$$s^2 = \langle n^2 \rangle - \tilde{n}^2$$

$$\frac{d}{dt} s^2 = \underbrace{\boxed{\frac{d}{dt}\langle n^2 \rangle}}_{\text{part A}} - \underbrace{\boxed{\frac{d}{dt}\tilde{n}^2}}_{\text{part B}}$$

part A:
$$\frac{d}{dt}\langle n^2 \rangle = \frac{d}{dt}\sum_{n=0}^{n=\infty} n^2 P_n.$$

$$= \sum n^2 [(n-1) k P_{n-1} - n k P_n])$$

$$= n^2 (n-1) k P_{n-1} + (n+1)^2 n k P_n + \cdots$$

0 when $n=1$

$$- n^3 k P_n - \cdots$$

$$= \sum [(n+1)^2 n - n^3] k P_n$$

$$= \sum [n(n^2+2n+1) - n n^2] k P_n$$

$$= \sum (2n^2+n) k P_n$$

$$= 2k\langle n^2 \rangle + k\hat{n}.$$

part B: $$\frac{d}{dt}\hat{n}^2 = 2\hat{n}\frac{d}{dt}(\hat{n})$$
$$= 2\hat{n}\, k\hat{n}$$
$$= 2k\hat{n}^2$$

part A - part B

$$\frac{d}{dt}s^2 = 2k\langle n^2\rangle + k\hat{n} - 2k\hat{n}^2$$
$$= 2k\left[\langle n^2\rangle - \hat{n}^2\right] + k\hat{n}$$
$$= 2ks^2 + k\hat{n}$$