

### HOMEWORK 3

10

- a. Possible values of  $T$  are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
- b. Possible values of  $X$  are: -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6.
- c. Possible values of  $U$  are: 0, 1, 2, 3, 4, 5, 6.
- d. Possible values of  $Z$  are: 0, 1, 2.

14.

- a. As the hint indicates, the sum of the probabilities must equal 1. Applied here, we get
 
$$\sum_{y=1}^5 p(y) = k[1 + 2 + 3 + 4 + 5] = 15k = 1 \Rightarrow k = \frac{1}{15}.$$
 In other words, the probabilities of the five  $y$ -values are  $\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{4}{15}, \frac{5}{15}$ .
- b.  $P(Y \leq 3) = P(Y = 1, 2, 3) = \frac{1}{15} + \frac{2}{15} + \frac{3}{15} = \frac{6}{15} = .4$ .
- c.  $P(2 \leq Y \leq 4) = P(Y = 2, 3, 4) = \frac{2}{15} + \frac{3}{15} + \frac{4}{15} = \frac{9}{15} = .6$ .
- d. Do the probabilities total 1? Let's check:  $\sum_{y=1}^5 \left( \frac{y^2}{50} \right) = \frac{1}{50}[1 + 4 + 9 + 16 + 25] = \frac{55}{50} \neq 1$ . No, that formula cannot be a pmf.

19

$$\begin{aligned}
 p(0) &= P(Y = 0) = P(\text{both arrive on Wed}) = (.3)(.3) = .09; \\
 p(1) &= P(Y = 1) = P((W, Th) \text{ or } (Th, W) \text{ or } (Th, Th)) = (.3)(.4) + (.4)(.3) + (.4)(.4) = .40; \\
 p(2) &= P(Y = 2) = P((W, F) \text{ or } (Th, F) \text{ or } (F, W) \text{ or } (F, Th) \text{ or } (F, F)) = .32; \\
 p(3) &= 1 - [.09 + .40 + .32] = .19.
 \end{aligned}$$

24

- a. Possible  $X$  values are those values at which  $F(x)$  jumps, and the probability of any particular value is the size of the jump at that value. Thus we have:

$x$	1	3	4	6	12
$p(x)$	.30	.10	.05	.15	.40

- b.  $P(3 \leq X \leq 6) = F(6) - F(3-) = .60 - .30 = .30$ ;  $P(4 \leq X) = 1 - P(X < 4) = 1 - F(4-) = 1 - .40 = .60$

30

- a.  $E(Y) = \sum_{y=0}^3 y \cdot p(y) = 0(.60) + 1(.25) + 2(.10) + 3(.05) = .60$ .
- b.  $E(100Y^2) = \sum_{y=0}^3 100y^2 \cdot p(y) = 0(.60) + 100(.25) + 400(.10) + 900(.05) = \$110$ .

39

From the table,  $E(X) = \sum xp(x) = 2.3$   
 $E(X^2) = 6.1$ , and  $V(X) = 6.1 - (2.3)^2 = .81$

Each lot weighs 5 lbs, so the number of pounds left =  $100 - 5X$ :  
 Thus the expected weight left is  $E(100 - 5X) = 100 - 5E(X) = 88.5$  lbs.  
 The variance of the weight left is  $V(100 - 5X) = V(-5X) = (-5)^2 V(X) = 25V(X) = 20.25$ .