

Comment on: "Analytical solution of the 2D classical Heisenberg model" by J. Curély

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1996 Europhys. Lett. 34 311

(<http://iopscience.iop.org/0295-5075/34/4/311>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 128.205.114.91

This content was downloaded on 18/05/2017 at 21:00

Please note that [terms and conditions apply](#).

You may also be interested in:

[Reply to Professor Leroyer's comment concerning the article "Analytical solution of the 2D classical Heisenberg model"](#)

J. Curély

[Correspondence between quantum Heisenberg models \(spin-1/2\) and bosonic models](#)

E A Pereira

[Renormalization Group for the Heisenberg Magnet in a Random Magnetic Field: The Zero-Temperature Fixed Point](#)

Y. S. Parmar

[On the critical behaviour of ferromagnets](#)

C Domb and D L Hunter

[High-temperature specific heat and susceptibility of the classical Heisenberg model](#)

G S Joyce and R G Bowers

[Spin glass behavior of the antiferromagnetic Heisenberg model on scale free network](#)

Tasrief Surungan, Freddy P Zen and Anthony G Williams

[On high temperature expansions for the Heisenberg model](#)

C Domb and D W Wood

[Cluster series for the infinite spin Heisenberg model](#)

G S Joyce and R G Bowers

## Comment on: “Analytical solution of the 2D classical Heisenberg model” by J. Curély

Y. LEROYER

*Centre de Physique Théorique et de Modélisation de Bordeaux  
19, rue du Solarium, 33175 Gradignan Cedex, France*

(received 22 January 1996; accepted 18 March 1996)

PACS. 75.10–b – General theory and models of magnetic ordering.

PACS. 75.10Hk – Classical spin models.

PACS. 75.50Gg – Ferrimagnetics.

J. Curély in paper [1] claims to have found an exact analytical solution of the classical two-dimensional Heisenberg model. I would like to make the following comments on this very surprising result:

- i) The expression for the partition function derived by the author in eqs. (3), (4) of the paper has been established by Joyce [2] in 1967. This work should have been mentioned.
- ii) The simplification of this expression rests on the affirmation (in the discussion between eq. (4) and eq. (5)) that “... a single (solution) allows to obtain the term of higher degree in the  $\lambda_\ell$  polynomial expansion of  $Z(0)$ ”. This assertion is not proved.
- iii) After making this hypothesis, the author obtains the following expression for the zero-field partition function:

$$Z_N(0) = [(4\pi)^2 \lambda_0(-\beta J_1) \lambda_0(-\beta J_2)]^{2N(2N+1)}. \quad (1)$$

Surprisingly, it is exactly the product of the one-dimensional partition functions along each spatial direction [2], [3]. By performing a high-temperature expansion of this expression, one observes that it is not correct. Setting  $J_1 = J_2 = J$ ,  $x = -\beta J$  and  $N_S = 2N(2N + 1)$ , eq. (1) may be expanded to yield

$$\frac{1}{(4\pi)^{N_S}} Z_N(0) = 1 + \frac{1}{3} x^2 N_S + x^4 \left[ \frac{1}{60} N_S + \frac{1}{36} N_S(2N_S - 1) \right] + O(x^6).$$

It is easy to derive the expansion of the full original partition function to this low order. At the 4th order, one gets an extra term,  $\frac{1}{27} x^4 N_S$  coming from the one-loop diagrams, which, of course, cannot be treated by the factorized partition function of eq. (1).

iv) Similarly, the expression for the susceptibility (eq. (9) of the paper) can be re-expressed in a factorized form (for  $G = G'$ ):

$$\frac{3k_B T}{N_S G^2} \chi = \frac{1 + u_1}{1 - u_1} \frac{1 + u_2}{1 - u_2},$$

which is again the product of one-dimensional susceptibilities. That this trivial result is not correct can be checked by comparing its high-temperature expansion to the result published in the well-known review article of Rushbrooke *et al.* [4]. Upon setting  $G = G'$ ,  $J_1 = J_2 = J$ ,  $x = -\beta J$  and  $u(x) = \mathcal{L}(2x)$  (the factor 2 takes into account the definition of the coupling in ref. [4]), eq. (9) of Cur  ly becomes

$$\frac{3k_{\text{B}}T}{N_{\text{S}}G^2}\chi = \left(\frac{1+u}{1-u}\right)^2 = 1 + \frac{8}{3}x + \frac{32}{9}x^2 + \frac{128}{45}x^3 + \dots,$$

whereas the result of Rushbrooke *et al.* is

$$\frac{3k_{\text{B}}T}{N_{\text{S}}G^2}\chi = 1 + \frac{8}{3}x + \frac{16}{3}x^2 + \frac{448}{45}x^3 + \dots$$

I conclude that the 2D Heisenberg model is still unsolved.

#### REFERENCES

- [1] CUR  LY J., *Europhys. Lett.*, **32** (1995) 529.
- [2] JOYCE G. S., *Phys. Rev.*, **155** (1967) 478.
- [3] FISHER M. E., *Am. J. Phys.*, **32** (1964) 343.
- [4] RUSHBROOKE G. S., BAKER G. A. and WOOD P. J., *Phase Transitions and Critical Phenomena*, edited by DOMB and GREEN, Vol. **3** (Academic Press), p. 245 and appendix III p. 345.