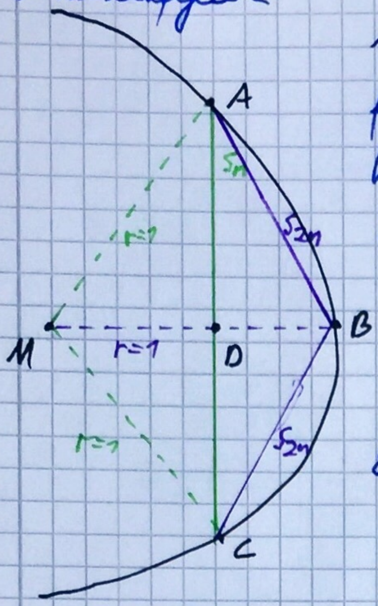
Bonusaufgabe



Man betrachte eine Kante s_n und füge die Ecke B ein, um s_n in 2 Kanten zu unterteilen.

Es gilt: $\overline{AD}^2 + \overline{MD}^2 = 1$

$$\Leftrightarrow \overline{MD} = \sqrt{1 - \left(\frac{s_n}{2}\right)^2} = \frac{1}{2}\sqrt{4 - s^2}$$

Es gilt: $\overline{AD}^2 + \overline{DB}^2 = AB^2$

$$\Leftrightarrow \overline{DB} = \sqrt{s_{2n}^2 - \left(\frac{s_n}{2}\right)^2} = \frac{1}{2}\sqrt{4s_{2n}^2 - s_n^2}$$

Mit $\overline{MD} + \overline{DB} = 1$:

$$\frac{1}{2}\sqrt{4 - s_n^2} + \frac{1}{2}\sqrt{4s_{2n}^2 - s_n^2} = 1$$

$$\Leftrightarrow 2 - \sqrt{4 - s_n^2} = \sqrt{4s_{2n}^2 - s_n^2}$$

$$\Rightarrow 4 - 4\sqrt{4 - s_n^2} + 4 - s_n^2 = 4s_{2n}^2 - s_n^2$$

$$\Leftrightarrow s_{2n}^2 = 2 \cancel{4} - \sqrt{4 - s_n^2}$$

$$\Rightarrow s_{2n} = \sqrt{2 - \sqrt{4 - s_n^2}}$$

$$t_{2n} = \frac{2s_{2n}}{\sqrt{4 - s_{2n}^2}} \overset{(s.o.)}{\Leftrightarrow} \frac{2\sqrt{2 - \sqrt{4 - s_n^2}}}{\sqrt{4 - (2 - \sqrt{4 - s_n^2})}}$$

$\overset{(1)}{=}$ n. Seite

$$t_n = \frac{2s_n}{\sqrt{4 - s_n^2}}$$

$$\Leftrightarrow t_n^2 (4 - s_n^2) = 4 s_n^2$$

$$\Leftrightarrow 4t_n^2 = 4s_n^2 + t_n^2 s_n^2$$

$$\Leftrightarrow (2t_n)^2 = (4 + t_n^2) s_n^2$$

$$\Leftrightarrow s_n = \frac{2t_n}{\sqrt{4 + t_n^2}} \quad (1)$$

$$= \frac{2\sqrt{2-\sqrt{4-\frac{4t_n^2}{4+t_n^2}}}}{\sqrt{2+\sqrt{4-\frac{4t_n^2}{4+t_n^2}}}}$$

$$= \frac{2\sqrt{2^2-4-\frac{4t_n^2}{4+t_n^2}}}{2+\sqrt{4-\frac{4t_n^2}{4+t_n^2}}}$$

$$= \frac{2\,\frac{2t_n}{\sqrt{4+t_n^2}}}{2+\sqrt{4-\frac{4t_n^2}{4+t_2^2}}}$$

$$= \frac{2t_n}{\left(1+\sqrt{1-\frac{t_n^2}{4+t_n^2}}\right)\sqrt{4+t_n^2}}$$

$$= \frac{2t_n}{\sqrt{4+t_n^2}+\sqrt{4+t_n^2-\frac{4t_n^2}{4+t_n^2}-\frac{t_n^4}{4+t_n^2}}}$$

$$= \frac{2t_n}{\sqrt{4+t_n^2}+\sqrt{4}}$$

$$= \frac{2t_n}{\sqrt{4+t_n^2}+2} \quad \blacksquare$$