Kalman filtering for GNSS positioning and sensor integration: part I

AAE4203 – Guidance and Navigation

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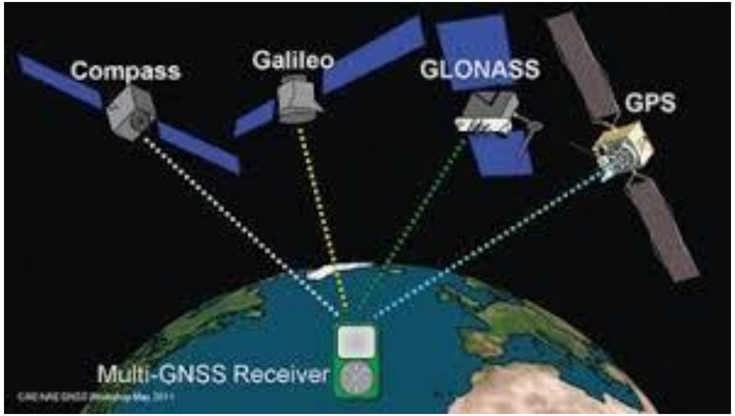
Outline

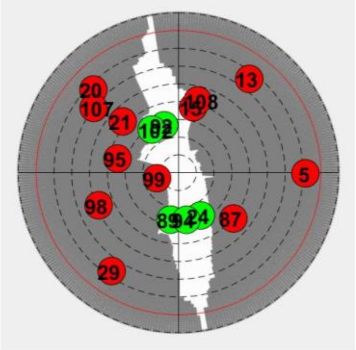
- Overview About State Estimation
- Kalman Filtering
- GNSS Doppler based Velocity Estimation
- Principle of Kalman Filtering
- GNSS Positioning with Kalman Filtering
  - Loosely Coupled GNSS Positioning
  - Tightly Coupled GNSS Positioning

GNSS Benefits

> GNSS provides a high long-term position accuracy with errors limited to a few meters (stand-alone), while user equipment is available for less than $100 (€80).

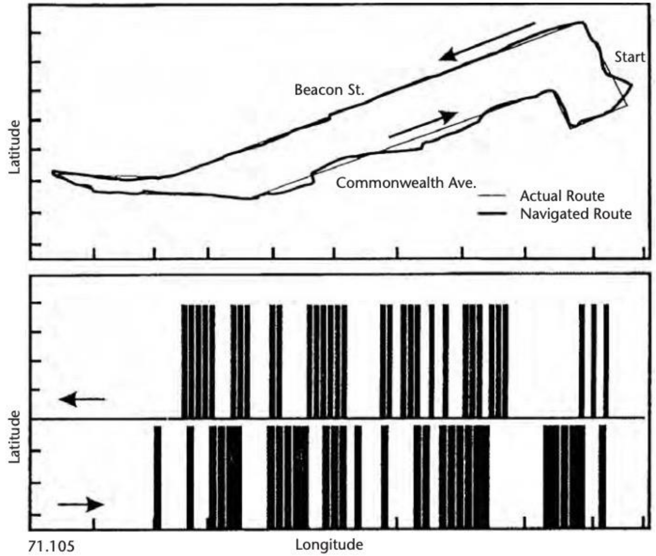


GPS Drawbacks



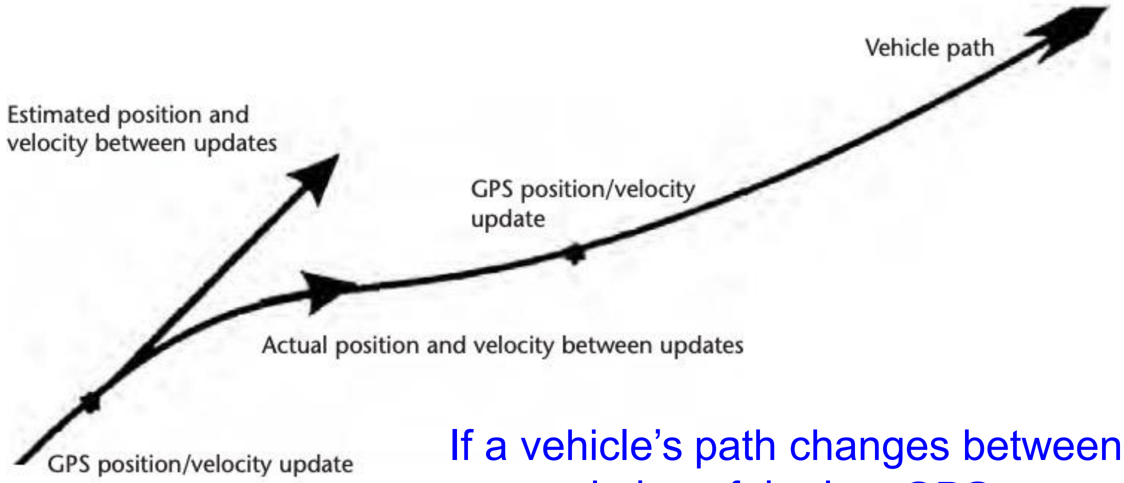
> One primary concern with using GPS as a stand-alone source for navigation is **signal interruption**.

If the number of usable satellites is less than three, some receivers have the option of not producing a solution or extrapolating the last position and velocity solution forward in what is called *dead-reckoning* (DR) navigation. Inertial navigation systems (INSs) can be used as a flywheel to provide navigation during shading outages.



GPS Drawbacks

> The **low update rate** of the GPS observations in some equipment is also of concern in real-time applications, especially those related to vehicle control.



If a vehicle's path changes between updates, the extrapolation of the last GPS measurement produces an error in the estimated and true position.

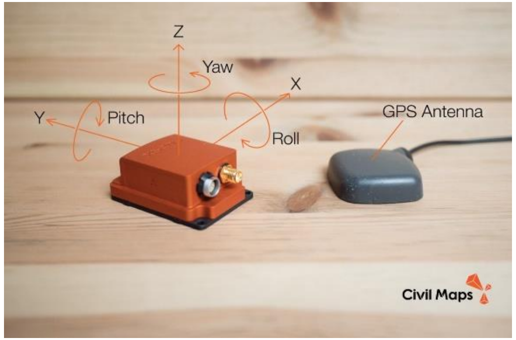
Framework of Sensors to Navigation

	Sensing	→	Measuring	→	Measurement	→	Model	→	Estimate
GNSS	GNSS Signal	→	Signal Defined Receiver	→	Pseudorange	→	Least Square Estimation	→	Point Position
INS	Raw measurements (voltage, delays, etc)	→	Accelerometer/ Gyroscope Models	→	Acceleration & Angular Velocity	→	INS Mechanization	→	Relative Position
Visual Navigation	Images	→	Features Detection & Tracking	→	Associated Features	→	Least Square Estimation	→	Relative Position

Sensor Integration Based on Kalman Filter

How the Sensor Fusion Problem Looks like…

- GPS provide the position in x, y, z
- IMU provide the linear angular velocity in x, y, z
- GPS Doppler provide velocity in x, y, z
- Visual positioning provide relative motion $\Delta x, \Delta y, \Delta z$



How to achieve this sensor fusion by combining the positioning from different sources?

State Estimation Methods

- First Order-Markov Assumption
- Gaussian noise assumption

Maximum A Posterior (MAP)

Filtering-based State Estimation

- First Order-Markov Assumption

Optimization-based State Estimation

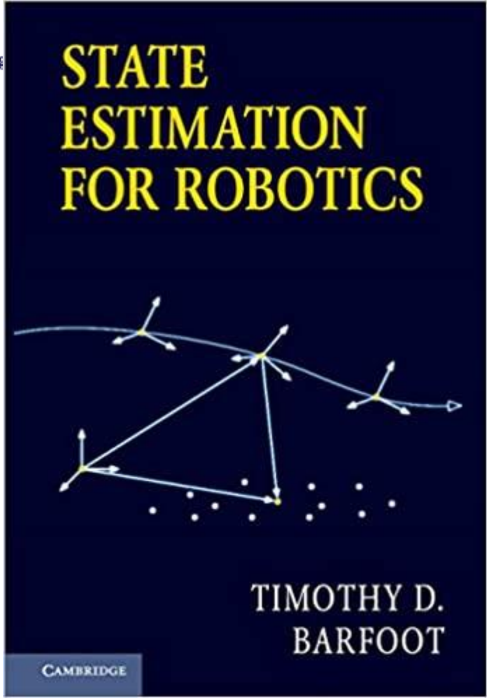
- Gaussian noise assumption

Kalman Filtering

Extended Kalman Filtering

Unscented Kalman Filtering

Factor Graph Optimization



Basics for Probabilistic

Event A and B. $P(\mathrm{A})$ denotes the probability that the event A happens.

$$P(\mathrm{B}|\mathrm{A}) = \frac{P(\mathrm{AB})}{P(\boldsymbol{A})} \quad \Rightarrow \quad P(\mathrm{AB}) = P(\mathrm{A}|\mathrm{B})P(\mathrm{B}) \quad \Rightarrow \quad P(\mathrm{B}|\mathrm{A}) = \frac{P(\mathrm{A}|\mathrm{B})P(\mathrm{B})}{P(\boldsymbol{A})}$$

Event z denotes the measurements. $P(z)$ denotes the probability that the event A happens.

$$P(\mathbf{x}_k|\mathbf{z}_1, \quad \ldots, \quad \mathbf{z}_k) = \frac{P(\mathbf{z}_0, \quad \ldots, \quad \mathbf{z}_k|\mathbf{x}_k)P(\mathbf{x}_k)}{P(\mathbf{z}_0, \quad \ldots, \quad \mathbf{z}_k)}$$

The probabilistic view of the state estimation is: given a set of measurements $(\mathbf{z}_0, \quad \ldots, \quad \mathbf{z}_k)$, can we find a best state $\mathbf{x}_k$ to maximize the conditional probabilistic $P(\mathbf{x}_k|\mathbf{z}_1, \quad \ldots, \quad \mathbf{z}_k)$?

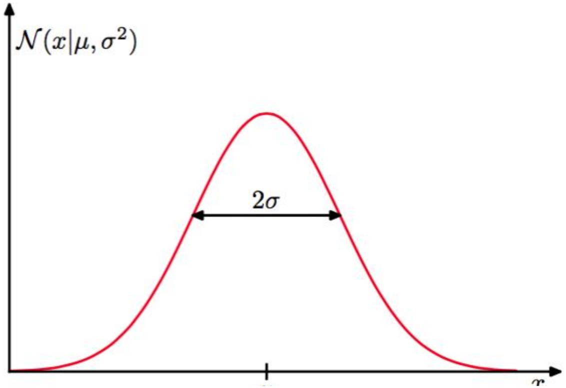
How do we describe the uncertainty with Gaussian Noise?

> Gaussian distribution

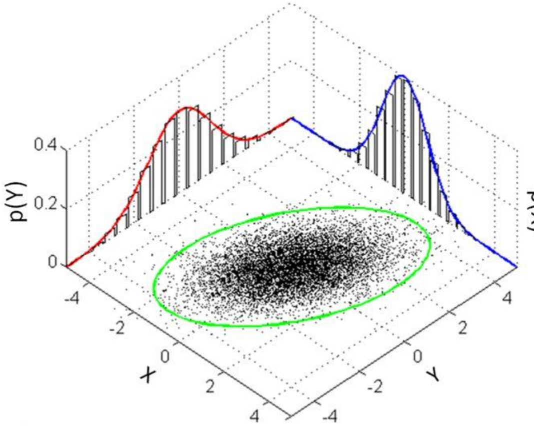
$$p(x) \sim N(\mu, \sigma^2)$$

1D (univariate)

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



2D+ (multi variate) $$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\mathbf{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^t \mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$



How to understand the $P(\mathbf{z}_0, \quad \ldots, \quad \mathbf{z}_k|\mathbf{x}_k)$

Observation function for pseudorange (code) measurement

$$\underbrace{\rho^s_{r,t}}_{\text{Pseudorange}} = \underbrace{r^s_{r,t}}_{\text{Range distance}} + c(\delta_{r,t} - \delta^s_{r,t}) + I^s_{r,t} + T^s_{r,t} + \varepsilon^s_{r,t}$$

Range distance: $||\mathbf{p}^{G,s}_t - \mathbf{p}^G_{r,t}||$

$\delta_{r,t}$: Receiver clock Bias (1~2m)

$\delta^s_{r,t}$: Satellite clock bias

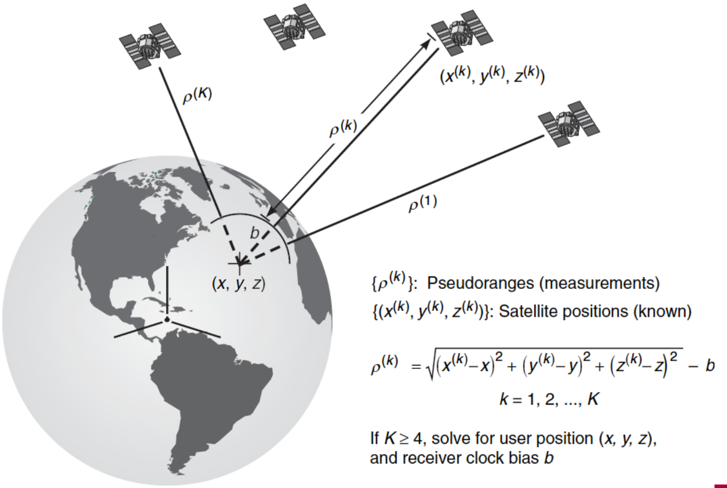
$I^s_{r,t}$: ionospheric delay Distance (1~2m)

$T^s_{r,t}$: tropospheric delay Distance (1~2m)

$\varepsilon^s_{r,t}$: multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)

$\rho^s_{r,t} \Rightarrow z^s_{r,t}$ $\quad$ $\mathbf{p}^G_{r,t} \Rightarrow \mathbf{x}^G_{r,t}$

$$P(\mathbf{x}_k|\mathbf{z}_1, \quad \ldots, \quad \mathbf{z}_k) = \frac{P(\mathbf{z}_0, \quad \ldots, \quad \mathbf{z}_k|\mathbf{x}_k)P(\mathbf{x}_k)}{P(\mathbf{z}_0, \quad \ldots, \quad \mathbf{z}_k)}$$

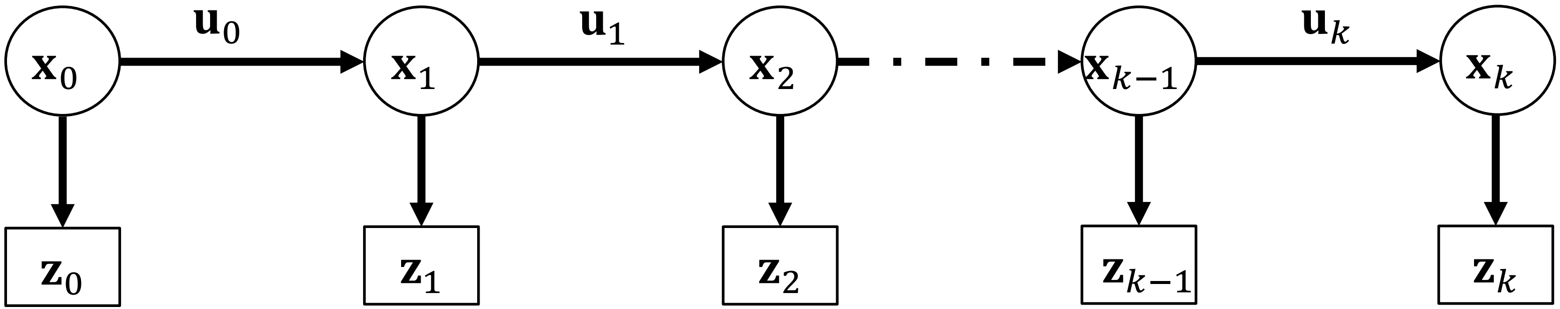


Formulate the $P(\mathbf{x}_k|\mathbf{z}_1, \quad \ldots, \quad \mathbf{z}_k)$ for the GNSS pseudorange measurements!

Estimation Formulation

$\mathbf{u}_i$: IMU Measurement

$\mathbf{z}_i$: GNSS measurement



States set

$\chi = \{\mathbf{x}_o, \mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k\}$

The **maximum a posteriori (MAP)** estimate is given by

Optimal State Set

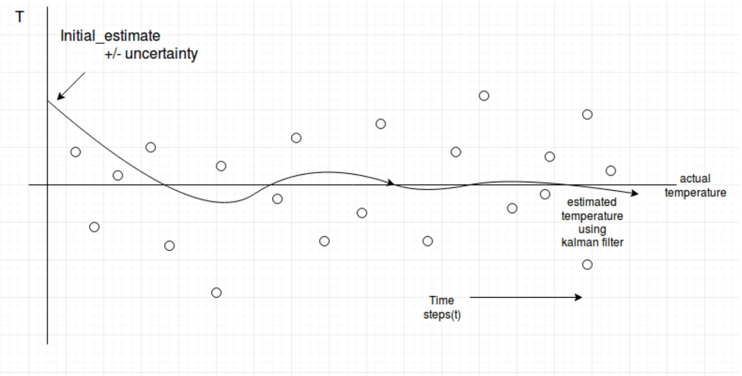
$$\hat{\chi} = arg \max_{\chi}(P(\chi|\mathbf{Z}, \mathbf{U})) \quad P(\chi|\mathbf{Z}, \mathbf{U}) = \prod_k P(\mathbf{z}_k|\mathbf{x}_k)\, P(\mathbf{x}_0) \prod_k P(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{u}_{k-1})$$

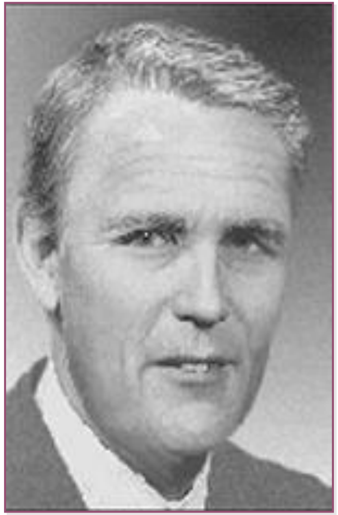
Bayesian theory

Kalman Filter

> In early 1960, American engineer R.E. Kalman discovered a linear minimum variance ESTIMATION method——Kalman Filter. Soon in space technology (such as flying Navigation system, missile guidance, and determination of satellite orbit and attitude) has been applied.

> Optimal combination of MEASUREMENT and PROPAGATION





Rudolf Emil Kalman

http://www.cs.unc.edu/~welch/kalman/media/pdf/Kalman1960.pdf

A New Approach to Linear Filtering and Prediction Problems[1]

R. E. KALMAN

Research Institute for Advanced Study,[2]
Baltimore, Md.

