2.38

a. There are 6 75W bulbs and 9 other bulbs. So, P(select exactly 2 75W bulbs) = P(select exactly 2 75W bulbs and 1 other bulb) = $\dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \dfrac{(15)(9)}{455} = .2967$.

b. P(all three are the same rating) = P(all 3 are 40W or all 3 are 60W or all 3 are 75W) =
$\dfrac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \dfrac{4+10+20}{455} = .0747$.

c. P(one of each type is selected) = $\dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = \dfrac{120}{455} = .2637$.

d. It is necessary to examine at least six bulbs if and only if the first five light bulbs were all of the 40W or 60W variety. Since there are 9 such bulbs, the chance of this event is
$\dfrac{\binom{9}{5}}{\binom{15}{5}} = \dfrac{126}{3003} = .042$.

2.100

a. First, P(both +) = P(carrier $\cap$ both +) + P(not a carrier $\cap$ both +) =
P(carrier)P(both + | carrier) + P(not a carrier)P(both + | not a carrier). Assuming independence of the tests, this equals $(.01)(.90)^2 + (.99)(.05)^2 = .010575$.
Similarly, P(both –) = $(.01)(.10)^2 + (.99)(.95)^2 = .893575$.
Therefore, P(tests agree) = .010575 + .893575 = .90415.

b. From the first part of **a**, P(carrier | both +) = $\dfrac{P(\text{carrier} \cap \text{both } +)}{P(\text{both } +)} = \dfrac{(.01)(.90)^2}{.010575} = .766$.

3.50

Let X be the number of faxes, so $X \sim \text{Bin}(25, .25)$.

a. $P(X \le 6) = B(6;25,.25) = .561$.

b. $P(X = 6) = b(6;25,.25) = .183$.

c. $P(X \ge 6) = 1 - P(X \le 5) = 1 - B(5;25,.25) = .622$.

d. $P(X > 6) = 1 - P(X \le 6) = 1 - .561 = .439$.

3.81

Let $X \sim \text{Poisson}(\mu = 20)$.

a. $P(X \le 10) = F(10; 20) = .011$.

b. $P(X > 20) = 1 - F(20; 20) = 1 - .559 = .441$.

c. $P(10 \le X \le 20) = F(20; 20) - F(9; 20) = .559 - .005 = .554$;
$P(10 < X < 20) = F(19; 20) - F(10; 20) = .470 - .011 = .459$.

d. $E(X) = \mu = 20$, so $\sigma = \sqrt{20} = 4.472$. Therefore, $P(\mu - 2\sigma < X < \mu + 2\sigma) =$
$P(20 - 8.944 < X < 20 + 8.944) \quad = P(11.056 < X < 28.944) = P(X \le 28) - P(X \le 11) =$
$F(28; 20) - F(11; 20) = .966 - .021 = .945$.

4.45

With $\mu = .500$ inches, the acceptable range for the diameter is between .496 and .504 inches, so unacceptable bearings will have diameters smaller than .496 or larger than .504.
The new distribution has $\mu = .499$ and $\sigma = .002$.

$P(X < .496 \text{ or } X > .504) = P\left(Z < \dfrac{.496 - .499}{.002}\right) + P\left(Z > \dfrac{.504 - .499}{.002}\right) = P(Z < -1.5) + P(Z > 2.5) =$

$\Phi(-1.5) + [1 - \Phi(2.5)] = .073$. 7.3% of the bearings will be unacceptable.

4.106

a. $F(x) = 0$ for $x < 1$ and $F(x) = 1$ for $x > 3$. For $1 \le x \le 3$, $F(x) = \int_1^x \frac{3}{2} \cdot \frac{1}{y^2}\, dy = 1.5\left(1 - \frac{1}{x}\right)$.

b. $P(X \le 2.5) = F(2.5) = 1.5(1 - .4) = .9$; $P(1.5 \le X \le 2.5) = F(2.5) - F(1.5) = .4$.

c. $E(X) = = \int_1^3 x \cdot \frac{3}{2} \cdot \frac{1}{x^2}\, dx = \frac{3}{2}\int_1^3 \frac{1}{x}\, dx = 1.5\ln(x)\big]_1^3 = 1.648$.

