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Critical temperature for the 2D Heisenberg model

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Abstract. Series, in powers of inverse coordination number, are derived for the critical temperature of 2D classical and quantum Heisenberg models from high-temperature series-expansion results. The results show convincingly that a phase transition occurs at a non-zero temperature.

1. Introduction

The two-dimensional Heisenberg model is one of the most problematic models of the theory of second-order phase transitions. In fact, answers given by several authors on the question whether there is a phase transition at a nonzero temperature $v_c > 0$ are different. One fact is clear: there is no long-range order (Mermin and Wagner 1966) therefore the phase occurring at the temperature $v < v_c$ (if $v_c > 0$) is not a ferromagnetic one.

Spin-wave theory gives $v_c = 0$. However, the applicability of that approach is unfounded as pointed out by Stanley and Kaplan (1966). Using the simple ratio method for evaluating the high-temperature series-expansion results for the classical two-dimensional Heisenberg model, they arrived at the conclusion that there is a phase transition, $v_c > 0$. Because their conclusion was not convincing enough, Brezin and Zinn-Justin (1976) examined the question using a field-theoretical method. They arrived at the conclusion that $v_c = 0$. The same was obtained by Migdal (1975) on the basis of the renormalisation group method realised in real space. Forgacs and Zawadowski (1978) have refined Migdal's scheme but have obtained the same results: $v_c = 0$. We mention also the work by Polyakov (1976). We re-examined the problem on the basis of the results obtained by Stanley and Kaplan for the classical spins, and by Baker *et al* (1970) for the $S = \frac{1}{2}$ case, our conclusion is that $v_c > 0$.

2. Results and conclusion

Our results are obtained from the series of $\theta = zJ/k_B v$ in powers of $1 - \chi^{-1}$. The expansion can be obtained as follows.

As is known (Baker *et al* 1970), inverse susceptibility χ^{-1} can be expanded into powers of the dimensionless high-temperature parameter $\theta = zJ/k_B v$ in the form

$$\chi^{-1} = 1 - \sum_{m=1}^{\infty} a_m \theta^m. \qquad (1)$$

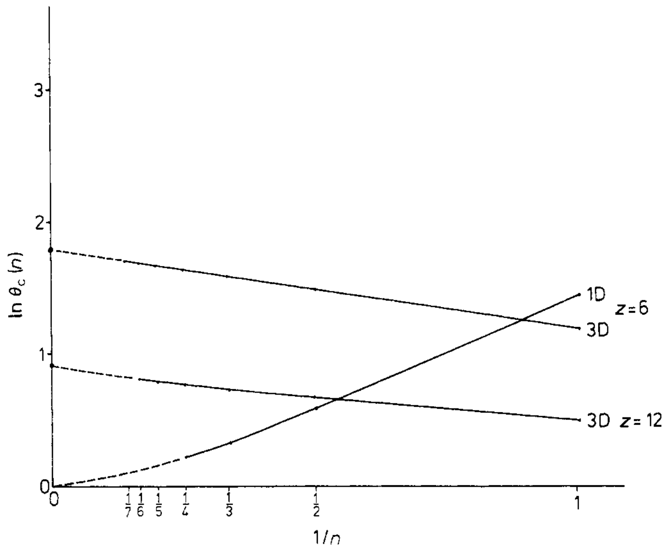

Figure 1. Values of $\ln \theta_c^{(n)}$ as a function of $1/n$ for the one- and three-dimensional Heisenberg model at $z = 6$ and 12. Extrapolated values are zero and the same as obtained by Baker, Eve and Rushbrooke, respectively.

From that expansion, θ can be expressed as

$$\theta = \sum_{n=1}^{\infty} A_n(1 - \chi^{-1})^n$$

where the coefficients A_n can be calculated from a_m with the use of Lagrange inversion formulae.

If we define the dimensionless inverse critical temperature $\theta_c = zJ/k_B v_c$ by the zero value of the inverse susceptibility $\chi^{-1} = 0$, then we get from equation 1,

$$\theta_c = \sum_{m=1}^{\infty} A_m. \tag{2}$$

Because a_m in equation 1 is determined (Baker *et al* 1970; Stanley and Kaplan 1966) up to a finite value of n, m, we can get approximations for θ_c of finite order only

$$\theta_c^{(n)} = \sum_{m=1}^{n} A_m.$$

Therefore, the best we can do is to rescale $\theta_c^{(n)}$ and n to obtain the most linear dependence and then to extrapolate.