The classical filtering and prediction problem is re-examined using the Bode-Shannon representation of random processes and the "state transition" method of analysis of dynamic systems. New results are:

(1) The formulation and methods of solution of the problem apply without modification to stationary and nonstationary statistics and to growing-memory and infinite-memory filters.

(2) A nonlinear difference (or differential) equation is derived for the covariance matrix of the optimal estimation error. From the solution of this equation the coefficients of the difference (or differential) equation of the optimal linear filter are obtained without further calculations.

(3) The filtering problem is shown to be the dual of the noise-free regulator problem.

The new method developed here is applied to two well-known problems, confirming and extending earlier results.

The discussion is largely self-contained and proceeds from first principles; basic concepts of the theory of random processes are reviewed in the Appendix.

Introduction

References

Conclusions

Transactions of the ASME–Journal of Basic Engineering, 82 (Series D): 35-45. Copyright © 1960 by ASME

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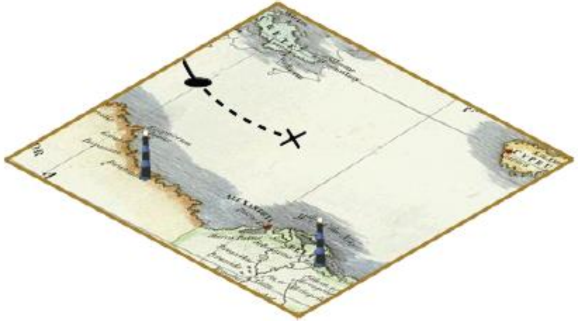
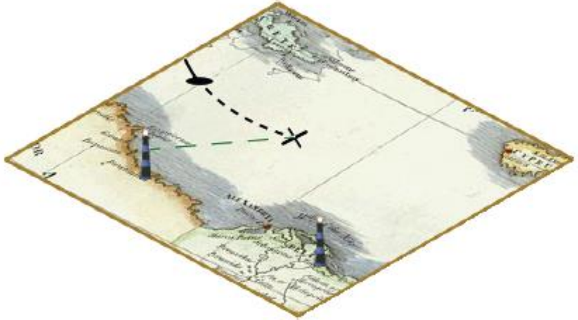
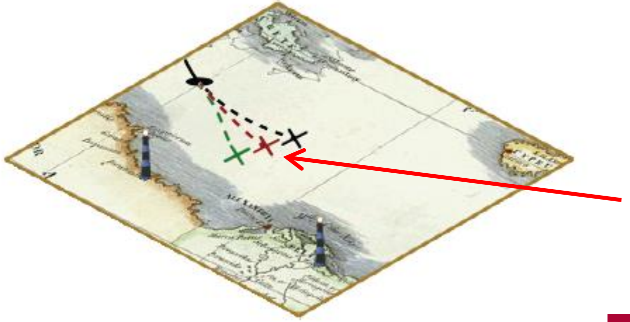
Kalman Filter——Navigation application using dead reckoning and visual measurement to landmark

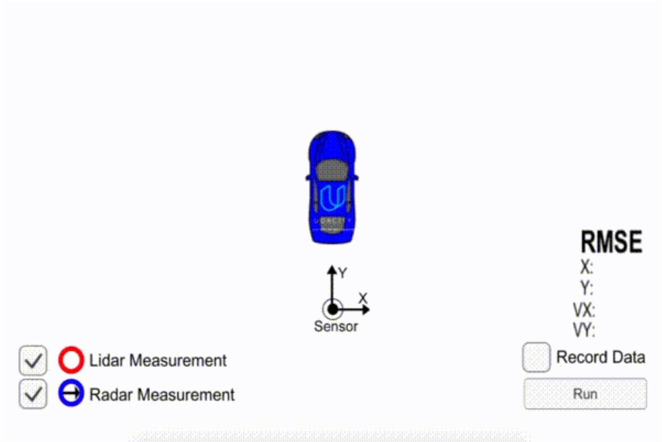


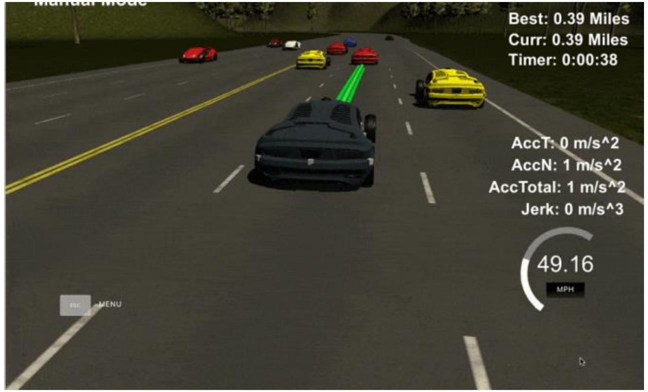


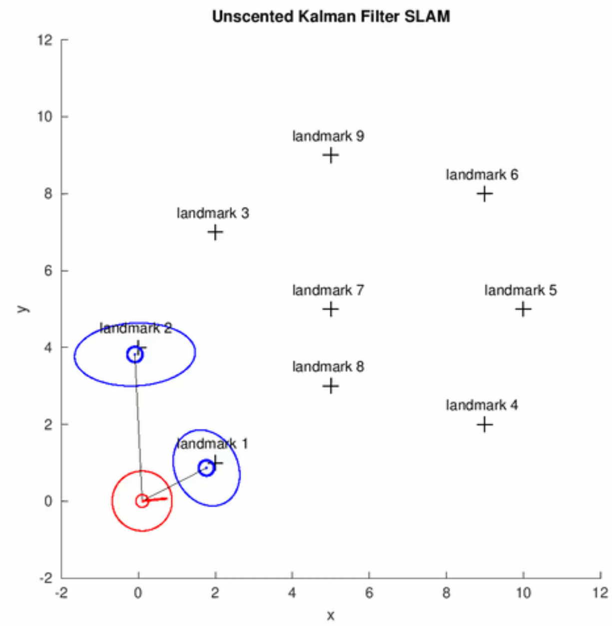


Predicted and corrected position of the ship

Examples of Kalman Filter in Navigation



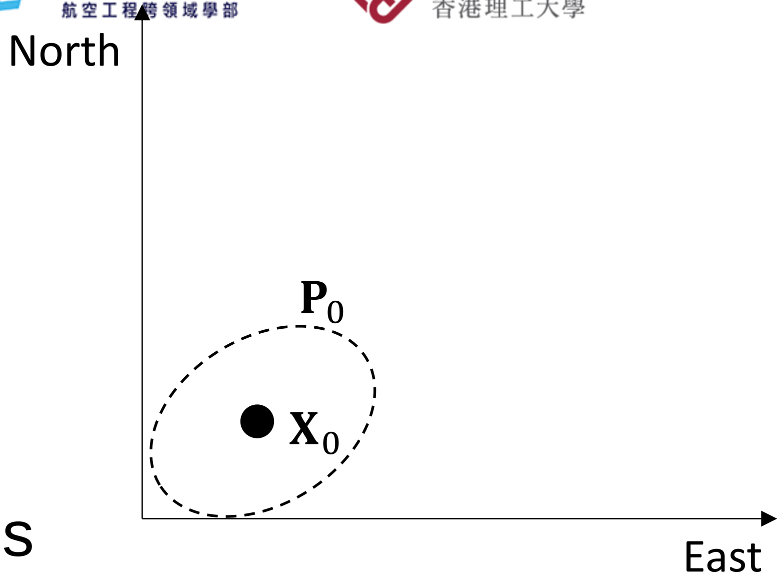




Definitions

- **X**, The *state vector* is the set of parameters describing a system, known as *states*, which the Kalman filter estimates.

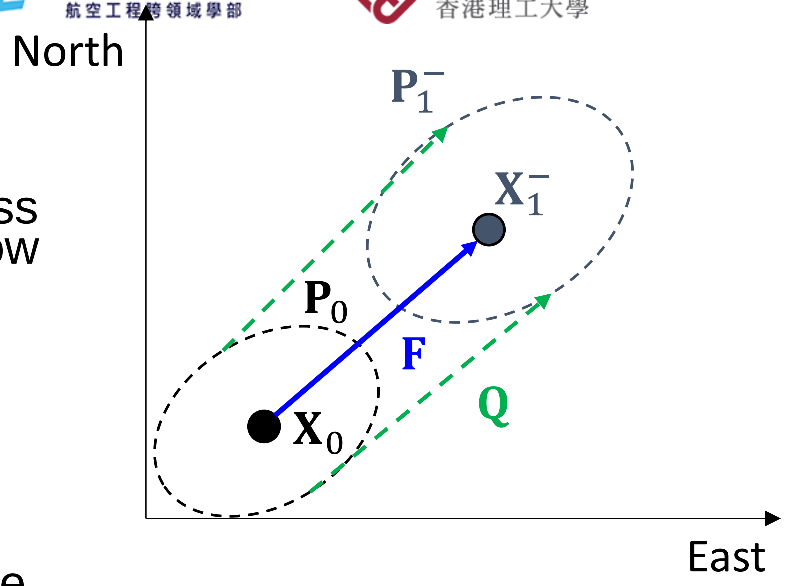
- **P**, Associated with the state vector is an *error covariance matrix*. This represents the uncertainties in the Kalman filter's state estimates and the degree of correlation between the errors in those estimates.



Definitions

> $\mathbf{F}$, The *system model*, also known as the process model or time-propagation model, describes how the Kalman filter states and error covariance matrix vary with time. (The system model is deterministic for the states as it is based on known properties of the system.)

> $\mathbf{Q}$, A state uncertainty should also be increased with time to account for unknown changes in the system that cause the state estimate to go out of date in the absence of new measurement information. This variation in the true values of the states is known as *system noise* or *process noise*.

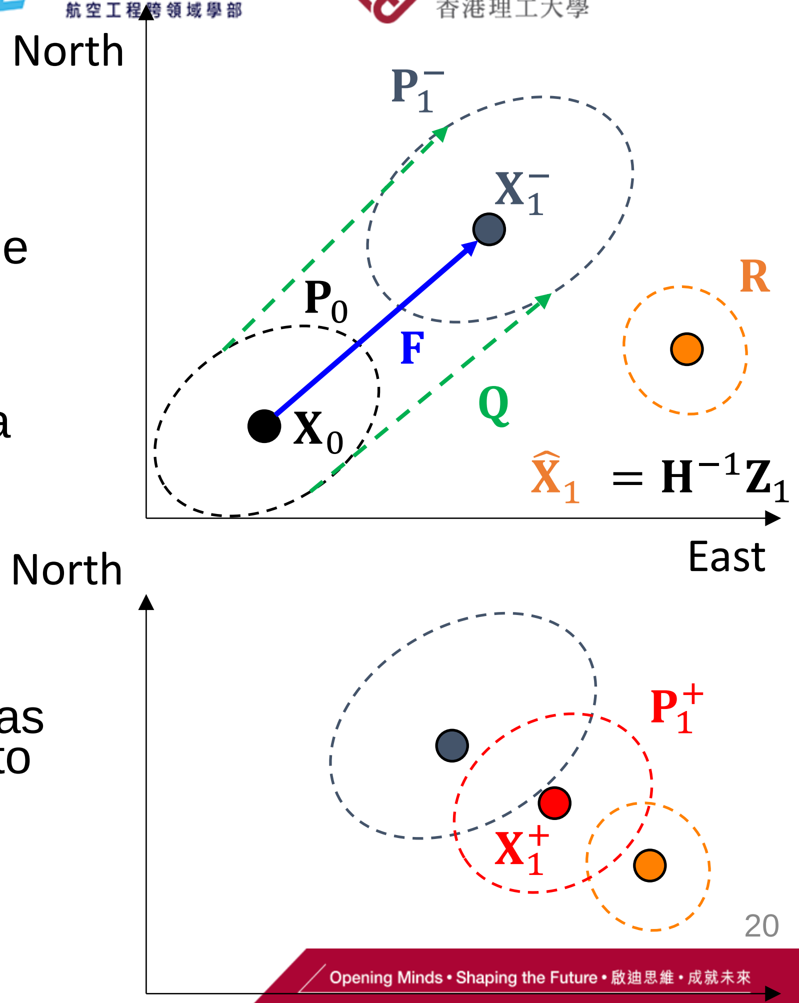


Definitions

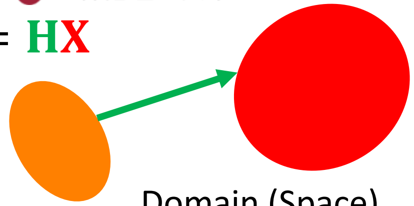
> $\mathbf{Z}$, The *measurement vector* is a set of simultaneous measurements of properties of the system which are functions of the state vector.

> $\mathbf{R}$, Associated with the measurement vector is a *measurement noise covariance* matrix which describes the statistics of the noise on the measurements.

> **H, Z=H(X)=HX**, The *measurement model* describes how the measurement vector varies as a function of the true state vector (as opposed to the state vector estimate) in the absence of measurement noise.



Key Equations of Kalman Filter



State Propagation

> $\mathbf{X}_t^{-} = \mathbf{F}\mathbf{X}_{t-1}^{+}$

> $\mathbf{P}_t^{-} = \mathbf{F}\mathbf{P}_{t-1}^{+}\mathbf{F}^{\mathrm{T}} + \mathbf{Q}$

Measurement Update

> $\Delta\mathbf{Z}_t = \mathbf{Z}_t - \mathbf{H}\mathbf{X}_t^{-}$

> $\mathbf{S}_t = \mathbf{H}\mathbf{P}_t^{-}\mathbf{H}^{\mathrm{T}} + \mathbf{R}$

> $\mathbf{K}_t = \mathbf{P}_t^{-}\mathbf{H}^{\mathrm{T}}\mathbf{S}_t^{-}$

> $\mathbf{X}_t^{+} = \mathbf{X}_t^{-} + \mathbf{K}_t\Delta\mathbf{Z}_t$

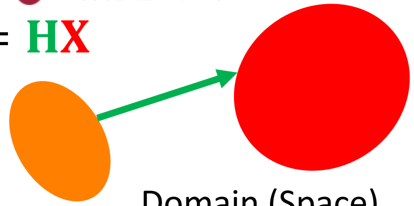
> $\mathbf{P}_t^{+} = (\mathbf{I} - \mathbf{K}_t\mathbf{H})\mathbf{P}_t^{-}$

Measurement Innovation, $\Delta\mathbf{Z}_t$

- $\Delta\mathbf{Z}_t = \mathbf{Z}_t - \mathbf{H}\mathbf{X}_t^-$
- Meaning: The difference between propagation and measurement in the domain of $\mathbf{Z}$ (measurement)
- $\Delta\mathbf{Z}_t = 0$, What does it mean?
- $\Delta\mathbf{Z}_t \neq 0$, What shall we do?

Kalman Filter Gain, $\mathbf{K}_t$

$\mathbf{Z} = \mathbf{H}\mathbf{X}$



Domain (Space) Transformation

$\mathbf{Z}^{\mathrm{T}} = \mathbf{X}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}$

> $\mathbf{K}_t = \mathbf{P}_{t-1}^{-}\mathbf{H}^{\mathrm{T}}\mathbf{S}_t^{-1}$

> $\mathbf{S}_t = \mathbf{H}\mathbf{P}_{t-1}^{-}\mathbf{H}^{\mathrm{T}} + \mathbf{R}$

If we forget there is no denominator in Matrix…

> $\mathbf{K}_t = \dfrac{\mathbf{P}_{t-1}^{-}\mathbf{H}^{\mathrm{T}}}{\mathbf{H}\mathbf{P}_{t-1}^{-}\mathbf{H}^{\mathrm{T}} + \mathbf{R}}$

The propagated state covariance (uncertainty) in the domain of the transpose of **Z** (measurement)

> $\mathbf{K}_t = \dfrac{\left(\mathbf{F}\mathbf{P}_{t-1}^{+}\mathbf{F}^{\mathrm{T}} + \mathbf{Q}\right)\mathbf{H}^{\mathrm{T}}}{\mathbf{H}\mathbf{P}_{t-1}^{-}\mathbf{H}^{\mathrm{T}} + \mathbf{R}}$

The propagated state covariance (uncertainty) in the domain of **Z** (measurement)
\+ measurement covariance (uncertainty) **R**

The updated state covariance (uncertainty)
considering (+) propagation covariance (uncertainty)

Kalman Filter Gain, $\mathbf{K}_t$

- $\mathbf{K}_t = \dfrac{(\mathbf{F}\mathbf{P}_{t-1}^{+}\mathbf{F}^{\mathrm{T}}+\mathbf{Q})\mathbf{H}^{\mathrm{T}}}{\mathbf{H}\mathbf{P}_{t-1}^{-}\mathbf{H}^{\mathrm{T}}+\mathbf{R}}$
- $\mathbf{R} \gg \mathbf{Q}$, what does it mean?
- $\mathbf{R} \ll \mathbf{Q}$, what does it mean?
- $\mathbf{R} = \mathbf{Q}$, what does it mean?

Simplified the Models to Identity Matrix, $\mathbf{I}$

$$\mathbf{F} = \mathbf{I}$$
$$\mathbf{H} = \mathbf{I}$$
$$\mathbf{B} = \mathbf{I}$$

> $\mathbf{X}_t^- = \mathbf{F}\mathbf{X}_{t-1}^+ + \mathbf{B}\mathbf{U}_t$
>
> $\mathbf{P}_t^- = \mathbf{F}\mathbf{P}_{t-1}^+\mathbf{F}^{\mathrm{T}} + \mathbf{Q}$
>
> $\Delta\mathbf{Z}_t = \mathbf{Z}_t - \mathbf{H}\mathbf{X}_t^-$
>
> $\mathbf{S}_t = \mathbf{H}\mathbf{P}_{t-1}^-\mathbf{H}^{\mathrm{T}} + \mathbf{R}$
>
> $\mathbf{K}_t = \mathbf{P}_{t-1}^-\mathbf{H}^{\mathrm{T}}\mathbf{S}_t^{-1}$
>
> $\mathbf{X}_t^+ = \mathbf{X}_t^- + \mathbf{K}_t\Delta\mathbf{Z}_t$
>
> $\mathbf{P}_t^+ = (\mathbf{I} - \mathbf{K}_t\mathbf{H})\mathbf{P}_{t-1}^-$

$\rightarrow$

> $\mathbf{X}_t^- = \mathbf{X}_{t-1}^+ + \mathbf{U}_t$
>
> $\mathbf{P}_t^- = \mathbf{P}_{t-1}^+ + \mathbf{Q}$
>
> $\Delta\mathbf{Z}_t = \mathbf{Z}_t - \mathbf{X}_t^-$
>
> $\mathbf{S}_t = \mathbf{P}_{t-1}^- + \mathbf{R}$
>
> $\mathbf{K}_t = \mathbf{P}_{t-1}^-\mathbf{S}_t^{-1}$
>
> $\mathbf{X}_t^+ = \mathbf{X}_t^- + \mathbf{K}_t\Delta\mathbf{Z}_t$
>
> $\mathbf{P}_t^+ = (\mathbf{I} - \mathbf{K}_t)\mathbf{P}_{t-1}^-$

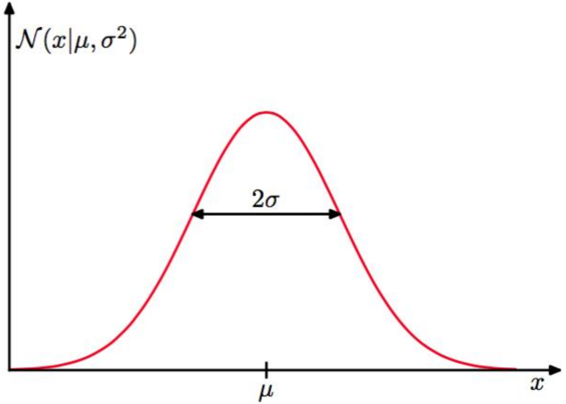
How do we describe the uncertainty?

