 0.00218869 | 0.0022121  | 0.00223142 |
| K          | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train      | 0.00185008  | 0.0020543   | 0.00217239 | 0.00226463 | 0.00232872 | 0.00238251 | 0.0024219  | 0.00246108 | 0.00248745 |
| test       | 0.00224783  | 0.00232007  | 0.00239111 | 0.00245001 | 0.00249705 | 0.0025281  | 0.00255824 | 0.00259124 | 0.0026105  |
| train      | 0.00185008  | 0.0020543   | 0.00217239 | 0.00226463 | 0.00232872 | 0.00238251 | 0.0024219  | 0.0024     |            |

| K(1-8GeV) | 1           | 2                | 3          | 4                | 5          | 6                | 7          | 8                                       | 9          |
|-----------|-------------|------------------|------------|------------------|------------|------------------|------------|---|------------|
| train     | 0.000167512 | 0.00127461       | 0.00186697 | 0.00217873       | 0.0023716  | 0.00251314       | 0.00261206 | 0.00269206                              | 0.00274934 |
| test      | 0.00713487  | 0.00546611       | 0.00479444 | 0.00445914       | 0.00425308 | 0.00411693       | 0.0040229  | 0.00393649                              | 0.00386971 |
|           | 1.0         |                  |            |                  |            | a =              | 4.0        |   |            |
| K         | 10          | 15               | 20         | 25               | 30         | 35               | 40         | 45                                      | 50         |
| train     | 0.00279886  | 15<br>0.00297606 | 0.00307552 | 25<br>0.00314302 | 0.00319729 | 35<br>0.00324315 | 0.0032849  | <ul><li>45</li><li>0.00331633</li></ul> | 0.00334734 |

| K(8-30GeV)     | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| train          | 0.000181709 | 0.000465415 | 0.000670725 | 0.000757814 | 0.000823976 | 0.000874241 | 0.000905492 | 0.00093569  | 0.000956104 |
| test           | 0.00123261  | 0.00116839  | 0.00118223  | 0.00118437  | 0.00120691  | 0.00121293  | 0.0012195   | 0.00122356  | 0.00121785  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.000971656 | 0.0010342   | 0.00107081  | 0.00110394  | 0.00112393  | 0.00114347  | 0.00116314  | 0.00117265  | 0.00117831  |
| test           | 0.00121005  | 0.00122068  | 0.00122325  | 0.00122588  | 0.00123664  | 0.00124391  | 0.00125185  | 0.00125678  | 0.00126782  |
| K(30-120GeV)   | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 6.9858e-05  | 0.000290774 | 0.000433925 | 0.000525355 | 0.000610605 | 0.000684531 | 0.000720869 | 0.000754558 | 0.000806407 |
| test           | 0.000430359 | 0.00053906  | 0.00063721  | 0.000702636 | 0.0007597   | 0.000813533 | 0.000863437 | 0.000902884 | 0.000936993 |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.000839236 | 0.000975099 | 0.00105271  | 0.00111733  | 0.00116059  | 0.00119417  | 0.00122086  | 0.00124625  | 0.00126633  |
| test           | 0.000955883 | 0.00106006  | 0.00113597  | 0.0011917   | 0.00124331  | 0.00127427  | 0.00128916  | 0.00130791  | 0.00132057  |
| K(120—1000GeV) | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 3.7378e-05  | 0.00074841  | 0.00116207  | 0.00141609  | 0.00159915  | 0.00178494  | 0.00194892  | 0.00207582  | 0.00219633  |
| test           | 0.000868915 | 0.00119154  | 0.00146561  | 0.00164299  | 0.00182176  | 0.00199025  | 0.00211618  | 0.00223842  | 0.00234032  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00228187  | 0.00265053  | 0.00286673  | 0.00303886  | 0.00315509  | 0.00325239  | 0.00331142  | 0.00338653  | 0.00342682  |
| test           | 0.00242526  | 0.00269964  | 0.00293023  | 0.00309771  | 0.0032217   | 0.00330442  | 0.00338677  | 0.00347265  | 0.00351634  |

### dPhis+theta+dTh13 weighted marker 29

| K(overall)         | 1   | 2                       | 3                        | 4                        | 5                        | 6                                     | 7                        | 8                        | 9                        |
|--------------------|---|-------------------------|--------------------------|--------------------------|--------------------------|---------------------------------------|--------------------------|--------------------------|--------------------------|
| train              | 0.000807493<br>(need to retrain<br>1 and 2) | 0.000807493             | 0.00121402               | 0.00146892               | 0.00165234               | 0.00178644                            | 0.00188632               | 0.00198001               | 0.00206136               |
| test               | 0.00245523                                  | 0.00245523              | 0.00238918               | 0.00237406               | 0.00240589               | 0.00243886                            | 0.00246978               | 0.00249729               | 0.0025365                |
| K                  | 10  | 15                      | 20                       | 25                       | 30                       | 35                                    | 40                       | 45                       | 50                       |
| train              | 0.00212802                                  | 0.00235065              | 0.00250051               | 0.00260303               | 0.00268265               | 0.00274714                            | 0.00279427               | 0.00283114               |                          |
| test               | 0.00256919                                  | 0.00268446              | 0.00277658               | 0.00284226               | 0.00289906               | 0.00294106                            | 0.00296987               | 0.0029955                |                          |
|                    |   |                         |                          |                          |                          |                                       |                          |                          |                          |
|                    |   |                         |                          |                          |                          |                                       |                          |                          |                          |
| K(1-8GeV)          | 1   | 2                       | 3                        | 4                        | 5                        | 6                                     | 7                        | 8                        | 9                        |
| K(1-8GeV)<br>train | 1<br>0.00123881                             | <b>2</b> 0.00123881     | 3<br>0.00185262          | <b>4</b> 0.00219191      | <b>5</b> 0.00239933      | <ul><li>6</li><li>0.0025453</li></ul> | <b>7</b> 0.00264847      | 8<br>0.00272329          | 9<br>0.00278711          |
|                    |   |                         |                          |                          |                          |                                       |                          |                          |                          |
| train              | 0.00123881                                  | 0.00123881              | 0.00185262               | 0.00219191               | 0.00239933               | 0.0025453                             | 0.00264847               | 0.00272329               | 0.00278711               |
| train<br>test      | 0.00123881<br>0.0056878                     | 0.00123881<br>0.0056878 | 0.00185262<br>0.00496438 | 0.00219191<br>0.00459083 | 0.00239933<br>0.00438942 | 0.0025453<br>0.00422488               | 0.00264847<br>0.00411306 | 0.00272329<br>0.00401583 | 0.00278711<br>0.00395112 |

