Homework 1: question 5

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Wei-Ting Lin

psID: 1192656

The Birth-Death-Immigration model

$$\frac{dn}{dt} = (\lambda - \mu)n - v$$

1. Show that the mean of the of the corresponding stochastic model, ?n, equals the solution of this differential equation, with an appropriate initial condition.

2. Give an example to show that this is not true for more general birth and death processes with immigration, and illustrate the example numerically.

clean workspace and set parameters

```
clear;clc
```

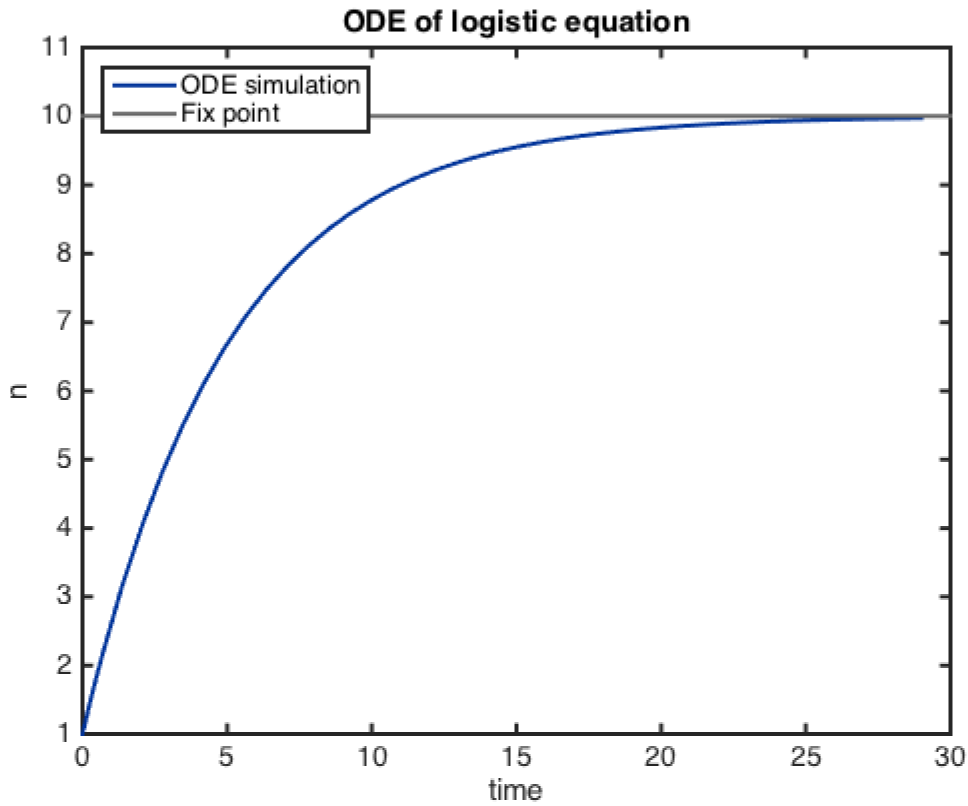
Parameters

```
global rB   rD   Im
    rB = 1; rD = 1.2; Im = 2;
% equilibrium:
    n_fp = Im/(rD - rB);
% simulation
    tlim = 30;
    n0 = 1;
    it = 100; % interation of simulation
```

