

# A Comprehensive Molecular-Based Study of the Stability of Clathrate Hydrates

## 1 ABSTRACT

Clathrate hydrates are crystalline compounds of significant technological and environmental significance. They are formed when small hydrophobic molecules are dissolved in water at high pressure and low temperature. They have been studied by experiment for decades, and more recently they have been examined by molecular simulation. Despite this attention, there have been no comprehensive studies performed to determine thermodynamic stability of the phases from molecular models, examining all important factors. There are many to consider, including the hydrate crystal structure, the chemical species, concentration, and distribution of the solute, conditions of temperature and pressure, nuclear quantum effects, and the impact of co-solutes. This work examines all of these factors via molecular simulation. The results are needed to provide a solid foundation for studies of other important phenomena for these systems, such as nucleation and growth processes. Results from these studies might also provide understanding needed to formulate better mitigation strategies for preventing clathrate formation, and conversely, better ways to use clathrates in technological applications. To this end, a simulation method toward methodical design of secondary solutes is proposed for development and application.

## 2 BACKGROUND AND MOTIVATION

Clathrate hydrates are crystalline compounds formed from water and small hydrophobic solutes (*e.g.* methane) in which the water molecules form a lattice of hydrogen-bonded cage-like structures, each surrounding one or perhaps a few of the solute molecules. The compounds are non-stoichiometric, in that some of the cages may be vacant or have multiple occupancy at equilibrium; however there is a minimum number that must be occupied to ensure stability of the phase. Formation of clathrates is promoted by high pressure and low temperature. Hydrates appear in several structures, each of which combines two or more cage types to form a unit cell, and differing in the size, shape, and number of cages in the repeat unit. The most common structures<sup>1, 2</sup> are: structure I (sI, with 2 small + 6 large cages formed from 46 water molecules in the unit cell);<sup>3</sup> structure II (sII, 16+8 cages from 136 water molecules);<sup>4</sup> and (less common) structure H (sH, 3+2+1 cages from 34 waters).<sup>5</sup> The structure adopted by a clathrate hydrate generally depends on the size of the solute molecule.

Clathrates are important for both technological and environmental reasons. From a technological standpoint, the formation of clathrates is a major concern in oil and gas industry, where undesired formation in pipelines produces solid plugs that halt flow. Consequently significant effort is directed toward flow assurance by prevention or control of the formation of clathrate hydrates. A more positive technological issue is in the exploration of clathrates as a means for storage and transportation of gaseous fuels, particularly methane and even hydrogen, or for sequestration of pollutants and greenhouse gases, particularly carbon dioxide.<sup>6</sup> Natural stores of methane in permafrost or seafloor hydrates are huge,<sup>7</sup> and recovery of this resource, particularly in conjunction with a complementary process to sequester CO<sub>2</sub>, could provide a long-term energy source to

meet worldwide demand. Other uses envisioned for clathrate hydrates include gas separation,<sup>8</sup> desalination,<sup>9</sup> and refrigeration.<sup>10, 11</sup>

The thermodynamic behavior of clathrate hydrates is for many purposes adequately described by the van der Waals-Platteeuw (vdWP) model.<sup>12, 13</sup> This is a statistical-mechanical theory that treats the hydrate as individual cages housing guest molecules that do not interact across cages. It further assumes single occupancy of the cages, that occupancy of one cage has no effect on others, and that quantum effects can be neglected. Various extensions have relaxed one or more of these assumptions. However, selective removal of approximations might in fact be counterproductive, as a molecular simulation studies<sup>14, 15</sup> have found that the vdWP model gains some accuracy through a cancellation of errors. Lattice dynamics, or harmonic treatments, have also been examined.<sup>16-21</sup> These approaches are limited to the range of (low) temperatures where the harmonic approximation is correct, and moreover they have difficulty capturing the effects of cage vacancies, which are prevalent in these systems.