> Gaussian distribution

$$p(x) \sim N(\mu, \sigma^2)$$

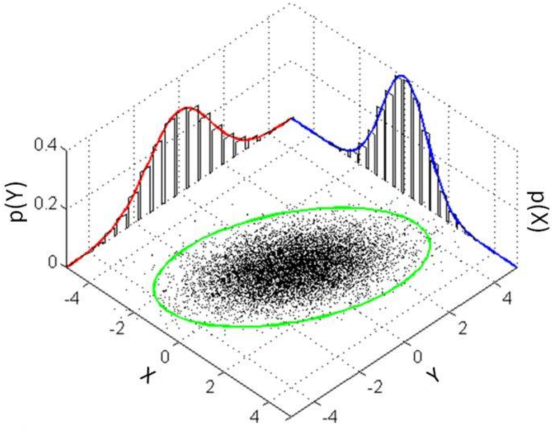
1D (univariate)

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

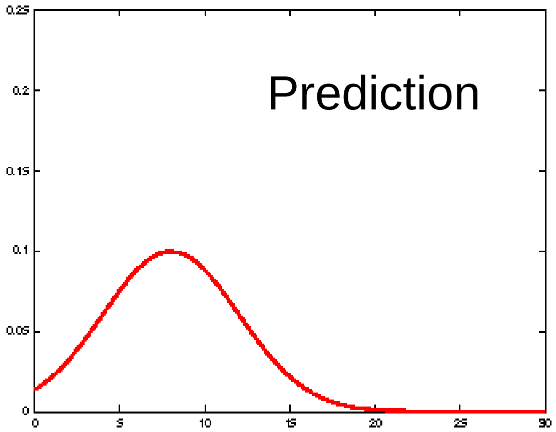


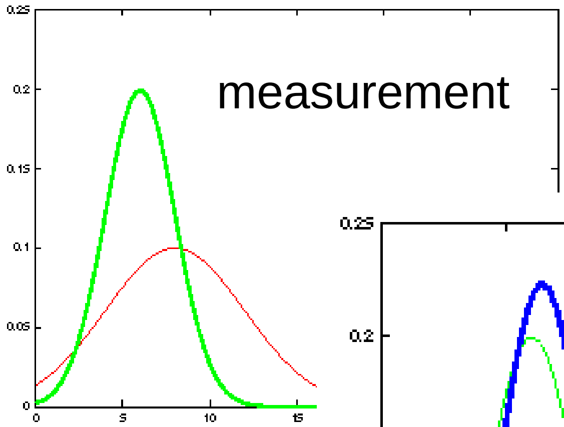
2D+ (multi variate)

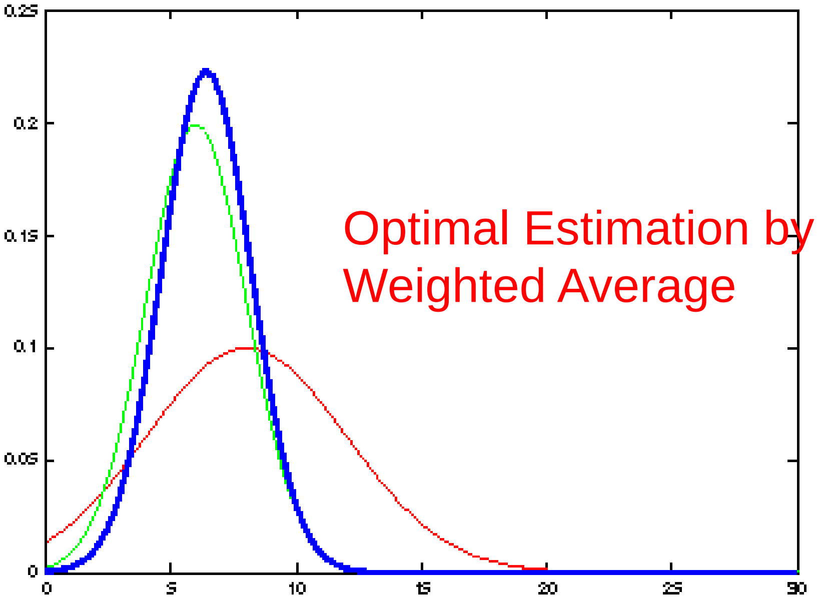
$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\mathbf{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^t \mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$



What Kalman Filter tries to achieve?







Fusion of Gaussian distribution in 1D

> Distribution of two measurements

$$\hat{q}_1 = q_1 \text{ with variance } \sigma_1^2$$
$$\hat{q}_2 = q_2 \text{ with variance } \sigma_2^2$$

> Weighted least-square

$$S = \sum_{i=1}^{n} w_i(\hat{q} - q_i)^2$$

> Finding minimum error

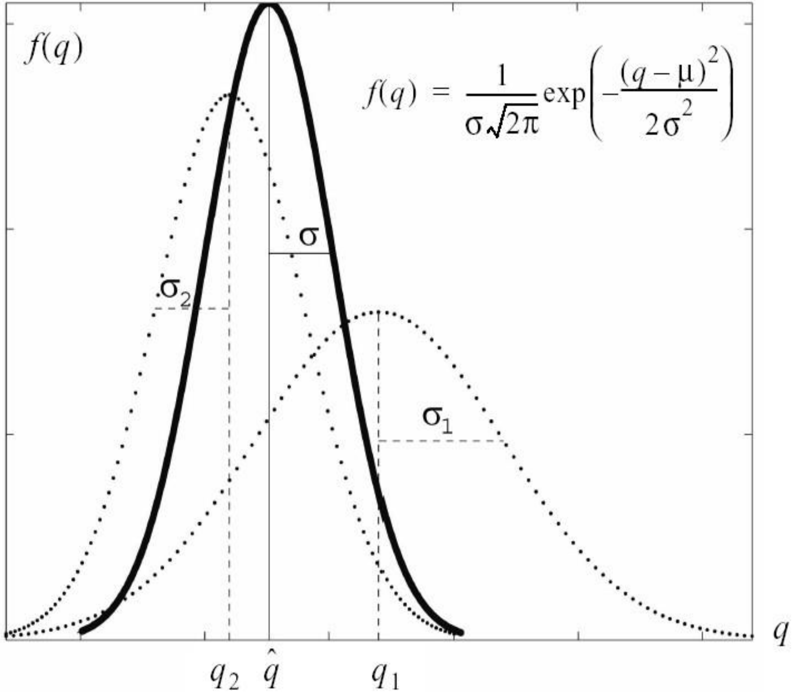
If $w_i = 1/\sigma_i^2$

$$\frac{\partial S}{\partial \hat{q}} = \frac{\partial}{\partial \hat{q}} \sum_{i=1}^{n} w_i(\hat{q} - q_i)^2 = 2\sum_{i=1}^{n} w_i(\hat{q} - q_i) = 0$$

After some calculation and rearrangements

$$\hat{q} = q_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}(q_2 - q_1)$$

Kalman Gain



Question: Why Kalman Filter is Optimal?

> What does it mean to Optimal?

- Maximum the Probability of the Estimation!

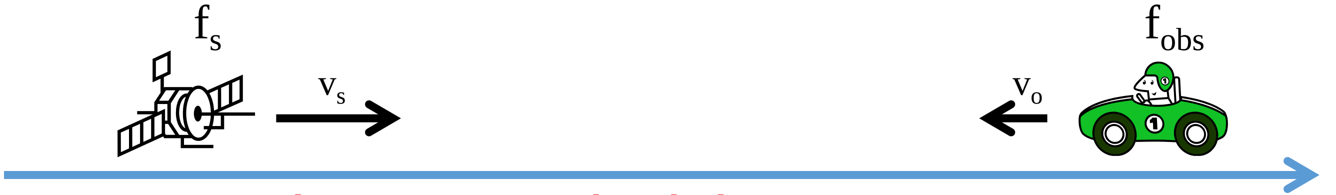
> What assumptions are made in Kalman Filter so that it can achieve Optimal?

A. Gaussian Random Variable (Gaussian Noise)
B. 1st Order of Markov Chain

Kalman Filter in GNSS

- **Example of the GNSS loosely-coupled pseudorange/Doppler integration using Kalman filter (this is done through the handwriting in visualizer)**
- Example of the GNSS tightly-coupled pseudorange/Doppler integration using Kalman filter

Doppler Effect



One dimension is assumed. Right direction is positive.

+

> Receiver is set in the car.

> Received frequency is

$$f_{obs} = f_s \frac{cs - v_o}{cs - v_s}$$

> “cs” is speed of light.

> Doppler frequency “f_D” is equal to “$f_{obs} - f_{source}$”

> FLL (frequency lock loop) tries to estimate “f_D”.

> Once we can estimate “f_D”, “v_o” can be resolved.

Velocity Estimation

> Velocity estimation in GPS is just same as shown in the previous slide.

> The differences are as follows.

*** 3 dimension velocity (v_x, v_y, v_z) have to be estimated.**

*** Frequency in the receiver is based on on-board clock.**

*** Measurement is pseudorange rate, which calculated from Doppler frequency AND satellite velocity.**

> 4 unknown variables (v_x, v_y, v_z, f_{clk}) have to be estimated using at least 4 visible satellites. DOP is also important.

> Velocity estimation is same as position estimation.

Receiver Velocity Estimation from the Doppler Measurements

Measurements from Doppler

$$\boldsymbol{y} = (-\lambda_i D_{r,i}^1, -\lambda_i D_{r,i}^2, -\lambda_i D_{r,i}^3, \ldots, -\lambda_i D_{r,i}^m)^T$$

Observation function

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{pmatrix} \dot{r}_r^1 + cd\dot{t}_r - cd\dot{T}^1 \\ \dot{r}_r^2 + cd\dot{t}_r - cd\dot{T}^2 \\ \dot{r}_r^3 + cd\dot{t}_r - cd\dot{T}^3 \\ \vdots \\ \dot{r}_r^m + cd\dot{t}_r - cd\dot{T}^m \end{pmatrix} \quad \boldsymbol{H} = \begin{pmatrix} -\boldsymbol{e}_r^{1T} & 1 \\ -\boldsymbol{e}_r^{2T} & 1 \\ -\boldsymbol{e}_r^{3T} & 1 \\ \vdots & \vdots \\ -\boldsymbol{e}_r^{mT} & 1 \end{pmatrix} \qquad \text{(F.6.28)}$$

The range-rage $\dot{r}_r^s$ between the receiver and the satellite in these equations is derived from:

$$\dot{r}_r^s = \boldsymbol{e}_r^{sT}\left(\boldsymbol{v}^s(t^s) - \boldsymbol{v}_r\right) + \frac{\omega_e}{c}\left(v_y^s x_r + y^s v_{x,r} - v_x^s y_r - x^s v_{y,r}\right) \qquad \text{(F.6.29)}$$

Can you try to formulate the steps for GNSS velocity estimation similar to the position estimation? ☺

Q&A

Thank you for your attention ☺

Q&A

Dr. Weisong Wen

If you have any questions or inquiries, please feel free to contact me.

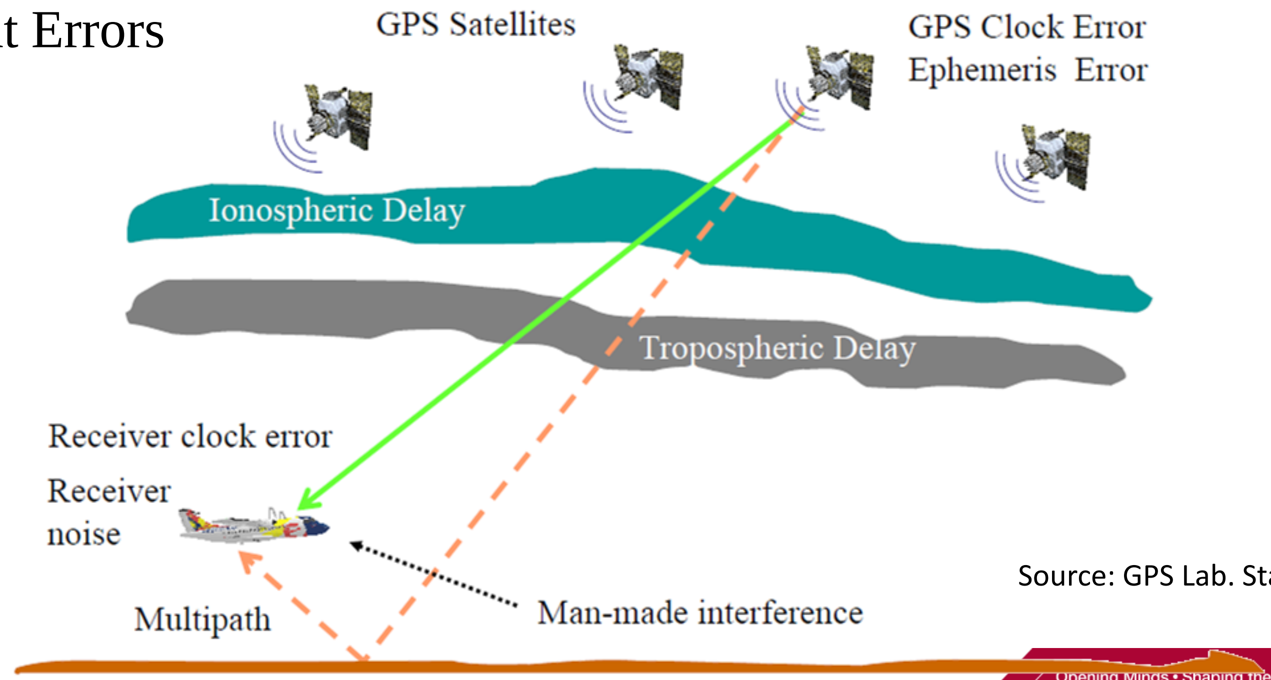
Email: welson.wen@polyu.edu.hk

Source of Measurements Errors

> Control Segment Errors

> Signal Propagation Modeling Errors

> Measurement Errors



Source: GPS Lab. Stanford Univ.

Why we discuss about measurement errors ?

- Back to bias and noise errors discussion, noise errors of pseudo-range can be mitigated to some degree using **carrier phase smoothing technique**.

- On the other hand, you have to estimate **bias errors** as accurate as possible **by yourself** to improve positioning performance.

- All kinds of improved techniques are essentially same in terms of estimating or eliminating bias or noise errors.

Observation Model for Pseudorange/Carrier Measurements

Observation function for pseudorange (code) measurement

$$\underbrace{\rho^s_{r,t}}_{\text{Pseudorange}} = \underbrace{r^s_{r,t}}_{\text{Range distance}} + c\big(\underbrace{\delta_{r,t}}_{\text{Receiver clock Bias (1\sim2m)}} - \underbrace{\delta^s_{r,t}}_{\text{Satellite clock bias}}\big) + \underbrace{I^s_{r,t}}_{\text{ionospheric delay Distance (1\sim2m)}} + \underbrace{T^s_{r,t}}_{\text{tropospheric delay Distance (1\sim2m)}} + \varepsilon^s_{r,t}$$

$\varepsilon^s_{r,t}$: multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)

Range distance: $||\mathbf{p}^{G,s}_t - \mathbf{p}^G_{r,t}||$

Observation function for carrier-phase measurement

$$\underbrace{\varphi^s_{r,t}}_{\text{Carrier-phase range}} = \underbrace{r^s_{r,t}}_{\text{Range distance}} + c\big(\underbrace{\delta_{r,t}}_{\text{Receiver clock Bias (1\sim2m)}} - \underbrace{\delta^s_{r,t}}_{\text{Satellite clock bias}}\big) + \underbrace{I^s_{r,t}}_{\text{ionospheric delay Distance (1\sim2m)}} + \underbrace{T^s_{r,t}}_{\text{tropospheric delay Distance (1\sim2m)}} + \varepsilon^s_{r,t} + \underbrace{N^s_{r,t}}_{\text{Ambiguity}}$$

$\varepsilon^s_{r,t}$: multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)

Range distance: $||\mathbf{p}^{G,s}_t - \mathbf{p}^G_{r,t}||$

To use the carrier-phase measurements, the ambiguity need to be resolved.

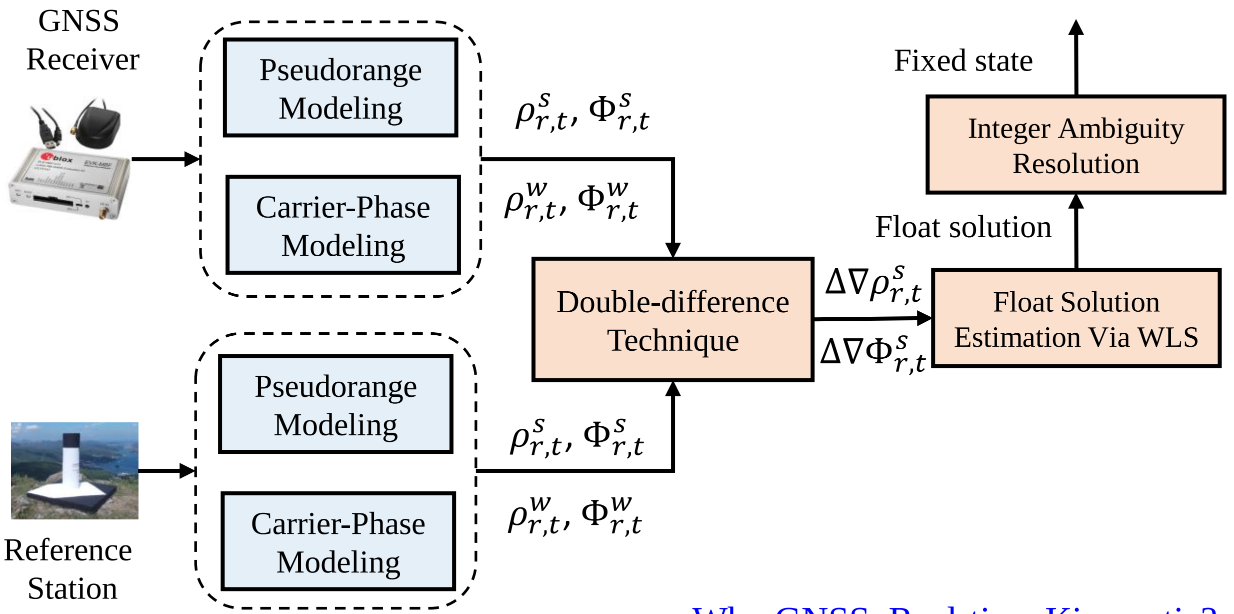
For each carrier-phase measurement, you got an unknown variable $N^s_{r,t}$!

