Satellite Navigation: Basics and Single Point Positioning

AAE4203 – Guidance and Navigation

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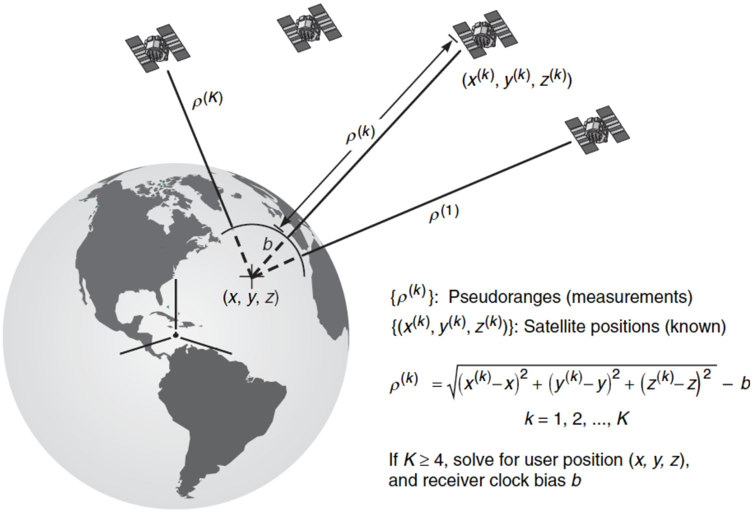
Week 3

GNSS Positioning Performance

Positioning Performance of GNSS

Positioning Accuracy =
Measurements Accuracy × DOP

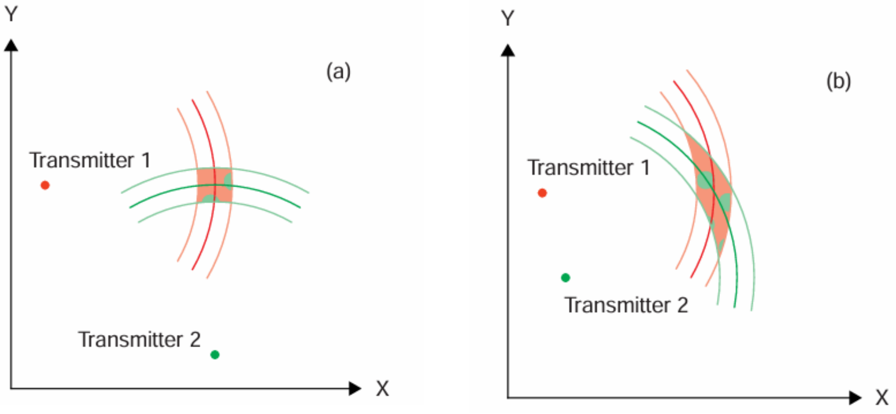


- If the measurements errors are zero, the calculated user position is true.
- However, if the measurements include errors, the positioning accuracy depends on measurement accuracy as well as dilution of precision (DOP)

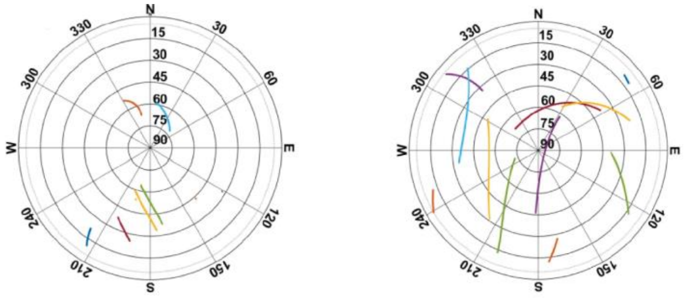
Interpretation of DOP

DOP, or geometric dilution of precision (GDOP), is used to specify the error propagation as a mathematical effect of satellite geometry on positioning accuracy.

- When visible satellites are close together, the geometry is weak, and the DOP value is high;
- When they are far apart, the geometry is strong, and the DOP value is low.



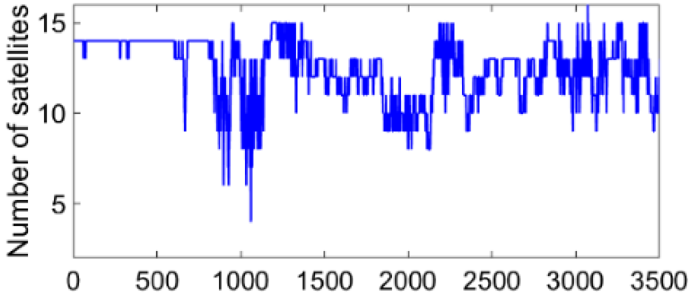
Richard B. Langley (May 1999)

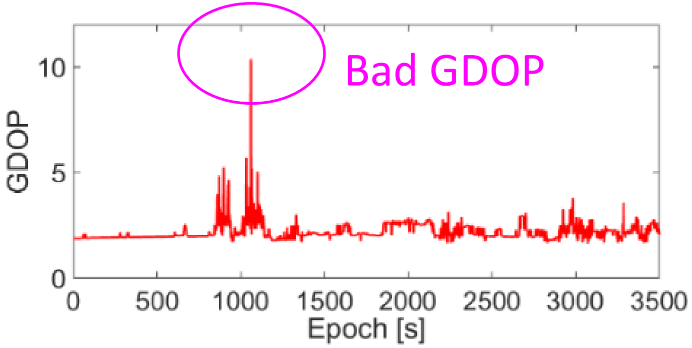


Interpretation of DOP

DOP Value	Rating	Description
< 1	Ideal	Highest possible confidence level to be used for applications
1–2	Excellent	Measurements are considered accurate enough to meet all but the most sensitive applications
2–5	Good	Measurements could be used to make reliable in-route navigation suggestions
5–10	Moderate	Measurements could be used for calculations, but quality could still be improved
10–20	Fair	Measurements should be discarded or used only to indicate a very rough estimate of the current location
> 20	Poor	Measurements should be discarded

Lower DOP values result in better accuracy





Calculation of DOP

$$\Delta \boldsymbol{p} = \left(\boldsymbol{G}^T \boldsymbol{G}\right)^{-1} \boldsymbol{G}^T \Delta \boldsymbol{\rho} \qquad \boldsymbol{Q} = \left(\boldsymbol{G}^T \boldsymbol{G}\right)^{-1}$$

Mapping matrix from *pseudorange* domain to *positioning* domain

For the case of 4 parameters:

$$\boldsymbol{P} = (x, y, z, t)^{\mathrm{T}}$$

The elements of $\boldsymbol{Q}$ are designated as

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 & \sigma_{zt} \\ \sigma_{xt} & \sigma_{yt} & \sigma_{zt} & \sigma_t^2 \end{bmatrix}$$

To correspond to the <u>local east-north-up</u> coordinate system, $\boldsymbol{Q}$ can be designated as

$$Q = \begin{bmatrix} \sigma_e^2 & \sigma_{en} & \sigma_{eu} & \sigma_{et} \\ \sigma_{en} & \sigma_n^2 & \sigma_{nu} & \sigma_{nt} \\ \sigma_{eu} & \sigma_{nu} & \sigma_u^2 & \sigma_{ut} \\ \sigma_{et} & \sigma_{nt} & \sigma_{ut} & \sigma_t^2 \end{bmatrix}$$

Calculation of DOP

DOP value can be divided into

- Position dilution of precision (PDOP)
- Horizontal dilution of precision (HDOP)
- Vertical dilution of precision (VDOP)
- Time dilution of precision (TDOP)
- Geometric dilution of precision (GDOP)

$$\text{GDOP}=\sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2+\sigma_t^2}$$

$$\text{PDOP}=\sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2}$$

$$\text{TDOP}=\sqrt{\sigma_t^2}$$

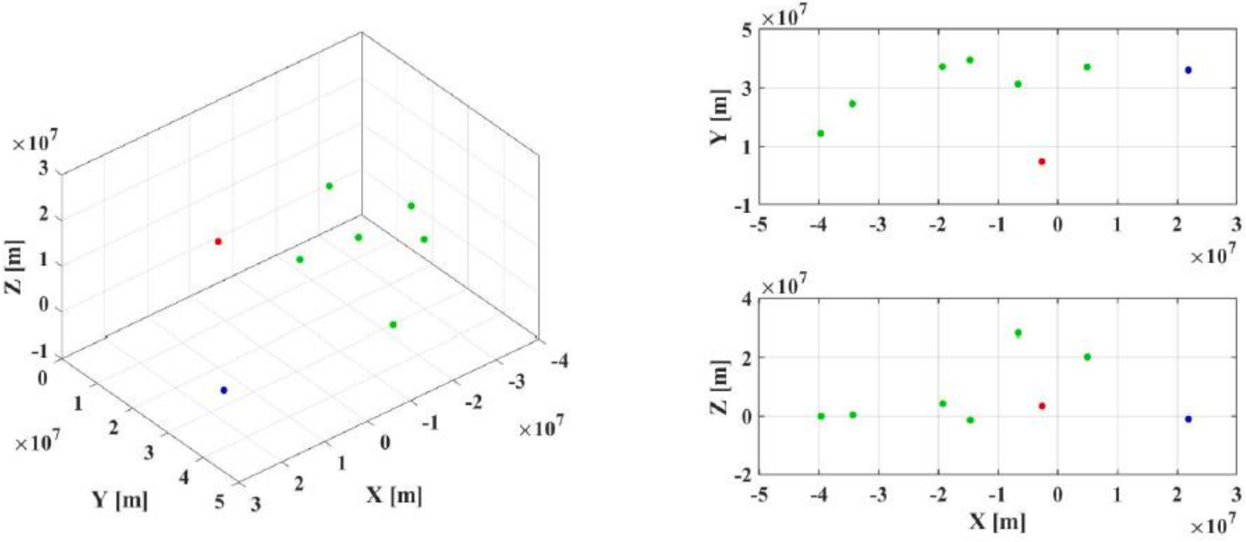
$$\text{HDOP}=\sqrt{\sigma_\mathrm{e}^2+\sigma_\mathrm{n}^2}$$

$$\text{VDOP}=\sqrt{\sigma_\mathrm{u}^2}$$

$$\boxed{\begin{aligned}&\text{HDOP}^2+\text{VDOP}^2=\text{PDOP}^2\\&\text{PDOP}^2+\text{TDOP}^2=\text{GDOP}^2\end{aligned}}$$

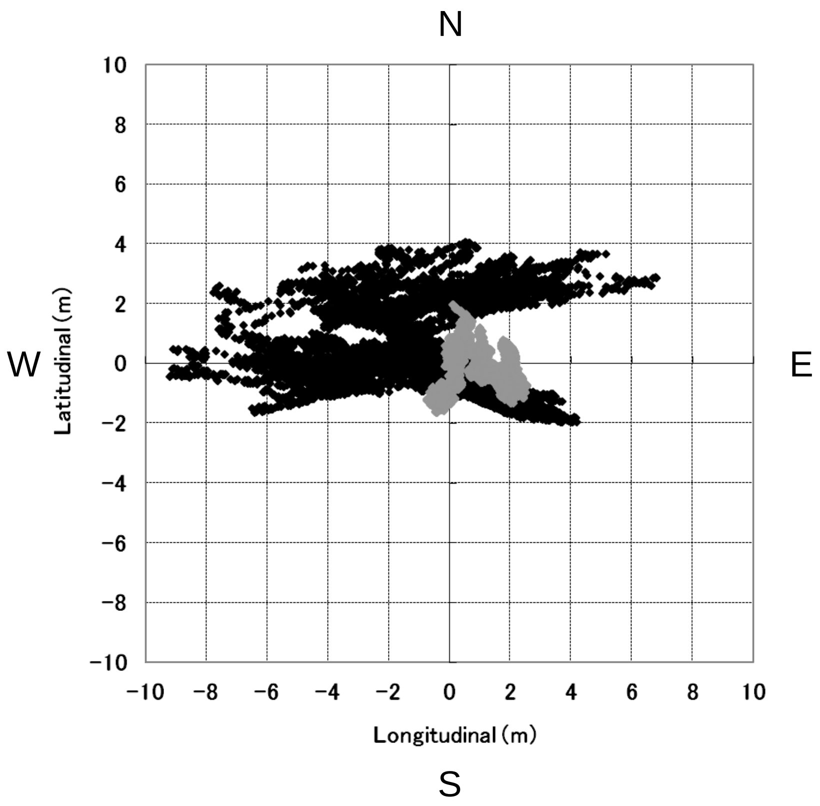
Satellite Geometry

Relative position between the user and the GNSS satellites affects the positioning accuracy



The red, blue, and green points denote the user, the GNSS satellite with the highest contribution of the PDOP, and the other GNSS satellites, respectively.

All Satellites VS. East Visible Satellites



- Only east-side satellites are used in the dark color plots. (average = 8.7 m)
- All satellites are used in the light color plots. (average = 4.6 m)

Why we learn measurements and errors ?

- Needless to say, "position, velocity and time" are important for users.
- The ability to improve final performance of the above outputs strongly depends on how can we estimate or possibly mitigate measurement errors.
- Measurement errors strongly depends on the environment and receiver performance.

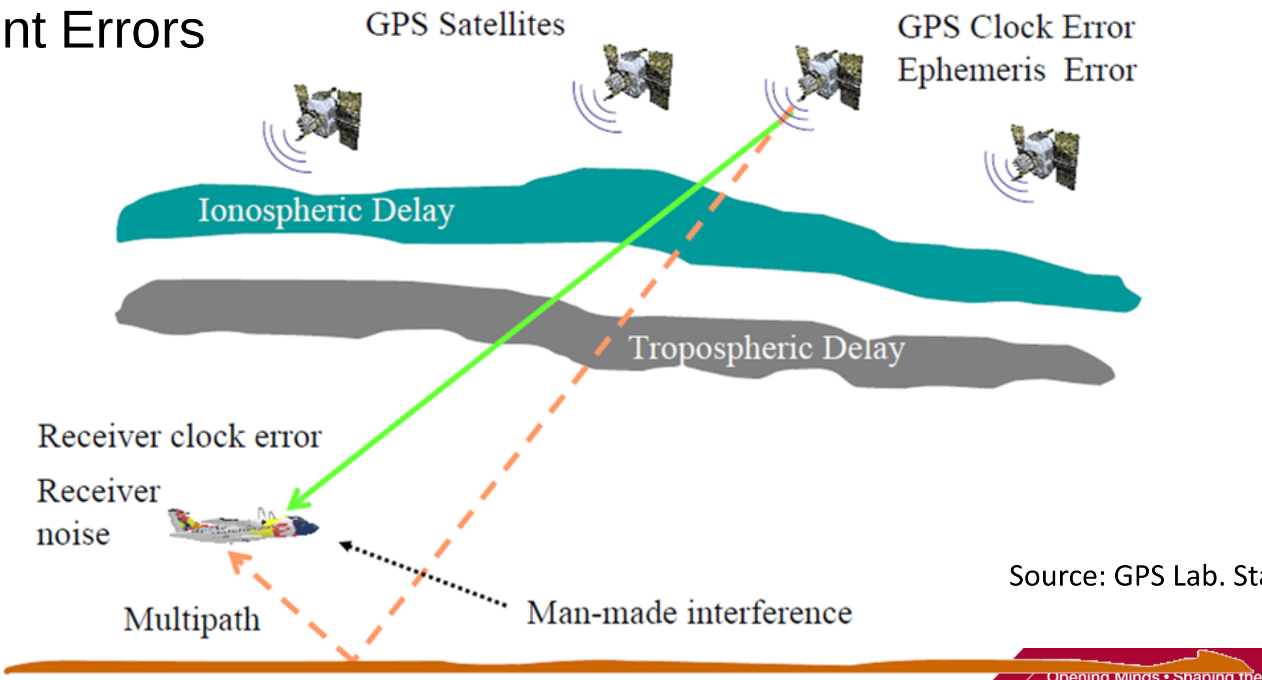
Noise and Bias

- Errors are often categorized as noise and bias.

- #1 Errors in the parameter values broadcast by a satellite in its navigation message for which the Control Segment is responsible
- #2 Uncertainties associated with the propagation medium that affect the travel time of the signal from a satellite to the receiver
- #3 Receiver noise which affects the precision of a measurement, and interference from signals reflected from surfaces in the vicinity of the antenna

Source of Measurements Errors

- Control Segment Errors
- Signal Propagation Modeling Errors
- Measurement Errors



Source: GPS Lab. Stanford Univ.

