In[ ]:=

Out[ ]=

In[323]:=

Setup

goodlabel

In[324]:=
```
goodlabel::usage =
  "Evaluate[goodlabel[xlabel,ylabel, xstyle, ystyle]] makes plot labels
    with the desired style. Labels should be passed as strings.";
goodlabel[x_, y_] := {Frame → {{True, False}, {True, False}}, FrameLabel → {x, y}};
goodlabel[x_, y_, style_] := goodlabel[x, y, style, style];
goodlabel[x_, y_, xstyle_, ystyle_] := {Frame → {{True, False}, {True, False}},
  FrameLabel → {Style[x, xstyle], Style[y, ystyle]}};
```

sims

In[ ]:=
```
(* Note that scale parameter c in this data set
 represents the scale parameter for each of two lineages*)
```

In[328]:=
```
sims =
  SemanticImport[FileNameJoin[{NotebookDirectory[], "MH_2D_discrete.txt"}]];
```

In[329]:=
```
sims = sims[All, <|#, "dhomLow" → #hom - #homLow, "dhomHigh" → #homHigh - #hom|> &];
```

In[330]:=
```
αlist = sims[All, #alpha &] // DeleteDuplicates // Normal // Sort;
coallist = sims[All, #coal &] // DeleteDuplicates // Normal // Sort;
μlist = sims[All, #mu &] // DeleteDuplicates // Normal // Sort;
```

In[•]:=
```
sims[All, #c &]
```

Out[•]=

10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0

showing 1–120 of 408

In[•]:=
```
sims[All, #mu &]
```

Out[•]=

0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	1.0

showing 1–120 of 408

In[•]:=
```
μlist
```

Out[•]=
```
{0.001, 0.01, 0.1, 1.}
```

In[ ]:=
```
sims[All, #mu &]
```

Out[ ]=

0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	0.001	0.01	0.1
0.001	0.01	0.1	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	0.001
0.01	0.1	0.001	0.01	0.1	1.0

showing 1–120 of 408

In[336]:=
```
clist = sims[All, #c &] // DeleteDuplicates // Normal // Sort;
```

In[337]:=
```
cdfsims = SemanticImport[FileNameJoin[{NotebookDirectory[], "CDF_data.txt"}]];
```

SemanticImport: File not found during SemanticImport.

In[338]:=
```
cdfsims =
  cdfsims [All, <|#, "dcdfLow" → #cdf - #cdfLow, "dcdfHigh" → #cdfHigh - #cdf|> &];
```

functions

In[339]:=
```
fLT[x_, μ_, α_, Dα_] := NIntegrate[(Cos[k x] / π)/(μ + Dα k^α), {k, 0, ∞}];
fLT[x_, μ_, α_] := fLT[x, μ, α, 10^α];
```

In[341]:=
```
coeff[c_, μ_, α_, Dα_] := 1 / (2/c + (μ/Dα)^(1/α) / (α Sin[π/α] μ));
coeff[c_, μ_, α_] := coeff[c, μ, α, 10^α];
```

In[343]:=
```
xscale[μ_, α_, Dα_] := (Dα/μ)^(1/α);
xscale[μ_, α_] := xscale[μ, α, 10^α];
```

In[345]:=
```
homscale[coal_, μ_, α_, Dα_] := (1/π) / (Csc[π/α]/α + 2 (Dα/μ)^(2/α) μ/coal);
homscale[coal_, μ_, α_] := homscale[coal, μ, α, 10^α/2];
```

In[347]:=
```
approxhomscale[coal_, μ_, α_, Dα_] := coal / (4 * Pi (Dα/μ)^(2/α) μ);
approxhomscale[coal_, μ_, α_] := approxhomscale[coal, μ, α, 10^α];
```

In[349]:=
```
Series[homscale[1/ρ, μ, α, Dα], {α, 1, 1}]
```

Out[•]= $(\alpha - 1) + O[\alpha - 1]^2$

In[350]:=

In[351]:=
```
xlist = 10^Range[-14,8.5,.25];
```

In[352]:=
```
xlist
```

Out[•]=
$\{1. \times 10^{-14}, 1.77828 \times 10^{-14}, 3.16228 \times 10^{-14}, 5.62341 \times 10^{-14}, 1. \times 10^{-13},$
$1.77828 \times 10^{-13}, 3.16228 \times 10^{-13}, 5.62341 \times 10^{-13}, 1. \times 10^{-12}, 1.77828 \times 10^{-12},$
$3.16228 \times 10^{-12}, 5.62341 \times 10^{-12}, 1. \times 10^{-11}, 1.77828 \times 10^{-11}, 3.16228 \times 10^{-11},$
$5.62341 \times 10^{-11}, 1. \times 10^{-10}, 1.77828 \times 10^{-10}, 3.16228 \times 10^{-10}, 5.62341 \times 10^{-10},$
$1. \times 10^{-9}, 1.77828 \times 10^{-9}, 3.16228 \times 10^{-9}, 5.62341 \times 10^{-9}, 1. \times 10^{-8}, 1.77828 \times 10^{-8},$
$3.16228 \times 10^{-8}, 5.62341 \times 10^{-8}, 1. \times 10^{-7}, 1.77828 \times 10^{-7}, 3.16228 \times 10^{-7}, 5.62341 \times 10^{-7},$
$1. \times 10^{-6}, 1.77828 \times 10^{-6}, 3.16228 \times 10^{-6}, 5.62341 \times 10^{-6}, 0.00001, 0.0000177828,$
$0.0000316228, 0.0000562341, 0.0001, 0.000177828, 0.000316228, 0.000562341,$
$0.001, 0.00177828, 0.00316228, 0.00562341, 0.01, 0.0177828, 0.0316228,$
$0.0562341, 0.1, 0.177828, 0.316228, 0.562341, 1., 1.77828, 3.16228, 5.62341,$
$10., 17.7828, 31.6228, 56.2341, 100., 177.828, 316.228, 562.341, 1000., 1778.28,$
$3162.28, 5623.41, 10\,000., 17\,782.8, 31\,622.8, 56\,234.1, 100\,000., 177\,828.,$
$316\,228., 562\,341., 1. \times 10^{6}, 1.77828 \times 10^{6}, 3.16228 \times 10^{6}, 5.62341 \times 10^{6}, 1. \times 10^{7},$
$1.77828 \times 10^{7}, 3.16228 \times 10^{7}, 5.62341 \times 10^{7}, 1. \times 10^{8}, 1.77828 \times 10^{8}, 3.16228 \times 10^{8}\}$

In[353]:=
```
αlistnew = {.5, 1.0, 1.25, 1.5, 2.0}
```

Out[•]= {0.5, 1., 1.25, 1.5, 2.}

In[354]:=
```
intvals = Quiet[Association @@ Table[
    α -> Association @@ Table[x -> NIntegrate[(k * BesselJ[0, k x] * Exp[-10^-16 * k^2])/(1 + k^α),
        {k, 0, ∞}], {x, xlist}], {α, αlistnew[[ ;; ]]}]];
```

In[355]:=

In[356]:=
```
xmaxes = AssociationThread[αlistnew, {100 000, 100 000, 100 000, 100 000, 100 000}];
```

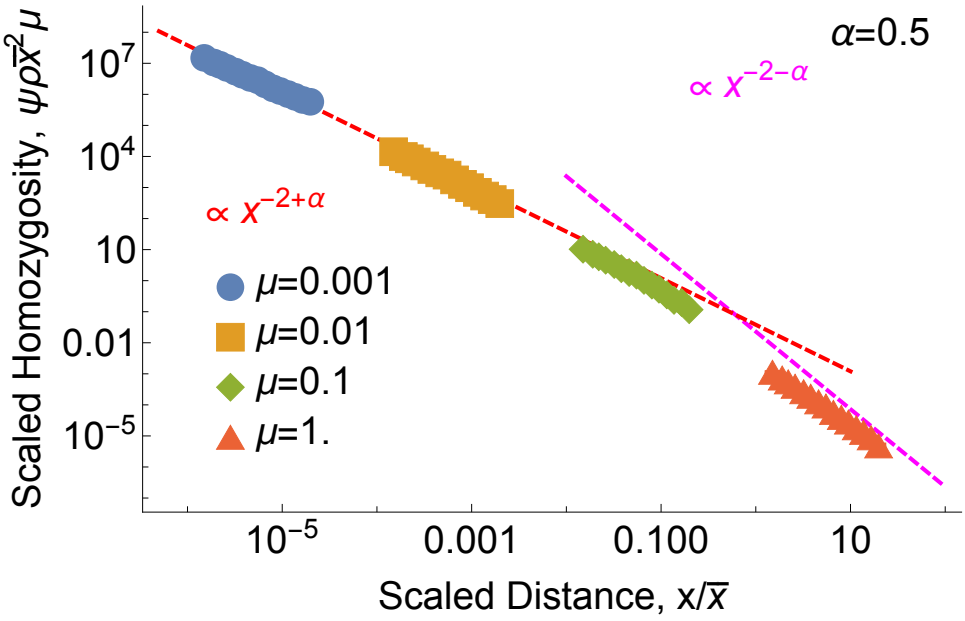
Plots

α=0.5

In[357]:=

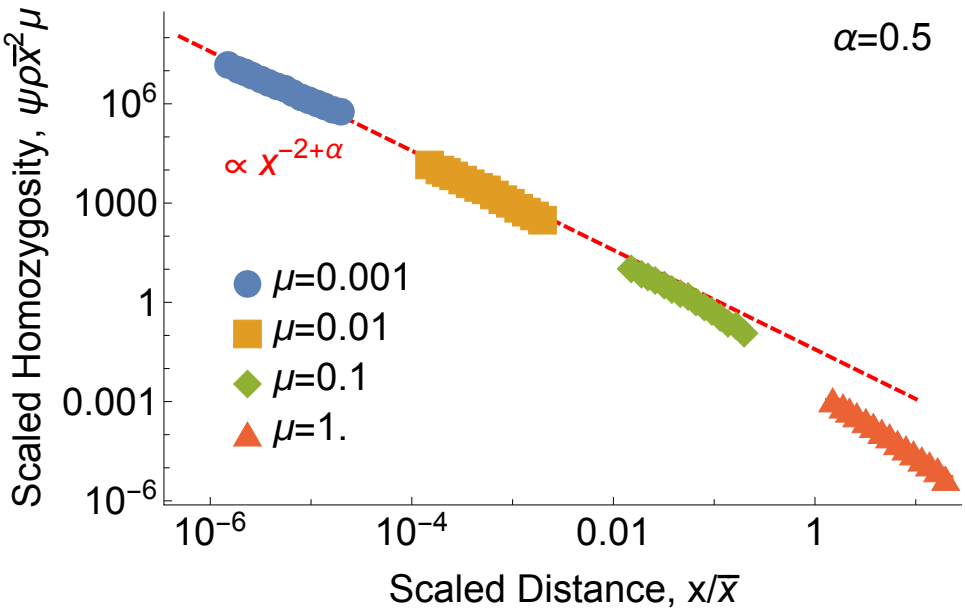
In[•]:=
```
Module[{α = 0.5, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
     ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
      approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[
  LogLogPlot[(0.724) * (2^-α * Gamma[α + 1]/Gamma[α/2 + 1]^2) *
    Sin[Pi * α/2] * Gamma[α/2 + 1]^2 * (2^(1-α) * Pi^2)^-1 * x^(-2-α),
   {x, .01, 100}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[Gamma[1 - α/2] * (Gamma[α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, .0000005, 10}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .6}]],
    Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["α=0.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



In[•]:=
```
Module[{α = 0.5, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[
  LogLogPlot[Gamma[1 - α/2] * (Gamma[α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, .0000005, 10}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .7}]],
    Text[Style["α=0.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

