HOMEWORK 8

Chapter 6

28.

a. $\left(\frac{x_1}{\theta}\exp\left[-x_1^2/2\theta\right]\right)\ldots\left(\frac{x_n}{\theta}\exp\left[-x_n^2/2\theta\right]\right)=\left(x_1\ldots x_n\right)\frac{\exp\left[-\Sigma x_i^2/2\theta\right]}{\theta^n}$. The natural log of the likelihood function is $\ln\left(x_i\ldots x_n\right)-n\ln\left(\theta\right)-\frac{\Sigma x_i^2}{2\theta}$. Taking the derivative with respect to θ and equating to 0 gives $-\frac{n}{\theta}+\frac{\Sigma x_i^2}{2\theta^2}=0$, so $n\theta=\frac{\Sigma x_i^2}{2}$ and $\theta=\frac{\Sigma x_i^2}{2n}$. The mle is therefore $\hat{\theta}=\frac{\Sigma X_i^2}{2n}$, which is identical to the unbiased estimator suggested in Exercise 15.

Chapter 7

3.

a. A 90% confidence interval will be narrower. The z critical value for a 90% confidence level is 1.645, smaller than the z of 1.96 for the 95% confidence level, thus producing a narrower interval.

b. Not a correct statement. Once and interval has been created from a sample, the mean μ is either enclosed by it, or not. We have 95% confidence in the general procedure, under repeated and independent sampling.

c. Not a correct statement. The interval is an estimate for the population mean, not a boundary for population values.

d. Not a correct statement. In theory, if the process were repeated an infinite number of times, 95% of the intervals would contain the population mean μ. We *expect* 95 out of 100 intervals will contain μ, but we don't know this to be true.

4.

a. $58.3\pm\frac{1.96(3)}{\sqrt{25}}=58.3\pm 1.18=(57.1,59.5)$.

c. $58.3\pm\frac{2.58(3)}{\sqrt{100}}=58.3\pm .77=(57.5,59.1)$.

e. $n=\left[\frac{2(2.58)3}{1}\right]^2=239.62\ \text{Z}\ \ 240$.

6.

a. $8439\pm\frac{(1.645)(100)}{\sqrt{25}}=8439\pm 32.9=$ (8406.1, 8471.9).

b. $1-\alpha=.92\Rightarrow\alpha=.08\Rightarrow\alpha/2=.04$ so $z_{\alpha/2}=z_{.04}=1.75$.