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Minireview

Phase diagram of the classical Heisenberg model in a trimodal random field distribution

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HIGHLIGHTS

- The reentrant phenomena depend on the trimodal distribution of the random field.
- Phase diagram in the plane T versus alpha exhibits double reentrant.
- An amorphous classical Heisenberg model (RFHM) with disorderly bond site model was investigated.
- The nature of the phase transition in the RFHM was investigated, as a function of the connectivity.
- Effective field theory (EFT) using clusters of two spins.

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ABSTRACT

The classical spin 1/2 Heisenberg model on a simple cubic lattice, with fluctuating bond interactions between nearest neighbors and in the presence of a random magnetic field, is investigated by effective field theory based on two-spin cluster. The random field is drawn from the asymmetric and anisotropic trimodal probability distribution. The fluctuating bond is extracted from the symmetric and anisotropic bimodal probability. We estimate the transition temperatures, and the phase diagram in the T_c- h, T_c- p and $T_c - \alpha$ planes. We observe that the temperature of the tricritical point decreases with the increase of disorder in exchange interactions until the system ceases to display tricritical behavior. The disorder of the interactions and reentrant phenomena depends on the trimodal distribution of the random field.

Contents

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1. Introduction

Over the last few decades researches in structurally disordered model have proposed that magnetic long range order might emerge in amorphous systems. There are two basic types of disorder in magnetic models: bond and site disorder [1]. The randomness of the applied magnetic field is a type of site disorder. The random field model (RFM), introduced by Imry and Ma [2], has been widely employed to describe the critical behavior of amorphous magnetic systems. It has become the subject of experimental and theoretical interest [3], since it was shown that materials such as the diluted antiferromagnets $Fe_x\ Zn_{1-x}F_2$ [4,5], $Rb_2Co_xMg_{1-x}F_4$ [6,7] and $Co_xZn_{1-x}F_2$ [7] in a uniform magnetic field correspond to a ferromagnet in a random uniaxial magnetic field and can be described by the RFM [8,9].

A problem associated with the ferromagnetic model in a random field is the survival of the tricritical point [10–14]. Depending on the choice of the random-field distribution, for example when it is given by a symmetric double-delta functions [15], the mean-field approximation gives rise to a tricritical point. In contrast, no tricritical point occurs when a Gaussian function is chosen [16]. On the basis of the central limit theorem, some arguments can be used to support the physical relevance of the Gaussian distribution. There is large possibility that the tricritical point is produced by the doubled δ-function being a simple product of the mean-field approximation [15,17].

The Random Field Heisenberg Model (RFHM) can also be used to describe the essential physics of a class of experimentally accessible disordered systems. The RFHM exhibits tricritical behavior when the random field is governed by a bimodal distribution within the framework of the effective field theory [18]. The behavior is qualitatively similar to that obtained for the Random Field Ising Model (RFIM) [19].

Otherwise, a recent work by Fytas and Martin-Mayor [20] asserts that the ferro-paramagnetic transition of the 3D RFIM with nearest-neighbor interactions is continuous for different types of random field probability distribution. Picco and Sourlas [21] studied for numerical simulations the critical behavior of the diluted antiferromagnet in a field in three dimensions. Their results are fully compatible with the prediction of the perturbative renormalization group (PRG) that the RFIM and it belong to the same universality class.

On the other hand, in the mean field renormalization group (MFRG) frame work the trimodal distribution (TD), introduced in the literature by Mattis [22] to simulate the Gaussian distribution, leads to a tricritical behavior as well as a second order phase transition only for certain values of the critical value of the parameter that governs the random field [23].

Mattis suggested that for the particular case, $p = 1/3$ (where p is a parameter related to a trimodal distribution), which may be considered as a good approximation to the Gaussian distribution [22]. This, in turn, indicated that the two models should be in the same universality class. Many studies of RFIM, using the mean-field and the renormalization-group approaches, have been conducted providing evidence for the critical aspects of the $p = 1/3$ model [24] and also proposed several approximations of its phase diagram for a range of values of p [19,25,26].

Ümit Akinci recently studied the effect of the trimodal random magnetic field distribution on the phase diagrams of the anisotropic quantum Heisenberg model for three-dimensional lattices with effective field theory (EFT) for the two spin cluster [27]. He detailed the behavior of the tricritical points with random magnetic field distribution and a network anisotropy. In this work we focus on the combined effect of the disorder in the field and disorder in the exchange interaction. That is called a disorderly bond site model.