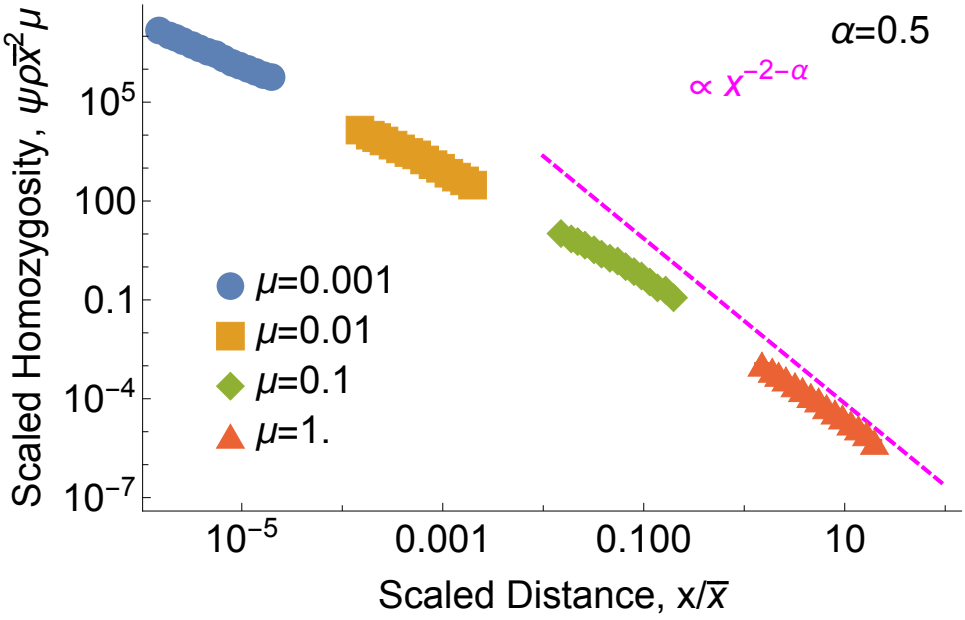
Out[•]=



In[363]:=

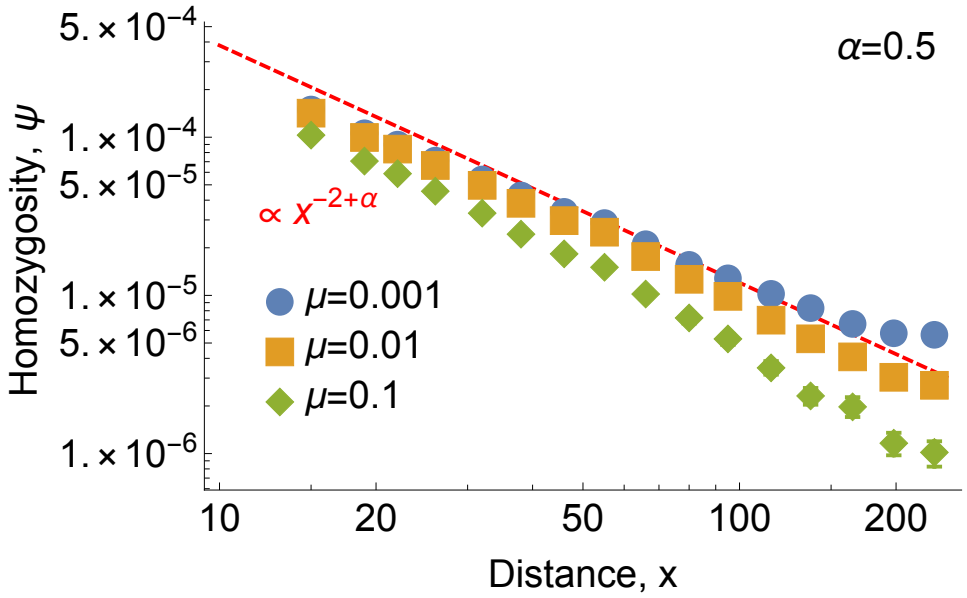
In[•]:=
```
Module[{α = 0.5, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 220 &]], {μ, μvals}];
 Show[
  LogLogPlot[(0.724) * (2^-α * Gamma[α + 1]/Gamma[α/2 + 1]^2) *
    Sin[Pi * α/2] * Gamma[α/2 + 1]^2 * (2^(1-α) * Pi^2)^-1 * x^(-2-α),
   {x, .01, 100}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["α=0.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄²μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



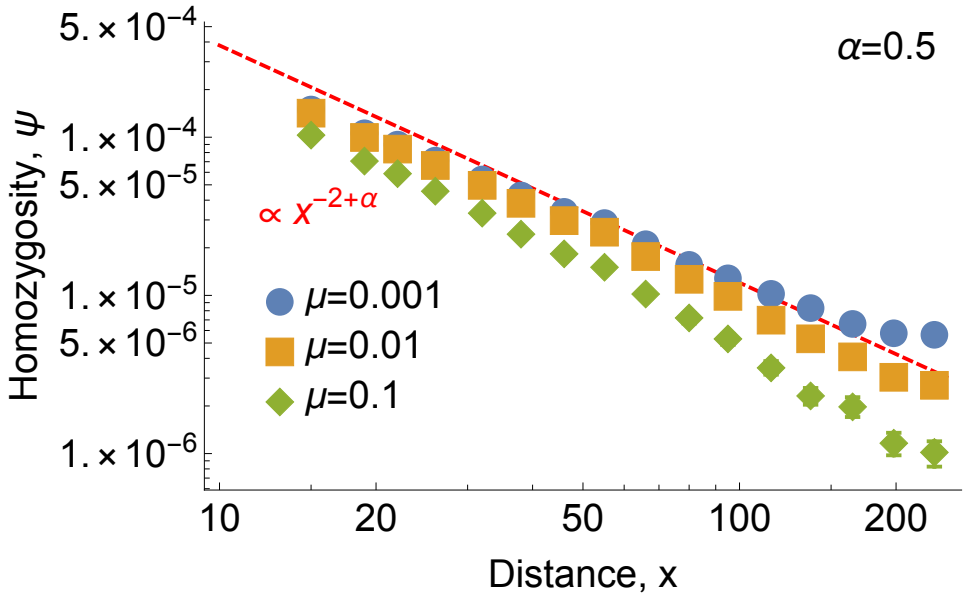
In[•]:=
```
Module[{α = 0.5, coal = 1, plotpoints, μvals = μlist[[1 ;; 3]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@
    sims[Select[#alpha == α && #coal == coal && #c == 10 &&
       #mu == μ && #x0 > 0 && #x0 < 250 &]], {μ, μvals}];
 Show[ LogLogPlot[coal * ((10) ^ .5)^-1 * Gamma[ 1 - α / 2] * (Gamma[ α / 2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, 10, 250}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .6}]],
    Text[Style["α=0.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Distance, x", " Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



In[•]:=
```
Module[{α = 0.5, coal = 1, plotpoints, μvals = μlist[[1 ;; 3]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@
    sims[Select[#alpha == α && #coal == coal && #c == 10 &&
       #mu == μ && #x0 > 0 && #x0 < 250 &]], {μ, μvals}];
 Show[ LogLogPlot[coal * (10^.5)^-1 * Gamma[ 1 - α/2] * (Gamma[ α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, 10, 250}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .6}]],
    Text[Style["α=0.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Distance, x", "Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=

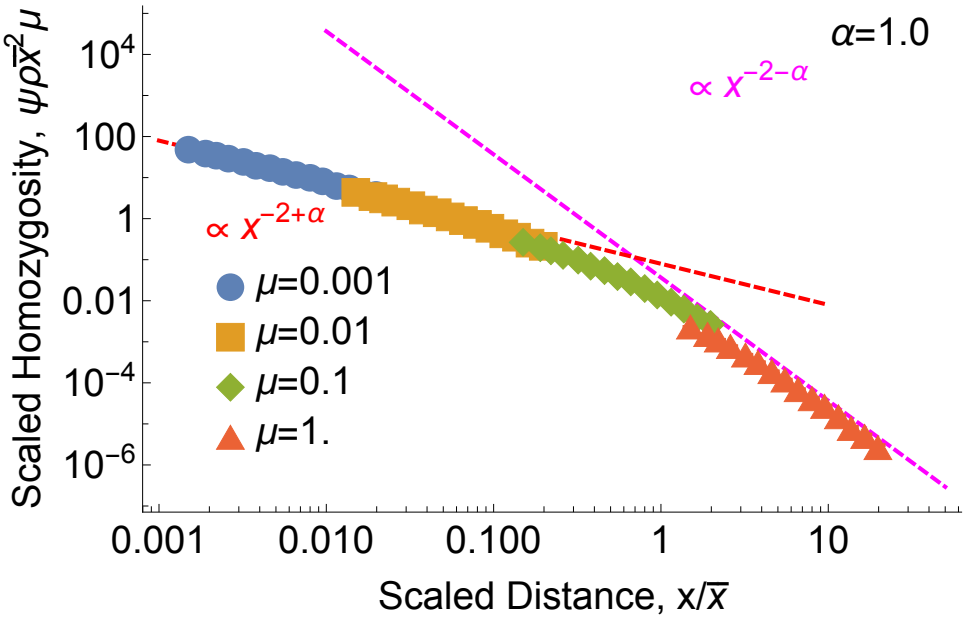


α=1.0

In[367]:=

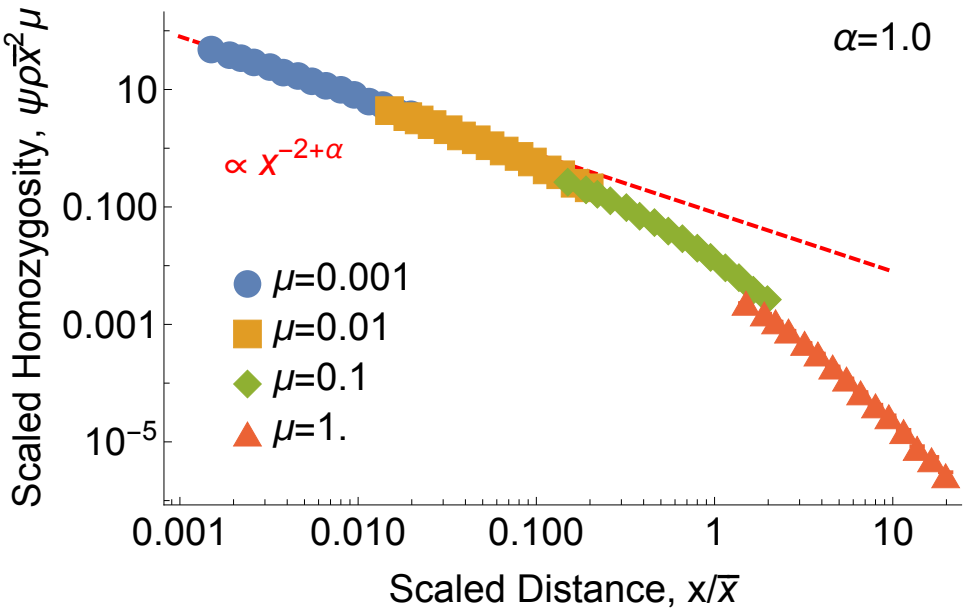
In[•]:=
```
Module[{α = 1.0, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[
  LogLogPlot[(0.724) * (2^-α * Gamma[α + 1]/Gamma[α/2 + 1]^2) *
    Sin[Pi * α/2] * Gamma[α/2 + 1]^2 * (2^(1-α) * Pi^2)^-1 * x^(-2-α),
   {x, .01, 50}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[Gamma[1 - α/2] * (Gamma[α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, .001, 10}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .57}]],
    Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["α=1.0", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