| K(8-30GeV)     | 1           | 2           | 3           | 4           | 5           | 6           | 7          | 8          | 9           |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|------------|-------------|
| train          | 0.000476732 | 0.000476732 | 0.000689687 | 0.000820104 | 0.000906986 | 0.00096223  | 0.00101073 | 0.00104404 | 0.00107479  |
| test           | 0.00126447  | 0.00126447  | 0.00125459  | 0.00125901  | 0.0012631   | 0.00127777  | 0.00128287 | 0.00129039 | 0.00129047  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40         | 45         | 50          |
| train          | 0.00109933  | 0.00116946  | 0.00122107  | 0.00124474  | 0.00126159  | 0.00127877  | 0.00129169 | 0.0012981  |             |
| test           | 0.00130741  | 0.00134202  | 0.00135489  | 0.00136712  | 0.00137825  | 0.0013845   | 0.00139599 | 0.00139962 |             |
| K(30-120GeV)   | 1           | 2           | 3           | 4           | 5           | 6           | 7          | 8          | 9           |
| train          | 0.000344692 | 0.000344692 | 0.00050223  | 0.000620992 | 0.000706517 | 0.000787264 | 0.00086244 | 0.00091821 | 0.000973259 |
| test           | 0.000694452 | 0.000694452 | 0.000833819 | 0.000900999 | 0.000980488 | 0.00106187  | 0.00111421 | 0.00116799 | 0.00121159  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40         | 45         | 50          |
| train          | 0.00102495  | 0.00116651  | 0.00126516  | 0.00134223  | 0.00141905  | 0.00145736  | 0.00149231 | 0.00150962 |             |
| test           | 0.00123439  | 0.00136728  | 0.00147235  | 0.00154346  | 0.00158396  | 0.00161882  | 0.00163619 | 0.00165925 |             |
| K(120—1000GeV) | 1           | 2           | 3           | 4           | 5           | 6           | 7          | 8          | 9           |
| train          | 0.000960514 | 0.000960514 | 0.00148164  | 0.00183716  | 0.00213007  | 0.00234232  | 0.00249019 | 0.00266647 | 0.0028148   |
| test           | 0.00159308  | 0.00159308  | 0.00192982  | 0.00216182  | 0.00238544  | 0.0025719   | 0.00273135 | 0.00286434 | 0.00301828  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40         | 45         | 50          |
| train          | 0.00292873  | 0.00334559  | 0.00362993  | 0.0038151   | 0.00395951  | 0.00408712  | 0.00417198 | 0.0042436  |             |
| test           | 0.0031383   | 0.00353309  | 0.00380153  | 0.00398177  | 0.00413638  | 0.0042494   | 0.0043202  | 0.00437987 |             |

### dPhis+theta+FR1 weighted marker 33

| K(overall) | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|------------|-------------|-------------|------------|------------|------------|------------|------------|------------|------------|
| train      | 0.000114752 | 0.000828712 | 0.00121606 | 0.00144119 | 0.00159645 | 0.00172623 | 0.00182058 | 0.0018916  | 0.00195435 |
| test       | 0.00267553  | 0.00233646  | 0.00229647 | 0.00228071 | 0.0023005  | 0.00232443 | 0.00234923 | 0.0023708  | 0.00239495 |
| K          | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train      | 0.00201706  | 0.00222383  | 0.00235668 | 0.00245513 | 0.00252822 | 0.00259694 | 0.00264285 | 0.00268663 |            |
| test       | 0.00242188  | 0.00251816  | 0.00258621 | 0.00264568 | 0.00270345 | 0.00275318 | 0.00278864 | 0.0028299  |            |

| K(1-8GeV) | 1           | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|-----------|-------------|------------|------------|------------|------------|------------|------------|------------|------------|
| train     | 0.000151065 | 0.00122223 | 0.0017671  | 0.00207465 | 0.00227112 | 0.00239897 | 0.00249671 | 0.00257114 | 0.00263746 |
| test      | 0.00711913  | 0.0053067  | 0.00469169 | 0.00432974 | 0.00410323 | 0.00396303 | 0.00385498 | 0.00378999 | 0.00373029 |
| K         | 10          | 15         | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train     | 0.00268788  | 0.00283262 | 0.00292305 | 0.00298555 | 0.00303449 | 0.00307278 | 0.00310265 | 0.00313072 |            |
| test      | 0.00366817  | 0.00352962 | 0.00344903 | 0.00341704 | 0.00339825 | 0.0033819  | 0.00337289 | 0.00336965 |            |

