(a)

$$\Gamma(\ell,z) = \delta_{\ell,0} + z\sum_{\ell'} p(\ell-\ell')\,\Gamma(\ell',z)$$

$$\hat{\Gamma}(k,z) = \mathcal{F}(\delta_{\ell,0}) + z\,\mathcal{F}\Big(\underbrace{\sum_{\ell'} p(\ell-\ell')\,\Gamma(\ell',z)}\Big)$$

$$\Downarrow$$

$$1$$

This is the discrete version of Theorem 2.2 from Box 2A.

$$\boxed{\begin{aligned} &\mathcal{F}\Big(\sum_{\ell'} p(\ell-\ell')\,\Gamma(\ell',z)\Big) \\ &= \sum_{\ell} e^{ik\ell} \sum_{\ell'} p(\ell-\ell')\,\Gamma(\ell',z) \\ &= \sum_{\ell}\sum_{\ell'} e^{ik(\ell-\ell')}\, e^{ik\ell'}\, p(\ell-\ell')\,\Gamma(\ell',z) \\ &= \underbrace{\sum_{\ell'}} e^{ik(\ell-\ell')}\, p(\ell-\ell') \times \sum_{\ell} e^{ik\ell'}\,\Gamma(\ell',z) \\ &= \lambda(k)\,\hat{\Gamma}(k,z) \end{aligned}}$$

↳ since we are suming $\ell' = -\infty$ to $\ell' = \infty$ it's the same as $(\ell-\ell') = -\infty$ to $(\ell-\ell') = \infty$

$$\hat{\Gamma}(k,z) = 1 + z\,\lambda(k)\,\hat{\Gamma}(k,z$$