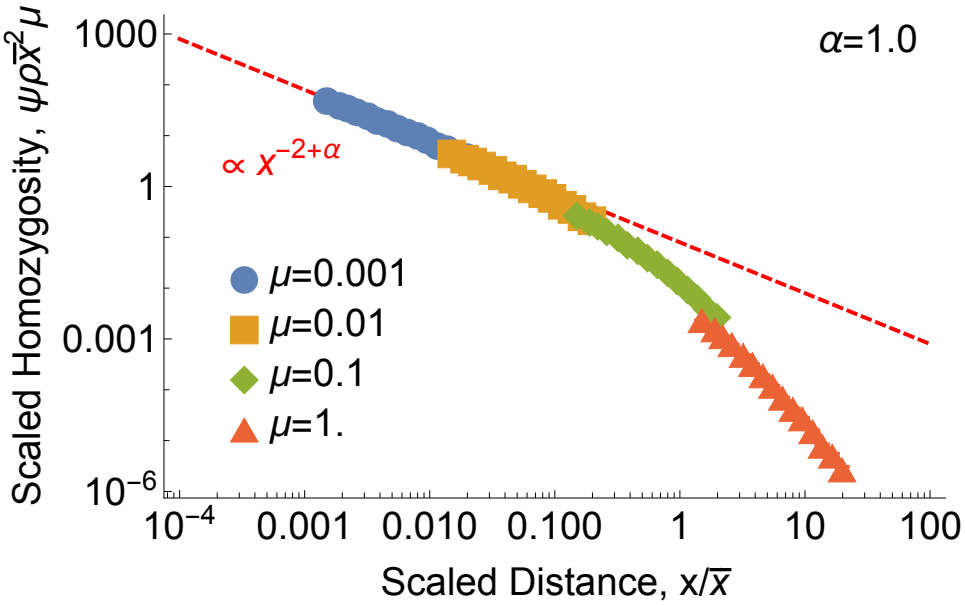
In[•]:=
```
Module[{α = 1.0, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[
  LogLogPlot[Gamma[1 - α/2] * (Gamma[α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, .001, 10}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .7}]],
    Text[Style["α=1.0", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



In[•]:=
```
Module[{α = 1.0, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[
  LogLogPlot[Gamma[1 - α/2] * (Gamma[α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, .0001, 100}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .7}]],
    Text[Style["α=1.0", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

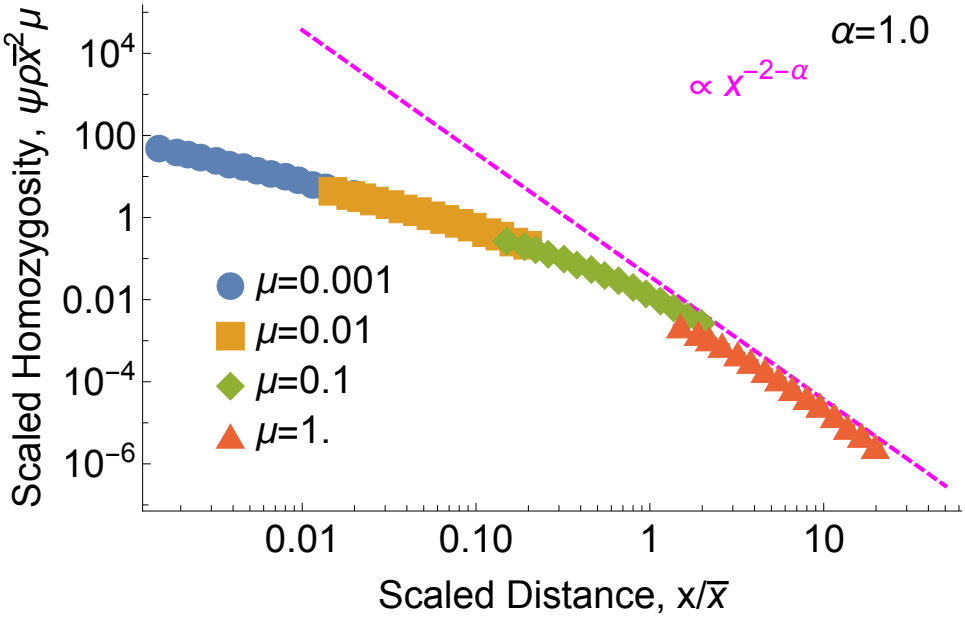
Out[•]=



In[•]:=

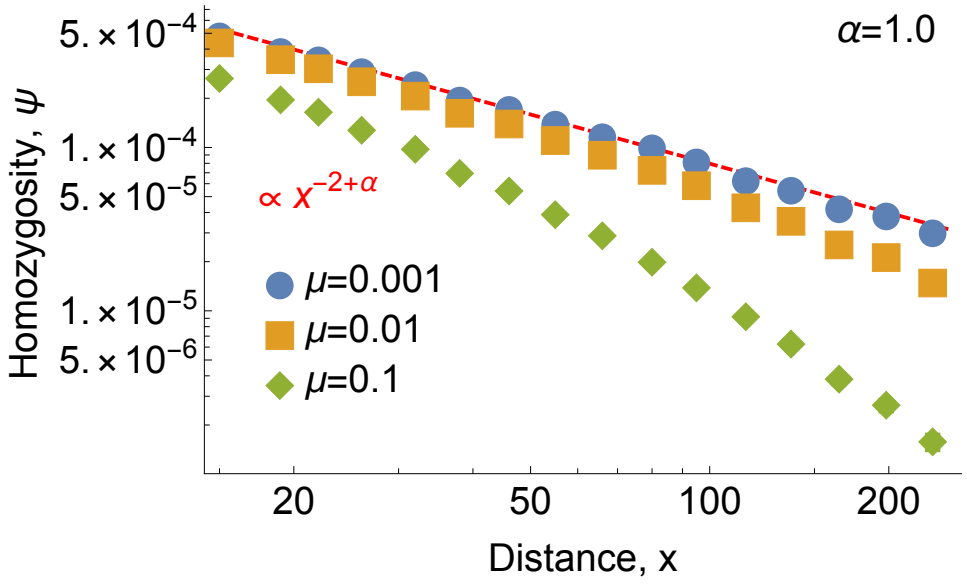
In[•]:=
```
Module[{α = 1.0, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[
  LogLogPlot[(0.724) * (2^-α * Gamma[α + 1]/Gamma[α/2 + 1]^2) *
    Sin[Pi * α/2] * Gamma[α/2 + 1]^2 * (2^(1-α) * Pi^2)^-1 * x^(-2-α),
   {x, .01, 50}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["α=1.0", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



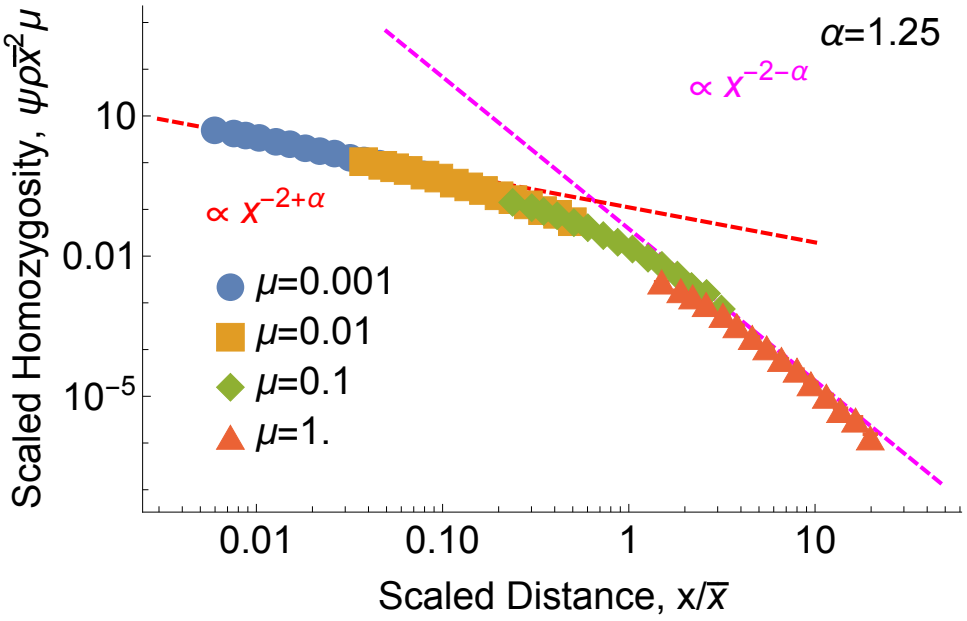
In[•]:=
```
Module[{α = 1.0, coal = 1, plotpoints, μvals = μlist[[1 ;; 3]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@
    sims[Select[#alpha == α && #coal == coal && #c == 10 &&
       #mu == μ && #x0 > 0 && #x0 < 250 &]], {μ, μvals}];
 Show[ LogLogPlot[coal * ((10) ^ 1)^-1 * Gamma[ 1 - α / 2] * (Gamma[ α / 2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, 15, 250}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .6}]],
    Text[Style["α=1.0", 20], Scaled[{.9, .95}]]},
  goodlabel["Distance, x", " Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