| K(8-30GeV)     | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9          |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|
| train          | 0.000186721 | 0.000523551 | 0.000713915 | 0.000824    | 0.000903153 | 0.000964097 | 0.00100018  | 0.00102693  | 0.00104442 |
| test           | 0.00122351  | 0.00120477  | 0.0012168   | 0.00121743  | 0.00122949  | 0.00125266  | 0.00126055  | 0.00127542  | 0.00127935 |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50         |
| train          | 0.00107291  | 0.00114984  | 0.00117878  | 0.00119571  | 0.00122142  | 0.00124056  | 0.00125395  | 0.00126935  |            |
| test           | 0.00128945  | 0.0013243   | 0.00133621  | 0.00133886  | 0.00133881  | 0.00135334  | 0.00135894  | 0.00136556  |            |
| K(30-120GeV)   | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9          |
| train          | 7.31432e-05 | 0.000381833 | 0.000564364 | 0.000689982 | 0.000788815 | 0.000854691 | 0.000903253 | 0.000951809 | 0.00100057 |
| test           | 0.000566963 | 0.000726971 | 0.000841777 | 0.000909826 | 0.000977666 | 0.00103857  | 0.0010901   | 0.00113058  | 0.00116609 |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50         |
| train          | 0.00103477  | 0.001176    | 0.00128142  | 0.00135348  | 0.00140537  | 0.00145085  | 0.00148118  | 0.00151017  |            |
| test           | 0.00120808  | 0.0013375   | 0.0014194   | 0.00146643  | 0.0015165   | 0.00157604  | 0.00159643  | 0.00164282  |            |
| K(120—1000GeV) | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9          |
| train          | 6.32095e-05 | 0.000987343 | 0.00150549  | 0.00180181  | 0.00200952  | 0.00223022  | 0.00239138  | 0.00250407  | 0.00260321 |
| test           | 0.00115594  | 0.00156368  | 0.0018909   | 0.00211256  | 0.00232204  | 0.0024661   | 0.00260116  | 0.0026907   | 0.00279454 |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50         |
| train          | 0.00271888  | 0.00311131  | 0.00336931  | 0.00357162  | 0.00371192  | 0.00385628  | 0.00394858  | 0.00403513  |            |
| test           | 0.00290099  | 0.00322275  | 0.00344988  | 0.00363631  | 0.00380535  | 0.00393014  | 0.00403495  | 0.00413504  |            |

#### dPhis+theta+bend\_1+FR1 weighted marker 24

| K(overall) | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|------------|-------------|-------------|------------|------------|------------|------------|------------|------------|------------|
| train      | 7.26377e-05 | 0.000714256 | 0.00105996 | 0.0012622  | 0.00141964 | 0.00152781 | 0.00162871 | 0.00170525 | 0.00178074 |
| test       | 0.00244133  | 0.00214049  | 0.00208023 | 0.00207417 | 0.00208704 | 0.00210859 | 0.00213982 | 0.00216688 | 0.00219594 |
| K          | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train      | 0.00184139  | 0.00204725  | 0.00218363 | 0.00227619 | 0.00234292 | 0.00239863 | 0.00244538 | 0.00249066 |            |
| test       | 0.0022198   | 0.00233547  | 0.00240123 | 0.00246397 | 0.00251345 | 0.00255587 | 0.00259289 | 0.00262107 |            |

| K(1-8GeV) | 1           | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|-----------|-------------|------------|------------|------------|------------|------------|------------|------------|------------|
| train     | 9.89024e-05 | 0.0011967  | 0.00175397 | 0.00205353 | 0.00226495 | 0.00239749 | 0.00249538 | 0.00257665 | 0.00264402 |
| test      | 0.00682156  | 0.00525553 | 0.00464606 | 0.00432138 | 0.00412924 | 0.00399678 | 0.00390588 | 0.00383243 | 0.00377092 |
| K         | 10          | 15         | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train     | 0.00269339  | 0.0028631  | 0.00297502 | 0.00305225 | 0.00311498 | 0.00316375 | 0.00320634 | 0.00324438 |            |
| test      | 0.00372051  | 0.00358967 | 0.00353623 | 0.00351691 | 0.00350526 | 0.00350046 | 0.00350399 | 0.00351265 |            |

| K(8-30GeV)     | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| train          | 0.00012228  | 0.000423489 | 0.000589406 | 0.000684434 | 0.000750589 | 0.000808654 | 0.000844941 | 0.000879626 | 0.000896621 |
| test           | 0.00108145  | 0.00106705  | 0.0010852   | 0.0010962   | 0.00109715  | 0.00110933  | 0.00111964  | 0.00111601  | 0.00112181  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.000924148 | 0.000986997 | 0.00102776  | 0.00105636  | 0.00107542  | 0.00109391  | 0.00110815  | 0.00112305  |             |
| test           | 0.00113202  | 0.00115499  | 0.00116221  | 0.00117496  | 0.00118995  | 0.0012013   | 0.00121288  | 0.00121466  |             |
| K(30-120GeV)   | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 5.39312e-05 | 0.000295461 | 0.000444937 | 0.000549002 | 0.000624426 | 0.000680979 | 0.000724932 | 0.000776434 | 0.00082169  |
| test           | 0.000418756 | 0.000546509 | 0.000636465 | 0.000704931 | 0.000769623 | 0.000810051 | 0.000877028 | 0.000911362 | 0.000953129 |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.000860814 | 0.000998672 | 0.00110141  | 0.00115186  | 0.00119644  | 0.00123791  | 0.00126614  | 0.00129189  |             |
| test           | 0.000991214 | 0.00112993  | 0.00121652  | 0.00127156  | 0.00130742  | 0.00133249  | 0.0013527   | 0.00137568  |             |
| K(120—1000GeV) | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 2.92036e-05 | 0.000765554 | 0.001177    | 0.00143113  | 0.00165724  | 0.00181225  | 0.00199742  | 0.00211462  | 0.00225667  |
| test           | 0.000857874 | 0.00119072  | 0.00146044  | 0.00167336  | 0.00183974  | 0.00199049  | 0.00211914  | 0.0022503   | 0.0023661   |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00236395  | 0.00274297  | 0.00298731  | 0.00316456  | 0.00328154  | 0.00337777  | 0.00346239  | 0.0035475   |             |
| test           | 0.0024549   | 0.00283406  | 0.00302983  | 0.00320348  | 0.00333906  | 0.00345558  | 0.00355047  | 0.003617    |             |

