STAT W1211 Sec 3 Introduction to Statistics (Fall 2012)
Quiz 2

Problem 1: Sampling Distributions

The time to complete a medical scanning procedure has a normal distribution with mean value 30 min and standard deviation 4 min. We have a simple random sample of 25 scans.

a What is the distribution of the sample mean completion time?

Solution: *Since the population is normal, we know that the sample mean is also a normal random variable. Besides, the expectation is 30, and standard deviation is* $\frac{4}{\sqrt{25}} = 0.8$.

$$\bar{X} \sim N(30, 0.8^2)$$

b If the time to completion is not normal, what conclusion can we reach about the distribution of the sample mean?

Solution: *The expectation of the sample mean is still 30, and the standard deviation is still 0.8. These two properties hold for any population distribution. But the form of distribution of the sample mean is not normal anymore. When* n *is large, say more than 30, we could use Central Limit Theorem and approximate the true distribution with normal distribution.*

Problem 2: Point Estimation

a Why is unbiasedness of an estimator for parameter θ not necessarily a guarantee that it will give an estimate close to the actual value of θ?

Solutions: *Unbiasedness only guarantees that in long run (i.e., sample size is large) the estimator will yield an estimate that is close to the true value of* θ. *However, in the absence of large sample, the estimator might fluctuate around the true value of* θ *due to randomness. That is why in addition to unbiasedness, we also look for estimators with small variance.*

b If $\hat{\alpha}$ is an unbiased estimator of a parameter α and β is a n unbiased estimator of a parameter β, give an unbiased estimator of $3\alpha - 2\beta$. Justify your answer.

Solution: *Since* $\hat{\alpha}$ *and* $\hat{\beta}$ *are both unbiased, we have*

$$E(\hat{\alpha}) = \alpha \text{ and } E(\hat{\beta}) = \beta$$

It is natural to propose $3\hat{\alpha} - 2\hat{\beta}$ as our estimator for $3\alpha - 2\beta$, and to show that it is unbiased

$$E(3\hat{\alpha} - 2\hat{\beta}) = 3E(\hat{\alpha}) - 2E(\hat{\beta}) = 3\alpha - 2\beta$$

The first equality results from linearity of expectation.