S1211Q Introduction to Statistics
Lecture 2

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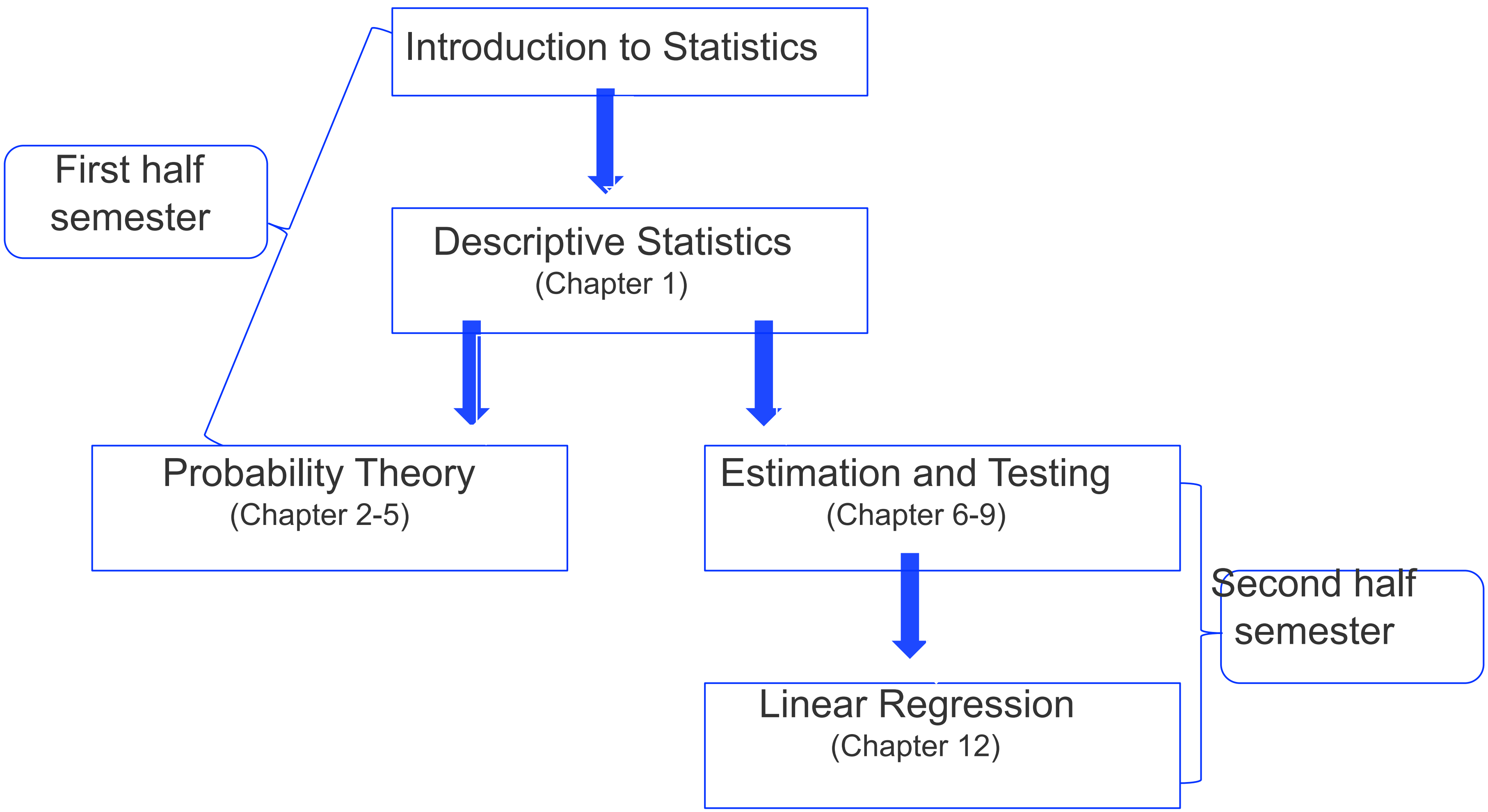
Overview of the course



Basic concepts

- **Population**: the whole class of individuals which an investigator is interested in.
- **Census**: the desired information is available for all objects in the population.
- **Sample**: a subset (part) of the population which is examined or observed.
- **Sample Size**: the number of observations in a single sample.
- **Variable**: any characteristic whose value may change from one object to another in the population, including *univariate*, *bivariate*, *multivariate*.