### dPhis+theta+dTh13+FR1 weighted marker 25

| K(overall) | 1          | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|------------|------------|-------------|------------|------------|------------|------------|------------|------------|------------|
| train      | 5.772e-05  | 0.000785819 | 0.0011806  | 0.00143627 | 0.00161803 | 0.00175161 | 0.00184983 | 0.00194044 | 0.00202459 |
| test       | 0.00271318 | 0.00235695  | 0.00230438 | 0.00231266 | 0.0023437  | 0.00237891 | 0.00241196 | 0.00244983 | 0.00248575 |
| K          | 10         | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
|            |            |             |            |            |            |            |            |            |            |
| train      | 0.00208766 | 0.00232251  | 0.00248694 | 0.00259126 | 0.00266563 | 0.0027229  | 0.00276526 | 0.00280128 |            |

| K(1-8GeV) | 1           | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|-----------|-------------|------------|------------|------------|------------|------------|------------|------------|------------|
| train     | 5.91528e-05 | 0.00118764 | 0.00175296 | 0.0020824  | 0.0022815  | 0.00242267 | 0.00252659 | 0.00261093 | 0.00268184 |
| test      | 0.00739461  | 0.00550147 | 0.00481123 | 0.00445075 | 0.00422974 | 0.00408316 | 0.00397268 | 0.00388223 | 0.00381948 |
| K         | 10          | 15         | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train     | 0.00274029  | 0.00292891 | 0.00303923 | 0.00312192 | 0.00318099 | 0.00323733 | 0.00328728 | 0.00332963 |            |
| test      | 0.00377783  | 0.00365261 | 0.00359824 | 0.00357708 | 0.00357073 | 0.00357419 | 0.00358071 | 0.00359228 |            |

| K(8-30GeV)     | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| train          | 0.000101818 | 0.000445202 | 0.000631486 | 0.000749661 | 0.000834376 | 0.000887715 | 0.000919217 | 0.000954613 | 0.000988911 |
| test           | 0.00115196  | 0.00112137  | 0.0011364   | 0.00114436  | 0.00117119  | 0.0011957   | 0.00119816  | 0.00120992  | 0.00120933  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00101217  | 0.00108523  | 0.00112383  | 0.0011479   | 0.00117578  | 0.00118862  | 0.00119587  | 0.00121167  |             |
| test           | 0.00121839  | 0.00125768  | 0.00127513  | 0.0012879   | 0.00129534  | 0.00130184  | 0.00130926  | 0.00131757  |             |
| K(30-120GeV)   | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 5.14934e-05 | 0.000353198 | 0.00050479  | 0.000636153 | 0.000736396 | 0.000808444 | 0.000877416 | 0.000935246 | 0.000987005 |
| test           | 0.000511071 | 0.000655257 | 0.000791137 | 0.000909201 | 0.000980595 | 0.00103547  | 0.00109868  | 0.00114831  | 0.00119041  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00102022  | 0.00119878  | 0.0013015   | 0.00138251  | 0.00144491  | 0.00149031  | 0.00151153  | 0.00152453  |             |
| test           | 0.00122238  | 0.0013587   | 0.00143904  | 0.00151463  | 0.00156223  | 0.00160678  | 0.00162934  | 0.00165057  |             |
| K(120—1000GeV) | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 3.14333e-05 | 0.000950921 | 0.00149876  | 0.00186505  | 0.00215136  | 0.00237337  | 0.00253072  | 0.00268582  | 0.00283682  |
| test           | 0.0011202   | 0.00156232  | 0.00189902  | 0.00215817  | 0.00238681  | 0.00257606  | 0.00273559  | 0.002896    | 0.00303907  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00295069  | 0.00337213  | 0.0037099   | 0.00390248  | 0.0040294   | 0.00412504  | 0.00419835  | 0.00425788  |             |
| test           | 0.00315018  | 0.00358043  | 0.00384916  | 0.00403912  | 0.00418994  | 0.00427249  | 0.00434059  | 0.00441288  |             |

### dPhis+theta+dTh13+ bend\_1 weighted marker 26

| K(overall) | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|------------|-------------|-------------|------------|------------|------------|------------|------------|------------|------------|
| train      | 5.19696e-05 | 0.000697575 | 0.00107443 | 0.00130661 | 0.00147693 | 0.00159257 | 0.00168939 | 0.0017805  | 0.00185771 |
| test       | 0.00256592  | 0.00226129  | 0.00218487 | 0.00217071 | 0.00218898 | 0.00220043 | 0.0022355  | 0.00226729 | 0.00229577 |
| K          | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train      | 0.00192144  | 0.00214375  | 0.00227608 | 0.00237613 | 0.00244956 | 0.00249698 | 0.00254238 | 0.00258117 |            |
|            |             |             |            |            |            |            |            |            |            |

| K(1-8GeV) | 1           | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|-----------|-------------|------------|------------|------------|------------|------------|------------|------------|------------|
| train     | 5.72662e-05 | 0.00123741 | 0.00186269 | 0.00219082 | 0.00240854 | 0.00254903 | 0.00265155 | 0.00273766 | 0.00280227 |
| test      | 0.00728033  | 0.00565674 | 0.00496845 | 0.00462567 | 0.00441044 | 0.00425458 | 0.00415945 | 0.00407662 | 0.00401177 |
| K         | 10          | 15         | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train     | 0.00286477  | 0.00306736 | 0.00318602 | 0.00327106 | 0.00334096 | 0.00339297 | 0.00343831 | 0.00348125 |            |
| test      | 0.00395984  | 0.00382722 | 0.00376683 | 0.00375569 | 0.00374695 | 0.00375402 | 0.00375905 | 0.00376458 |            |

