HOMEWORK 7

3.

a. We use the sample mean, $\bar{x} = 1.3481$.

b. Because we assume normality, the mean = median, so we also use the sample mean $\bar{x} = 1.3481$. We could also easily use the sample median.

c. We use the 90th percentile of the sample:
$\hat{\mu} + (1.28)\hat{\sigma} = \bar{x} + 1.28s = 1.3481 + (1.28)(.3385) = 1.7814$.

d. Since we can assume normality,

$$P(X < 1.5) \approx P\left(Z < \frac{1.5 - \bar{x}}{s}\right) = P\left(Z < \frac{1.5 - 1.3481}{.3385}\right) = P(Z < .45) = .6736.$$

e. The estimated standard error of $\bar{x} = \dfrac{\hat{\sigma}}{\sqrt{n}} = \dfrac{s}{\sqrt{n}} = \dfrac{.3385}{\sqrt{16}} = .0846.$

4.

a. $E(\bar{X} - \bar{Y}) = E(\bar{X}) - E(\bar{Y}) = \mu_1 - \mu_2$; $\bar{x} - \bar{y} = 8.141 - 8.575 = -.434$.

b. $V(\bar{X} - \bar{Y}) = V(\bar{X}) + V(\bar{Y}) = \sigma_{\bar{X}}^2 + \sigma_{\bar{Y}}^2 = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$ $\sigma_{\bar{X}-\bar{Y}} = \sqrt{V(\bar{X} - \bar{Y})} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$. The estimate would be $s_{\bar{X}-\bar{Y}} = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}} = \sqrt{\dfrac{1.66^2}{27} + \dfrac{2.104^2}{20}} = .5687$.

c. $\dfrac{s_1}{s_2} = \dfrac{1.660}{2.104} = .7890.$

d. $V(X - Y) = V(X) + V(Y) = \sigma_1^2 + \sigma_2^2 = 1.66^2 + 2.104^2 = 7.1824.$

5. Let θ = the total audited value. Three potential estimators of θ are $\hat{\theta}_1 = N\bar{X}$, $\hat{\theta}_2 = T - N\bar{D}$, and $\hat{\theta}_3 = T \cdot \dfrac{\bar{X}}{\bar{Y}}$. From the data, $\bar{y}$ = 374.6, $\bar{x}$ = 340.6, and $\bar{d}$ = 34.0. Knowing N = 5,000 and T = 1,761,300, the three corresponding estimates are $\hat{\theta}_1 = (5{,}000)(340.6) = 1{,}703{,}000$, $\hat{\theta}_2 = 1{,}761{,}300 - (5{,}000)(34.0) = 1{,}591{,}300$, and $\hat{\theta}_3 = 1{,}761{,}300\left(\dfrac{340.6}{374.6}\right) = 1{,}601{,}438.281$.

11.

a. $E\left(\frac{X_1}{n_1}-\frac{X_2}{n_2}\right)=\frac{1}{n_1}E(X_1)-\frac{1}{n_2}E(X_2)=\frac{1}{n_1}(n_1p_1)-\frac{1}{n_2}(n_2p_2)=p_1-p_2$.

b. $V\left(\frac{X_1}{n_1}-\frac{X_2}{n_2}\right)=V\left(\frac{X_1}{n_1}\right)+V\left(\frac{X_2}{n_2}\right)=\left(\frac{1}{n_1}\right)^2V(X_1)+\left(\frac{1}{n_2}\right)^2V(X_2)=$
$\frac{1}{n_1^2}(n_1p_1q_1)+\frac{1}{n_2^2}(n_2p_2q_2)=\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}$, and the standard error is the square root of this quantity.

c. With $\hat{p}_1=\frac{x_1}{n_1}$, $\hat{q}_1=1-\hat{p}_1$, $\hat{p}_2=\frac{x_2}{n_2}$, $\hat{q}_2=1-\hat{p}_2$, the estimated standard error is $\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$.

d. $(\hat{p}_1-\hat{p}_2)=\frac{127}{200}-\frac{176}{200}=.635-.880=-.245$

e. $\sqrt{\frac{(.635)(.365)}{200}+\frac{(.880)(.120)}{200}}=.041$

15.

a. $E(X^2)=2\theta$ implies that $E\left(\frac{X^2}{2}\right)=\theta$. Consider $\hat{\theta}=\frac{\sum X_i^2}{2n}$. Then
$E(\hat{\theta})=E\left(\frac{\sum X_i^2}{2n}\right)=\frac{\sum E(X_i^2)}{2n}=\frac{\sum 2\theta}{2n}=\frac{2n\theta}{2n}=\theta$, implying that $\hat{\theta}$ is an unbiased estimator for θ.

b. $\sum x_i^2=1490.1058$, so $\hat{\theta}=\frac{1490.1058}{20}=74.505$.