**5.46**

**a.** The sampling distribution of $\bar{X}$ is centered at $E(\bar{X}) = \mu = 12$ cm, and the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \frac{\sigma_{\bar{X}}}{\sqrt{n}} = \frac{.04}{\sqrt{16}} = .01$ cm.

**b.** With $n = 64$, the sampling distribution of $\bar{X}$ is still centered at $E(\bar{X}) = \mu = 12$ cm, but the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \frac{\sigma_{\bar{X}}}{\sqrt{n}} = \frac{.04}{\sqrt{64}} = .005$ cm.

**c.** $\bar{X}$ is more likely to be within .01 cm of the mean (12 cm) with the second, larger, sample. This is due to the decreased variability of $\bar{X}$ that comes with a larger sample size.

**5.50**

**a.** $P(9{,}900 \le \bar{X} \le 10{,}200) \approx P\left(\frac{9{,}900-10{,}000}{500/\sqrt{40}} \le Z \le \frac{10{,}200-10{,}000}{500/\sqrt{40}}\right)$
$= P(-1.26 \le Z \le 2.53) = \Phi(2.53) - \Phi(-1.26) = .9943 - .1038 = .8905$.

**b.** According to the guideline given in Section 5.4, $n$ should be greater than 30 in order to apply the CLT, thus using the same procedure for $n = 15$ as was used for $n = 40$ would not be appropriate.

**5.53**

**a.** With the values provided,

$$P(\bar{X} \ge 51) = P\left(Z \ge \frac{51-50}{1.2/\sqrt{9}}\right) = P(Z \ge 2.5) = 1 - .9938 = .0062 .$$

**b.** Replace $n = 9$ by $n = 40$, and

$$P(\bar{X} \ge 51) = P\left(Z \ge \frac{51-50}{1.2/\sqrt{40}}\right) = P(Z \ge 5.27) \approx 0 .$$

**5.60**

$Y$ is normally distributed with $\mu_Y = \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{3}(\mu_3 + \mu_4 + \mu_5) = -1$, and

$\sigma_Y^2 = \frac{1}{4}\sigma_1^2 + \frac{1}{4}\sigma_2^2 + \frac{1}{9}\sigma_3^2 + \frac{1}{9}\sigma_4^2 + \frac{1}{9}\sigma_5^2 = 3.167 \Rightarrow \sigma_Y = 1.7795$.

Thus, $P(0 \le Y) = P\left(\frac{0-(-1)}{1.7795} \le Z\right) = P(.56 \le Z) = .2877$ and

$P(-1 \le Y \le 1) = P\left(0 \le Z \le \frac{2}{1.7795}\right) = P(0 \le Z \le 1.12) = .3686$.

**6.3**

**a.** We use the sample mean, $\bar{x} = 1.3481$.

**b.** Because we assume normality, the mean = median, so we also use the sample mean $\bar{x} = 1.3481$. We could also easily use the sample median.

**c.** We use the 90th percentile of the sample: $\hat{\mu} + (1.28)\sigma = \bar{x} + 1.28s = 1.3481 + (1.28)(.3385) = 1.7814$.

**d.** Since we can assume normality,
$$P(X < 1.5) \approx P\left(Z < \frac{1.5 - \bar{x}}{s}\right) = P\left(Z < \frac{1.5 - 1.3481}{.3385}\right) = P(Z < .45) = .6736.$$

**e.** The estimated standard error of $\bar{x} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{.3385}{\sqrt{16}} = .0846.$

**6.5**

Let $\theta$ = the total audited value. Three potential estimators of $\theta$ are $\hat{\theta}_1 = N\bar{X}$, $\hat{\theta}_2 = T - N\bar{D}$, and $\hat{\theta}_3 = T \cdot \frac{\bar{X}}{\bar{Y}}$.
From the data, $\bar{y}$ = 374.6, $\bar{x}$ = 340.6, and $\bar{d}$ = 34.0. Knowing $N$ = 5,000 and $T$ = 1,761,300, the three corresponding estimates are $\hat{\theta}_1 = (5{,}000)(340.6) = 1{,}703{,}000$, $\hat{\theta}_2 = 1{,}761{,}300 - (5{,}000)(34.0) = 1{,}591{,}300$, and $\hat{\theta}_3 = 1{,}761{,}300\left(\frac{340.6}{374.6}\right) = 1{,}601{,}438.281$.

**6.28**

**a.** $\left(\frac{x_1}{\theta}\exp\left[-x_1^2/2\theta\right]\right)\ldots\left(\frac{x_n}{\theta}\exp\left[-x_n^2/2\theta\right]\right) = (x_1 \ldots x_n)\frac{\exp\left[-\Sigma x_i^2/2\theta\right]}{\theta^n}$. The natural log of the likelihood function is $\ln(x_i \ldots x_n) - n\ln(\theta) - \frac{\Sigma x_i^2}{2\theta}$. Taking the derivative with respect to $\theta$ and equating to 0 gives $-\frac{n}{\theta} + \frac{\Sigma x_i^2}{2\theta^2} = 0$, so $n\theta = \frac{\Sigma x_i^2}{2}$ and $\theta = \frac{\Sigma x_i^2}{2n}$. The mle is therefore $\hat{\theta} = \frac{\Sigma X_i^2}{2n}$, which is identical to the unbiased estimator suggested in Exercise 15.

**b.** For $x > 0$ the cdf of $X$ is $F(x;\,\theta) = P(X \le x) = 1 - \exp\left[\frac{-x^2}{2\theta}\right]$. Equating this to .5 and solving for $x$ gives the median in terms of $\theta$: $.5 = \exp\left[\frac{-x^2}{2\theta}\right] \Rightarrow x = \tilde{\mu} = \sqrt{-2\theta\ln(.5)} = \sqrt{1.3863\theta}$. The mle of $\tilde{\mu}$ is therefore $\sqrt{1.3863\hat{\theta}}$.

**7.3**

**a.** A 90% confidence interval will be narrower. The *z* critical value for a 90% confidence level is 1.645, smaller than the *z* of 1.96 for the 95% confidence level, thus producing a narrower interval.

**b.** Not a correct statement. Once and interval has been created from a sample, the mean $\mu$ is either enclosed by it, or not. We have 95% confidence in the general procedure, under repeated and independent sampling.

**c.** Not a correct statement. The interval is an estimate for the population mean, not a boundary for population values.

**d.** Not a correct statement. In theory, if the process were repeated an infinite number of times, 95% of the intervals would contain the population mean $\mu$. We *expect* 95 out of 100 intervals will contain $\mu$, but we don't know this to be true.

**7.6**

**a.** $8439 \pm \frac{(1.645)(100)}{\sqrt{25}} = 8439 \pm 32.9 =$ (8406.1, 8471.9).

**b.** $1-\alpha = .92 \Rightarrow \alpha = .08 \Rightarrow \alpha/2 = .04$ so $z_{\alpha/2} = z_{.04} = 1.75$.