Probability

- What are random variables? Example: coin tosses.
- To describe random variables: *distribution*. This course will cover a variety of commonly used probability distributions.
  - Discrete distributions: Binomial, Poisson, etc.
  - Continuous distributions: Exponential, Normal (Gaussian), etc.
- Conditional probability.

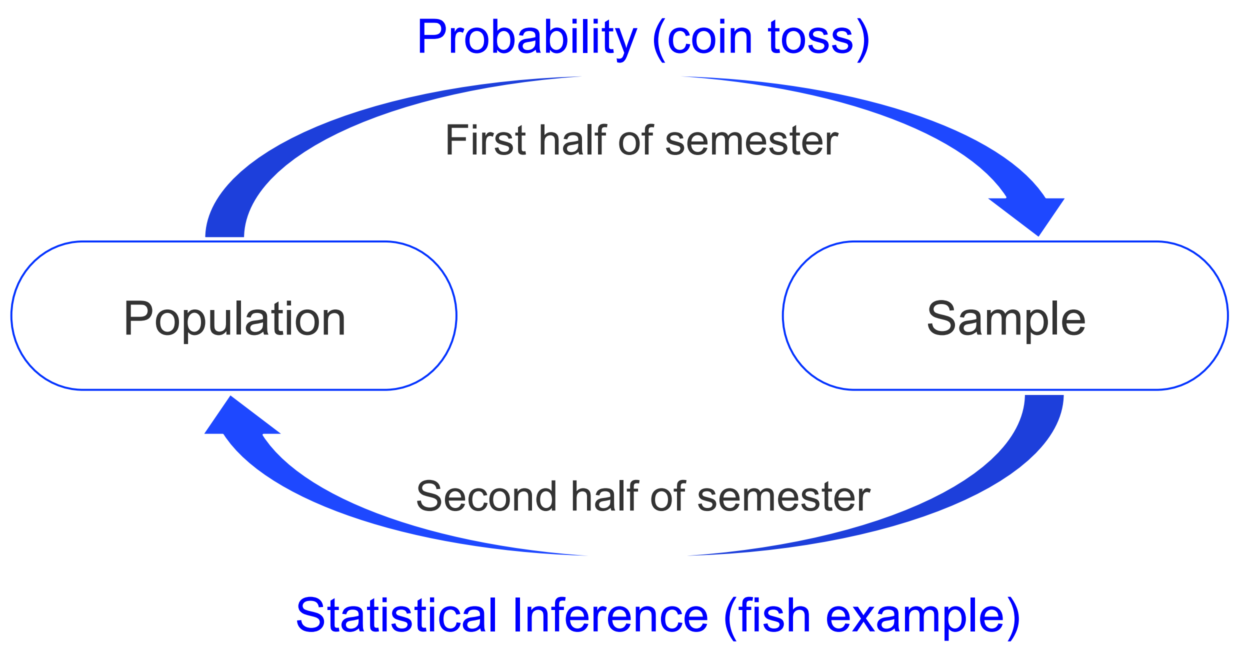
Statistical Inference

- Estimation:
  - Point estimation. Example: What is the total number of fish in a lake?
  - Interval estimation.

- Hypothesis testing:
  - One sample testing.
  - Two sample testing. Example: Is there a significant improvement in the new drug?

- Estimation and hypothesis testing are just two different ways of looking at the same problem.

Probability and Inference



Descriptive Statistics

- Pictorial methods:
  - Stem-and-Leaf Displays.
  - Dotplots.
  - Histograms.

- All these methods convey information about the following aspects of the data:
  - Identification of a typical or representative value
  - Extent of spread about the typical value
  - Presence of any gaps in the data
  - Extent of symmetry in the distribution of values
  - Number and location of peaks
  - Presence of any outlying values

Stem-and-Leaf displays

- Steps for constructing a Stem-and-Leaf Display:
  1. Select one or more leading digits for the stem values. The trailing digits become the leaves.
  2. List possible stem values in a vertical column.
  3. Record the leaf for every observation beside the corresponding stem value.
  4. Indicate the units for stems and leaves someplace in the display.

