In[97]:=

Out[97]=

In[98]:=

Setup

goodlabel

In[99]:=
```
goodlabel::usage =
  "Evaluate[goodlabel[xlabel,ylabel, xstyle, ystyle]] makes plot labels
    with the desired style. Labels should be passed as strings.";
goodlabel[x_, y_] := {Frame → {{True, False}, {True, False}}, FrameLabel → {x, y}};
goodlabel[x_, y_, style_] := goodlabel[x, y, style, style];
goodlabel[x_, y_, xstyle_, ystyle_] := {Frame → {{True, False}, {True, False}},
  FrameLabel → {Style[x, xstyle], Style[y, ystyle]}};
```

sims

In[102]:=
```
sims = SemanticImport[FileNameJoin[{NotebookDirectory[], "MH_data.txt"}]];
```

In[103]:=
```
sims = sims[All, <|#, "dhomLow" → #hom - #homLow, "dhomHigh" → #homHigh - #hom|> &];
```

In[104]:=
```
αlist = sims[All, #alpha &] // DeleteDuplicates // Normal // Sort;
coallist = sims[All, #coal &] // DeleteDuplicates // Normal // Sort;
μlist = sims[All, #mu &] // DeleteDuplicates // Normal // Sort;
```

In[106]:=
```
αlist
```

Out[106]=
```
{0.25, 0.5, 0.75, 1., 1.25, 1.45, 1.65, 1.85, 2.05}
```

In[107]:=
```
clist = sims[All, #c &] // DeleteDuplicates // Normal // Sort;
```

In[108]:=
```
cdfsims = SemanticImport[FileNameJoin[{NotebookDirectory[], "CDF_data.txt"}]];
```

In[109]:=
```
cdfsims =
  cdfsims [All, <|#, "dcdfLow" → #cdf - #cdfLow, "dcdfHigh" → #cdfHigh - #cdf|> &];
```

functions

In[110]:=
```
fLT[x_, μ_, α_, Dα_] := NIntegrate[Cos[k x] / π / (μ + Dα k^α), {k, 0, ∞}];
fLT[x_, μ_, α_] := fLT[x, μ, α, 250^α / 2];
```

In[112]:=
```
coeff[c_, μ_, α_, Dα_] := 1 / (2/c + (μ/Dα)^(1/α) / (α Sin[π/α] μ));
coeff[c_, μ_, α_] := coeff[c, μ, α, 250^α / 2];
```

In[114]:=
```
xscale[μ_, α_, Dα_] := (Dα/μ)^(1/α);
xscale[μ_, α_] := xscale[μ, α, 250^α / 2];
```

In[116]:=
```
homscale[coal_, μ_, α_, Dα_] := 1 / π / (Csc[π/α]/α + 2 (Dα/μ)^(1/α) μ/coal);
homscale[coal_, μ_, α_] := homscale[coal, μ, α, 250^α / 2];
```

In[118]:=
```
approxhomscale[coal_, μ_, α_, Dα_] := coal / (2 * Pi (Dα/μ)^(1/α) μ);
approxhomscale[coal_, μ_, α_] := approxhomscale[coal, μ, α, 250^α / 2];
```

In[120]:=
```
Series[homscale[1/ρ, μ, α, Dα], {α, 1, 1}]
```

Out[120]=
$$(\alpha - 1) + O[\alpha - 1]^2$$

In[121]:=
```
exphom[x_, coal_, μ_, Dα_: 250^2 / 2] := Exp[-Sqrt[μ/Dα] x] / (1 + 4 Sqrt[μ Dα] / coal)
```

In[122]:=
```
xlist = 10^Range[-6, 5.5, .33];
```

In[123]:=
```
xlist
```

Out[123]=
$$\{1. \times 10^{-6}, 2.13796 \times 10^{-6}, 4.57088 \times 10^{-6}, 9.77237 \times 10^{-6}, 0.000020893, 0.0000446684,$$
$$0.0000954993, 0.000204174, 0.000436516, 0.000933254, 0.00199526, 0.0042658,$$
$$0.00912011, 0.0194984, 0.0416869, 0.0891251, 0.190546, 0.40738, 0.870964,$$
$$1.86209, 3.98107, 8.51138, 18.197, 38.9045, 83.1764, 177.828, 380.189,$$
$$812.831, 1737.8, 3715.35, 7943.28, 16\,982.4, 36\,307.8, 77\,624.7, 165\,959.\}$$

In[124]:=
```
intvals = Quiet[Association @@
    Table[α -> Association @@ Table[x -> NIntegrate[Cos[k x] / (1 + k^α), {k, 0, ∞}], {x, xlist}],
     {α, αlist[[ ;; -2]]}]];
```

In[125]:=

In[126]:=
```
xmaxes = AssociationThread[αlist, {6400, 3200, 1600, 400, 200, 100, 30, 20, 8}];
```

In[127]:=

In[128]:=

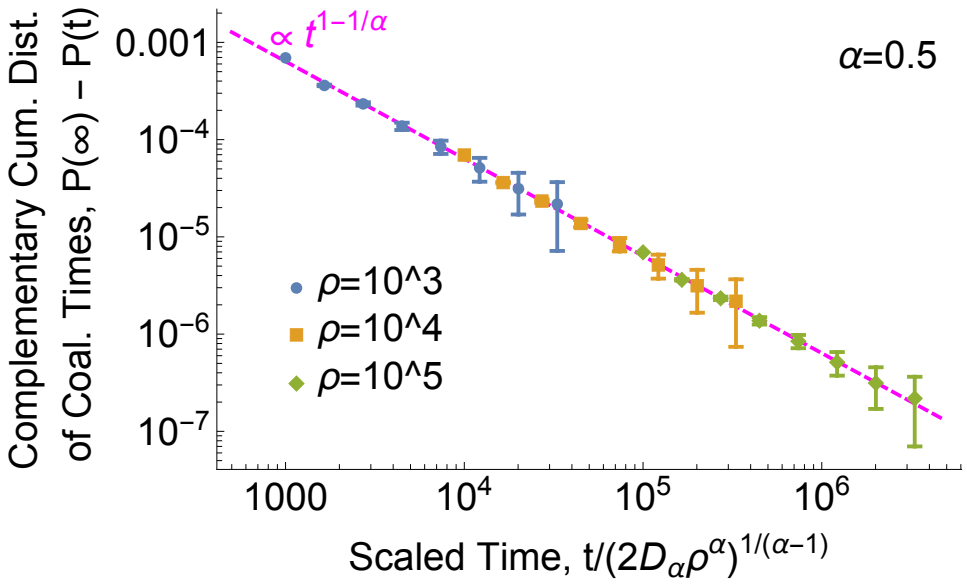
Plots

α=0.5

In[129]:=
```
(* note that coal here is 1/rho, i.e. 1 over the population density*)
```

In[130]:=
```
Module[{α = 0.5, coal = .001, plotcdf, D = .5},
 dummy = cdfsims[Select[#alpha == α && #coal < 10 * coal &]];
 dummy = dummy[
   All, <|#, "maxcdfrho1000" → Max[dummy[Select[#coal == .001  &], "cdf"]]|> &];
 dummy = dummy[All, <|#, "maxcdfrho10000" →
       Max[dummy[Select[#coal == .0001  &], "cdf"]]|> &];
 dummy = dummy[All, <|#, "maxcdfrho100000" →
       Max[dummy[Select[#coal == .00001  &], "cdf"]]|> &];
 dummy = dummy[All, <|#, "maxcdf" → KroneckerDelta[#coal, .001] * #maxcdfrho1000 +
        KroneckerDelta[#coal, .0001] * #maxcdfrho10000 +
        KroneckerDelta[#coal, .00001] * #maxcdfrho100000 |> &];
 dummy = dummy[All, <|#, "compcdf" → #maxcdf - #cdf|> &];
 dummy = dummy[All, <|#, "scaledT" → (2 * #coal^(-α) * D)^(1 / (1 - α)) * #T|> &];
 dummy = dummy[All, <|#, "scaledcompcdf" → #compcdf|> &];
 dummy = dummy[All, <|#, "scaleddcdfLow" → #dcdfLow|> &];
 dummy = dummy[All, <|#, "scaleddcdfHigh" → #dcdfHigh|> &];
 plotcdf = dummy;
 plotpoints =
  Table[{#scaledT, Around[#scaledcompcdf, {#scaleddcdfHigh, #scaleddcdfLow}]} & /@
    dummy[Select[#alpha == α && #coal == coal && #T < 50 &]],
   {coal, {.001, .0001, .00001}}];
 (*Prediction continues to work at T >50, but error bars get very large*)
 Show[LogLogPlot[(3.141 * ((1 / α - 1) / (Gamma[1 / α + 1])))^-1 * x^(1-1/α), {x, 500, 5 000 000},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], ListLogLogPlot[plotpoints ,
   PlotRange → All, PlotMarkers → Automatic,
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["ρ=" <> ToString[#], 20] & /@ {"10^3", "10^4", "10^5"},
     Scaled[{.2, .3}]]], Epilog → {Text[Style["α=0.5", 20], Scaled[{.9, .9}]],
    Text[Style["∝ t^(1-1/α)", Magenta, 20], Scaled[{.15, .95}]]},
  goodlabel["Scaled Time, t/(2D_α ρ^α)^(1/(α-1))", "Complementary Cum. Dist.
of Coal. Times, P(∞) - P(t)", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[130]=



In[131]:=

In[132]:=
```
1 / (1 + Pi * (2^(3.5 / 2) / Gamma[.25]) * .01^-1 * .5 * (.5)^(1 - .5))
```

Out[132]= 0.00961124

In[133]:=
```
dummy = cdfsims[Select[#alpha == .5 && #coal < 100 &]];
dummy = dummy[
   All, <|#, "maxcdfrho1000" → Max[dummy[Select[#coal == .001  &], "cdf"]]|> &];
dummy = dummy[All, <|#, "maxcdfrho10000" →
      Max[dummy[Select[#coal == .0001  &], "cdf"]]|> &];
dummy = dummy[All, <|#, "maxcdfrho100000" →
      Max[dummy[Select[#coal == .00001  &], "cdf"]]|> &];
```

In[137]:=
```
Null
```

In[138]:=
```
Null
```

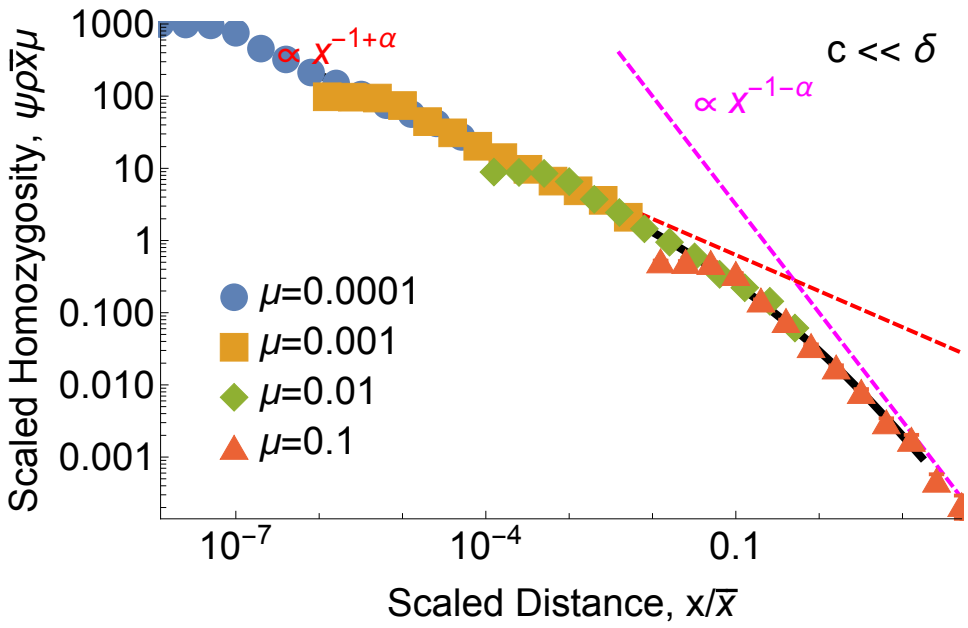
In[139]:=
```
maxcdf
```

Out[139]= maxcdf

In[140]:=

In[141]:=
```
Module[{α = 0.5, coal = .01, plotpoints, μvals = μlist[[1 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha/2],
    (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/approxhomscale[#coal, #mu, #alpha, #c^#alpha/2]} & /@ sims[Select[
    #alpha == α && #coal == coal && #c == .2 && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]}
   , PlotRange → {{10^-8, 20}, {10^-3, 10^3.5}}],
  LogLogPlot[Sin[Pi * α/2] * Gamma[α + 1] * (2 * Pi)^-1 * x^(-1-α),
   {x, .004, 50}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[Sin[Pi * α/2] * Gamma[1 - α] * (2 * Pi)^-1 * x^(-1+α),
   {x, .0001, 50}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["∝ x^(-1+α)", Red, 20], Scaled[{.22, .95}]],
    Text[Style["∝ x^(-1-α)", Magenta, 20], Scaled[{.74, .85}]],
    Text[Style["c << δ", Black, 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

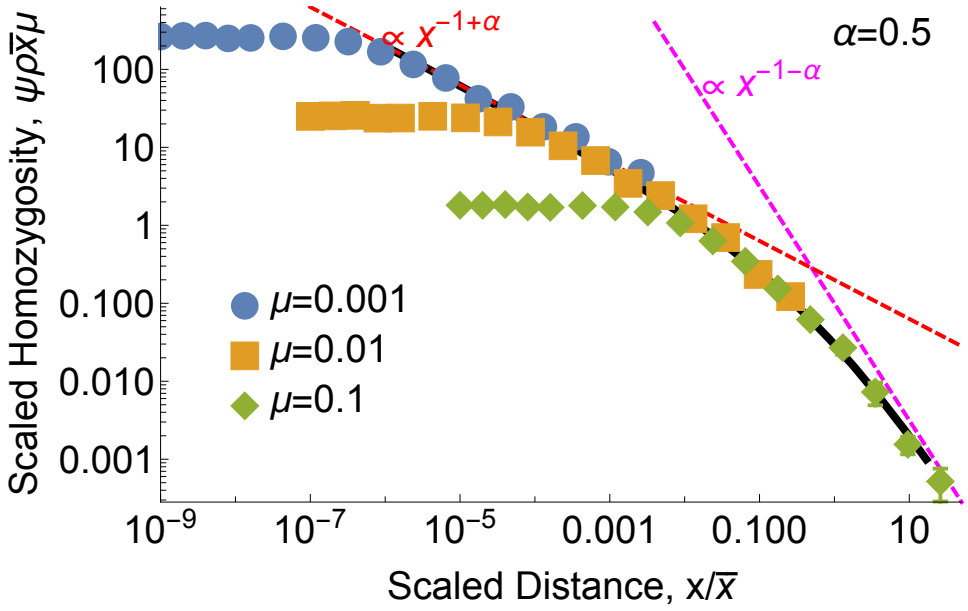
Out[141]=



In[142]:=

In[143]:=
```
Module[{α = 0.5, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha/2],
    (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/approxhomscale[#coal, #mu, #alpha, #c^#alpha/2]} & /@ sims[Select[
    #alpha == α && #coal == coal && #c == 250 && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]}
   , PlotRange → {{10^-8, 20}, {10^-3, 10^3.5}}],
  LogLogPlot[Sin[Pi * α/2] * Gamma[α + 1] * (2 * Pi)^-1 * x^(-1-α),
   {x, .004, 50}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[Sin[Pi * α/2] * Gamma[1 - α] * (2 * Pi)^-1 * x^(-1+α),
   {x, .0000001, 50}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=0.5", 20], Scaled[{.9, .95}]],
    Text[Style["∝ x^(-1+α)", Red, 20], Scaled[{.35, .95}]],
    Text[Style["∝ x^(-1-α)", Magenta, 20], Scaled[{.75, .85}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

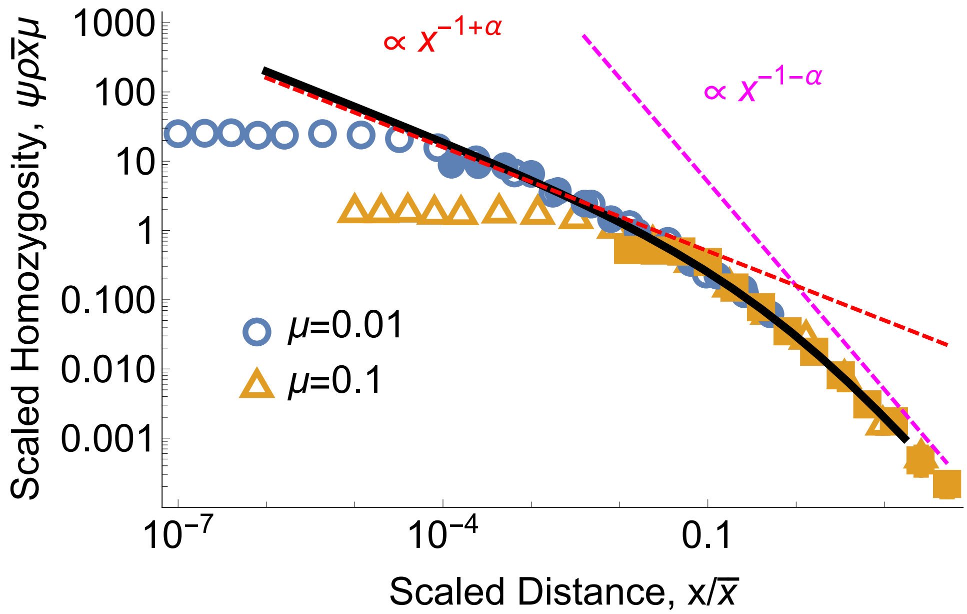
Out[143]=



In[144]:=

In[145]:=
```
Module[{α = 0.5, coal = .01, plotpoints, μvals = μlist[[3 ;; 4]]},
 plotpoints = Table[{#x0/xscale[#mu, #alpha, #c^#alpha/2],
    (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha/2]} & /@ sims[Select[
    #alpha == α && #coal == coal && #mu == μ && #c == 250 && #x0 > 0 &]], {μ, μvals}];
 plotpoints1 = Table[{#x0/xscale[#mu, #alpha, #c^#alpha/2],
    (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha/2]} & /@
   sims[Select[#alpha == α && #coal == coal && #mu == μ && #c == 250 && #x0 > 0 &]], {μ,
   μvals}];
 plotpoints2 = Table[{#x0/xscale[#mu, #alpha, #c^#alpha/2],
    (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/
     approxhomscale[#coal, #mu, #alpha, #c^#alpha/2]} & /@
   sims[Select[#alpha == α && #coal == coal && #c == .2 && #mu == μ && #x0 > 0 &]], {μ,
   μvals}];
 Show[ListLogLogPlot[plotpoints, PlotMarkers → {"OpenMarkers", 12},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  ListLogLogPlot[plotpoints2, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>],
  ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}]],
  Joined → True, PlotStyle → {Black, Thickness[.01]}
  , PlotRange → {{10^-8, 1}, {10^-3, 10^3.5}}],
  LogLogPlot[(2 * Pi)^-1 * x^-1-α, {x, .004, 50},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], LogLogPlot[(2 * Pi)^-1 * x^-1+α,
   {x, .000001, 50}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  Epilog → {Text[Style["∝ x^-1+α", Red, 20], Scaled[{.35, .95}]],
    Text[Style["∝ x^-1-α", Magenta, 20], Scaled[{.75, .85}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[145]=



