3.10

a. Possible values of T are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

b. Possible values of X are: –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6.

c. Possible values of U are: 0, 1, 2, 3, 4, 5, 6.

d. Possible values of Z are: 0, 1, 2.

3.19

$p(0) = P(Y = 0) = P(\text{both arrive on Wed}) = (.3)(.3) = .09$;
$p(1) = P(Y = 1) = P((\text{W,Th}) \text{ or } (\text{Th,W}) \text{ or } (\text{Th,Th})) = (.3)(.4) + (.4)(.3) + (.4)(.4) = .40$;
$p(2) = P(Y = 2) = P((\text{W,F}) \text{ or } (\text{Th,F}) \text{ or } (\text{F,W}) \text{ or } (\text{F,Th}) \text{ or } (\text{F,F})) = .32$;
$p(3) = 1 - [.09 + .40 + .32] = .19$.

3.24

a. Possible X values are those values at which $F(x)$ jumps, and the probability of any particular value is the size of the jump at that value. Thus we have:

x	1	3	4	6	12
$p(x)$	.30	.10	.05	.15	.40

b. $P(3 \le X \le 6) = F(6) - F(3-) = .60 - .30 = .30$; $P(4 \le X) = 1 - P(X < 4) = 1 - F(4-) = 1 - .40 = .60$.

3.30

a. $E(Y) = \sum_{y=0}^{3} y \cdot p(y) = 0(.60) + 1(.25) + 2(.10) + 3(.05) = .60$.

b. $E(100Y^2) = \sum_{y=0}^{3} 100y^2 \cdot p(y) = 0(.60) + 100(.25) + 400(.10) + 900(.05) = \$110$.

3.33

a. $E(X^2) = \sum_{x=0}^{1} x^2 \cdot p(x) = 0^2(1-p) + 1^2(p) = p$.

b. $V(X) = E(X^2) - [E(X)]^2 = p - [p]^2 = p(1-p)$.

c. $E(X^{79}) = 0^{79}(1 - p) + 1^{79}(p) = p$. In fact, $E(X^n) = p$ for any non-negative power n.

3.37

Using the hint, $E(X) = \sum_{x=1}^{n} x \cdot \left(\frac{1}{n}\right) = \frac{1}{n}\sum_{x=1}^{n} x = \frac{1}{n}\left[\frac{n(n+1)}{2}\right] = \frac{n+1}{2}$. Similarly,
$E(X^2) = \sum_{x=1}^{n} x^2 \cdot \left(\frac{1}{n}\right) = \frac{1}{n}\sum_{x=1}^{n} x^2 = \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6}\right] = \frac{(n+1)(2n+1)}{6}$, so
$V(X) = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}$.

3.47

a. $B(4;15,.3) = .515$.

b. $b(4;15,.3) = B(4;15,.3) - B(3;15,.3) = .219$.

c. $b(6;15,.7) = B(6;15,.7) - B(5;15,.7) = .012$.

d. $P(2 \le X \le 4) = B(4;15,.3) - B(1;15,.3) = .480$.

e. $P(2 \le X) = 1 - P(X \le 1) = 1 - B(1;15,.3) = .965$.

f. $P(X \le 1) = B(1;15,.7) = .000$.

g. $P(2 < X < 6) = P(2 < X \le 5) = B(5;15,.3) - B(2;15,.3) = .595$.

3.49

Let X be the number of "seconds," so $X \sim \text{Bin}(6, .10)$.

a. $P(X = 1) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{6}{1}(.1)^1(.9)^5 = .3543$.

b. $P(X \ge 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - \left[\binom{6}{0}(.1)^0(.9)^6 + \binom{6}{1}(.1)^1(.9)^5\right] = 1 - [.5314 + .3543] =$ $.1143$.

c. Either 4 or 5 goblets must be selected.

Select 4 goblets with zero defects: $P(X = 0) = \binom{4}{0}(.1)^0(.9)^4 = .6561$.

Select 4 goblets, one of which has a defect, and the 5th is good: $\left[\binom{4}{1}(.1)^1(.9)^3\right] \times .9 = .26244$

So, the desired probability is .6561 + .26244 = .91854.

3.50

Let X be the number of faxes, so $X \sim \text{Bin}(25, .25)$.

a. $P(X \le 6) = B(6;25,.25) = .561$.

b. $P(X = 6) = b(6;25,.25) = .183$.

c. $P(X \ge 6) = 1 - P(X \le 5) = 1 - B(5;25,.25) = .622$.

d. $P(X > 6) = 1 - P(X \le 6) = 1 - .561 = .439$.

3.81

Let $X \sim \text{Poisson}(\mu = 20)$.

a. $P(X \le 10) = F(10; 20) = .011$.

b. $P(X > 20) = 1 - F(20; 20) = 1 - .559 = .441$.

c. $P(10 \le X \le 20) = F(20; 20) - F(9; 20) = .559 - .005 = .554$;
$P(10 < X < 20) = F(19; 20) - F(10; 20) = .470 - .011 = .459$.

d. $E(X) = \mu = 20$, so $\sigma = \sqrt{20} = 4.472$. Therefore, $P(\mu - 2\sigma < X < \mu + 2\sigma) =$
$P(20 - 8.944 < X < 20 + 8.944) = P(11.056 < X < 28.944) = P(X \le 28) - P(X \le 11) =$
$F(28; 20) - F(11; 20) = .966 - .021 = .945$.