- R demo for Stem-and-Leaf:
  - Command: `>stem(x)`
  - Option: `scale=`…, scale has to be a positive number. It controls the plot length. A value of scale=2 will cause the plot to be roughly twice as long as the default (=1).

Dotplots

- When to use? *When the data is reasonably small or there are relatively few distinct data values, and have ties.*
- Each observation is represented by a dot above the corresponding location on a horizontal measurement scale. When a value occurs more than once, there is a dot for each occurrence, and these dots are stacked vertically.
- There are no built-in functions in R that can plot dotplots. One has to write his own function to do the job.
  - R demo. Define new functions.
  - Function: `>dotplot(x)`

More basic concepts

- **Discrete Variable**: Its set of possible values is either finite or else can be listed in an infinite sequence. (Gender, Age, etc.)

- **Continuous Variable**: Its possible values consist of an entire interval on the real number line. (Height, Weight, etc.)

- **Frequency**: Number of times a value occurs in the data set.

- **Relative Frequency**: Frequency/(Sample size).

Histogram

- Most commonly used tool in descriptive statistics.

- Histogram for discrete data:
  - Determine the frequency and relative frequency of each *x* value.
  - Mark possible x values on a horizontal scale.
  - Above each value, draw a rectangle whose height is the relative frequency (or the frequency) of that value.

- Histogram for continuous data:
  - Divide the range of the data into classes (5-10) of *equal width*. (It can also be unequal.)
  - Determine the frequency and relative frequency for each class.
  - Mark the class boundaries on a horizontal measurement axis.
  - Above each class interval, draw a rectangle whose height is the corresponding relative frequency (or frequency).

Constructing histogram

- Example: The maximum daily temperature in degrees Fahrenheit measured from May to September 1973 at La Guardia Airport. (154 observations)

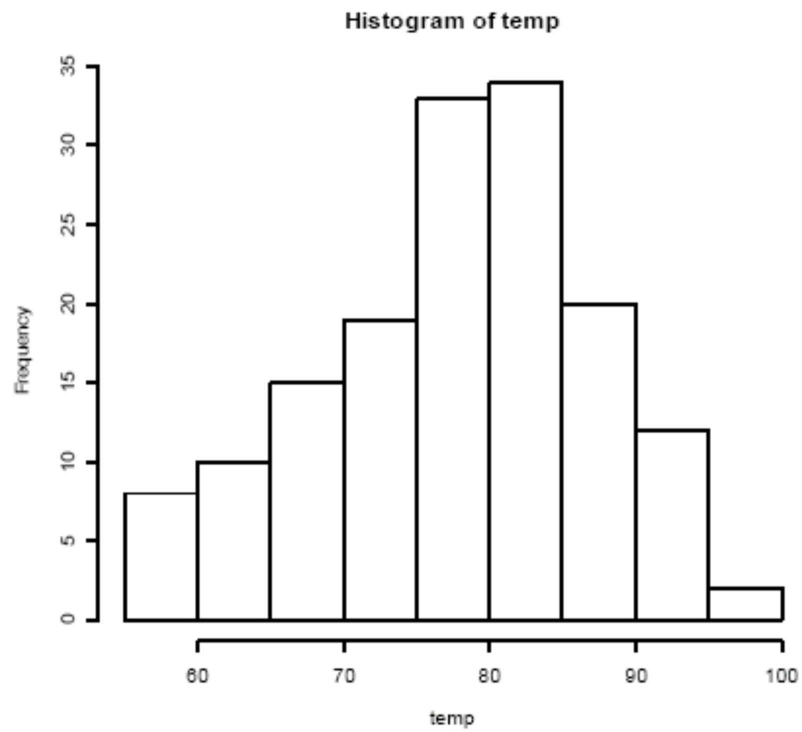
  Data

  {67 72 74 62 56 66 65 59 61 69 74 69 66 68 58 64 66 57 68 62 59 73 61 61 57 58 57 67 81 79 76 78 74 67 84 85 79 82 87 90 87 93 92 82 80 79 77 72 65 73 76 77 76 76 76 75 78 73 80 77 83 84 85 81 84 83 83 88 92 92 89 82 73 81 91 80 81 82 84 87 85 74 81 82 86 85 82 86 88 86 83 81 81 81 82 86 85 87 89 90 90 92 86 86 82 80 79 77 79 76 78 78 77 72 75 79 81 86 88 97 94 96 94 91 92 93 93 87 84 80 78 75 73 81 76 77 71 71 78 67 76 68 82 64 71 81 69 63 70 77 75 76 68}