In[146]:=
```
Charting`ResolvePlotTheme[Automatic, Plot]
```

Out[146]=
```
{GridLinesStyle → Directive[■],
 Method → {AxisPadding → Scaled[0.02], DefaultBoundaryStyle → Automatic,
   DefaultGraphicsInteraction → {Version → 1.2, TrackMousePosition → {True, False},
     Effects → {Highlight → {ratio → 2}, HighlightPoint → {ratio → 2},
       Droplines → {freeformCursorMode → True, placement → {x → All, y → None}}}},
   DefaultMeshStyle → AbsolutePointSize[6], DefaultPlotStyle →
    {Directive[■, AbsoluteThickness[1.6]],
     Directive[■, AbsoluteThickness[1.6]], Directive[■, AbsoluteThickness[1.6]],
     Directive[■, AbsoluteThickness[1.6]], Directive[■, AbsoluteThickness[1.6]],
     Directive[■, AbsoluteThickness[1.6]], Directive[■, AbsoluteThickness[1.6]],
     Directive[■, AbsoluteThickness[1.6]], Directive[■, AbsoluteThickness[1.6]],
     Directive[■, AbsoluteThickness[1.6]], Directive[■, AbsoluteThickness[1.6]],
     Directive[■, AbsoluteThickness[1.6]], Directive[■, AbsoluteThickness[1.6]],
     Directive[■, AbsoluteThickness[1.6]], Directive[■, AbsoluteThickness[1.6]]},
   DomainPadding → Scaled[0.02], RangePadding → Scaled[0.05]}}
```

In[147]:=
```
clist
```

Out[147]=
```
{0.2, 179.675, 250.}
```

In[148]:=

In[149]:=

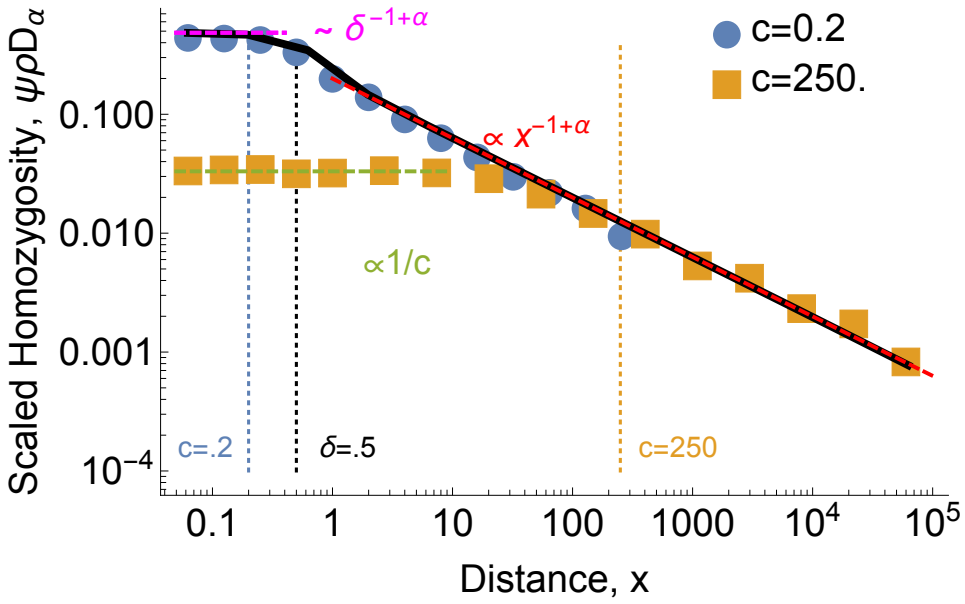
In[150]:=

In[151]:=
```
linec250 = Line[{{250, -1}, {250, 1}}];
```

In[152]:=
```
Module[{α = 0.5, D = (250)^(.5)/2, coal = .01,
  plotpoints, cvals = {0.2, 250.}, μvals = μlist[[2 ;; 2]]},
```

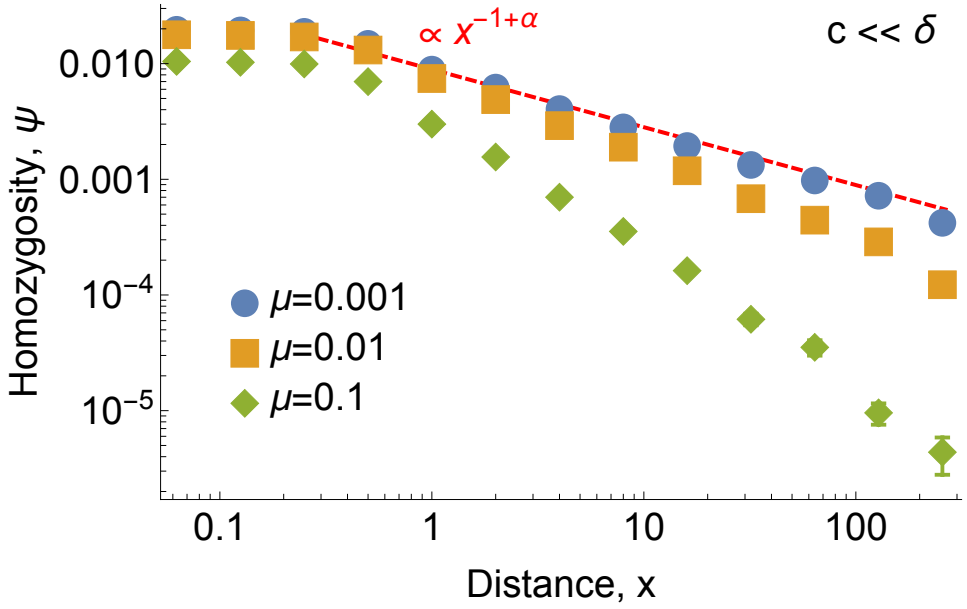
```
xlist2 = 10^Range[-11,5,.5];
intvalsfinitedelta =
 Quiet[Association @@ Table[α -> Association @@ Table[x -> NIntegrate[
          Cos[k x] * Exp[-.125 * k^2 / ((D / .0001)^(1 / .5))^2] / (1 + k^α), {k, 0, ∞}],
        {x, xlist2}], {α, αlist[[;; -2]]}]];
plotpoints = Table[{#x0, Around[#coal^-1 * (#c)^(#alpha) / 2 * #hom, {#coal^-1 *
          (#c)^(#alpha) / 2 * #dhomLow, #coal^-1 * (#c)^(#alpha) / 2 * #dhomHigh}]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == .001 && #c == c &&
        #x0 > 0 && #x0 < 100 000 &]], {c, cvals}];
Show[LogLogPlot[coal^-1 * D / (1 + Pi * (2^(3.5 / 2) / Gamma[.25]) *
        D * coal^-1 * (.5)^(1 - α)), {x, .05, .4},
  PlotStyle → {Magenta, Dashed, Thickness[.005]}], ParametricPlot[
  {{Log[.5], Log[u]}}, {u, .0001, .42}, PlotStyle → {Black, Dotted}],
 ParametricPlot[{{Log[.2], Log[u]}}, {u, .0001, .42}, PlotStyle → {■, Dotted}],
 ParametricPlot[{{Log[250], Log[u]}}, {u, .0001, .42}, PlotStyle → {■, Dotted}],
 ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
  IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
  PlotLegends → Placed[Style["c=" <> ToString[#], 20] & /@ cvals, {.8, .9}]],
 ListLogLogPlot[Table[{x * (D / .0001)^(1 / .5), coal^-1 * D *
     intvalsfinitedelta[α][x] / (2 * Pi * (D / .0001)^(1 / .5) * coal^-1 * .0001)},
   {x, Select[xlist2, # ≤ .00001 &]}], Joined → True,
  PlotStyle → {Black, Thickness[.01]}
 ], LogLogPlot[(Pi^2 / 6) * D * Gamma[1 + 1 / .5] / (250 * Pi),
  {x, .05, 10}, PlotStyle → {■, Dashed, Thickness[.005]}],
 LogLogPlot[(Gamma[1 - α] * Sin[Pi * α / 2] / (2 * Pi)) * x^-1+α, {x, 1, 100 000},
  PlotStyle → {Red, Dashed, Thickness[.005]}], LogLogPlot[
  coal^-1 * D / (1 + Pi * (2^(3.5 / 2) / Gamma[.25]) * D * coal^-1 * (.5)^(1 - α)),
  {x, .05, .4}, PlotStyle → {Magenta, Dashed, Thickness[.005]}], LogLogPlot[
  coal^-1 * D / (1 + Pi * (2^(3.5 / 2) / Gamma[.25]) * D * coal^-1 * (.5)^(1 - α)),
  {x, .05, .4}, PlotStyle → {Magenta, Dashed, Thickness[.005]},
  PlotRange → {{10^-8, 1}, {10^-4, 100}}],
 Epilog → {Text[Style[" ∝ x^-1+α ", Red, 18], Scaled[{.48, .74}]],
   Text[Style[" ∝1/c ", ■, 18], Scaled[{.30, .48}]],
   Text[Style[" ~ δ^-1+α ", Magenta, 18], Scaled[{.27, .96}]],
   Text[Style[" δ=.5 ", Black, 15], Scaled[{.24, .1}]],
   Text[Style[" c=.2 ", ■, 15], Scaled[{.06, .1}]],
   Text[Style[" c=250 ", ■, 15], Scaled[{.66, .1}]]},
 goodlabel["Distance, x", "Scaled Homozygosity, ψρD_α", 20],
 FrameStyle → Directive[20, Black], ImageSize → 500,
 PlotRange → All, Axes → False]]
```

Out[152]=


Scaled Homozygosity, ψρD_α
0.100
0.010
0.001
10^−4
~ δ^−1+α
∝ x^−1+α
∝1/c
c=0.2
c=250.
c=.2
δ=.5
c=250
0.1
1
10
100
1000
10^4
10^5
Distance, x

In[153]:=
```
Module[{α = 0.5, D = (.2)^(.5)/2, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@ sims[Select[
      #alpha == α && #coal == coal && #c == .2 && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[LogLogPlot[coal * (Gamma[1 - .5] * Sin[3.141/4]/(2 * 3.141 * D)) * x^(-1+α),
   {x, .25, 265}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style[" ∝ x^(-1+α) ", Red, 20], Scaled[{.4, .95}]],
    Text[Style["c << δ", Black, 20], Scaled[{.9, .95}]]},
  goodlabel["Distance, x", "Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

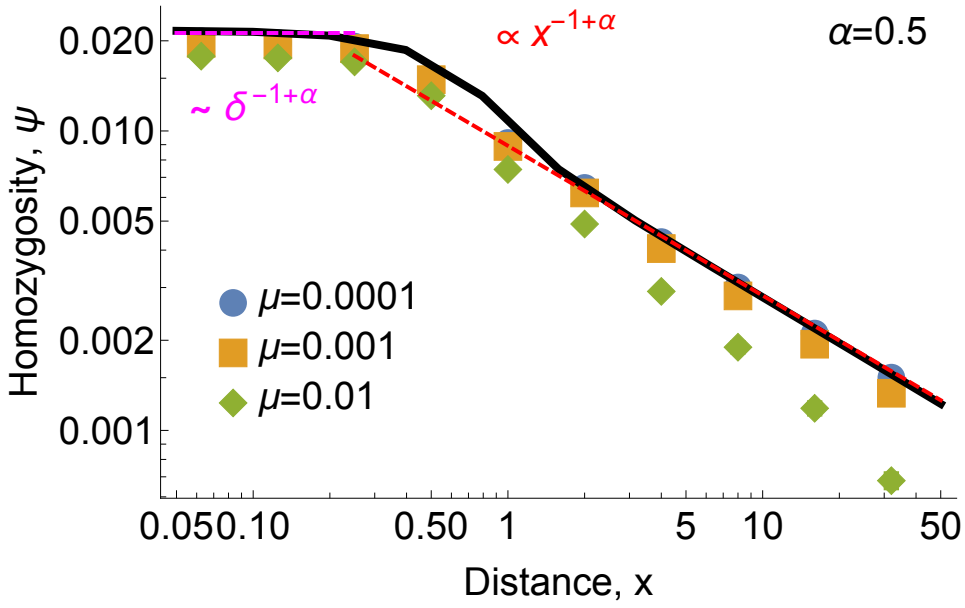
Out[153]=



In[154]:=

In[155]:=
```
Module[{α = 0.5, D = (.2)^(.5)/2,
  coal = .01, plotpoints, μvals = μlist[[;; 3]]}, plotpoints =
  Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@ sims[Select[#alpha == α &&
        #coal == coal && #c == .2 && #mu == μ && #x0 > 0 && #x0 < 50 &]], {μ, μvals}];
 xlist2 = 10^Range[-8,0,.3];
 intvalsfinitedelta =
  Quiet[Association @@ Table[α -> Association @@ Table[x -> NIntegrate[
          Cos[k x] * Exp[-.125 * k^2/((D/.0001)^(1/.5))^2]/(1 + k^α), {k, 0, ∞}],
        {x, xlist2}], {α, αlist[[;; -2]]}]];
 Show[
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  ListLogLogPlot[Table[{x * (D/.0001)^(1/.5), intvalsfinitedelta[α][x]/
      (2 * Pi * (D/.0001)^(1/.5) * coal^-1 * .0001)}, {x, Select[xlist2, # ≤ .00001 &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]}
   , PlotRange → {{10^-8, 1}, {10^-6, 10}}],
  LogLogPlot[coal * (Gamma[1 - .5] * Sin[3.141/4]/(2 * 3.141 * D)) * x^(-1+α),
   {x, .25, 50}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  LogLogPlot[1/(1 + Pi * (2^(3.5/2)/Gamma[.25]) * coal^-1 * D * (.5)^(1 - α)),
   {x, .05, .25}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  Epilog → {Text[Style["α=0.5", 20], Scaled[{.9, .95}]],
    Text[Style[" ∝ x^(-1+α) ", Red, 20], Scaled[{.5, .95}]],
    Text[Style[" ~ δ^(-1+α) ", Magenta, 20], Scaled[{.12, .8}]]},
  goodlabel["Distance, x", "Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[155]=