The focus of clathrate modeling is moving away from thermodynamics and equilibrium, and toward considerations of dynamical behavior, in particular nucleation and growth.<sup>2</sup> This redirection is connected to a strategy of flow assurance geared toward managing and controlling, rather than preventing, unwanted clathrate formation. Molecular simulation has emerged as a key tool toward developing understanding needed to model and control clathrate formation processes. This development, however, is severely hindered by a lack of detailed information about the thermodynamic behavior of the model systems. This problem has been highlighted in a recent (2011) review<sup>2</sup>:

*Clearly, one must also know the stability boundary for clathrate hydrates to correctly determine the driving force and mechanism for hydrate formation. However, determination of the equilibrium phase boundary for clathrate hydrates from molecular simulations is a major challenge, and is thus, with a few exceptions, neglected despite its critical importance. ... Due to the difficulty in obtaining such data, most studies published on the dynamics and stability of clathrate hydrates have assumed a temperature, pressure, and hydrate composition (typically at full occupancy) for the simulations. However, without a more physical basis for determining this information, one is left to make inconclusive hypotheses about the mechanism, since the thermodynamic state for the system has not been defined.*

Recent molecular simulation studies have examined aspects of the free energy and phase equilibria of clathrate hydrate models. Wierchowski and Monson<sup>22, 23</sup> computed a complete sI-liquid-vapor-solid phase diagram for an idealized system using a primitive model for water plus a hard-sphere solute, demonstrating the qualitative features needed to yield clathrate phase behavior. In subsequent work<sup>14</sup> they examined a more realistic model formed from TIP4P water and a Lennard-Jones methane solute, demonstrating how the hydrate free energy could be computed, and testing assumptions underlying the vdWP theory; however this work did not examine the thermodynamic phase behavior. A step in this direction was taken by Conde and Vega,<sup>24</sup> who computed direct three-phase coexistence for the sI methane hydrate at three pressures and 100% occupancy, examining several realistic models for water. At about the same time, Jensen *et al.*<sup>25</sup> reported a rigorous, free-energy based molecular simulation study of this system, accounting also for partial occupancy of the cages.

These recent studies of realistic models of methane hydrates have been very encouraging, showing good agreement with experiment. There remains, however, much more to do. Accordingly, we propose a comprehensive and rigorous free-energy-based molecular simulation study of clathrate phase equilibria considering most of the relevant factors, including pressure, temperature, solute concentration and distribution, solute type, hydrate structure, and (where appropriate) nuclear quantum effects.

### **3 METHODS**

#### **3.1 Calculation of free energies and coexistence lines**

Calculation of the free energy starts with a specification of the crystal structure, with all cages occupied by a solute molecule and the whole configuration relaxed to a local minimum. Alternatively, we might start with all cages vacant which, although thermodynamically unstable, presents a mechanically stable structure; this approach is advantageous in that it provides a common starting point for any solute species and thus need not be repeated when considering different solutes. We then proceed to evaluate the change in free energy with temperature, starting from the limiting behavior at  $T \rightarrow 0$ , where a harmonic analysis yields the free energy.<sup>26</sup>

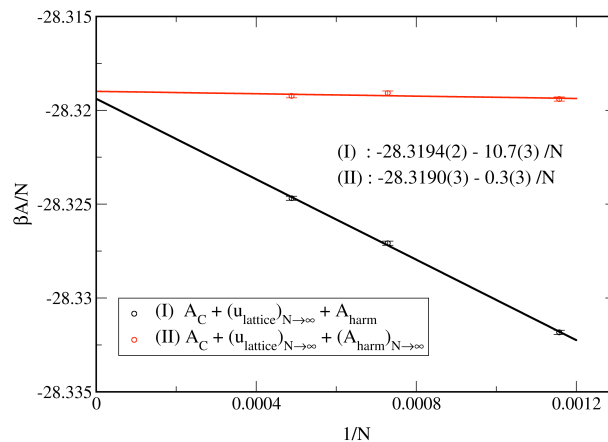
To evaluate the free energy with increasing temperature, we turn to the recently-introduced harmonically targeted temperature perturbation (HTTP) method,<sup>27</sup> which provides what is arguably the most efficient means for computing the difference in free energy of a crystalline system at two different temperatures by molecular simulation. The approach implements the general targeted free-energy perturbation suggested by Jarzynski.<sup>28</sup> As such, temperature perturbation is accompanied by perturbation in the atom positions, applying the harmonic approxi-