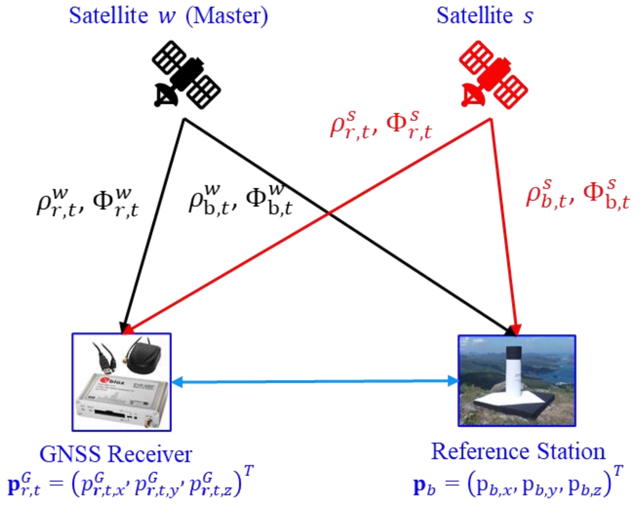
RTK (Real Time Kinematic)

- The concept of RTK is same as DGPS.
- RTK uses carrier phase measurements. DGPS uses pseudo-range measurements.
- GPS receiver is able to measure 1/100 of wavelength of L1 frequency (19 cm).
- If you have high-end receiver, you know your position within 1-2cm accuracy as long as you have 5 or more LOS satellites.

Overview of GNSS Real-time Kinematic

WLS*: Weighted Least Squares
DD *: Double-difference





$\rho^s_{r,t}$: Pseudorange measurement
$\Phi^s_{r,t}$: Carrier-phase measurement
$\Delta\nabla\rho^s_{r,t}$: DD Pseudorange measurement
$\Delta\nabla\Phi^s_{r,t}$: DD Carrier-phase measurement

Why GNSS Real-time Kinematic?

- Remove the error from receiver/satellite clock bias, atmosphere error using double-difference technique.
- Use the high-accuracy carrier-phase measurements.

Observation Model for Pseudorange/Carrier Measurements

Observation function for pseudorange (code) measurement

$$\underbrace{\rho^{s}_{r,t}}_{\text{Pseudorange}} = \underbrace{r^{s}_{r,t}}_{\text{Range distance}} + c\big(\underbrace{\delta_{r,t}}_{\text{Receiver clock Bias (1~2m)}} - \underbrace{\delta^{s}_{r,t}}_{\text{Satellite clock bias}}\big) + \underbrace{I^{s}_{r,t}}_{\text{ionospheric delay Distance (1~2m)}} + \underbrace{T^{s}_{r,t}}_{\text{tropospheric delay Distance (1~2m)}} + \underbrace{\varepsilon^{s}_{r,t}}_{\text{multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)}}$$

Range distance: $||\mathbf{p}^{G,s}_{t} - \mathbf{p}^{G}_{r,t}||$

Observation function for carrier-phase measurement

$$\underbrace{\psi^{s}_{r,t}}_{\text{Carrier-phase range}} = \underbrace{r^{s}_{r,t}}_{\text{Range distance}} + c\big(\underbrace{\delta_{r,t}}_{\text{Receiver clock Bias (1~2m)}} - \underbrace{\delta^{s}_{r,t}}_{\text{Satellite clock bias}}\big) + \underbrace{I^{s}_{r,t}}_{\text{ionospheric delay Distance (1~2m)}} + \underbrace{T^{s}_{r,t}}_{\text{tropospheric delay Distance (1~2m)}} + \underbrace{\varepsilon^{s}_{r,t}}_{\text{multipath effects, NLOS receptions, receiver noise, antenna phase-related noise (0~100m)}} + \underbrace{N^{s}_{r,t}}_{\text{Ambiguity}}$$

Range distance: $||\mathbf{p}^{G,s}_{t} - \mathbf{p}^{G}_{r,t}||$

To use the carrier-phase measurements, the ambiguity need to be resolved.

Single Difference Pseudorange Measurements

Single difference between the GNSS receiver and the reference station to remove the atmosphere errors:

$$\rho_{r,t}^{s} = r_{r,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) + I_{r,t}^{s} + T_{r,t}^{s} + \varepsilon_{r,t}^{s}$$

$$\rho_{b,t}^{s} = r_{b,t}^{s} + c\left(\delta_{b,t} - \delta_{b,t}^{s}\right) + I_{b,t}^{s} + T_{b,t}^{s} + \varepsilon_{b,t}^{s}$$

$$\Delta\rho_{r,t}^{s} = \rho_{r,t}^{s} - \rho_{b,t}^{s} = r_{r,t}^{s} - r_{b,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{s}\right)$$

Satellite s

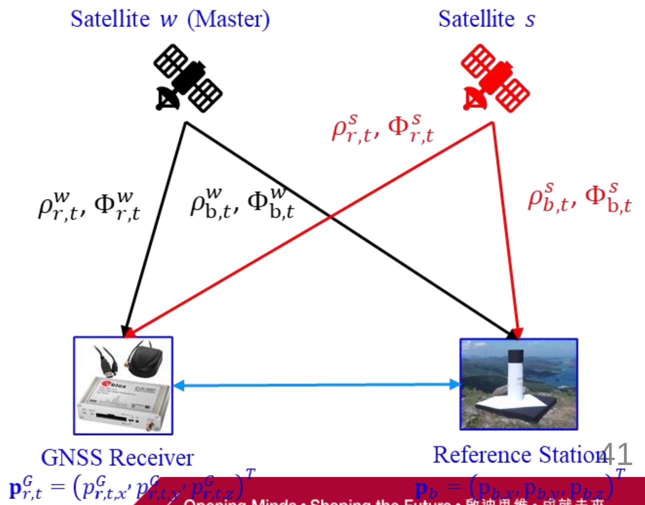
$$\rho_{r,t}^{w} = r_{r,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) + I_{r,t}^{w} + T_{r,t}^{w} + \varepsilon_{r,t}^{w}$$

$$\rho_{b,t}^{w} = r_{b,t}^{w} + c\left(\delta_{b,t} - \delta_{b,t}^{w}\right) + I_{b,t}^{w} + T_{b,t}^{w} + \varepsilon_{b,t}^{w}$$

$$\Delta\rho_{r,t}^{w} = \rho_{r,t}^{w} - \rho_{b,t}^{w} = r_{r,t}^{w} - r_{b,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{w}\right)$$

Satellite w (Master)

Assumption: GNSS receiver and the reference station are close with the same atmosphere errors



Double Difference Pseudorange Measurements

Second difference between the master satellite and the satellite s to remove the atmosphere errors:

$$\Delta\rho_{r,t}^{s} = \rho_{r,t}^{s} - \rho_{b,t}^{s} = r_{r,t}^{s} - r_{b,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{s}\right)$$

Satellite s

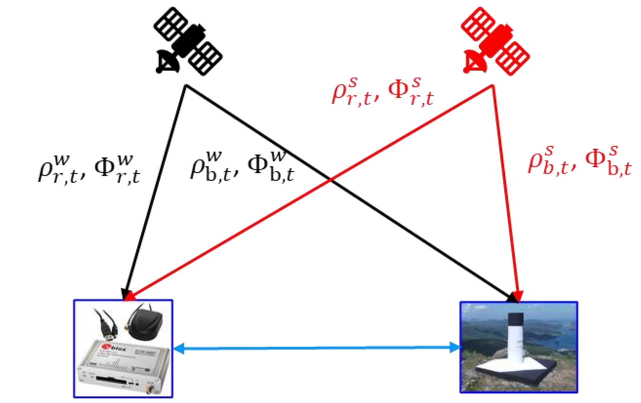
$$\Delta\rho_{r,t}^{w} = \rho_{r,t}^{w} - \rho_{b,t}^{w} = r_{r,t}^{w} - r_{b,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{w}\right)$$

Satellite w (Master)

$$\Delta\nabla\rho_{r,t}^{s} = \Delta\rho_{r,t}^{s} - \Delta\rho_{r,t}^{w} = \rho_{r,t}^{s} - \rho_{b,t}^{s} - \rho_{r,t}^{w} - \rho_{b,t}^{w}$$

DD measurements

Assumption: GNSS receiver and the reference station are close with the same atmosphere errors



Single Difference Carrier-phase Measurements

Single difference between the GNSS receiver and the reference station to remove the atmosphere errors:

$$\psi^s_{r,t} = r^s_{r,t} + c\left(\delta_{r,t} - \delta^s_{r,t}\right) + I^s_{r,t} + T^s_{r,t} + \varepsilon^s_{r,t} + N^s_{r,t}$$

$$\psi^s_{b,t} = r^s_{b,t} + c\left(\delta_{b,t} - \delta^s_{b,t}\right) + I^s_{b,t} + T^s_{b,t} + \varepsilon^s_{b,t} + N^s_{b,t}$$

Satellite s

$$\Delta\psi^s_{r,t} = \psi^s_{r,t} - \psi^s_{b,t} = r^s_{r,t} - r^s_{b,t} + c\left(\delta_{r,t} - \delta^s_{r,t}\right) - c\left(\delta_{b,t} - \delta^s_{b,t}\right) + N^s_{r,t} - N^s_{b,t}$$

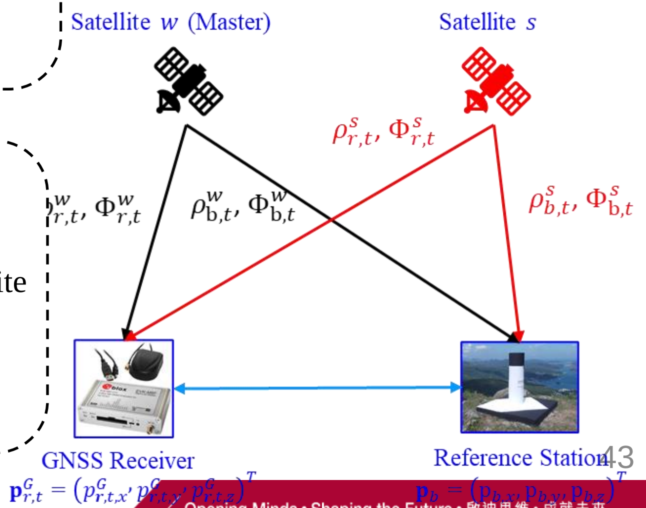
$$\psi^w_{r,t} = r^w_{r,t} + c\left(\delta_{r,t} - \delta^w_{r,t}\right) + I^w_{r,t} + T^w_{r,t} + \varepsilon^w_{r,t} + N^w_{r,t}$$

$$\psi^w_{b,t} = r^w_{b,t} + c\left(\delta_{b,t} - \delta^w_{b,t}\right) + I^w_{b,t} + T^w_{b,t} + \varepsilon^w_{b,t} + N^w_{b,t}$$

Satellite w

$$\Delta\psi^w_{r,t} = \psi^w_{r,t} - \psi^w_{b,t} = r^w_{r,t} - r^w_{b,t} + c\left(\delta_{r,t} - \delta^w_{r,t}\right) - c\left(\delta_{b,t} - \delta^w_{b,t}\right) + N^w_{r,t} - N^w_{b,t}$$

Assumption: GNSS receiver and the reference station are close with the same atmosphere errors



Double Difference Pseudorange Measurements

Second difference between the master satellite and the satellite s to remove the atmosphere errors:

Assumption: GNSS receiver and the reference station are close with the same atmosphere errors

Satellite s

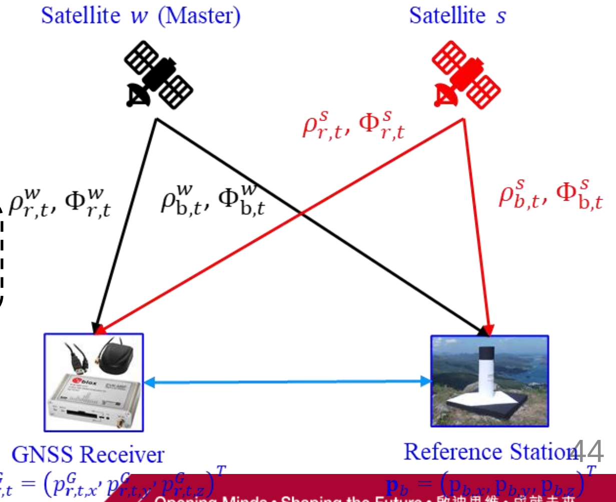
$$\Delta\psi_{r,t}^{s} = \psi_{r,t}^{s} - \psi_{b,t}^{s} = r_{r,t}^{s} - r_{b,t}^{s} + c\left(\delta_{r,t} - \delta_{r,t}^{s}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{s}\right) + N_{r,t}^{s} - N_{b,t}^{s}$$

Satellite w (Master)

$$\Delta\psi_{r,t}^{w} = \psi_{r,t}^{w} - \psi_{b,t}^{w} = r_{r,t}^{w} - r_{b,t}^{w} + c\left(\delta_{r,t} - \delta_{r,t}^{w}\right) - c\left(\delta_{b,t} - \delta_{b,t}^{w}\right) + N_{r,t}^{w} - N_{b,t}^{w}$$

DD measurements

$$\Delta\nabla\psi_{r,t}^{s} = \Delta\psi_{r,t}^{s} - \Delta\psi_{r,t}^{w} = \rho_{r,t}^{s} - \rho_{b,t}^{s} - \rho_{r,t}^{w} - \rho_{b,t}^{w} + \underbrace{\left(N_{r,t}^{s} - N_{b,t}^{s}\right) - \left(N_{r,t}^{w} - N_{b,t}^{w}\right)}_{\text{DD Ambiguity: } \Delta\nabla N_{r,t}^{s}}$$



GNSS Real-time Kinematic Positioning: Float Solution Estimation

Estimate the float solution via weighted least square positioning

$$\begin{bmatrix} \mathbf{p}_{r,t}^{G} \\ \Delta\nabla N_{r,t}^{1} \\ \Delta\nabla N_{r,t}^{2} \\ \dots \\ \Delta\nabla N_{r,t}^{m-1} \end{bmatrix} = \left({\mathbf{G}_t^G}^T \mathbf{W}_t \mathbf{G}_t^G \right)^{-1} {\mathbf{G}_t^G}^T \mathbf{W}_t \begin{bmatrix} \Delta\nabla \rho_{r,t}^{1} \\ \Delta\nabla \rho_{r,t}^{2} \\ \vdots \\ \Delta\nabla \rho_{r,t}^{m-1} \\ \Delta\nabla \psi_{r,t}^{1} \\ \Delta\nabla \psi_{r,t}^{2} \\ \dots \\ \Delta\nabla \psi_{r,t}^{m-1} \end{bmatrix}$$

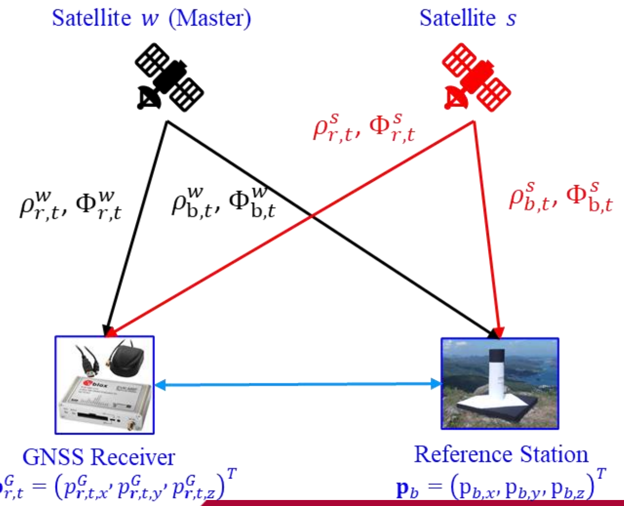
Output $\Longrightarrow$

$\mathbf{p}_{r,t}^{G}$: Float solution of position of GNSS receiver
$\Delta\nabla N_{r,t}^{1}, \Delta\nabla N_{r,t}^{2}, \dots$: Float ambiguity
$\left({\mathbf{G}_t^G}^T \mathbf{W}_t \mathbf{G}_t^G \right)^{-1}$: Covariance matrix

$\mathbf{p}_{r,t}^{G}$: Position of GNSS receiver
$\mathbf{W}_t$: Weighting matrix
m: number of satellite
$\mathbf{G}_t^G$: Observation matrix
$\Delta\nabla \rho_{r,t}^{m-1}$: DD pseudorange measurements
$\Delta\nabla \psi_{r,t}^{m-1}$: DD carrier-phase measurements

How to formulate the linear least square problem of the GNSS-RTK

Formulate it!



GNSS Real-time Kinematic Positioning: Ambiguity Resolution

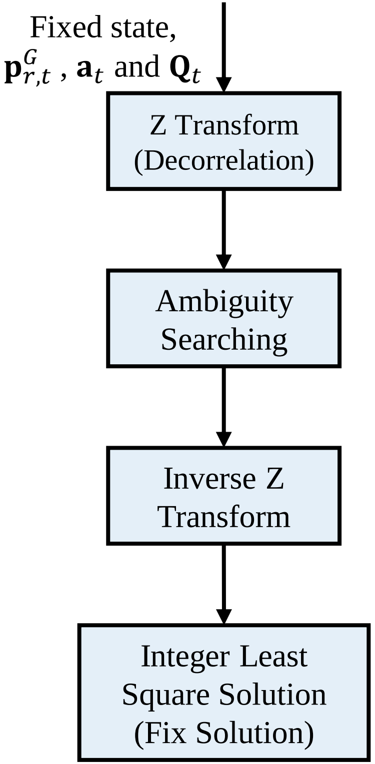
$\mathbf{p}_{r,t}^{G}$: Float solution of position of GNSS receiver

$\mathbf{a}_t = \Delta\nabla N_{r,t}^{1}, \Delta\nabla N_{r,t}^{2}, \ldots$: Float ambiguity

$\mathbf{Q}_t = \left(\mathbf{G}_t^{G\,T}\mathbf{W}_t\mathbf{G}_t^{G}\right)^{-1}$: Covariance matrix

$$\mathbf{G}_t^{G} = \begin{bmatrix} \frac{p_{t,x}^{G,1} - p_{r,t,x}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,y}^{G,1} - p_{r,t,y}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,z}^{G,1} - p_{r,t,z}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{p_{t,x}^{G,m-1} - p_{r,t,x}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,y}^{G,m-1} - p_{r,t,y}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,z}^{G,m-1} - p_{r,t,z}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & 0 & \cdots & 0 \\ \frac{p_{t,x}^{G,1} - p_{r,t,x}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,y}^{G,1} - p_{r,t,y}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,z}^{G,1} - p_{r,t,z}^{G}}{\|\mathbf{p}_t^{G,1} - \mathbf{p}_{r,t}^{G}\|} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{p_{t,x}^{G,m-1} - p_{r,t,x}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,y}^{G,m-1} - p_{r,t,y}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & \frac{p_{t,z}^{G,m-1} - p_{r,t,z}^{G}}{\|\mathbf{p}_t^{G,m-1} - \mathbf{p}_{r,t}^{G}\|} & 0 & \cdots & 1 \end{bmatrix}$$

The correct fixed solution relies on the accuracy of float solution $\mathbf{p}_{r,t}^{G}$ and the covariance matrix $\mathbf{Q}_t$.



