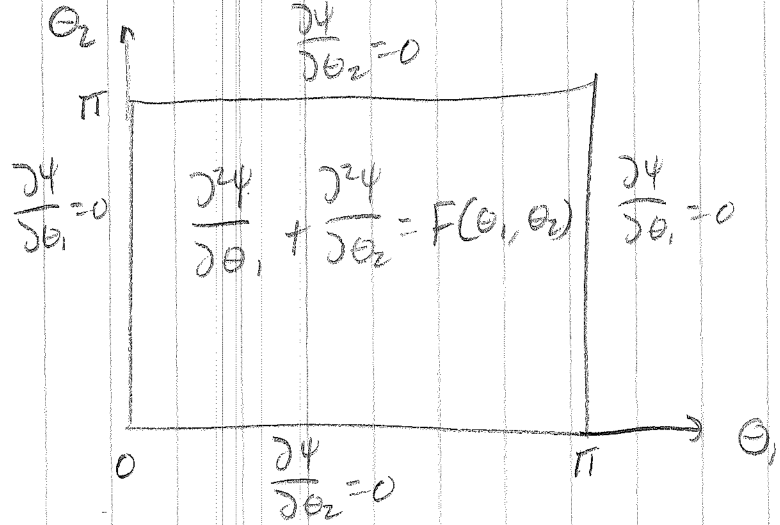
Solve



$F(\theta_1,\theta_2)$ is forcing supplied by Dave

Choose θ_2 as basis direction, & create bases functions $B_n(\theta_2)$

Let: $\psi = \sum_n A_n(\theta_1)\, B_n(\theta_2)$

Sub into PDE

$$\sum_n \left[\frac{d^2 A_n}{d\theta_1^2} B_n + \frac{d^2 B_n}{d\theta_2^2} A_n\right] = F(\theta_1,\theta_2)$$

$$\sum_n \left[\frac{d^2 A_n}{d\theta_1^2} + \frac{A_n}{B_n}\frac{d^2 B_n}{d\theta_2^2}\right] B_n = F(\theta_1,\theta_2)$$

For sepability, require $\frac{1}{B_n}\frac{d^2 B_n}{d\theta_2^2} = K_n$ ← Constant

$$\boxed{\sum_n \left[\frac{d^2 A_n}{d\theta_1^2} + K_n A_n\right] B_n = F(\theta_1, \theta_2)}$$

$\frac{\partial \psi}{\partial \theta_2} = 0 \quad @ \theta = 0 \Rightarrow \sum_n A_n(\theta_1) \frac{dB_n}{d\theta_2}\Big|_{\theta=0} = 0$

$\frac{\partial \psi}{\partial \theta_2} = 0 \quad @ \theta = \pi \Rightarrow \sum_n A_n(\theta_1) \frac{dB_n}{d\theta_2}\Big|_{\theta=\pi} = 0$

<u>Basis</u> <u>functions</u> <u>satisfy</u>:

$$\boxed{\begin{aligned} &\frac{d^2 B_n}{d\theta_2^2} = K_n B_n \\ &\left.\begin{aligned} &\frac{dB_n}{d\theta_2} = 0 \quad @ \theta = 0 \\ &\frac{dB_n}{d\theta_2} = 0 \quad @ \theta = \pi \end{aligned}\right\} \end{aligned}}$$

This is a Sturm-Liouville problem

⇒ ① Eigenvalues K_n are real

② Eigenvectors form a basis w/ inner product $\langle f, g\rangle = \int_0^\pi f g \, d\theta_2$

③ 1 eigenvector per eigenvalue

④ Eigenvectors of distinct eigenvalues are ⊥ to one another using inner product $\langle f, g\rangle$ above

Perform eigensearch:

$K_n = 0$:

$$\frac{d^2 B_n}{d\theta_z^2} = 0 \Rightarrow B_n = C\theta_z + D$$

BCS: $\frac{dB_n}{d\theta_z} = 0 \Rightarrow C = 0 \Rightarrow B_n = D \equiv \text{Constant}$

@ $\theta_z = 0, \pi$

$$\Rightarrow \boxed{B_n = 1 \text{ is nontrivial eigenfunction for } K_n = 0}$$

Positive eigenvalues: Let $K_n = \gamma_n^2$, $\gamma_n > 0$ (avoid repetitive eigenvalues!)