| K(8-30GeV)     | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| train          | 9.99708e-05 | 0.000399508 | 0.000575274 | 0.000694707 | 0.00077624  | 0.00083118  | 0.000862363 | 0.000893816 | 0.0009151   |
| test           | 0.00114782  | 0.00110811  | 0.00111124  | 0.00111307  | 0.00112577  | 0.00114445  | 0.00114403  | 0.00114322  | 0.00114564  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.000929617 | 0.00100975  | 0.00106291  | 0.0010914   | 0.00110561  | 0.00112393  | 0.00113734  | 0.00114961  |             |
| test           | 0.00114476  | 0.00118155  | 0.00120327  | 0.00121617  | 0.00122092  | 0.00123372  | 0.00124676  | 0.00124689  |             |
| K(30-120GeV)   | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 4.58846e-05 | 0.00028109  | 0.000414516 | 0.000505221 | 0.000571419 | 0.000659132 | 0.000714983 | 0.000779293 | 0.000832743 |
| test           | 0.000417332 | 0.000513937 | 0.000610912 | 0.000677911 | 0.00075685  | 0.00080923  | 0.000862802 | 0.000910377 | 0.000960557 |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.000871744 | 0.000983396 | 0.00107146  | 0.00112629  | 0.00117488  | 0.00120839  | 0.00124328  | 0.00126526  |             |
| test           | 0.00100524  | 0.00111678  | 0.00120144  | 0.00125766  | 0.00128392  | 0.00132249  | 0.00135162  | 0.00135973  |             |
| K(120—1000GeV) | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 1.95788e-05 | 0.000701377 | 0.00115765  | 0.00147607  | 0.00173426  | 0.00188723  | 0.00205045  | 0.00220378  | 0.00234543  |
| test           | 0.000812127 | 0.00122143  | 0.00151379  | 0.00172361  | 0.00191031  | 0.00203665  | 0.00219763  | 0.0023415   | 0.00245495  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00245974  | 0.00286916  | 0.00309626  | 0.00328785  | 0.0034206   | 0.00349273  | 0.00356659  | 0.00363075  |             |
| test           | 0.00256029  | 0.0029193   | 0.00315115  | 0.00335231  | 0.00349265  | 0.00359701  | 0.00366733  | 0.00374225  |             |

#### dPhis+theta+dTh13+ bend\_1+FR1 weighted marker 27

| K(overall) | 1           | 2           | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|------------|-------------|-------------|------------|------------|------------|------------|------------|------------|------------|
| train      | 3.81526e-05 | 0.000696435 | 0.0010741  | 0.00129765 | 0.00145079 | 0.00157178 | 0.00167794 | 0.00176969 | 0.00184027 |
| test       | 0.00251116  | 0.00221709  | 0.00214681 | 0.00214803 | 0.00216229 | 0.00219214 | 0.00221862 | 0.0022496  | 0.00227759 |
| K          | 10          | 15          | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train      | 0.0019061   | 0.00212222  | 0.00226712 | 0.00236319 | 0.0024347  | 0.00249266 | 0.00254652 | 0.00258959 |            |
|            |             |             |            |            |            |            |            |            |            |

| K(1-8GeV) | 1           | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          |
|-----------|-------------|------------|------------|------------|------------|------------|------------|------------|------------|
| train     | 3.84383e-05 | 0.00119673 | 0.001797   | 0.00212804 | 0.00233734 | 0.00248439 | 0.00259641 | 0.0026805  | 0.00275047 |
| test      | 0.00715137  | 0.00553694 | 0.00487503 | 0.00455544 | 0.00432813 | 0.00418865 | 0.00409131 | 0.00400829 | 0.00395554 |
| K         | 10          | 15         | 20         | 25         | 30         | 35         | 40         | 45         | 50         |
| train     | 0.00281793  | 0.00301318 | 0.00313758 | 0.00322992 | 0.00330479 | 0.00336742 | 0.00342577 | 0.00347637 |            |
| test      | 0.00390641  | 0.00378258 | 0.00373478 | 0.00372353 | 0.00372534 | 0.00373499 | 0.00374625 | 0.00376424 |            |

| K(8-30GeV)     | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| train          | 6.88033e-05 | 0.000390502 | 0.000562844 | 0.000660135 | 0.00071714  | 0.000766815 | 0.00080392  | 0.000833491 | 0.000863269 |
| test           | 0.00101548  | 0.000994954 | 0.00100771  | 0.00101895  | 0.00104062  | 0.00105253  | 0.00106723  | 0.00107642  | 0.0010814   |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00088645  | 0.000953108 | 0.0010079   | 0.00103658  | 0.00105834  | 0.00107847  | 0.00109195  | 0.00110822  |             |
| test           | 0.00109245  | 0.00113297  | 0.00116019  | 0.00117233  | 0.00118406  | 0.00118717  | 0.00119598  | 0.00120286  |             |
| K(30-120GeV)   | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 3.84256e-05 | 0.00027827  | 0.000398325 | 0.000504638 | 0.000589681 | 0.000675456 | 0.000733381 | 0.000778382 | 0.000823461 |
| test           | 0.000432713 | 0.000565993 | 0.000654185 | 0.000722939 | 0.000764574 | 0.000832327 | 0.000881455 | 0.000918741 | 0.000965988 |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.000861811 | 0.000983118 | 0.00109529  | 0.00116877  | 0.00121447  | 0.00125078  | 0.00127495  | 0.00129595  |             |
| test           | 0.00099887  | 0.00112386  | 0.00121009  | 0.00126798  | 0.00130381  | 0.00133549  | 0.00136034  | 0.00138156  |             |
| K(120—1000GeV) | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           | 9           |
| train          | 1.73777e-05 | 0.000741573 | 0.00123394  | 0.001526    | 0.00173963  | 0.00190886  | 0.0020885   | 0.00226009  | 0.00237565  |
| test           | 0.000827028 | 0.00122471  | 0.00151312  | 0.00174467  | 0.00194834  | 0.00211372  | 0.00224184  | 0.00238356  | 0.00248504  |
| K              | 10          | 15          | 20          | 25          | 30          | 35          | 40          | 45          | 50          |
| train          | 0.00248705  | 0.00288537  | 0.00313046  | 0.00328992  | 0.00340902  | 0.0035027   | 0.00359958  | 0.0036688   |             |
| test           | 0.00259531  | 0.00295078  | 0.00320186  | 0.00337725  | 0.00354912  | 0.00364869  | 0.00372837  | 0.00379334  |             |