  Draw a histogram.

Example cont.

Class	Count	Percent
55-59.9	8	5.2
60-64.9	10	6.5
65-69.9	15	9.8
65-74.9	19	12.4
75-79.9	33	21.6
80-84.9	34	22.2
85-89.9	20	13.1
90-94.9	12	7.9
95-99.9	2	1.3



- R demo. `>hist(x)` (option: `breaks=`...)

Unequal class widths

- For unequal class widths, the rectangle height is determined by the formula,

$$\text{rectangle height} = \frac{\text{relative frequency of the class}}{\text{class width}}$$

  the resulting rectangle height is called *densities*.

- The areas under densities sum up to 1.

- In R, we use `breaks` to define the unequal class widths. Setting `freq` = FALSE to switch from frequencies to densities.

Examining distributions

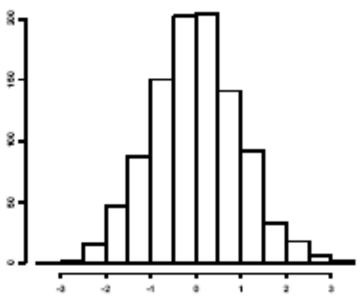
- When examining a distribution, look at its shape, center and spread. Look for clear deviations from the overall shape.

- We are interested in whether it is symmetric or skewed, as well as the number of modes.

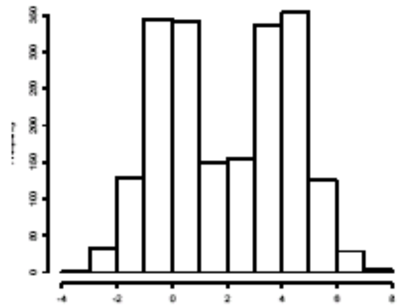
- Outliers are observations that lie outside of the overall pattern of a distribution.

Examining distributions

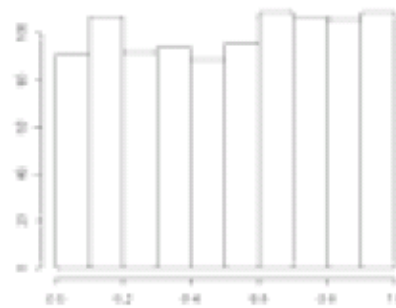
(a) Symmetric, unimodal



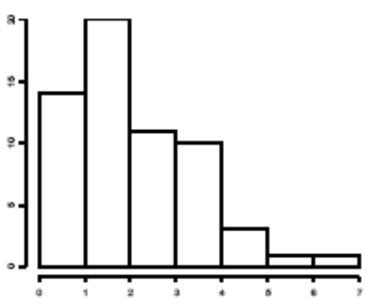
(b) bimodal



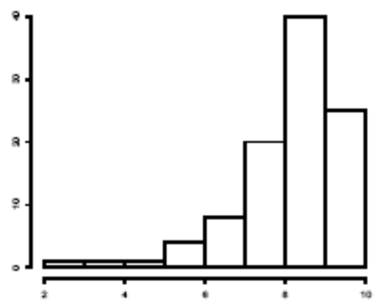
(c) Uniform



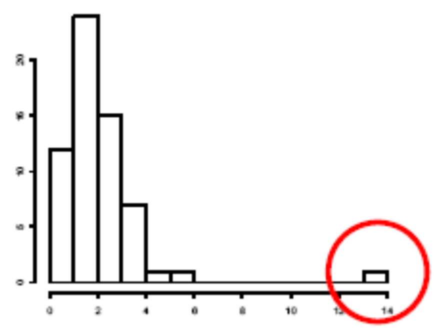
(d) right skewed



(e) left skewed



(f) Outlier



Examining a new data set

1. Examine each variable by itself.
2. Study the relationship between variables.

For both steps 1 and 2 we want to:

- Display the data graphically.
- Summarize the data numerically (Statistics).
- Construct a mathematical model.

