2.38

a. There are 6 75W bulbs and 9 other bulbs. So, P(select exactly 2 75W bulbs) = P(select exactly 2 75W bulbs and 1 other bulb) = $\frac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \frac{(15)(9)}{455} = .2967$.

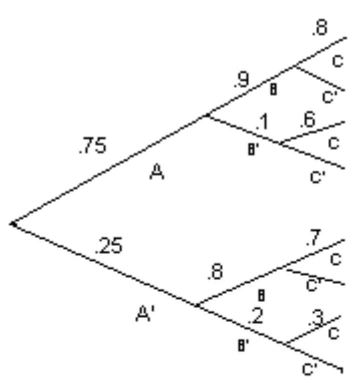
b. P(all three are the same rating) = P(all 3 are 40W or all 3 are 60W or all 3 are 75W) = $\frac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \frac{4+10+20}{455} = .0747$.

c. P(one of each type is selected) = $\frac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = \frac{120}{455} = .2637$.

d. It is necessary to examine at least six bulbs if and only if the first five light bulbs were all of the 40W or 60W variety. Since there are 9 such bulbs, the chance of this event is $\frac{\binom{9}{5}}{\binom{15}{5}} = \frac{126}{3003} = .042$.

2.63

e.



f. From the top path of the tree diagram, $P(A \cap B \cap C) = (.75)(.9)(.8) = .54$.

g. Event $B \cap C$ occurs twice on the diagram: $P(B \cap C) = P(A \cap B \cap C) + P(A' \cap B \cap C) = .54 + (.25)(.8)(.7) = .68$.

h. $P(C) = P(A \cap B \cap C) + P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B' \cap C) = .54 + .045 + .14 + .015 = .74$.

i. Rewrite the conditional probability first: $P(A \mid B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{.54}{.68} = .7941$.

2.93

Apply the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow .626 = P(A) + P(B) - .144$. Apply independence: $P(A \cap B) = P(A)P(B) = .144$.
So, $P(A) + P(B) = .770$ and $P(A)P(B) = .144$.
Let $x = P(A)$ and $y = P(B)$. Using the first equation, $y = .77 - x$, and substituting this into the second equation yields $x(.77 - x) = .144$ or $x^2 - .77x + .144 = 0$. Use the quadratic formula to solve:

$$x = \frac{.77 \pm \sqrt{(-.77)^2 - (4)(1)(.144)}}{2(1)} = \frac{.77 \pm .13}{2} = .32 \text{ or } .45.$$ Since $x = P(A)$ is assumed to be the larger probability, $x = P(A) = .45$ and $y = P(B) = .32$.

2.101

Let $A = 1^{st}$ functions, $B = 2^{nd}$ functions, so $P(B) = .9$, $P(A \cup B) = .96$, $P(A \cap B) = .75$. Use the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow .96 = P(A) + .9 - .75 \Rightarrow P(A) = .81$.

Therefore, $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{.75}{.81} = .926$.

3.52

Let X be the number of students who want a new copy, so $X \sim \text{Bin}(n = 25, p = .3)$.

a. $E(X) = np = 25(.3) = 7.5$ and $SD(X) = \sqrt{np(1-p)} = \sqrt{25(.3)(.7)} = 2.29$.

b. Two standard deviations from the mean converts to $7.5 \pm 2(2.29) = 2.92$ & 12.08. For X to be more than two standard deviations from the means requires $X < 2.92$ or $X > 12.08$. Since X must be a non-negative integer, $P(X < 2.92 \text{ or } X > 12.08) = 1 - P(2.92 \le X \le 12.08) = 1 - P(3 \le X \le 12) =$

$$1 - \sum_{x=3}^{12} \binom{25}{x} (.3)^x (.7)^{25-x} = 1 - .9736 = .0264.$$

3.80

Solutions are found using the cumulative Poisson table, $F(x; \mu) = F(x; 4)$.