ODE

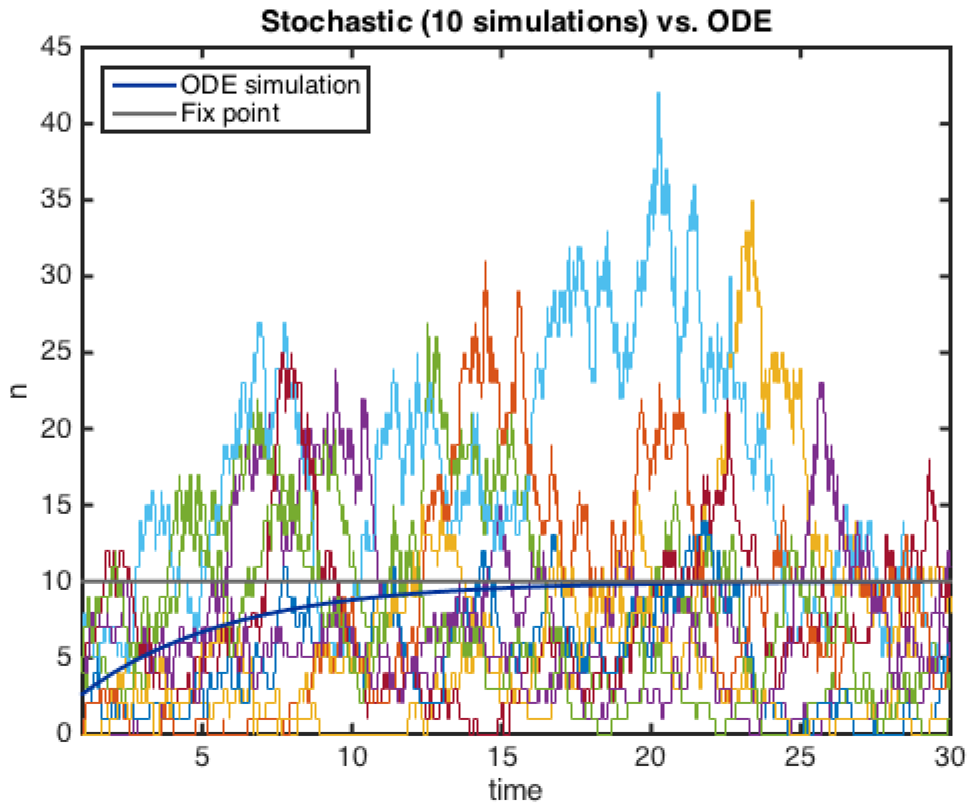
```
    rng(123); % set seed
```

```
    [t1, ns1] = ode45('BDIODE',[1 tlim],  1);
% Figure
    myplot(t1-1, ns1, 'L'); hold on  % -1 so t starts at 0
    hline(n_fp);
        xlabel('time');
        ylabel('n');
        legend('ODE simulation','Fix point', 'Location','northwest')
        title('ODE of logistic equation')
```



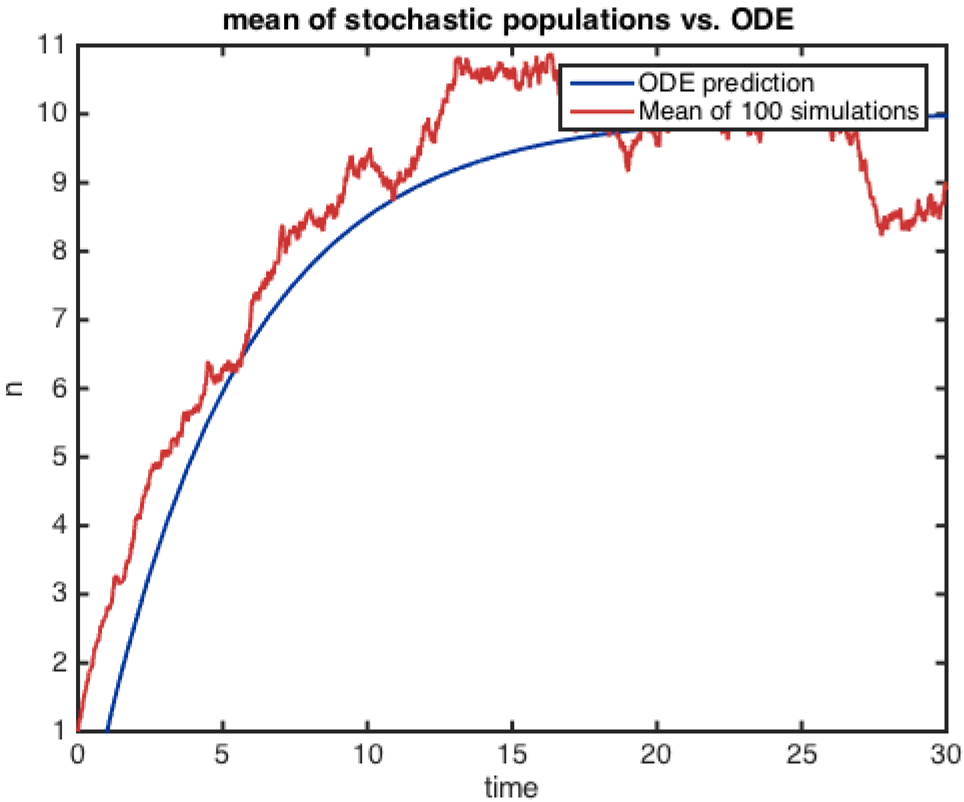
Gillespie Simulation

```
rng(123)
for i=1:10
    [t, x] = BDIGillespie(n0, tlim);
    stairs(t,x); hold on
end
% re-plot so they lay on top
    myplot(t1-1, ns1, 'L'); hold on  % -1 so t starts at 0
    hline(n_fp);
    xlim([1 tlim]);
     xlabel('time');
      ylabel('n');
    title('Stochastic (10 simulations) vs. ODE')
```



