29.

a. There are 26 letters, so allowing repeats there are $(26)(26) = (26)^2 = 676$ possible 2-letter domain names. Add in the 10 digits, and there are 36 characters available, so allowing repeats there are $(36)(36) = (36)^2 = 1296$ possible 2-character domain names.

b. By the same logic as part **a**, the answers are $(26)^3 = 17{,}576$ and $(36)^3 = 46{,}656$.

c. Continuing, $(26)^4 = 456{,}976$; $(36)^4 = 1{,}679{,}616$.

d. P(4-character sequence is already owned) = 1 – P(4-character sequence still available) = 1 – $97{,}786/(36)^4 = .942$.

38

a. There are 6 75W bulbs and 9 other bulbs. So, P(select exactly 2 75W bulbs) = P(select exactly 2 75W bulbs and 1 other bulb) = $\dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \dfrac{(15)(9)}{455} = .2967$.

b. P(all three are the same rating) = P(all 3 are 40W or all 3 are 60W or all 3 are 75W) =

$$\frac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \frac{4+10+20}{455} = .0747.$$

c. P(one of each type is selected) = $\dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = \dfrac{120}{455} = .2637$.

d. It is necessary to examine at least six bulbs if and only if the first five light bulbs were all of the 40W or 60W variety. Since there are 9 such bulbs, the chance of this event is

$$\frac{\binom{9}{5}}{\binom{15}{5}} = \frac{126}{3003} = .042.$$

45.

a. $P(A) = .106 + .141 + .200 = .447$, $P(C) = .215 + .200 + .065 + .020 = .500$, and $P(A \cap C) = .200$.

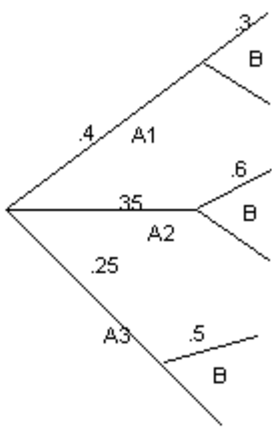
b. $P(A|C) = \dfrac{P(A\cap C)}{P(C)} = \dfrac{.200}{.500} = .400$. If we know that the individual came from ethnic group 3, the probability that he has Type A blood is .40. $P(C|A) = \dfrac{P(A\cap C)}{P(A)} = \dfrac{.200}{.447} = .447$. If a person has Type A blood, the probability that he is from ethnic group 3 is .447.

c. Define D = "ethnic group 1 selected." We are asked for $P(D|B')$. From the table, $P(D \cap B')$ = .082 + .106 + .004 = .192 and $P(B') = 1 - P(B) = 1 - [.008 + .018 + .065] = .909$. So, the desired probability is $P(D|B') = \dfrac{P(D \cap B')}{P(B')} = \dfrac{.192}{.909} = .211$.

56.

$$P(A \mid B) + P(A' \mid B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} = \frac{P(A \cap B) + P(A' \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

59 The required probabilities appear in the tree diagram below.



$.4 \times .3 = .12 = P(A_1 \cap B) = P(A_1)P(B \mid A_1)$

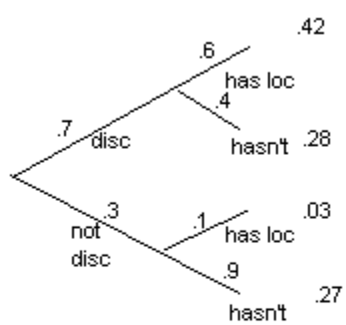
$.35 \times .6 = .21 = P(A_2 \cap B)$

$.25 \times .5 = .125 = P(A_3 \cap B)$

a. $P(A_2 \cap B) = .21$.

b. By the law of total probability, $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .455$.

c. Using Bayes' theorem, $P(A_1 \mid B) = \dfrac{P(A_1 \cap B)}{P(B)} = \dfrac{.12}{.455} = .264$; $P(A_2 \mid B) = \dfrac{.21}{.455} = .462$; $P(A_3 \mid B) = 1 - .264 - .462 = .274$. Notice the three probabilities sum to 1.



60 The tree diagram below shows the probability for the four disjoint options; e.g., P(the flight is discovered and has a locator) = P(discovered)P(locator | discovered) = (.7)(.6) = .42.

a. P(not discovered | has locator) = $\dfrac{P(\text{not discovered} \cap \text{has locator})}{P(\text{has locator})} = \dfrac{.03}{.03+.42} = .067$.

b. P(discovered | no locator) = $\dfrac{P(\text{discovered} \cap \text{no locator})}{P(\text{no locator})} = \dfrac{.28}{.55} = .509$.

74 Using subscripts to differentiate between the selected individuals,
$P(O_1 \cap O_2) = P(O_1)P(O_2) = (.45)(.45) = .2025$.
P(two individuals match) = $P(A_1 \cap A_2) + P(B_1 \cap B_2) + P(AB_1 \cap AB_2) + P(O_1 \cap O_2) =$
$.40^2 + .11^2 + .04^2 + .45^2 = .3762$.

78 P(at least one opens) = 1 – P(none open) = $1 - (.05)^5 = .99999969$.
P(at least one fails to open) = 1 – P(all open) = $1 - (.95)^5 = .2262$.