Table 1. Values of $1/\ln \theta_c(n)$ as a function of n at several values of S and z.

	n	1	2	3	4	5	6
$S = \frac{1}{2}$	$z = 4$	2·466	1·650	1·386	1·266	1·202	1·160
$S = \frac{1}{2}$	$z = 6$	3·476	2·204	1·790	1·588	1·470	1·393
$S = \infty$	$z = 4$	2·466	1·757	1·526	1·409	1·336	—
$S = \infty$	$z = 6$	6·487	4·230	3·476	3·090	2·850	—

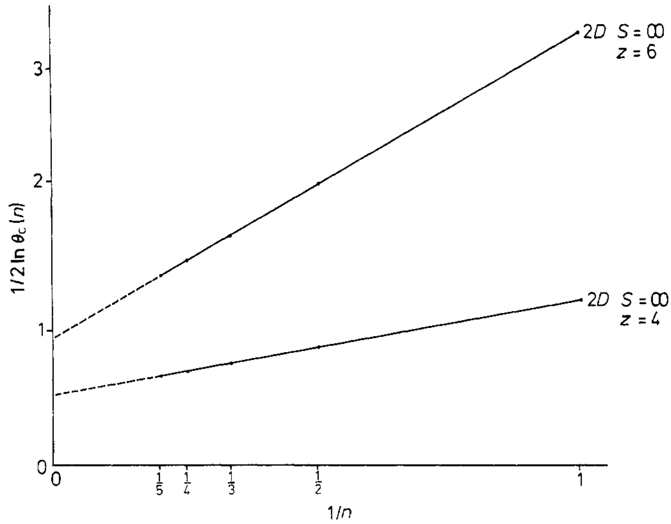

Figure 2. Values of $1/\ln\,\theta_c^{(n)}$ as a function of $1/n$ for the two-dimensional, $S = \infty$ Heisenberg model at $z = 4$ and 6.

The simple method explained above is controlled in cases of one- and three-dimensional Heisenberg models. The results (figure 1) are excellent. In fact, it is confirmed that $\nu_c = 0$ for the one-dimensional model and the value of the critical temperature obtained by extrapolation agrees very well with that obtained by Baker *et al* using Padé approximants (1970) for $D = 3$. Notice that the linearity can be improved by slight modifications of the rescalings.

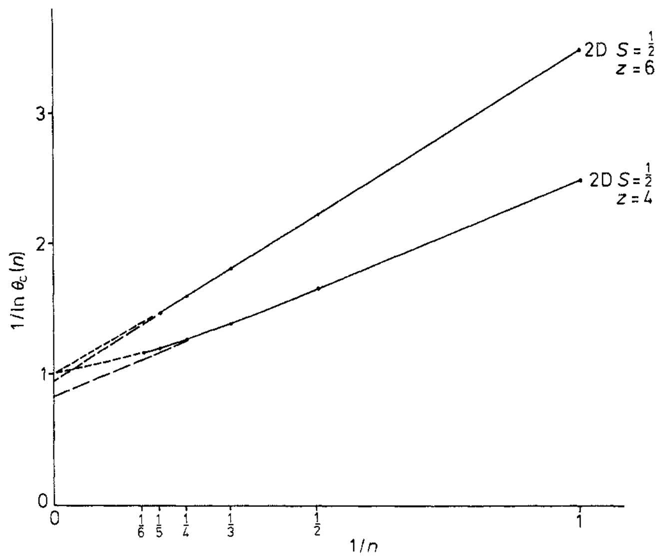

Figure 3. Values of $1/\ln\,\theta_c^{(n)}$ as a function of $1/n$ for the two-dimensional $S = \frac{1}{2}$ Heisenberg model at $z = 4$ and 6.

For the two-dimensional case, the values of approximants of θ_c, $\theta_c^{(n)}$ are listed in table 1, and $1/\ln \theta_c^{(n)}$ as functions of $1/n$ are shown in cases of $s = \infty$ and $\frac{1}{2}$ and $z = 4$ and 6 by figures 2 and 3.

The results are, in our opinion, astonishingly good. In fact, the convergence of approximants is nearly ideal for the classical spin cases and is even more convincing than the linear one in cases $S = \frac{1}{2}$. If we are to be objective we have to state, with respect to the results above, that there is a phase transition in the two-dimensional Heisenberg model.

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