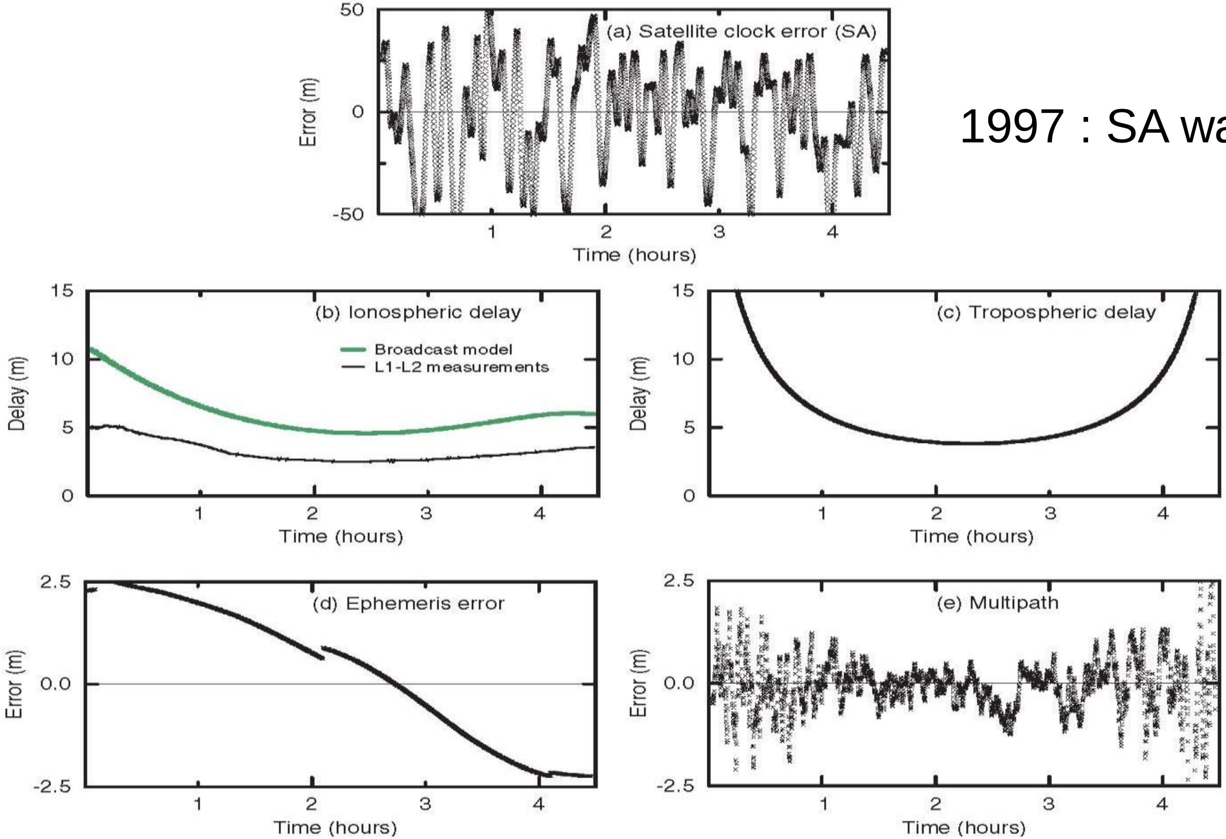
Typical pseudo-range measurement errors for L1 receiver

Error Source	RMS Range Error
Satellite clock and ephemeris parameters	3 m (SIS URE)
Atmospheric propagation modeling	5 m
Receiver noise and multipath	1 m
User range error (URE)	6 m

Total RMS Range Error = SIS+ URE

URE : User Range Error
SIS : Signal-in-Space

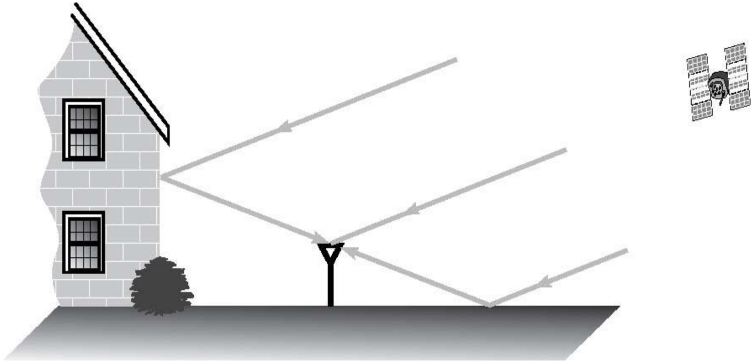
Measurement Error : Empirical Data



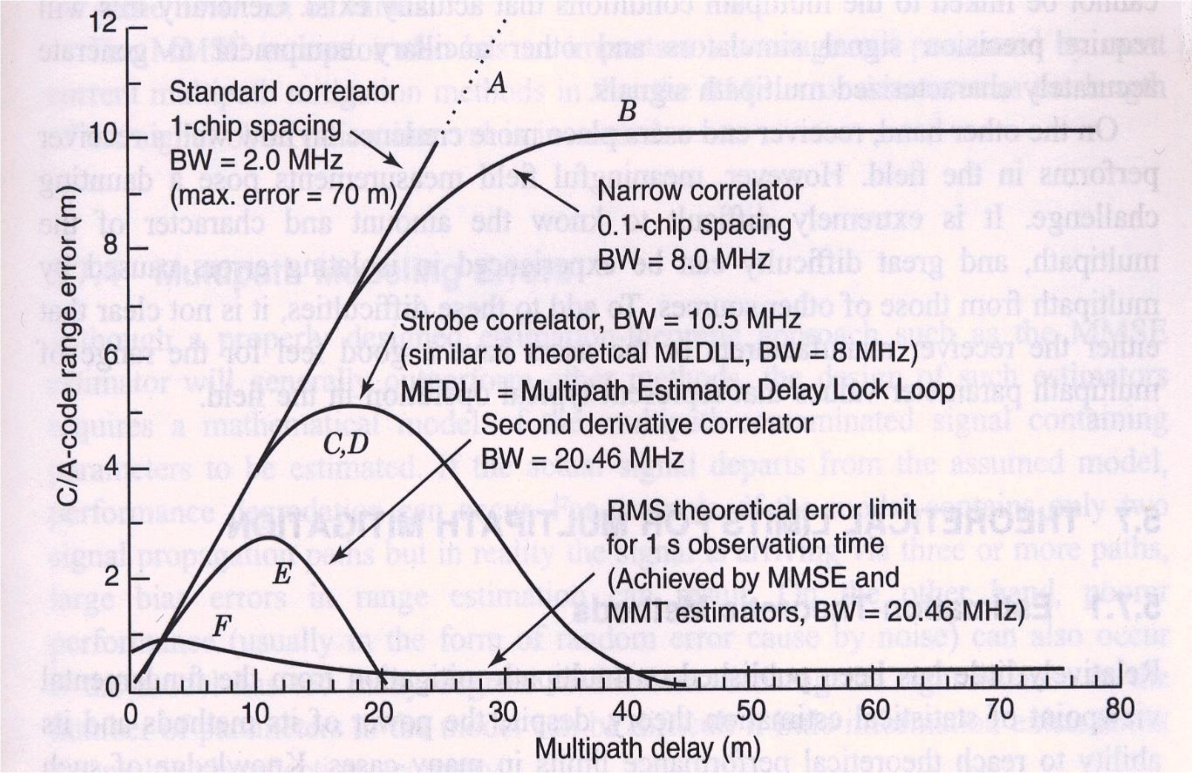
1997 : SA was activated.

Measurement Errors - Multipath

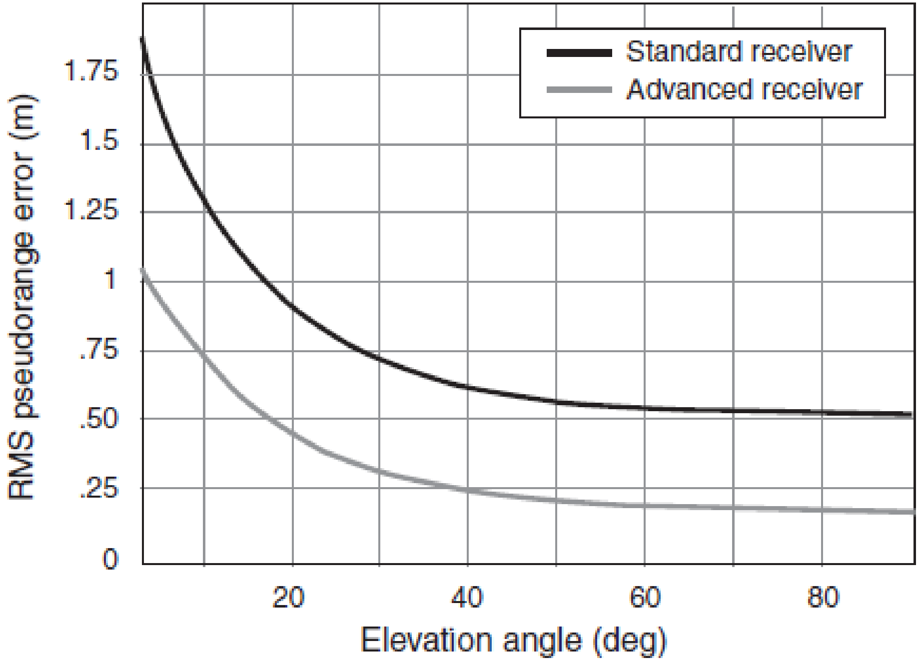
- Multipath refers to the phenomenon of a signal reaching an antenna via two or more paths.
- The range measurement error due to multipath depends on the strength of the reflected signal and the delay between direct and reflected signals.
- Mitigation of multipath errors: Antenna or Receiver

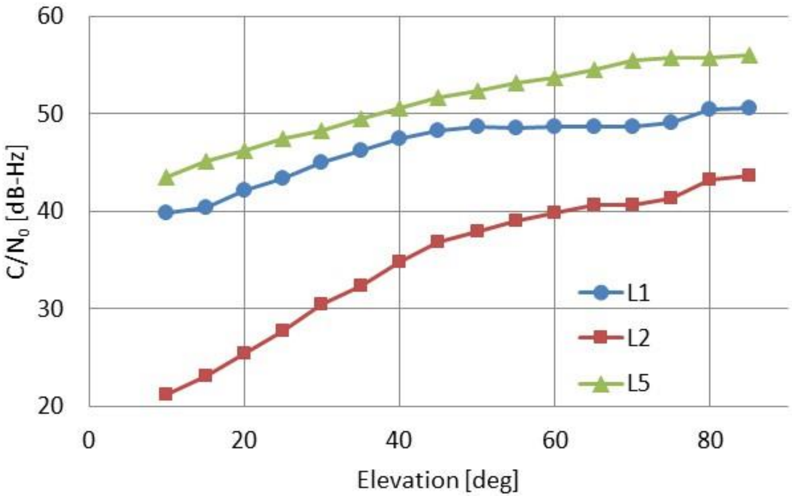


Multipath Mitigation Technique (Receiver inside)



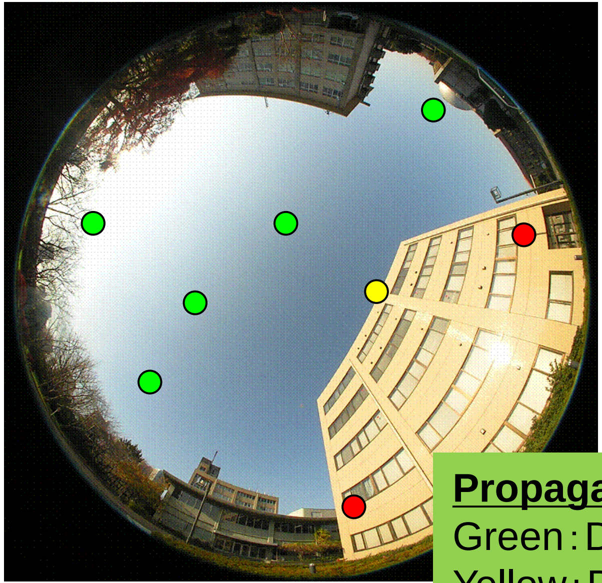
Receiver thermal noise for two types of receiver



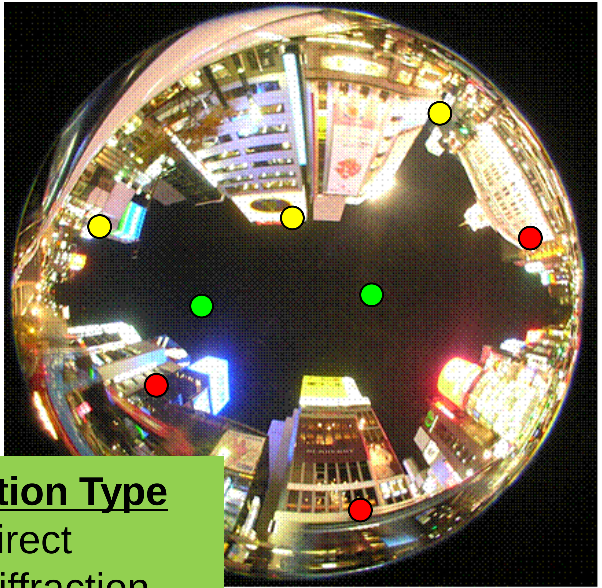


Received Signal Strength (C/N_0) and Elevation

Sky Views in two different places（same constellation but different performance）



Campus

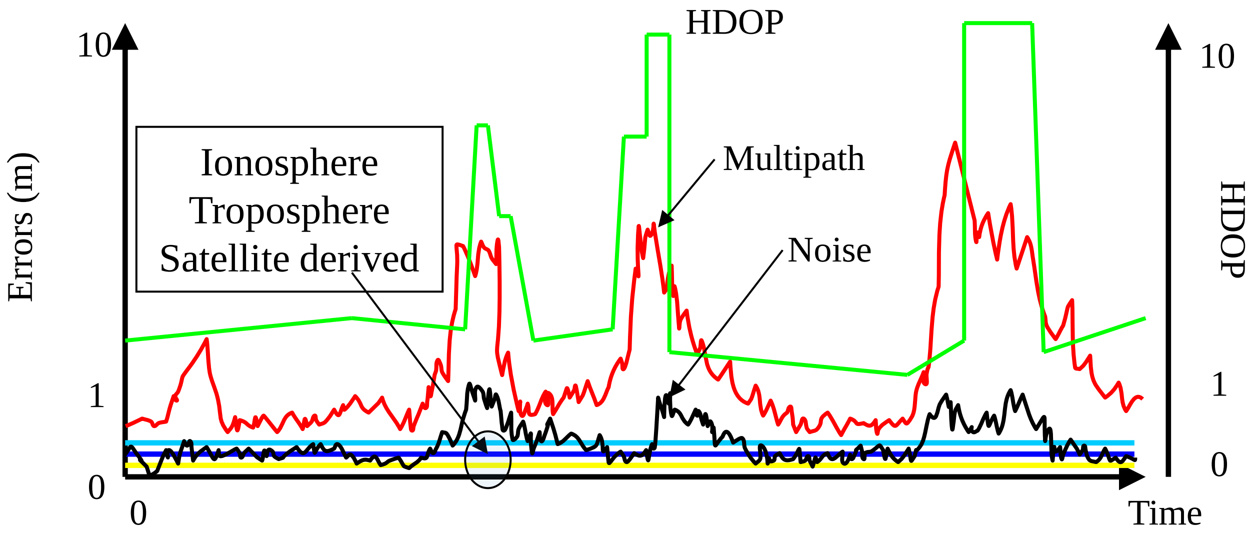


Urban Canyon

Propagation Type

Green : Direct
Yellow : Diffraction
Red : Masking +Reflection

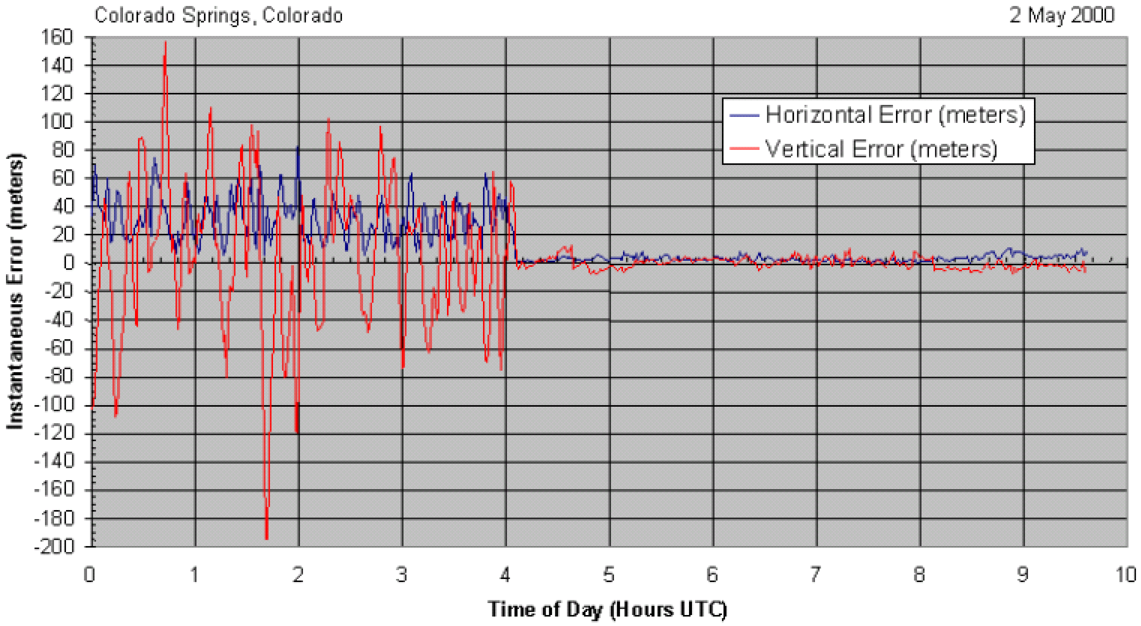
Temporal Measurements Errors and DOP Variation (sub-urban)



GPS Measurement Errors

Source	Potential error size	Error mitigation using single point positioning
Satellite clock model	2 m (rms)	→
Satellite ephemeris prediction	2 m (rms) along the LOS	→
Ionospheric delay	2-10 m (zenith) Obliquity factor 3 at 5°	1-5 m (single-freq.) within 0.5m (dual-freq.)
Tropospheric delay	2.3-2.5m (zenith) Obliquity factor 10 at 5°	0.1-1 m
Multipath (open sky)	Code : 0.5-1 m Carrier : 0.5-1 cm	→
Receiver Noise	Code : 0.25-0.5 m (rms) Carrier : 1-2 mm (rms)	→

History…Deactivation of the artificial distortion of the signal



On September 18, 2007, the US DoD reported that with the next generation of GPS satellites (GPS III), satellite navigation signals can no longer be artificially distorted

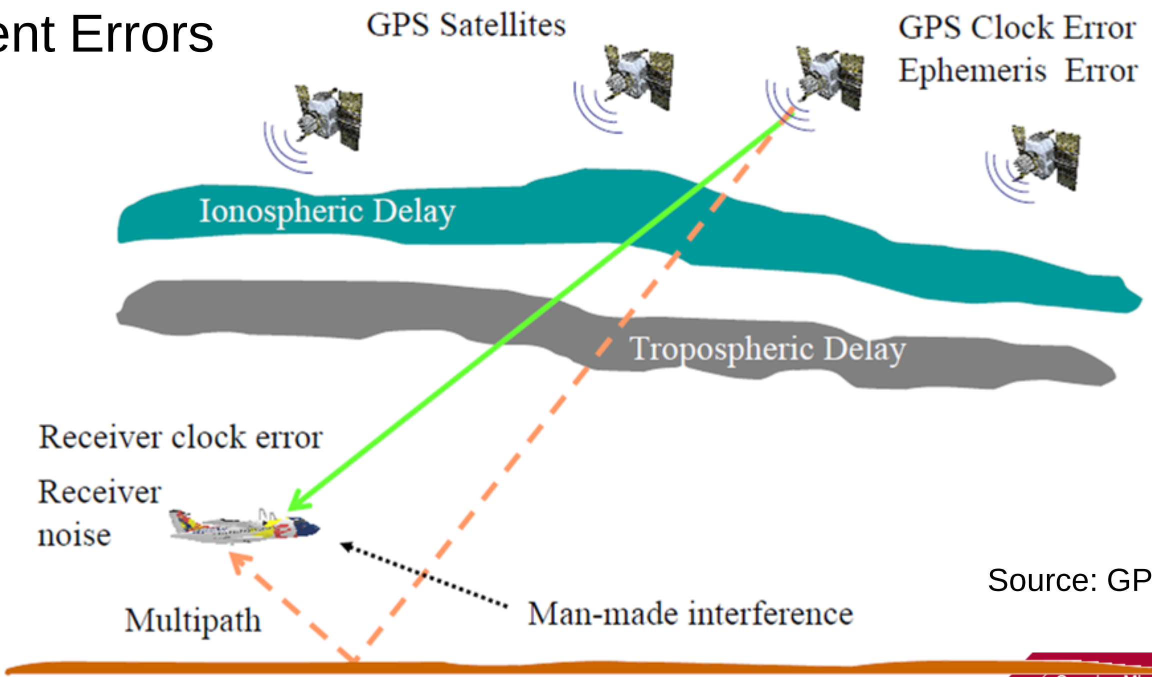
Improved GNSS Positioning

Source of Measurements Errors

> Control Segment Errors

> Signal Propagation Modeling Errors

> Measurement Errors



Source: GPS Lab. Stanford Univ.

