

$$\text{let } \langle x^2 \rangle_{\text{avg}} = y$$

$$\frac{dy}{dt} = 2ky + k\bar{n}$$

$$\left(\frac{d}{dt} \langle n^2 \rangle_{\text{AV}} = 2k \langle n^2 \rangle_{\text{AV}} + k\bar{n} \right)$$

$$\rightarrow \frac{dy}{dt} + p(t)y = q(t)$$

$$p(t) = -2k$$

$$q(t) = k\bar{n}$$

$$y(t) = \frac{\int \mu(t) q(t) \cdot dt + B}{\mu(t)} \quad \rightarrow \text{constant}$$

$$\mu(t) = e^{\int p(t) dt + C} = A \cdot e^{\int -2k \cdot dt} = A \cdot e^{-2kt}$$

$$y(t) = \frac{\int A \cdot e^{-2kt} \cdot k\bar{n} \cdot dt + B}{A \cdot e^{-2kt}}$$

$$\bar{n} = n_0 e^{kt}$$

$$= \frac{\int A \cdot k \cdot n_0 \cdot e^{-kt} \cdot dt + B}{A \cdot e^{-2kt}}$$

$$= \frac{-A n_0 e^{-kt} + B}{A \cdot e^{-2kt}}$$

$$= -n_0 e^{kt} + \frac{B}{A} e^{2kt}$$

$$= A' e^{2kt} - n_0 e^{kt}$$