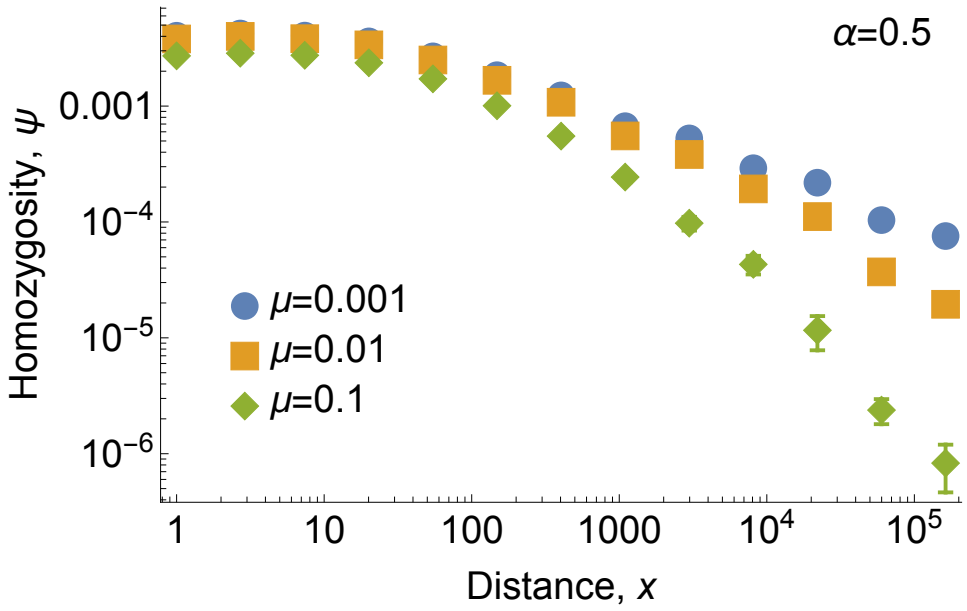


In[156]:=
```
μlist[[2 ;; 3]]
```

Out[156]=
```
{0.001, 0.01}
```

In[157]:=
```
Module[{α = 0.5, coal = 1, plotpoints, μvals = μlist[[2 ;; 4]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=0.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Distance, x", "Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False ]]
```

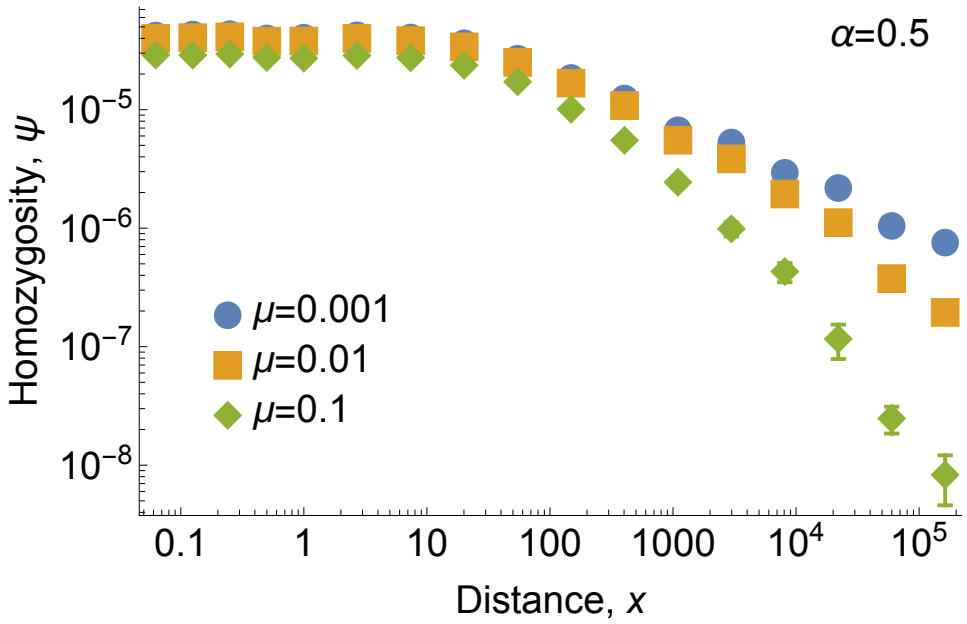
Out[157]=



In[158]:=

In[159]:=
```
Module[{α = 0.5, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]},
 plotpoints = Table[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@ sims[Select[
      #alpha == α && #coal == coal && #c > 100 && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=0.5", 20], Scaled[{.9, .95}]]},
  goodlabel["Distance, x", "Homozygosity, ψ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False ]]
```

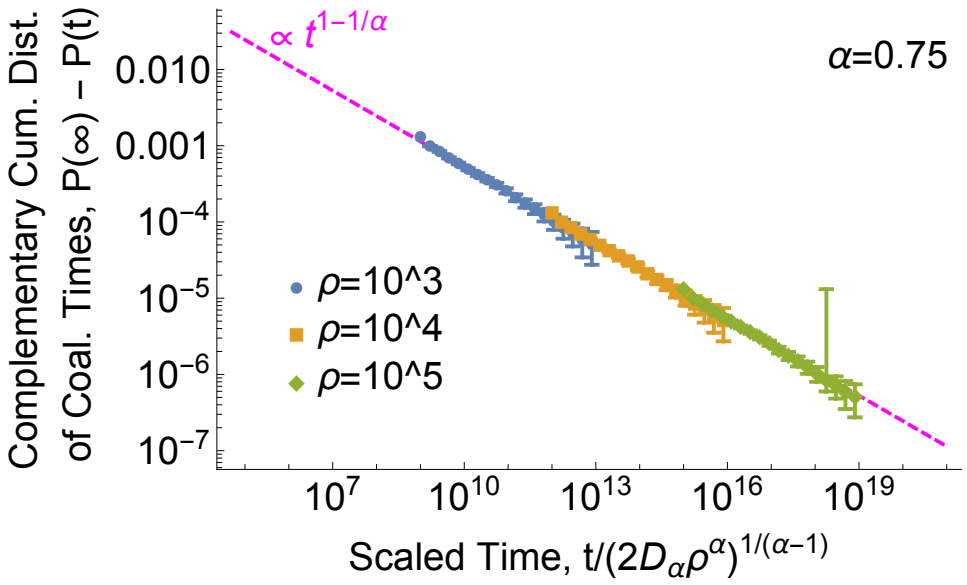
Out[159]=



α=.75

In[160]:=
```
Module[{α = 0.75, coal = .001, plotcdf, c = 1, D = .5},
 dummy = cdfsims[Select[#alpha == α && #coal < 10 * coal &]];
 dummy = dummy[
   All, <|#, "maxcdfrho1000" → Max[dummy[Select[#coal == .001  &], "cdf"]]|> &];
 dummy = dummy[All, <|#, "maxcdfrho10000" →
       Max[dummy[Select[#coal == .0001  &], "cdf"]]|> &];
 dummy = dummy[All, <|#, "maxcdfrho100000" →
       Max[dummy[Select[#coal == .00001  &], "cdf"]]|> &];
 dummy = dummy[All, <|#, "maxcdf" → KroneckerDelta[#coal, .001] * #maxcdfrho1000 +
        KroneckerDelta[#coal, .0001] * #maxcdfrho10000 +
        KroneckerDelta[#coal, .00001] * #maxcdfrho100000 |> &];
 dummy = dummy[All, <|#, "compcdf" → #maxcdf - #cdf|> &];
 dummy = dummy[All, <|#, "scaledT" → (2 * #coal^(-α) * D)^(1 / (1 - α)) * #T|> &];
 dummy = dummy[All, <|#, "scaledcompcdf" → #compcdf|> &];
 dummy = dummy[All, <|#, "scaleddcdfLow" → #dcdfLow|> &];
 dummy = dummy[All, <|#, "scaleddcdfHigh" → #dcdfHigh|> &];
 plotcdf = dummy;
 plotpoints =
  Table[{#scaledT, Around[#scaledcompcdf, {#scaleddcdfHigh, #scaleddcdfLow}]} & /@
    dummy[Select[#alpha == α && #coal == coal && #T < 10^4 &]],
   {coal, {.001, .0001, .00001}}];

 Show[LogLogPlot[(3.141 * ((1 / α - 1) / (Gamma[1 / α + 1])))^-1 * x^(1-1/α), {x, 50 000, 10^21}],
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], ListLogLogPlot[plotpoints ,
   PlotRange → All, PlotMarkers → Automatic,
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["ρ=" <> ToString[#], 20] & /@ {"10^3", "10^4", "10^5"},
     Scaled[{.2, .3}]]], Epilog → {Text[Style["α=0.75", 20], Scaled[{.9, .9}]],
    Text[Style["∝ t^(1-1/α)", Magenta, 20], Scaled[{.15, .95}]]},
  goodlabel["Scaled Time, t/(2D_α ρ^α)^(1/(α-1))", "Complementary Cum. Dist.
of Coal. Times, P(∞) - P(t)", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[160]=

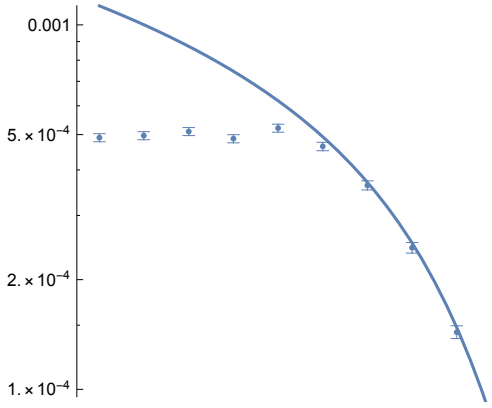


In[161]:=

α=1

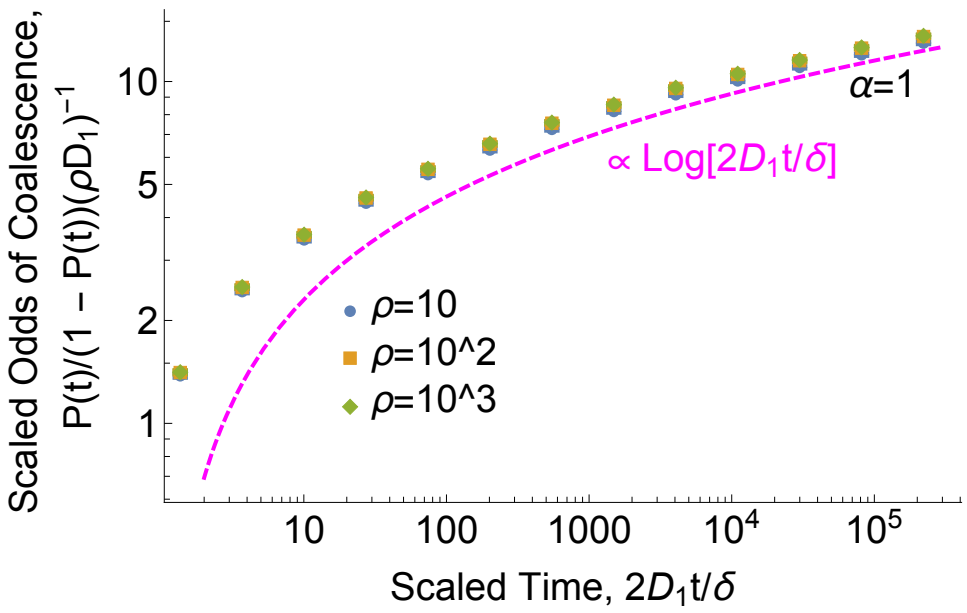
In[162]:=
```
Module[{α = 1.0, coal = 0.1, μ = 0.01, plotds},
 plotds = sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]];
 Show[ListLogLogPlot[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@ plotds],
  LogLogPlot[.05 fLT[x, μ, α], {x, 1, 5 × 10^5}],
  PlotRange → {All, {-4, -3} Log[10]}]]
```

Out[162]=



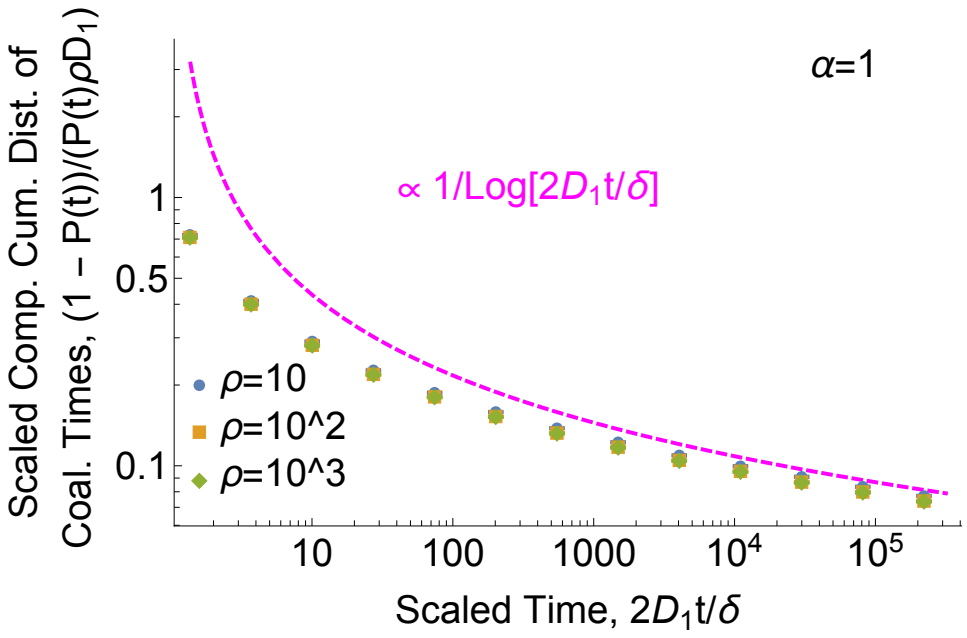
In[163]:=
```
Module[{α = 1.00, coal = 1, plotcdf, c = 1, D = .5},
 dummy = cdfsims[Select[#alpha == α && #coal ≥ .001 * coal && #T > 1 &]];
 maxcdf = Max[dummy[All, "cdf"]];
 dummy = dummy[All, <|#, "compcdf" → 1 - #cdf|> &];
 dummy = dummy[All, <|#, "scaledT" → (.5 * (2 * D)^-1) * #T|> &];
 dummy =
  dummy[All, <|#, "scaledcompcdf" → ((2 * Pi * #coal^-1 * D) * (#cdf) / #compcdf)|> &];
 dummy = dummy[All, <|#, "scaleddcdfLow" →
       ((2 * Pi * #coal^-1 * D) * #dcdfLow / #compcdf)|> &];
 dummy = dummy[All, <|#, "scaleddcdfHigh" →
       ((2 * Pi * #coal^-1 * D) * #dcdfHigh / #compcdf)|> &];
 dummy = dummy[Select[#T < 1 000 000 &]];
 plotcdf = dummy;
 plotpoints =
  Table[{#scaledT, Around[#scaledcompcdf, {#scaleddcdfHigh, #scaleddcdfLow}]} & /@
    dummy[Select[#alpha == α && #coal == coal &]], {coal, { .1, .01, .001}}];



 Show[LogLogPlot[Log[x], {x, 2, 10^5.5},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], ListLogLogPlot[plotpoints ,
   PlotRange → All, PlotMarkers → Automatic,
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["ρ=" <> ToString[#], 20] & /@ { "10", "10^2", "10^3"},
     Scaled[{.32, .3}]]], Epilog → {Text[Style["α=1", 20], Scaled[{.9, .85}]],
    Text[Style["∝ Log[2D1t/δ]", Magenta, 20], Scaled[{.7, .7}]], ImageSize → 500 },
  goodlabel["Scaled Time, 2D1t/δ", "Scaled Odds of Coalescence,
P(t)/(1 - P(t))(ρD1)^-1", 20], FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[163]=


Scaled Odds of Coalescence,
P(t)/(1 – P(t))(ρD1)^-1
10
5
2
1
10
100
1000
10^4
10^5
Scaled Time, 2D1t/δ
α=1
∝ Log[2D1t/δ]
ρ=10
ρ=10^2
ρ=10^3