In this work, we study an amorphous classical Heisenberg model (RFHM) with a probability distribution function for the exchange interaction and a trimodal distribution random field distribution for clusters containing two spins on a simple cubic lattice. The calculation is carried out within the effective field theory (EFT) approximation. An analytical expression for the second order phase transition line is obtained, and the existence of tricritical points, reentrant phenomena and topology of the phase diagram are investigated.

2. Model and calculations

The system under investigation is the amorphous classical Heisenberg ferromagnet in a version of a n-vector model in a trimodal random field. The model Hamiltonian then reads

$$-\beta\mathcal{H} = \sum_{(i,j)} K_{ij}\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i \mathbf{h}_i\mathbf{S}_i \tag{1}$$

where K_{ij} ($\equiv J_{ij}/k_BT$, k_B is the Boltzmann constant and T the absolute temperature) is the ferromagnetic exchange interaction between nearest neighbors on a simple cubic lattice. The summation is carried out only over pairs of nearest-neighboring sites (i, j). The quantities $\mathbf{S}_i$ are isotropically interacting n-dimensional classical spins of magnitude n localized at the sites i, and the Cartesian components of $\mathbf{S}_i$ obey the normalization condition [28],

$$\sum_{\nu}^{n} \left(S_i^{\nu}\right)^2 = n, \tag{2}$$

the exchange interaction between the spins is quenched, uncorrelated random variables, which is assumed to be distributed according to the probability distribution function

$$P\left(K_{ij}\right) = \frac{1}{2}\left[\delta\left(K_{ij} - K - \alpha K\right) + \delta\left(K_{ij} - K + \alpha K\right)\right], \tag{3}$$

and the reduced random magnetic field h_i at site i comply to the following trimodal distribution,

$$P\left(h_i\right) = p\delta\left(h_i\right) + \frac{1-p}{2}\left[\delta\left(h_i+h\right)+\delta\left(h_i-h\right)\right], \tag{4}$$

where h ($\equiv \mu_B H/k_B T$, μ_B is the Bohr magneton and H is the random magnetic field) defines the disorder strength and $p \in (0, 1)$. Clearly, for $p = 1$ one switches to the pure classical Heisenberg ferromagnet, whereas for $p = 0$ the well-known bimodal distribution is recovered. The Hamiltonian (1) reduces to the well-known $S = 1/2$ Ising, classical planar (XY), classical Heisenberg and spherical models for $n = 1, 2, 3$ and ∞, respectively. In this work, we follow the EFT procedure (see Refs. [29–32]) to study the critical properties of the amorphous ferromagnet. The random field distribution is described by the Hamiltonian given by Eq. (1) and we employed the axial approximation [33]. The Hamiltonian for a cluster with two spins can be written as

$$H = K_{12}\mathbf{S}_1 \cdot \mathbf{S}_2 + a_1 S_1^1 + a_2 S_2^1 \tag{5}$$

where $a_l = h_l + \sum_{j\neq l}^{Z-1} K_{lj} S_j^1,\ (l = 1, 2)$ and Z is the lattice coordination number.

In order to obtain the average magnetization per spin $m = \frac{1}{2}\left\langle\left(S_1^1+S_2^1\right)\right\rangle$ for a two-spin cluster, we employ the equation (see Refs. [34–37])

$$m = \left\langle \prod_{K\neq 1,2}^{z-1}\left(\Upsilon_x + S_k^1\Phi_x\right)\prod_{l\neq 1,2}^{z-1}\left(\Upsilon_y + S_l^1\Phi_y\right)\right\rangle g_n(X,Y)\,|_{X=h_1,Y=h_2} \tag{6}$$

where $\Upsilon_\nu = \cosh(K_{l,j}D_\nu)$, $\Phi_\nu = \sinh(K_{l,j}D_\nu)$, $(\nu = x, y;\ l = 1, 2)$, and $X(Y) = x(y) + h_1(h_2)$. D_ν $\left(\equiv \frac{\partial}{\partial\nu}\right)$ is the differential operator [27] which satisfies the mathematical relation $2\sinh\left(aD_x + bD_y\right) g_n\left(X, Y\right)|_{X=h_1,Y=h_2} = g_n\left(a+h_1, b+h_2\right) - g_n\left(-a+h_1, -b+h_2\right)$, where $g_n(X, Y)$ is given by

