例 6.4

$$E(\bar{X}) = \mu$$

$$V(\bar{X}) = \frac{\sigma^2}{n} = E(\bar{X}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left[\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}\right]$$

$$= \frac{1}{n}E\left\{\sum_{i=1}^{n}X_i^2 - n\bar{X}^2\right\}$$

$$= \frac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 + n\mu^2)$$

$$= \frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2) = E\left[\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}\right]$$

$$= \frac{1}{n-1}E\left[\sum_{i=1}^{n}X_i^2 - n\bar{X}^2\right]$$

$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \sigma^2$$

∴ $\hat{\theta}_2$ 為不偏估計量

$\hat{\theta}_1$ 為偏誤估計量