$$\frac{d^2 B_n}{d\theta_z^2} - \gamma_n^2 B_n = 0$$

$$B_n = E \sinh \gamma_n \theta_z + F \cosh \gamma_n \theta_z$$

$$\frac{dB_n}{d\theta_z} = (E \cosh \gamma_n \theta_z)\gamma_n + F\gamma_n \sinh \gamma_n \theta_z$$

@ $\Theta_z = 0$, $\frac{dB_n}{d\Theta_z} = 0 \Rightarrow E = 0$

$\Rightarrow \frac{dB_n}{d\Theta_z} = F\gamma_n \sinh\gamma_n\Theta_z$ @ $\Theta_z = \pi$, $\frac{dB_n}{d\Theta_z} = 0 \Rightarrow F = 0$

$\boxed{\text{Conclusion: No Positive eigenvalues}}$

Negative Eigenvalues: Let $K_n = -\gamma_n^2$, $\gamma_n > 0$ (Avoid repetitive eigenvalues!)

$$\frac{d^2B_n}{d\Theta_z^2} + \gamma_n^2 B_n = 0$$

$$\Rightarrow B_n = G\sin\gamma_n\Theta_z + H\cos\gamma_n\Theta_z$$

$$\frac{dB_n}{d\Theta_z} = G\gamma_n\cos\gamma_n\Theta_z - H\gamma_n\sin\gamma_n\Theta_z$$

$\frac{dB_n}{d\Theta_z} = 0$ @ $\Theta_z = 0 \Rightarrow G = 0$

$\Rightarrow B_n = H\cos\gamma_n\Theta_z$

$\frac{dB_n}{d\Theta_z} = 0$ @ $\Theta_z = \pi \Rightarrow \sin\gamma_n\pi = 0$

$\Rightarrow \gamma_n\pi = n\pi \Rightarrow \gamma_n = n$

$n = 1, 2, 3, \ldots$

$$\Rightarrow \boxed{K_n = -n^2,\ B_n = \cos n\Theta_z}$$

So, we have:

$$B_n = \begin{cases} 1 & K_n = 0, \ n = 0 \\ \cos n\theta_2 & K_n = -n^2, \ n = 1, 2, 3, \ldots \end{cases}$$

$$\langle B_n, B_m \rangle = \int_0^{\pi} B_n B_m \, d\theta_2 = 0 \quad \text{for } n \neq m.$$

$$\langle B_n, B_n \rangle = ?$$

$n=0$:

$$\langle B_0, B_0 \rangle = \int_0^{\pi} 1 \cdot 1 \, d\theta_2 = \pi$$

$n>0$:

$$\langle B_n, B_n \rangle = \int_0^{\pi} \cos^2 n\theta_2 \, d\theta_2 = \int_0^{\pi} \left(\frac{1}{2} + \frac{1}{2}\cos 2n\theta_2\right) d\theta_2 = \frac{\pi}{2} + \frac{1}{4n} \sin 2n\theta_2 \Big|_0^{\pi}$$

$$= \frac{\pi}{2}$$

So: Eigenvalue/vector summary

$$B_n = \begin{cases} 1 & K_0 = 0, \quad n = 0 \\ \cos n\theta_z & K_n = -n^2, \quad n > 0 \end{cases}$$

$$\langle B_n, B_m \rangle = \begin{cases} 0, & n \neq m \\ \pi, & n = m = 0 \\ \frac{\pi}{2}, & n = m \geq 1 \end{cases}$$

$$\langle f, g \rangle = \int_0^{\pi} f\, g\, d\theta_z$$ is inner product.

Back to pages ① + ②: Rewrite expressions using basis, page ⑥

$$\psi = \sum_n A_n(\theta_1) B_n(\theta_2) \Leftarrow \text{page ①}$$

$$\boxed{\psi = A_0(\theta_1) + \sum_{n=1}^{\infty} A_n(\theta_1) \cos n\theta_2} \quad i$$

$$\sum_n \left[\frac{d^2 A_n}{d\theta_1^2} + K_n A_n \right] B_n = F(\theta_1, \theta_2) \Leftarrow \text{Page ②}$$

$$\Rightarrow \boxed{\frac{d^2 A_0}{d\theta_1^2} + \sum_{n=1}^{\infty} \left[\frac{d^2 A_n}{d\theta_1^2} - n^2 A_n \right] \cos n\theta_2 = F(\theta_1, \theta_2)} \quad ii)$$