mation to target the displacements. Specifically, all atom positions are scaled away from their respective lattice sites in proportion to the square root of the temperature ratios. For a perfectly harmonic system, this means that the ratio of energy to temperature,  $U/T$  would be unchanged, and the perturbation average would indicate a zero free-energy difference; when the exact harmonic component is added to this result (which emerges as a Jacobian for the coordinate transformation), the total free energy is obtained. **Thus the quantity measured by the Monte Carlo process is strictly the change in the anharmonic contribution to the free energy; consequently, sampling noise that normally accompanies the harmonic contribution—the largest component of the free energy—is eliminated.** Changes for large differences in temperature are computed by summing results for smaller temperature perturbations, and overlap sampling<sup>29,30</sup> is applied to span each temperature interval to promote accuracy and efficiency in the calculations.

In the context of the HTTP method, one can compute the absolute, classical free energy of a target system by computing the free energy difference down to very low temperature  $T_e$ , where the harmonic approximation becomes highly accurate. The small low-temperature anharmonic correction can be evaluated<sup>27</sup> by a harmonic perturbation, or extrapolation. If the system is not thermodynamically stable at low temperature, constraints can stabilize it and enable calculation of a low-temperature reference free energy. This approach was demonstrated, and the method was generalized to allow for rigid-molecule rotation, in an application to calculate the pressure-temperature coexistence line between the  $\alpha$  and  $\beta$  crystalline forms of  $N_2$ .<sup>31</sup>

An important feature of this method for calculating solid-phase free energies is demonstrated in Figure 1.<sup>31</sup> Here we plot the the free energy (over  $kT$ ) per molecule for  $\alpha$ - $N_2$  as computed from simulations of different system sizes, at density  $0.0230\text{\AA}^{-3}$  and temperature 30K.

The system size is given on the  $x$ -axis, which shows the reciprocal of the number of molecules  $N$  in the system; thus the infinite-system behavior is obtained by the  $y$ -intercept. Two lines are presented. The black line labeled (II) shows the free energy in which the lattice energy is given for the infinite system, while the harmonic and anharmonic contributions are both included in the  $N$



**Figure 1. Dependence of free-energy contributions on the size of the simulated system. From Ref. 31.**

dependence. This is the free energy that would be computed using almost any conventional molecular simulation technique (*e.g.*, the very popular Frenkel-Ladd method<sup>32</sup>). The red line labeled (I) shows the free energy in which both the lattice and the harmonic contributions are given for the infinite system, and only the anharmonic contribution is included in the  $N$  dependence. This is the free energy computed by the HTTP method. The free energy given in this fashion has almost no system size dependence at all (indeed the slope of the curve is statistically zero). This means that we can work with small numbers of unit cells in our calculation using HTTP, and be confident that we represent the large-system behavior. Such an economy is particularly important when using the computationally expensive molecular potentials and path integrals that are contemplated as part of this study. In this respect it will be of interest to observe whether this system-size independence of the anharmonic contribution persists for water, and whether we need apply an Ewald sum only for the lattice energy and harmonic analysis. One might also take note in the figure of the high precision of the results obtained using HTTP—in this application the precision of the free energies likely exceeds the accuracy of the  $N_2$  model potential itself.<sup>33</sup>

Having established the free energy of the fully occupied hydrate at the target temperature, we can then conduct additional molecular simulations to gauge the dependence on composition and/or pressure. Simulation in an isobaric<sup>34</sup> osmotic ensemble is appropriate for this purpose. The fundamental equation for such an ensemble is:<sup>35</sup>

$$d(\beta N_w \mu_w) = H d\beta - \beta V dP - N_s d\beta \mu_s + \beta \mu_w dN_w \quad (1)$$

where  $H$  is the enthalpy,  $\beta$  is the reciprocal temperature,  $P$  and  $V$  are the pressure and volume, and  $\mu$  and  $N$  are the chemical potential and number of molecules; subscripts  $S$  and  $W$  indicate solute and water, respectively. Here we fix the number of water molecules and allow the number of solutes to fluctuate under the influence of an imposed solute chemical potential; in some circumstances we may have two or more solute species, and the formalism is easily extended to accommodate this. Equation (1) suggests that the free energy for this ensemble can be evaluated from the full-occupancy state by integration along  $\mu_s$ . The initial condition for the integration can be determined from the free energy evaluated via the HTTP series (yielding  $-PV + N_w \mu_w + N_s \mu_s$ ), along with Widom insertions to measure  $\mu_s$ . Wierzchowski and Monson<sup>14</sup> took an alternative approach in their studies, starting instead from the metastable zero-occupancy system of cages. Another strategy could have only the smaller cages or only the larger cages occupied, depending on the solute. We could turn to such approaches for some systems if we run into difficulties with the planned method (*e.g.*, for systems having multiple-occupancy cages we might find no suitable single-occupancy minimum on which to form a harmonic free energy), or as a consistency check.