Why we discuss about measurement errors ?

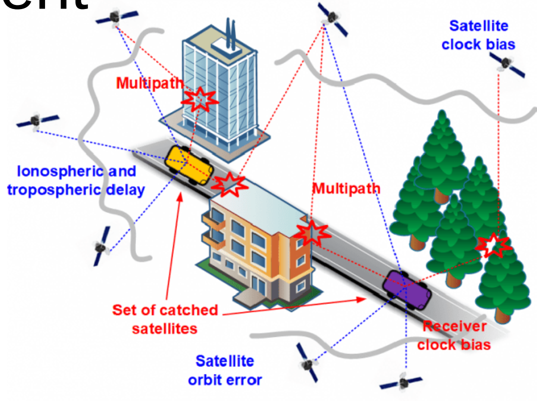
- Back to bias and noise errors discussion, noise errors of pseudo-range can be mitigated to some degree using **carrier phase smoothing technique**.

- On the other hand, you have to estimate **bias errors** as accurate as possible **by yourself** to improve positioning performance.

- All kinds of improved techniques are essentially same in terms of estimating or eliminating bias or noise errors.

Improved GNSS

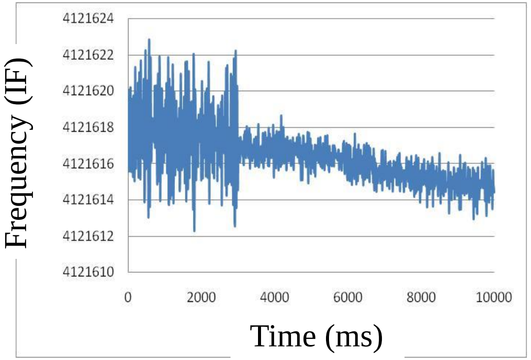
- Positioning Smoothing by Carrier measurement



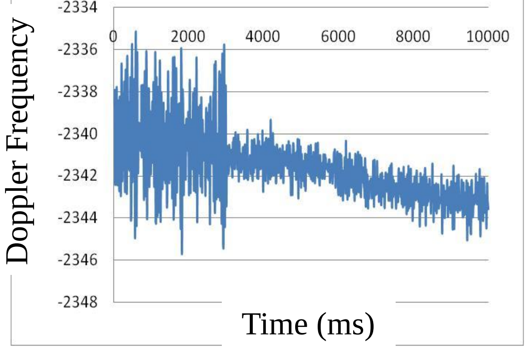
- DGNSS (Differential GPS) and
  RTK (Real Time Kinematic)
  are powerful method for error mitigation.

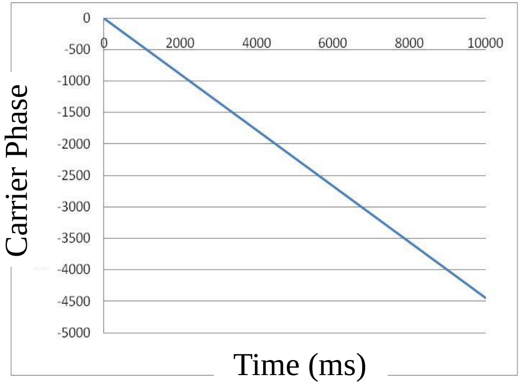
- DGNSS uses the fact that the most of error sources change slowly in the time domain if the distance between reference and user is approx. within 100km.

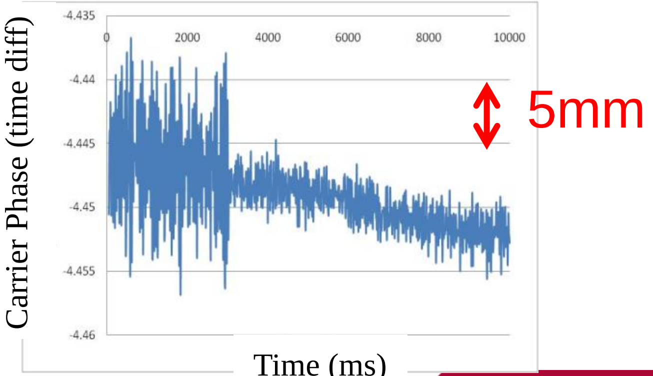
Real Carrier Phase Measurements



IF=4.123968MHz







Carrier-phase is very accurate with 5 mm level accuracy!

Observation Model for Carrier phase Measurements

Observation function for carrier phase measurement

$$\varphi_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) - I_{r,t}^{s} + T_{r,t}^{s} + \varepsilon_{r,t}^{s} + \lambda N_{r,t}^{s}$$

Carrier phase

Range distance

Receiver clock Bias (1~2m)

Satellite clock bias

ionospheric delay Distance (1~2m)

tropospheric delay Distance (1~2m)

multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)

Ambiguity

$$\sqrt{(x_t^s - x_{r,t})^2 + (y_t^s - y_{r,t})^2 + (z_t^s - z_{r,t})^2}$$

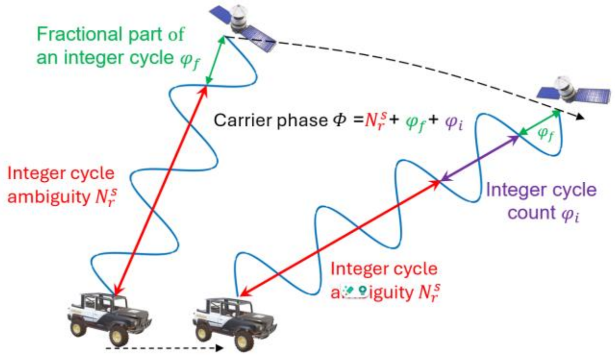
To use the carrier-phase measurements, the ambiguity need to be resolved.

For each carrier-phase measurement, you got an unknown variable N_r^s!

Constant integer ambiguity:

The variable $N_{r,t}^{s}$ tend to be constant within consecutive epochs

$$N_{r,1}^{s} = N_{r,2}^{s} = N_{r,3}^{s} = N_{r,4}^{s} = N_{r,5}^{s} = \ldots$$



Observation Model for Pseudorange/Carrier Measurements

Observation function for pseudo-range (code) measurement

$$\underbrace{\rho_{r,t}^{s}}_{\text{Pseudorange}} = \underbrace{r_{r,t}^{s}}_{\text{Range distance}} + c\left(\underbrace{\delta_{r,t}}_{\text{Receiver clock Bias (1\sim2m)}} - \underbrace{\delta_{r,t}^{s}}_{\text{Satellite clock bias}}\right) + \underbrace{I_{r,t}^{s}}_{\text{ionospheric delay Distance (1\sim2m)}} + \underbrace{T_{r,t}^{s}}_{\text{tropospheric delay Distance (1\sim2m)}} + \underbrace{\varepsilon_{r,t}^{s}}_{\text{multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0\sim100m)}}$$

Range distance:

$$\sqrt{(x_t^s - x_{r,t})^2 + (y_t^s - y_{r,t})^2 + (z_t^s - z_{r,t})^2}$$

Observation function for carrier-phase measurement

$$\underbrace{\varphi_{r,t}^{s}}_{\text{Carrier phase}} = \underbrace{r_{r,t}^{s}}_{\text{Range distance}} + c\left(\underbrace{\delta_{r,t}}_{\text{Receiver clock Bias (1\sim2m)}} - \underbrace{\delta_{r,t}^{s}}_{\text{Satellite clock bias}}\right) - \underbrace{I_{r,t}^{s}}_{\text{ionospheric delay Distance (1\sim2m)}} + \underbrace{T_{r,t}^{s}}_{\text{tropospheric delay Distance (1\sim2m)}} + \underbrace{\varepsilon_{r,t}^{s}}_{\text{multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0\sim100m)}} + \underbrace{\lambda N_{r,t}^{s}}_{\text{Ambiguity}}$$

Range distance:

$$\sqrt{(x_t^s - x_{r,t})^2 + (y_t^s - y_{r,t})^2 + (z_t^s - z_{r,t})^2}$$

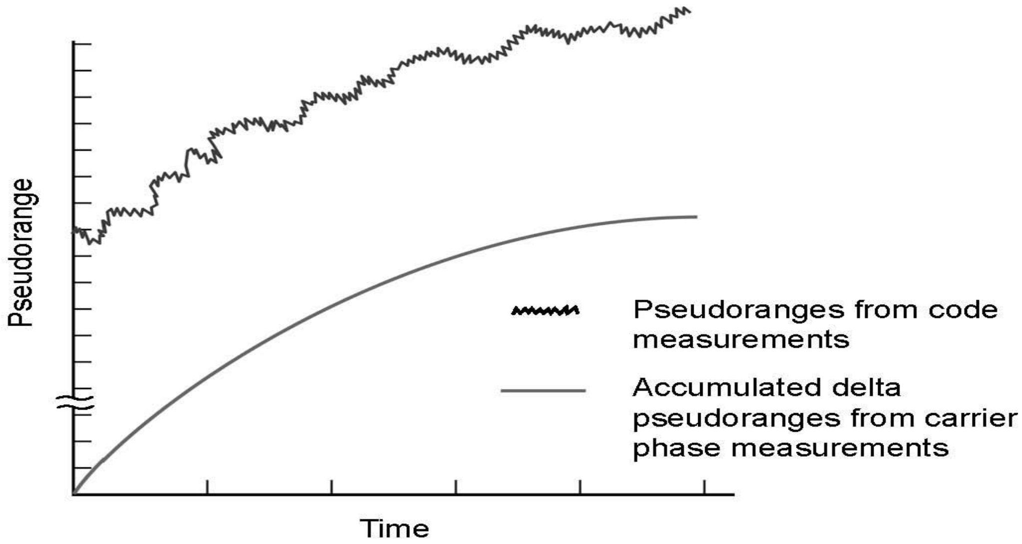
To use the carrier-phase measurements, the ambiguity need to be resolved.

For each carrier-phase measurement, you got an unknown variable N_r^s!

Positioning Smoothing by Carrier measurement

Combining Code and Carrier Measurements

Carrier phase measurement can be used to smooth pseudo-range Measurement.



The code-based measurements are noisy. The carrier-based estimates are precise but ambiguous, and the plot starts arbitrarily at zero value.

Formula of Carrier Smoothing

Phase-smoothed pseudorange aims to combine the pseudorange and carrier phase measurements to obtain an observation with no integer ambiguity and very low noise:

$$\bar{\rho}_{r,t+1}^{s} = \frac{1}{M}\rho_{r,t+1}^{s} + \frac{M-1}{M}\left[\bar{\rho}_{r,t}^{s} + \left[\varphi_{r,t+1}^{s} - \varphi_{r,t}^{s}\right]\right]$$

Can we remove the unknown variable N_r^s within two consecutive epochs?

Formulate it!

Formula of Carrier Smoothing

Tips: time-difference for pseudorange and carrier-phase measurements

The undifferenced and uncombined GNSS pseudorange and carrier phase measurements for a satellite at epoch t and t+1 are shown below:

$$\rho_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) + I_{r,t}^{s} + T_{r,t}^{s} + \varepsilon_{r,t}^{s} \quad (1)$$

$$\rho_{r,t+1}^{s} = r_{r,t+1}^{s} + c\left(\delta_{r,t+1} - \delta_{r,t+1}^{s}\right) + I_{r,t+1}^{s} + T_{r,t+1}^{s} + \varepsilon_{r,t+1}^{s} \quad (2)$$

$$\varphi_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) - I_{r,t}^{s} + T_{r,t}^{s} + E_{r,t}^{s} + \lambda N_{r,t}^{s} \quad (3)$$

$$\varphi_{r,t+1}^{s} = r_{r,t+1}^{s} + c\left(\delta_{r,t+1} - \delta_{r,t+1}^{s}\right) - I_{r,t+1}^{s} + T_{r,t+1}^{s} + E_{r,t+1}^{s} + \lambda N_{r,t+1}^{s} \quad (4)$$

Formula of Carrier Smoothing

Step 1: Difference the carrier phase measurements of the same satellite at t+1 and t epochs

Assume that the integer ambiguity are constant if no cycle slip occurs and the atmospheric delays in adjacent epochs are almost equal, $N^s_{r,t+1} = N^s_{r,t}, I^s_{r,t} \approx I^s_{r,t+1}, T^s_{r,t} \approx T^s_{r,t+1}$

$$D^s_{r,t+1} = \varphi^s_{r,t+1} - \varphi^s_{r,t} = r^s_{r,t+1} - r^s_{r,t} + c(\delta_{r,t+1} - \delta^s_{r,t+1} - \delta_{r,t} + \delta^s_{r,t}) + E^s_{r,t+1} - E^s_{r,t} \quad (5)$$

Step 2: Difference the code measurements of the same satellite at t+1 and t epochs

$$\rho^s_{r,t+1} - \rho^s_{r,t} = r^s_{r,t+1} - r^s_{r,t} + c(\delta_{r,t+1} - \delta^s_{r,t+1} - \delta_{r,t} + \delta^s_{r,t}) + \varepsilon^s_{r,t+1} - \varepsilon^s_{r,t} \quad (6)$$

Step 3: Move the term in (5) to express $r^s_{r,t+1}$

$$r^s_{r,t+1} = D^s_{r,t+1} + r^s_{r,t} - c(\delta_{r,t+1} - \delta^s_{r,t+1} - \delta_{r,t} + \delta^s_{r,t}) - E^s_{r,t+1} + E^s_{r,t} \quad (7)$$

Formula of Carrier Smoothing

Step 4: Use $r_{r,t+1}^{s}$ to express the $\rho_{r,t+1}^{s}$

$$\rho_{r,t+1}^{s} = D_{r,t+1}^{s} + \rho_{r,t}^{s} \tag{8}$$

Step 5: Express the $\rho_{r,t+1}^{s}$ between epoch t+1 and each epoch from 1 to t+1

$$\begin{cases} \rho_{r,\mathrm{t}+1}^{s} = \rho_{r,1}^{s} + D_{r,1,t+1}^{s} = \rho_{r,1}^{s} + \varphi_{r,t+1}^{s} - \varphi_{r,1}^{s} \\ \rho_{r,\mathrm{t}+1}^{s} = \rho_{r,2}^{s} + D_{r,2,t+1}^{s} = \rho_{r,1}^{s} + \varphi_{r,t+1}^{s} - \varphi_{r,2}^{s} \\ \vdots \\ \rho_{r,\mathrm{t}+1}^{s} = \rho_{r,t}^{s} + D_{r,\mathrm{t},t+1}^{s} = \rho_{r,t}^{s} + \varphi_{r,t+1}^{s} - \varphi_{r,t}^{s} \\ \rho_{r,\mathrm{t}+1}^{s} = \rho_{r,t+1}^{s} + D_{r,\mathrm{t},t+1}^{s} = \rho_{r,t+1}^{s} + \varphi_{r,t+1}^{s} - \varphi_{r,t+1}^{s} \end{cases} \tag{9}$$