$$g_n\left(X,Y\right) = \sinh\left(X+Y\right)\left[\cosh\left(X+Y\right)+\exp\left(-2K_{12}\right)T_n\left(K_{12}\right)\cosh\left(X-Y\right)\right]^{-1} \tag{7}$$

with $T_n\left(K_{12}\right) = \left(1-\tanh_{\left(\frac{n}{2}-1\right)}\left(nK_{12}\right)\right) \times \left(1+\tanh_{\left(\frac{n}{2}-1\right)}\left(nK_{12}\right)\right)^{-1}$. Here $\tanh(\frac{n}{2}-1)(X)$ denotes the generalized hyperbolic tangent defined by $\tanh_{\left(\frac{n}{2}-1\right)}\left(X\right) = \frac{I_{\frac{n}{2}}(X)}{I_{\left(\frac{n}{2}-1\right)}(X)}$, and $I_n\left(X\right)$ is a modified Bessel function of the first kind. Eq. (6) is exact and will be applied here as the basis for our formalism, since it yields the cluster magnetization and the corresponding multi-spin correlation functions associated with various sites for the cluster under consideration. Here we apply the EFT approximation on both sides of Eq. (6), i.e., the thermal and random average (denoted by $\langle\ldots\rangle_c$), along with the decoupling procedure which ignores all high-order spin correlations, namely $\left\langle S_i^1 S_j^1\ldots S_n^1\right\rangle_c \approx \left\langle S_i^1\right\rangle_c \cdot \left\langle S_j^1\right\rangle_c \cdots \left\langle S_n^1\right\rangle_c$, with $i \neq j \neq \ldots \neq p$. Based on this approximation and in the random characteristic of the system, it is necessary to perform the random average in Eq. (6) by means of the integral

$$\bar{m} = \int \mathrm{d}K_{ij}\mathrm{d}h_i P\left(K_{ij}\right)P\left(h_i\right)m. \tag{8}$$

By using the properties of the differential operator and assuming translational invariance; the magnetization for the n-vector model in a random field (RFNVM), on a simple cubic lattice, is given by

$$\bar{m} = \sum_{l=0}^{4} A_{2l+1}\left(K, n, \alpha, h\right) m^{-2l+1} \tag{9}$$

in which, in order to satisfy the time reversal symmetry of the Ising model as well the properties of the operator technique, the coefficients $A_{2l}\left(K, n, \alpha, h\right)$ (even) have been set equal to zero.

3. Results

In this section we discuss the phase transition, reentrant phenomenon and tricritical point (TCP) of the diagrams generated from the analytical expressions using the RFNVM, where only the case $n = 3$ should be studied, corresponding to the 1/2 spin Heisenberg classic model. Near the second order phase transition $\bar{m} \approx 0$, and thus Eq. (9) can be rewritten as

$$\bar{m}^2 = -\frac{A_1\left(K,\alpha,h\right)-1}{A_3\left(K,\alpha,h\right)}. \tag{10}$$

When the magnetization continuously decreases to zero the seconder-order transition line is obtained from the simultaneous solution of the equations

$$A_1\left(K,\alpha,h\right) = 1, \qquad A_3\left(K,\alpha,h\right) < 0. \tag{11}$$

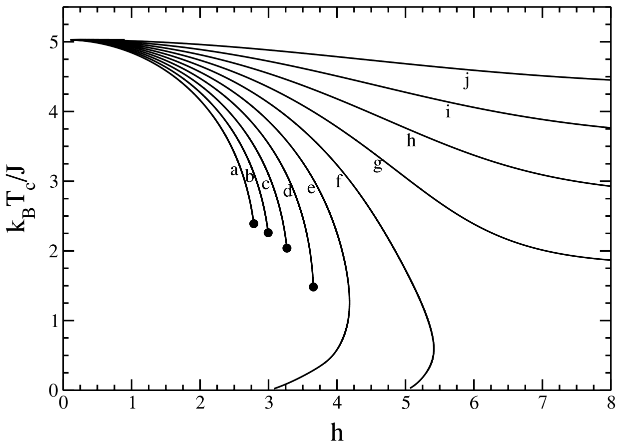

Fig. 1. Phase diagram in the plane T versus h. The solid lines represent continuous transitions and also show the reentrant behavior. The black points indicate tricritical points. The line is the solution of $A_1 - 1 = 0$ and $A_3 > 0$; (a) $p = 0.0$, (b) $p = 0.1$, (c) $p = 0.2$, (d) $p = 0.3$, (e) $p = 0.4$, (f) $p = 0.5$, (g) $p = 0.6$, (h) $p = 0.7$, (i) $p = 0.8$, (j) $p = 0.9$.

