4.2

$f(x) = \frac{1}{10}$ for $-5 \le x \le 5$ and $= 0$ otherwise

a. $P(X < 0) = \int_{-5}^{0} \frac{1}{10} dx = .5$.

b. $P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} \frac{1}{10} dx = .5$.

c. $P(-2 \le X \le 3) = \int_{-2}^{3} \frac{1}{10} dx = .5$.

d. $P(k < X < k + 4) = \int_{k}^{k+4} \frac{1}{10} dx = \frac{1}{10} x \Big]_{k}^{k+4} = \frac{1}{10}[(k+4) - k] = .4$.

4.4

a. $\int_{-\infty}^{\infty} f(x;\theta)dx = \int_{0}^{\infty} \frac{x}{\theta^2} e^{-x^2/2\theta^2} dx = -e^{-x^2/2\theta^2} \Big]_{0}^{\infty} = 0 - (-1) = 1$

b. $P(X \le 200) = \int_{-\infty}^{200} f(x;\theta)dx = \int_{0}^{200} \frac{x}{\theta^2} e^{-x^2/2\theta^2} dx = -e^{-x^2/2\theta^2} \Big]_{0}^{200} \approx -.1353 + 1 = .8647$.

$P(X < 200) = P(X \le 200) \approx .8647$, since X is continuous.

$P(X \ge 200) = 1 - P(X < 200) \approx .1353$.

c. $P(100 \le X \le 200) = \int_{100}^{200} f(x;\theta)dx = -e^{-x^2/20{,}000} \Big]_{100}^{200} \approx .4712$.

d. For $x > 0$, $P(X \le x) = \int_{-\infty}^{x} f(y;\theta)dy = \int_{0}^{x} \frac{y}{\theta^2} e^{-y^2/2\theta^2} dx = -e^{-y^2/2\theta^2} \Big]_{0}^{x} = 1 - e^{-x^2/2\theta^2}$.

4.12

a. $P(X < 0) = F(0) = .5$.

b. $P(-1 \le X \le 1) = F(1) - F(-1) = .6875$.

c. $P(X > .5) = 1 - P(X \le .5) = 1 - F(.5) = 1 - .6836 = .3164$.

d. $f(x) = F'(x) = \frac{d}{dx}\left(\frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right)\right) = 0 + \frac{3}{32}\left(4 - \frac{3x^2}{3}\right) = .09375\left(4 - x^2\right)$.

e. By definition, $F(\tilde{\mu}) = .5$. $F(0) = .5$ from **a** above, which is as desired.

4.23

With X = temperature in °C, the temperature in °F equals $1.8X + 32$, so the mean and standard deviation in °F are $1.8\mu_X + 32 = 1.8(120) + 32 = 248$°F and $|1.8|\sigma_X = 1.8(2) = 3.6$°F. Notice that the additive constant, 32, affects the mean but does <u>not</u> affect the standard deviation.

4.28

a. $P(0 \le Z \le 2.17) = \Phi(2.17) - \Phi(0) = .4850$.

b. $\Phi(1) - \Phi(0) = .3413$.

c. $\Phi(0) - \Phi(-2.50) = .4938$.

d. $\Phi(2.50) - \Phi(-2.50) = .9876$.

e. $\Phi(1.37) = .9147$.

f. $P(-1.75 < Z) + [1 - P(Z < -1.75)] = 1 - \Phi(-1.75) = .9599$.

g. $\Phi(2) - \Phi(-1.50) = .9104$.

h. $\Phi(2.50) - \Phi(1.37) = .0791$.

4.36

a. $P(X < 1500) = P(Z < 3) = \Phi(3) = .9987$; $P(X \ge 1000) = P(Z \ge -.33) = 1 - \Phi(-.33) = 1 - .3707 = .6293$.

b. $P(1000 < X < 1500) = P(-.33 < Z < 3) = \Phi(3) - \Phi(-.33) = .9987 - .3707 = .6280$

c. From the table, $\Phi(z) = .02 \Rightarrow z = -2.05 \Rightarrow x = 1050 - 2.05(150) = 742.5$ μm. The smallest 2% of droplets are those smaller than 742.5 μm in size.

d. P(at least one droplet in 5 that exceeds 1500 μm) = 1 – P(all 5 are less than 1500 μm) = $1 - (.9987)^5 = 1 - .9935 = .0065$.

4.45

With μ = .500 inches, the acceptable range for the diameter is between .496 and .504 inches, so unacceptable bearings will have diameters smaller than .496 or larger than .504.

The new distribution has $\mu = .499$ and $\sigma = .002$.

$P(X < .496 \text{ or } X > .504) = P\left(Z < \frac{.496-.499}{.002}\right) + P\left(Z > \frac{.504-.499}{.002}\right) = P(Z < -1.5) + P(Z > 2.5) =$

$\Phi(-1.5) + [1 - \Phi(2.5)] = .073$. 7.3% of the bearings will be unacceptable.

5.7

a. $p(1,1) = .030$.

b. $P(X \le 1 \text{ and } Y \le 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .120$.

c. $P(X = 1) = p(1,0) + p(1,1) + p(1,2) = .100$; $P(Y = 1) = p(0,1) + \ldots + p(5,1) = .300$.

d. $P(\text{overflow}) = P(X + 3Y > 5) = 1 - P(X + 3Y \le 5) = 1 - P((X,Y)=(0,0) \text{ or } \ldots\text{or } (5,0) \text{ or } (0,1) \text{ or } (1,1) \text{ or } (2,1)) = 1 - .620 = .380$.

e. The marginal probabilities for X (row sums from the joint probability table) are $p_X(0) = .05$, $p_X(1) = .10$, $p_X(2) = .25$, $p_X(3) = .30$, $p_X(4) = .20$, $p_X(5) = .10$; those for Y (column sums) are $p_Y(0) = .5$, $p_Y(1) = .3$, $p_Y(2) = .2$. It is now easily verified that for every (x,y), $p(x,y) = p_X(x) \cdot p_Y(y)$, so X and Y are independent.

5.22

a. $E(X+Y) = \sum\sum (x+y)p(x,y) = (0 + 0)(.02) + (5 + 0)(.04) + \ldots + (10 + 15)(.01) = 14.10$. Note: It can be shown that $E(X + Y)$ always equals $E(X) + E(Y)$, so in this case we could also work out the means of X and Y from their marginal distributions: $E(X) = 5.55$, $E(Y) = 8.55$, so $E(X + Y) = 5.55 + 8.55 = 14.10$.

b. For each coordinate, we need the maximum; e.g., max(0,0) = 0, while max(5,0) = 5 and max(5,10) = 10. Then calculate the sum: $E(\max(X,Y)) = \sum\sum \max(x,y)\cdot p(x,y) =$ $\max(0,0)(.02) + \max(5,0)(.04) + \ldots + \max(10,15)(.01) = 0(.02) + 5(.04) + \ldots + 15(.01) = 9.60$.