Formula of Carrier Smoothing

Step 6: calculate the average value of $\rho_{r,t+1}^{s}$

$$\bar{\rho}_{r,t+1}^{s} = \frac{1}{t+1}\sum_{i=1}^{t+1}\rho_{r,i}^{s} - \frac{1}{t+1}\sum_{i=1}^{t+1}\varphi_{r,i}^{s} + \varphi_{r,t+1}^{s}$$

$$= \frac{1}{t+1}\sum_{i=1}^{t}\rho_{r,i}^{s} + \frac{\rho_{r,t+1}^{s}}{t+1} + \varphi_{r,t+1}^{s} - \frac{1}{t+1}\sum_{i=1}^{t}\varphi_{r,i}^{s} - \frac{\varphi_{r,t+1}^{s}}{t+1}$$

$$= \frac{1}{t+1}\sum_{i=1}^{t}\rho_{r,i}^{s} + \frac{\rho_{r,t+1}^{s}}{t+1} - \frac{1}{t+1}\sum_{i=1}^{t}\varphi_{r,i}^{s} + \frac{t\varphi_{r,t+1}^{s}}{t+1}$$

$$= \frac{\rho_{r,t+1}^{s}}{t+1} + \frac{t}{t+1}\left(\frac{1}{t}\sum_{i=1}^{t}\rho_{r,i}^{s} - \frac{1}{t}\sum_{i=1}^{t}\varphi_{r,i}^{s} + \varphi_{r,t+1}^{s}\right)$$

$$= \frac{\rho_{r,t+1}^{s}}{t+1} + \frac{t}{t+1}\left(\frac{1}{t}\sum_{i=1}^{t}\rho_{r,i}^{s} + \varphi_{r,t+1}^{s} + \varphi_{r,t}^{s} - \varphi_{r,t}^{s} - \frac{1}{t}\sum_{i=1}^{t}\varphi_{r,i}^{s}\right)$$

$$= \frac{\rho_{r,t+1}^{s}}{t+1} + \frac{t}{t+1}\left(\frac{1}{t}\sum_{i=1}^{t}\rho_{r,i}^{s} - \frac{1}{t}\sum_{i=1}^{t}\varphi_{r,i}^{s} + \varphi_{r,t}^{s} + \varphi_{r,t+1}^{s} - \varphi_{r,t}^{s}\right)$$

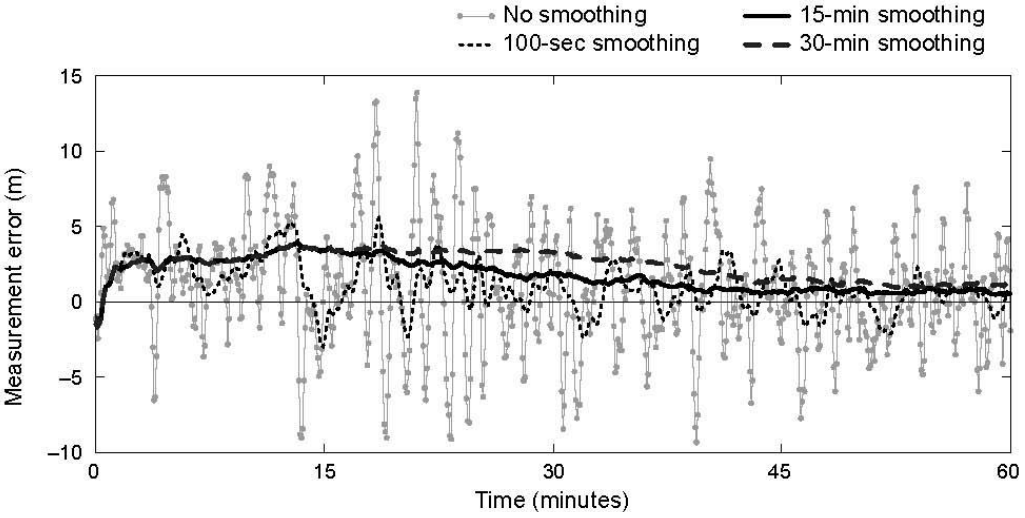
$$= \frac{\rho_{r,t+1}^{s}}{t+1} + \frac{t}{t+1}\left(\bar{\rho}_{r,t}^{s} + \varphi_{r,t+1}^{s} - \varphi_{r,t}^{s}\right)$$

$$\bar{\rho}_{r,t+1}^{s} = \frac{1}{M}\rho_{r,t+1}^{s} + \frac{M-1}{M}\left[\bar{\rho}_{r,t}^{s} + \left[\varphi_{r,t+1}^{s} - \varphi_{r,t}^{s}\right]\right] \text{(M=t+1)}$$

Carrier-smoothed pseudo-ranges with different filter lengths

$$\bar{\rho}_{r,t+1}^{s} = \frac{1}{M}\rho_{r,t+1}^{s} + \frac{M-1}{M}\left[\bar{\rho}_{r,t}^{s} + \left[\varphi_{r,t+1}^{s} - \varphi_{r,t}^{s}\right]\right]$$

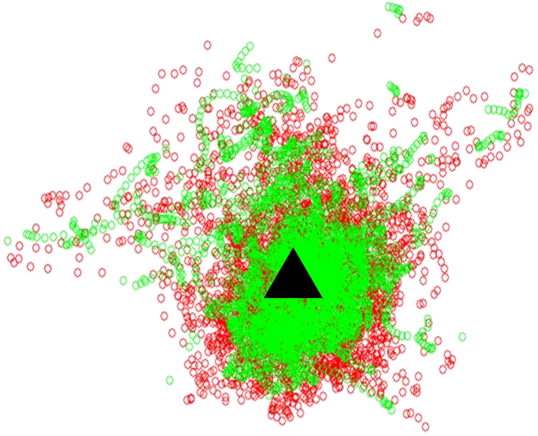
Larger window M tends to provide better accuracy?



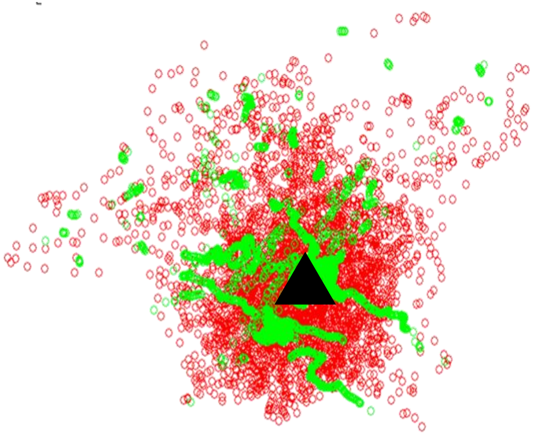
- Carrier-smoothed pseudoranges are better than the pseudorange-only measurements.
- Too large window M can even cause worse accuracy.
- A larger window means more trust is placed on the smoothing components.

Carrier Smoothing Result using different M values

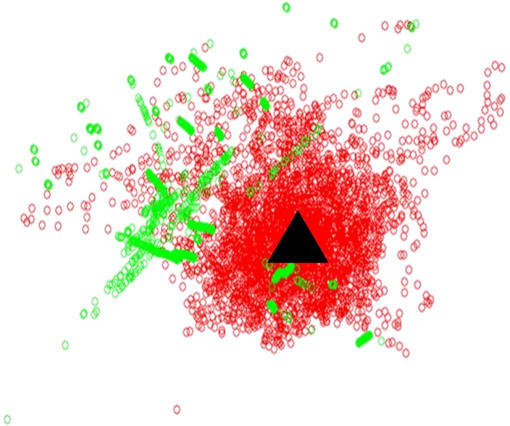
- M=10



- M=100



- M=5000



○ Single point positioning
▲ True position
○ Single point positioning with Carrier smoothing

Problem of Carrier Smoothing

Phase cycle slips

- Cycle slips, which are sudden changes in the carrier phase measurements, can cause discontinuities in the phase-smoothed pseudorange.
- This can degrade the smoothing performance and introduce errors in the positioning solution.

Window size (M)

- As a larger M also increases the time delay and the sensitivity to ionospheric effects.
- The optimal window size depends on the specific application requirements and the dynamic characteristics of the GNSS receiver.

Gross error

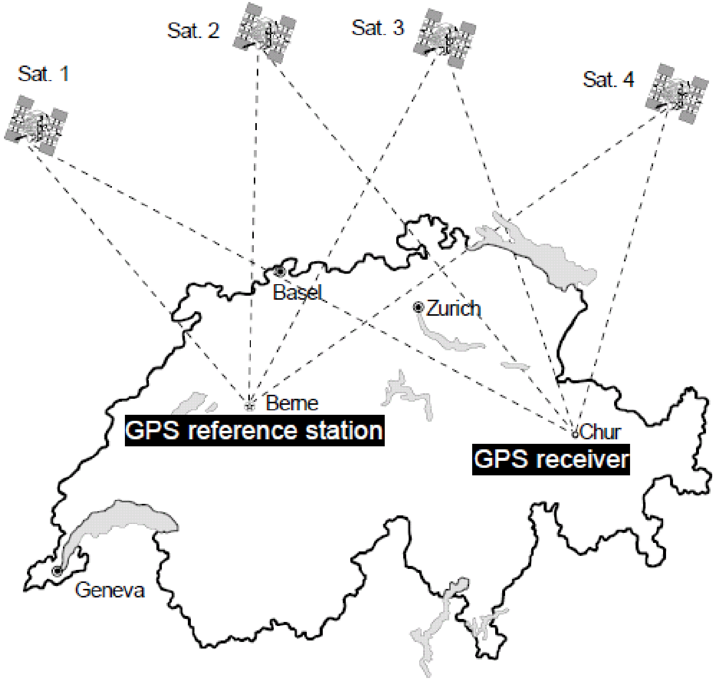
- Observations with gross errors, such as those caused by multipath or interference, can adversely impact the smoothed results.
- Robust outlier detection methods are necessary to remove these gross errors before smoothing.

Ionosphere accumulation

- As the M increases, the difference between the smoothed value and the actual ionospheric effect also increases.
- The impact of ionospheric effects on the smoothed results becomes more significant with larger smoothing windows.

Differential GNSS

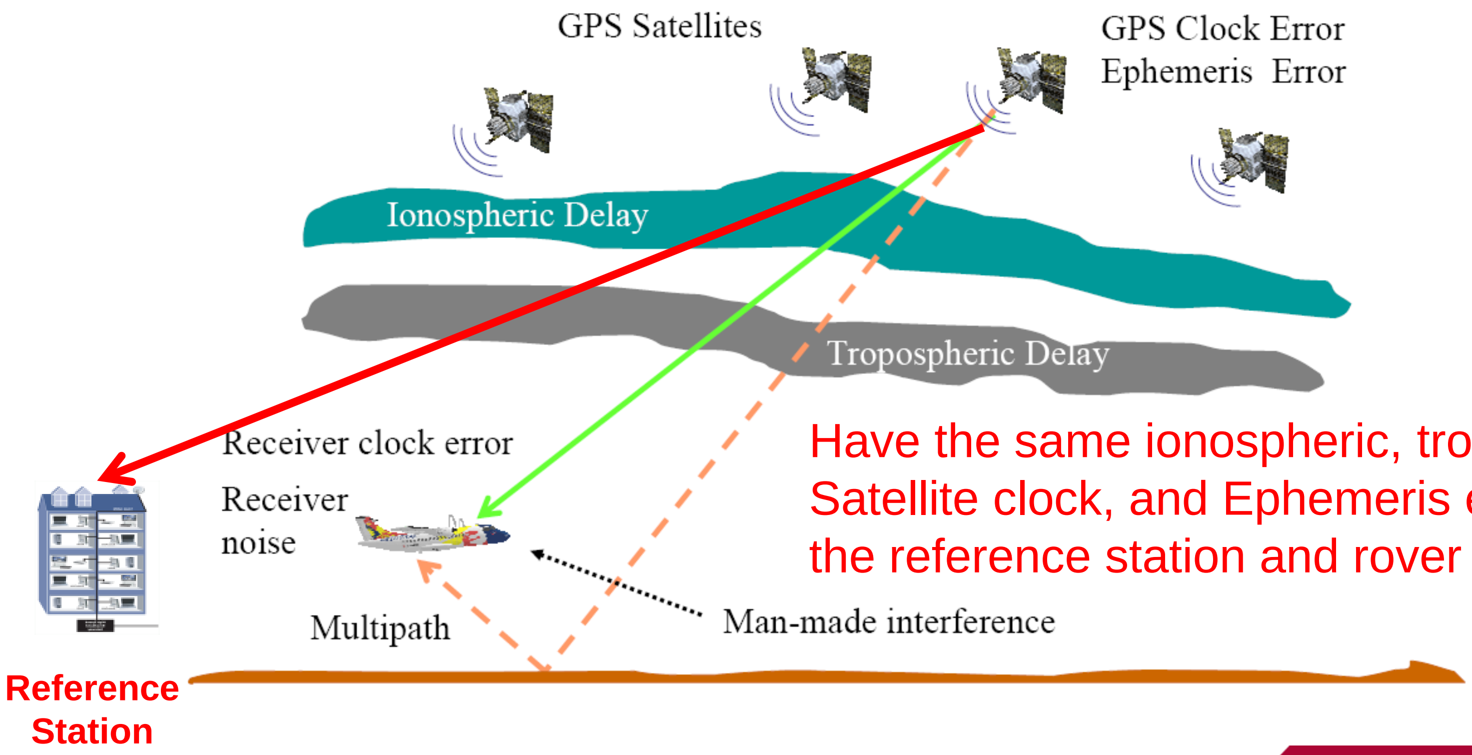
Architecture of DGNSS



- Determination of the correction value s at the reference station
- Transmission of the correction values from the reference station to the GP S user
- Compensation for the determined pse udo-ranges to correct the calculated position of the GPS user

Correction [prn] = Pseudo-range[prn] – True-range [prn]

Principle of DGNSS



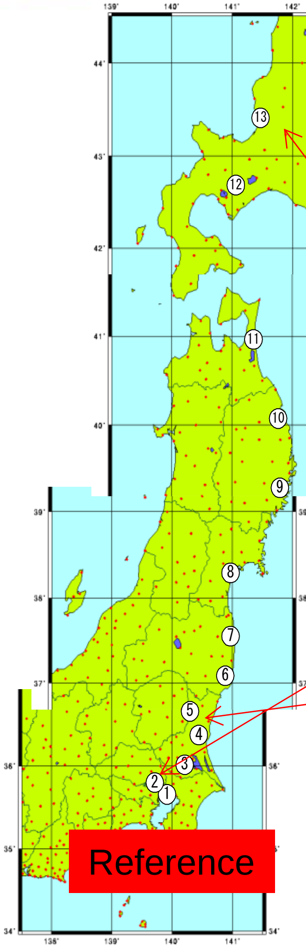
Have the same ionospheric, tropospheric, Satellite clock, and Ephemeris error between the reference station and rover station

DGNSS mitigates …

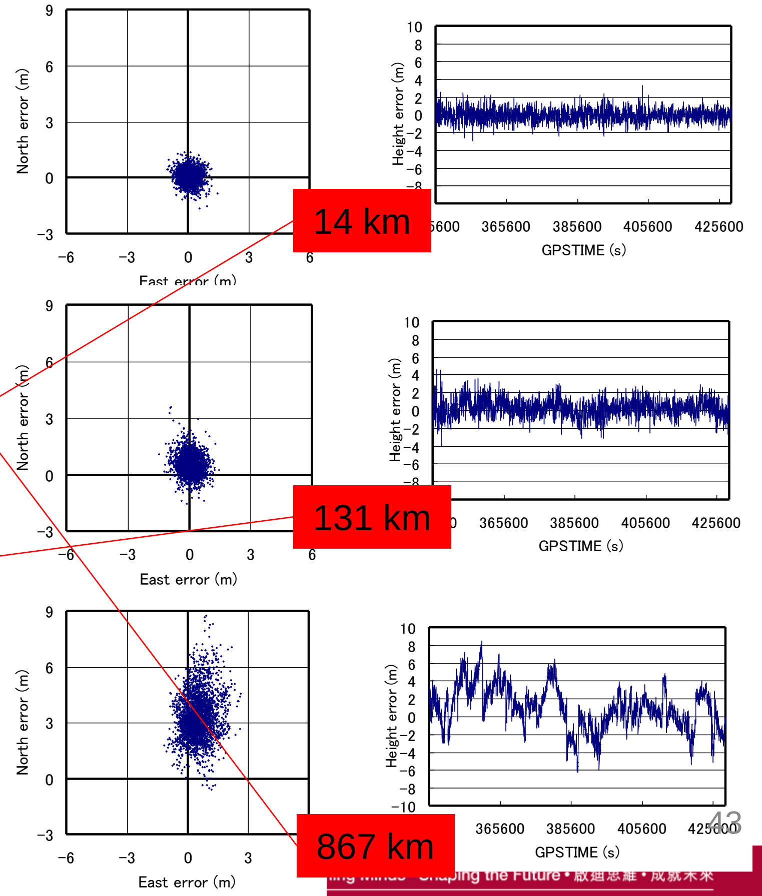
Source	Potential error using SPP	Error mitigation using DGPS
Satellite clock model	2 m (rms)	0.0 m
Satellite ephemeris prediction	2 m (rms) along the LOS	0.1 m (rms)
Ionospheric delay	2-10 m (zenith) Obliquity factor 3 at 5°	0.2 m (rms)
Tropospheric delay	2.3-2.5m (zenith) Obliquity factor 10 at 5°	0.2 m (rms) + altitude effect
Multipath (open sky)	Code : 0.5-1 m Carrier : 0.5-1 cm	Not helpful
Receiver Noise	Code : 0.25-0.5 m (rms) Carrier : 1-2 mm (rms)	Not helpful

rms: root mean square

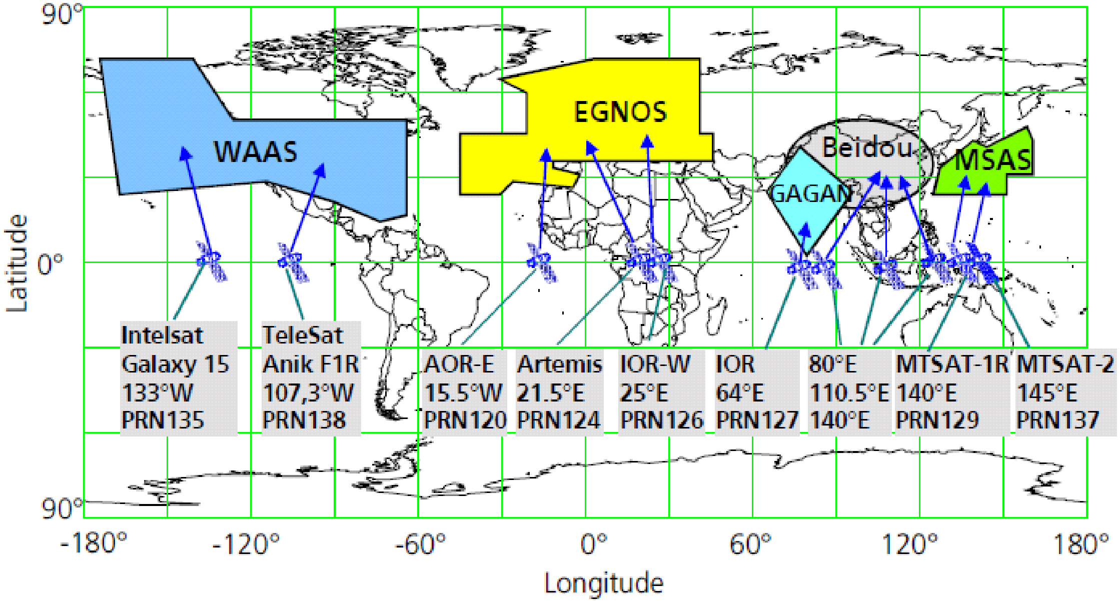
Limitation of DGNSS



number	name	type
①	千葉市川	基準局
②	足立	未知点
③	阿見	未知点
④	水戸	未知点
⑤	大田原	未知点
⑥	いわき	未知点
⑦	小高	未知点
⑧	利府	未知点
⑨	釜石	未知点
⑩	久慈	未知点
⑪	六ヶ所	未知点
⑫	大滝	未知点
⑬	厚田	未知点



Satellite-Based Augmentation System (SBAS)



Without the installation of the reference stations, you can use correction data.
through the SBAS satellite such as the Multi-functional Transport Satellite (MTSAT) in Japan. Under quiet ionospheric conditions, the performance is generally good within 1-2 m.