Coexistence with the gas phase and with liquid water are determined via the usual conditions of equality of temperature, pressure, and chemical potentials. For the gas, we can use the virial equation of state to determine the thermodynamic properties as a function of temperature, pressure (via the density) and composition. Mayer sampling Monte Carlo<sup>36</sup> has been shown to be effective in computing virial coefficients for alkanes and similar molecules,<sup>37-42</sup> sufficient to provide an accurate equation of state at all conditions of interest to this study. For the liquid, properties can be determined via an overlap-sampling<sup>43, 44</sup> (Bennett's method<sup>30</sup>) implementation of molecule insertion/deletion. Previous studies<sup>29</sup> have shown this to be an effective method for computing the chemical potential of liquid water.

Once a single temperature/pressure/composition condition is identified at which the gas, liquid, and/or hydrate phases are equally stable, Gibbs-Duhem integration<sup>35, 45</sup> can trace out stability boundaries from it in the  $P$ - $T$ -composition space, producing a range of coexistence conditions in a very efficient manner.

### 3.2 Nuclear quantum effects

At the relatively low temperatures of interest to this study, and noting that some of the solutes have low mass, it is appropriate to consider the importance of nuclear quantum effects in determining stability. Even the water molecules can display the quantum effects at these conditions, given their small moment of inertia. However, Conde and Vega<sup>46</sup> examined nuclear quantum effects for empty hydrate cages, and found they could largely be neglected for temperatures above 150 K. Still, this is something to keep watch on in our studies. One convenient approach to this end is via semiclassical methods. In the Feynman-Hibbs treatment an effective potential is defined via a series expansion in Planck's constant,<sup>42, 47-49</sup> yielding a form that can then be used as

if it were a classical potential. This can be helpful in assessing the importance of quantum nuclear effects without invoking expensive techniques. However, in studies of light molecules,  $H_2$  in particular, more rigorous methods are needed.

Path integral Monte Carlo (PIMC) is a means for capturing nuclear quantum effects that matches well with the other methods used in the proposed work.<sup>50</sup> The approach is based on Feynman's path-integral formulation<sup>51</sup> of quantum mechanics. Space limitations preclude a detailed review here, but the net effect on the simulation is simple to describe. Each molecule to be treated quantum mechanically is replaced by number of replicas, or "beads" (on the order of 10 to 500, depending on temperature), which are joined in sequence by a harmonic potential coupled to their positions and orientations<sup>52,53</sup> such that they form a closed ring. This assembly represents the single molecule it replaces, and has the effect of making its position more "fuzzy", or diffuse. Each bead in the ring interacts with a single counterpart on the ring representing each of the other molecules being treated this way (molecules not treated for nuclear quantum effects are represented by a single bead which interacts with all beads on the path-integral molecules), and the interaction energy for the pair is given by the average of the interactions across the beads in the ring. The intra-ring harmonic potential becomes stronger at higher temperature and for more massive molecules, so in these limits the ring effectively collapses into a single molecule, reducing to classical behavior.