ODE vs. stochastic mean

```
    tsample = 0 : 0.01 : tlim;
    xsample = nan(it, length(tsample));
    rng(123)
    for i = 1 : it
            [t, x] = BDIGillespie(n0, tlim);
            xsample(i, :) =  fixsample(t,x, tsample);
    end
% plot ODE vs. stochastic mean
    figure
            myplot(t1, ns1, 'L'); hold on
            myplot(tsample, mean(xsample), 'L', 2);
            legend('ODE prediction', ['Mean of ' num2str(it) '
 simulations'])
            xlabel('time')
            ylabel('n')
            title('mean of stochastic populations vs. ODE ')
```



General case

an example is a logistic equation; can be viewed as a special case of Birth-Death-Imigration process with immigration = 0

$$\frac{dn}{dt} = rn(1 - \frac{n}{K})$$

, where r = b1 - d1, K = r / (d2 - b2).

I chose the parameters so the system has the same fix point as the first case

Parameters

```
global b1 b2 d1 d2 r K
b1 = 11; b2 = 1;
d1 = 1;  d2 = 2;
r = b1 -d1;          % the intrinsic growth rate
K = r/(d2-b2);   % the carrying capacity; also is the theoretical
fixed point
tlim =5;
```

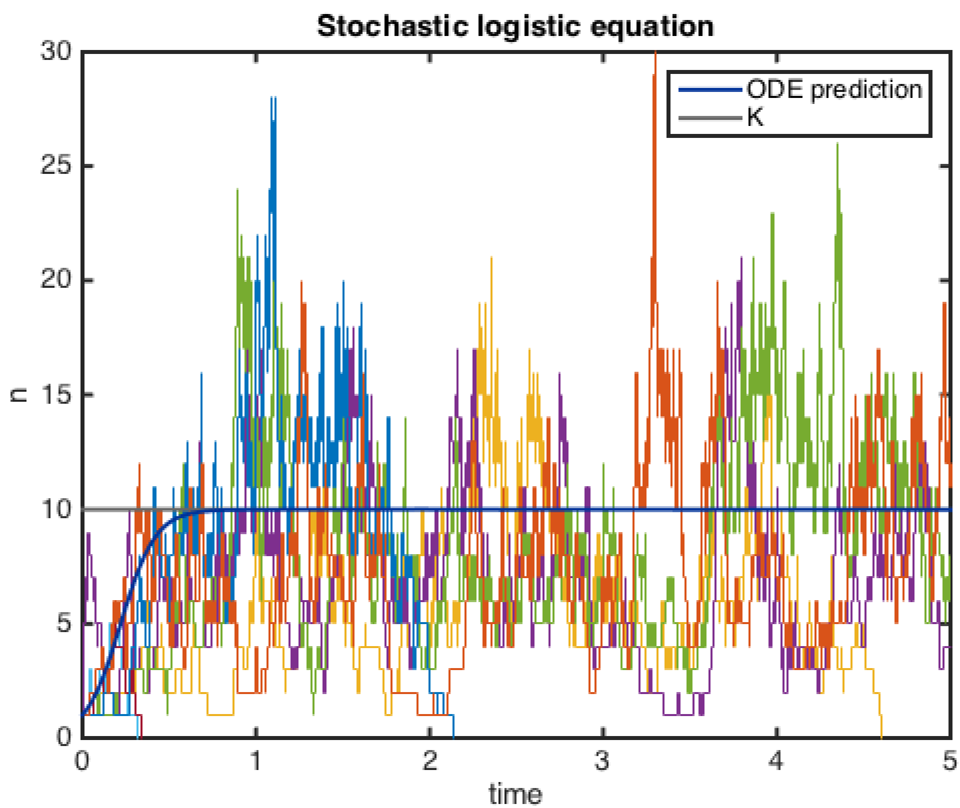
ODE

```
rng(1)
```

```
[t3, ns3] = ode45('logisticODE',[1 tlim+1],  n0);
```