In[•]:=
```
Module[{α = 1.25, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
     ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
      approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[
  LogLogPlot[(0.724) * (2^-α * Gamma[α + 1]/Gamma[α/2 + 1]^2) *
    Sin[Pi * α/2] * Gamma[α/2 + 1]^2 * (2^(1-α) * Pi^2)^-1 * x^(-2-α),
   {x, .05, 50}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[Gamma[1 - α/2] * (Gamma[α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, .003, 10}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .6}]],
    Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["α=1.25", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

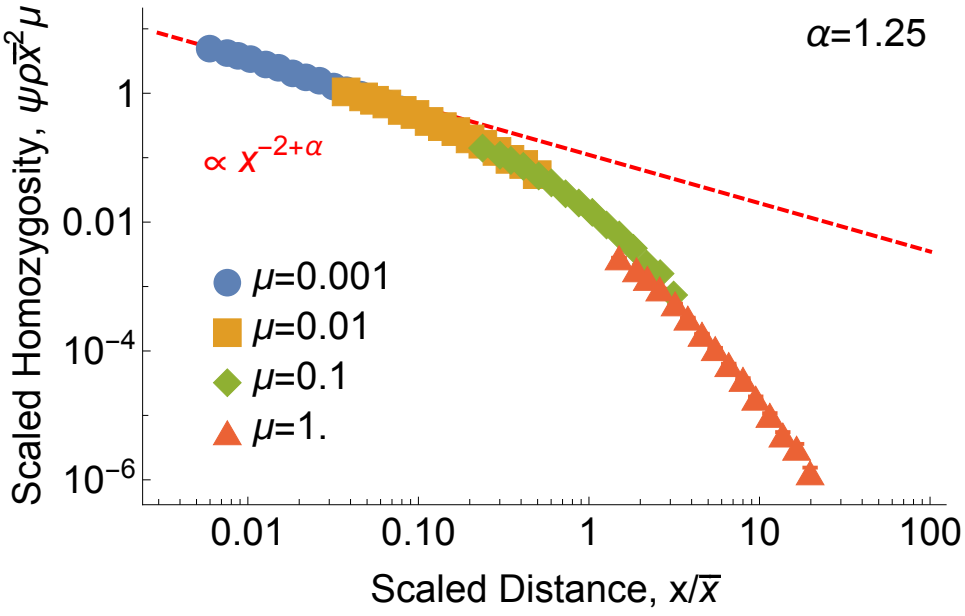
Out[•]=



In[•]:=

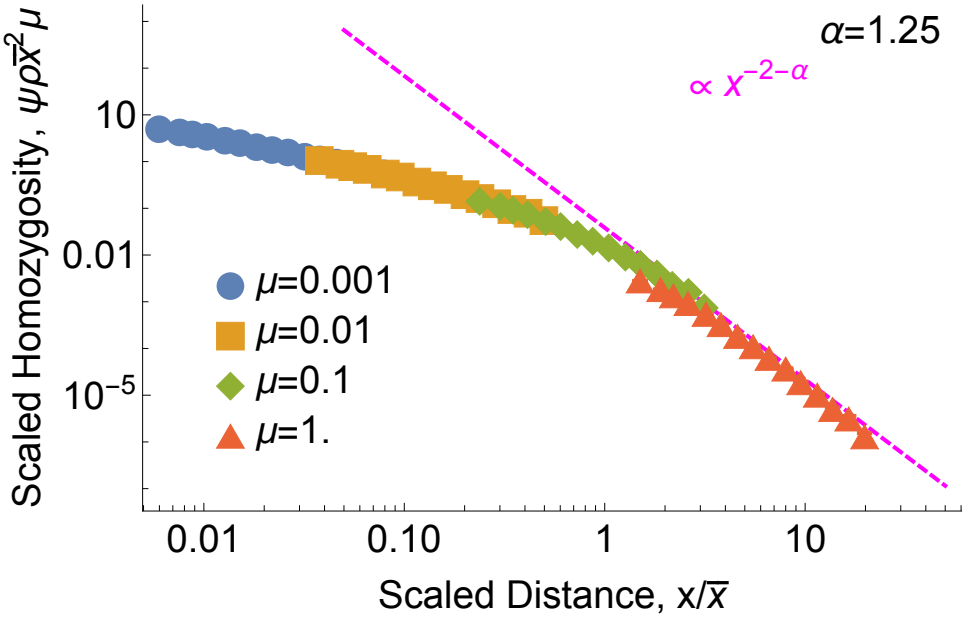
In[•]:=
```
Module[{α = 1.25, coal = 1, plotpoints, μvals = μlist[[1 ;;]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 & ]], {μ, μvals}];
 Show[
  LogLogPlot[Gamma[1 - α/2] * (Gamma[α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, .003, 100}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .7}]],
    Text[Style["α=1.25", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



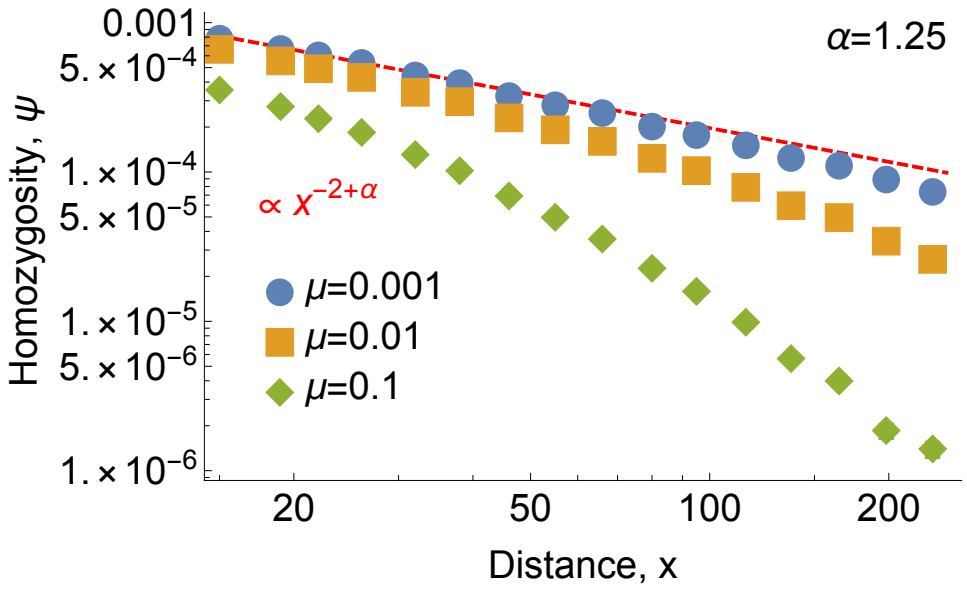
In[•]:=
```
Module[{α = 1.25, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[
  LogLogPlot[(0.724) * (2^-α * Gamma[α + 1]/Gamma[α/2 + 1]^2) *
    Sin[Pi * α/2] * Gamma[α/2 + 1]^2 * (2^(1-α) * Pi^2)^-1 * x^(-2-α),
   {x, .05, 50}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["α=1.25", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



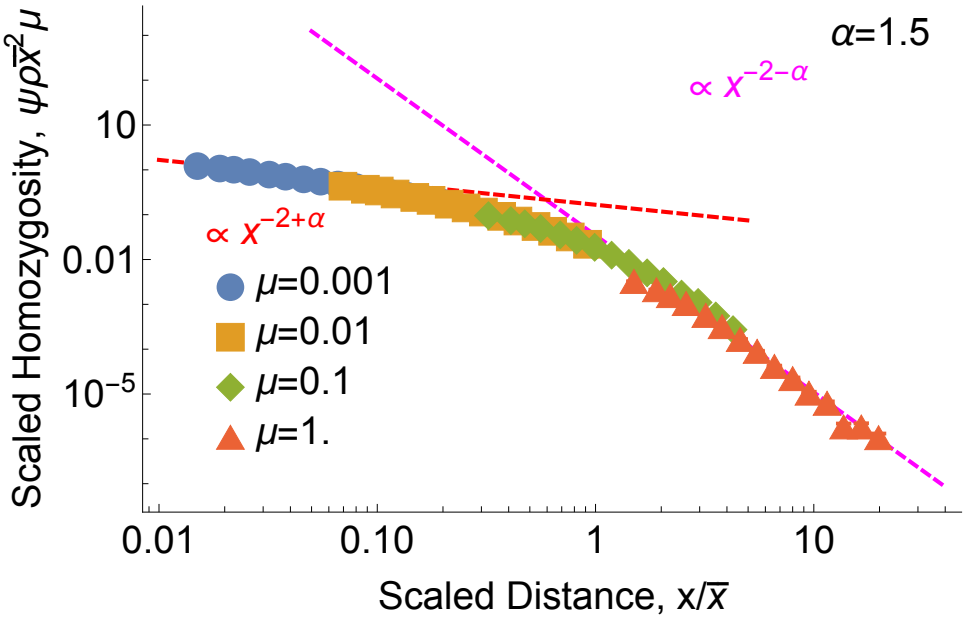
In[•]:=
```
Module[{α = 1.25, coal = 1, plotpoints, μvals = μlist[[1 ;; 3]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@
    sims[Select[#alpha == α && #coal == coal && #c == 10 &&
       #mu == μ && #x0 > 0 && #x0 < 250 &]], {μ, μvals}];
 Show[ LogLogPlot[coal * ((10) ^ 1.25)^-1 * Gamma[ 1 - α / 2] * (Gamma[ α / 2] * 2^(α+1) * Pi)^-1 *
    x^(-2+α), {x, 15, 250}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .6}]],
    Text[Style["α=1.25", 20], Scaled[{.9, .95}]]},
  goodlabel["Distance, x", " Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