#### dPhis+theta+dPhiSum3

| k             | 1           | 2           | 3           | 4           | 5           | 6           | 7           | 8           |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| train         | 0.000173378 | 0.000898661 | 0.00127605  | 0.00151907  | 0.00168583  | 0.00181372  | 0.00190758  | 0.0019785   |
| test          | 0.00290756  | 0.00251034  | 0.00242445  | 0.00241031  | 0.00242718  | 0.002443    | 0.00246962  | 0.00248495  |
| Train1-8      | 0.000260811 | 0.00136209  | 0.00193215  | 0.00224735  | 0.00245001  | 0.00259744  | 0.00269786  | 0.00277425  |
| Test1-8       | 0.00773645  | 0.00572551  | 0.00497046  | 0.00462695  | 0.00439637  | 0.00424664  | 0.00413542  | 0.00405371  |
| Train8-30     | 0.00027177  | 0.000585601 | 0.000811253 | 0.000958956 | 0.00101694  | 0.0010607   | 0.00110806  | 0.00113883  |
| Test8-30      | 0.00142295  | 0.00136947  | 0.00136608  | 0.00137922  | 0.00137846  | 0.00138251  | 0.00140196  | 0.00140664  |
| Train30-120   | 9.75E-05    | 0.000399533 | 0.000575278 | 0.000689472 | 0.000793208 | 0.000848759 | 0.000913145 | 0.000958067 |
| Test30-120    | 0.000640602 | 0.000808223 | 0.000934591 | 0.00100362  | 0.00107197  | 0.00112541  | 0.00117181  | 0.0011944   |
| Train120-1000 | 8.24E-05    | 0.00103627  | 0.00148119  | 0.00181131  | 0.00206141  | 0.00227707  | 0.00241606  | 0.00252649  |
| Test120-1000  | 0.0011774   | 0.00159053  | 0.00189561  | 0.00209721  | 0.00230907  | 0.00245319  | 0.00259266  | 0.00269572  |
|               |             |             |             |             |             |             |             |             |

2/24/2017 weishi@rice.edu 38

# Tune Scale Frac when k=4, dPhis+theta+bend\_1+FR1

| Scale             | 0                     | 0.61        | 0.62        | 0.63        | 0.64        | 0.65                 | 0.7                   | 0.75        | 0.8         | 0.85        | 0.9         | 0.95             | 1           |
|-------------------|-----------------------|-------------|-------------|-------------|-------------|----------------------|-----------------------|-------------|-------------|-------------|-------------|------------------|-------------|
| Frac              |                       |             |             |             |             |                      |                       |             |             |             |             |                  |             |
| train             | 0.00115444            | 0.00127317  | 0.00127317  | 0.00127317  | 0.00127205  | 0.00127205           | 0.00125961            | 0.00127142  | 0.0012622   | 0.0012503   | 0.00123737  | 0.00121511       | 0.00117735  |
| test              | 0.00193683            | 0.00210214  | 0.00210214  | 0.00210214  | 0.0020944   | 0.0020944            | 0.00205614            | 0.00209088  | 0.00207417  | 0.00204699  | 0.0020257   | 0.00200334       | 0.00195252  |
| Train1-<br>8      | 0.00194561            | 0.00208354  | 0.00208354  | 0.00208354  | 0.00206749  | 0.00206749           | 0.00203562            | 0.00207168  | 0.00205353  | 0.00202487  | 0.00201127  | 0.00199914       | 0.00196419  |
| Test1-8           | 0.00411105            | 0.00437369  | 0.00437369  | 0.00437369  | 0.00430556  | 0.00430556           | 0.00424837            | 0.00434412  | 0.00432138  | 0.00426636  | 0.00423452  | 0.00420019       | 0.0041213   |
| Train8-<br>30     | 0.000647514           | 0.0006876   | 0.0006876   | 0.0006876   | 0.00068423  | 0.00068423           | 0.000679781           | 0.000683418 | 0.000684434 | 0.000675688 | 0.000674294 | 0.00067146       | 0.000651809 |
| Test8-<br>30      | 0.00101022            | 0.00110844  | 0.00110844  | 0.00110844  | 0.00111266  | 0.00111266           | 0.00109876            | 0.00110472  | 0.0010962   | 0.00109778  | 0.00108567  | 0.00106981       | 0.00104576  |
| Train3<br>0-120   | 5.07E-04              | 0.000554101 | 0.000554101 | 0.000554101 | 5.47E-04    | 5.47E-04             | 0.000544083           | 0.000553031 | 0.000549002 | 0.000546359 | 0.000532228 | 0.000522622      | 0.000523127 |
| Test30-<br>120    | 0.000669573           | 0.000713521 | 0.000713521 | 0.000713521 | 0.000737045 | 0.00073704           | 0.000712268           | 0.00071577  | 0.000704931 | 0.000713775 | 0.000689143 | 0.000685072      | 0.000667237 |
| Train120<br>-1000 | 2/24/2017<br>1.23E-03 | 0.00143444  | 0.00143444  | 0.00143444  | 1.45E-03    | weishi@r<br>1.45E-03 | ice.edu<br>0.00144505 | 0.00144277  | 0.00143113  | 0.00142566  | 0.0014065   | 39<br>0.00135377 | 0.00127544  |