Find eqns for A_0, A_n:

Use orthogonality of eigenvectors, page ⑥

Inner product of eqn i) w.r.t. basis functions

$B_0 = 1$:

$$\left\langle \frac{d^2A_0}{d\theta_1^2} + \sum_{n=1}^{\infty}\left[\frac{d^2A_n}{d\theta_1^2} - n^2A_n\right]\cos n\theta_2,\ 1\right\rangle = \langle F(\theta_1,\theta_2), 1\rangle$$

$$\langle f,g\rangle = \int_0^{\pi} fg\, d\theta_2 \Rightarrow$$

$$\frac{d^2A_0}{d\theta_1^2}\langle 1,1\rangle + \sum_{n=1}^{\infty}\left(\frac{d^2A_n}{d\theta_1^2} - n^2A_n\right)\langle \cos n\theta_2, 1\rangle = \langle F(\theta_1,\theta_2), 1\rangle$$

$\langle \cos n\theta_2, 1\rangle = 0$ from page ⑥

$$\Rightarrow \boxed{\frac{d^2A_0}{d\theta_1^2} = \frac{\langle F(\theta_1,\theta_2), 1\rangle}{\langle 1,1\rangle}}$$

Let: $B_m = \cos m\theta_2$, $m = 1, 2, 3, \ldots$ Take inner product of eqn ii) of page ⑦:

$$\left\langle \frac{d^2 A_0}{d\theta_1^2} + \sum_{n=1}^{\infty}\left(\frac{d^2 A_n}{d\theta_1^2} - n^2 A_n\right)\cos n\theta_2,\ \cos m\theta_2 \right\rangle = \langle F(\theta_1, \theta_2), \cos m\theta_2 \rangle$$

$$\langle f, g \rangle = \int_0^{\pi} fg \, d\theta_2 \Rightarrow$$

$$\frac{d^2 A_0}{d\theta_1^2}\langle 1, \cos m\theta_2 \rangle + \sum_{n=1}^{\infty}\left(\frac{d^2 A_n}{d\theta_1^2} - n^2 A_n\right)\langle \cos n\theta_2, \cos m\theta_2 \rangle = \langle F(\theta_1, \theta_2), \cos m\theta_2 \rangle$$

$\langle 1, \cos m\theta_2 \rangle = 0 \quad (m > 0)$

$\langle \cos n\theta_2, \cos m\theta_2 \rangle = 0$ for $n \neq m$; $\neq 0$ for $n = m$

$$\Rightarrow \frac{d^2 A_m}{d\theta_1^2} - m^2 A_m = \frac{\langle F(\theta_1, \theta_2), \cos m\theta_2 \rangle}{\langle \cos m\theta_2, \cos m\theta_2 \rangle}$$

m is dummy variable, so can rewrite m ≡ n:

$$\Rightarrow \boxed{\frac{d^2 A_n}{d\theta_1^2} - n^2 A_n = \frac{\langle F(\theta_1,\theta_2), \cos n\theta_2 \rangle}{\langle \cos n\theta_2, \cos n\theta_2 \rangle}, \quad n = 1, 2, 3 \ldots}$$

Find BCS:

$$@\ \theta_1 = 0, \pi, \quad \frac{d\psi}{d\theta_1} = 0$$

From page 7:

$$\psi = A_0(\theta_1) + \sum_{n=1}^{\infty} A_n(\theta_1) \cos n\theta_2$$

$$\frac{\partial \psi}{\partial \theta_1} = \frac{dA_0}{d\theta_1} + \sum_{n=1}^{\infty} \frac{dA_n}{d\theta_1} \cos n\theta_2$$

$$\Rightarrow @\ \theta = 0, \ \frac{\partial \psi}{\partial \theta_1} = 0 \quad \Rightarrow \quad \boxed{0 = \left.\frac{dA_0}{d\theta_1}\right|_{\theta_1=0} + \sum_{n=1}^{\infty} \left.\frac{dA_n}{d\theta_1}\right|_{\theta_1=0} \cos(n\theta_2)}$$

Take inner product of boxed expression at bottom of page (10) w.r.t. $\underline{1}$:

$$0 = \left.\frac{dA_0}{d\theta_1}\right|_{\theta_1=0} \langle 1, 1\rangle + \sum_{n=1}^{\infty} \left.\frac{dA_n}{d\theta_1}\right|_{\theta_1=0} \langle \cos(n\theta_2), 1\rangle$$

