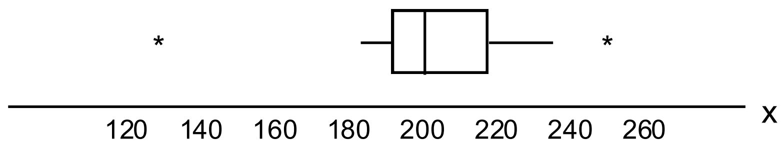
1.57

**a.** $f_s = 216.8 - 196.0 = 20.8$
inner fences: $196 - 1.5(20.8) = 164.6$, $216.8 + 1.5(20.8) = 248$
outer fences: $196 - 3(20.8) = 133.6$, $216.8 + 3(20.8) = 279.2$
Of the observations listed, 125.8 is an extreme low outlier and 250.2 is a mild high outlier.

**b.** A boxplot of this data appears below. There is a bit of positive skew to the data but, except for the two outliers identified in part (a), the variation in the data is relatively small.




1.78

**a.** Since the constant $\bar{x}$ is subtracted from each $x$ value to obtain each $y$ value, and addition or subtraction of a constant doesn't affect variability, $s_y^2 = s_x^2$ and $s_y = s_x$.

**b.** Let $c = 1/s$, where $s$ is the sample standard deviation of the $x$'s (and also, by part (a), of the $y$'s). Then $z_i = cy_i \Rightarrow s_z^2 = c^2 s_y^2 = (1/s)^2 s^2 = 1$ and $s_z = 1$. That is, the "standardized" quantities $z_1, \ldots, z_n$ have a sample variance and standard deviation of 1.

2.93

Apply the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow .626 = P(A) + P(B) - .144$. Apply independence: $P(A \cap B) = P(A)P(B) = .144$.
So, $P(A) + P(B) = .770$ and $P(A)P(B) = .144$.
Let $x = P(A)$ and $y = P(B)$. Using the first equation, $y = .77 - x$, and substituting this into the second equation yields $x(.77 - x) = .144$ or $x^2 - .77x + .144 = 0$. Use the quadratic formula to solve:

$$x = \frac{.77 \pm \sqrt{(-.77)^2 - (4)(1)(.144)}}{2(1)} = \frac{.77 \pm .13}{2} = .32 \text{ or } .45.$$ Since $x = P(A)$ is assumed to be the larger probability, $x = P(A) = .45$ and $y = P(B) = .32$.

2.104

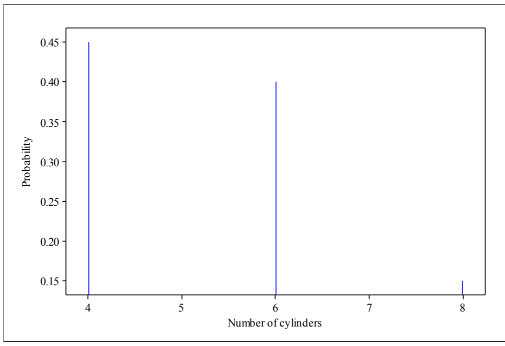
Let $B$ denote the event that a component needs rework. By the law of total probability,
$P(B) = \sum P(A_i)P(B \mid A_i) = (.50)(.05) + (.30)(.08) + (.20)(.10) = .069$.
Thus, $P(A_1 \mid B) = \frac{(.50)(.05)}{.069} = .362$, $P(A_2 \mid B) = \frac{(.30)(.08)}{.069} = .348$, and $P(A_3 \mid B) = .290$.

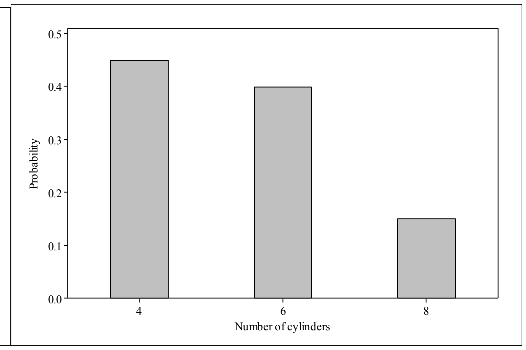
3.11

**a.** As displayed in the chart, $p(4) = .45$, $p(6) = .40$, $p(8) = .15$, and $p(x) = 0$ otherwise.

| $x$ | 4 | 6 | 8 |
|---|---|---|---|
| $p(x)$ | .45 | .40 | .15 |

**b.**




**c.** $P(X \geq 6) = .40 + .15 = .55$; $P(X > 6) = P(X = 8) = .15$.

3.95

**a.** We'll find $p(1)$ and $p(4)$ first, since they're easiest, then $p(2)$. We can then find $p(3)$ by subtracting the others from 1.
$p(1) = P(\text{exactly one suit}) = P(\text{all } \spadesuit) + P(\text{all ⍰}) + P(\text{all ⍰}) + P(\text{all ⍰}) =$

$4 \cdot P(\text{all } \spadesuit) = 4 \cdot \dfrac{\binom{13}{5}\binom{39}{0}}{\binom{52}{5}} = .00198$, since there are 13 ♠s and 39 other cards.