Flowchart of DGNSS positioning using Iterative LS

Step 1: Form the DD pseudorange measurement function

➢ The raw measurement function at rover station *r* and base station *b* can be written as follows:

$$\rho_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) + I_{r,t}^{s} + T_{r,t}^{s} + \varepsilon_{r,t}^{s} \quad (1)$$

$$\rho_{b,t}^{s} = r_{b,t}^{s} + c\left(\delta_{b,t} - \delta_{b,t}^{s}\right) + I_{b,t}^{s} + T_{b,t}^{s} + \varepsilon_{b,t}^{s} \quad (2)$$

➢ Make inter-station difference and the single-differenced (SD) measurement function can be shown as follows:

$$\rho_{r,t}^{s} - \rho_{b,t}^{s} = r_{r,t}^{s} - r_{b,t}^{s} + c\left(\delta_{r,t} - \delta_{b,t}\right) + \varepsilon_{r} \quad (3)$$

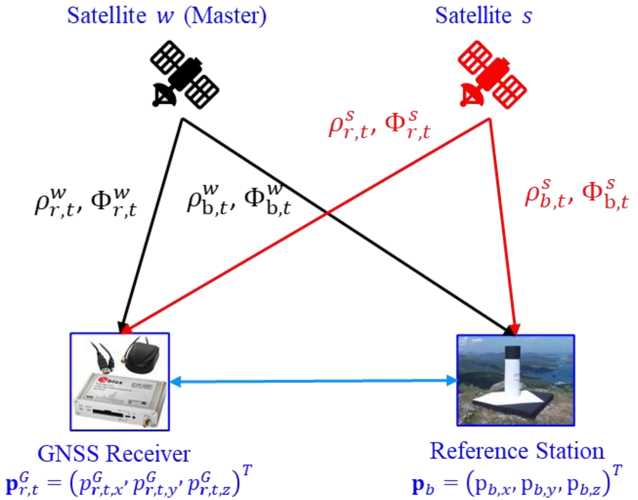
$$\rho_{r,t}^{sf} - \rho_{b,t}^{sf} = r_{r,t}^{sf} - r_{b,t}^{sf} + c\left(\delta_{r,t} - \delta_{b,t}\right) + \varepsilon_{b} \quad (4)$$

where sf denotes the reference satellite

➢ Make inter-satellite difference and the double-differenced (DD) measurement function can be shown as follows:

$$\rho_{r,t}^{s} - \rho_{b,t}^{s} - \rho_{r,t}^{sf} + \rho_{b,t}^{sf} = r_{r,t}^{s} - r_{b,t}^{s} - r_{r,t}^{sf} + r_{b,t}^{sf} + \varepsilon \quad (5)$$

Assumption: GNSS receiver and the reference station are close with the same atmosphere errors



Flowchart of DGNSS positioning using Iterative LS

Step 2: Linearize the DD pseudorange measurement function

$$\nabla\Delta P_{r,t}^{s}=\rho_{r,t}^{s}-\rho_{b,t}^{s}-\rho_{r,t}^{sf}+\rho_{b,t}^{sf}=r_{r,t}^{s}-r_{b,t}^{s}-r_{r,t}^{sf}+r_{b,t}^{sf}+\varepsilon$$

$$=\sqrt{(x_t^s-x_{r,t})^2+(y_t^s-y_{r,t})^2+(z_t^s-z_{r,t})^2}-\sqrt{(x_t^{sf}-x_{r,t})^2+(y_t^{sf}-y_{r,t})^2+(z_t^{sf}-z_{r,t})^2}-\sqrt{(x_t^s-x_{b,t})^2+(y_t^s-y_{b,t})^2+(z_t^s-z_{b,t})^2}+\sqrt{(x_t^{sf}-x_{b,t})^2+(y_t^{sf}-y_{b,t})^2+(z_t^{sf}-z_{b,t})^2}+\varepsilon$$

Linearization

$$=\boxed{\rho_0-\frac{(x^s-x_0)}{\rho_0}(x_{r,t}-x_0)-\frac{(y^s-y_0)}{\rho_0}(y_{r,t}-y_0)-\frac{(z^s-z_0)}{\rho_0}(z_{r,t}-z_0)}\boxed{+\rho_0^f+\frac{(x^{sf}-x_0)}{\rho_0^f}(x_{r,t}-x_0)+\frac{(y^{sf}-y_0)}{\rho_0^f}(y_{r,t}-y_0)+\frac{(z^{sf}-z_0)}{\rho_0^f}(z_{r,t}-z_0)}$$

$$-\sqrt{(x_t^s-x_{b,t})^2+(y_t^s-y_{b,t})^2+(z_t^s-z_{b,t})^2}+\sqrt{(x_t^{sf}-x_{b,t})^2+(y_t^{sf}-y_{b,t})^2+(z_t^{sf}-z_{b,t})^2}+\varepsilon$$

$$=\left[\left(\frac{(x^{sf}-x_0)}{\rho_0^f}-\frac{(x^s-x_0)}{\rho_0}\right)\ \left(\frac{(y^{sf}-y_0)}{\rho_0^f}-\frac{(y^s-y_0)}{\rho_0}\right)\ \left(\frac{(z^{sf}-z_0)}{\rho_0^f}-\frac{(z^s-z_0)}{\rho_0}\right)\right]\left[(x_{r,t}-x_0)\ \ (y_{r,t}-y_0)\ \ (z_{r,t}-z_0)\right]^{\mathrm{T}}+b+\varepsilon \qquad (6)$$

with $b=\rho_0-\rho_0^f-\sqrt{(x_t^s-x_{b,t})^2+(y_t^s-y_{b,t})^2+(z_t^s-z_{b,t})^2}+\sqrt{(x_t^{sf}-x_{b,t})^2+(y_t^{sf}-y_{b,t})^2+(z_t^{sf}-z_{b,t})^2}$;

Flowchart of DGNSS positioning using Iterative LS

Step 3: Form the error function

- Accordingly, the error equation v can be written as:

$$v = f\left(x_{r,t}, y_{r,t}, z_{r,t}\right) - \nabla\Delta P_{r,t}^{s}$$

$$= \left[\left(\frac{(x^{sf}-x_0)}{\rho_0^{f}} - \frac{(x^{s}-x_0)}{\rho_0}\right) \quad \left(\frac{(y^{sf}-y_0)}{\rho_0^{f}} - \frac{(y^{s}-y_0)}{\rho_0}\right) \quad \left(\frac{(z^{sf}-z_0)}{\rho_0^{f}} - \frac{(z^{s}-z_0)}{\rho_0}\right)\right]\left[(x_{r,t}-x_0) \quad (y_{r,t}-y_0) \quad (z_{r,t}-z_0)\right]^{T} - \Delta\rho \quad (7)$$

with $\Delta\rho = \nabla\Delta P_{r,t}^{s} - b$.

- Assuming that n+1 satellites are observed, the DD function is established:

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} \left(\frac{(x^{sf}-x_0)}{\rho_0^{f1}} - \frac{(x^{s1}-x_0)}{\rho_0^{1}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{f1}} - \frac{(y^{s1}-y_0)}{\rho_0^{1}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{f1}} - \frac{(z^{s1}-z_0)}{\rho_0^{1}}\right) \\ \left(\frac{(x^{sf}-x_0)}{\rho_0^{f2}} - \frac{(x^{s2}-x_0)}{\rho_0^{2}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{f2}} - \frac{(y^{s2}-y_0)}{\rho_0^{2}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{f2}} - \frac{(z^{s2}-z_0)}{\rho_0^{2}}\right) \\ & \vdots & \\ \left(\frac{(x^{sf}-x_0)}{\rho_0^{fn}} - \frac{(x^{sn}-x_0)}{\rho_0^{n}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{fn}} - \frac{(y^{sn}-y_0)}{\rho_0^{n}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{fn}} - \frac{(z^{sn}-z_0)}{\rho_0^{n}}\right) \end{bmatrix} \begin{bmatrix} x_{r,t} - x_0 \\ y_{r,t} - y_0 \\ z_{r,t} - z_0 \end{bmatrix} - \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \vdots \\ \Delta\rho_n \end{bmatrix} \quad (8)$$

$$\boldsymbol{V} = \boldsymbol{G}\boldsymbol{\Delta p} - \boldsymbol{\Delta\rho}$$

Flowchart of DGNSS positioning using Iterative LS

Step 4: DGNSS Positioning using Iterative LS

- Based on the principle of LS, the solution can be calculated as follows:

$$\Delta \boldsymbol{p} = \begin{bmatrix} x_{r,t} - x_0 \\ y_{r,t} - y_0 \\ z_{r,t} - z_0 \end{bmatrix} = \left(\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}\right)^{-1}\boldsymbol{G}^{\mathrm{T}}\Delta\boldsymbol{\rho} \qquad (5)$$

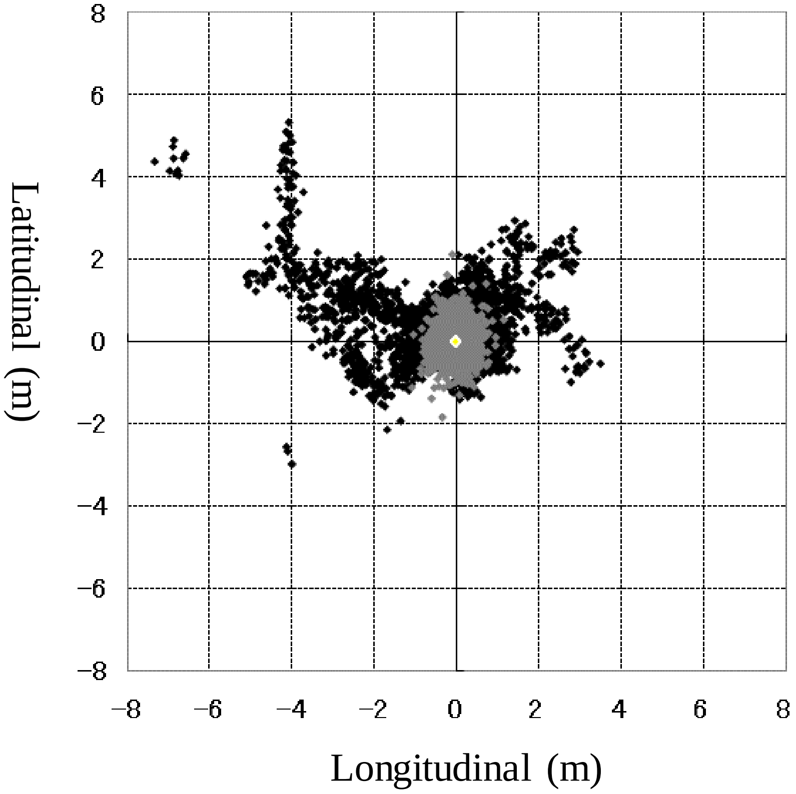
- Therefore, the rover receiver position can be obtained:

$$\boldsymbol{P}_{\boldsymbol{1}} = \boldsymbol{P}_{\boldsymbol{0}} + \Delta \boldsymbol{p} \qquad (6)$$

- Repeat steps 1-4 and set $\boldsymbol{P}_{\boldsymbol{0}} = \boldsymbol{P}_{\boldsymbol{i}}$ until the difference between the calculated receiver coordinates $\boldsymbol{P}_{\boldsymbol{i+1}}$ and the previously calculated coordinates $\boldsymbol{P}_{\boldsymbol{i}}$ is within the threshold, thus obtaining the final receiver coordinates：

$$\boldsymbol{P}_{\boldsymbol{final}} = \boldsymbol{P}_{\boldsymbol{i}} + \Delta \boldsymbol{p} \qquad (7)$$

DGNSS and Real Time Kinematic (RTK) Performance



- Single Point Positioning
- DGNSS
- RTK

Rooftop (Lab.)
15s interval
24 hours
Reference : Ichikawa

The DGNSS is still
not accurate enough!

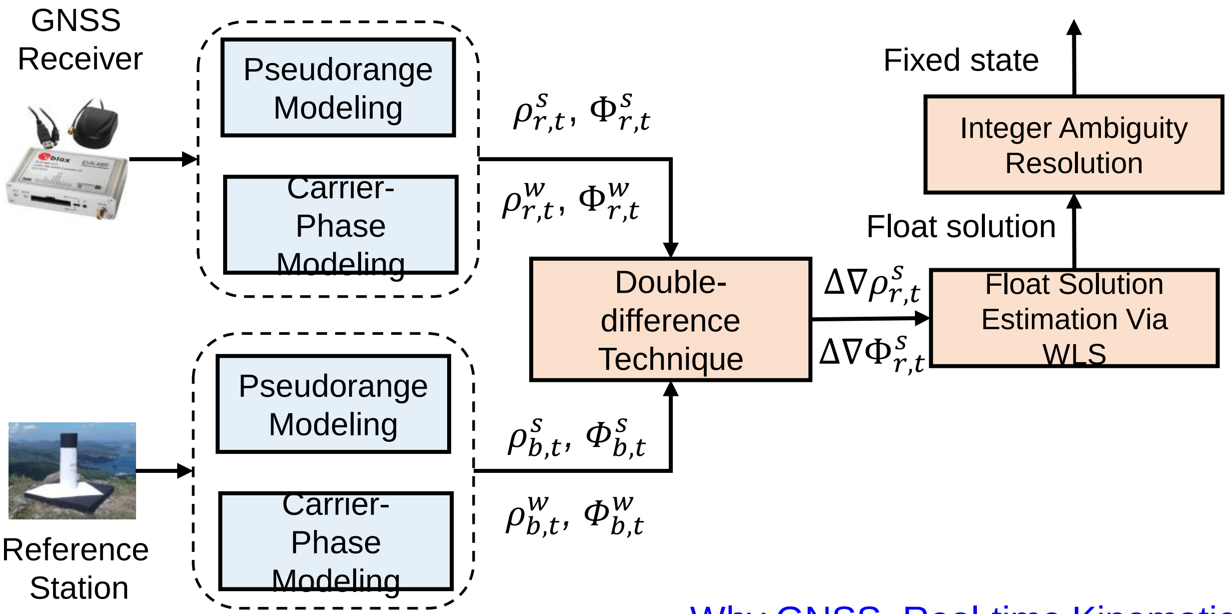
Real Time Kinematic positioning

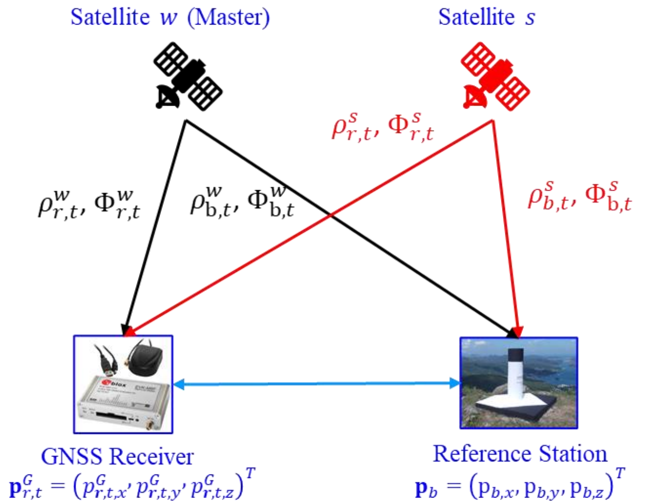
RTK

- The concept of RTK is the same as DGNSS.
- RTK uses pseudo-range and carrier phase measurements, while DGNSS only uses pseudo-range measurements.
- GNSS receiver is able to measure 1/100 of wavelength of L1 frequency (19 cm).
- If you have a high-end receiver, you know your position within 1-2 cm accuracy as long as you have 5 or more LOS satellites.