In[164]:=
```
Module[{α = 1.00, coal = 1, plotcdf, c = 1, D = .5},
 dummy = cdfsims[Select[#alpha == α && #coal ≥ .001 * coal && #T > 1 &]];
 maxcdf = Max[dummy[All, "cdf"]];
 dummy = dummy[All, <|#, "compcdf" → 1 - #cdf|> &];
 dummy = dummy[All, <|#, "scaledT" → (.5 * (2 * D)^-1) * #T|> &];
 dummy =
  dummy[All, <|#, "scaledcompcdf" → (2 * Pi * #coal^-1 * D)^-1 * (#compcdf / #cdf)|> &];
 dummy = dummy[All, <|#, "scaleddcdfLow" →
       ((2 * Pi * #coal^-1 * D)^-1 * #dcdfLow / #cdf)|> &];
 dummy = dummy[All, <|#, "scaleddcdfHigh" →
       ((2 * Pi * #coal^-1 * D)^-1 * #dcdfHigh / #cdf)|> &];
 dummy = dummy[Select[#T < 1 000 000 &]];
 plotcdf = dummy;
 plotpoints =
  Table[{#scaledT, Around[#scaledcompcdf, {#scaleddcdfHigh, #scaleddcdfLow}]} & /@
    dummy[Select[#alpha == α && #coal == coal &]], {coal, { .1, .01, .001}}];


 Show[LogLogPlot[1 / Log[x], {x, 1, 10^5.5},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], ListLogLogPlot[plotpoints ,
   PlotRange → All, PlotMarkers → Automatic,
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["ρ=" <> ToString[#], 20] & /@ { "10", "10^2", "10^3"},
     Scaled[{.12, .2}]]], Epilog → {Text[Style["α=1", 20], Scaled[{.85, .95}]], Text[
     Style["∝ 1/Log[2D1t/δ]", Magenta, 20], Scaled[{.45, .7}]], ImageSize → 500 },
  goodlabel["Scaled Time, 2D1t/δ", "Scaled Comp. Cum. Dist. of
Coal. Times, (1 - P(t))/(P(t)ρD1)", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

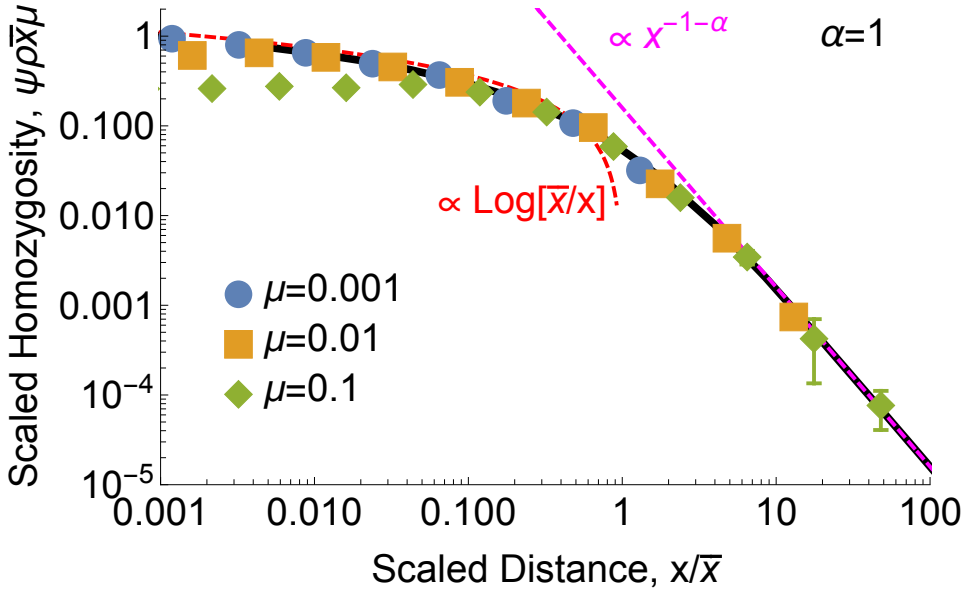
Out[164]=



In[165]:=

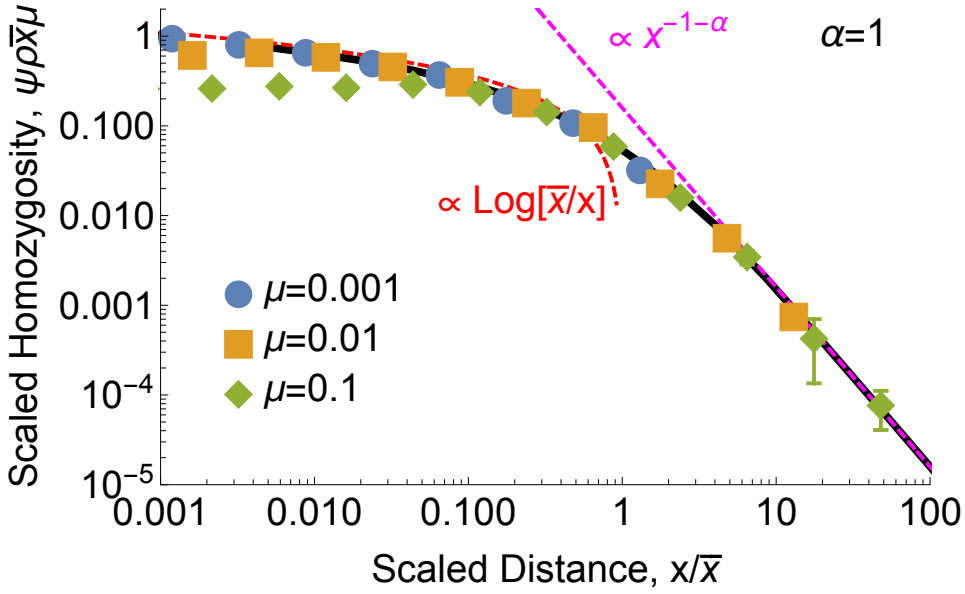
In[166]:=
```
Module[{α = 1.0, coal = .01, plotpoints,
  μvals = μlist[[2 ;; 4]], yscale = 80 000}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.001, 100}, {10^-5, 2}}], LogLogPlot[(2 * Pi)^-1 * Log[1 / x],
   {x, .001, 5}, PlotStyle → {Red, Dashed, Thickness[.005]}] ,
  LogLogPlot[Sin[Pi * α / 2] * Gamma[α + 1] * (2 * Pi)^-1 * x^-1-α,
   {x, .001, 5000}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=1", 20], Scaled[{.9, .95}]],
    Text[Style["∝ x^-1-α", Magenta, 20], Scaled[{0.66, .95}]],
    Text[Style["∝ Log[x̄/x]", Red, 20], Scaled[{0.47, .6}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[166]=



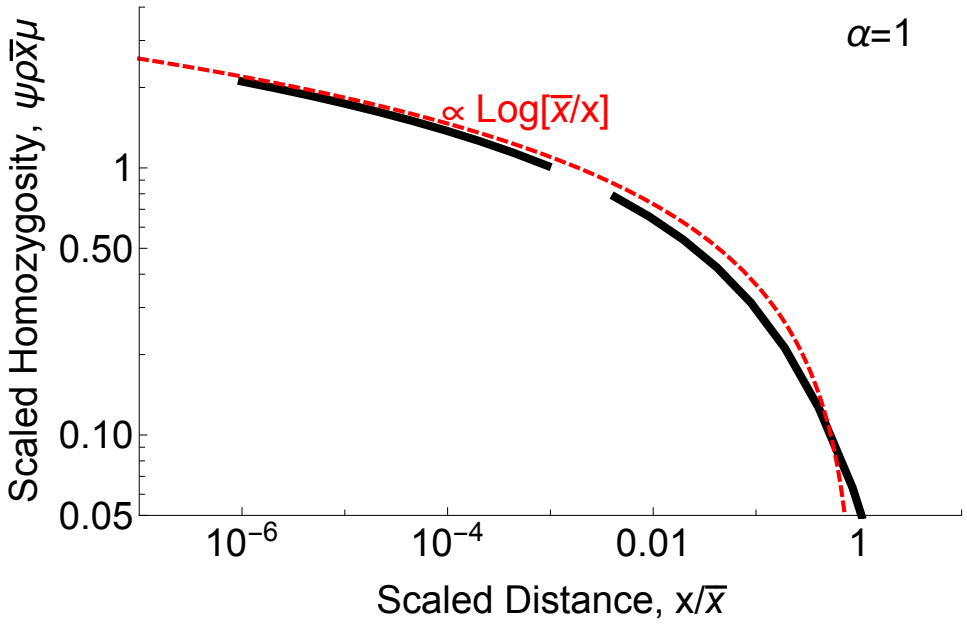
In[167]:=
```
Module[{α = 1.0, coal = .01, plotpoints,
  μvals = μlist[[2 ;; 4]], yscale = 80 000}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.001, 100}, {10^-5, 2}}], LogLogPlot[(2 * Pi)^-1 * Log[1 / x],
   {x, .001, 5}, PlotStyle → {Red, Dashed, Thickness[.005]}] ,
  LogLogPlot[Sin[Pi * α / 2] * Gamma[α + 1] * (2 * Pi)^-1 * x^(-1-α),
   {x, .001, 5000}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=1", 20], Scaled[{.9, .95}]],
    Text[Style["∝ x^(-1-α)", Magenta, 20], Scaled[{0.66, .95}]],
    Text[Style["∝ Log[x̄/x]", Red, 20], Scaled[{0.47, .6}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[167]=



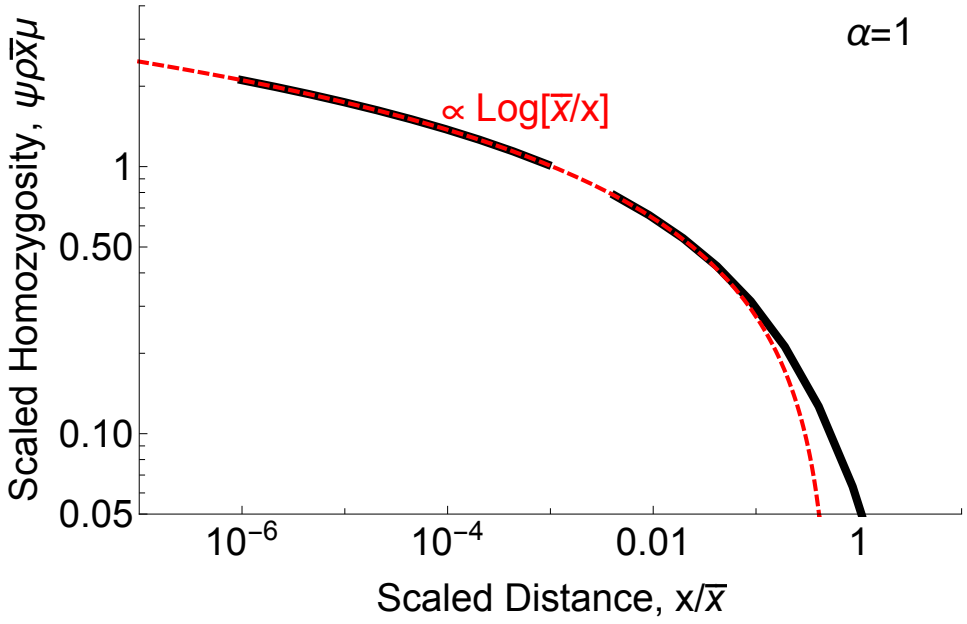
In[168]:=
```
Module[{α = 1.0, coal = .01, plotpoints,
  μvals = μlist[[2 ;; 4]], yscale = 80 000}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.0000001, 10}, {.05, 4}}], LogLogPlot[(2 * Pi)^-1 * Log[1 / x],
   {x, .0000001, 10}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  Epilog → {Text[Style["α=1", 20], Scaled[{.9, .95}]],
    Text[Style["∝ Log[x̄/x]", Red, 20], Scaled[{0.47, .8}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[168]=



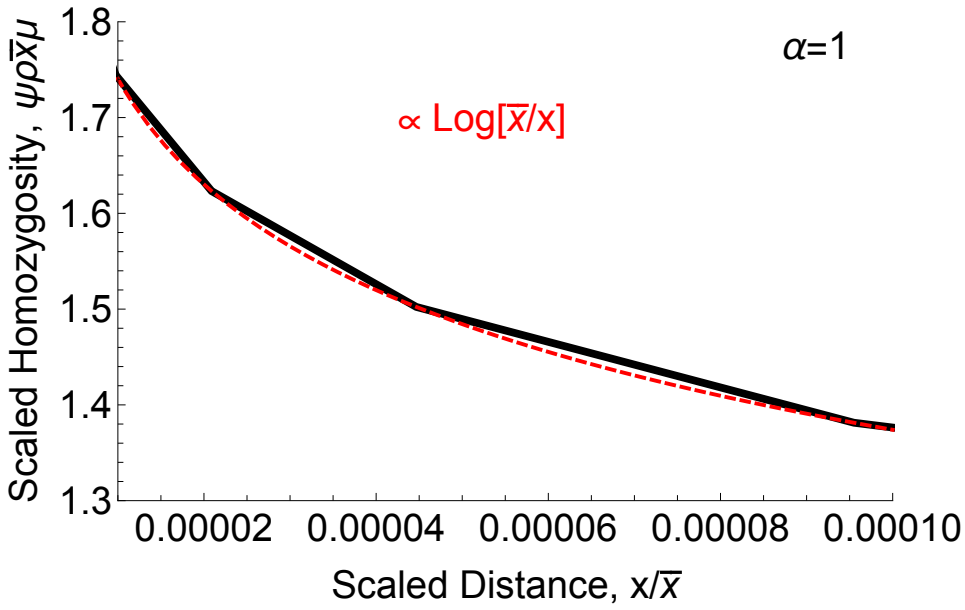
In[169]:=
```
Module[{α = 1.0, coal = .01, plotpoints,
  μvals = μlist[[2 ;; 4]], yscale = 80 000}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.0000001, 10}, {.05, 4}}],
  LogLogPlot[(2 * Pi)^-1 * (Log[1 / x] - 0.5772156649),
   {x, .0000001, 10}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  Epilog → {Text[Style["α=1", 20], Scaled[{.9, .95}]],
    Text[Style["∝ Log[x̄/x]", Red, 20], Scaled[{0.47, .8}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[169]=



In[170]:=
```
Module[{α = 1.0, coal = .01, plotpoints,
  μvals = μlist[[2 ;; 4]], yscale = 80 000}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.00001, .0001}, {1.3, 1.8}}],
  Plot[(2 * Pi)^-1 * (Log[1 / x] - 0.5772156649), {x, .00001, .0001},
   PlotStyle → {Red, Dashed, Thickness[.005]}],
  Epilog → {Text[Style["α=1", 20], Scaled[{.9, .95}]],
    Text[Style["∝ Log[x̄/x]", Red, 20], Scaled[{0.47, .8}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[170]=



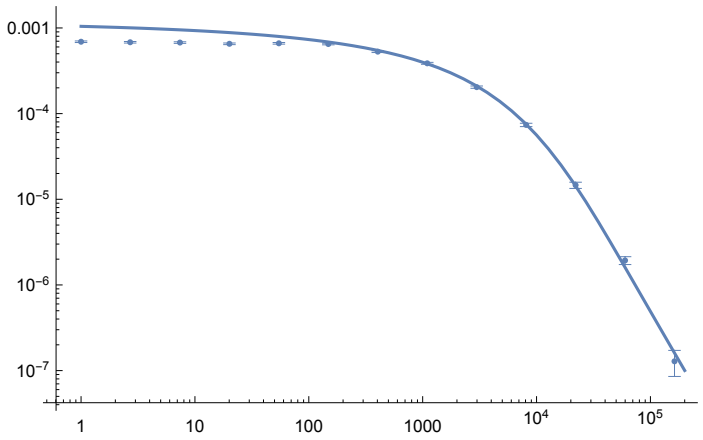
$\alpha = 1.25$

In[171]:=
```
250^1.25 / 2
```

Out[171]=
```
497.044
```

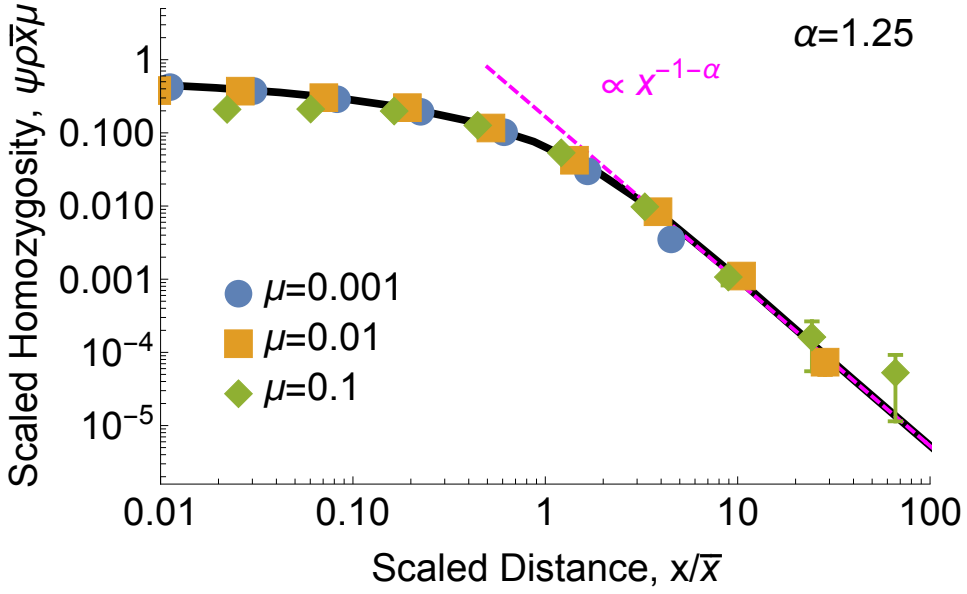
In[172]:=
```
Module[{α = 1.25, coal = 0.1, μ = 0.01, plotds},
 plotds = sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]];
 Show[ListLogLogPlot[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@ plotds],
  LogLogPlot[coeff[coal, μ, α] × fLT[x, μ, α], {x, 1, 2 × 10^5}], PlotRange → All]]
```