a. $P(X \le 4) = F(4; 4) = .629$, while $P(X < 4) = P(X \le 3) = F(3; 4) = .434$.

b. $P(4 \le X \le 8) = F(8; 4) - F(3; 4) = .545$.

c. $P(X \ge 8) = 1 - P(X < 8) = 1 - P(X \le 7) = 1 - F(7; 4) = .051$.

d. For this Poisson model, $\mu = 4$ and so $\sigma = \sqrt{4} = 2$. The desired probability is $P(X \le \mu + \sigma) = P(X \le 4 + 2) = P(X \le 6) = F(6; 4) = .889$.

3.97

e. From the description, $X \sim \text{Bin}(15, .75)$. So, the pmf of X is $b(x; 15, .75)$.

f. $P(X > 10) = 1 - P(X \le 10) = 1 - B(10;15, .75) = 1 - .314 = .686$.

g. $P(6 \le X \le 10) = B(10; 15, .75) - B(5; 15, .75) = .314 - .001 = .313$.

h. $\mu = (15)(.75) = 11.75$, $\sigma^2 = (15)(.75)(.25) = 2.81$.

i. Requests can all be met if and only if $X \le 10$, and $15 - X \le 8$, i.e. iff $7 \le X \le 10$. So, $P(\text{all requests met}) = P(7 \le X \le 10) = B(10; 15, .75) - B(6; 15, .75) = .310$.

4.28.

a. $P(0 \le Z \le 2.17) = \Phi(2.17) - \Phi(0) = .4850$.

b. $\Phi(1) - \Phi(0) = .3413$.

c. $\Phi(0) - \Phi(-2.50) = .4938$.

d. $\Phi(2.50) - \Phi(-2.50) = .9876$.

e. $\Phi(1.37) = .9147$.

f. $P(-1.75 < Z) + [1 - P(Z < -1.75)] = 1 - \Phi(-1.75) = .9599$.

g. $\Phi(2) - \Phi(-1.50) = .9104$.

h. $\Phi(2.50) - \Phi(1.37) = .0791$.

i. $1 - \Phi(1.50) = .0668$.

j. $P(|Z| \le 2.50) = P(-2.50 \le Z \le 2.50) = \Phi(2.50) - \Phi(-2.50) = .9876$.

4.29

a. .9838 is found in the 2.1 row and the .04 column of the standard normal table so $c = 2.14$.

b. $P(0 \le Z \le c) = .291 \Rightarrow \Phi(c) - \Phi(0) = .2910 \Rightarrow \Phi(c) - .5 = .2910 \Rightarrow \Phi(c) = .7910 \Rightarrow$ from the standard normal table, $c = .81$.

c. $P(c \le Z) = .121 \Rightarrow 1 - P(Z < c) = .121 \Rightarrow 1 - \Phi(c) = .121 \Rightarrow \Phi(c) = .879 \Rightarrow c = 1.17$.

d. $P(-c \le Z \le c) = \Phi(c) - \Phi(-c) = \Phi(c) - (1 - \Phi(c)) = 2\Phi(c) - 1 = .668 \Rightarrow \Phi(c) = .834 \Rightarrow$
$c = 0.97$.

e. $P(c \le |Z|) = 1 - P(|Z| < c) = 1 - [\Phi(c) - \Phi(-c)] = 1 - [2\Phi(c) - 1] = 2 - 2\Phi(c) = .016 \Rightarrow \Phi(c) = .992 \Rightarrow c = 2.41$.