Overview of GNSS Real-time Kinematic

WLS*: Weighted Least Squares
DD *: Double-difference





$\rho^s_{r,t}$: Pseudorange measurement
$\Phi^s_{r,t}$: Carrier-phase measurement
$\Delta\nabla\rho^s_{r,t}$: DD Pseudorange measurement
$\Delta\nabla\Phi^s_{r,t}$: DD Carrier-phase measurement

Why GNSS Real-time Kinematic?

- Remove the error from receiver/satellite clock bias, atmosphere error using the double-difference technique.
- Use the high-accuracy carrier-phase measurements.

Observation Model for Pseudorange/Carrier Measurements

Observation function for pseudo-range (code) measurement

$$\underbrace{\rho_{r,t}^{s}}_{\text{Pseudorange}} = \underbrace{r_{r,t}^{s}}_{\text{Range distance}} + c(\underbrace{\delta_{r,t}}_{\text{Receiver clock Bias (1~2m)}} - \underbrace{\delta_{r,t}^{s}}_{\text{Satellite clock bias}}) + \underbrace{I_{r,t}^{s}}_{\text{ionospheric delay Distance (1~2m)}} + \underbrace{T_{r,t}^{s}}_{\text{tropospheric delay Distance (1~2m)}} + \varepsilon_{r,t}^{s}$$

$\varepsilon_{r,t}^{s}$: multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)

Range distance:

$$\sqrt{(x_t^s - x_{r,t})^2 + (y_t^s - y_{r,t})^2 + (z_t^s - z_{r,t})^2}$$

Observation function for carrier-phase measurement

$$\underbrace{\varphi_{r,t}^{s}}_{\text{Carrier phase}} = \underbrace{r_{r,t}^{s}}_{\text{Range distance}} + c(\underbrace{\delta_{r,t}}_{\text{Receiver clock Bias (1~2m)}} - \underbrace{\delta_{r,t}^{s}}_{\text{Satellite clock bias}}) - \underbrace{I_{r,t}^{s}}_{\text{ionospheric delay Distance (1~2m)}} + \underbrace{T_{r,t}^{s}}_{\text{tropospheric delay Distance (1~2m)}} + \varepsilon_{r,t}^{s} + \underbrace{\lambda N_{r,t}^{s}}_{\text{Ambiguity}}$$

$\varepsilon_{r,t}^{s}$: multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)

Range distance:

$$\sqrt{(x_t^s - x_{r,t})^2 + (y_t^s - y_{r,t})^2 + (z_t^s - z_{r,t})^2}$$

To use the carrier-phase measurements, the ambiguity need to be resolved.

For each carrier-phase measurement, you got an unknown variable N_r^s!

GNSS Real-time Kinematic Positioning: Float Solution Estimation

Estimate the float solution via weighted least square positioning

$$\begin{bmatrix} p_{x,r,t} \\ p_{y,r,t} \\ p_{z,r,t} \\ \nabla\Delta N_{r,t}^{s1} \\ \nabla\Delta N_{r,t}^{s2} \\ \vdots \\ \nabla\Delta N_{r,t}^{sn} \end{bmatrix} = \left(\boldsymbol{G}^{\mathrm{T}}\boldsymbol{W}_{DD}\boldsymbol{G}\right)^{-1}\boldsymbol{G}^{T}\boldsymbol{W}_{DD}\Delta\boldsymbol{\rho} \xrightarrow{\text{Output}}$$

$\boldsymbol{p}_{r,t}$: Float solution of position of GNSS receiver

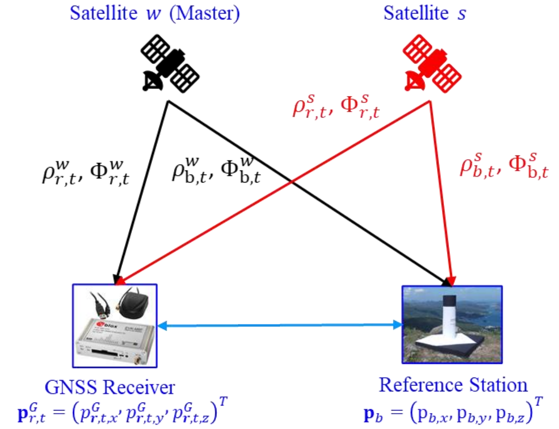
$\Delta\nabla N_{r,t}^{s1}, \Delta\nabla N_{r,t}^{s2}$, …: Float ambiguities

$\left(\boldsymbol{G}^{\mathrm{T}}\boldsymbol{W}_{DD}\boldsymbol{G}\right)^{-1}$: Covariance matrix

$\boldsymbol{p}_{r,t}$: Position of GNSS receiver
$\boldsymbol{W}_{DD}$: Weighting matrix
n: Number of satellite
$\boldsymbol{G}$: Design matrix
$\Delta\nabla\rho_{r,t}^{s}$: DD pseudorange measurements
$\Delta\nabla\psi_{r,t}^{s}$: DD carrier-phase measurements

How to formulate the linear least square problem of the GNSS-RTK

Formulate it!



Flowchart of RTK positioning using Iterative LS

Step 1: Form the DD pseudorange and carrier phase measurement function

- The raw measurement function at rover station r and base station b can be written as follows:

$$\rho^s_{r,t} = r^s_{r,t} + c\left(\delta_{r,t} - \delta^s_{r,t}\right) + I^s_{r,t} + T^s_{r,t} + \varepsilon^s_{r,t} \qquad (1)$$

$$\rho^s_{b,t} = r^s_{b,t} + c\left(\delta_{b,t} - \delta^s_{b,t}\right) + I^s_{b,t} + T^s_{b,t} + \varepsilon^s_{b,t} \qquad (2)$$

$$\varphi^s_{r,t} = r^s_{r,t} + c\left(\delta_{r,t} - \delta^s_{r,t}\right) - I^s_{r,t} + T^s_{r,t} + E^s_{r,t} + \lambda N^s_{r,t} \qquad (3)$$

$$\varphi^s_{b,t} = r^s_{b,t} + c\left(\delta_{r,t} - \delta^s_{r,t}\right) - I^s_{b,t} + T^s_{b,t} + E^s_{b,t} + \lambda N^s_{b,t} \qquad (4)$$

- Make inter-station difference and the SD measurement function can be shown as follows:

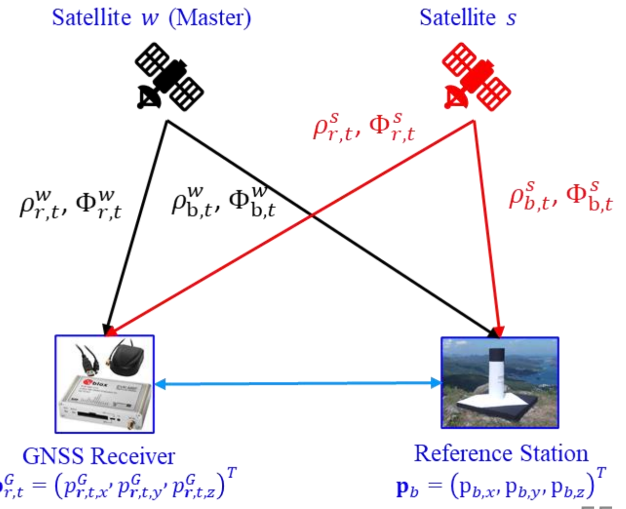
$$\rho^s_{r,t} - \rho^s_{b,t} = r^s_{r,t} - r^s_{b,t} + c\left(\delta_{r,t} - \delta_{b,t}\right) + \nabla\varepsilon^s_{r,t} \qquad (5)$$

$$\rho^{sf}_{r,t} - \rho^{sf}_{b,t} = r^{sf}_{r,t} - r^{sf}_{b,t} + c\left(\delta_{r,t} - \delta_{b,t}\right) + \nabla\varepsilon^{sf}_{r,t} \qquad (6)$$

$$\varphi^s_{r,t} - \varphi^s_{b,t} = r^s_{r,t} - r^s_{b,t} + c\left(\delta_{r,t} - \delta_{b,t}\right) + \lambda N^s_{r,t} - \lambda N^s_{b,t} + \nabla E^s_{r,t} \qquad (7)$$

$$\varphi^{sf}_{r,t} - \varphi^{sf}_{b,t} = r^{sf}_{r,t} - r^{sf}_{b,t} + c\left(\delta_{r,t} - \delta_{b,t}\right) + \lambda N^{sf}_{r,t} - \lambda N^{sf}_{b,t} + \nabla E^{sf}_{r,t} \qquad (8)$$

Assumption: GNSS receiver and the reference station are close with the same atmosphere errors



Flowchart of RTK positioning using Iterative LS

Step 1: Form the DD pseudorange and carrier phase measurement function

- Make inter-satellite difference and the double-differenced function can be shown as follows:

$$\nabla\Delta P_{r,t}^{s} = \rho_{r,t}^{s} - \rho_{b,t}^{s} - \rho_{r,t}^{sf} + \rho_{b,t}^{sf} = r_{r,t}^{s} - r_{b,t}^{s} - r_{r,t}^{sf} + r_{b,t}^{sf} + \nabla\Delta\varepsilon_{r,t}^{s} \quad (9)$$

$$\nabla\Delta\varphi_{r,t}^{s} = \varphi_{r,t}^{s} - \varphi_{b,t}^{s} - \varphi_{r,t}^{sf} + \varphi_{b,t}^{sf} = r_{r,t}^{s} - r_{b,t}^{s} - r_{r,t}^{sf} + r_{b,t}^{sf} + \lambda\nabla\Delta N_{r,t}^{s} + \nabla\Delta E_{r,t}^{s} \quad (10)$$

where $\lambda\nabla\Delta N_{r,t}^{s} = \lambda N_{r,t}^{s} - \lambda N_{b,t}^{s} - \lambda N_{r,t}^{sf} + \lambda N_{b,t}^{sf}$.

Step 2: Linearize the DD pseudorange measurement function

$$\nabla\Delta P_{r,t}^{s} = \left[\left(\frac{(x^{sf}-x_0)}{\rho_0^{f}} - \frac{(x^{s}-x_0)}{\rho_0}\right) \quad \left(\frac{(y^{sf}-y_0)}{\rho_0^{f}} - \frac{(y^{s}-y_0)}{\rho_0}\right) \quad \left(\frac{(z^{sf}-z_0)}{\rho_0^{f}} - \frac{(z^{s}-z_0)}{\rho_0}\right) \right] \left[(x_{r,t}-x_0) \quad (y_{r,t}-y_0) \quad (z_{r,t}-z_0) \right]^{T} + b + \varepsilon \quad (11)$$

$$\nabla\Delta\varphi_{r,t}^{s} = \left[\left(\frac{(x^{sf}-x_0)}{\rho_0^{f}} - \frac{(x^{s}-x_0)}{\rho_0}\right) \quad \left(\frac{(y^{sf}-y_0)}{\rho_0^{f}} - \frac{(y^{s}-y_0)}{\rho_0}\right) \quad \left(\frac{(z^{sf}-z_0)}{\rho_0^{f}} - \frac{(z^{s}-z_0)}{\rho_0}\right) \quad \lambda \right] \left[(x_{r,t}-x_0) \quad (y_{r,t}-y_0) \quad (z_{r,t}-z_0) \quad \nabla\Delta N_{r,t}^{s} \right]^{T} + b + \varepsilon \quad (12)$$

with $b = \rho_0 - \rho_0^{f} - \sqrt{(x_t^{s}-x_{b,t})^2 + (y_t^{s}-y_{b,t})^2 + (z_t^{s}-z_{b,t})^2} + \sqrt{(x_t^{sf}-x_{b,t})^2 + (y_t^{sf}-y_{b,t})^2 + (z_t^{sf}-z_{b,t})^2}$;

Flowchart of RTK positioning using LS

Step 3: Form the error function

➢ Accordingly, the pseudorange error equation v can be written as:

$$v = f(x_{r,t}, y_{r,t}, z_{r,t}) - \nabla\Delta P^{s}_{r,t}$$
$$= \left[\left(\frac{(x^{sf}-x_0)}{\rho_0^f} - \frac{(x^s-x_0)}{\rho_0}\right) \quad \left(\frac{(y^{sf}-y_0)}{\rho_0^f} - \frac{(y^s-y_0)}{\rho_0}\right) \quad \left(\frac{(z^{sf}-z_0)}{\rho_0^f} - \frac{(z^s-z_0)}{\rho_0}\right) \right] \left[(x_{r,t}-x_0) \quad (y_{r,t}-y_0) \quad (z_{r,t}-z_0) \right]^{\mathrm{T}} - \Delta\rho \tag{13}$$

with $\Delta\rho = \nabla\Delta P^{s}_{r,t} - b$.

➢ Accordingly, the carrier phase error equation V can be written as:

$$V = f(x_{r,t}, y_{r,t}, z_{r,t}) - \nabla\Delta\varphi^{s}_{r,t}$$
$$= \left[\left(\frac{(x^{sf}-x_0)}{\rho_0^f} - \frac{(x^s-x_0)}{\rho_0}\right) \quad \left(\frac{(y^{sf}-y_0)}{\rho_0^f} - \frac{(y^s-y_0)}{\rho_0}\right) \quad \left(\frac{(z^{sf}-z_0)}{\rho_0^f} - \frac{(z^s-z_0)}{\rho_0}\right) \quad \lambda \right] \left[(x_{r,t}-x_0) \quad (y_{r,t}-y_0) \quad (z_{r,t}-z_0) \quad \nabla\Delta N^{s}_{r,t} \right]^{\mathrm{T}} - \Delta\sigma \tag{14}$$

with $\Delta\sigma = \nabla\Delta\varphi^{s}_{r,t} - b$.

Flowchart of RTK positioning using LS

Step 3: Form the error function

➢Assuming that n+1 satellites are observed, the DD function is established:

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} \left(\frac{(x^{sf}-x_0)}{\rho_0^{f1}} - \frac{(x^{s1}-x_0)}{\rho_0^{1}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{f1}} - \frac{(y^{s1}-y_0)}{\rho_0^{1}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{f1}} - \frac{(z^{s1}-z_0)}{\rho_0^{1}}\right) & 0 & 0 & \cdots 0 \\ \left(\frac{(x^{sf}-x_0)}{\rho_0^{f2}} - \frac{(x^{s2}-x_0)}{\rho_0^{2}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{f2}} - \frac{(y^{s2}-y_0)}{\rho_0^{2}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{f2}} - \frac{(z^{s2}-z_0)}{\rho_0^{2}}\right) & 0 & 0 & \cdots 0 \\ & \vdots & & & & \\ \left(\frac{(x^{sf}-x_0)}{\rho_0^{fn}} - \frac{(x^{sn}-x_0)}{\rho_0^{n}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{fn}} - \frac{(y^{sn}-y_0)}{\rho_0^{n}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{fn}} - \frac{(z^{sn}-z_0)}{\rho_0^{n}}\right) & 0 & 0 & \cdots 0 \\ \left(\frac{(x^{sf}-x_0)}{\rho_0^{f1}} - \frac{(x^{s1}-x_0)}{\rho_0^{1}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{f1}} - \frac{(y^{s1}-y_0)}{\rho_0^{1}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{f1}} - \frac{(z^{s1}-z_0)}{\rho_0^{1}}\right) & \lambda_{s1} & 0 & \cdots 0 \\ \left(\frac{(x^{sf}-x_0)}{\rho_0^{f2}} - \frac{(x^{s2}-x_0)}{\rho_0^{2}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{f2}} - \frac{(y^{s2}-y_0)}{\rho_0^{2}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{f2}} - \frac{(z^{s2}-z_0)}{\rho_0^{2}}\right) & 0 & \lambda_{s1} & \cdots 0 \\ & \vdots & & & & \\ \left(\frac{(x^{sf}-x_0)}{\rho_0^{fn}} - \frac{(x^{sn}-x_0)}{\rho_0^{n}}\right) & \left(\frac{(y^{sf}-y_0)}{\rho_0^{fn}} - \frac{(y^{sn}-y_0)}{\rho_0^{n}}\right) & \left(\frac{(z^{sf}-z_0)}{\rho_0^{fn}} - \frac{(z^{sn}-z_0)}{\rho_0^{n}}\right) & 0 & 0 & \cdots \lambda_{sn} \end{bmatrix} \begin{bmatrix} x_{r,t} - x_0 \\ y_{r,t} - y_0 \\ z_{r,t} - z_0 \\ \nabla\Delta N_{r,t}^{s1} \\ \nabla\Delta N_{r,t}^{s2} \\ \vdots \\ \nabla\Delta N_{r,t}^{sn} \end{bmatrix} - \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \vdots \\ \Delta\rho_n \\ \Delta\sigma_1 \\ \Delta\sigma_2 \\ \vdots \\ \Delta\sigma_n \end{bmatrix} \quad (15)$$

$$\boldsymbol{V} = \boldsymbol{G}\boldsymbol{\Delta p} - \boldsymbol{\Delta\rho}$$