Out[172]=



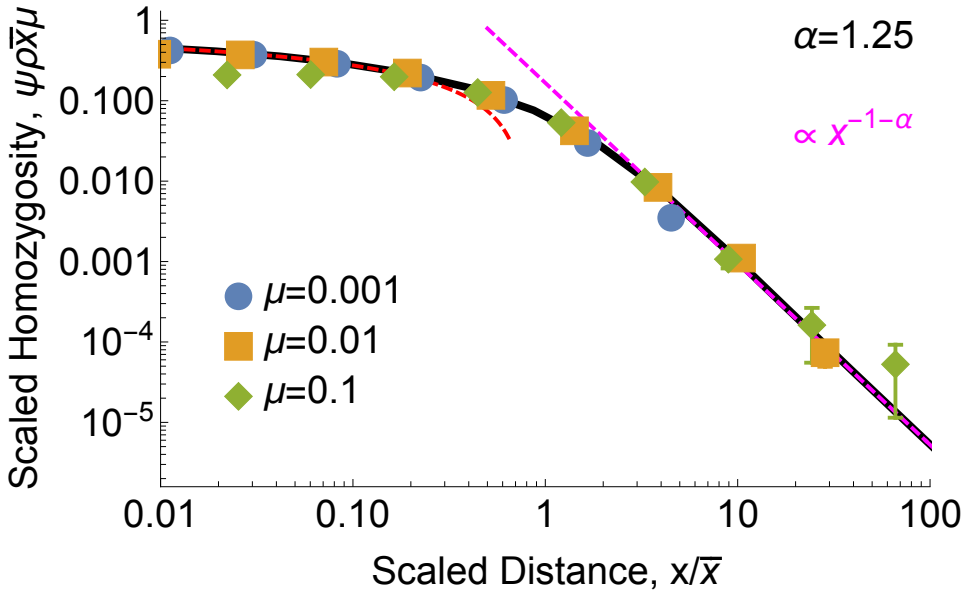
In[173]:=
```
Module[{α = 1.25, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0/xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 100}, {10^-5.8, 5}}],
  LogLogPlot[Sin[Pi * α/2] * Gamma[α + 1] * (2 * Pi)^-1 * x^(-1-α),
   {x, .5, 200}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=1.25", 20], Scaled[{.9, .95}]],
    Text[Style["∝ x^(-1-α)", Magenta, 20], Scaled[{0.65, .85}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[173]=



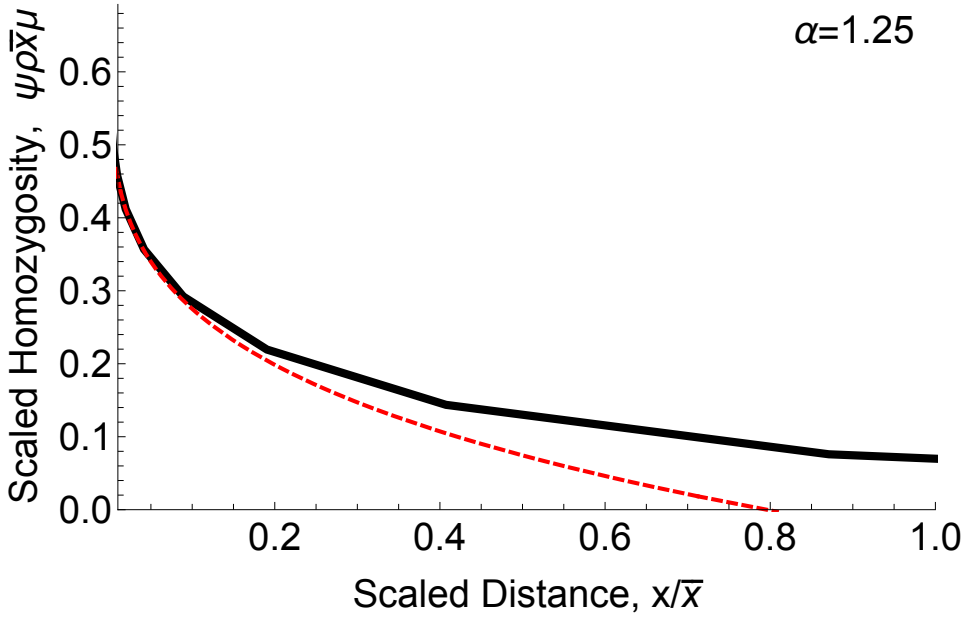
In[174]:=
```
Module[{α = 1.25, coal = .01, D = 250^(α) / 2,
  plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 100}, {10^-5.8, All}}],
  LogLogPlot[Sin[Pi * α / 2] * Gamma[α + 1] * (2 * Pi)^-1 * x^(-1-α),
   {x, .5, 200}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[(1 + α * Gamma[1 - α] * Sin[Pi / α] / (Gamma[1 - α / 2] * Gamma[α / 2]) * x^(α-1)) /
    (2 * α * Sin[Pi / α]), {x, .001, 5}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=1.25", 20], Scaled[{.9, .95}]],
    Text[Style["∝ x^(-1-α)", Magenta, 20], Scaled[{0.9, .75}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[174]=



In[175]:=
```
Module[{α = 1.25, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ 3 * xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 1}, {10^-4.5, All}}],
  Plot[(1 + α * Gamma[1 - α] * Sin[Pi / α] / (Gamma[1 - α / 2] * Gamma[α / 2]) * x^(α-1)) /
    (2 * α * Sin[Pi / α]), {x, .001, 20}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  Epilog → {Text[Style["α=1.25", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

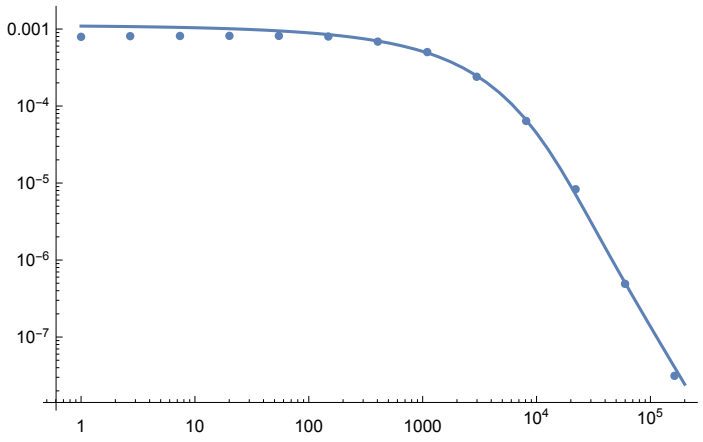
Out[175]=



α=1.45

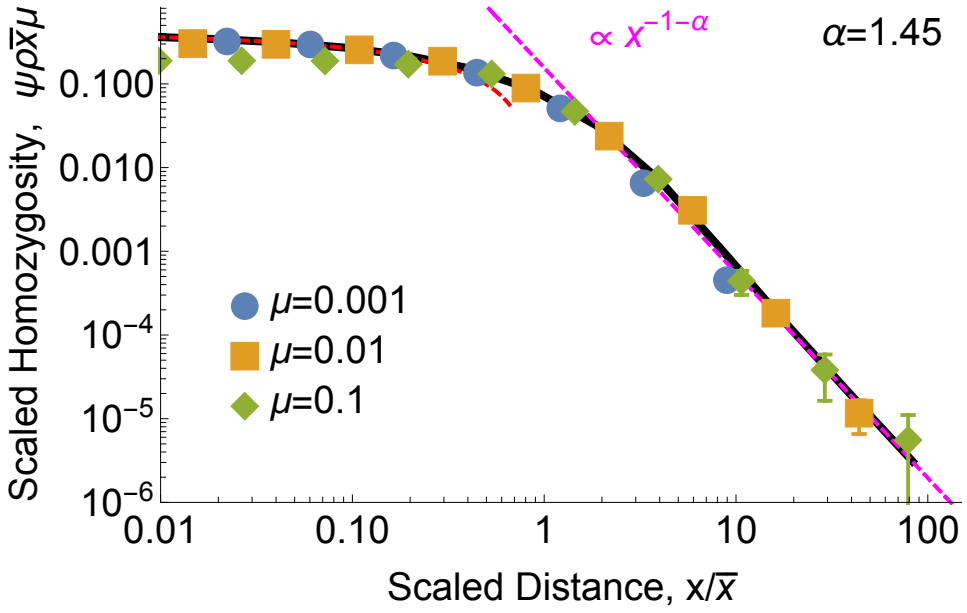
In[176]:=
```
Module[{α = 1.45, coal = 0.1, μ = 0.01, plotds},
 plotds = sims[Select[#alpha == α && #coal == coal && #mu == μ &]];
 Show[ListLogLogPlot[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@ plotds],
  LogLogPlot[coeff[coal, μ, α] × fLT[x, μ, α], {x, 1, 2 × 10^5}],
  PlotRange → All]]
```

Out[176]=



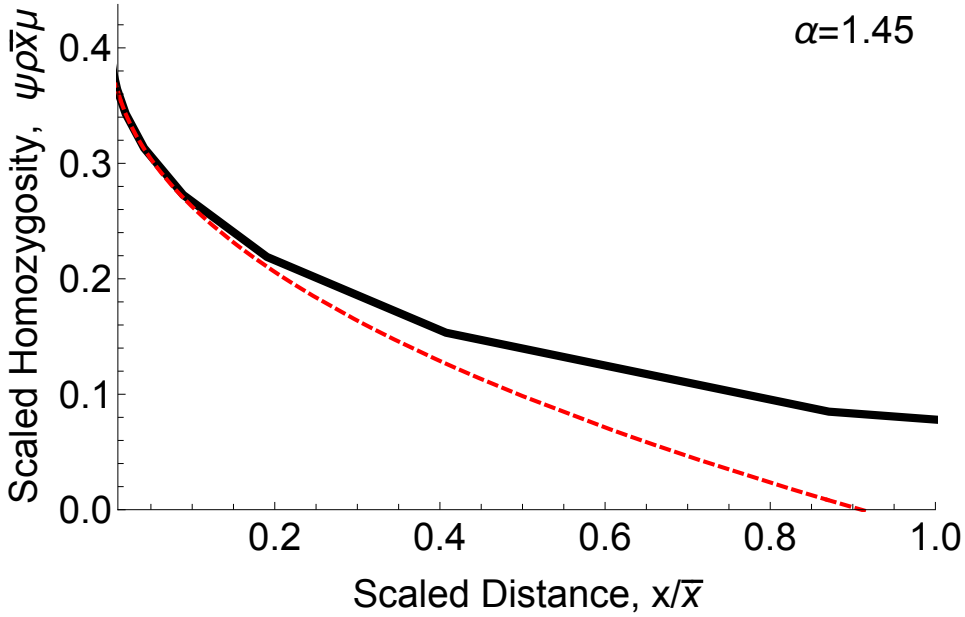
In[177]:=
```
Module[{α = 1.45, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0/xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ 1.0 * xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 150}, {10^-6, All}}],
  LogLogPlot[Sin[Pi * α/2] * Gamma[α + 1] * (2 * Pi)^-1 * x^(-1-α),
   {x, .1, 200}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[(1 + α * Gamma[1 - α] * Sin[Pi/α] / (Gamma[1 - α/2] * Gamma[α/2]) * x^(α-1)) /
    (2 * α * Sin[Pi/α]), {x, .001, 5000},
   PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=1.45", 20], Scaled[{.9, .95}]],
    Text[Style["∝ x^(-1-α)", Magenta, 20], Scaled[{0.61, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[177]=



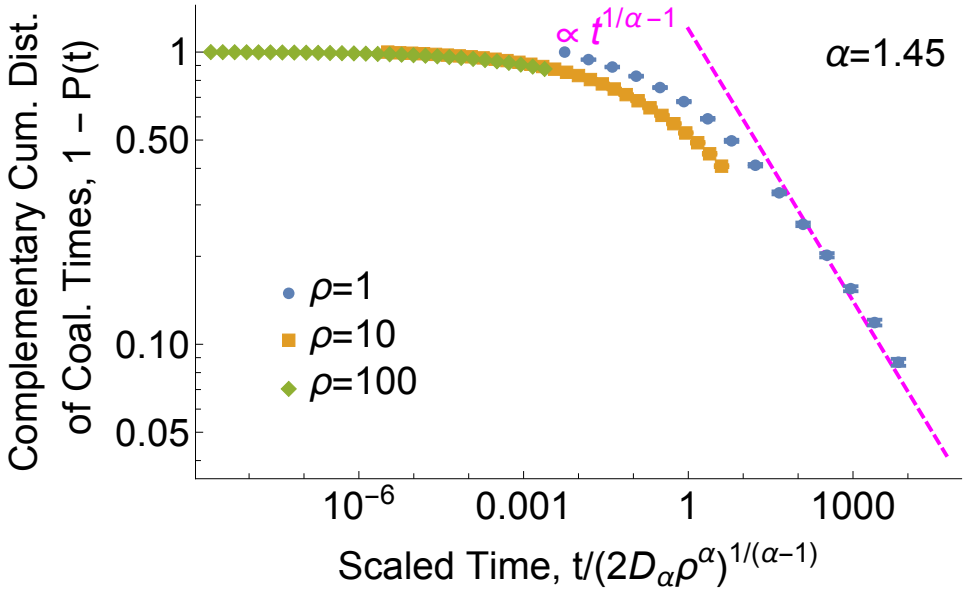
In[178]:=
```
Module[{α = 1.45, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ 3 * xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 1}, {10^-4.5, All}}],
  Plot[(1 + α * Gamma[1 - α] * Sin[Pi / α] / (Gamma[1 - α / 2] * Gamma[α / 2]) * x^(α-1)) /
    (2 * α * Sin[Pi / α]), {x, .001, 20}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  Epilog → {Text[Style["α=1.45", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[178]=



In[179]:=
```
Module[{α = 1.45, coal = .01, plotcdf, c = 5, D = 5^1.45/2},
 dummy = cdfsims[Select[#alpha == α && #coal <= 100 * coal &]];
 maxcdf = Max[dummy[All, "cdf"]];
 dummy = dummy[All, <|#, "compcdf" → 1 - #cdf|> &];
 dummy = dummy[All, <|#, "scaledT" → (2 * #coal^(-α) * D)^(1 / (1 - α)) * #T|> &];
 dummy = dummy[All, <|#, "scaleddcdfLow" → #dcdfLow|> &];

 dummy = dummy[All, <|#, "scaleddcdfHigh" → #dcdfHigh|> &];
 plotcdf = dummy;
 dummy = dummy[All, <|#, "scaledcompcdf" → #compcdf|> &];
 plotpoints =
  Table[{#scaledT, Around[#scaledcompcdf, {#scaleddcdfHigh, #scaleddcdfLow}]} & /@
    dummy[Select[#alpha == α && #coal == coal &]], {coal, {1, .1, .01}}];

 Show[LogLogPlot[(α * Sin[Pi/α]) * x^(1/α-1), {x, 1, 50 000},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], ListLogLogPlot[plotpoints ,
   PlotRange → All, PlotMarkers → Automatic,
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["ρ=" <> ToString[#], 20] & /@ {"1", "10", "100"},
     Scaled[{.2, .3}]]], Epilog → {Text[Style["α=1.45", 20], Scaled[{.9, .9}]],
    Text[Style["∝ t^(1/α-1)", Magenta, 20], Scaled[{.55, .95}]]},
  goodlabel["Scaled Time, t/(2D_α ρ^α)^(1/(α-1))", "Complementary Cum. Dist.
of Coal. Times, 1 - P(t)", 20], FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[179]=



α=1.65

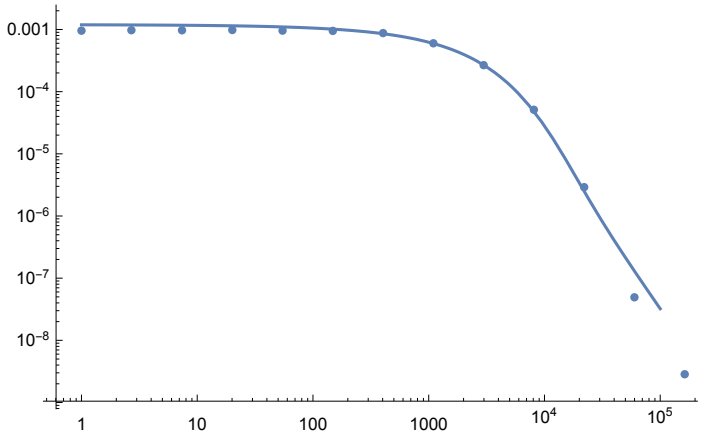
In[180]:=
```
Module[{α = 1.65, coal = 0.1, μ = 0.01, plotds},
 plotds = sims[Select[#alpha == α && #coal == coal && #mu == μ &]];
 Show[ListLogLogPlot[{#x0, Around[#hom, {#dhomLow, #dhomHigh}]} & /@ plotds],
  LogLogPlot[coeff[coal, μ, α] × fLT[x, μ, α], {x, 1, 10^5}],
  PlotRange → All]]
```

NIntegrate: DoubleExponentialOscillatory has failed to converge for the integrand $\frac{0.31831\,\text{Cos}[1.00024\,k]}{0.01+4524.5\,(0.+k)^{1.65}}$ over {0, ∞}.

DoubleExponentialOscillatory obtained 0.023814792218728027` and 2.0195266425804596`*^-6 for the integral and error estimates.