4.105

a. $P(X > 100) = 1 - \Phi\left(\frac{100-96}{14}\right) = 1 - \Phi(.29) = 1 - .6141 = .3859.$

b. $P(50 < X < 80) = \Phi\left(\frac{80-96}{14}\right) - \Phi\left(\frac{50-96}{14}\right) = \Phi(-1.5) - \Phi(-3.29) = .1271 - .0005$ $= .1266.$

c. Notice that a and b are the 5^{th} and 95^{th} percentiles, respectively. From the standard normal table, $\Phi(z) = .05 \Rightarrow z = -1.645$, so -1.645 is the 5^{th} percentile of the standard normal distribution. By symmetry, the 95^{th} percentile is $z = 1.645$. So, the desired percentiles of this distribution are $a = 96 + (-1.645)(14) = 72.97$ and $b = 96 + (1.645)(14) = 119.03$. The interval (72.97, 119.03) contains the central 90% of all grain sizes.

4.106

d. $F(x) = 0$ for $x < 1$ and $F(x) = 1$ for $x > 3$. For $1 \le x \le 3$,
$F(x) = \int_1^x \frac{3}{2}\cdot\frac{1}{y^2}\,dy = 1.5\left(1 - \frac{1}{x}\right).$

e. $P(X \le 2.5) = F(2.5) = 1.5(1 - .4) = .9$; $P(1.5 \le X \le 2.5) = F(2.5) - F(1.5) = .4.$

f. $E(X) = \int_1^3 x\cdot\frac{3}{2}\cdot\frac{1}{x^2}\,dx = \frac{3}{2}\int_1^3 \frac{1}{x}\,dx = 1.5\ln(x)\Big]_1^3 = 1.648.$

g. $E(X^2) = \int_1^3 x^2\cdot\frac{3}{2}\cdot\frac{1}{x^2}\,dx = \frac{3}{2}\int_1^3 dx = 3$, so $V(X) = E(X^2) - [E(X)]^2 = .284$ and $\sigma = .553$.

h. From the description, $h(x) = 0$ if $1 \le x \le 1.5$; $h(x) = x - 1.5$ if $1.5 \le x \le 2.5$ (one second later), and $h(x) = 1$ if $2.5 \le x \le 3$. Using those terms,
$E[h(X)] = \int_1^3 h(x)\,dx = \int_{1.5}^{2.5} (x - 1.5)\cdot\frac{3}{2}\cdot\frac{1}{x^2}\,dx + \int_{2.5}^{3} 1\cdot\frac{3}{2}\cdot\frac{1}{x^2}\,dx = .267.$

5.22

a. $E(X+Y) = \sum\sum(x+y)p(x,y) = (0+0)(.02) + (5+0)(.04) + \ldots + (10+15)(.01) = 14.10.$
Note: It can be shown that $E(X + Y)$ always equals $E(X) + E(Y)$, so in this case we could also work out the means of X and Y from their marginal distributions: $E(X) = 5.55$, $E(Y) = 8.55$, so $E(X + Y) = 5.55 + 8.55 = 14.10$.

b. For each coordinate, we need the maximum; e.g., max(0,0) = 0, while max(5,0) = 5 and max(5,10) = 10. Then calculate the sum: $E(\max(X,Y)) = \sum\sum \max(x,y)\cdot p(x,y) =$ max(0,0)(.02) + max(5,0)(.04) + … + max(10,15)(.01) = 0(.02) + 5(.04) + … + 15(.01) = 9.60.

c. If $X > 15$, then more people want new copies than the bookstore carries. At the other end, though, there are $25 - X$ students wanting used copies; if $25 - X > 15$, then there aren't enough used copies to meet demand.

The inequality $25 - X > 15$ is the same as $X < 10$, so the bookstore can't meet demand if either $X > 15$ or $X < 10$. All 25 students get the type they want iff $10 \le X \le 15$:

$P(10 \le X \le 15) = \sum_{x=10}^{15} \binom{25}{x} (.3)^x (.7)^{25-x} = .1890.$

d. The bookstore sells X new books and $25 - X$ used books, so total revenue from these 25 sales is given by $h(X) = 100(X) + 70(25 - X) = 30X + 1750$. Using linearity/rescaling properties, expected revenue equals $E(h(X)) = E(30X + 1750) = 30\mu + 1750 = 30(7.5) + 1750 = \$1975$.