Flowchart of RTK positioning using LS

Step 4: Establish the weight matrix of the DD measurement

The relationship between DD and raw measurements is shown as follows：

$$\begin{bmatrix} \nabla\Delta P_{r,t}^{s1} \\ \nabla\Delta P_{r,t}^{s2} \\ \vdots \\ \nabla\Delta P_{r,t}^{sn} \\ \nabla\Delta \varphi_{r,t}^{s1} \\ \nabla\Delta \varphi_{r,t}^{s2} \\ \vdots \\ \nabla\Delta \varphi_{r,t}^{sn} \end{bmatrix} = \begin{bmatrix} \rho_{r,t}^{s1} - \rho_{b,t}^{s1} - \rho_{r,t}^{sf} + \rho_{b,t}^{sf} \\ \rho_{r,t}^{s2} - \rho_{b,t}^{s2} - \rho_{r,t}^{sf} + \rho_{b,t}^{sf} \\ \vdots \\ \rho_{r,t}^{sn} - \rho_{b,t}^{sn} - \rho_{r,t}^{sf} + \rho_{b,t}^{sf} \\ \varphi_{r,t}^{s1} - \varphi_{b,t}^{s1} - \varphi_{r,t}^{sf} + \varphi_{b,t}^{sf} \\ \varphi_{r,t}^{s2} - \varphi_{b,t}^{s2} - \varphi_{r,t}^{sf} + \varphi_{b,t}^{sf} \\ \vdots \\ \varphi_{r,t}^{sn} - \varphi_{b,t}^{sn} - \varphi_{r,t}^{sf} + \varphi_{b,t}^{sf} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \rho_{r,t}^{sf} \\ \rho_{r,t}^{s1} \\ \rho_{r,t}^{s2} \\ \vdots \\ \rho_{r,t}^{sn} \\ \rho_{b,t}^{sf} \\ \rho_{b,t}^{s1} \\ \rho_{b,t}^{s2} \\ \vdots \\ \rho_{b,t}^{sn} \\ \varphi_{r,t}^{sf} \\ \varphi_{r,t}^{s1} \\ \varphi_{r,t}^{s2} \\ \vdots \\ \varphi_{r,t}^{sn} \\ \varphi_{b,t}^{sf} \\ \varphi_{b,t}^{s1} \\ \varphi_{b,t}^{s2} \\ \vdots \\ \varphi_{b,t}^{sn} \end{bmatrix} = \boldsymbol{J} \begin{bmatrix} \rho_{r,t}^{sf} \\ \rho_{r,t}^{s1} \\ \rho_{r,t}^{s2} \\ \vdots \\ \rho_{r,t}^{sn} \\ \rho_{b,t}^{sf} \\ \rho_{b,t}^{s1} \\ \rho_{b,t}^{s2} \\ \vdots \\ \rho_{b,t}^{sn} \\ \varphi_{r,t}^{sf} \\ \varphi_{r,t}^{s1} \\ \varphi_{r,t}^{s2} \\ \vdots \\ \varphi_{r,t}^{sn} \\ \varphi_{b,t}^{sf} \\ \varphi_{b,t}^{s1} \\ \varphi_{b,t}^{s2} \\ \vdots \\ \varphi_{b,t}^{sn} \end{bmatrix} \quad (16)$$

With $\boldsymbol{J} = \begin{bmatrix} \boldsymbol{F} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{F} \end{bmatrix}$; $\boldsymbol{F} = [-\boldsymbol{e_n} \quad \boldsymbol{I_n} \quad \boldsymbol{e_n} \quad -\boldsymbol{I_n}]$

Flowchart of RTK positioning using LS

Step 4: Establish the weight matrix of the DD measurement

- Based on the heteroskedasticity, assume that the raw observations are independent of each other, and their variance matrix is shown below:

$$\boldsymbol{Q}_{\mathrm{raw}} = diag(q_{\rho_{r,t}^{sf}} \ q_{\rho_{r,t}^{s1}} \ \cdots \ q_{\rho_{r,t}^{sn}} \ q_{\rho_{b,t}^{sf}} \ q_{\rho_{b,t}^{s1}} \cdots q_{\rho_{b,t}^{sn}} \ q_{\varphi_{r,t}^{sf}} \ q_{\varphi_{r,t}^{s1}} \cdots q_{\varphi_{r,t}^{sn}} \ q_{\varphi_{b,t}^{sf}} \ q_{\varphi_{b,t}^{s1}} \cdots q_{\varphi_{b,t}^{sn}}) \tag{17}$$

where q denotes the variance of the raw measurement.

- According to the covariance propagation rate

$$\boldsymbol{Q}_{DD} = \boldsymbol{J} * \boldsymbol{Q}_{raw}\boldsymbol{J}^{\mathrm{T}} \tag{18}$$

- Therefore, the weighting matrix of DD measurement can be shown as follows：

$$\boldsymbol{W}_{DD} = \boldsymbol{Q}_{DD}^{-1} \tag{19}$$

where $\boldsymbol{W}_{DD}$ denote the weight of DD measurement .

Flowchart of RTK positioning using LS

Step 4: Calculate the RTK float solution using LS

- Based on heteroskedasticity, the solution can be directly calculated as follows:

$$\Delta\boldsymbol{p} = \begin{bmatrix} x_{r,t} - x_0 \\ y_{r,t} - y_0 \\ z_{r,t} - z_0 \\ \nabla\Delta N_{r,t}^{s1} \\ \nabla\Delta N_{r,t}^{s2} \\ \vdots \\ \nabla\Delta N_{r,t}^{sn} \end{bmatrix} = \left(\boldsymbol{G}^T\boldsymbol{W}_{\boldsymbol{DD}}\boldsymbol{G}\right)^{-1}\boldsymbol{G}^T\boldsymbol{W}_{\boldsymbol{DD}}\Delta\boldsymbol{\rho} \quad (20)$$

- The current rover position and DD float ambiguities can be obtained:

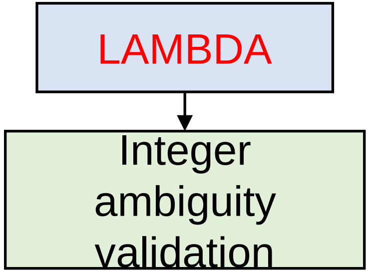
$$\boldsymbol{P}_{float} = \boldsymbol{P}_{\boldsymbol{0}} + \Delta\boldsymbol{p} \quad (21)$$

- The variance matrix of estimated parameters can be obtained:

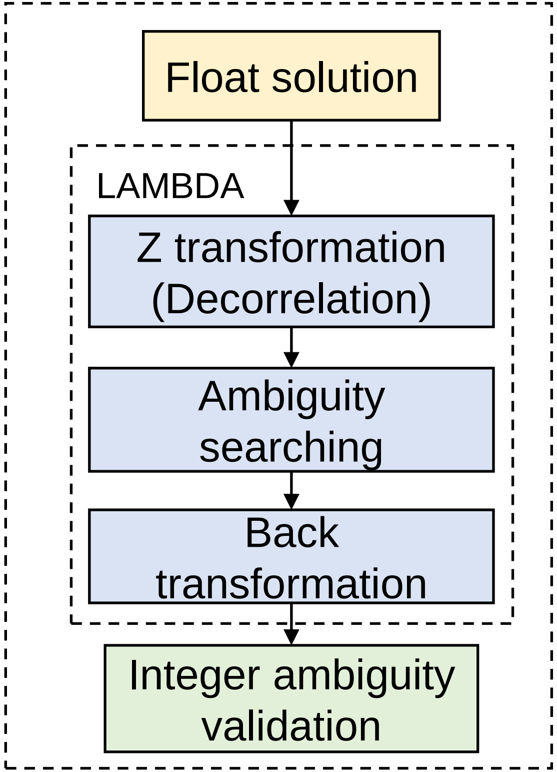
$$\boldsymbol{Q}_{\Delta\boldsymbol{p}} = \left(\boldsymbol{G}^T\boldsymbol{W}_{\boldsymbol{DD}}\boldsymbol{G}\right)^{-1} \quad (22)$$

Flowchart of RTK positioning using LS

Step 5: Ambiguity resolution using Least-squares Ambiguity Decorrelation Adjustment (LAMBDA)



Steps:
- ambiguity parameters are decorrelated by multidimensional integer transform.
- the optimal integer ambiguity solution is efficiently searched in the decorrelation space.
- the obtained ambiguity solution is transformed back into the original ambiguity space to obtain the ambiguity integer solution.



$\boldsymbol{P}_{\text{float}}$, $\hat{\boldsymbol{a}}$ and $\boldsymbol{Q}_{\hat{a}}$

Larger covariance matrix $Q_{\hat{a}}$, larger searching space

$$\hat{\mathbf{z}}_{\hat{a}} = \mathbf{Z}^T\hat{\boldsymbol{a}}$$
$$\mathbf{Q}_{\hat{a},z} = \mathbf{Z}^T\boldsymbol{Q}_{\hat{a}}\mathbf{Z}$$

The distance between the candidate ambiguity vector and the float ambiguity vector is the smallest

$$\breve{z}_{\hat{a}} = \min_{z\in Z^{m-1}} \|\hat{\mathbf{z}}_{\hat{a}} - \mathbf{Z}_t\|^2_{\mathbf{Q}_{\hat{a},z}}$$

$$\breve{\boldsymbol{a}} = \mathbf{Z}^{-T}\breve{z}_{\hat{a}}$$

$\boldsymbol{Q}_{\hat{a}}$: covariance matrix of ambiguity
$\hat{\boldsymbol{a}}$: float ambiguity
$\breve{\boldsymbol{a}}$: integer ambiguity

Flowchart of RTK positioning using LS

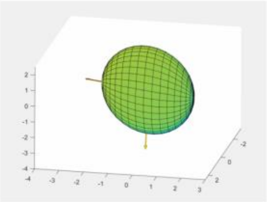
Step 4: Ambiguity resolution using LAMBDA

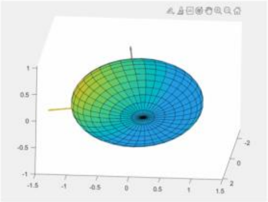
Practice

Download LAMBDA source code: [LAMBDA and Ps-LAMBDA software packages, ambiguity estimation, GNSS - Global Navigation Satellite Systems Research Centre | Curtin University, Perth, Australia]

$$\hat{\boldsymbol{a}} = [5.45, 3.10, 2.97]^{\mathrm{T}}$$

$$\boldsymbol{Q}_{\hat{a}} = \begin{bmatrix} 6.290 & 5.978 & 0.544 \\ 5.978 & 6.292 & 2.340 \\ 0.544 & 2.340 & 6.288 \end{bmatrix}$$





Visualization of $Q_{\hat{a}}$ before (left) and after (right) decorrelation

① Decorrelation

$$\boldsymbol{Q}_{\hat{a}} = \begin{bmatrix} 4.476 & 0.334 & 0.230 \\ 0.334 & 1.146 & 0.082 \\ 0.230 & 0.082 & 0.626 \end{bmatrix} \qquad Z = \begin{bmatrix} -2 & 3 & 1 \\ 3 & -3 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$\hat{\boldsymbol{z}}_{\hat{a}} = \boldsymbol{Z}^{\mathrm{T}}\, \hat{\boldsymbol{a}} = \begin{bmatrix} -2 & 3 & 1 \\ 3 & -3 & -1 \\ -1 & 1 & 0 \end{bmatrix}^{\mathrm{T}} [5.45, 3.10, 2.97]^{\mathrm{T}} = [-4.57, 10.02, 2.35]^{\mathrm{T}}$$

https://blog.csdn.net/Gou_Hailong/article/details/129364756

Flowchart of RTK positioning using LS

Step 4: Ambiguity resolution using LAMBDA

Practice

② Searching

Two ambiguity candidate groups are searched: $\breve{\boldsymbol{z}}_{\hat{\boldsymbol{a}}} = \begin{bmatrix} -5 & -4 \\ 10 & 10 \\ 2 & 2 \end{bmatrix}$

The distance between the candidate ambiguity vector and the float ambiguity vector: $s = [0.2183 \quad 0.3073]$

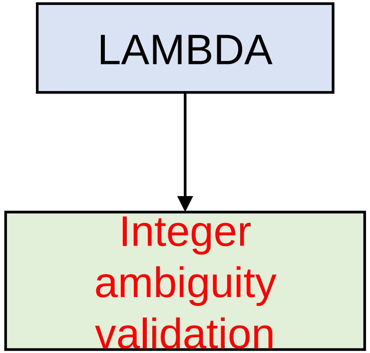
③ Back transformation

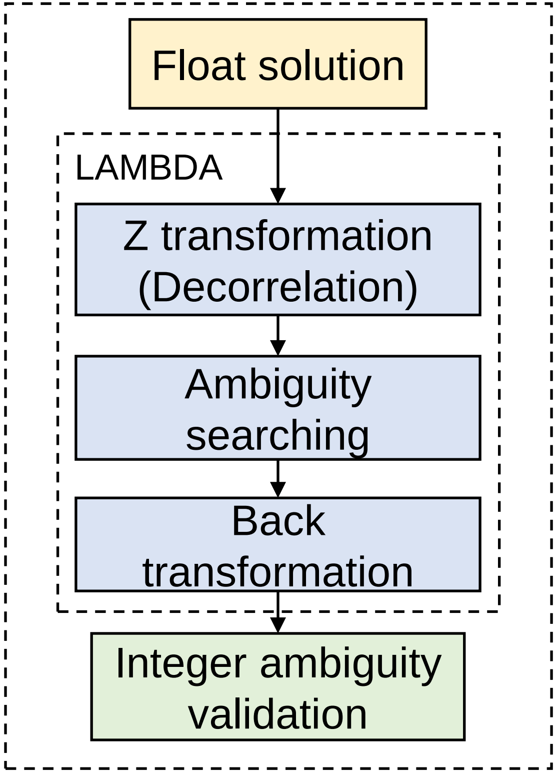
$$\breve{\boldsymbol{a}} = \mathbf{Z}^{-T}\breve{\boldsymbol{z}}_{\hat{\boldsymbol{a}}} = \begin{bmatrix} -2 & 3 & 1 \\ 3 & -3 & -1 \\ -1 & 1 & 0 \end{bmatrix}^{-\mathbf{T}} \begin{bmatrix} -5 & -4 \\ 10 & 10 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} -5 & -4 \\ 10 & 10 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 3 & 4 \\ 4 & 4 \end{bmatrix}$$

Therefore, [5,3,4] is the optimal integer ambiguity vector; [6,4,4] is the suboptimal integer ambiguity vector

Flowchart of RTK positioning using LS

Step 5: Ambiguity resolution using LAMBDA





Integer ambiguity validation

Nominator component $O_1 = (\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}_l)\boldsymbol{Q}_{\hat{a}}^{-1}(\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}_l)$

Denominator component $O_2 = (\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}_u)\boldsymbol{Q}_{\hat{a}}^{-1}(\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}_u)$

Ratio test $O = {}^{O_1}/_{O_2} > k$ (k can be set to 3)

$\breve{\boldsymbol{a}}_u$: optimal integer ambiguity vector

$\breve{\boldsymbol{a}}_l$: suboptimal integer ambiguity vector

$\hat{\boldsymbol{b}}$: float positioning state

$\breve{\boldsymbol{b}}$: fix positioning state

If ratio test passes, $\breve{\boldsymbol{b}} = \hat{\boldsymbol{b}} - \boldsymbol{Q}_{\hat{b}\hat{a}}\boldsymbol{Q}_{\hat{a}\hat{a}}^{-1}(\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}})$

Flowchart of RTK positioning using LS

Step 5: Obtain the RTK fix solution

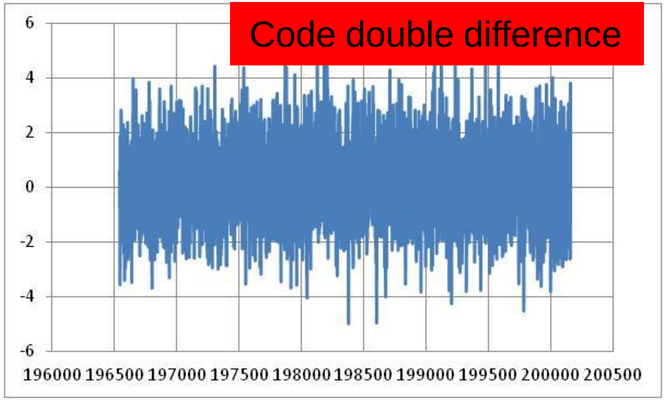
- If passing the ratio test, update the fix solution; otherwise, output the float solution $\boldsymbol{P}_{float}$:

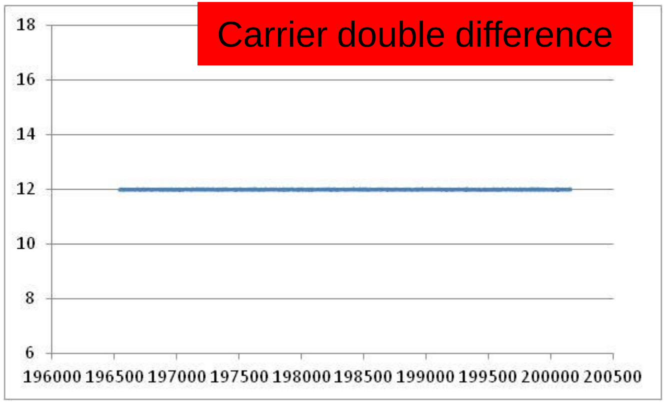
$$\boldsymbol{P}_{final} = \begin{cases} \boldsymbol{P}_{float}, O = {}^{O_1}/_{O_2} \leq k \\ \boldsymbol{P}_{fix}, \quad O = {}^{O_1}/_{O_2} > k \end{cases} \quad (23)$$

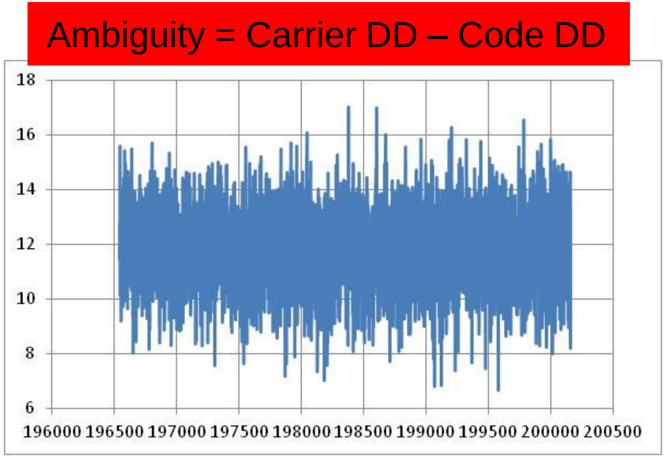
The correct fixed solution $\boldsymbol{P}_{fix}$ relies on the accuracy of float solution of positioning state $\boldsymbol{P}_{float}$ and the ambiguity covariance matrix $\boldsymbol{Q}_{\hat{\boldsymbol{a}}}$.