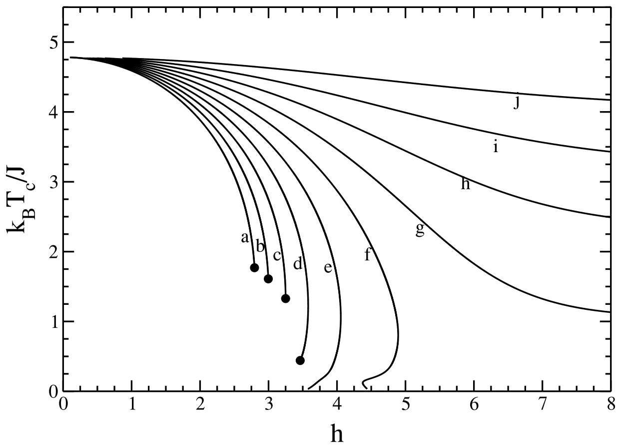

Fig. 2. Same as Fig. 1 for $\alpha = 0.5$.

On the other hand, the r.h.s. of Eq. (10) must be positive, otherwise the phase transition is of first-order. In the present work, we have restricted our calculation to the second-order transition, including the TCP. The tricritical points are obtained by solving the equations

$$A_1(K, \alpha, h) = 1, \qquad A_3(K, \alpha, h) = 0. \tag{12}$$

Fig. 1 shows the second order phase diagram for $\alpha = 0$ and for ten different values of p. For $\alpha = 0$, the curves for $p = 0, 0.1, 0.2$ and 0.3 exhibit tricritical point (TCP). The observed tricritical points at low p values gradually decrease and reduce to $k_BT_c/J = 0$ at a certain h_c. For $p = 0.4$ and $p = 0.5$ no TCP is obtained, but they present critical field (h_c). For $p = 0.6, 0.7, 0.8$ and 0.9 the curves no longer intersect the x-axis, i.e. there is no h_c value. For some values of p the system exhibits second order reentrant behavior. In other words, the system changes from a disordered phase at zero temperature to an ordered phase with a second order transition. When the temperature increases, it passes again to another disordered phase characterized by a second order transition.

For $\alpha = 0.5$ (Fig. 2), beyond the disturbance caused by the random field the system also suffers a disturbance in the exchange interactions. This perturbation decreases the value of the temperature T_t and increases the value of the random field h_t in the tricritical point, as shown in Table 1. There exists h_c for larger values of p, since the system has disorder in exchange interactions, it is easier to destroy the order and therefore the same as a high value for p there is still H_c field.

For $\alpha = 1$ (Fig. 3) there is no tricritical points for any p. On the other hand, there are critical random fields for more values of p.

Something out in these curves is that the reentrant behavior is related to the position of the critical field. Note that if the critical field tends to be a discrete function of concentration, it behaves similar to that obtained for the critical concentration of the Mixed-Bond Ising Model [38]. To obtain an estimation of the critical field behavior as a function of p, since it is not feasible to calculate the limit analytically, we plot curves h as function of p for low temperature (estimation for $t = 0$). This

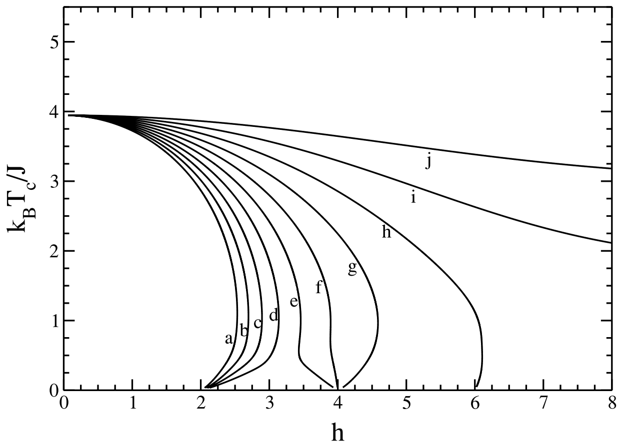

Fig. 3. Same as Fig. 1 for $\alpha = 1$.

