S1211Q Introduction to Statistics
Lecture 16

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Review of Estimations

- Estimation is an educated guess of the unknown parameters.
- All the discussions are based on the premise of Simple Random Sample.
- There are two types of Estimations: Point Estimation and Interval Estimation.

Point Estimation

- An estimator is a statistic, i.e., a function of the sample.
- We want our estimator to be truthful and stable.

Unbiasedness

- If $\hat{\theta}$ is an estimator of θ, then it is an unbiased estimator if $E(\hat{\theta}) = \theta$. This guarantee that in long term, the estimator yields the parameter truthfully.
- Under the assumption of Simple Random Sample, sample mean is the unbiased estimator of population mean and sample variance is the unbiased estimator of population variance.

Standard Error and Estimated Standard Error

- We also want to know the variability of our estimators (because they are RV's). We use their standard deviation as a measure, and we typically call it the standard error of the estimators.
- But sometimes the standard error involves unknown parameters and we need to find the estimated standard error by plugging in the estimators of the parameters.

Confidence Interval

- Confidence Interval is random, the parameter is fixed!
- The general strategy is to find a pivotal quantity and derive the CI from there.
- We have worked out the formula of CI's for sample mean μ in the following scenarios.

Normal Distribution, Known σ, Any Sample Size

- Under these assumptions, a $100(1-\alpha)\%$ CI of sample mean μ is given by

$$\left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

General Distribution, Unknown σ, Large Sample

- Under these assumptions, an approximate $100(1-\alpha)\%$ CI of sample mean μ is given by

$$\left(\bar{x} - z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}\right)$$

- Notice a special case if the distribution is Bernoulli, we have a more accurate but very complicated formula.

Normal Distribution, Unknown σ, Any Sample Size

- Under these assumptions, a $100(1-\alpha)\%$ CI of sample mean μ is given by

$$\left(\bar{x} - t_{\alpha/2,n-1} \cdot \frac{\hat{\sigma}}{\sqrt{n}}, \bar{x} + t_{\alpha/2,n-1} \cdot \frac{\hat{\sigma}}{\sqrt{n}}\right)$$

- Here we utilize the *t* distribution.