0 from orthogonality

$$\Rightarrow \boxed{\left.\frac{dA_0}{d\theta_1}\right|_{\theta_1=0} = 0}$$

Take inner product of same expression w.r.t. $\cos(m\theta_2)$, $m>0$:

$$0 = \left.\frac{dA_0}{d\theta_1}\right|_{\theta_1=0} \langle 1, \cos(m\theta_2)\rangle + \sum_{n=1}^{\infty} \left.\frac{dA_n}{d\theta_1}\right|_{\theta_1=0} \langle \cos(n\theta_2), \cos(m\theta_2)\rangle$$

0 from orthogonality

0 except when $n=m$

$$0 = \left.\frac{dA_m}{d\theta_1}\right|_{\theta_1=0} \Rightarrow \text{Since index is dummy, } \boxed{\left.\frac{dA_n}{d\theta_1}\right|_{\theta_1=0} = 0}$$

Same process/results hold @ $\theta = \pi$.

Summarizing eqns + BCs thus far:

$$\psi = A_0(\theta_1) + \sum_{n=1}^{\infty} A_n(\theta_1)\cos n\theta_2 \quad ,$$

$$\boxed{\begin{gathered} \frac{d^2 A_0}{d\theta_1^2} = \frac{\langle F(\theta_1,\theta_2), 1\rangle}{\langle 1, 1\rangle} \\ \frac{dA_0}{d\theta_1} = 0 \quad @ \ \theta = 0, \pi \end{gathered}}$$

$$\boxed{\begin{gathered} \frac{d^2 A_n}{d\theta_1^2} - n^2 A_n = \frac{\langle F(\theta_1,\theta_2), \cos n\theta_2\rangle}{\langle \cos n\theta_2, \cos n\theta_2\rangle} \quad , \quad n > 0 \\ \frac{dA_n}{d\theta_1} = 0 \quad @ \ \theta = 0, \pi \end{gathered}}$$

$$\langle f, g\rangle = \int_0^{\pi} f g \, d\theta_2$$

Rewrite eqns, page (12); using $\int_0^{\pi} f, g \, d\theta_2 = \langle f, g \rangle$

$$\frac{\langle F(\theta_1, \theta_2), 1 \rangle}{\underbrace{\langle 1, 1 \rangle}_{= \pi, \text{ page } 6}} = \frac{1}{\pi} \langle F(\theta_1, \theta_2), 1 \rangle = \frac{1}{\pi} \int_0^{\pi} F(\theta_1, \theta_2) \, d\theta_2$$

$$\frac{\langle F(\theta_1, \theta_2), \cos n\theta_2 \rangle}{\underbrace{\langle \cos n\theta_2, \cos n\theta_2 \rangle}_{= \frac{\pi}{2}, \text{ page } 6}} = \frac{2}{\pi} \int_0^{\pi} F(\theta_1, \theta_2) \cos n\theta_2 \, d\theta_2$$

Then:

$$\boxed{\begin{aligned} &\frac{d^2A_0}{d\theta_1^2} = H_0(\theta_1) \\ &\frac{dA_0}{d\theta_1} = 0 \quad @\ \theta = 0, \pi \end{aligned}}$$

$$\boxed{\begin{aligned} &\frac{d^2A_n}{d\theta_1^2} - n^2 A_n = H_n(\theta_1) \\ &\frac{dA_n}{d\theta_1} = 0 \quad @\ \theta_1 = 0, \pi \end{aligned}}$$

$$\Bigg\rangle \ \boxed{\psi = A_0(\theta_1) + \sum_{n=1}^{\infty} A_n(\theta_1) \cos n\theta_2}$$

Where:

$$H_0(\theta_1) = \frac{\langle F(\theta_1, \theta_2), 1 \rangle}{\langle 1, 1 \rangle} = \frac{1}{\pi} \int_0^{\pi} F(\theta_1, \theta_2)\, d\theta_2$$

$$H_n(\theta_1) = \frac{\langle F(\theta_1, \theta_2), \cos n\theta_2 \rangle}{\langle \cos n\theta_2, \cos n\theta_2 \rangle} = \frac{2}{\pi} \int_0^{\pi} F(\theta_1, \theta_2) \cos n\theta_2\, d\theta_2$$

Solve BVPs, page 14.