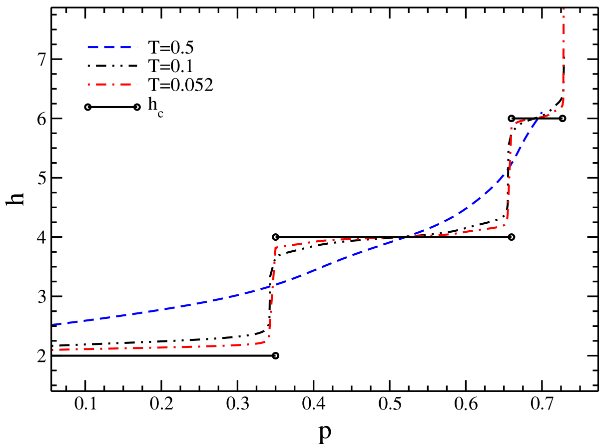

Fig. 4. Critical random magnetic fields as function of p and $\alpha = 1$.

Table 1
Localization of the tricritical points T_t, h_t for selected values of p and α.

	p	h_t	T_t
$\alpha = 0$	0	2.784(1)	2.390(1)
	0.1	2.994(1)	2.260(1)
	0.2	3.268(14)	2.039(1)
	0.3	3.654(1)	1.482(1)
$\alpha = 0.5$	0	2.795(1)	1.769(1)
	0.1	2.997(1)	1.610(1)
	0.2	3.249(1)	1.328(1)
	0.3	3.462(1)	0.441(1)

result is shown in Fig. 4. The values of critical random field h_c were 2, 4 or 6 and 3.5, 4.5 or 5.5 for $\alpha = 1$ and $\alpha = 0.5$ respectively.

Fig. 4 shows the plot of the transition temperature as a function of α for various selected values of the random field h and $p = 1/3$. Most curves tend to $\alpha = 1$ when t goes to 0 except the h values between 0.5 and 1.5 that feature a distinctive behavior. In bimodal distribution the curves do not tend to $\alpha = 1$, but show the same reentrant behavior [11].

Fig. 5 exhibits the transition temperature as a function of the α for $p = 1/3$ and various selected values of the random field. For all values of h the reentrant phenomenon happens. In this diagram, we see two differences compared to that observed for the bimodal distribution. The curves in the interval $1 < h < 1.7$ exhibit double reentrant and it tended to $\alpha = 1$ when t goes to zero.

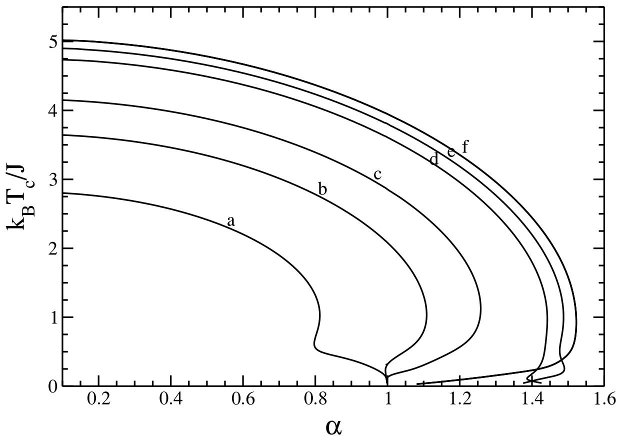

Fig. 5. Phase diagram in the plane T versus α. The solid lines represent continuous transitions and also show the reentrant behavior. (a) $h = 3.5$, (b) $h = 3.0$, (c) $h = 2.5$, (d) $h = 2.0$, (e) $h = 1.5$, (f) $h = 1.0$, (g) $h = 0.0$.

4. Conclusion

We show how the tricritical behavior disappears when we pass from a bimodal probability distribution to a trimodal one by means of the parameter p. As predicted, for $p = 1/3$ there are no critical points for any value of disorder in the exchange interaction which corroborates the results for the Gaussian distribution. On the other hand, for $\alpha = 1$ the diagram does not show tricritical point for any value of p. This demonstrates that as more random variables are incorporated to the model, the more difficult becomes the existence of tricritical points in the model. Thus, it is expected that disordered materials do not show actual tricritical points because they are under the influence of a large number of random variables.

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