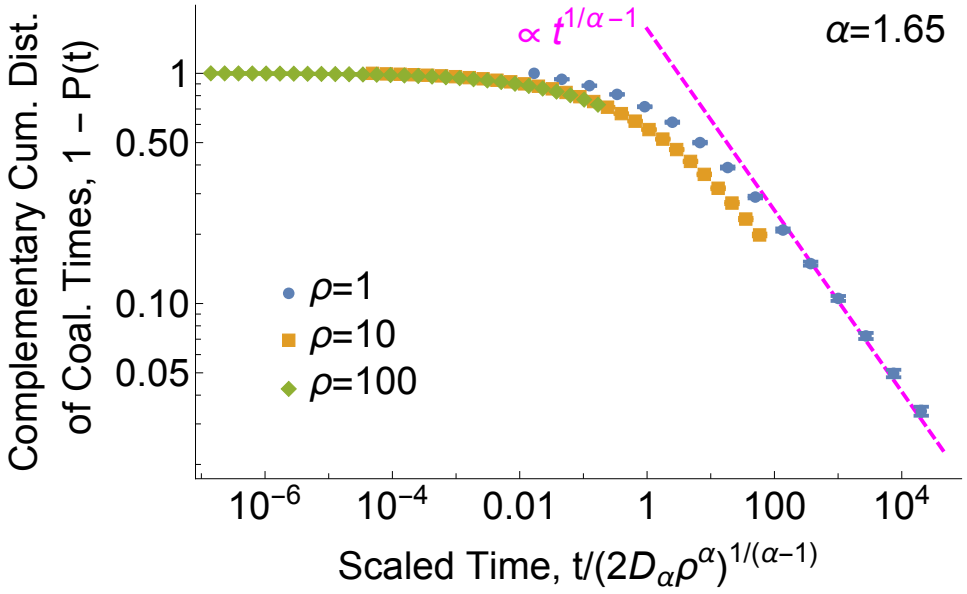
NIntegrate: NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in k near {k} = {283.95}.
NIntegrate obtained 0.02381457069255994` and 5.711868463930635`*^-7 for the integral and error estimates.

Out[180]=



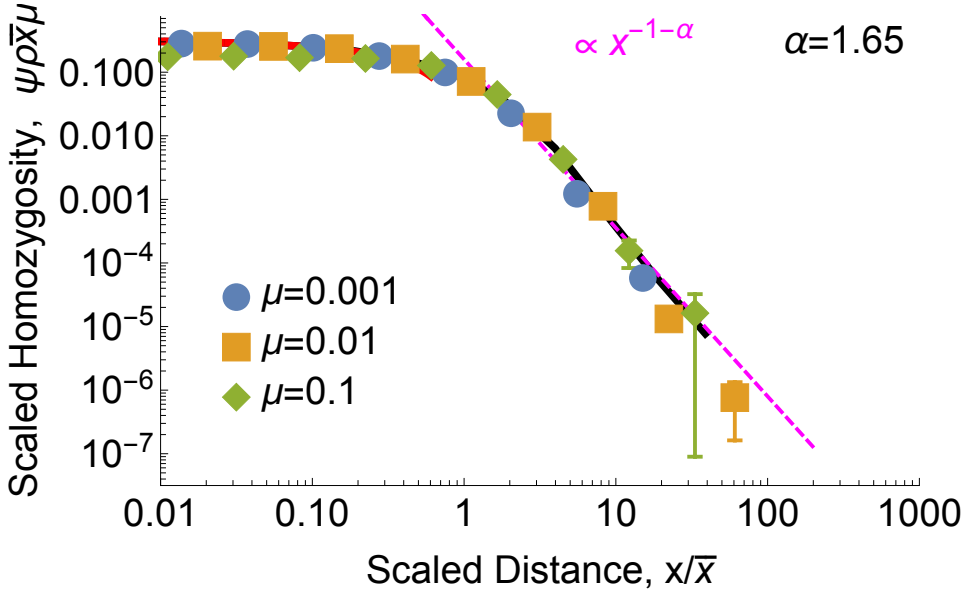
In[181]:=
```
Module[{α = 1.65, coal = .01, plotcdf, c = 5, D = 5^1.65/2},
 dummy = cdfsims[Select[#alpha == α && #coal <= 100 * coal &]];
 maxcdf = Max[dummy[All, "cdf"]];
 dummy = dummy[All, <|#, "compcdf" → 1 - #cdf|> &];
 dummy = dummy[All, <|#, "scaledT" → (2 * #coal^(-α) * D)^(1 / (1 - α)) * #T|> &];
 dummy = dummy[All, <|#, "scaleddcdfLow" → #dcdfLow|> &];

 dummy = dummy[All, <|#, "scaleddcdfHigh" → #dcdfHigh|> &];
 plotcdf = dummy;
 dummy = dummy[All, <|#, "scaledcompcdf" → #compcdf|> &];
 plotpoints =
  Table[{#scaledT, Around[#scaledcompcdf, {#scaleddcdfHigh, #scaleddcdfLow}]} & /@
    dummy[Select[#alpha == α && #coal == coal &]], {coal, {1, .1, .01}}];

 Show[LogLogPlot[(α * Sin[Pi/α]) * x^(1/α-1), {x, 1, 50 000},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], ListLogLogPlot[plotpoints ,
   PlotRange → All, PlotMarkers → Automatic,
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["ρ=" <> ToString[#], 20] & /@ {"1", "10", "100"},
     Scaled[{.2, .3}]]], Epilog → {Text[Style["α=1.65", 20], Scaled[{.9, .95}]],
    Text[Style["∝ t^(1/α-1)", Magenta, 20], Scaled[{.50, .95}]]},
  goodlabel["Scaled Time, t/(2D_α ρ^α)^(1/(α-1))", "Complementary Cum. Dist.
of Coal. Times, 1 - P(t)", 20], FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[181]=



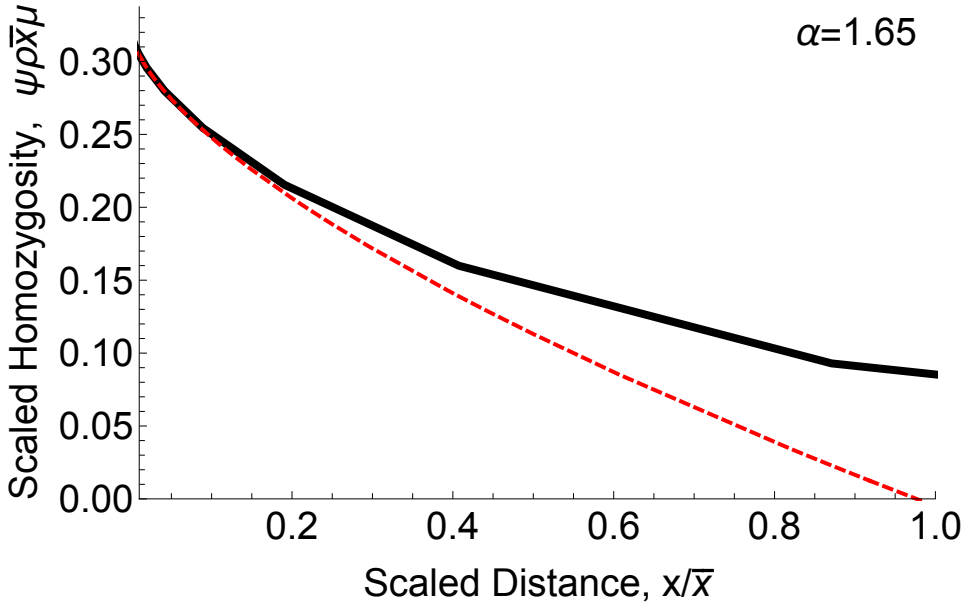
In[182]:=
```
Module[{α = 1.65, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0 / xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}] / approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ 10 * xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 1000}, {10^-7.5, All}}],
  LogLogPlot[(1 + α * Gamma[1 - α] * Sin[Pi / α] / (Gamma[1 - α / 2] * Gamma[α / 2]) * x^(α-1)) /
    (2 * α * Sin[Pi / α]), {x, .001, 500}, PlotStyle → {Red, Dashed, Thickness[.01]}],
  LogLogPlot[(2 * Pi)^-1 * x^(-1-α), {x, .01, 200},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=1.65", 20], Scaled[{.9, .95}]],
    Text[Style[" ∝ x^(-1-α)", Magenta, 20], Scaled[{0.61, 0.95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity,  ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[182]=



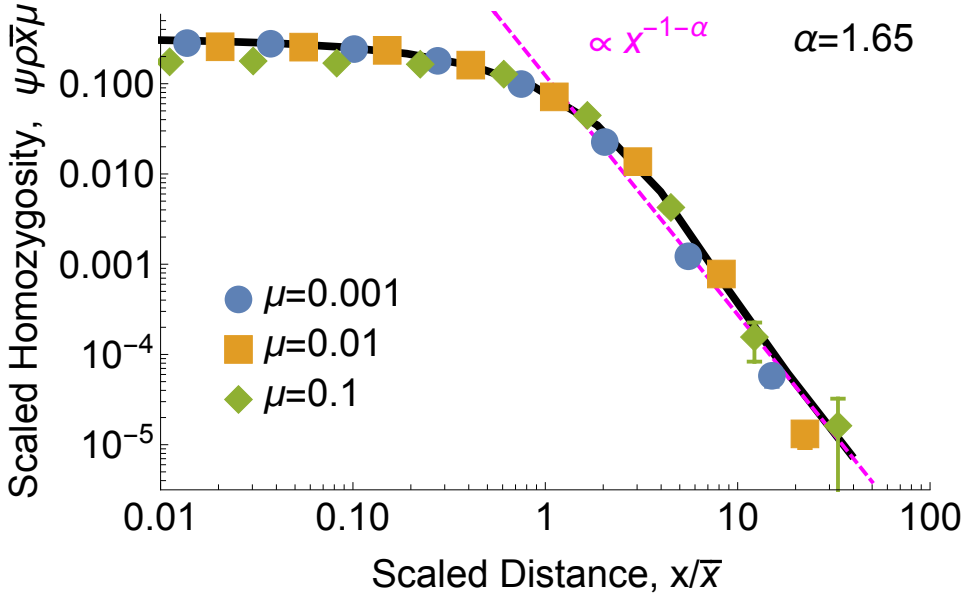
In[183]:=
```
Module[{α = 1.65, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0/xscale[#mu, #alpha], (2 * Pi)^-1 * Around[#hom, {#dhomLow, #dhomHigh}]/approxhomscale[#coal, #mu, #alpha]} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ 3 * xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 1}, {10^-4.5, All}}],
  Plot[(1 + α * Gamma[1 - α] * Sin[Pi/α]/(Gamma[1 - α/2] * Gamma[α/2]) * x^(α-1))/
    (2 * α * Sin[Pi/α]), {x, .001, 20}, PlotStyle → {Red, Dashed, Thickness[.005]}],
  Epilog → {Text[Style["α=1.65", 20], Scaled[{.9, .95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[183]=



In[184]:=
```
Module[{α = 1.65, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0/xscale[#mu, #alpha], Around[#hom, {#dhomLow, #dhomHigh}]/((2 * Pi) * approxhomscale[#coal, #mu, #alpha])} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ 1000 * xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 100}, {10^-5.5, All}}],
  LogLogPlot[Sin[Pi * α/2] * Gamma[α + 1] * (2 * Pi)^-1 * x^(-1-α),
   {x, .01, 50}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=1.65", 20], Scaled[{.9, .95}]],
    Text[Style[" ∝ x^(-1-α)", Magenta, 20], Scaled[{0.62, 0.95}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

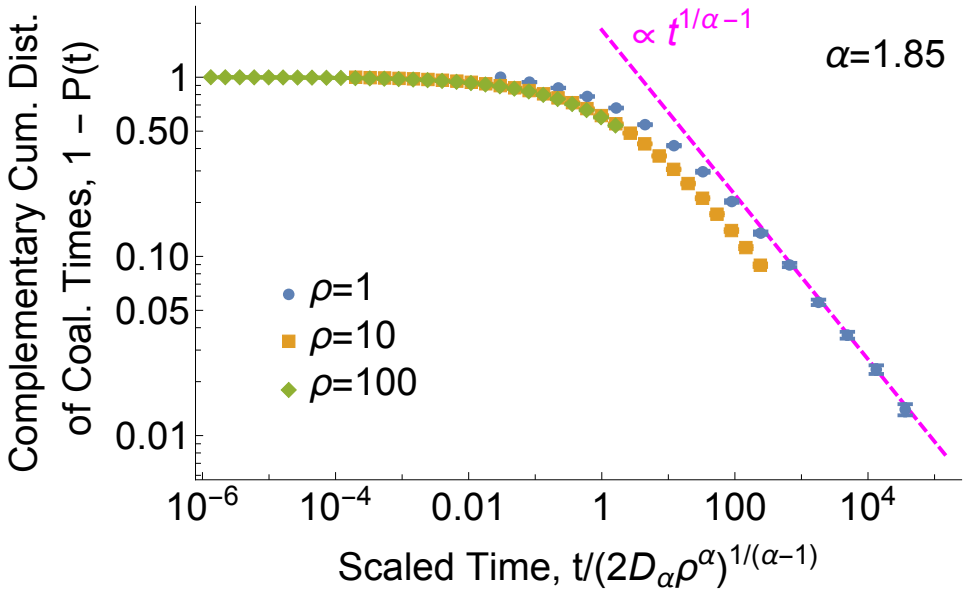
Out[184]=



α=1.85

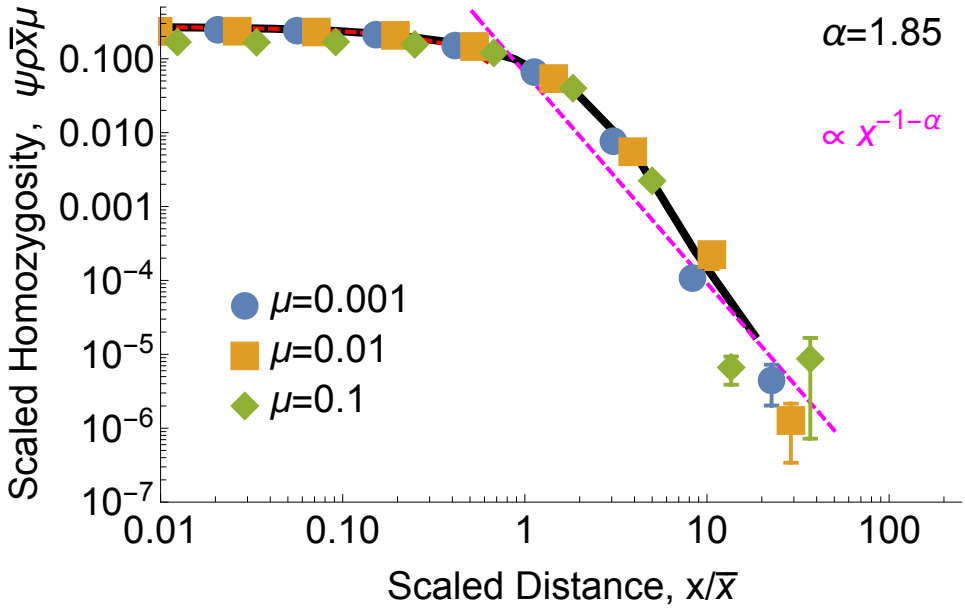
In[185]:=
```
Module[{α = 1.85, coal = .01, plotcdf, c = 5, D = 5^1.85/2},
 dummy = cdfsims[Select[#alpha == α && #coal <= 100 * coal &]];
 maxcdf = Max[dummy[All, "cdf"]];
 dummy = dummy[All, <|#, "compcdf" → 1 - #cdf|> &];
 dummy = dummy[All, <|#, "scaledT" → (2 * #coal^(-α) * D)^(1 / (1 - α)) * #T|> &];
 dummy = dummy[All, <|#, "scaleddcdfLow" → #dcdfLow|> &];

 dummy = dummy[All, <|#, "scaleddcdfHigh" → #dcdfHigh|> &];
 plotcdf = dummy;
 dummy = dummy[All, <|#, "scaledcompcdf" → #compcdf|> &];
 plotpoints =
  Table[{#scaledT, Around[#scaledcompcdf, {#scaleddcdfHigh, #scaleddcdfLow}]} & /@
    dummy[Select[#alpha == α && #coal == coal &]], {coal, {1, .1, .01}}];

 Show[LogLogPlot[(α * Sin[Pi/α]) * x^(1/α-1), {x, 1, 150000},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], ListLogLogPlot[plotpoints ,
   PlotRange → All, PlotMarkers → Automatic,
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["ρ=" <> ToString[#], 20] & /@ {"1", "10", "100"},
     Scaled[{.2, .3}]]], Epilog → {Text[Style["α=1.85", 20], Scaled[{.9, .9}]],
    Text[Style["∝ t^(1/α-1)", Magenta, 20], Scaled[{.65, .95}]]},
  goodlabel["Scaled Time, t/(2D_α ρ^α)^(1/(α-1))", "Complementary Cum. Dist.
of Coal. Times, 1 - P(t)", 20], FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[185]=



In[186]:=
```
Module[{α = 1.85, coal = .01, plotpoints, μvals = μlist[[2 ;; 4]]}, plotpoints =
  Table[{#x0/xscale[#mu, #alpha], Around[#hom, {#dhomLow, #dhomHigh}]/(2 * Pi * approxhomscale[#coal, #mu, #alpha])} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 > 0 &]], {μ, μvals}];
 Show[ListLogLogPlot[Table[{x, (2 * Pi)^-1 * intvals[α][x]},
    {x, Select[xlist, # ≤ xmaxes[α] &]}],
   Joined → True, PlotStyle → {Black, Thickness[.01]},
   PlotRange → {{.01, 250}, {10^-7, .5}}],
  LogLogPlot[Sin[Pi * α/2] * Gamma[α + 1] * (2 * Pi)^-1 * x^(-1-α),
   {x, .01, 50}, PlotStyle → {Magenta, Dashed, Thickness[.005]}],
  LogLogPlot[(1 + α * Gamma[1 - α] * Sin[Pi/α]) / (Gamma[1 - α/2] * Gamma[α/2]) * x^(α-1)) /
    (2 * α * Sin[Pi/α]), {x, .001, 500 000},
   PlotStyle → {Red, Dashed, Thickness[.005]}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=1.85", 20], Scaled[{.9, .95}]],
    Text[Style["∝ x^(-1-α)", Magenta, 20], Scaled[{0.9, .75}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity, ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

