HOMEWORK 4

47.

a. $B(4;15,.3) = .515$.

b. $b(4;15,.3) = B(4;15,.3) - B(3;15,.3) = .219$.

f. $P(X \le 1) = B(1;15,.7) = .000$.

g. $P(2 < X < 6) = P(2 < X \le 5) = B(5;15,.3) - B(2;15,.3) = .595$.

49. Let X be the number of "seconds," so $X \sim \text{Bin}(6, .10)$.

a. $P(X = 1) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{6}{1} (.1)^1 (.9)^5 = .3543$.

b. $P(X \ge 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - \left[\binom{6}{0}(.1)^0(.9)^6 + \binom{6}{1}(.1)^1(.9)^5\right] = 1 - [.5314 + .3543] = .1143$.

c. Either 4 or 5 goblets must be selected.

Select 4 goblets with zero defects: $P(X = 0) = \binom{4}{0}(.1)^0(.9)^4 = .6561$.

Select 4 goblets, one of which has a defect, and the 5th is good: $\left[\binom{4}{1}(.1)^1(.9)^3\right] \times .9 = .26244$

So, the desired probability is $.6561 + .26244 = .91854$.

50. Let X be the number of faxes, so $X \sim \text{Bin}(25, .25)$.

a. $P(X \le 6) = B(6;25,.25) = .561$.

b. $P(X = 6) = b(6;25,.25) = .183$.

c. $P(X \ge 6) = 1 - P(X \le 5) = 1 - B(5;25,.25) = .622$.

d. $P(X > 6) = 1 - P(X \le 6) = 1 - .561 = .439$.

58. Let p denote the actual proportion of defectives in the batch, and X denote the number of defectives in the sample.

a. If the actual proportion of defectives is p, then $X \sim \text{Bin}(10, p)$, and the batch is accepted iff $X \le 2$. Using the binomial formula, $P(X \le 2) =$

$$\binom{10}{0} p^0 (1-p)^{10} + \binom{10}{1} p^1 (1-p)^9 + \binom{10}{2} p^2 (1-p)^8 = [(1-p)^2 + 10p(1-p) + 45p^2](1-p)^8.$$

Values for this expression are tabulated below.

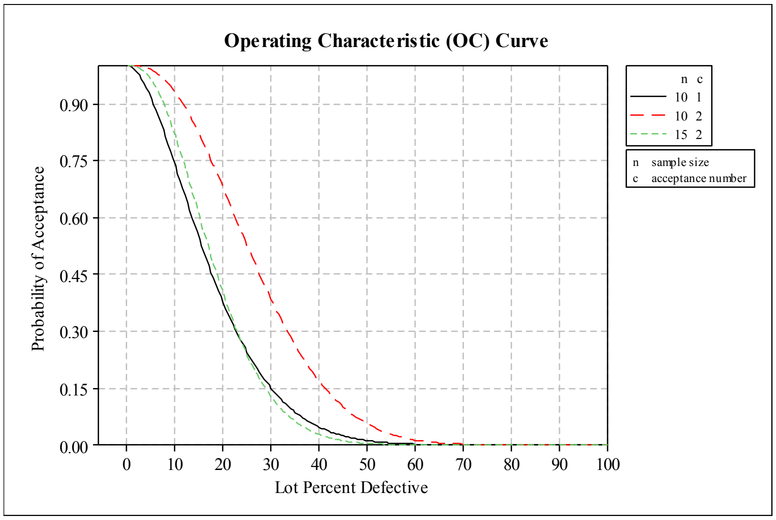
p:	.01	.05	.10	.20	.25
$P(X \le 2)$:	.9999	.9885	.9298	.6778	.5256

b. The polynomial function listed in part **a** is graphed below.

c. Replace "2" with "1," and the shipment is accepted iff $X \le 1$ and the probability of this event is given by $P(X \le 1) = \binom{10}{0}p^0(1-p)^{10} + \binom{10}{1}p^1(1-p)^9 = (1+9p)(1-p)^9$. Values for this new expression are tabulated below.

p:	.01	.05	.10	.20	.25
$P(X \le 1)$:	.9957	.9139	.7361	.3758	.2440

This operating characteristic (OC) curve is also graphed below.



d. Now $n = 15$, and $P(X \le 2) = \binom{15}{0}p^0(1-p)^{15} + \binom{15}{1}p^1(1-p)^{14} + \binom{15}{2}p^2(1-p)^{13}$. Values for this function are tabulated below. The corresponding OC curve is also presented above.

p:	.01	.05	.10	.20	.25
$P(X \le 2)$:	.9996	.9638	.8159	.3980	.2361

e. The exercise says the batch is acceptable iff $p \le 10$, so we want P(accept) to be high when p is less than .10 and low when p is greater than .10. The plan in **d** seems most satisfactory in these respects.

68.

a. There are 20 items total, 12 of which are "successes" (two slots). Among these 20 items, 6 have been randomly selected to be put under the shelf. So, the random variable X is hypergeometric, with $N = 20$, $M = 12$, and $n = 6$.

b. $P(X = 2) = \dfrac{\binom{12}{2}\binom{20-12}{6-2}}{\binom{20}{6}} = \dfrac{\binom{12}{2}\binom{8}{4}}{\binom{20}{6}} = \dfrac{(66)(70)}{(38760)} = .1192.$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = \dfrac{\binom{12}{0}\binom{8}{6}}{\binom{20}{6}} + \dfrac{\binom{12}{1}\binom{8}{5}}{\binom{20}{6}} + .1192 =$

$.0007 + .0174 + .1192 = .1373$.
$P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(X = 0) + P(X = 1)] = 1 - [.0007 + .0174] = .9819$.

c. $E(X) = n \cdot \frac{M}{N} = 6 \cdot \frac{12}{20} = 6 \cdot (.6) = 3.6$; $V(X) = \left(\frac{20-6}{20-1}\right) \cdot 6(.6)(1-.6) = 1.061$; $\sigma = 1.030$.

81. Let $X \sim$ Poisson($\mu = 20$).

a. $P(X \leq 10) = F(10; 20) = .011$.

b. $P(X > 20) = 1 - F(20; 20) = 1 - .559 = .441$.

c. $P(10 \leq X \leq 20) = F(20; 20) - F(9; 20) = .559 - .005 = .554$;
$P(10 < X < 20) = F(19; 20) - F(10; 20) = .470 - .011 = .459$.

d. $E(X) = \mu = 20$, so $\sigma = \sqrt{20} = 4.472$. Therefore, $P(\mu - 2\sigma < X < \mu + 2\sigma) =$
$P(20 - 8.944 < X < 20 + 8.944) = P(11.056 < X < 28.944) = P(X \leq 28) - P(X \leq 11) =$
$F(28; 20) - F(11; 20) = .966 - .021 = .945$.

86.

a. $P(X = 4) = \frac{e^{-5}5^4}{4!} = .175$.

b. $P(X \geq 4) = 1 - P(X \leq 3) = 1 - F(3; 5) = 1 - .265 = .735$.

c. Arrivals occur at the rate of 5 per hour, so for a 45-minute period the mean is $\mu = (5)(.75) =$ 3.75, which is the expected number of arrivals in a 45-minute period.