$\hat{\mathbf{z}}_t = \mathbf{Z}^T\mathbf{a}_t \qquad \mathbf{Q}_{t,z} = \mathbf{Z}^T\mathbf{Q}_t\mathbf{Z}$

Larger covariance matrix $\mathbf{Q}_t$, larger searching space

$\breve{z}_t = \min_{z \in Z^{m-1}} \|\hat{\mathbf{z}}_t - \mathbf{Z}_t\|_{\mathbf{Q}_{t,z}}^2$

$\breve{\mathbf{a}}_t = \mathbf{Z}^{-T}\breve{z}_t$

$\mathbf{p}_{r,t}^{G} = \mathbf{p}_{r,t}^{G} - \mathbf{Q}_{t,pa}\mathbf{Q}_{t,a}^{-1}(\mathbf{a}_t - \breve{\mathbf{a}}_t)$

Statistical Methods for Estimation and Optimization

Statistical method

States set

$$\boldsymbol{\chi} = \{\mathbf{x}_o, \mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k\}$$

$$\mathbf{x} = \begin{bmatrix} x_0 \\ \vdots \\ x_m \end{bmatrix}$$

Measurement set

$$\mathbf{Z} = \{\mathbf{z}_o, \mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_k\}$$

$$\mathbf{z} = \begin{bmatrix} z_0 \\ \vdots \\ z_n \end{bmatrix}$$

Frequentist → **Maximum Likelihood Estimation (MLE)**

$$P(\mathbf{z}|\mathbf{x})$$

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}\in\mathbf{X}} P_{(\mathbf{Z}|\chi)}(\mathbf{z}|\mathbf{x})$$

event-centric estimation, fully based on data. Should be used in Big Data application.

Bayesians → **Maximum a Posterior Estimation (MAP)**

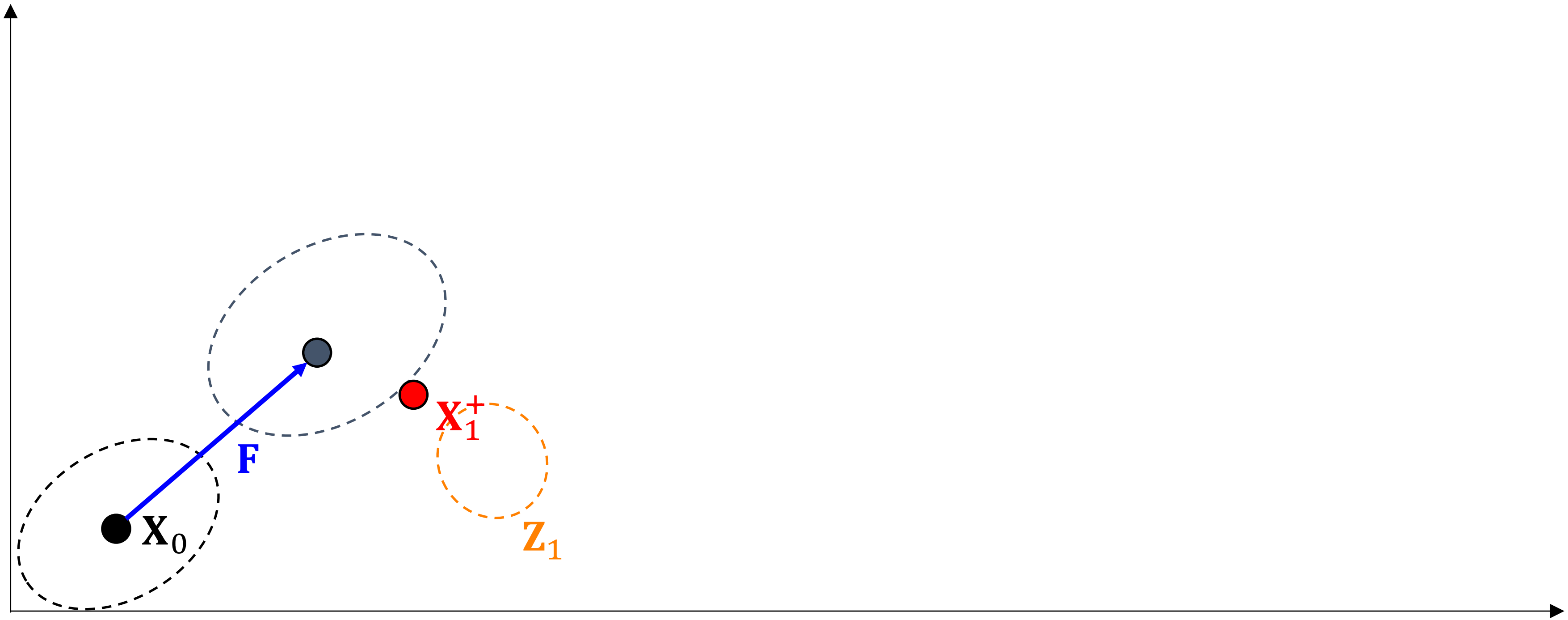
$$P(\mathbf{x}|\mathbf{z}) = \frac{P(\mathbf{z}|\mathbf{x})P(\mathbf{x})}{P(\mathbf{z})}$$

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}\in\mathbf{X}} P_{(\mathbf{Z}|\chi)}(\mathbf{x}|\mathbf{z})$$

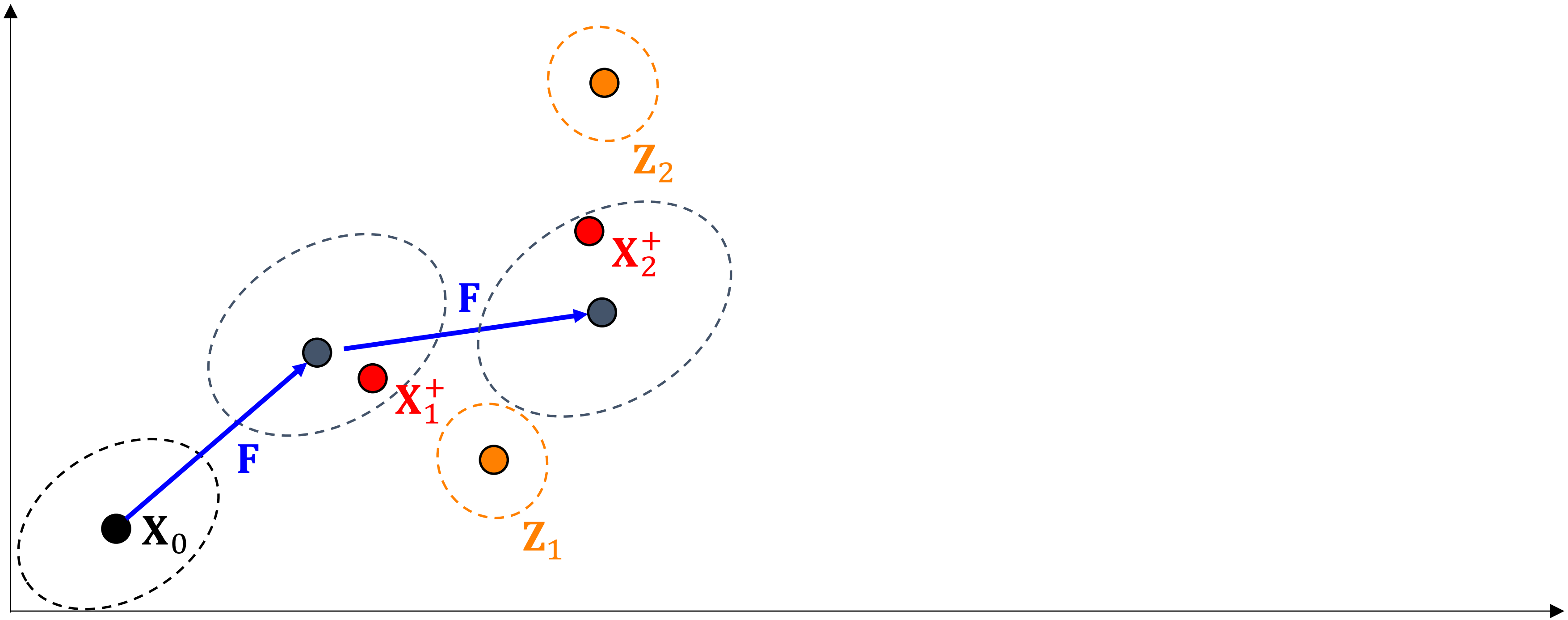
observer-centric estimation, required a knowledgeful observer (good prior-information)

Optimization, i.e., Kalman filter, factor graph optimization, etc.

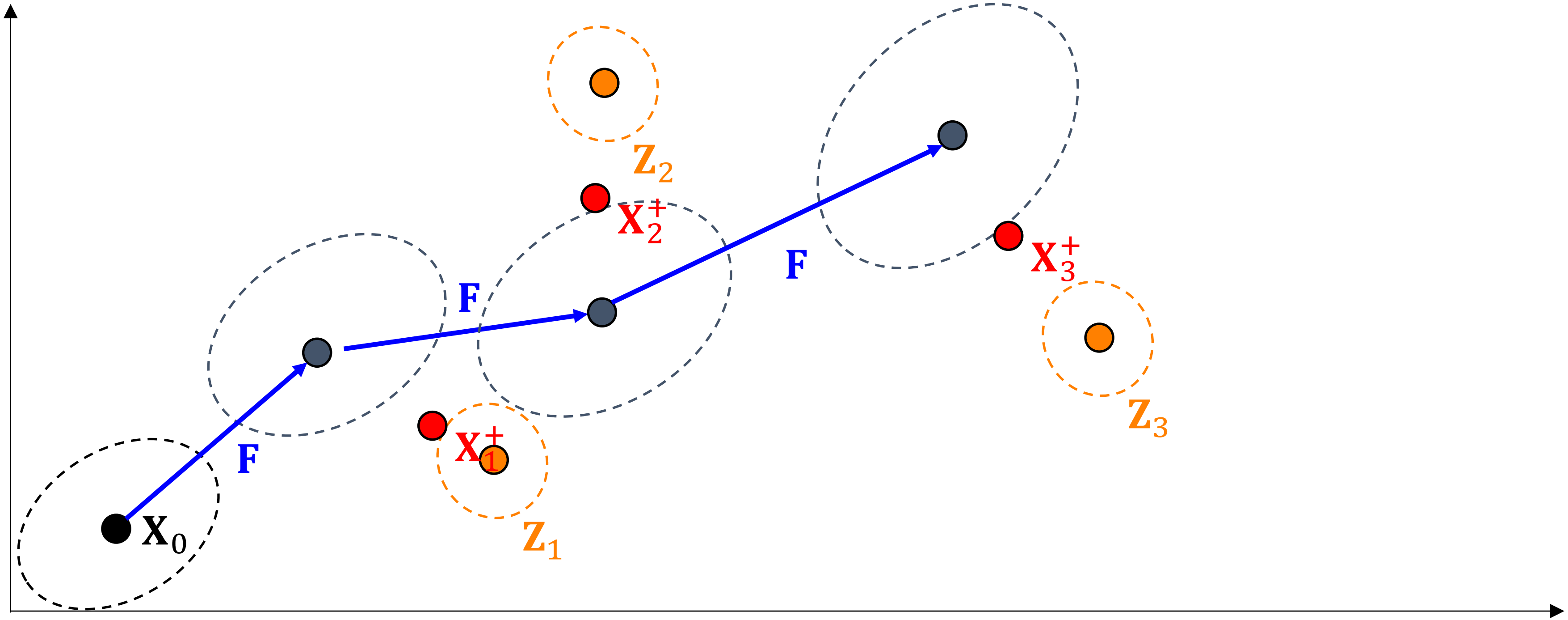
Batch (including all the data in the past) optimization



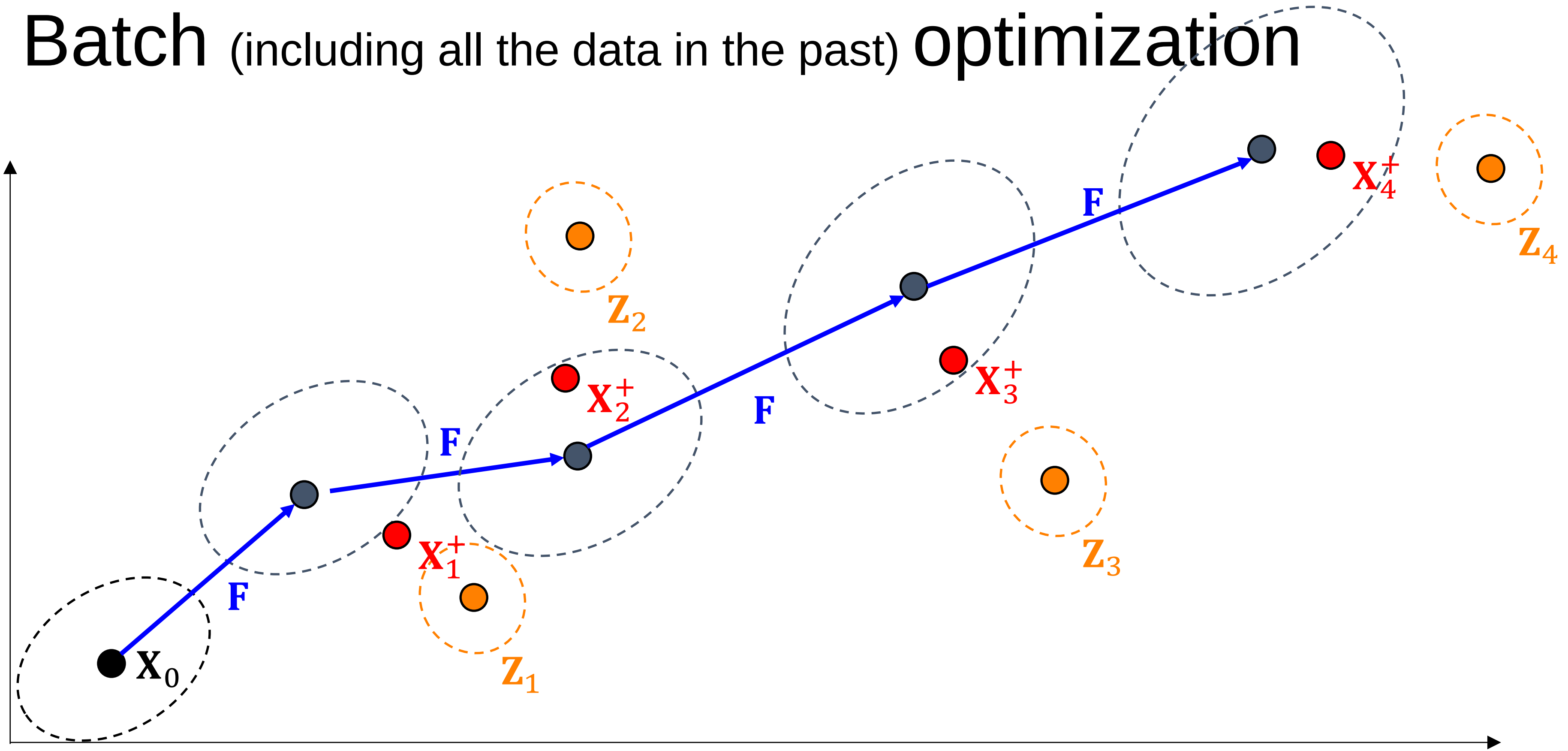
Batch (including all the data in the past) optimization