| Track |     |     |     | $\Delta \varphi$ $\Delta \theta$ |     |     |     |     | Bits |     |     |     |     |     |
|-------|-----|-----|-----|----------------------------------|-----|-----|-----|-----|------|-----|-----|-----|-----|-----|
| mode  | 1-2 | 1-3 | 1-4 | 2-3                              | 2-4 | 3-4 | +/- | 1-2 | 1-3  | 1-4 | 2-3 | 2-4 | 3-4 | DIG |
| 15    | 7   |     |     | 5                                |     | 6   | 2** |     |      |     |     |     |     | 20  |
| 14    | 7   |     |     | 5                                |     |     | 2   |     | 3    |     |     |     |     | 17  |
| 13    | 7   |     |     |                                  | 5   |     | 2   |     |      | 3   |     |     |     | 17  |
| 12    | 9   |     |     |                                  |     |     | 1   | 3   |      |     |     |     |     | 13  |
| 11    |     | 7   |     |                                  |     | 5   | 2   |     |      | 3   |     |     |     | 17  |
| 10    |     | 9   |     |                                  |     |     | 1   |     | 3    |     |     |     |     | 13  |
| 9     |     |     | 9   |                                  |     |     | 1   |     |      | 3   |     |     |     | 13  |
| 7     |     |     |     | 7                                |     | 6   | 2   |     |      |     |     | 3   |     | 18  |
| 6     |     |     |     | 9                                |     |     | 1   |     |      |     | 3   |     |     | 13  |
| 5     |     |     |     |                                  | 9   |     | 1   |     |      |     |     | 3   |     | 13  |
| 3     |     |     |     |                                  |     | 9   | 1   |     |      |     |     |     | 3   | 13  |

| Track    | ] | Bend | l (CI | LCT | )   |   | F | R |   | θ | Md | Σ  | Σ  | Bits |
|----------|---|------|-------|-----|-----|---|---|---|---|---|----|----|----|------|
| mode     | 1 | 2    | 3     | 4   | +/- | 1 | 2 | 3 | 4 | V | Mu | 1  | 1  | DILS |
| 15       |   |      |       |     |     | 1 |   |   |   | 5 | 4  | 10 | 20 | 30   |
| 14       | 2 |      |       |     | 1   | 1 |   |   |   | 5 | 4  | 13 | 17 | 30   |
| 13       | 2 |      |       |     | 1   | 1 |   |   |   | 5 | 4  | 13 | 17 | 30   |
| 12       | 2 | 2    |       |     | 2   | 1 | 1 |   |   | 5 | 4  | 17 | 13 | 30   |
| 11       | 2 |      |       |     | 1   | 1 |   |   |   | 5 | 4  | 13 | 17 | 30   |
| 10       | 2 |      | 2     |     | 2   | 1 | 1 |   |   | 5 | 4  | 17 | 13 | 30   |
| 9        | 2 |      |       | 2   | 2   | 1 |   |   | 1 | 5 | 4  | 17 | 13 | 30   |
| 7        |   | 2    |       |     | 1   |   |   |   |   | 5 | 4  | 12 | 18 | 30   |
| 6        |   | 2    | 2     |     | 2   |   | 1 | 1 |   | 5 | 4  | 17 | 13 | 30   |
| 5        |   | 2    |       | 2   | 2   |   | 1 |   | 1 | 5 | 4  | 17 | 13 | 30   |
| <b>3</b> |   |      | 2     | 2   | 2   |   |   | 1 | 1 | 5 | 4  | 17 | 13 | 30   |

## Modes

| Mode # | Definiition in code | Stations |
|--------|---------------------|----------|
| 15     | 1+2+4+8             | 1,2,3,4  |
| 14     | 2+4+8               | 1,2,3    |
| 13     | 1+4+8               | 1,2,4    |
| 12     | 4+8                 | 1,2      |
| 11     | 1+2+8               | 1,3,4    |
| 10     | 2+8                 | 1,3      |
| 9      | 1+8                 | 1,4      |
| 8      | 8                   | 1        |
| 7      | 1+2+4               | 2,3,4    |
| 6      | 2+4                 | 2,3      |
| 5      | 1+4                 | 2,4      |
| 4      | 4                   | 2        |
| 3      | 1+2                 | 3,4      |
| 2      | 2                   | 3        |
| 1      | 1                   | 4        |

#### Euclidean distance

x: input variables(dimension d)

of train event

y: input variables of test event

f: polynomial kernel weight

function

w<sub>i</sub>: weight of j in train sample

$$R_{ ext{rescaled}} = \left(\sum_{i=1}^d rac{1}{w_i^2} |x_i - y_i|^2
ight)^{\!\!rac{1}{2}}$$