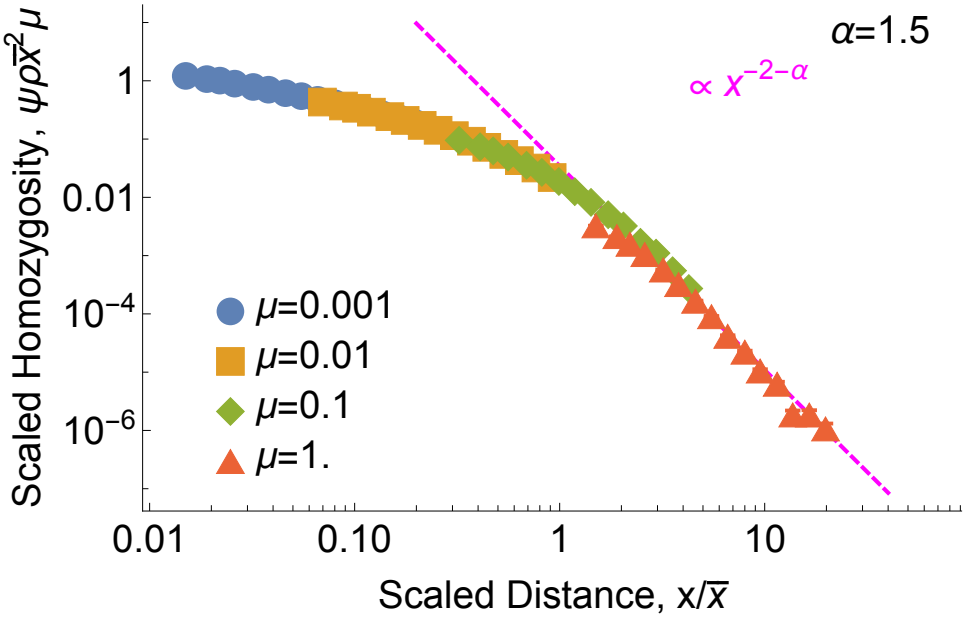
In[•]:=
```
Module[{α = 1.5, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 & ]], {μ, μvals}];
 Show[
  LogLogPlot[(0.724) * (2^-α * Gamma[α + 1]/Gamma[α/2 + 1]^2) *
    Sin[Pi * α/2] * Gamma[α/2 + 1]^2 * (2^(1-α) * Pi^2)^-1 * x^(-2-α),
   {x, .05, 40}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[Gamma[1 - α/2] * (Gamma[α/2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, .01, 5}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .56}]],
    Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["α=1.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



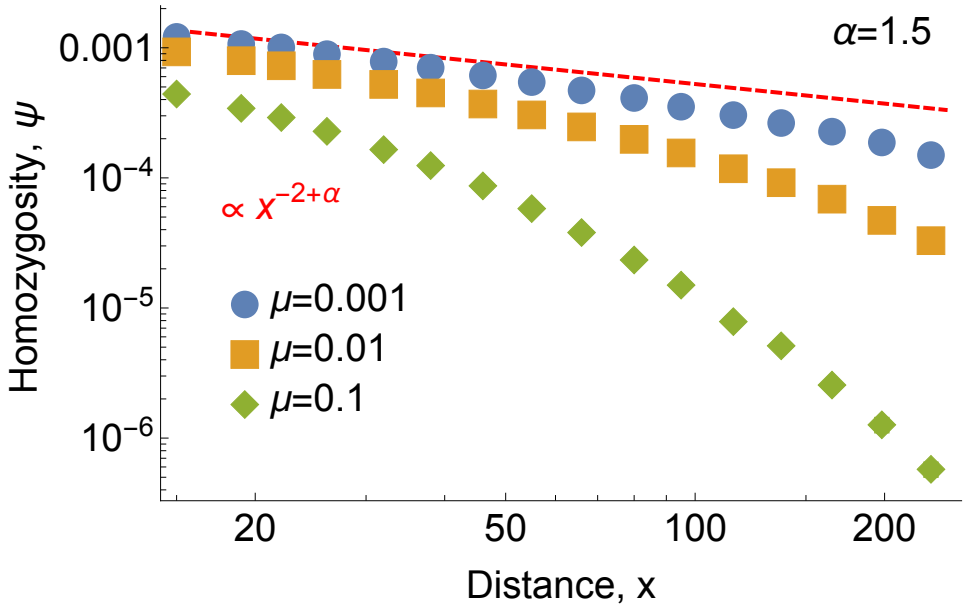
In[•]:=
```
Module[{α = 1.5, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 & ]], {μ, μvals}];
 Show[
  LogLogPlot[(0.724) * (2^-α * Gamma[α + 1]/Gamma[α/2 + 1]^2) *
    Sin[Pi * α/2] * Gamma[α/2 + 1]^2 * (2^(1-α) * Pi^2)^-1 * x^(-2-α),
   {x, .2, 40}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .25}]],
  Epilog → {Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["α=1.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → {{-4.5, 4.33}, {-16, 2}}, Axes → False]]
```

Out[•]=



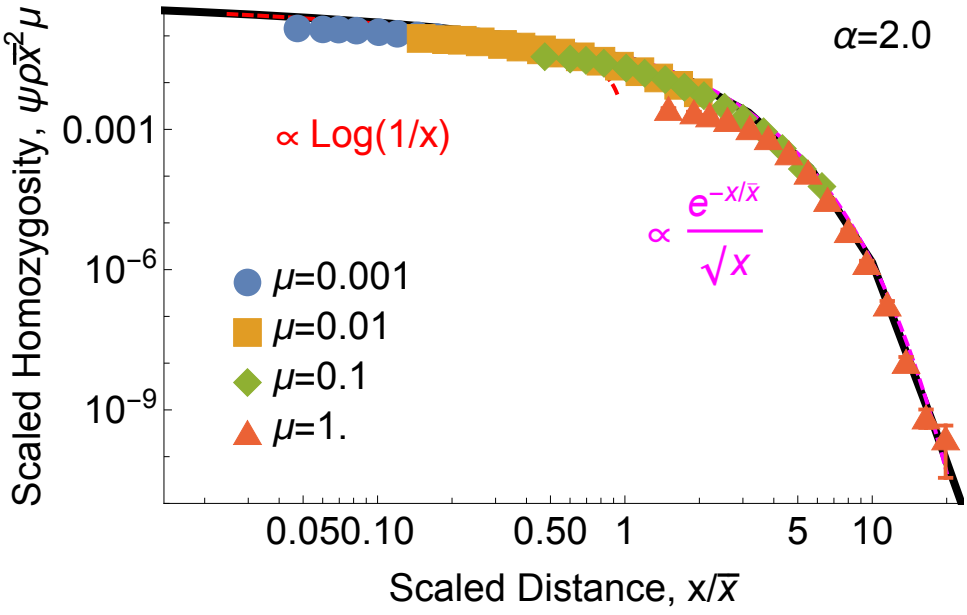
In[•]:=
```
Module[{α = 1.5, coal = 1, plotpoints, μvals = μlist[[1 ;; 3]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@
    sims[Select[#alpha == α && #coal == coal && #c == 10 &&
       #mu == μ && #x0 > 0 && #x0 < 250 &]], {μ, μvals}];
 Show[ LogLogPlot[coal * ((10) ^ 1.5)^-1 * Gamma[ 1 - α / 2] * (Gamma[ α / 2] * 2^(α+1) * Pi)^-1 * x^(-2+α),
   {x, 15, 250}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-2+α)", Red, 20], Scaled[{.15, .6}]],
    Text[Style["α=1.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Distance, x", " Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



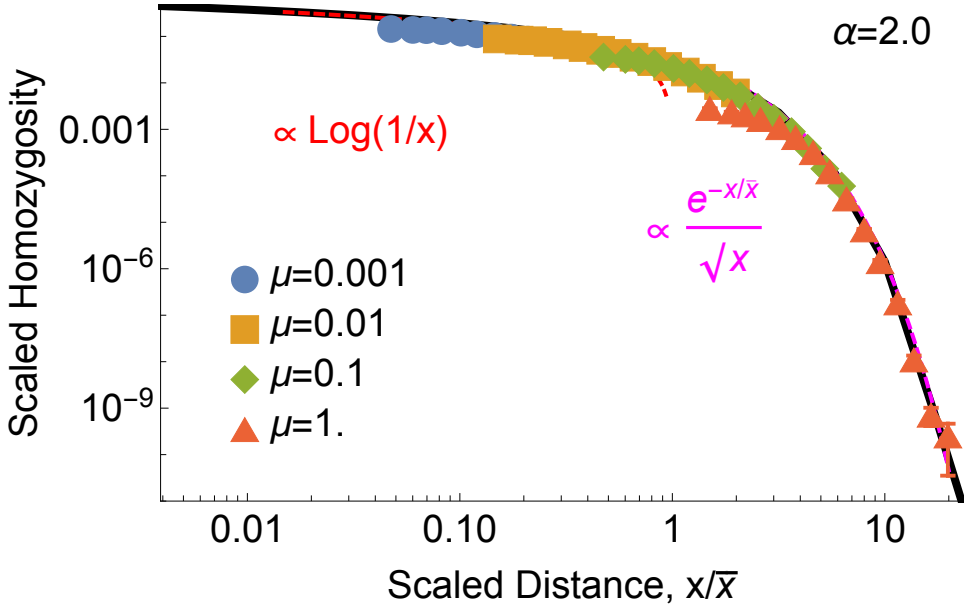
In[•]:=
```
Module[{α = 2.0, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    ((4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}])/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 && #mu == μ &&
      #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (4 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]}
   , PlotRange → {{10^-3, 10 000}, {10^-11, .35}}],
  LogLogPlot[(4 * Sqrt[2 * Pi])^-1 * Exp[-x]/Sqrt[x], {x, 1, 20},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], LogLogPlot[(4 * Pi)^-1 * Log[1/x],
   {x, .025, 100 000}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ Log(1/x)", Red, 20], Scaled[{.25, .75}]],
    Text[Style["∝ e^(-x/x̄)/√x", Magenta, 20], Scaled[{.68, .55}]],
    Text[Style["α=2.0", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



In[•]:=
```
Module[{α = 2.0, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha],
    (4 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/approxhomscale[#coal, #mu, #alpha, #c^#alpha]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == 10 &&
      #mu == μ && #x0 > 0 && #x0 < 200 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (4 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]}
   , PlotRange → {{10^-3, 10 000}, {10^-11, .45}}],
  LogLogPlot[(4 * Sqrt[2 * Pi])^-1 * Exp[-x] / Sqrt[x], {x, 1, 20},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], LogLogPlot[(4 * Pi)^-1 * Log[1 / x],
   {x, .015, 100 000}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ Log(1/x)", Red, 20], Scaled[{.25, .75}]],
    Text[Style["∝ e^(-x/x̄)/√x", Magenta, 20], Scaled[{.68, .55}]],
    Text[Style["α=2.0", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