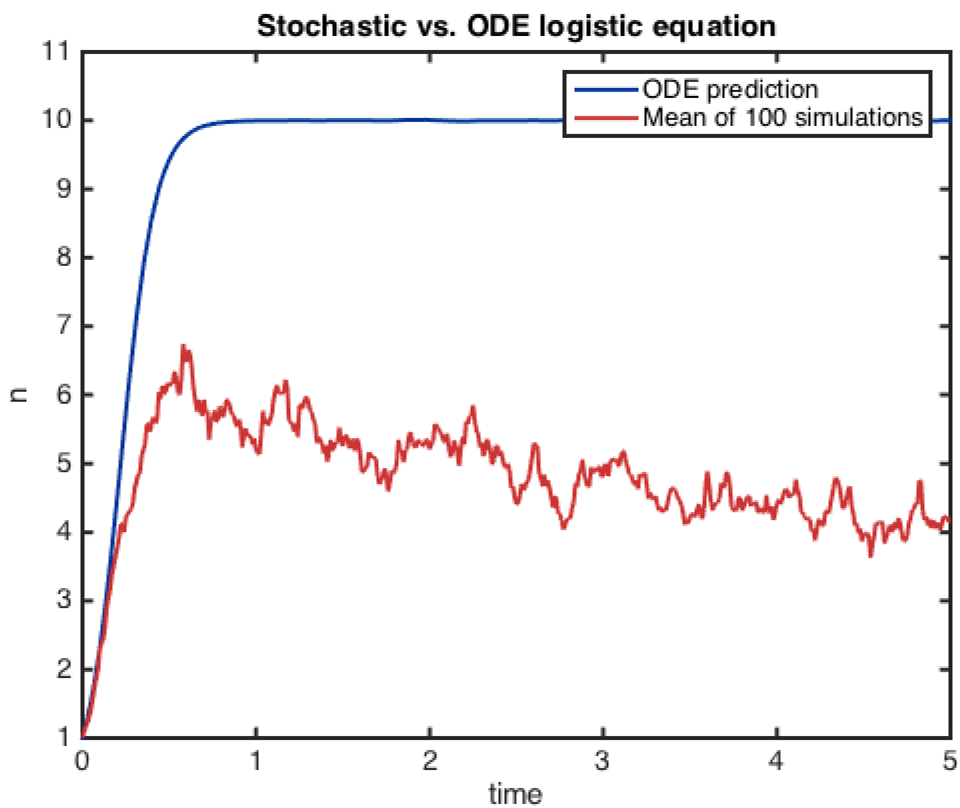
Gillespie simulation

```
    figure
         myplot(t3-1, ns3, 'L'); hold on
         hline(K);
% do simulation
for i=1:7
    [t, x] = logisticGillespie(n0, tlim);
     stairs(t, x); hold on  % -1 s
end
% plot3_2_3
      hline(K);
      myplot(t3-1, ns3, 'L'); % plot again so this line is on top
       ylimits = ylim;
       axis([ 0  tlim ylimits])
        legend('ODE prediction', 'K')
        xlabel('time')
        ylabel('n')
        title('Stochastic logistic equation')
```



Mean of stochastic populations vs. ODE prediction

```
    tsample = 0 : 0.01 : tlim;
    xsample = nan(it, length(tsample));
    for i = 1 : it
            [t, x] = logisticGillespie(n0, tlim);
            xsample(i, :) =  fixsample(t,x, tsample);
    end
% plot
    figure
        myplot(t3-1, ns3, 'L'); hold on
        myplot(tsample, mean(xsample), 'L', 2);
            legend('ODE prediction', ['Mean of ' num2str(it) '
 simulations'])
            xlabel('time')
            ylabel('n')
            title('Stochastic vs. ODE logistic equation')
```



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