Batch (including all the data in the past) optimization

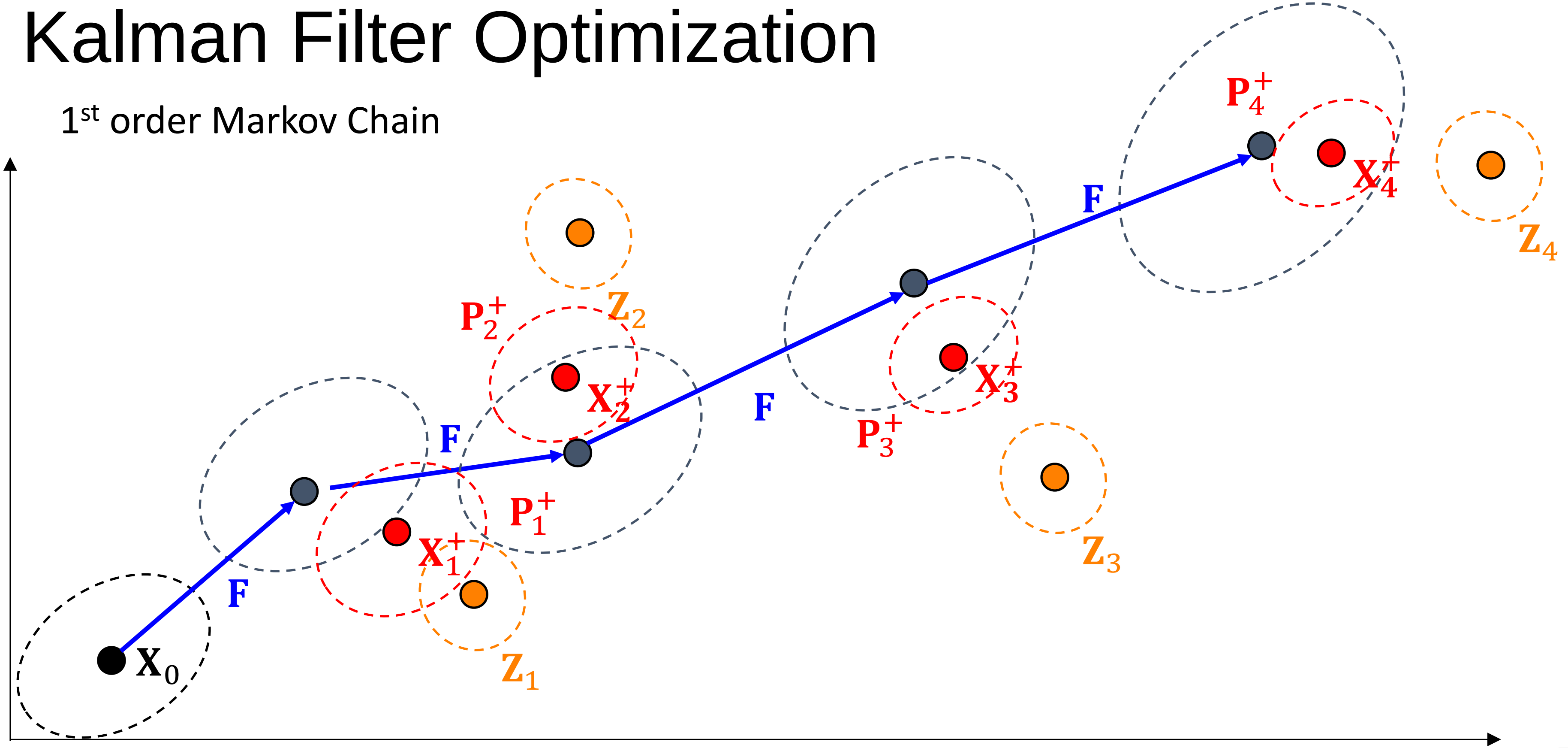


Batch (including all the data in the past) optimization

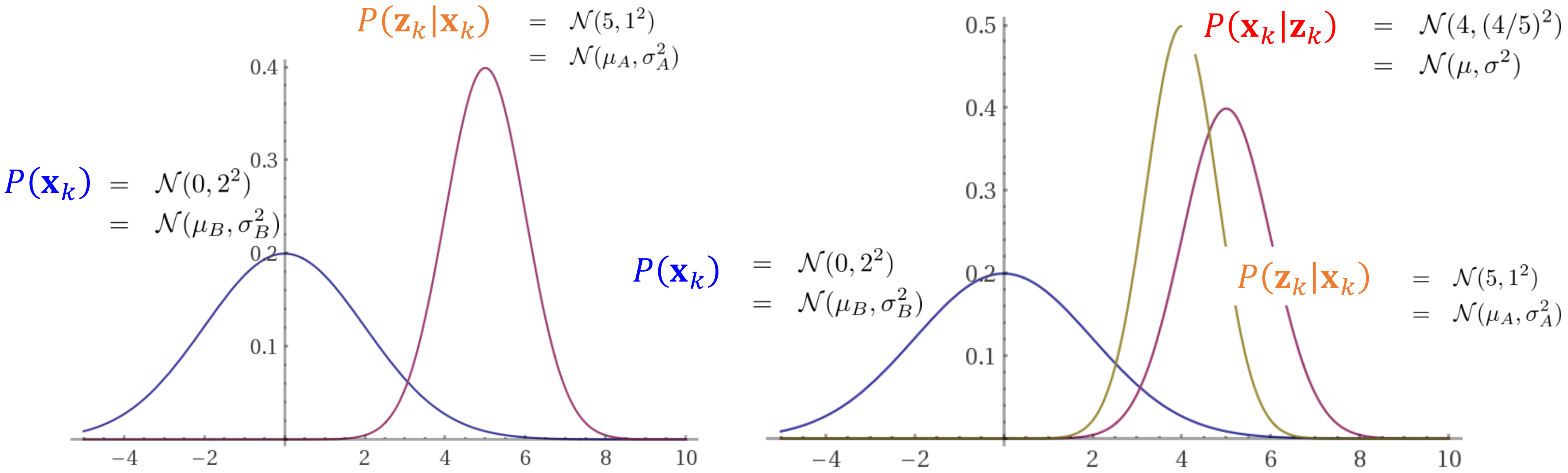


Kalman Filter Optimization

1st order Markov Chain



Kalman Filter with 1D state——Update step

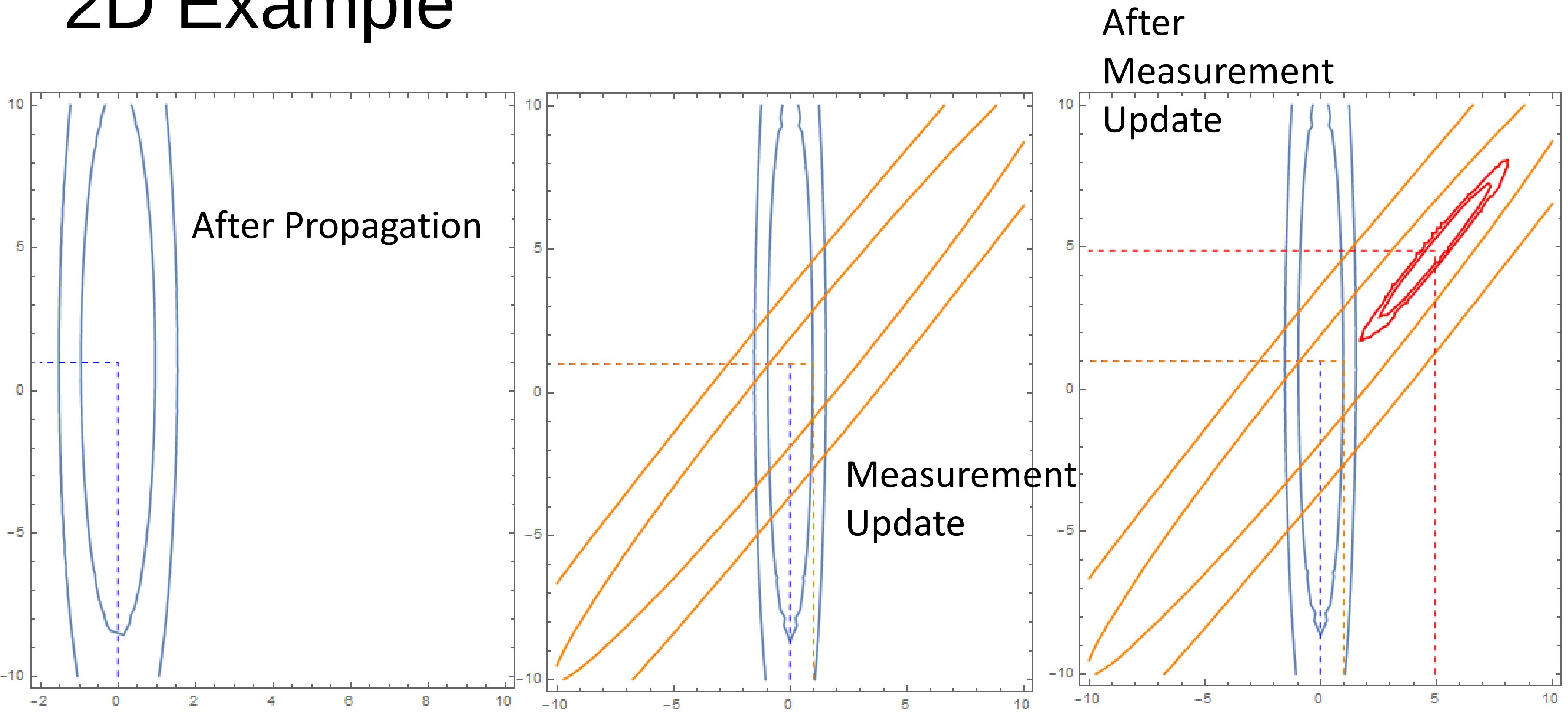


Kalman Gain: specifies how much effect will the measurement have in the posterior, compared to the prediction prior.
Which one do you trust more, your prior or your measurement ?

$$\mu = \mu_B + \boxed{\frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2}}(\mu_A - \mu_B)$$

$$\sigma^2 = \sigma_B^2 - \boxed{\frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2}}\sigma_B^2$$

2D Example


After Propagation
Measurement Update
After Measurement Update

Kalman filter in GNSS

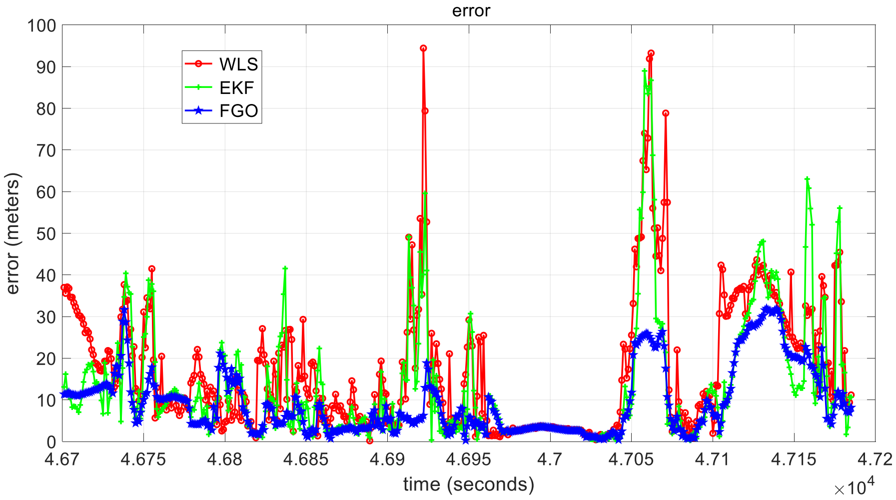
- Example of the GNSS loosely-coupled pseudorange/Doppler integration using Kalman filter
- **Example of the GNSS tightly-coupled pseudorange/Doppler integration using Kalman filter**

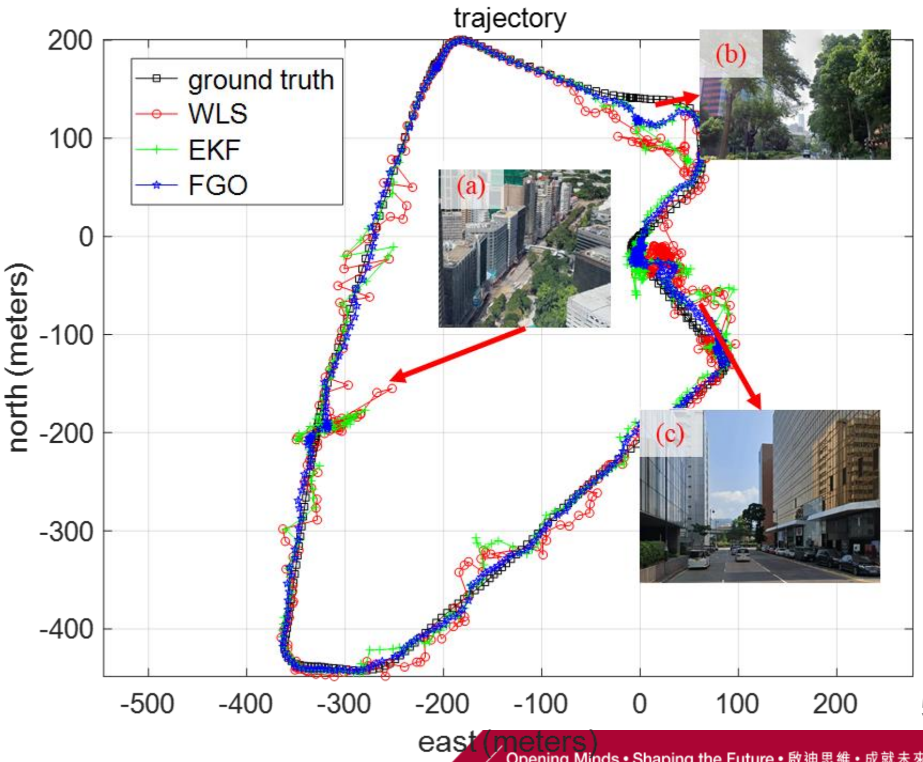
Evaluation of GNSS Positioning

WLS*: weighted least square with pseudorange
EKF*: Pseudorange/Doppler fusion with extended Kalman filter
FGO*: Pseudorange/Doppler fusion with factor graph optimization

GNSS positioning performance using the three listed methods

All data	WLS	EKF	FGO
MEAN (m)	17.39	13.61	9.45
STD (m)	16.01	15.19	8.06
MAX (m)	94.43	88.97	31.94
Availability	100%	100%	100%



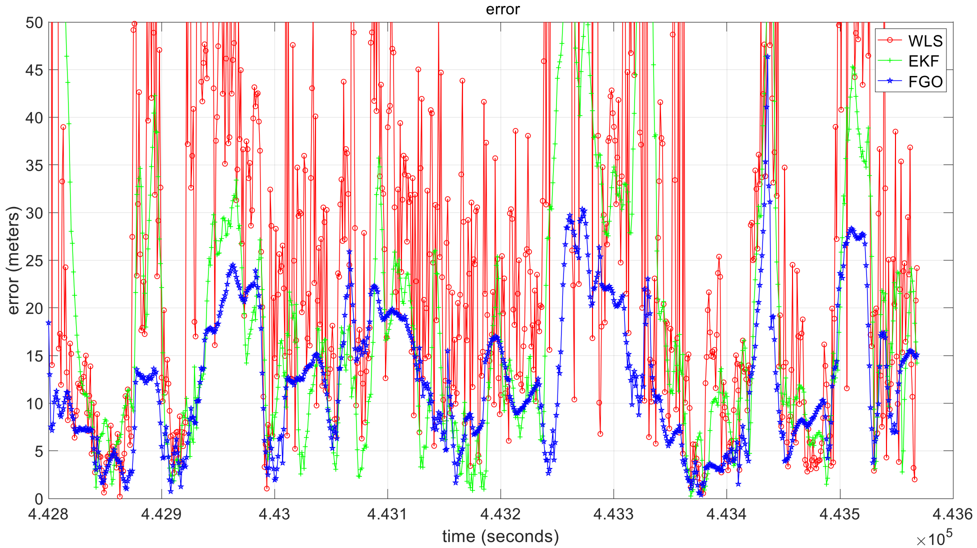


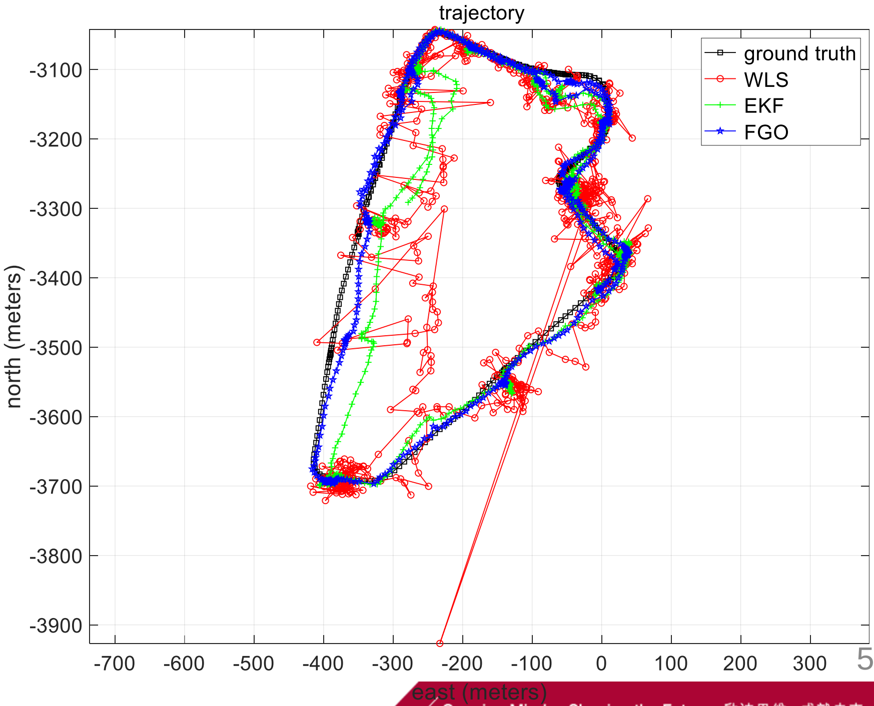
Evaluation with Huawei P40 Pro

Huawei P40 Pro Phone

WLS*: weighted least square with pseudorange
EKF*: Pseudorange/Doppler fusion with extended Kalman filter
FGO*: Pseudorange/Doppler fusion with factor graph optimization

All data	WLS	EKF	FGO
MEAN (m)	31.98	19.84	12.541
STD (m)	38.22	15.78	7.48
MAX (m)	701.7	77.28	46.36





Supplementary: GNSS/INS Integration Using Kalman Filtering

Inertial navigation system

$\hat{\mathbf{a}}_t$, $\hat{\boldsymbol{\omega}}_t$ are the raw accelerometer and gyroscope measurements in the body frame

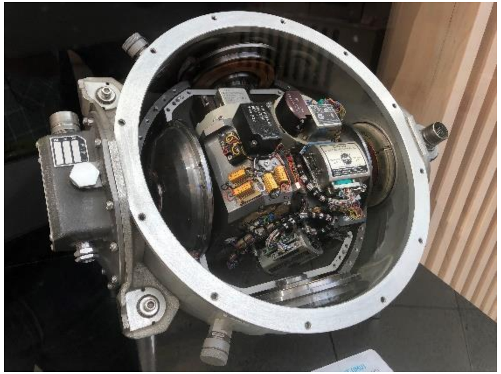
$\mathbf{a}_t$, $\boldsymbol{\omega}_t$ are expected measurements

The cap ^ denotes the noisy measurement or estimation of a certain quantity

$$\hat{\mathbf{a}}_t = \mathbf{a}_t + \mathbf{R}_w^t \mathbf{g}^w + \mathbf{b}_{a_t} + \mathbf{n}_a \quad (1)$$

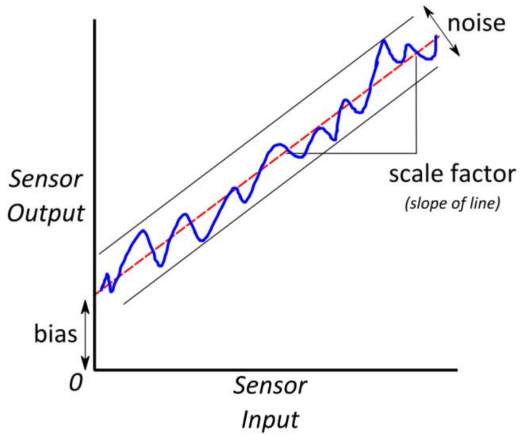
$$\hat{\boldsymbol{\omega}}_t = \boldsymbol{\omega}_t + \mathbf{b}_{\omega_t} + \mathbf{n}_\omega \quad (2)$$

$$\mathbf{n}_a \sim \mathcal{N}(0, \sigma_a^2), \mathbf{n}_\omega \sim \mathcal{N}(0, \sigma_\omega^2)$$



Error analysis of inertial navigation system

- The errors of **accelerometer and gyroscope** can be divided into: **deterministic error & random error.**
- Deterministic errors can be calibrated in advance including bias, scale...
- Random error usually assumes that noise obeys Gaussian distribution, including Gaussian white noise, bias random walk...