Out[186]=



α=2.05 (*F* distribution)

Null

In[187]:=
```
df = 0.5 Module[{α = 2.05}, Moment[FRatioDistribution[2 α, 2 α], 2]]
```

Out[187]= 59.5476

In[188]:=
```
250^2.05 / 2
```

Out[188]= 41 185.8

In[189]:=

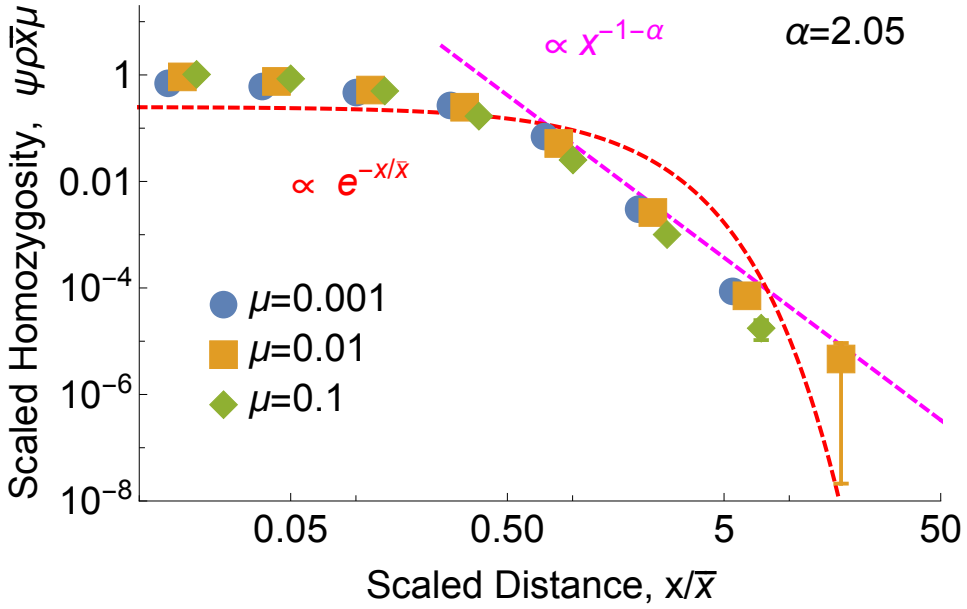
In[190]:=
```
Sin[Pi * 2.05 / 2]
```

Out[190]= −0.0784591

In[191]:=

In[192]:=
```
Module[{α = 2.05, coal = .01, D = (179.675)^2/2, plotpoints, μvals = μlist[[2 ;; 4]]},
 plotpoints = Table[{#x0/Sqrt[D/#mu], Around[#hom, {#dhomLow, #dhomHigh}]/(2 * Pi * approxhomscale[#coal, #mu, 2.00, D])} & /@
    sims[Select[#alpha == α && #coal == coal && #mu == μ && #x0 ≥ 1 && #x0 < 50 000 &]],
   {μ, μvals}];
 Show[LogLogPlot[(Gamma[2 * α]/(4 * Gamma[α]^2)) *
    (.25 * α * (2 * α - 1) * (α - 2)^-1 * (α - 1)^-2 + α^2/(2 * α - 2)^2)^(-α/2) * x^(-1-α),
   {x, .25, 55}, PlotStyle → {{Magenta, Dashed, Thickness[.005]}},
   PlotRange → {{.01, 50}, {10^-8, 20}}], LogLogPlot[Exp[-x]/(4), {x, .0001, 55},
   PlotStyle → {{Red, Dashed, Thickness[.005]}, {Red, Dotted, Thickness[.005]}},
   PlotRange → {{.01, 10}, {10^-8, 20}}],
  ListLogLogPlot[plotpoints, PlotMarkers → {Automatic, 15},
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["μ=" <> ToString[#], 20] & /@ μvals, {.2, .3}]],
  Epilog → {Text[Style["α=2.05", 20], Scaled[{.88, .95}]],
    Text[Style["∝ x^(-1-α)", Magenta, 20], Scaled[{.58, 0.93}]],
    Text[Style["  ∝  e^(-x/x̄)", Red, 20], Scaled[{.25, .65}]]},
  goodlabel["Scaled Distance, x/x̄", "Scaled Homozygosity,  ψρx̄μ", 20],
  FrameStyle → Directive[20, Black], ImageSize → 500]]
```

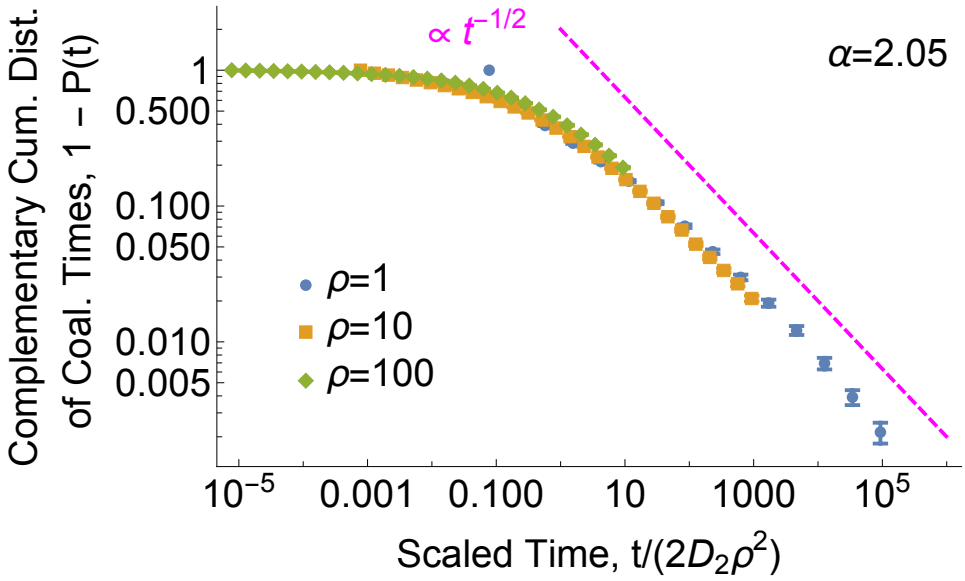
Out[192]=



All

In[193]:=
```
Module[{α = 2.05, coal = .01, plotcdf, c = 3.59, D = (3.59)^2/2},
 dummy = cdfsims[Select[#alpha == α && #coal <= 100 * coal &]];
 maxcdf = Max[dummy[All, "cdf"]];
 dummy = dummy[All, <|#, "compcdf" → 1 - #cdf|> &];
 dummy = dummy[All, <|#, "scaledT" → (2 * #coal^(-2) * D)^(1/(1 - 2)) * #T|> &];
 dummy = dummy[All, <|#, "scaleddcdfLow" → #dcdfLow|> &];

 dummy = dummy[All, <|#, "scaleddcdfHigh" → #dcdfHigh|> &];
 plotcdf = dummy;
 dummy = dummy[All, <|#, "scaledcompcdf" → #compcdf|> &];
 plotpoints =
  Table[{#scaledT, Around[#scaledcompcdf, {#scaleddcdfHigh, #scaleddcdfLow}]} & /@
    dummy[Select[#alpha == α && #coal == coal &]], {coal, {1, .1, .01}}];

 Show[LogLogPlot[(2 * Sin[Pi/2]) * x^(-1/2), {x, 1, 1000000},
   PlotStyle → {Magenta, Dashed, Thickness[.005]}], ListLogLogPlot[plotpoints ,
   PlotRange → All, PlotMarkers → Automatic,
   IntervalMarkersStyle → <|"WhiskerStyle" → Thick, "FenceStyle" → Thick|>,
   PlotLegends → Placed[Style["ρ=" <> ToString[#], 20] & /@ {"1", "10", "100"},
     Scaled[{.2, .3}]]], Epilog → {Text[Style["α=2.05", 20], Scaled[{.9, .9}]],
    Text[Style["∝ t^(-1/2)", Magenta, 20], Scaled[{.35, .95}]]},
  goodlabel["Scaled Time, t/(2D2ρ^2)", "Complementary Cum. Dist.
of Coal. Times, 1 - P(t)", 20], FrameStyle → Directive[20, Black], ImageSize → 500,
  PlotRange → All, Axes → False]]
```