## 4 SYSTEMS AND PHENOMENA FOR STUDY

### 4.1 Methane

Our initial focus of the study will be on methane hydrates, which adopt the sI form. We plan to use the models employed by Conde and Vega,<sup>24</sup> who examined three-phase coexistence using a direct method, and Jensen *et al.*,<sup>25</sup> who performed detailed free-energy calculations. We aim to reproduce those results using completely different methods, both to verify those calculations and ours, and, we expect, to produce results of greater precision (this may be expected from our experience<sup>27</sup> in reproducing Frenkel-Ladd calculations). Previous studies employ the TIP4P/Ice model,<sup>54</sup> which was formulated to describe the behavior of crystalline phases of water. This is also the potential that was employed by Walsh *et al.*<sup>55, 56</sup> in their microsecond-scale molecular dynamics studies of hydrate nucleation processes. Again following the previous studies,<sup>24, 25</sup> we will model methane as a Lennard-Jones site, with Lorentz-Berthelot mixing rules for the water-methane interactions. It is possible that we will want to adjust these models (particularly methane) as we generate detailed coexistence data, to improve comparison with experiment. Calculations will start at 250K and 50MPa, matching the conditions used by Walsh *et al.*<sup>55</sup> We will determine free energies separately for each phase as described in Sec. 3.1. With a three-phase coexistence point established, we can then apply a variant of Gibbs-Duhem integration to trace the three-phase line in temperature-pressure-composition space.<sup>35, 57</sup> For the methane hydrate, we can employ a simplification that assumes that the gas and liquid are pure methane and water, respectively (and thus relaxing the dilute-component chemical potential equalities), but if needed for this or other systems we can also work with the more rigorous coexistence criteria.

In this course of this work we will study system-size effects to determine if previous observations about the insensitivity of the anharmonic contribution to the free energy with system size carries over to the methane hydrate system. If so, we may then in further studies work with small systems and perhaps also dispense with expensive Ewald sums when computing the anharmonic contribution via molecular simulation.

Going further with the methane system, it would be of interest to establish also the phase diagram for the sII structure. Although this form is not normally observed for methane hydrates in nature, there is evidence from experiment<sup>58</sup> and simulation<sup>55, 59</sup> that these structures play a role in the nucleation and growth of the sI methane hydrate. While the relevance of sII is more likely as a polycrystal with sI, it is nevertheless of value to have a baseline picture of the stability of the bulk sII phase, relative to sI, water, and methane gas.

## **4.2 Carbon dioxide**

Carbon dioxide also forms a sI hydrate. Hydrates of carbon dioxides are of interest for sequestration, and various strategies may be considered to use them for this purpose.<sup>6</sup> The viability of each approach depends on the conditions at which the CO<sub>2</sub> hydrate is stable (particularly with respect to the methane hydrate, which is of interest to strategies that couple CO<sub>2</sub> sequestration with CH<sub>4</sub> liberation), as well as how quickly it forms (which can determine the ability of hydrates to self-seal when injected in deep-sea geological structures). No simulation studies exist to establish stability boundaries for any model of CO<sub>2</sub> hydrates, and without this information computational studies of formation kinetics cannot proceed.

We expect to continue use of the TIP4P/Ice model for water, while modeling CO<sub>2</sub> initially with the EPM2 model of Harris and Yung,<sup>60</sup> which is a rigid linear three-site model with point-charge electrostatics. This model accurately reproduces the vapor-liquid coexistence curve for CO<sub>2</sub>, and we have used it previously in modeling solubility in supercritical CO<sub>2</sub> via the virial equation of state.<sup>61</sup> We will also examine other CO<sub>2</sub> models, complementing a study of H<sub>2</sub>O-CO<sub>2</sub> models at higher temperatures.<sup>62</sup> With results for the methane hydrate system in hand (Sec. 4.1), we should be able to develop phase diagrams for the CO<sub>2</sub> hydrate somewhat more easily. In particular, the hydrate free energy can be evaluated via a thermodynamic pathway that transforms the methane hydrate into a CO<sub>2</sub> hydrate. This is accomplished via a series of semigrand Monte Carlo simulations<sup>35, 63</sup> in which the chemical potential difference  $\mu_{\text{CO}_2} - \mu_{\text{CH}_4}$  is imposed and guest molecules fluctuate in identity between the two species. A potential advantage in following such a path is that it yields information about the multicomponent CO<sub>2</sub>-CH<sub>4</sub> hydrate system as a byproduct of the calculation, which may be of interest in coupled sequestration-liberation scenarios.

### 4.3 Hydrogen

Clathrate hydrates of molecular hydrogen are of interest for energy storage.<sup>64, 65</sup> These substances exhibit qualitative differences from clathrates formed by methane or carbon dioxide. Hydrogen hydrates adopt a multiple-occupancy sII structure, with one H<sub>2</sub> molecule in the small cage, and up to four in the large cage. The low molecular weight of H<sub>2</sub> causes quantum effects to be quite significant,<sup>66-71</sup> and the strong confinement imposed by the clathrate structure leads to quantization of translational degrees of freedom and coupling with (already-quantized) rotation.

