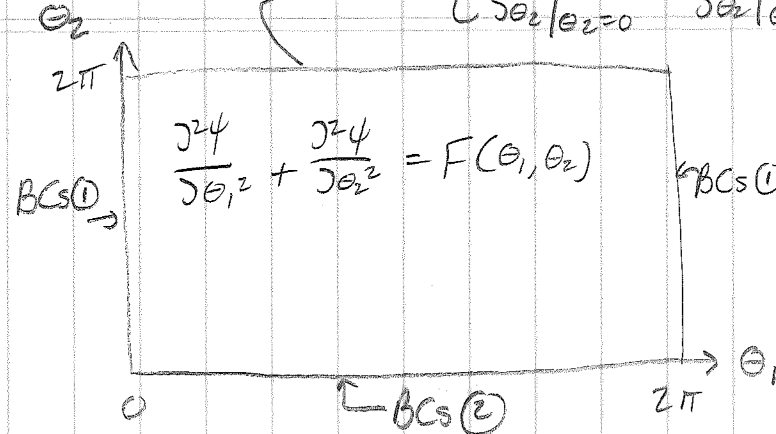
Summary:

Soln to:

$$\text{BCs (2)} = \begin{cases} \psi\big|_{\theta_2=0} = \psi\big|_{\theta_2=2\pi} \\ \dfrac{\partial\psi}{\partial\theta_2}\Big|_{\theta_2=0} = \dfrac{\partial\psi}{\partial\theta_2}\Big|_{\theta_2=2\pi} \end{cases}$$



$$\frac{\partial^2\psi}{\partial\theta_1^2} + \frac{\partial^2\psi}{\partial\theta_2^2} = F(\theta_1,\theta_2)$$

$$\text{BCs (1)} = \begin{cases} \psi\big|_{\theta_1=0} = \psi\big|_{\theta_1=2\pi} \\ \dfrac{\partial\psi}{\partial\theta_1}\Big|_{\theta_1=0} = \dfrac{\partial\psi}{\partial\theta_1}\Big|_{\theta_1=2\pi} \end{cases}$$

$$\psi = A_0(\theta_1) + \sum_{n=1}^{\infty} A_{n_s}(\theta_1)\sin(n\theta_2) + \sum_{n=1}^{\infty} A_{n_c}(\theta_1)\cos(n\theta_2)$$

$$\boxed{\text{Solvability} \Rightarrow \int_0^{2\pi} H_0(\xi)\,d\xi = 0}$$

Must Be Satisfied for soln. to exist!

$$A_0(\theta_1) = D + \int_0^{\theta_1} H_0(\xi)(\theta_1-\xi)\,d\xi - \frac{\theta_1}{2\pi}\int_0^{2\pi} H_0(\xi)(2\pi-\xi)\,d\xi$$

$$A_{n_{s,c}} = E_{n_{s,c}}\sinh(n\theta_1) + F_{n_{s,c}}\cosh(n\theta_1) + \frac{1}{n}\boxed{\int_0^{\theta_1} H_{n_{s,c}}(\xi)\sinh[n(\theta_1-\xi)]\,d\xi}$$

$A_{n_s} = A_{n_{s,c}}$, $A_{n_c} = A_{n_{s,c}}$

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$$F_{s,c} = \frac{\sinh(2\pi n)}{2n\,[1-\cosh(2\pi n)]}\boxed{\int_0^{2\pi} H_{n_{s,c}}(\xi)\cosh[n(2\pi-\xi)]\,d\xi} + \frac{1}{2n}\boxed{\int_0^{2\pi} H_{n_{s,c}}\sinh[n(2\pi-\xi)]\,d\xi}$$

$$E_{s,c} = F_{s,c}\frac{[1-\cosh(2\pi n)]}{\sinh(2\pi n)} - \frac{1}{n\sinh(2\pi n)}\int_0^{2\pi} H_{n_{s,c}}\sinh[n(2\pi-\xi)]\,d\xi$$

$$H_0(\theta_1) = \frac{1}{2\pi}\int_0^{2\pi} F(\theta_1,\theta_2)\,d\theta_2$$

$$H_{nc}(\theta_1) = \frac{1}{\pi}\int_0^{2\pi} F(\theta_1,\theta_2)\cos(n\theta_2)\,d\theta_2$$

$$H_{ns}(\theta_1) = \frac{1}{\pi}\int_0^{2\pi} F(\theta_1,\theta_2)\sin(n\theta_2)\,d\theta_2$$

Note:

$$\left.\begin{array}{l} A_{ns} = A_{ns,c} \Rightarrow E_{ns,c} \Rightarrow F_{ns,c} \Rightarrow H_{ns,c} \\ A_{nc} = A_{ns,c} \Rightarrow E_{ns,c} \Rightarrow F_{ns,c} \Rightarrow H_{ns,c} \end{array}\right\}$$ Notation