Out[193]=



In[194]:=

In[195]:= `lineStyle = {Thick, Black, Dashed};`

In[196]:= `lineStyle2 = {Thick, Black};`

In[197]:= `lineStyle3 = {Thick, Blue};`

In[198]:= `lineStyle4 = {Thick, Green};`

In[199]:= `lineStyle5 = {Thick, Red};`

In[200]:= `lineStyle6 = {Thick, Purple};`

In[201]:= `lineStyle7 = {Thick, Yellow};`

In[202]:= `lineStyle8 = {Thick, White};`

In[203]:= `line99 = Line[{{-.5, 0}, {-.5, 2}}];`

In[204]:= `line0 = Line[{{0, 0}, {0, 2}}];`

In[205]:= `line1 = Line[{{1, 0}, {1, 1}}];`

In[206]:= `line2 = Line[{{2, 0}, {2, 1}}];`

In[207]:= `line3 = Line[{{3, 0}, {3, 2}}];`

In[208]:=

In[209]:=

In[210]:= `line999 = Line[{{-.5, 0}, {-.5, 2}}];`

In[211]:= `line00 = Line[{{0, 0}, {0, 2}}];`

In[212]:= `line11 = Line[{{1, 1}, {1, 2}}];`

In[213]:= `line22 = Line[{{2, 0}, {2, 2}}];`

In[214]:= `line33 = Line[{{3, 0}, {3, 2}}];`

In[215]:= `line44 = Line[{{0, 2}, {3, 2}}];`

In[216]:= `line55 = Line[{{0, 0}, {3, 0}}];`

In[217]:= `line66 = Line[{{0, 1}, {3, 1}}];`

In[218]:= `line77 = Line[{{0, .5}, {1, .5}}];`

In[219]:= `line88 = Line[{{0, .5}, {3, .5}}];`

In[220]:=
```
line999 = Line[{{-.5, 0}, {-.5, -2}}];
```

In[221]:=
```
line000 = Line[{{0, 0}, {0, -2}}];
```

In[222]:=
```
line111 = Line[{{1, 0}, {1, -1}}];
```

In[223]:=
```
line222 = Line[{{2, 0}, {2, -1}}];
```

In[224]:=
```
line333 = Line[{{3, 0}, {3, -2}}];
```

In[225]:=
```
line444 = Line[{{0, -2}, {3, -2}}];
```

In[226]:=
```
line555 = Line[{{0, 0}, {3, 0}}];
```

In[227]:=
```
line666 = Line[{{0, -1}, {3, -1}}];
```

In[228]:=
```
line777 = Line[{{0, .3}, {1, .3}}];
```

In[229]:=
```
line888 = Line[{{0, .3}, {3, .3}}];
```

In[230]:=
```
line999 = Line[{{0, 0}, {3, 0}}];
```

In[231]:=
```
(D2 μ)^1/2
```

Out[231]=
$$\sqrt{\mu\, D_2}$$

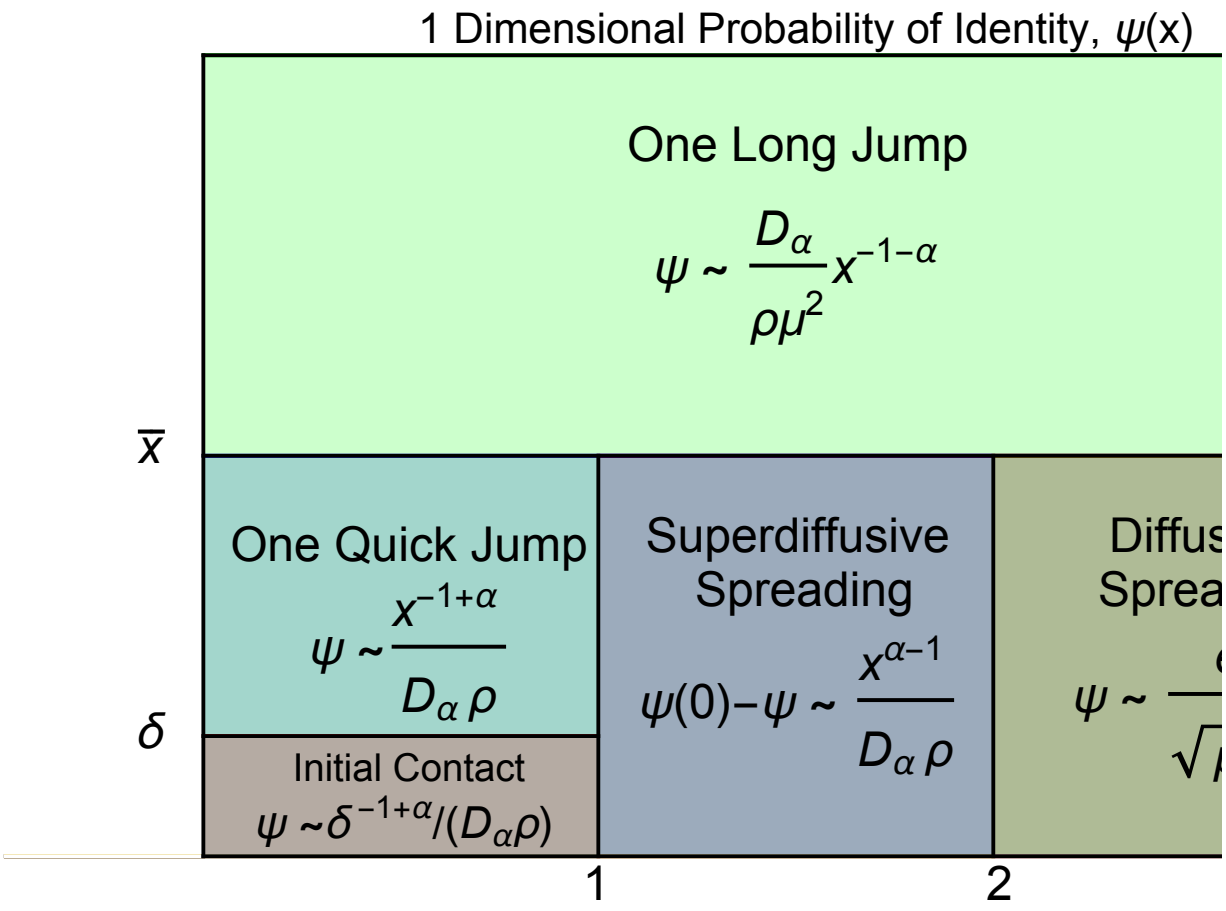
In[232]:=
```
(Dμ)^1/2
```

Out[232]=
$$\sqrt{D\mu}$$

In[233]:=

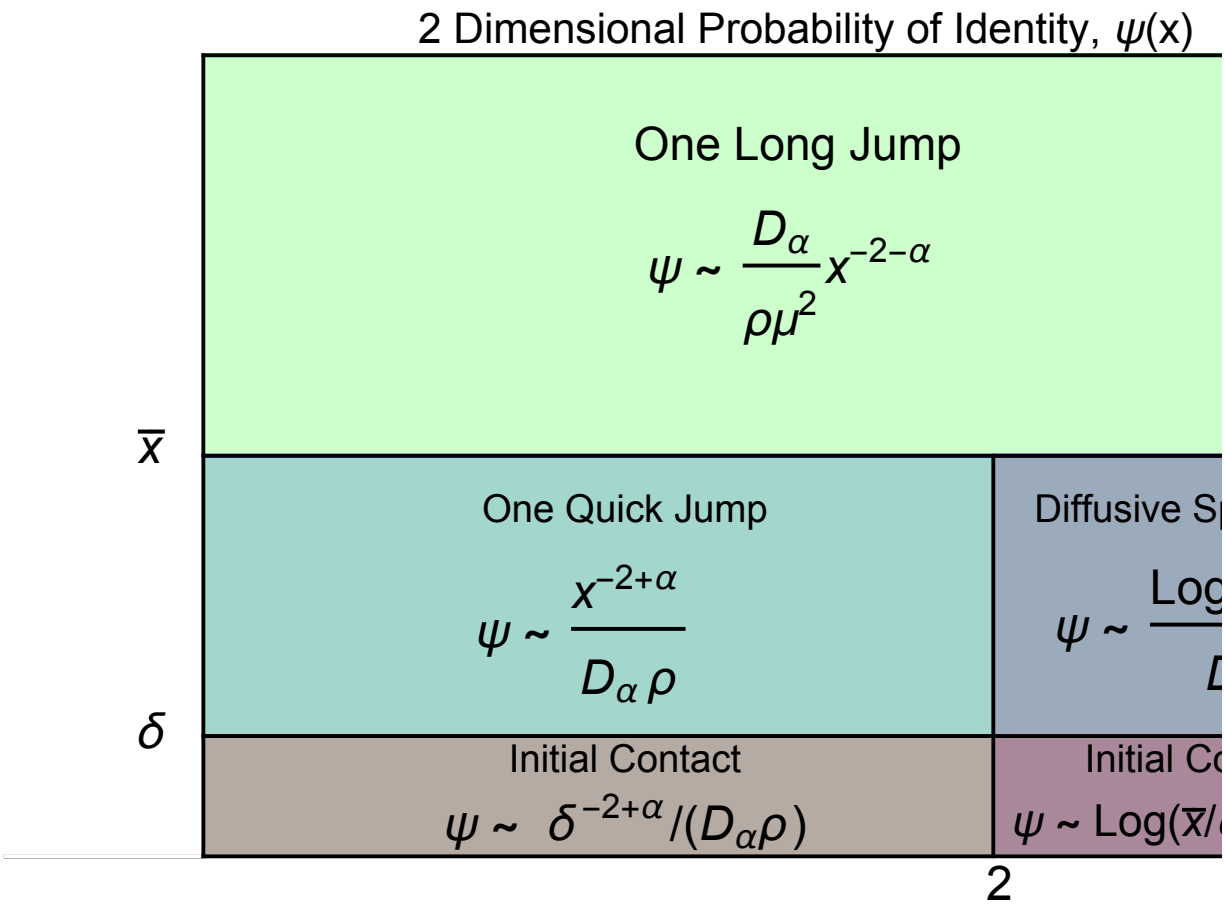
In[234]:=
```
Show[Plot[{0, UnitStep[x] , 2 * UnitStep[x] , .3 * UnitStep[x] - .3 * UnitStep[x - 1],
    UnitStep[x - 1] - UnitStep[x - 4],  UnitStep[x - 2], UnitStep[x - 3]},
  {x, -.5 , 3 }, LabelStyle → Directive[Bold, Black],
  PlotStyle → {Directive[lineStyle2], Directive[lineStyle3],
    Directive[lineStyle4], Directive[lineStyle5], Directive[lineStyle6],
    Directive[lineStyle7], Directive[lineStyle8], Directive[lineStyle8]},
  Epilog → {Directive[lineStyle2], Text[Style["1", Black, 25], Scaled[{.43, .020}]],
    Text[Style["2", Black, 25], Scaled[{.71, .020}]],
    Text[Style["x̄", Black, 25], Scaled[{.12, .51}]],
    Text[Style["δ", Black, 25], Scaled[{.12, .19}]],
    Text[Style["One Long Jump", Black, 25], Scaled[{.57, .85}]],
    Text[Style["ψ ~ D_α/(ρμ^2) x^(-1-α) ", Black, 25], Scaled[{.57, .70}]],
    Text[Style["Superdiffusive", Black, 25], Scaled[{.57, .41}]],
    Text[Style[" Spreading", Black, 25], Scaled[{.57, .35}]],
    Text[Style["ψ(0)-ψ ~ x^(α-1)/(D_α ρ)", Black, 25], Scaled[{.57, .22}]],
    Text[Style["ψ ~x^(-1+α)/(D_α ρ)", Black, 25], Scaled[{.30, .28}]] ,
    Text[Style["One Quick Jump", Black, 25], Scaled[{.3, .40}]],
    Text[Style["ψ ~δ^(-1+α)/(D_α ρ)", Black, 23], Scaled[{.296, .09}]],
    Text[Style["Initial Contact", Black, 20], Scaled[{.30, .15}]] ,
    Text[Style["  ψ ~ e^(-x/x̄)/Sqrt[μ D_2 ρ]", Black, 25], Scaled[{.85, .22}]],
    Text[Style["1 Dimensional Probability of Identity, ψ(x) ", Black, 22],
     Scaled[{.58, .98}]], Text[Style["Diffusive", Black, 25], Scaled[{.85, .41}]],
    Text[Style[" Spreading", Black, 25], Scaled[{.85, .35}]], line0,
    line1, line2, line3, line44, line66, line777, line999},
  ImageSize → 750, Filling -> Bottom], Frame → False,
 Axes → False, FrameTicksStyle →
  {{Directive[FontOpacity → 1, FontSize → 1], Automatic}, {Automatic, Automatic}}]
```

Out[234]=


1 Dimensional Probability of Identity, ψ(x)
One Long Jump
ψ ~ D_α / (ρμ^2) x^(−1−α)
One Quick Jump
ψ ~ x^(−1+α) / (D_α ρ)
Initial Contact
ψ ~δ^(−1+α)/(D_α ρ)
Superdiffusive Spreading
ψ(0)−ψ ~ x^(α−1) / (D_α ρ)
Diffus
Sprea
ψ ~
√
x̄
δ
1
2

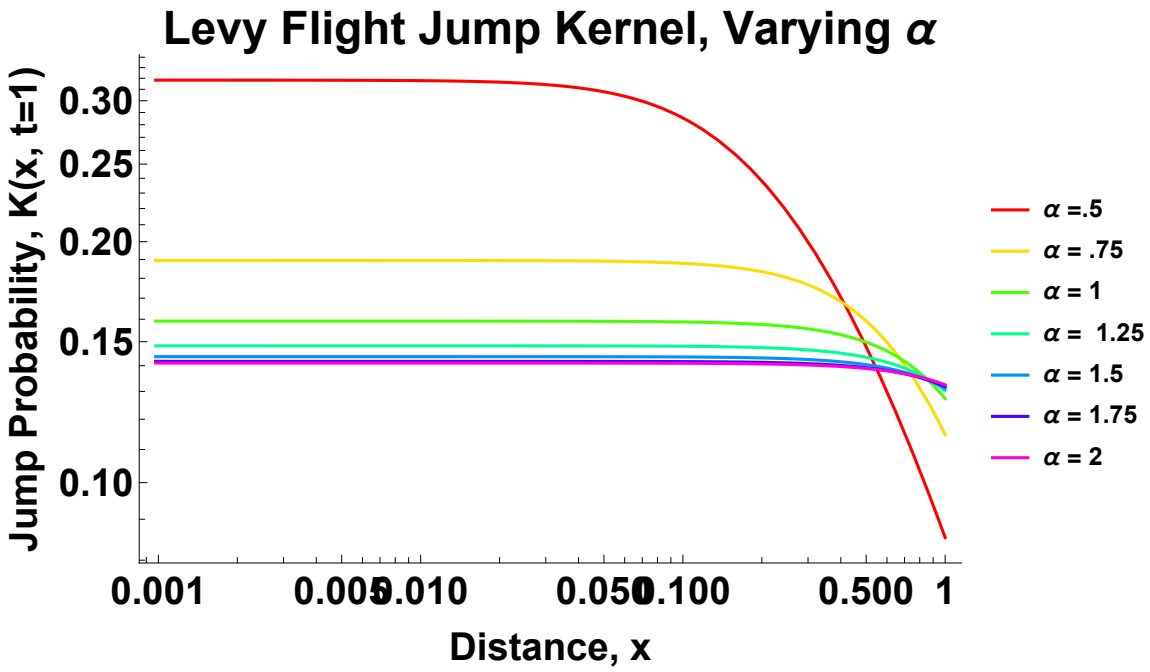
In[235]:=
```
Show[Plot[{0, UnitStep[x], 2 * UnitStep[x] - 2 * UnitStep[x - 3], .3 * UnitStep[x],
    UnitStep[x - 2], 0, 0 }, {x, -.5 , 3 }, LabelStyle → Directive[Bold, Black],
  PlotStyle → {Directive[lineStyle2], Directive[lineStyle3], Directive[lineStyle4],
    Directive[lineStyle5], Directive[lineStyle6], Directive[lineStyle8],
    Directive[lineStyle8] }, Epilog → {Directive[lineStyle2],
    Text[Style["x̄", Black, 25], Scaled[{.12, .51}]],
    Text[Style["δ", Black, 25], Scaled[{.12, .19}]],
    Text[Style["One Long Jump", Black, 24], Scaled[{.57, .85}]],
    Text[Style["ψ ~ D_α/(ρμ^2) x^(-2-α) ", Black, 25], Scaled[{.57, .70}]],
    Text[Style["One Quick Jump", Black, 20], Scaled[{.45, .44}]],
    Text[Style["ψ ~ x^(-2+α)/(D_α ρ)", Black, 25], Scaled[{.42, .3}]],
    Text[Style["ψ ~  δ^(-2+α)/(D_α ρ)", Black, 25], Scaled[{.45, .09}]],
    Text[Style["ψ ~ Log(x/x)/(D_2 ρ)", Black, 25], Scaled[{.85, .3}]],
    Text[Style["Diffusive Spreading", Black, 20], Scaled[{.85, .44}]],
    Text[Style["Initial Contact", Black, 20], Scaled[{.45, .16}]],
    Text[Style[" ψ ~ Log(x̄/δ)/(D_2 ρ)", Black, 23], Scaled[{.84, .09}]],
    Text[Style["Initial Contact", Black, 20], Scaled[{.85, .16}]],
    Text[Style[" 2 ", Black, 25], Scaled[{.71, .020}]],
    Text[Style["2 Dimensional Probability of Identity, ψ(x) ", Black, 22],
     Scaled[{.58, .98}]], line0, line2, line3, line44, line55, line66, line888},
  ImageSize → 750, Filling -> Bottom], Frame → False,
 Axes → False, FrameTicksStyle →
  {{Directive[FontOpacity → 1, FontSize → 1], Automatic}, {Automatic, Automatic}}]
```

Out[235]=


2 Dimensional Probability of Identity, ψ(x)
One Long Jump
ψ ~ D_α / (ρ μ^2) x^(−2−α)
One Quick Jump
ψ ~ x^(−2+α) / (D_α ρ)
Diffusive S
ψ ~ Log
Initial Contact
ψ ~ δ^(−2+α) / (D_α ρ)
Initial C
ψ ~ Log(x̄/
x̄
δ
2

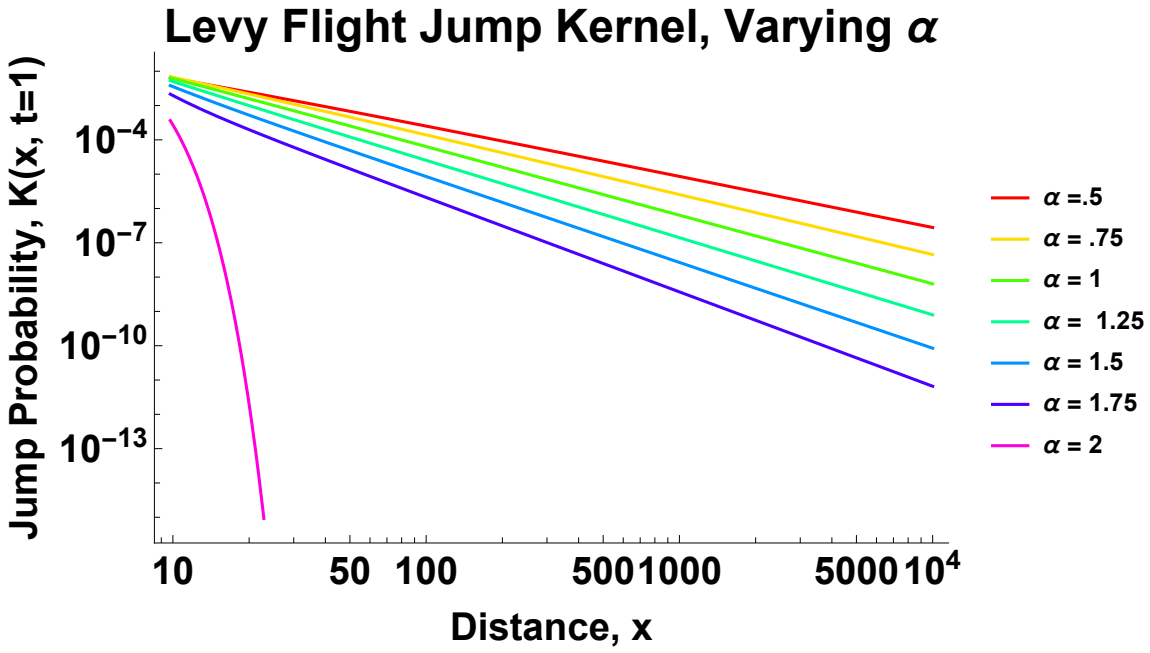
In[236]:=
```
Show[LogLogPlot[Table[PDF[StableDistribution[0, α, 0, 0, 2], x],
    {α, {.5, .75, 1, 1.25, 1.5, 1.75, 2}}] // Evaluate, {x, 0, 1},
  PlotStyle → Table[Hue[h - .1, 1, 1], {h, .1, 1.1 - 1 / 7, 1 / 7}] ,
  LabelStyle → Directive[Bold, Black],
  PlotLabel → Style["Levy Flight Jump Kernel, Varying α", FontSize → 24],
  PlotLegends → {"α =.5", "α = .75", "α = 1",
    "α =  1.25", "α = 1.5", "α = 1.75", "α = 2"}, Frame → True,
  FrameLabel → {"xlabel", "ylabel"}, FrameStyle → {{None, Black}, {None, Black}}],
 goodlabel["Distance, x", "Jump Probability, K(x, t=1)", 20],
 FrameStyle → Directive[20, Black], ImageSize → 500,
 PlotRange → All, Axes → False]
```

Out[236]=



In[237]:=
```
Show[LogLogPlot[Table[PDF[StableDistribution[0, α, 0, 0, 2], x],
    {α, {.5, .75, 1, 1.25, 1.5, 1.75, 2}}] // Evaluate, {x, 0, 10 000},
  PlotStyle → Table[Hue[h - .1, 1, 1], {h, .1, 1.1 - 1 / 7, 1 / 7}] ,
  LabelStyle → Directive[Bold, Black],
  PlotLabel → Style["Levy Flight Jump Kernel, Varying α", FontSize → 24],
  PlotLegends → {"α =.5", "α = .75", "α = 1",
    "α =  1.25", "α = 1.5", "α = 1.75", "α = 2"}, Frame → True,
  FrameLabel → {"xlabel", "ylabel"}, FrameStyle → {{None, Black}, {None, Black}}],
 goodlabel["Distance, x", "Jump Probability, K(x, t=1)", 20],
 FrameStyle → Directive[20, Black], ImageSize → 500,
 PlotRange → All, Axes → False]
```

Out[237]=



In[238]:=

In[239]:=

1D Asymptotics

In[240]:=
```
(*These are expressions for ψ(x)/(1-ψ(0)) *)
```

$x << \delta$

```
In[241]:= (2 * Pi * ρ * D)^-1 * Integrate[Exp[-k^2 * delta^2 / 2] / (k^a),
            {k, 0, Infinity}, Assumptions -> {x > 0, delta > 0, a > 0}]
```

$$\text{Out[241]= ConditionalExpression}\left[\frac{2^{-\frac{3}{2}-\frac{a}{2}}\,\text{delta}^{-1+a}\,\text{Gamma}\left[\frac{1}{2}-\frac{a}{2}\right]}{D\,\pi\,\rho},\ a<1\right]$$

$\delta << x << \overline{x}$

```
In[242]:= (4 * Pi * ρ * D)^-1 * (2 * Pi)^1/2 InverseFourierTransform[
            Abs[k]^(-a), k, x]
```

$$\text{Out[242]= }\frac{\text{Abs}[x]^{-1+a}\,\text{Gamma}[1-a]\,\text{Sin}\left[\frac{a\pi}{2}\right]}{2\,D\,\pi\,\rho}$$

$x >> \overline{x}$

```
In[243]:= (4 * Pi * ρ * mu * xbar)^-1 * xbar^(a + 1) * (2 * Pi)^1/2 InverseFourierTransform[
            Abs[k]^(a), k, x]
```

$$\text{Out[243]= }-\frac{\text{xbar}^{a}\,\text{Abs}[x]^{-1-a}\,\text{Gamma}[1+a]\,\text{Sin}\left[\frac{a\pi}{2}\right]}{2\,\text{mu}\,\pi\,\rho}$$

```
In[244]:= (*Note that xbar = (D/mu)^1/a*)
```