A_0:

$$\frac{d^2A_0}{d\theta_1^2} = W_0(\theta_1) \quad , \quad \frac{dA_0}{d\theta_1} = 0 \quad @\ \theta = 0, \pi$$

particular soln

$$A_0 = A_{0h} + A_{0p}$$

Homogeneous soln

<u>Homogeneous soln:</u>

$$\frac{d^2A_{0h}}{d\theta_1^2} = 0 \Rightarrow \quad A_{0h} = C\theta_1 + D$$, C + D arbitrary @ this point.

<u>Particular soln:</u> Use variation of parameters or... Laplace transforms w/ easy initial conditions.

$\hookrightarrow$

Use LaPlace; w $\frac{dA_{op}}{d\theta_1} = A_{op} = 0$ @ $\theta_1 = 0$ (Homogeneous soln compensates for these ICs!)

$$\mathcal{L}\left[\frac{d^2 A_{op}}{d\theta_1^2}\right] = \mathcal{L}\left[N_o(\theta_1)\right]$$

$$s^2 \bar{A}_{op}(s) = \bar{N}_o(s)$$

$$\bar{A}_{op}(s) = \frac{1}{s^2}\bar{N}_o(s)$$

Use convolution to invert: Let $\boxed{A_{op}(\theta_1) = \int_0^{\theta_1} f(\xi)\, g(\theta_1 - \xi)\, d\xi}$

$$\mathcal{L}\left[\int_0^{\theta_1} f(\xi)\, g(\theta_1 - \xi)\, d\xi\right] = F(s)\, G(s) = \bar{N}_o(s)\frac{1}{s^2}$$

$$F(s) = \bar{N}_o(s) \Rightarrow f(\theta_1) = \mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}[\bar{N}_o(s)] = N_o(\theta_1)$$

$$\boxed{G(s) = \frac{1}{s^2} \Rightarrow g(\theta_1) = \mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = \theta_1}$$

So...

$$\boxed{A_{op}(\theta_1) = \int_0^{\theta_1} H_o(\xi)(\theta_1 - \xi)\, d\xi}$$

Then:

$$A_o = A_{oh} + A_{op} = C\theta_1 + D + \int_0^{\theta_1} H_o(\xi)(\theta_1 - \xi)\, d\xi$$

Apply BC: $\frac{dA_o}{d\theta_1} = 0$ @ $\theta_1 = 0, \pi$

$$\frac{dA_o}{d\theta_1} = C + \frac{d}{d\theta_1}\int_0^{\theta_1} H_o(\xi)(\theta_1 - \xi)\, d\xi$$

Leibnitz rule: $\frac{d}{d\theta_1}\int_0^{\theta_1} H_o(\xi)(\theta_1 - \xi)\, d\xi = \int_0^{\theta_1} H_o(\xi)\, d\xi + \cancel{\frac{d\theta_1}{d\theta_1} H_o(\theta_1)(\theta_1 - \theta_1)}$ 0

So:

$$\frac{dA_0}{d\theta_1} = C + \int_0^{\theta_1} H_0(\xi)\, d\xi$$

$$\frac{dA_0}{d\theta_1} = 0 \quad @ \ \theta_1 = 0 \ \Rightarrow \ C = 0$$

$$\frac{dA_0}{d\theta_1} = 0 \quad @ \ \theta_1 = \pi \ \Rightarrow \ \boxed{\int_0^{\pi} H_0(\xi)\, d\xi = 0}$$

Solvability condition

Then:

$$\boxed{A_0 = D + \int_0^{\theta_1} H_0(\xi)(\theta_1 - \xi)\, d\xi \ , \ \text{where} \int_0^{\pi} H_0(\xi)\, d\xi = 0}$$

Soln

Solve:

$$\frac{d^2 A_n}{d\theta_1^2} - n^2 A_n = W_n(\theta_1)$$

$$\frac{dA_n}{d\theta_1} = 0 \quad @\ \theta_1 = 0, \pi$$

Soln:

$$A_n = A_{nh} + A_{np}$$

A_{np} ← Particular

A_{nh} ← Homogeneous

<u>Homogeneous</u>:

$$\frac{d^2 A_{nh}}{d\theta_1^2} - n^2 A_{nh} = 0 \quad \Rightarrow \quad \boxed{A_{nh} = E\sinh(n\theta_1) + F\cosh(n\theta_1)}$$