It was long thought that  $H_2$  was too small to form clathrate structures, and only recently has attention turned to this system as experiments have shown<sup>72-74</sup> that hydrogen can indeed form stable hydrates at sufficiently high pressure ( $\sim 200$  MPa). Interest has accelerated with the report<sup>75</sup> that hydrogen hydrates could be formed at moderate conditions of temperature and pressure (5 MPa and 280K) through the addition of tetrahydrofuran (THF) as a stabilizer. Offsetting this benefit is the reduced concentration of  $H_2$  in the resulting structure due to its displacement by THF in the large cage. This has led to examination of structures that use THF in nonstoichiometric amounts,<sup>76</sup> leaving more of the large cages for  $H_2$  occupancy; this compositional tuning strategy, however, does not seem to work,<sup>64</sup> and there may be value in developing a stronger understanding of the reasons for this failure via free-energy based molecular simulation. The effectiveness of THF as a stabilizer has spurred on numerous other experimental investigations,<sup>64</sup> examining the suitability of alternative stabilizing molecules, as well as the formation of other hydrate structures (particularly sH), and even non-water clathrates.

Our studies of hydrogen hydrates will use the TIP4P/Ice model for water, the  $H_2$ - $H_2O$  potential of Alavi *et al.*,<sup>77</sup> and the first-principles  $H_2$ - $H_2$  potential of Diep and Johnson.<sup>78</sup> Nuclear quantum effects in  $H_2$  will be examined both semi-classically and via path integrals (Sec. 3.2). Comparison of these approaches will identify the range of suitability of semiclassical methods. As with the  $CH_4$  and  $CO_2$  studies, we will characterize the full phase diagram, considering the sII and sH structures, the former for both pure  $H_2$  and with a THF stabilizer. With 136 waters in the unit cell, sII will present some challenge, and the effectiveness of the techniques presented in Sec. 3.1 to capture the large-system behavior with few unit cells will be particularly valuable. The distribution of  $H_2$  in the THF-stabilized system, and the behavior for nonstoichiometric THF, will be of particular interest.

#### 4.4 Optimizing Stabilizers and Destabilizers

Finally, we will consider a more speculative research direction, which aims to make use of the unique capabilities of molecular simulation in application to clathrate hydrates. Once we have established that we can characterize the phase behavior accurately via free-energy calculations, then with simulation, we can attempt a systematic design of secondary guest molecules that either stabilize or destabilize (depending on the application) the hydrate relative to the liquid and/or gas. In particular, we can do this by perturbing gross qualitative features of the secondary guest molecules, without regard to specifying a particular chemical identity. So, for example, we can start with a simple sphere, and consider at once the effects of (1) changing its size; (2) changing its shape (prolate or oblate distortion); (3) adding or altering its charge distribution; and (4) budding another sphere(s) from it. The derivative of the clathrate free energy with such changes can be computed, and used to evolve the molecule in a favorable direction, from which the process is repeated. As the solute evolves, we can consider other types of adjustments, such as making the molecule more rigid or flexible. Now, it is not enough to consider just the clathrate free energy, since a highly destabilizing molecule (for example) could find itself simply excluded from the structure. Instead it is necessary to consider the molecule in the context of the other phases as well. Thus we require simulation of the liquid, and evaluation of mixture virial coefficients including the co-guest, to determine its effect on those phases. Then the overall effect can be evaluated using a construction similar to that used to develop the Clapeyron equation. Thus we perform a type of Gibbs-Duhem integration that is directed to, for example, alter the co-guest in a way that lowers the coexistence pressure (in an application to hydrogen hydrates). Once the process has reached a satisfactory conclusion, one can then attempt to identify a real molecule that has the same qualitative features exhibited by the optimal stabilizer/destabilizer molecule,

for follow-up by laboratory researchers. Clearly some development will be needed to implement and tune this methodology, but any progress in formulating simulation methods to achieve this could prove quite useful in the long run.

Proposal Narrative = 3997 words.



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