$$p(4) = 4 \cdot P(2\ \spadesuit,\ 1\ \heartsuit,\ 1\ \diamondsuit,\ 1\ \clubsuit) = 4 \cdot \frac{\binom{13}{2}\binom{13}{1}\binom{13}{1}\binom{13}{1}}{\binom{52}{5}} = .26375.$$

$p(2) = P(\text{all } \heartsuit\text{s and } \spadesuit\text{s, with} \geq \text{one of each}) + \ldots + P(\text{all } \diamondsuit\text{s and } \clubsuit\text{s with} \geq \text{one of each}) =$
$\binom{4}{2} \cdot P(\text{all } \heartsuit\text{s and } \spadesuit\text{s, with} \geq \text{one of each}) =$
$6 \cdot [P(1\ \heartsuit \text{ and } 4\ \spadesuit) + P(2\ \heartsuit \text{ and } 3\ \spadesuit) + P(3\ \heartsuit \text{ and } 2\ \spadesuit) + P(4\ \heartsuit \text{ and } 1\ \spadesuit)] =$

$$6 \cdot \left[2 \cdot \frac{\binom{13}{4}\binom{13}{1}}{\binom{52}{5}} + 2 \cdot \frac{\binom{13}{3}\binom{13}{2}}{\binom{52}{5}}\right] = 6\left[\frac{18{,}590 + 44{,}616}{2{,}598{,}960}\right] = .14592.$$

Finally, $p(3) = 1 - [p(1) + p(2) + p(4)] = .58835$.

**b.** $\mu = \sum_{x=1}^{4} x \cdot p(x) = 3.114$; $\sigma^2 = \left[\sum_{x=1}^{4} x^2 \cdot p(x)\right] - (3.114)^2 = .405 \Rightarrow \sigma = .636$.

4.100

**a.** Clearly $f(x) \geq 0$. Now check that the function integrates to 1:

$$\int_0^\infty \frac{32}{(x+4)^3}\,dx = \int_0^\infty 32(x+4)^{-3}\,dx = -\frac{16}{(x+4)^2}\bigg|_0^\infty = 0 - -\frac{16}{(0+4)^2} = 1.$$

**b.** For $x \leq 0$, $F(x) = 0$. For $x > 0$,

$$F(x)=\int_{-\infty}^{x} f(y)dy=\int_0^x \frac{32}{(y+4)^3}dy=-\frac{1}{2}\cdot\frac{32}{(y+4)^2}\Bigg]_0^x=1-\frac{16}{(x+4)^2}.$$

**c.** $P(2 \le X \le 5) = F(5) - F(2) = 1-\frac{16}{81}-\left(1-\frac{16}{36}\right)=.247.$

**d.** $E(X)=\int_{-\infty}^{\infty} x\cdot f(x)dx=\int_{-\infty}^{\infty} x\cdot\frac{32}{(x+4)^3}dx=\int_0^{\infty}(x+4-4)\cdot\frac{32}{(x+4)^3}dx$

$=\int_0^{\infty}\frac{32}{(x+4)^2}dx-4\int_0^{\infty}\frac{32}{(x+4)^3}dx=8-4=4$ years.

**e.** $E\left(\frac{100}{X+4}\right)=\int_0^{\infty}\frac{100}{x+4}\cdot\frac{32}{(x+4)^3}dx=3200\int_0^{\infty}\frac{1}{(x+4)^4}dx=\frac{3200}{(3)(64)}=16.67.$

4.106

**a.** $F(x) = 0$ for $x < 1$ and $F(x) = 1$ for $x > 3$. For $1 \le x \le 3$,
$F(x)=\int_1^x \frac{3}{2}\cdot\frac{1}{y^2}dy=1.5\left(1-\frac{1}{x}\right).$

**b.** $P(X \le 2.5) = F(2.5) = 1.5(1 - .4) = .9$; $P(1.5 \le X \le 2.5) = F(2.5) - F(1.5) = .4$.

**c.** $E(X) = \int_1^3 x\cdot\frac{3}{2}\cdot\frac{1}{x^2}dx=\frac{3}{2}\int_1^3\frac{1}{x}dx=1.5\ln(x)\Big]_1^3=1.648.$

**d.** $E(X^2) = \int_1^3 x^2\cdot\frac{3}{2}\cdot\frac{1}{x^2}dx=\frac{3}{2}\int_1^3 dx=3$, so $V(X) = E(X^2) - [E(X)]^2 = .284$ and $\sigma$ =.553.

**e.** From the description, $h(x) = 0$ if $1 \le x \le 1.5$; $h(x) = x - 1.5$ if $1.5 \le x \le 2.5$ (one second later), and $h(x) = 1$ if $2.5 \le x \le 3$. Using those terms,
$E[h(X)]=\int_1^3 h(x)\,dx=\int_{1.5}^{2.5}(x-1.5)\cdot\frac{3}{2}\cdot\frac{1}{x^2}dx+\int_{2.5}^{3}1\cdot\frac{3}{2}\cdot\frac{1}{x^2}dx=.267.$