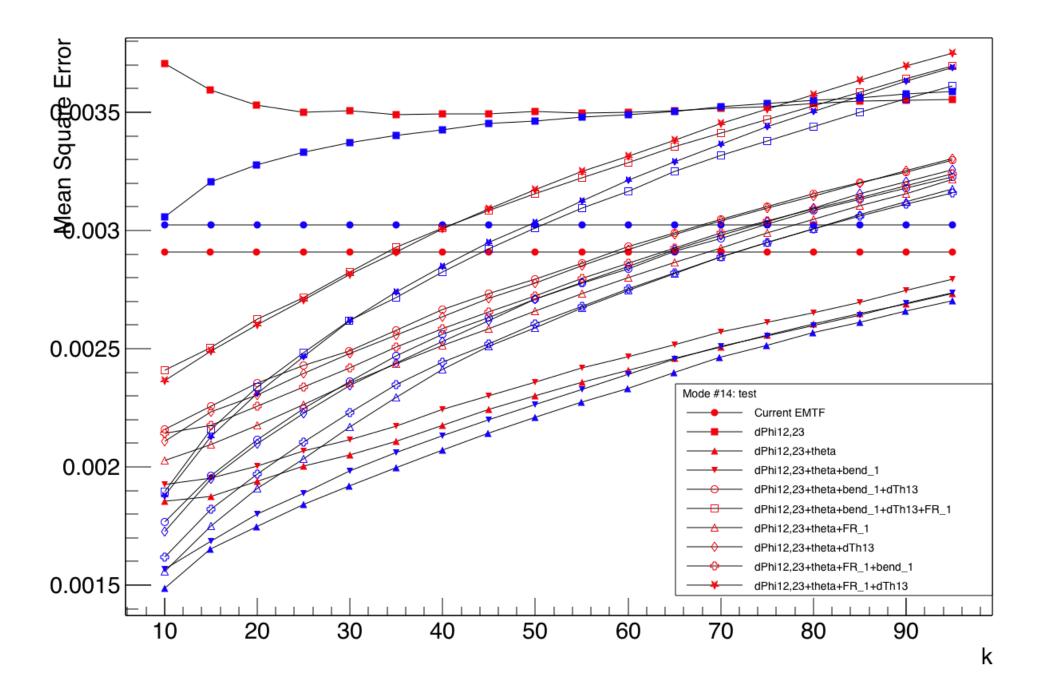
$$\langle t(i,V) \rangle = \frac{\sum_{j \in V} w_j t_j f(\operatorname{dis}(i,j))}{\sum_{j \in V} w_j f(\operatorname{dis}(i,j))}$$

#### **kNN**

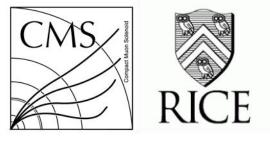
- Tune k and Scale frac
- Continuous/categorical variable: different metric(Euclidean/Hamming)
- When mixture of both kinds of variables, standardization or scale variables(called feature normalization)
- E.g. x'=(x-min)/(max-min); x'= (x-x\_mean)/x\_variance; assign weights to d(i,j)(TMVA adopts this)
- A non-parametric method Unlike other supervised learning algorithms, K-Nearest Neighbors doesn't learn an explicit mapping f from the training data
- Simply uses the training data at the test time to make predictions
- Need large dataset/cross validation dataset

#### KNN Regression

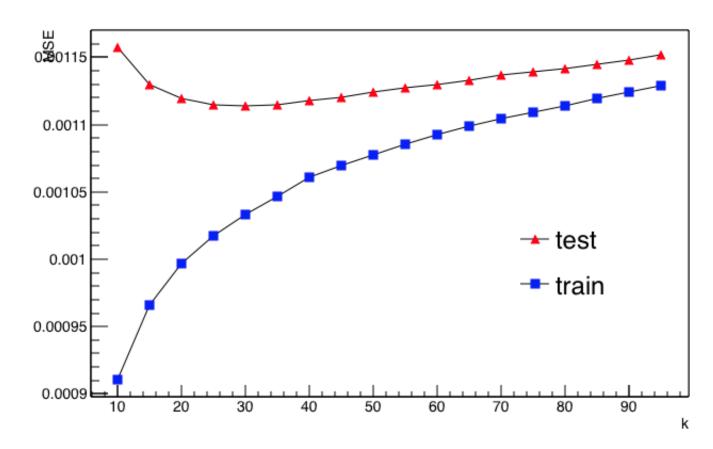
- For a test event, the algorithm finds the k-nearest neighbours using the input variables, where each training event contains a regression value. The predicted regression value for the test event is the weighted average of the regression values of the k-nearest neighbours
- The choice of the metric governs the performance of the nearest neighbour algorithm. When input variables have different units a variable that has a wider distribution contributes with a greater weight to the Euclidean metric. This feature is compensated by rescaling the variables using a scaling fraction determined by the option ScaleFrac.



### "nkNN=30:ScaleFrac=0.8:SigmaFact=1.0: Kernel=Gaus:UseKernel=F:UseWeight=T:! Trim"



MSE = Deviation^2/N



Mode: 15
Input variables:
dphi12, dphi23,
dphi34
Target variable:
1/Gen pT

#### Comments

- Maybe we don't need to choose the best model, instead, we can use weighted models for final pT LUT
- Maybe we can use different model for different input variables, some model may perform better than others in certain parameter region(eta)

# BDT parameters(mode 15) from Andrew B.(check)

#### For mode 15

- 400 trees
- Depths: 5
- 1/pT weight(0-120 GeV), Log2(pT) is better target than 1/pT
- For very high pT, unweighted events better(>120 GeV)
- Input variables: FR bits bring significant improvement at low and high pT
- In addition to track theta, FR 1, and dPhi 1-2, 2-3, and 3-4(LUT v1), add combinations of dPhis, and ring number of station 1
- https://indico.cern.ch/event/608207/contributions/2451751/subcontributions/2 18758/attachments/1402616/2142649/2017 01 26 Mode 15 BDT.pdf

# BDT good/bad

Good: little tuning required(simple)

- Bad:
- 1. will ignore non-discriminating variables as for each node splitting only the best discriminating variable is used
- 2. theoretically best performance on a given problem is generally inferior to other techniques like neural networks.

See TMVA tutorial