Common systematic errors in IMU
bias
noise
scale factor

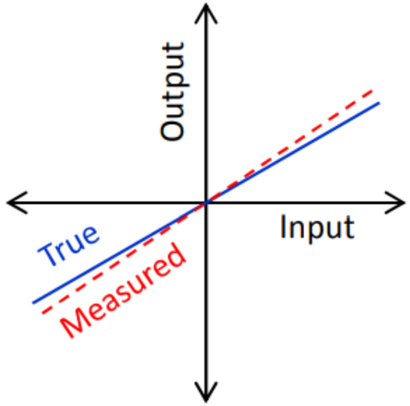
Deterministic error

(sourcing imperfectness of electrical/mechanical components)

> Bias: In theory, the output of the IMU sensor should be 0 when there is no external action. However, there is a bias $\mathbf{b}$ to the international data. Influence of accelerometer bias on orientation estimation:

$$\mathbf{V}_{\mathbf{error}} = \mathbf{b}_{\mathbf{a}} t, \mathbf{P}_{\mathbf{error}} = \frac{1}{2} \mathbf{b}_{\mathbf{a}} t^2$$

> Scale: The ratio between the actual value and the sensor output value.



Deterministic error (sourcing imperfectness of installation)

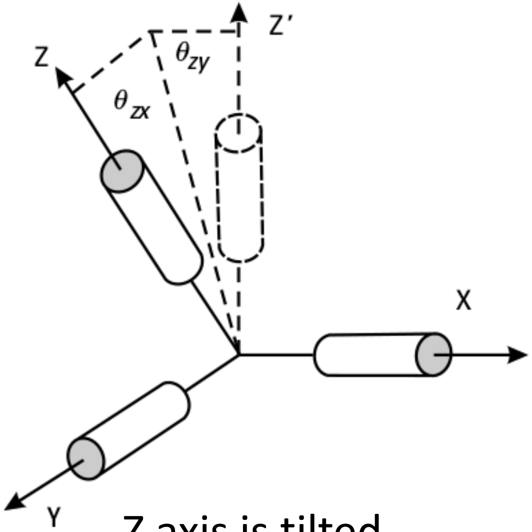
> Nonorthogonality/Misalignment Errors: When manufacturing multi-axis IMU sensors, due to the manufacturing process, the xyz axis may not be vertical.

Scale + Misalignment

$$\begin{bmatrix} l_{ax} \\ l_{ay} \\ l_{az} \end{bmatrix} = \begin{bmatrix} s_{xx} & m_{xy} & m_{xz} \\ m_{yz} & s_{yy} & m_{yz} \\ m_{zx} & m_{zy} & s_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Measured Acc

True Acc



Z axis is tilted

Deterministic error calibration method—Accelerometer

> The six-sided method means that the three axes of the accelerometer are placed horizontally up or down for a period of time, and data on the six sides are collected to complete the calibration.

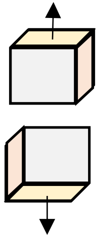
If the axes are orthogonal, it is easy to get bias and scale:

$$\mathbf{b_a} = \frac{\mathbf{l_f^{up}} + \mathbf{l_f^{down}}}{2}$$

$$\mathbf{s_a} = \frac{\mathbf{l_f^{up}} - \mathbf{l_f^{down}}}{2 \cdot \mathbf{g}} = \begin{bmatrix} s_{a,xx} \\ s_{a,yy} \\ s_{a,zz} \end{bmatrix}$$

$\mathbf{l_f^{up}}$

$\mathbf{l_f^{dowb}}$



$\mathbf{l}$ is the measured value of a certain axis of the accelerometer, $\mathbf{g}$ is the local gravity acceleration

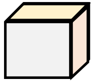
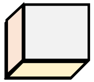
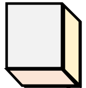
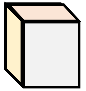
Deterministic error calibration method—Accelerometer

- When considering the inter-axis error, the relationship between the actual acceleration and the measured value is:

$$\begin{bmatrix} l_{ax} \\ l_{ay} \\ l_{az} \end{bmatrix} = \begin{bmatrix} s_{xx} & m_{xy} & m_{xz} \\ m_{yz} & s_{yy} & m_{yz} \\ m_{zx} & m_{zy} & s_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_{ax} \\ b_{ay} \\ b_{az} \end{bmatrix}$$

- When placed horizontally and statically on 6 sides, the theoretical value of acceleration is

$$\mathbf{a}_1 = \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -g \\ 0 \\ 0 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix}, \mathbf{a}_4 = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix}, \mathbf{a}_5 = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}, \mathbf{a}_6 = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$$

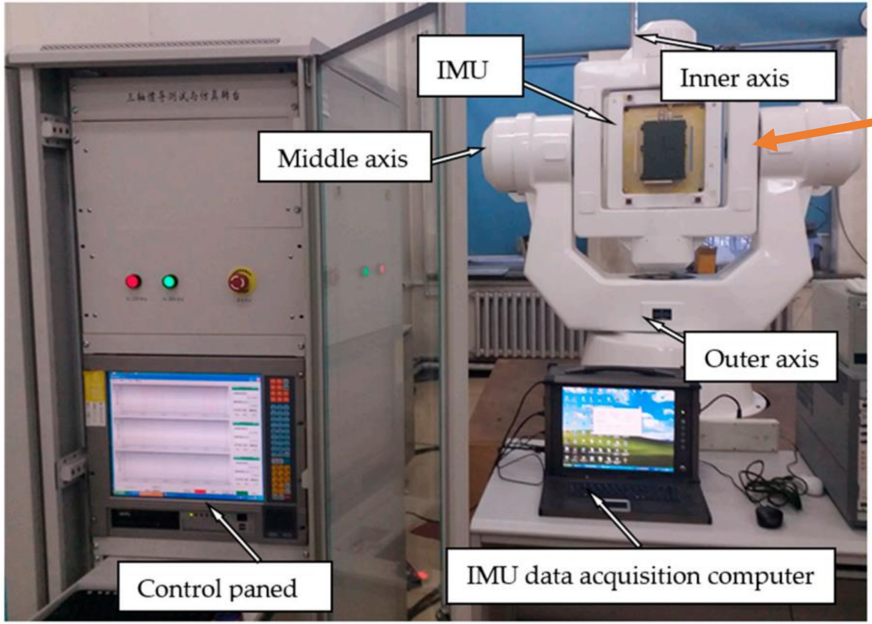
- Corresponding measurement value matrix $\mathbf{L}$

$$\mathbf{L} = [\boldsymbol{l}_1 \quad \boldsymbol{l}_2 \quad \boldsymbol{l}_3 \quad \boldsymbol{l}_4 \quad \boldsymbol{l}_5 \quad \boldsymbol{l}_6]$$

- 12 variables can be obtained by using least squares.

Deterministic error calibration method——Gyroscope

> Unlike the six-sided method of accelerometer, the true value of the gyroscope is provided by a high-precision turntable. The 6 faces in this refer to the clockwise and counterclockwise rotation of each axis



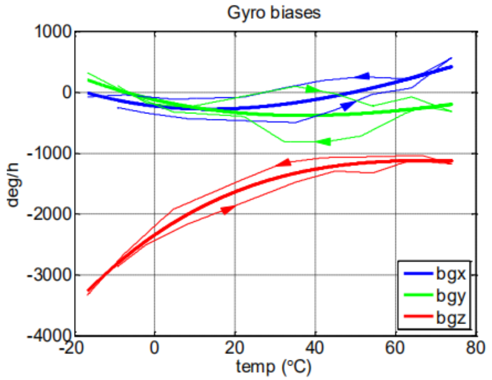
high-precision three-axis turntable

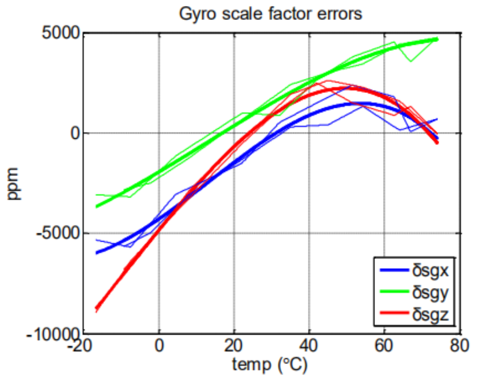
Random error – Unstableness of electrical and mechanical component due to temperature

> We can calibrate to do temperature compensation on the bias and scale estimated by the sensor, and to obtain the values of bias and scale at different temperatures and draw them into a curve.

> Soak method: control the temperature value of **the constant temperature room**, and then read the sensor value for calibration.

The thin solid lines are the results under separated heating and cooling processes The thick lines are the final curve fitted result .

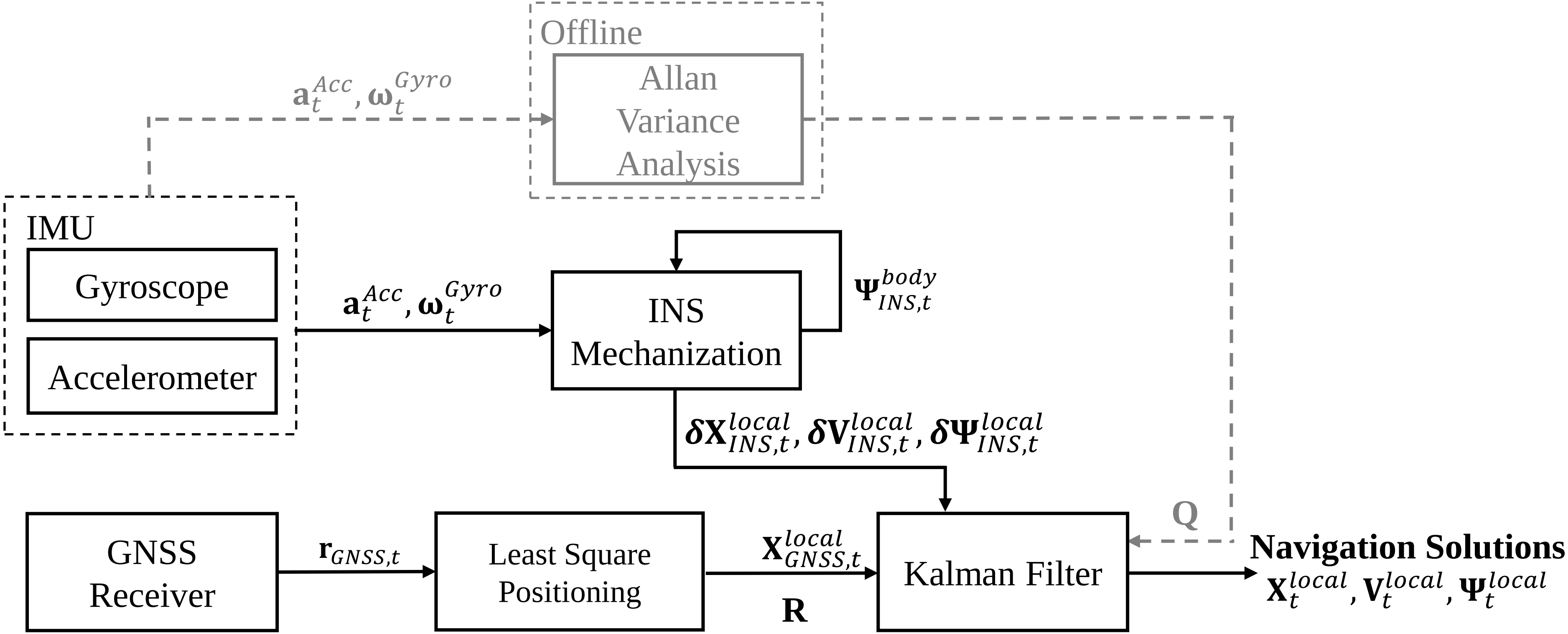




Nomenclature

- $\mathbf{a}_t^{Acc}$:measured 3-axis accelerations by the accelerometers at epoch t
- $\boldsymbol{\omega}_t^{Gyro}$:measured 3-axis rotation by the gyroscopes at epoch t
- $\mathbf{X}_{INS,t}^{body}$:estimated 3-axis position in body frame by INS at epoch t
- $\mathbf{V}_{INS,t}^{body}$:estimated 3-axis velocity in body frame by INS at epoch t
- $\boldsymbol{\Psi}_{INS,t}^{body}$:estimated 3-axis orientation in body frame (Euler angles) by INS at epoch t
- $\mathbf{B}_{a,t}^{body}$:estimated 3−axis biaes of accelerometers in body frame at epoch t
- $\mathbf{B}_{\omega,t}^{body}$:estimated 3−axis biaes of gyroscopes in body frame at epoch t
- W_{b_a}:estimated 3−axis random walk noise of accelerometers in body frame
- W_{b_a}:estimated 3−axis random walk noise of gyroscopes in body frame

Open Loop


Offline
Allan Variance Analysis
$\mathbf{a}_t^{Acc}, \boldsymbol{\omega}_t^{Gyro}$
IMU
Gyroscope
Accelerometer
$\mathbf{a}_t^{Acc}, \boldsymbol{\omega}_t^{Gyro}$
INS Mechanization
$\boldsymbol{\Psi}_{INS,t}^{body}$
$\boldsymbol{\delta X}_{INS,t}^{local}, \boldsymbol{\delta V}_{INS,t}^{local}, \boldsymbol{\delta \Psi}_{INS,t}^{local}$
GNSS Receiver
$\mathbf{r}_{GNSS,t}$
Least Square Positioning
$\mathbf{X}_{GNSS,t}^{local}$
$\mathbf{R}$
Kalman Filter
$\mathbf{Q}$
Navigation Solutions
$\mathbf{X}_t^{local}, \mathbf{V}_t^{local}, \boldsymbol{\Psi}_t^{local}$

INS Mechanization

Sola, Joan. "Quaternion kinematics for the error-state Kalman filter." *arXiv preprint arXiv:1711.02508* (2017).

The Euler angle rates obtained by angular velocity:

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \dot{\boldsymbol{\Psi}}_{INS,t}^{body} = \begin{bmatrix}1 & sin(\phi)\,tan(\theta) & cos(\phi)\,tan(\theta)\\ 0 & cos(\phi) & -sin(\phi)\\ 0 & sin(\phi)\,sec(\theta) & cos(\phi)\,sec(\theta)\end{bmatrix}\boldsymbol{\omega}_t^{Gyro} = \begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

$$\boldsymbol{\delta\Psi}_{INS,t}^{body} = \dot{\boldsymbol{\Psi}}_{INS,t}^{body}\Delta t = \begin{bmatrix}\delta\phi\\ \delta\theta\\ \delta\psi\end{bmatrix}$$

Rotate the Euler angles from Body to Local

$$\boldsymbol{\Psi}_{INS,t}^{local} = \mathbf{R}_{body}^{local}\boldsymbol{\Psi}_{INS,t}^{body}$$

Update the current Euler angles

$$\boldsymbol{\Psi}_{INS,t}^{body} = \boldsymbol{\Psi}_{INS,t-1}^{body} + \boldsymbol{\delta\Psi}_{INS,t}^{body}$$

$$\begin{aligned}\mathbf{R}_{body}^{local} &= \mathbf{R}(X,\phi)\mathbf{R}(Y,\theta)\mathbf{R}(Z,\psi)\\ &= \begin{bmatrix}1 & 0 & 0\\ 0 & cos(\phi) & -sin(\phi)\\ 0 & sin(\phi) & cos(\phi)\end{bmatrix}\begin{bmatrix}cos(\theta) & 0 & sin(\theta)\\ 0 & 1 & 0\\ -sin(\theta) & 0 & cos(\theta)\end{bmatrix}\begin{bmatrix}cos(\psi) & -sin(\psi) & 0\\ sin(\psi) & cos(\psi) & 0\\ 0 & 0 & 1\end{bmatrix}\end{aligned}$$

INS Mechanization

To remove the gravity from the acceleration

$$\mathbf{a}_{INS,t}^{local} = \mathbf{R}_{body}^{local}\mathbf{a}_{INS,t}^{body} - \mathbf{g}$$

To obtain the change of aircraft in terms of position, velocity and orientation

$$\boldsymbol{\delta}\mathbf{X}_{INS,t}^{local} = \mathbf{X}_{INS,t}^{local} - \mathbf{X}_{INS,t-1}^{local} = \mathbf{V}_{INS,t-1}^{local}\Delta t + \frac{\boldsymbol{a}_{INS,t}^{local}\Delta t^2}{2}$$

$$\boldsymbol{\delta}\mathbf{V}_{INS,t}^{local} = \mathbf{V}_{INS,t}^{local} - \mathbf{V}_{INS,t-1}^{local} = \boldsymbol{a}_{INS,t}^{local}\Delta t$$

$$\boldsymbol{\delta}\boldsymbol{\Psi}_{INS,t}^{local} = \boldsymbol{\Psi}_{INS,t}^{local} - \boldsymbol{\Psi}_{INS,t-1}^{local}$$

Kalman Filter——GNSS/INS(Open Loop)

System States:

$$\mathbf{X}_t = (\mathbf{X}_t^{local}, \mathbf{V}_t^{local}, \mathbf{\Psi}_t^{local})$$
$$\mathbf{X}_t^{local} = (x_t^{local}, y_t^{local}, z_t^{local})$$
$$\mathbf{V}_t^{local} = (vx_t^{local}, vy_t^{local}, vz_t^{local})$$
$$\mathbf{\Psi}_t^{local} = (\phi_{roll}, \theta_{pitch}, \psi_{yaw})$$

Propagation model:

$$\mathbf{X}_t^- = \mathbf{F}\mathbf{X}_{t-1}^+ + \mathbf{B}\mathbf{U}_t$$

$$\mathbf{F} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \quad \mathbf{U}_t = \begin{bmatrix} \delta\mathbf{X}_{INS,t}^{local} \\ \delta\mathbf{V}_{INS,t}^{local} \\ \delta\mathbf{\Psi}_{INS,t}^{local} \end{bmatrix}$$