d. $E(X^2) = = \int_1^3 x^2 \cdot \frac{3}{2} \cdot \frac{1}{x^2}\, dx = \frac{3}{2}\int_1^3 dx = 3$, so $V(X) = E(X^2) - [E(X)]^2 = .284$ and $\sigma = .553$.

e. From the description, $h(x) = 0$ if $1 \le x \le 1.5$; $h(x) = x - 1.5$ if $1.5 \le x \le 2.5$ (one second later), and $h(x) = 1$ if $2.5 \le x \le 3$. Using those terms,
$E[h(X)] = \int_1^3 h(x)\, dx = \int_{1.5}^{2.5} (x - 1.5) \cdot \frac{3}{2} \cdot \frac{1}{x^2}\, dx + \int_{2.5}^3 1 \cdot \frac{3}{2} \cdot \frac{1}{x^2}\, dx = .267$.

5.60

Y is normally distributed with $\mu_Y = \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{3}(\mu_3 + \mu_4 + \mu_5) = -1$, and

$\sigma_Y^2 = \frac{1}{4}\sigma_1^2 + \frac{1}{4}\sigma_2^2 + \frac{1}{9}\sigma_3^2 + \frac{1}{9}\sigma_4^2 + \frac{1}{9}\sigma_5^2 = 3.167 \Rightarrow \sigma_Y = 1.7795$.

Thus, $P(0 \le Y) = P\left(\frac{0-(-1)}{1.7795} \le Z\right) = P(.56 \le Z) = .2877$ and

$P(-1 \le Y \le 1) = P\left(0 \le Z \le \frac{2}{1.7795}\right) = P(0 \le Z \le 1.12) = .3686.$

6.11

a. $E\left(\frac{X_1}{n_1} - \frac{X_2}{n_2}\right) = \frac{1}{n_1}E(X_1) - \frac{1}{n_2}E(X_2) = \frac{1}{n_1}(n_1 p_1) - \frac{1}{n_2}(n_2 p_2) = p_1 - p_2$.

b. $V\left(\frac{X_1}{n_1} - \frac{X_2}{n_2}\right) = V\left(\frac{X_1}{n_1}\right) + V\left(\frac{X_2}{n_2}\right) = \left(\frac{1}{n_1}\right)^2 V(X_1) + \left(\frac{1}{n_2}\right)^2 V(X_2) =$

$\frac{1}{n_1^2}(n_1 p_1 q_1) + \frac{1}{n_2^2}(n_2 p_2 q_2) = \frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}$, and the standard error is the square root of this quantity.

c. With $\hat{p}_1 = \frac{x_1}{n_1}$, $\hat{q}_1 = 1 - \hat{p}_1$, $\hat{p}_2 = \frac{x_2}{n_2}$, $\hat{q}_2 = 1 - \hat{p}_2$, the estimated standard error is $\sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$.

d. $(\hat{p}_1 - \hat{p}_2) = \frac{127}{200} - \frac{176}{200} = .635 - .880 = -.245$

e. $\sqrt{\frac{(.635)(.365)}{200} + \frac{(.880)(.120)}{200}} = .041$

7.20

Because the sample size is so large, the simpler formula (7.11) for the confidence interval for p is sufficient:

$.15 \pm 2.58\sqrt{\frac{(.15)(.85)}{4722}} = .15 \pm .013 = (.137, .163).$

8.58

μ = the true average percentage of organic matter in this type of soil, and the hypotheses are H_0: $\mu = 3$ v. H_a: $\mu \ne 3$. With $n = 30$, and assuming normality, we use the t test: $t = \frac{\bar{x} - 3}{s / \sqrt{n}} = \frac{2.481 - 3}{.295} = \frac{-.519}{.295} = -1.759$. The P-value = 2[P($t >$ 1.759)] = 2(.041) = .082. At significance level .10, since .082 ≤ .10, we would reject H_0 and conclude that the true average percentage of organic matter in this type of soil is something other than 3. At significance level .05, we would not have rejected H_0.