In[•]:=

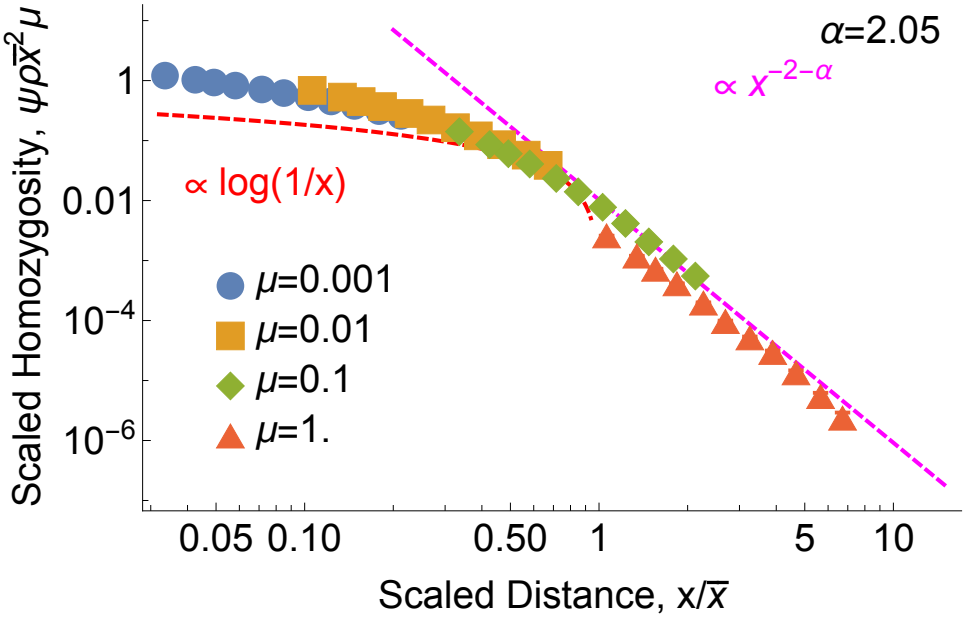
In[•]:=
```
(*For alpha = 2.05,
difference between (c^2) and ω^α is factor of (Sqrt[2*(α-2)*(α-1)/(α^2)])^α*)
```

In[•]:=
```
(*Extra factor of two for long distance expression since we draw from
 fisher dist for each lineage, i.e. prefactor is 2ω^α in power law*)
```

In[•]:=

In[•]:=
```
Module[{α = 2.05, coal = 1, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, 2.0, (#c)^2.00],
    (4*Pi)^-1*Around[#hom, {#dhomLow, #dhomHigh}]/approxhomscale[#coal, #mu, 2.0, (#c)^2.00]} & /@
   sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 && #x0 < 100 &]], {μ, μvals}];
 Show[LogLogPlot[(200)^-.05/2.05*(0.724)*2*(2*Pi)^-1*(1/4)*
    (Sqrt[2*(α-2)*(α-1)/(α^2)])^α*(1/α)^(-1-α)*x^(-2-α), {x, .2, 15},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], LogLogPlot[
   (4*Pi)^-1 Log[1/x], {x, .032, 1000}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ log(1/x)", Red, 20], Scaled[{.15, .65}]],
    Text[Style["∝ x^(-2-α)", Magenta, 20], Scaled[{.77, .85}]],
    Text[Style["α=2.05", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

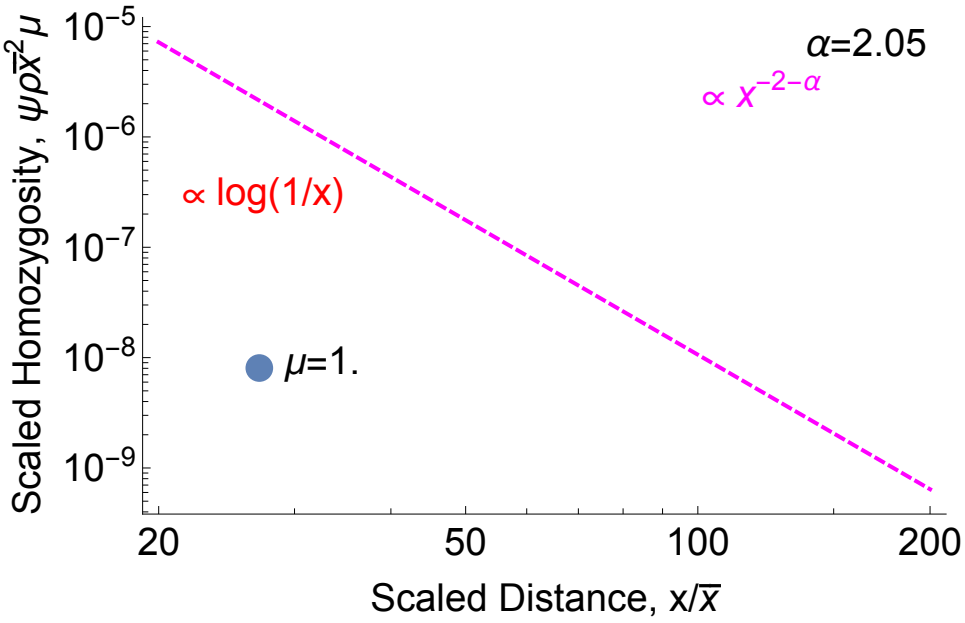
Out[•]=



In[•]:=

In[•]:=
```
Module[{α = 2.05, coal = 1, plotpoints, μvals = μlist[[4 ;; 4]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@
    sims[Select[#alpha == α && #coal == coal && #c == 10 &&
        #mu == μ && #x0 > 0 && #x0 < 160 &]], {μ, μvals}];
 Show[LogLogPlot[(0.724) * (2 * Pi)^-1 * (1 / 4) * (200)^2.05/2 *
    (Sqrt[2 * (α - 2) * (α - 1) / (α^2)])^α * (1 / α)^-1-α * x^-2-α,
   {x, 20, 200}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ log(1/x)", Red, 20], Scaled[{.15, .65}]],
    Text[Style["∝ x^-2-α", Magenta, 20], Scaled[{.77, .85}]],
    Text[Style["α=2.05", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄^2μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[•]=



In[•]:=
```
`
```

In[•]:=

In[•]:=
```
(*For alpha = 2.05,
difference between (c^2/2)^α and ω^α is factor of (Sqrt[2*(α-2)*(α-1)/(α^2)])^α*)
```

In[•]:=

```
In[•]:= Simplify[a^3 / ((a - 1)^2 * (a - 2)) + a^2 / (a - 1)^2]
```

Out[•]=
$$\frac{2\,a^2}{2 - 3\,a + a^2}$$

```
In[•]:=
```

```
In[•]:= (*add variance and mean^2 of one sided
         fisher distribution to get variance of two sided*)
```

```
In[•]:= FullSimplify[
          2 * (2 a)^2 * (2 + 2 a - 2) / (2 * (2 * a - 2)^2 * (2 * a - 4)) + (2 * a)^2 / (2 * a - 2)^2]
```

Out[•]=
$$\frac{2\,a^2}{2 - 3\,a + a^2}$$

```
In[•]:= FullSimplify[2 a^2 / (2 - 3 a + a^2) - 2 a^2 / ((a - 2) * (a - 1))]
```

Out[•]= 0

```
In[•]:= Simplify[2 * (2 a)^2 * (2 + 2 a - 2) / (2 * (2 * a - 2)^2 * (2 * a - 4))]
```

Out[•]=
$$\frac{a^3}{(-2 + a)\ (-1 + a)^2}$$

```
In[•]:= (4 * Pi)^-1 InverseHankelTransform[
          Abs[k]^(-a), k, x]
```

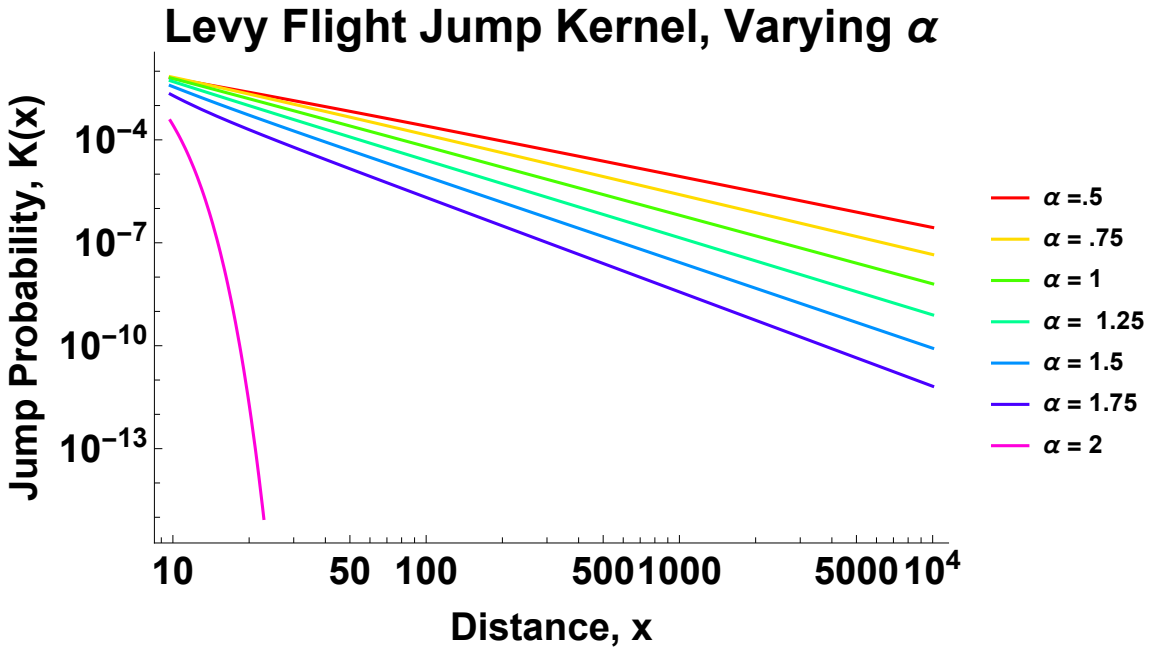
Out[•]=
$$\frac{2^{-1-a}\,x^{-2+a}\,\text{Gamma}\left[1 - \frac{a}{2}\right]}{\pi\,\text{Gamma}\left[\frac{a}{2}\right]}$$

```
In[•]:= 4 * N[Sum[n * Exp[-2 * n], {n, 1, 1 Infinity}]]
```