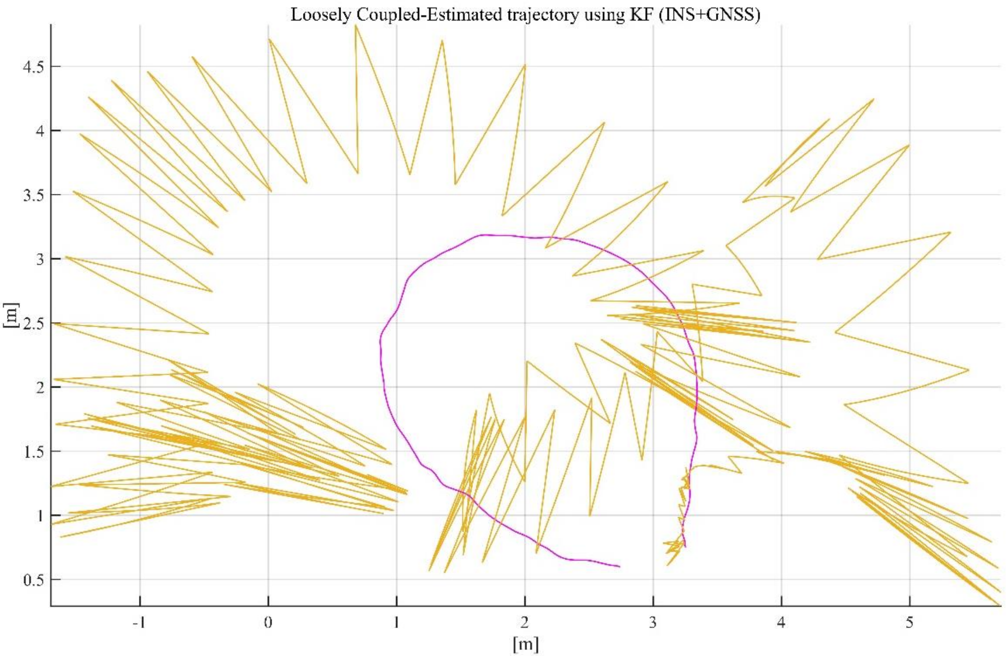
Measurement model:

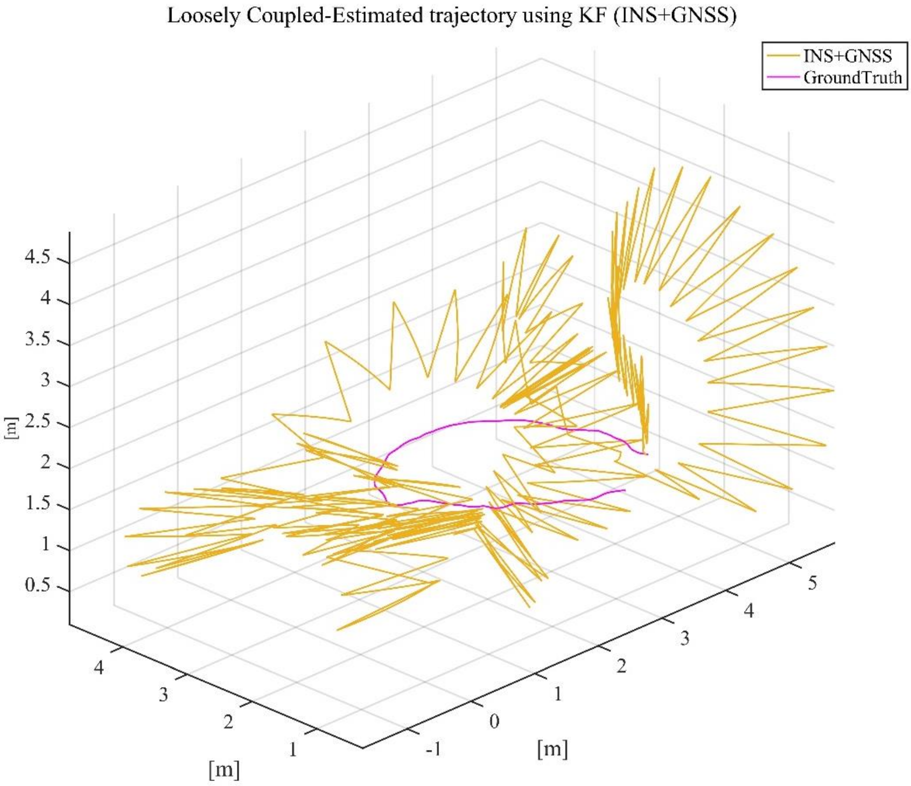
$$\Delta\mathbf{Z}_t = \mathbf{Z}_t - \mathbf{H}\mathbf{X}_t^-$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{3\times3} & \cdots & \mathbf{0} \\ \vdots & \mathbf{0} & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}$$

$$\mathbf{Z}_t = \mathbf{X}_{GNSS,t}^{local} = \begin{bmatrix} x_{GNSS,t}^{local} \\ y_{GNSS,t}^{local} \\ z_{GNSS,t}^{local} \end{bmatrix}$$

Open Loop


Loosely Coupled-Estimated trajectory using KF (INS+GNSS)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
[m]
-1
0
1
2
3
4
5
[m]


Loosely Coupled-Estimated trajectory using KF (INS+GNSS)
INS+GNSS
GroundTruth
4.5
4
3.5
3
2.5
2
1.5
1
0.5
[m]
4
3
2
1
[m]
-1
0
1
2
3
4
5
[m]

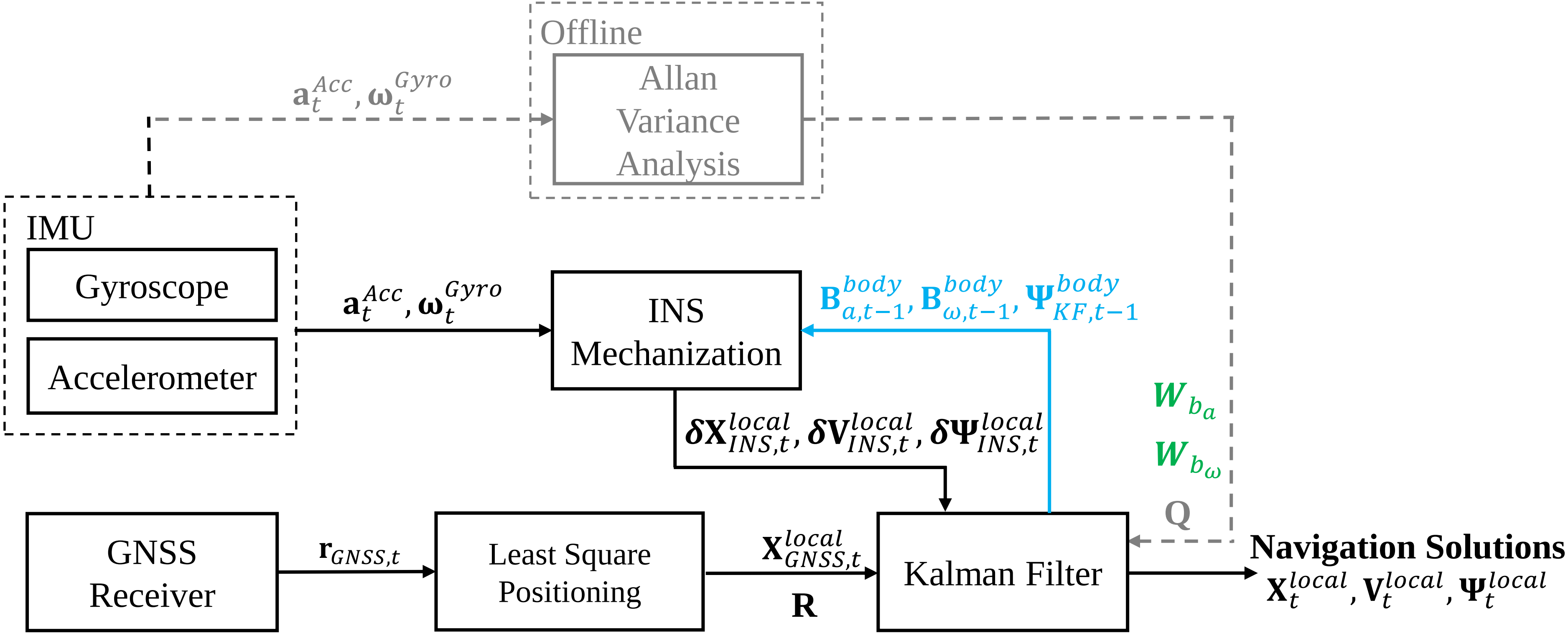
Closed-loop Correction

> The estimated position, velocity, and attitude errors are fed back to the inertial navigation processor, where they are used to correct the inertial navigation solution itself.

> any accelerometer and gyro errors estimated by the Kalman filter are fed back to correct the IMU measurements, as they are input to the inertial navigation equations.

> Unlike the position, velocity, and attitude corrections, the accelerometer and gyro corrections must be applied on every iteration

Closed Loop


Offline
Allan Variance Analysis
$\mathbf{a}_t^{Acc}, \boldsymbol{\omega}_t^{Gyro}$
IMU
Gyroscope
Accelerometer
$\mathbf{a}_t^{Acc}, \boldsymbol{\omega}_t^{Gyro}$
INS Mechanization
$\mathbf{B}_{a,t-1}^{body}, \mathbf{B}_{\omega,t-1}^{body}, \boldsymbol{\Psi}_{KF,t-1}^{body}$
$\delta\mathbf{X}_{INS,t}^{local}, \delta\mathbf{V}_{INS,t}^{local}, \delta\boldsymbol{\Psi}_{INS,t}^{local}$
W_{b_a}
W_{b_ω}
$\mathbf{Q}$
GNSS Receiver
$\mathbf{r}_{GNSS,t}$
Least Square Positioning
$\mathbf{X}_{GNSS,t}^{local}$
$\mathbf{R}$
Kalman Filter
Navigation Solutions
$\mathbf{X}_t^{local}, \mathbf{V}_t^{local}, \boldsymbol{\Psi}_t^{local}$

INS Mechanization

The Euler angle rates obtained by angular velocity:

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \dot{\boldsymbol{\Psi}}_{INS,t}^{body} = \begin{bmatrix}1 & sin(\phi)\,tan(\theta) & cos(\phi)\,tan(\theta)\\ 0 & cos(\phi) & -sin(\phi)\\ 0 & sin(\phi)\,sec(\theta) & cos(\phi)\,sec(\theta)\end{bmatrix}\left(\boldsymbol{\omega}_t^{Gyro} - \mathbf{B}_{\omega,t-1}^{body}\right)$$

$$\boldsymbol{\delta\Psi}_{INS,t}^{body} = \dot{\boldsymbol{\Psi}}_{INS,t}^{body}\Delta t = \begin{bmatrix}\delta\phi\\ \delta\theta\\ \delta\psi\end{bmatrix}$$

Update the current Euler angles

$$\boldsymbol{\Psi}_{INS,t}^{body} = \boldsymbol{\Psi}_{KF,t-1}^{body} + \boldsymbol{\delta\Psi}_{INS,t}^{body}$$

$$\boldsymbol{\Psi}_{KF,t-1}^{local} = \mathbf{R}_{body}^{local}\boldsymbol{\Psi}_{KF,t-1}^{body}$$

Rotate the Euler angles from Body to Local

$$\boldsymbol{\Psi}_{INS,t}^{local} = \mathbf{R}_{body}^{local}\boldsymbol{\Psi}_{INS,t}^{body}$$

$$\begin{aligned}\mathbf{R}_{body}^{local} &= \mathbf{R}(X,\phi)\mathbf{R}(Y,\theta)\mathbf{R}(Z,\psi)\\ &= \begin{bmatrix}1 & 0 & 0\\ 0 & cos(\phi) & -sin(\phi)\\ 0 & sin(\phi) & cos(\phi)\end{bmatrix}\begin{bmatrix}cos(\theta) & 0 & sin(\theta)\\ 0 & 1 & 0\\ -sin(\theta) & 0 & cos(\theta)\end{bmatrix}\begin{bmatrix}cos(\psi) & -sin(\psi) & 0\\ sin(\psi) & cos(\psi) & 0\\ 0 & 0 & 1\end{bmatrix}\end{aligned}$$

INS Mechanization

To remove the gravity from the acceleration

$$\mathbf{a}_{INS,t}^{local} = \mathbf{R}_{body}^{local}\left(\mathbf{a}_{INS,t}^{body} - \mathbf{B}_{a,t-1}^{body}\right) - \mathbf{g}$$

To obtain the change of aircraft in terms of position, velocity and orientation

$$\boldsymbol{\delta}\mathbf{X}_{INS,t}^{body} = \mathbf{V}_{INS,t-1}^{local}\Delta t + \frac{\boldsymbol{a}_{INS,t}^{local}\Delta t^2}{2}$$

$$\boldsymbol{\delta}\mathbf{V}_{INS,t}^{local} = \boldsymbol{a}_{INS,t}^{local}\Delta t$$

$$\boldsymbol{\delta\Psi}_{INS,t}^{local} = \boldsymbol{\Psi}_{KF,t-1}^{local} - \boldsymbol{\Psi}_{INS,t-1}^{local}$$

Kalman Filter——GNSS/INS (Closed Loop)

System States:

$$\mathbf{X}_t = \left(\mathbf{X}_t^{local}, \mathbf{V}_t^{local}, \mathbf{\Psi}_t^{local}, \mathbf{B}_{a,t}^{body}, \mathbf{B}_{\omega,t}^{body}\right)$$

$$\mathbf{X}_t^{local} = \left(x_t^{local}, y_t^{local}, z_t^{local}\right)$$

$$\mathbf{V}_t^{local} = \left(vx_t^{local}, vy_t^{local}, vz_t^{local}\right)$$

$$\mathbf{\Psi}_t^{local} = \left(\phi_{roll}, \theta_{pitch}, \psi_{yaw}\right)$$

$$\mathbf{B}_{a,t}^{body} = \left(b_{ax,t}^{body}, b_{ay,t}^{body}, b_{az,t}^{body}\right)$$

$$\mathbf{B}_{\omega,t}^{body} = \left(b_{\omega x,t}^{body}, b_{\omega y,t}^{body}, b_{\omega z,t}^{body}\right)$$

Measurement model:

$$\Delta\mathbf{Z}_t = \mathbf{Z}_t - \mathbf{H}\mathbf{X}_t^-$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \cdots & \mathbf{0} \\ \vdots & \mathbf{0} & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}$$

$$\mathbf{Z}_t = \mathbf{X}_{GNSS,t}^{local} = \begin{bmatrix} x_{GNSS,t}^{local} \\ y_{GNSS,t}^{local} \\ z_{GNSS,t}^{local} \end{bmatrix}$$

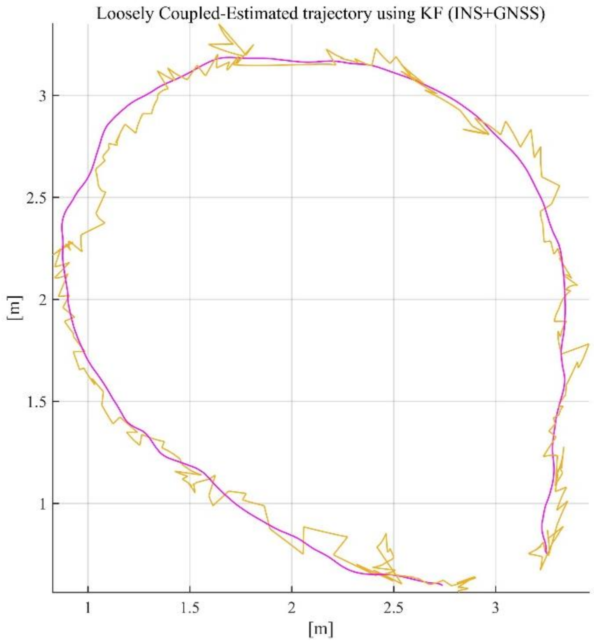
Propagation model:

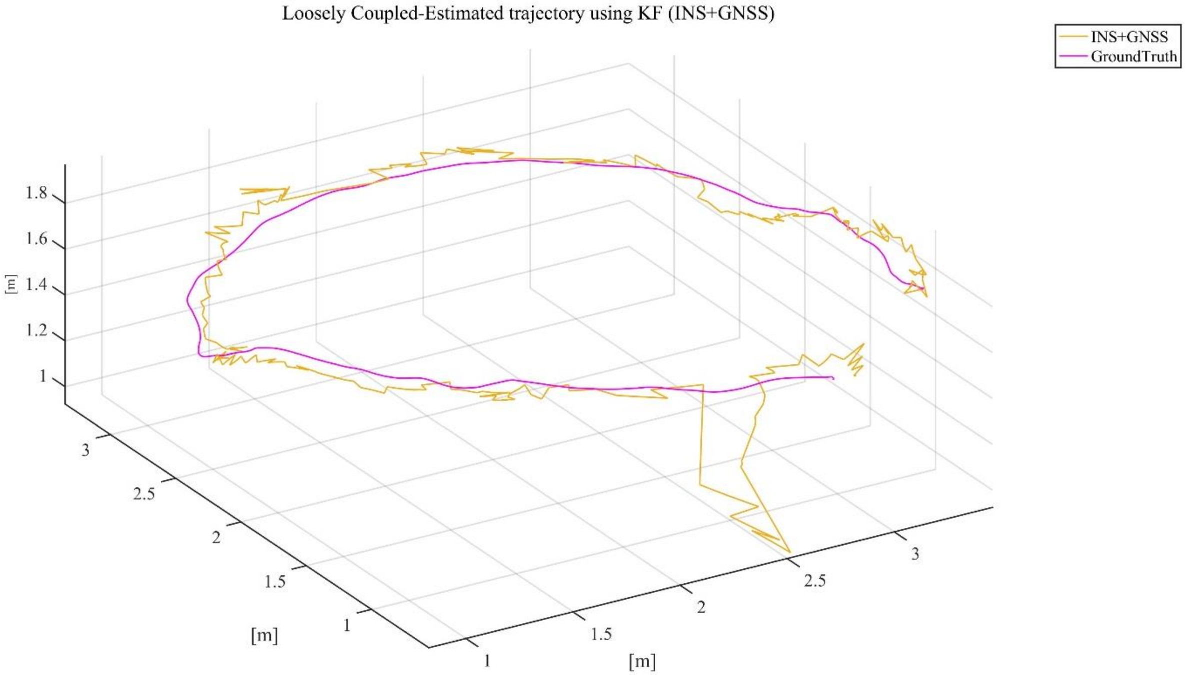
$$\mathbf{X}_t^- = \mathbf{F}\mathbf{X}_{t-1}^+ + \mathbf{B}\mathbf{U}_t$$

$$\mathbf{F} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$$

$$\mathbf{U}_t = \begin{bmatrix} \delta\mathbf{X}_{INS,t}^{local} \\ \delta\mathbf{V}_{INS,t}^{local} \\ \delta\mathbf{\Psi}_{INS,t}^{local} \\ W_{b_\omega} \\ W_{b_a} \end{bmatrix}$$

Closed Loop


Loosely Coupled-Estimated trajectory using KF (INS+GNSS)
3
2.5
2
1.5
1
[m]
1
1.5
2
2.5
3
[m]


Loosely Coupled-Estimated trajectory using KF (INS+GNSS)
INS+GNSS
GroundTruth
1.8
1.6
1.4
1.2
1
[m]
3
2.5
2
1.5
1
[m]
1
1.5
2
2.5
3
[m]

References

- Chapters 3 and 5, Paul D. Groves, *Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, 2nd Edition*, Artech House, 2013.