HOMEWORK 10

Chapter 8

6. H_0: $\mu = 40$ v. H_a: $\mu \neq 40$, where μ is the true average burn-out amperage for this type of fuse. The alternative reflects the fact that a departure from $\mu = 40$ in either direction is of concern. A type I error would say that one of the two concerns exists (either $\mu < 40$ or $\mu > 40$) when, in fact, the fuses are perfectly compliant. A type II error would be to fail to detect either of these concerns when one exists.

19.

a. Reject H_0 if either $z \geq 2.58$ or $z \leq -2.58$; $\frac{\sigma}{\sqrt{n}} = 0.3$, so $z = \frac{94.32 - 95}{0.3} = -2.27$. Since -2.27 is not in the rejection region, don't reject H_0.

b. $\beta(94) = \Phi\left(2.58 + \frac{1}{0.3}\right) - \Phi\left(-2.58 + \frac{1}{0.3}\right) = \Phi(5.91) - \Phi(.75) = .2266.$

c. $n = \left[\frac{1.20(2.58 + 1.28)}{95 - 94}\right]^2 = 21.46$, so use $n = 22$.

26. Reject H_0 if $z \geq 1.645$; $\frac{s}{\sqrt{n}} = .7155$, so $z = \frac{52.7 - 50}{.7155} = 3.77$. Since 3.77 is ≥ 1.645, reject H_0 at level .05 and conclude that true average penetration exceeds 50 mils.

27.

a. Using software, $\bar{x} = 0.75$, $\tilde{x} = 0.64$, $s = .3025$, $f_s = 0.48$. These summary statistics, as well as a box plot (not shown) indicate substantial positive skewness, but no outliers.

b. No, it is not plausible from the results in part **a** that the variable ALD is normal. However, since $n = 49$, normality is not required for the use of z inference procedures.

c. We wish to test H_0: $\mu \geq 1.0$ versus H_a: $\mu < 1.0$. The test statistic is $z = \frac{0.75 - 1.0}{.3025/\sqrt{49}} = -5.79$; at any reasonable significance level, we reject the null hypothesis. Yes, the data provides strong evidence that the true average ALD is less than 1.0.

d. $\bar{x} + z_{.05}\frac{s}{\sqrt{n}} = 0.75 + 1.645\frac{.3025}{\sqrt{49}} = 0.821$

29.

a. The hypotheses are H_0: $\mu = 200$ versus H_a: $\mu > 200$. H_0 will be rejected at level $\alpha = .05$ if $t \geq t_{.05,12-1} = t_{.05,11} = 1.796$. With the data provided, $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{249.7 - 200}{145.1/\sqrt{12}} = 1.19$. Since $1.19 < 1.796$, H_0 is not rejected at the $\alpha = .05$ level. We have insufficient evidence to conclude that the true average repair time exceeds 200 minutes.

37.

a. The parameter of interest is p = the proportion of the population of female workers that have BMIs of at least 30 (and, hence, are obese). The hypotheses are H_0: $p = .20$ versus H_a: $p > .20$.
With $n = 541$, $np_0 = 541(.2) = 108.2 \geq 10$ and $n(1 - p_0) = 541(.8) = 432.8 \geq 10$, so the "large-sample" z procedure is applicable. Hence, we will reject H_0 if $z \geq z_{.05} = 1.645$.

From the data provided, $\hat{p} = \frac{120}{541} = .2218$, so $z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{.2218 - .20}{\sqrt{.20(.80)/541}} = 1.27$.
Since 1.27 < 1.645, we fail to reject H_0 at the α = .05 level. We do not have sufficient evidence to conclude that more than 20% of the population of female workers is obese.

b. A Type I error would be to incorrectly conclude that more than 20% of the population of female workers is obese, when the true percentage is 20%. A Type II error would be to fail to recognize that more than 20% of the population of female workers is obese when that's actually true.

c. The question is asking for the chance of committing a Type II error when the true value of p is .25, i.e. $\beta(.25)$. Using the textbook formula,

$$\beta(.25) = \Phi\left[\frac{.20 - .25 + 1.645\sqrt{.20(.80)/541}}{\sqrt{.25(.75)/541}}\right] = \Phi(-1.166) \approx .121.$$

39.

1 p = true proportion of all donors with type A blood

2 H_0: p = .40

3 H_a: $p \neq .40$

4 $z = \frac{\hat{p} - p_o}{\sqrt{p_o(1-p_o)/n}} = \frac{\hat{p} - .40}{\sqrt{.40(.60)/n}}$

5 Reject H_0 if $z \geq 2.58$ or $z \leq -2.58$

6 $z = \frac{82/150 - .40}{\sqrt{.40(.60)/150}} = \frac{.147}{.04} = 3.667$

7 Reject H_0. The data does suggest that the percentage of all donors with type A blood differs from 40%. (at the .01 significance level). Since the z critical value for a significance level of .05 is less than that of .01, the conclusion would not change.

71. n= 47, $\bar{x} = 215$ mg, s = 235 mg, scope of values = 5 mg to 1,176 mg

a. No, the distribution does not appear to be normal. It appears to be skewed to the right, since 0 is less than one standard deviation below the mean. It is not necessary to assume normality if the sample size is large enough due to the central limit theorem. This sample size is large enough so we can conduct a hypothesis test about the mean.

b.

1 Parameter of interest: μ = true daily caffeine consumption of adult women.

2 H_0: μ = 200

3 H_a: μ > 200

4 $z = \frac{\bar{x} - 200}{s/\sqrt{n}}$

5 RR: $z \geq 1.282$ or if P-value $\leq .10$

6 $z = \frac{215 - 200}{235/\sqrt{47}} = .44$; P-value = 1 – $\Phi(.44)$ = .33

7 Fail to reject H_0, because .33 > .10. The data does not indicate that daily consumption of all adult women exceeds 200 mg.

76. A t test is appropriate. H_0: μ = 1.75 is rejected in favor of H_a: $\mu \neq 1.75$ if the P-value < .05. The computed test statistic is $t = \frac{1.89 - 1.75}{.42/\sqrt{26}} = 1.70$. Since the P-value is $2P(t>1.7) = 2(.051) = .102$ > .05, do not reject H_0; the data does not contradict prior research.
We assume that the population from which the sample was taken was approximately normally distributed.