Out[•]= 0.724062

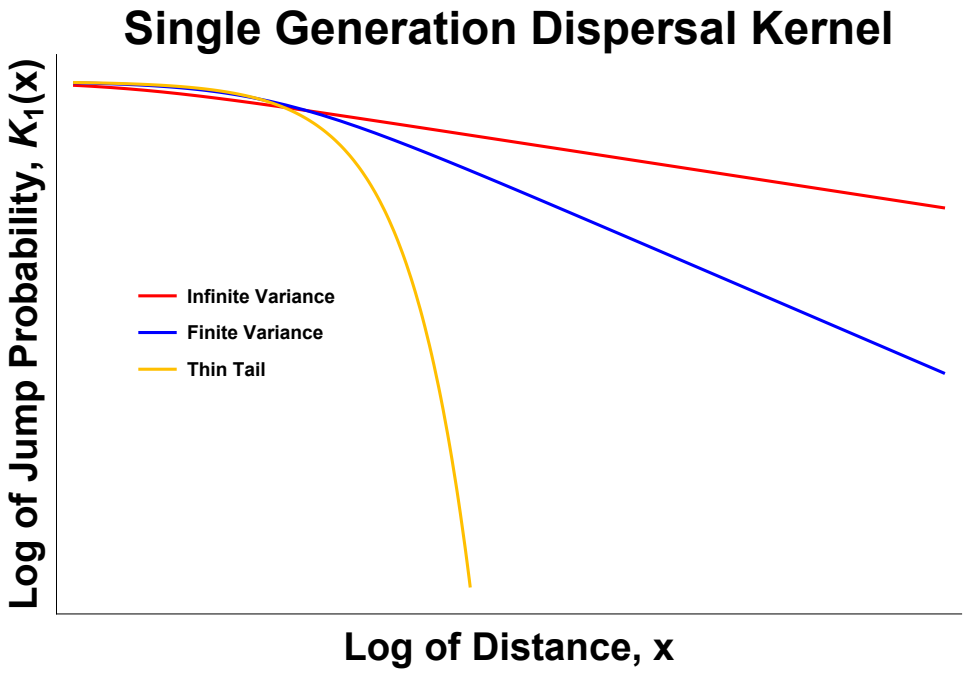
In[•]:=
```
Show[LogLogPlot[Table[PDF[StableDistribution[0, α, 0, 0, 2], x],
    {α, {.5, .75, 1, 1.25, 1.5, 1.75, 2}}] // Evaluate, {x, 0, 10 000},
  PlotStyle → Table[Hue[h - .1, 1, 1], {h, .1, 1.1 - 1 / 7, 1 / 7}] ,
  LabelStyle → Directive[Bold, Black],
  PlotLabel → Style["Levy Flight Jump Kernel, Varying α", FontSize → 24],
  PlotLegends → {"α =.5", "α = .75", "α = 1",
    "α =  1.25", "α = 1.5", "α = 1.75", "α = 2"}, Frame → True,
  FrameLabel → {"xlabel", "ylabel"}, FrameStyle → {{None, Black}, {None, Black}}],
 goodlabel["Distance, x", "Jump Probability, K(x)", 20],
 FrameStyle → Directive[20, Black], ImageSize → 500,
 PlotRange → All, Axes → False]
```

Out[•]=



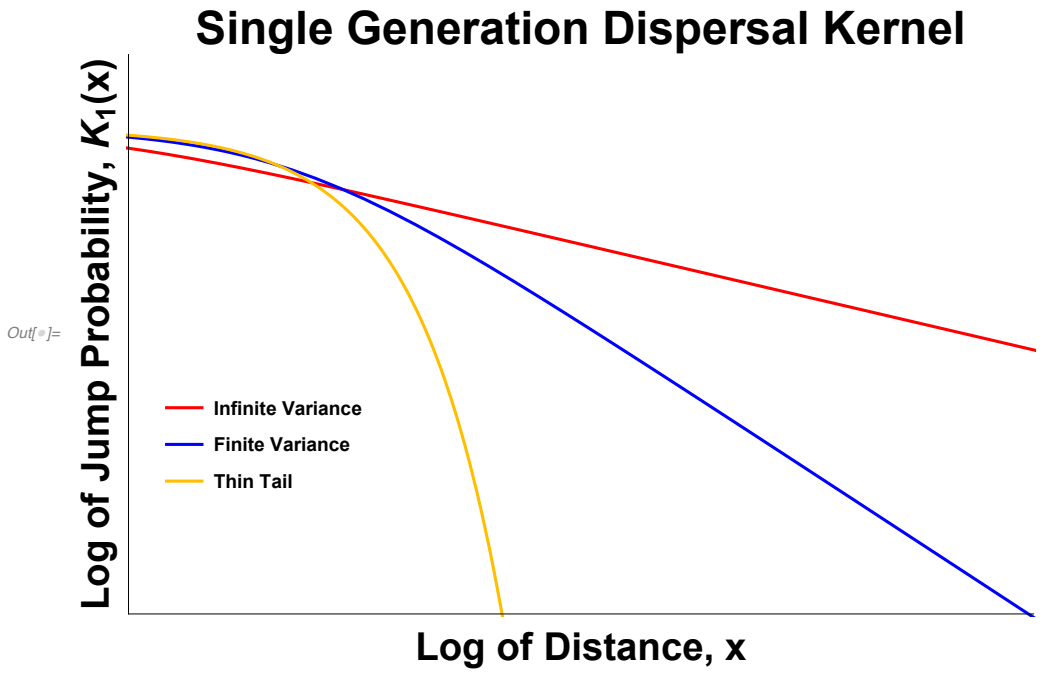
In[•]:=
```
Show[LogLogPlot[Table[PDF[MultivariateTDistribution[{{1, 0}, {0, 1}}, α], {x, 0}],
    {α, {.5, 5.0, 1000000}}] // Evaluate,
  {x, 0, 500}, PlotStyle → {Red, Blue, RGBColor[1, .75, 0]} ,
  LabelStyle → Directive[Bold, Black],
  PlotLabel → Style["Single Generation Dispersal Kernel", FontSize → 24],
  PlotLegends → Placed[LineLegend[{Red, Blue, RGBColor[1, .75, 0]},
     {"Infinite Variance", "Finite Variance", "Thin Tail"}], {.2, .5}],
  Frame → True, FrameLabel → {"xlabel", "ylabel"}, Ticks → {None, None},
  FrameTicks → {None, None},
  FrameTicksStyle → Directive[FontOpacity → 0, FontSize → 0],
  FrameStyle → {{None, Black}, {None, Black}}],
 goodlabel["Log of Distance, x", "Log of Jump Probability, K1(x)", 20],
 FrameStyle → Directive[20, Black], ImageSize → 500,
 PlotRange → All, Axes → False]
```

Out[•]=



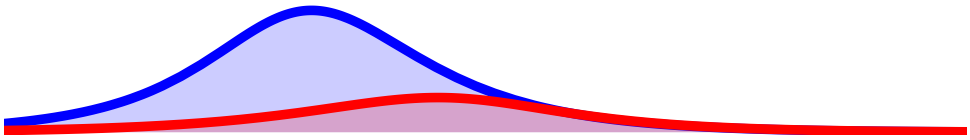
In[•]:=
```
Show[LogLogPlot[Table[PDF[MultivariateTDistribution[{{1, 0}, {0, 1}}, α], {x, 0}],
    {α, {.5, 5.0, 1000000}}] // Evaluate,
  {x, 0, 500}, PlotStyle → {Red, Blue, RGBColor[1, .75, 0]} ,
  LabelStyle → Directive[Bold, Black],
  PlotLabel → Style["Single Generation Dispersal Kernel", FontSize → 24],
  PlotLegends → Placed[LineLegend[{Red, Blue, RGBColor[1, .75, 0]},
     {"Infinite Variance", "Finite Variance", "Thin Tail"}], {.15, .3}],
  Frame → True, FrameLabel → {"xlabel", "ylabel"}, Ticks → {None, None},
  FrameTicks → {None, None},
  FrameTicksStyle → Directive[FontOpacity → 0, FontSize → 0],
  FrameStyle → {{None, Black}, {None, Black}}],
 goodlabel["Log of Distance, x", "Log of Jump Probability, K1(x)", 20],
 FrameStyle → Directive[20, Black], ImageSize → 500,
 PlotRange → {{0, 5}, {-30, 1}}, Axes → False]
```

Out[•]=



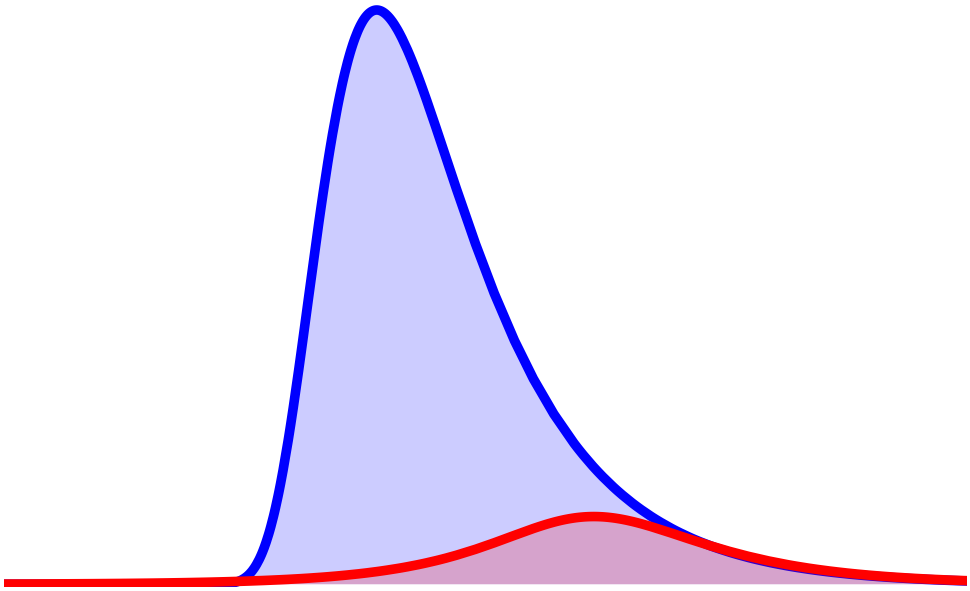
In[•]:=
```
Show[LogLinearPlot[
  Table[.12 * PDF[StableDistribution[0, α, 0, 0, t], 2], {α, {1.1}}] // Evaluate,
  {t, .1, 1000}, PlotStyle → {Blue, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{.1, 1000}, {.000, .05}}, Filling → Axis], LogLinearPlot[
  Table[.12 * PDF[StableDistribution[0, α, 0, 0, t], 6], {α, {0.9}}] // Evaluate,
  {t, .1, 1000}, PlotStyle → {Red, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black], PlotRange → {{1, 1000}, {.000, .1}},
  Filling → Axis], FrameStyle → Directive[20, Black],
 FrameTicksStyle → Directive[FontOpacity → 0, FontSize → 0], ImageSize → 500,
 Axes → False]
```

