Comments on hwk6

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Theory Problem

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- Joint density for no pooling H_1

$$p(y, \theta_1, \theta_2, \ldots, \theta_j | H_1) = (\frac{1}{2\pi A})^J \prod_{j=1}^{J} \frac{1}{2\pi\sigma_j} \exp(-\frac{\sum_{j=1}^{J} (y_j - \theta_j)^2}{2\sigma_j^2} - \frac{\sum_{j=1}^{J} \theta_j^2}{2A^2})$$

Theory Prob Cont'd

- The trick: completing the squares H_2

$$
\begin{aligned}
p(y|H_2) =& \frac{1}{(2\pi)^{J+1} A \prod \sigma_j} \int \exp[-(\sum \frac{1}{2\sigma_j^2} + \frac{1}{2A^2})\theta^2 \\
& + 2\sum \frac{y_j}{2\sigma_j^2}\theta - \sum \frac{y_j^2}{2\sigma_j^2}] d\theta \\
=& \frac{1}{(2\pi)^{J+1} A \prod \sigma_j} \exp(-\sum \frac{y_j^2}{2\sigma_j^2}) \exp(\frac{(\sum \frac{y_j}{2\sigma_j^2})^2}{\sum \frac{1}{2\sigma_j^2} + \frac{1}{2A^2}}) \\
& \times \int \exp\{-\frac{[\theta - \frac{\sum \frac{y_j}{2\sigma_j^2}}{\sum \frac{1}{2\sigma_j^2} + \frac{1}{2A^2}}]^2}{1/\sum \frac{1}{2\sigma_j^2} + \frac{1}{2A^2}}\} d\theta
\end{aligned}
$$

Theory Prob Cont'd

- The trick: completing the squares H_1
- Since the θ_j are independent in the join density, we can work on them separately

$$p(y|H_1) = \prod p(y_j|H_1) \quad \text{and}$$

$$p(y_j|H_1) = \frac{1}{(2\pi)A\sigma_j} \exp(-\frac{y_j^2}{2\sigma_j^2}) \exp(\frac{(\frac{y_j}{2\sigma_j^2})^2}{\frac{1}{2\sigma_j^2} + \frac{1}{2A^2}})$$

$$\times \int \exp\{-\frac{[\theta_j - \frac{\frac{y_j}{2\sigma_j^2}}{\frac{1}{2\sigma_j^2}+\frac{1}{2A^2}}]^2}{1 / \frac{1}{2\sigma_j^2} + \frac{1}{2A^2}}\} d\theta_j$$

Theory Prob Cont'd

- For fixed J, let $A \to \infty$,

$$\frac{p(y|H_2)}{p(y|H_1)} = O(A^{J-1}) \to \infty$$

- For fixed A, let $J \to \infty$,

$$\frac{p(y|H_2)}{p(y|H_1)} = O(C^{J-1})$$

  where C is a constant depending on A and σ. If $C < 1$, the limit goes to zero.