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Comment

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J. Curély in paper [1] claims to have found an exact analytical solution of the classical two-dimensional Heisenberg model. I would like to make the following comments on this very surprising result:

i) The expression for the partition function derived by the author in eqs. (3), (4) of the paper has been established by Joyce [2] in 1967. This work should have been mentioned.

ii) The simplification of this expression rests on the affirmation (in the discussion between eq. (4) and eq. (5)) that "... a single (solution) allows to obtain the term of higher degree in the λ_ℓ polynomial expansion of $Z(0)$". This assertion is not proved.

iii) After making this hypothesis, the author obtains the following expression for the zero-field partition function:

$$Z_N(0) = [(4\pi)^2 \lambda_0(-\beta J_1)\lambda_0(-\beta J_2)]^{2N(2N+1)} \, . \tag{1}$$

Surprisingly, it is exactly the product of the one-dimensional partition functions along each spatial direction [2], [3]. By performing a high-temperature expansion of this expression, one observes that it is not correct. Setting $J_1 = J_2 = J$, $x = -\beta J$ and $N_\mathrm{S} = 2N(2N+1)$, eq. (1) may be expanded to yield

$$\frac{1}{(4\pi)^{N_\mathrm{S}}} Z_N(0) = 1 + \frac{1}{3}x^2 N_\mathrm{S} + x^4 \left[\frac{1}{60}N_\mathrm{S} + \frac{1}{36}N_\mathrm{S}(2N_\mathrm{S}-1)\right] + \mathrm{O}(x^6) \, .$$

It is easy to derive the expansion of the full original partition function to this low order. At the 4th order, one gets an extra term, $\frac{1}{27}x^4 N_\mathrm{S}$ coming from the one-loop diagrams, which, of course, cannot be treated by the factorized partition function of eq. (1).

iv) Similarly, the expression for the susceptibility (eq. (9) of the paper) can be re-expressed in a factorized form (for $G = G'$):

$$\frac{3k_\mathrm{B}T}{N_\mathrm{S}G^2}\chi = \frac{1+u_1}{1-u_1}\frac{1+u_2}{1-u_2} \, ,$$

which is again the product of one-dimensional susceptibilities. That this trivial result is not correct can be checked by comparing its high-temperature expansion to the result published in the well-known review article of Rushbrooke *et al.* [4]. Upon setting $G = G'$, $J_1 = J_2 = J$, $x = -\beta J$ and $u(x) = \mathcal{L}(2x)$ (the factor 2 takes into account the definition of the coupling in ref. [4]), eq. (9) of Curély becomes

$$\frac{3k_{\mathrm{B}}T}{N_{\mathrm{S}}G^2}\chi = \left(\frac{1+u}{1-u}\right)^2 = 1 + \frac{8}{3}x + \frac{32}{9}x^2 + \frac{128}{45}x^3 + \cdots ,$$

whereas the result of Rushbrooke *et al.* is

$$\frac{3k_{\mathrm{B}}T}{N_{\mathrm{S}}G^2}\chi = 1 + \frac{8}{3}x + \frac{16}{3}x^2 + \frac{448}{45}x^3 + \cdots$$

I conclude that the 2D Heisenberg model is still unsolved.

REFERENCES

[1] Curély J., *Europhys. Lett.*, **32** (1995) 529.
[2] Joyce G. S., *Phys. Rev.*, **155** (1967) 478.
[3] Fisher M. E., *Am. J. Phys.*, **32** (1964) 343.
[4] Rushbrooke G. S., Baker G. A. and Wood P. J., *Phase Transistions and Critical Phenomena*, edited by Domb and Green, Vol. **3** (Academic Press), p. 245 and appendix III p. 345.