Out[•]=



In[•]:=
```
Show[LogLinearPlot[
  Table[PDF[StableDistribution[0, α, 0, 0, t], 5], {α, {2.0}}] // Evaluate,
  {t, .1, 1000}, PlotStyle → {Blue, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{.1, 1000}, {.0000, .05}}, Filling → Axis], LogLinearPlot[
  Table[PDF[StableDistribution[0, α, 0, 0, t], 30], {α, {1.1}}] // Evaluate,
  {t, .1, 1000}, PlotStyle → {Red, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{1, 1000}, {.0000, .1}}, Filling → Axis],
 FrameTicksStyle → Directive[FontOpacity → 0, FontSize → 0],
 ImageSize → 500, Axes → False]
```

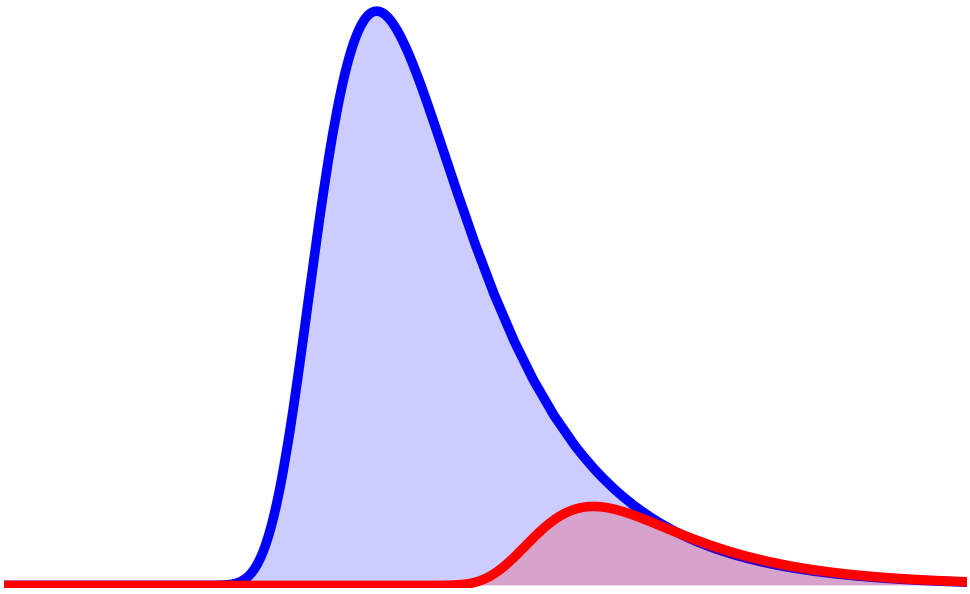
Out[•]=



In[•]:=
```
Show[LogLinearPlot[
  Table[PDF[StableDistribution[0, α, 0, 0, t], 5], {α, {2.0}}] // Evaluate,
  {t, .1, 1000}, PlotStyle → {Blue, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{.1, 1000}, {0, .05}}, Filling → Axis], LogLinearPlot[
  Table[1.1 * PDF[StableDistribution[0, α, 0, 0, t], 40], {α, {2.0}}] // Evaluate,
  {t, .1, 1000}, PlotStyle → {Red, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{1, 1000}, {0, .1}}, Filling → Axis], FrameTicksStyle →
  Directive[FontOpacity → 0, FontSize → 0], ImageSize → 500, Axes → False]
```

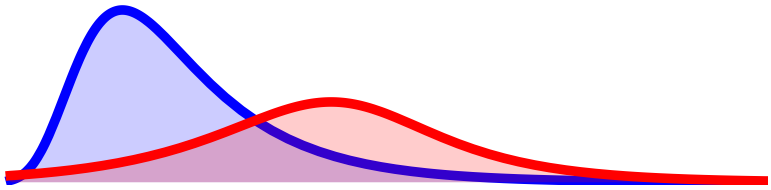
••• General: Exp[−39985.] is too small to represent as a normalized machine number; precision may be lost.

Out[•]=



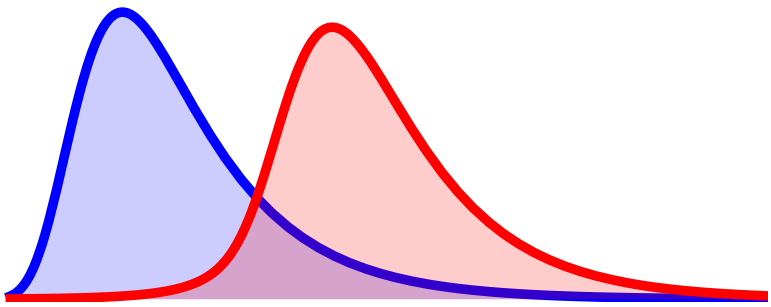
In[•]:=
```
Show[LogLinearPlot[
  Table[2.5 * .12 * PDF[StableDistribution[0, α, 0, 0, t], 5], {α, {2.0}}] // Evaluate,
  {t, 1.0, 5000}, PlotStyle → {Blue, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{.1, 5000}, {.000, .05}}, Filling → Axis], LogLinearPlot[
  Table[1.8 * 7.5 * .12 * PDF[StableDistribution[0, α, 0, 0, t], 35], {α, {0.9}}] //
   Evaluate, {t, 1.0, 5000}, PlotStyle → {Red, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black], PlotRange → {{.1, 5000}, {.000, .1}},
  Filling → Axis], FrameStyle → Directive[20, Black],
 FrameTicksStyle → Directive[FontOpacity → 0, FontSize → 0], ImageSize → 500,
 Axes → False]
```

Out[•]=



In[•]:=
```
Show[LogLinearPlot[
  Table[.5 * PDF[StableDistribution[0, α, 0, 0, t], 5], {α, {2.0}}] // Evaluate,
  {t, 1.0, 5000}, PlotStyle → {Blue, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{.1, 5000}, {.0000, .05}}, Filling → Axis], LogLinearPlot[
  Table[5.1 * PDF[StableDistribution[0, α, 0, 0, t], 50], {α, {1.7}}] // Evaluate,
  {t, 1.0, 5000}, PlotStyle → {Red, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{1, 5000}, {.0000, .1}}, Filling → Axis],
 FrameTicksStyle → Directive[FontOpacity → 0, FontSize → 0],
 ImageSize → 500, Axes → False]
```

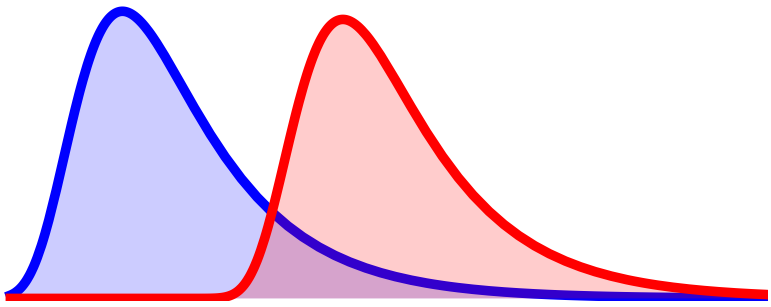
Out[•]=



In[•]:=
```
Show[LogLinearPlot[
  Table[.50 * PDF[StableDistribution[0, α, 0, 0, t], 5], {α, {2.0}}] // Evaluate,
  {t, 1, 5000}, PlotStyle → {Blue, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{.1, 5000}, {0, .05}}, Filling → Axis], LogLinearPlot[
  Table[5.3 * 1.1 * PDF[StableDistribution[0, α, 0, 0, t], 60], {α, {2.0}}] // Evaluate,
  {t, 1, 5000}, PlotStyle → {Red, Thickness[.01]} ,
  LabelStyle → Directive[Bold, Black],
  PlotRange → {{1, 5000}, {0, .1}}, Filling → Axis], FrameTicksStyle →
  Directive[FontOpacity → 0, FontSize → 0], ImageSize → 500, Axes → False]
```

... General: Exp[−899.687] is too small to represent as a normalized machine number; precision may be lost.

Out[•]=



2D Asymptotics

In[63]:=
```
(*These are expressions for ψ(x)/(1-ψ(0)) *)
```

x << δ

In[42]:=
```
(4 * Pi * ρ * D)^-1 * Integrate[k * Exp[-k^2 * delta^2/2] / (k^a),
  {k, 0, Infinity}, Assumptions -> {x > 0, delta > 0, a > 0}]
```

Out[42]=
$$\text{ConditionalExpression}\left[\frac{2^{-2-\frac{a}{2}}\,\text{delta}^{-2+a}\,\text{Gamma}\left[1-\frac{a}{2}\right]}{D\,\pi\,\rho},\ a<2\right]$$

$$\delta << x << \overline{x}$$

In[43]:=
```
(4 * Pi * ρ * D)^-1 * InverseHankelTransform[
  Abs[k]^(-a), k, x]
```

Out[43]=
$$\frac{2^{-1-a}\, x^{-2+a}\, \text{Gamma}\left[1-\frac{a}{2}\right]}{D\,\pi\,\rho\,\text{Gamma}\left[\frac{a}{2}\right]}$$

$$x >> \overline{x}$$

In[62]:=
```
(4 * Pi * ρ * mu * xbar^2)^-1 * xbar^(a + 2) * InverseHankelTransform[
  Abs[k]^(a), k, x]
```

Out[62]=
$$\frac{2^{-1+a}\, x^{-2-a}\, \text{xbar}^{a}\, \text{Gamma}\left[1+\frac{a}{2}\right]}{\text{mu}\,\pi\,\rho\,\text{Gamma}\left[-\frac{a}{2}\right]}$$

In[52]:=
```
(*Note that xbar = (D/mu)^(1/a)*)
```