E ← constant

F ← constant

Particular soln (Use variation of parameters or Laplace transforms w/ convolution)

Solve:

$$\frac{d^2 A_{np}}{d\theta_1^2} - n^2 A_{np} = H_n(\theta_1)$$

$$\frac{dA_n}{d\theta_1} = A_n = 0 \quad @ \ \theta_1 = 0$$ (Homogeneous soln compensates for the choice of BCs!)

$$\mathcal{L}\left[\frac{d^2 A_{np}}{d\theta_1^2} - n^2 A_{np}\right] = \mathcal{L}\left[H_n(\theta_1)\right]$$

$$\Rightarrow \quad s^2 \bar{A}_{np}(s) - n^2 \bar{A}_{np}(s) = \bar{H}_n(s)$$

$$\Rightarrow \quad \bar{A}_{np}(s) = \left(\frac{1}{s^2 - n^2}\right) \bar{H}_n(s)$$

Let:

$$\boxed{A_{np}(\Theta_1) = \int_0^{\Theta_1} f(\xi)\, g(\Theta_1 - \xi)\, d\xi}$$

$$\Rightarrow \mathcal{L}\left[A_{np}(\Theta_1)\right] = F(s)\, G(s) = \bar{H}_n(s)\left(\frac{1}{s^2 - n^2}\right)$$

Let $F(s) = \bar{H}_n(s) \Rightarrow \boxed{f(\Theta_1) = H_n(\Theta_1)}$,

$G(s) = \dfrac{1}{s^2 - n^2}$, $\mathcal{L}\left[\sinh(k\Theta_1)\right] = \dfrac{k}{s^2 - k^2}$

$$\Rightarrow \mathcal{L}\left[\frac{1}{k}\sinh(k\Theta_1)\right] = \frac{1}{s^2 - k^2}$$

If $k = n \Rightarrow \mathcal{L}\left[\frac{1}{n}\sinh(n\Theta_1)\right] = \dfrac{1}{s^2 - n^2}$

So, $\boxed{g(\Theta_1) = \frac{1}{n}\sinh(n\Theta_1)}$

Then:

$$\boxed{A_{np}(\theta_1) = \int_0^{\theta_1} H_n(\xi)\,\frac{1}{n}\sinh\left[n(\theta_1-\xi)\right]d\xi}$$

From page 19:

$$A_n = A_{nh} + A_{np}$$

$$\Rightarrow A_n = E\sinh(n\theta_1) + F\cosh(n\theta_1) + \frac{1}{n}\int_0^{\theta_1} H_n(\xi)\sinh\left[n(\theta_1-\xi)\right]d\xi$$

Find $E + F$ from BCs, page 19: $\frac{dA_n}{d\theta_1} = 0$ @ $\theta_1 = 0, \pi$

Leibnitz' Rule:

$$\frac{d}{d\theta_1}\left[\frac{1}{n}\int_0^{\theta_1} H_n(\xi)\sinh[n(\theta_1-\xi)]\,d\xi\right] = \int_0^{\theta_1} H_n(\xi)\cosh[n(\theta_1-\xi)]\,d\xi$$

$$+\ \frac{1}{n}\frac{d\theta_1}{d\theta_1} H_n(\xi)\cancel{\sinh[n(\theta_1-\xi)]}\Big|_{\xi=\theta_1}$$

$$\Rightarrow \frac{d}{d\theta_1}\left[\frac{1}{n}\int_0^{\theta_1} H_n(\xi)\sinh[n(\theta_1-\xi)]\,d\xi\right] = \int_0^{\theta_1} H_n(\xi)\cosh[n(\theta_1-\xi)]\,d\xi$$

So:

$$\frac{dA_n}{d\theta_1} = nE\cosh[n\theta_1] + F_n\sinh[n\theta_1] + \int_0^{\theta_1} H_n(\xi)\cosh[n(\theta_1-\xi)]\,d\xi$$

$$@\ \theta=0 \quad \frac{dA_n}{d\theta_1}=0 \quad \Rightarrow \boxed{E=0}$$

$$\Rightarrow \frac{dA_n}{d\theta_1} = F_n\sinh[n\theta_1] + \int_0^{\theta_1} H_n(\xi)\cosh[n(\theta_1-\xi)]\,d\xi$$