Describing distributions numerically

- For single variables, We are interested in summaries that provide information about the center and spread of the distribution.
- A statistic is a numerical summary of data.
- The two most common measures of center are the mean and median.
- “generous” vs. “selfish”.

Mean

- If we have *n* ,observations, their mean is defined by,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

or

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Ex. Calculate the mean of the data set: {1,2,3,4,5}.

$$\bar{x} = \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

Ex. Calculate the mean of the data set: {1,2,3,4,30}.

$$\bar{x} = \frac{1+2+3+4+30}{5} = \frac{40}{5} = 8$$

Mean cont.

- The mean is non-resistant, meaning that it is influenced by very large or very small data points that are extreme values for the data set.

Median

The median, written as M, is defined as the middle value of a data set.

1. List all *n* observations in order of size.
2. If *n* is odd, the median is the center value of the ordered list.
3. If *n* is even, the median is the average of the two center observations.

Median Cont.

<u>Ex.</u> Calculate the median of {6,2,5,19,12,10}.

2 5 6 10 12 19

M is the average of 6 and 10, hence M=8.

<u>Ex.</u> Calculate the median of {1,2,3,4,5} and {1,2,3,4,30}.

1 2 3 4 5 M=3.

1 2 3 4 30 M=3.

Median cont.

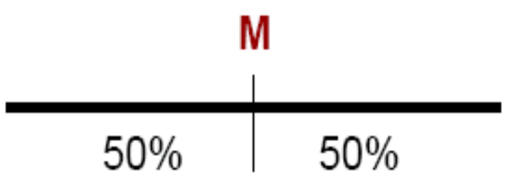
- The median is resistant (robust) to the extremes in the data set. Extremely large or small values do NOT influence the median.

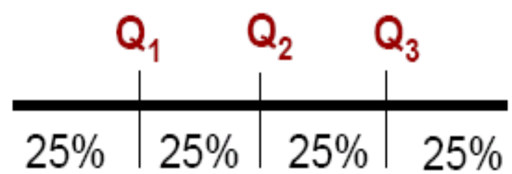
Measures of variability

- Mean and median provide measures of location (center).
- One also needs some measures of variability to further describe the spread of the data set.
- Commonly used numerical values that can summarize the spread of a distribution.
  - Range
  - Interquartile Range (IQR)
  - Standard deviation

Quartiles

- The median divides the data into two groups of equal size.
- The quartiles divide the data into four groups of equal size.





Quartiles cont.

To find the quartiles:

1. Find the median.
2. Find the first quartile (Q1, or the *lower fourth*) by finding the median of the lower half of the data.
3. Find the third quartile (Q3, or the *upper fourth*) by finding the median of the upper half of the data.

   (*When n is odd include the median in both halves in steps 2 and 3.*)

<u>Ex.</u> Find the quartiles for the data set {2,4,6,8,12,14,18,19,41}.

2 4 6 8 12 14 18 19 41

2 4 6 8 12

12 14 18 19 41

M = 12 Q1 = 6 Q3=18

IQR

- The Interquartile Range, IQR, is the distance between the first and third quartiles,

$$IQR = Q3 - Q1.$$

- The IQR measures the spread of the middle 50% of the data.

- An observation is a suspected outlier if it falls more than 1.5*IQR from the closest fourth. An outlier is extreme if it is more than 3*IQR from the nearest fourth, and it is mild otherwise.

<u>Ex.</u> Can any of the observations in the data set {2,4,6,8,12,14,18,19,41} be considered outliers?

Recall we had M = 12, Q1=6, Q3=18. Therefore, IQR = 18 – 6 = 12.

1.5*IQR = 1.5*12 = 18. Q3+18 = 36, Q1-18 = -12. Since 41 > 36, 41 is classified as a potential outlier.