Double Differenced Observation

(open sky condition : prn19->prn3 : 1 hour)

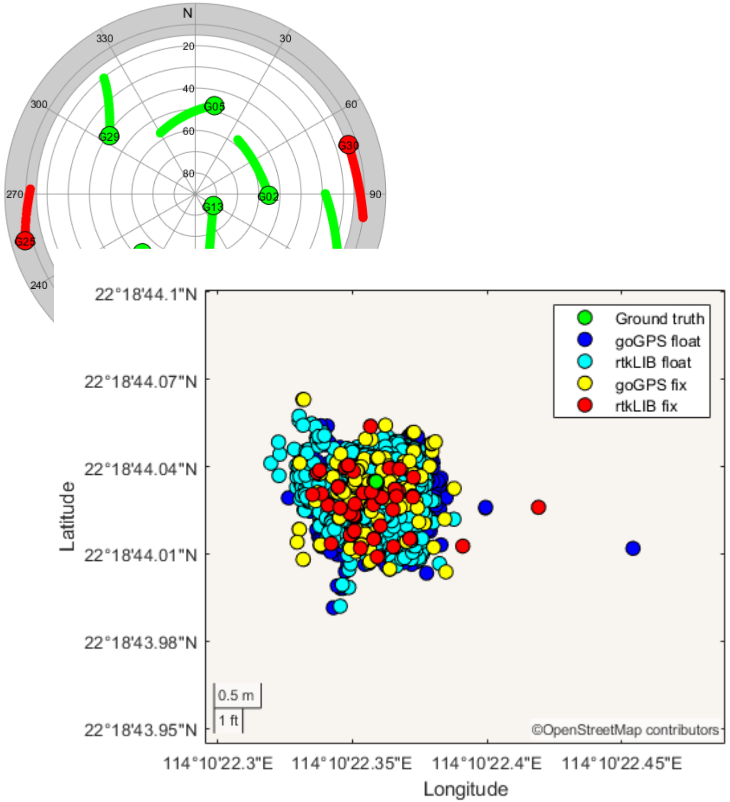






Average = 11.8
Std = 1.4

Test RTK Fix



Receiver: ublox F9P
Constellation: GPS
Frequency: L1 only
Epochs: 3562 (00:59:22); 3533 available
Elev. mask: 15°; SNR mask: 15

Method	Fixing Rate (%)	RMS 2D Error (m)		
		Fix	Float	All
goGPS	15.34	0.476	0.396	0.409
RTKLIB (2.4.3 b33)	20.10	0.197	0.386	0.356
WLS	100	0.805		

Multi-GNSS RTK Test using Car

Test	Schedule
1st	2014/8/13 13:07–13:32
2nd	2014/8/13 17:26–17:52
3rd	2014/8/13 22:26–22:50
4th	2014/8/14 8:36–9:02
5th	2014/8/14 12:07–12:35

* GPS/QZS/GLONASS/GALILEO/BeiDou are entirely used in this test
* Trimble SPS855 receiver was used
* RTK : Trimble and Laboratory engine

Summary of Test Results

Multi-GNSS RTK
NUS: number of satellites

	Average NUS	Fix rate
Test 1	12.3	58.7%
Test 2	12.3	75.4%
Test 3	13.6	65.5%
Test 4	12.4	60.0%
Test 5	14.2	70.5%

GPS VS. Multi-GNSS RTK
(using two same receivers : SPS855)

Test 5	Average NUS	Fix rate
GPS	5.8	26.8%
Multi-GNSS	14.2	70.5%

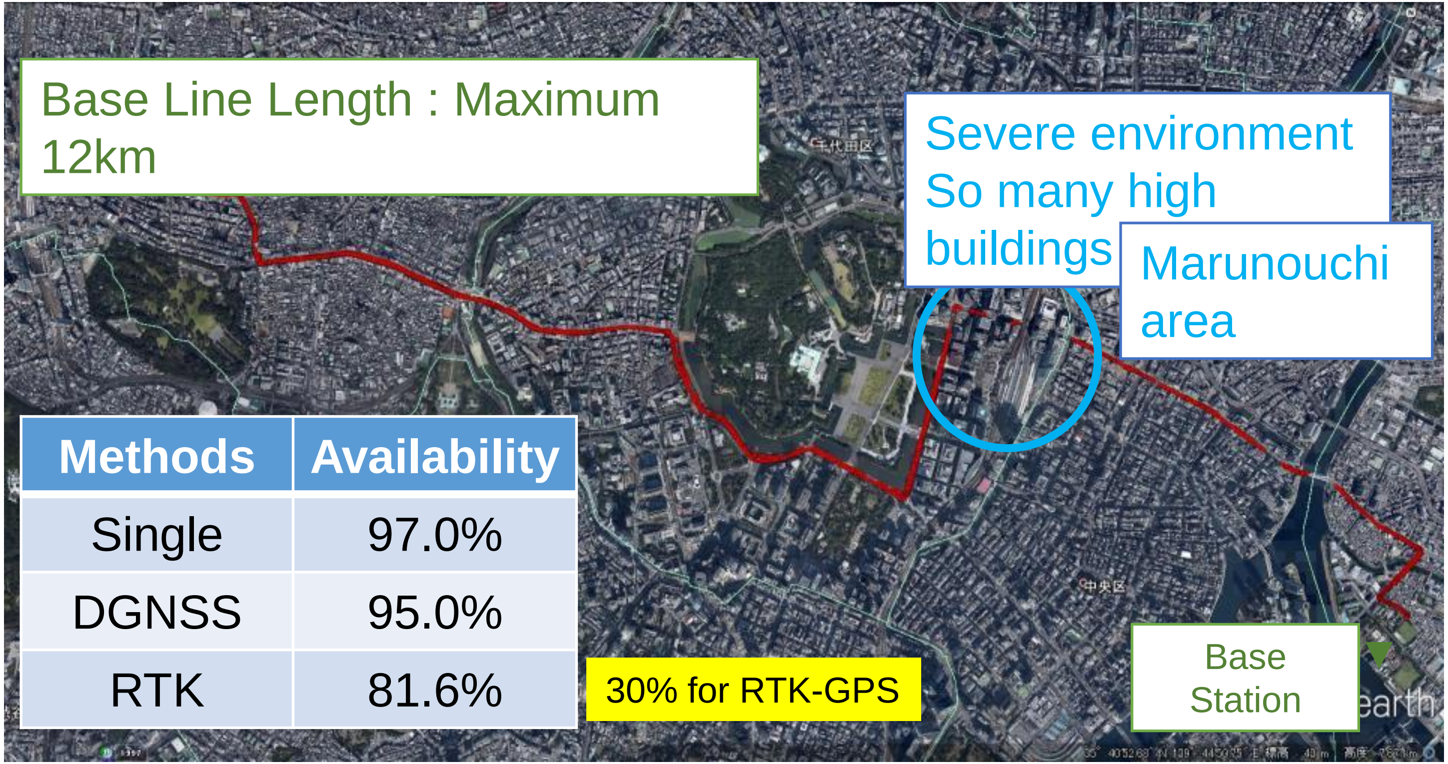
Fix rate comparison between GNSS combinations

Velocity : Doppler based velocity output
G:GPS J:QZSS C:BeiDou R:GLONASS

Test 3	G	GJ	GC	GR	GJC	GJCR
RTK FIX rate (%)	48.2	58.2	55.5	55.4	64.7	65.9

The reason for small contribution of BeiDou/GLONASS to RTK was just due to the shortage of high elevation those satellites

Height Determination using Automobile



Base Line Length : Maximum 12km

Severe environment
So many high buildings

Marunouchi area

Methods	Availability
Single	97.0%
DGNSS	95.0%
RTK	81.6%

30% for RTK-GPS

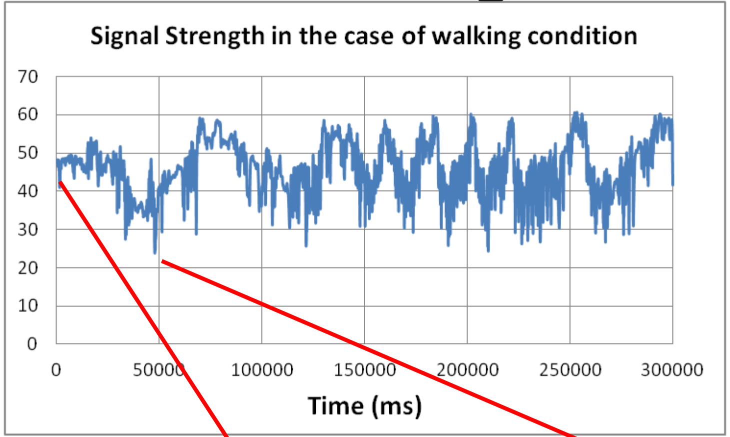
Base Station

Shinjuku

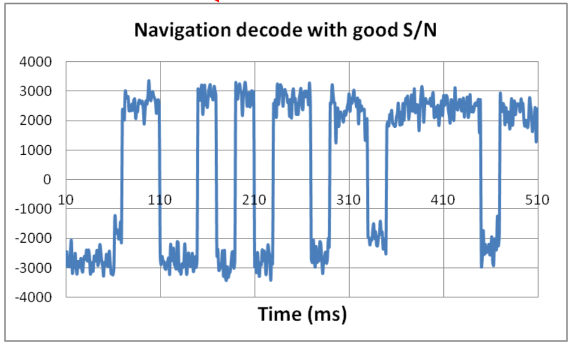
Route 20

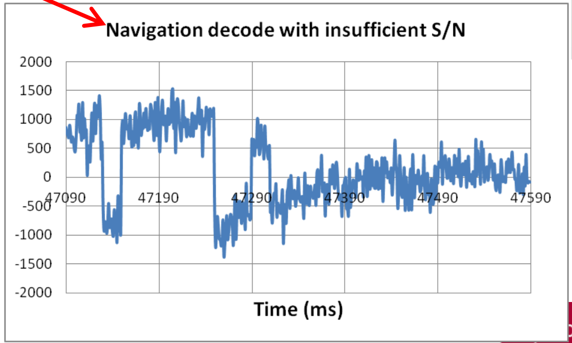
TUMSAT

How about indoor?
5 min IMES tracking in Lab.



Front-end

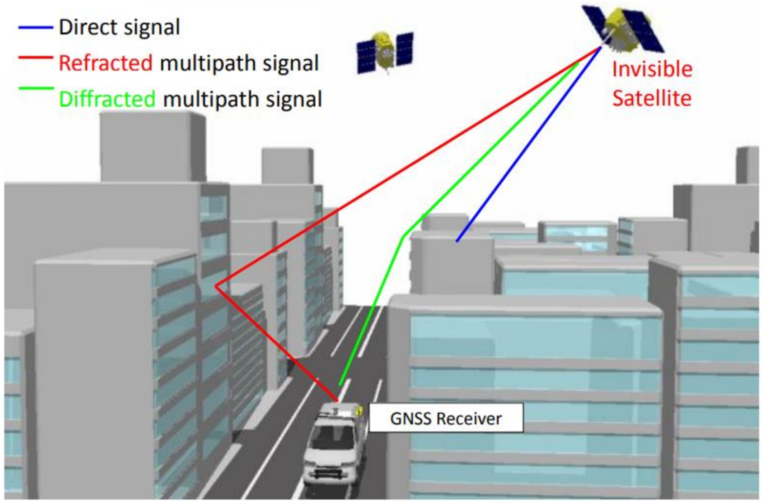




Improve GNSS Positioning

➢ <u>3D Mapping Aided (3DMA) GNSS</u>:

- Suzuki, Taro and Kubo, Nobuaki. GNSS positioning with multipath simulation using 3D surface model in urban canyon. (*ION GNSS+ 2012*). (GNSS NLOS exclusion causes poor satellite geometry)

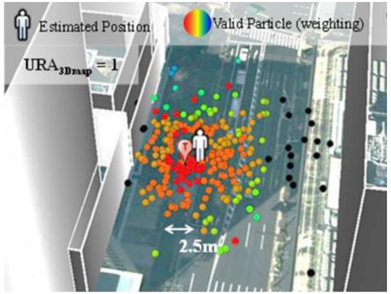


Prof. Kubo, 2012

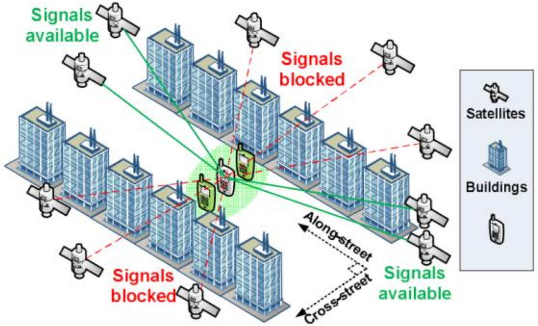
Improve GNSS Positioning

➢ 3D Mapping Aided (3DMA) GNSS:

- Wang, Lei, et al. "Urban positioning on a smartphone: Real-time shadow matching using GNSS and 3D city models." *Navigation, The Institute of Navigation,* 2013.(Relies on the satellite visibility classification and the initial guess of the receiver)
- Hsu, Li-Ta, et al. “3D building model-based pedestrian positioning method using GPS/GLONASS/QZSS and its reliability calculation.” *GPS solutions,* 2016.(Relies on the initial guess of the receiver and causes high computation load)



Hsu, 2016



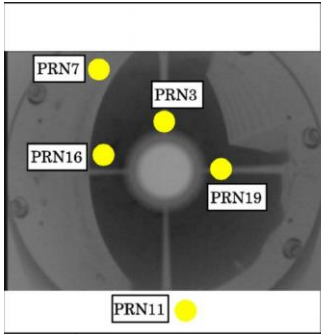
Wang, 2013

Improve GNSS Positioning

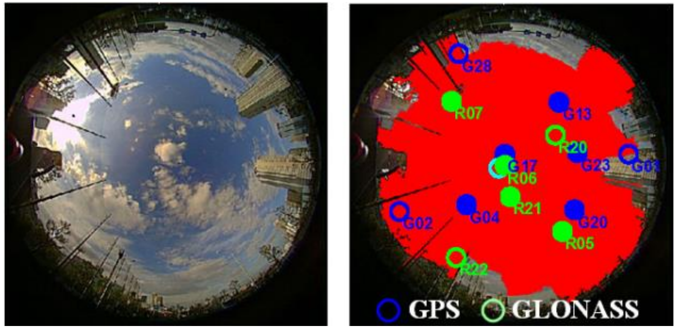
- **Camera-aided GNSS Positioning**:

- Meguro, Jun-ichi, et al. "GPS multipath mitigation for urban area using omnidirectional infrared camera." *IEEE Transactions on Intelligent Transportation Systems*, 2013. (NLOS exclusion cause poor geometry)

- Suzuki, Taro and Kubo, Nobuaki, "N-LOS GNSS Signal Detection Using Fish-Eye Camera for Vehicle Navigation in Urban Environments," (*ION GNSS+* 2014), *Tampa, Florida*, September 2014. (NLOS detection and exclusion with monocular camera, cause poor satellite geometry)



Meguro, 2013



Kubo, 2014

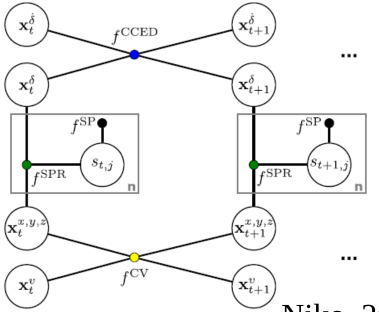
Improve GNSS Positioning

➢ **Robust Model-aided GNSS Positioning:**

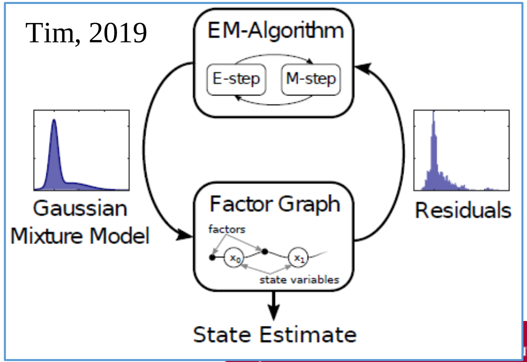
- Sünderhauf, et al. "Switchable constraints and incremental smoothing for online mitigation of non-line-of-sight and multipath effects. *IEEE IV* 2013. (Relies on the initial guess of the prior factor)

- Pfeifer, Tim, et al. "Dynamic Covariance Estimation—A parameter free approach to robust Sensor Fusion." *IEEE MFI* 2017. (Relies on the percentage of healthy measurements)

- Pfeifer, Tim, and Peter Protzel. "Expectation-maximization for adaptive mixture models in graph optimization." *ICRA,* 2019. (Relies on the initial guess of the state estimation)



Niko, 2013



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Q&A

Thank you for your attention ☺

Q&A

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If you have any questions or inquiries, please feel free to contact me.

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