@ $\theta = \pi$, $\frac{dA_n}{d\theta_1} = 0$

$$\Rightarrow \quad 0 = F_n \sinh[n\pi] + \int_0^{\pi} H_n(\xi) \cosh[n(\pi - \xi)]\, d\xi$$

$$\Rightarrow \quad \boxed{F = -\frac{1}{n \sinh[n\pi]} \int_0^{\pi} H_n(\xi) \cosh[n(\pi - \xi)]\, d\xi}$$

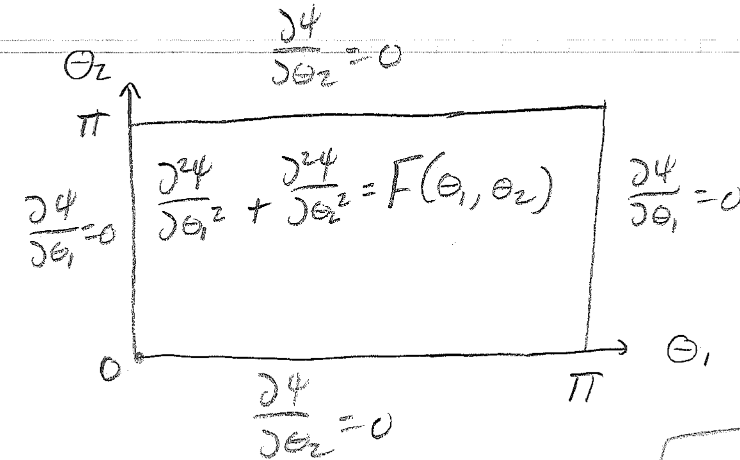
Then:

$$A_n = F\cosh(n\theta_1) + \frac{1}{n}\int_0^{\theta_1} H_n(\xi)\sinh[n(\theta_1-\xi)]\,d\xi$$

$$\Rightarrow A_n = \frac{-\cosh(n\theta_1)}{n\sinh[n\pi]} + \frac{1}{n}\int_0^{\theta_1} H_n(\xi)\sinh[n(\theta_1-\xi)]\,d\xi$$

Summary:

Soln to:



$$\psi = A_0(\Theta_1) + \sum_{n=1}^{\infty} A_n(\Theta_1) \cos(n\Theta_2)$$

Must be satisfied for soln to exist!

$$\boxed{\text{Solvability} \Rightarrow \int_0^{\pi} H_0(\xi)\, d\xi = 0}$$

$$A_0(\Theta_1) = D + \int_0^{\Theta_1} H_0(\xi)(\Theta_1 - \xi)\, d\xi$$

$D \equiv$ Arbitrary Constant

$$A_n(\Theta_1) = \frac{1}{n} \int_0^{\Theta_1} H_n(\xi) \sinh[n(\Theta_1 - \xi)]\, d\xi \; - \; \frac{\cosh(n\Theta_1)}{n \sinh[n\pi]}$$

$$H_0(\Theta_1) = \frac{1}{\pi} \int_0^{\pi} F(\Theta_1, \Theta_2)\, d\Theta_2, \quad H_n(\Theta_1) = \frac{2}{\pi} \int_0^{\pi} F(\Theta_1, \Theta_2) \cos n\Theta_2\, d\Theta_2$$

Need: $\frac{\partial \psi}{\partial \theta_1}, \frac{\partial \psi}{\partial \theta_2}$:

page (23) - written below

$$\frac{\partial \psi}{\partial \theta_1} = \frac{dA_0}{d\theta_1} + \sum_{n=1}^{\infty} \frac{dA_n}{d\theta_1} \cos(n\theta_2)$$

page (18) - written below

$$\frac{dA_0}{d\theta_1} = \int_0^{\theta_1} H_0(\xi)\, d\xi$$

$$\frac{dA_n}{d\theta_1} = \int_0^{\theta_1} H_n(\xi) \cosh[n(\theta_1 - \xi)]\, d\xi - \frac{\sinh[n\theta_1]}{\sinh[n\pi]} \int_0^{\pi} H_n(\xi) \cosh[n(\pi - \xi)]\, d\xi$$

$$\frac{\partial \psi}{\partial \theta_2} = -\sum_{n=1}^{\infty} n A_n(\theta_1) \sin(n\theta_2)$$