3.80

Solutions are found using the cumulative Poisson table, $F(x; \mu) = F(x; 4)$.

a. $P(X \le 4) = F(4; 4) = .629$, while $P(X < 4) = P(X \le 3) = F(3; 4) = .434$.

b. $P(4 \le X \le 8) = F(8; 4) - F(3; 4) = .545$.

c. $P(X \ge 8) = 1 - P(X < 8) = 1 - P(X \le 7) = 1 - F(7; 4) = .051$.

d. For this Poisson model, $\mu = 4$ and so $\sigma = \sqrt{4} = 2$. The desired probability is $P(X \le \mu + \sigma) = P(X \le 4 + 2) = P(X \le 6) = F(6; 4) = .889$.

3.97

a. From the description, $X \sim \text{Bin}(15, .75)$. So, the pmf of X is $b(x; 15, .75)$.

b. $P(X > 10) = 1 - P(X \le 10) = 1 - B(10;15, .75) = 1 - .314 = .686$.

c. $P(6 \le X \le 10) = B(10; 15, .75) - B(5; 15, .75) = .314 - .001 = .313$.

d. $\mu = (15)(.75) = 11.75$, $\sigma^2 = (15)(.75)(.25) = 2.81$.

e. Requests can all be met if and only if $X \le 10$, and $15 - X \le 8$, i.e. iff $7 \le X \le 10$. So, $P(\text{all requests met}) = P(7 \le X \le 10) = B(10; 15, .75) - B(6; 15, .75) = .310$.

4.28

a. $P(0 \le Z \le 2.17) = \Phi(2.17) - \Phi(0) = .4850$.

b. $\Phi(1) - \Phi(0) = .3413$.

c. $\Phi(0) - \Phi(-2.50) = .4938$.

d. $\Phi(2.50) - \Phi(-2.50) = .9876$.

e. $\Phi(1.37) = .9147$.

f. $P(-1.75 < Z) + [1 - P(Z < -1.75)] = 1 - \Phi(-1.75) = .9599$.

g. $\Phi(2) - \Phi(-1.50) = .9104$.

h. $\Phi(2.50) - \Phi(1.37) = .0791$.

i. $1 - \Phi(1.50) = .0668$.

j. $P(|Z| \le 2.50) = P(-2.50 \le Z \le 2.50) = \Phi(2.50) - \Phi(-2.50) = .9876$.

4.29

a. .9838 is found in the 2.1 row and the .04 column of the standard normal table so $c = 2.14$.

b. $P(0 \le Z \le c) = .291 \Rightarrow \Phi(c) - \Phi(0) = .2910 \Rightarrow \Phi(c) - .5 = .2910 \Rightarrow \Phi(c) = .7910 \Rightarrow$ from the standard normal table, $c = .81$.

c. $P(c \le Z) = .121 \Rightarrow 1 - P(Z < c) = .121 \Rightarrow 1 - \Phi(c) = .121 \Rightarrow \Phi(c) = .879 \Rightarrow c = 1.17$.

d. $P(-c \le Z \le c) = \Phi(c) - \Phi(-c) = \Phi(c) - (1 - \Phi(c)) = 2\Phi(c) - 1 = .668 \Rightarrow \Phi(c) = .834 \Rightarrow$ $c = 0.97$.

e. $P(c \le |Z|) = 1 - P(|Z| < c) = 1 - [\Phi(c) - \Phi(-c)] = 1 - [2\Phi(c) - 1] = 2 - 2\Phi(c) = .016 \Rightarrow \Phi(c) = .992 \Rightarrow$ $c = 2.41$.

4.105

a. $P(X > 100) = 1 - \Phi\left(\frac{100-96}{14}\right) = 1 - \Phi(.29) = 1 - .6141 = .3859.$

b. $P(50 < X < 80) = \Phi\left(\frac{80-96}{14}\right) - \Phi\left(\frac{50-96}{14}\right) = \Phi(-1.5) - \Phi(-3.29) = .1271 - .0005 = .1266.$

c. Notice that a and b are the 5th and 95th percentiles, respectively. From the standard normal table, $\Phi(z) = .05 \Rightarrow z = -1.645$, so -1.645 is the 5th percentile of the standard normal distribution. By symmetry, the 95th percentile is $z = 1.645$. So, the desired percentiles of this distribution are $a = 96 + (-1.645)(14) = 72.97$ and $b = 96 + (1.645)(14) = 119.03$. The interval (72.97, 119.03) contains the central 90% of all grain sizes.

4.106

a. $F(x) = 0$ for $x < 1$ and $F(x) = 1$ for $x > 3$. For $1 \le x \le 3$, $F(x) = \int_1^x \frac{3}{2} \cdot \frac{1}{y^2}\, dy = 1.5\left(1 - \frac{1}{x}\right)$.

b. $P(X \le 2.5) = F(2.5) = 1.5(1 - .4) = .9$; $P(1.5 \le X \le 2.5) = F(2.5) - F(1.5) = .4$.

c. $E(X) = = \int_1^3 x \cdot \frac{3}{2} \cdot \frac{1}{x^2}\, dx = \frac{3}{2}\int_1^3 \frac{1}{x}\, dx = 1.5\ln(x)\Big]_1^3 = 1.648.$

d. $E(X^2) = = \int_1^3 x^2 \cdot \frac{3}{2} \cdot \frac{1}{x^2}\, dx = \frac{3}{2}\int_1^3 dx = 3$, so $V(X) = E(X^2) - [E(X)]^2 = .284$ and $\sigma = .553$.

e. From the description, $h(x) = 0$ if $1 \le x \le 1.5$; $h(x) = x - 1.5$ if $1.5 \le x \le 2.5$ (one second later), and $h(x) = 1$ if $2.5 \le x \le 3$. Using those terms,
$E[h(X)] = \int_1^3 h(x)\, dx = \int_{1.5}^{2.5} (x - 1.5) \cdot \frac{3}{2} \cdot \frac{1}{x^2}\, dx + \int_{2.5}^3 1 \cdot \frac{3}{2} \cdot \frac{1}{x^2}\, dx = .267$.

5.59

a. $E(X_1 + X_2 + X_3) = 180$, $V(X_1 + X_2 + X_3) = 45$, $SD(X_1 + X_2 + X_3) = \sqrt{45} = 6.708$.

$P(X_1 + X_2 + X_3 \le 200) = P\left(Z \le \frac{200-180}{6.708}\right) = P(Z \le 2.98) = .9986$.

$P(150 \le X_1 + X_2 + X_3 \le 200) = P(-4.47 \le Z \le 2.98) \approx .9986$.

b. $\mu_{\bar{X}} = \mu = 60$ and $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{\sqrt{15}}{\sqrt{3}} = 2.236$, so

$P(\bar{X} \ge 55) = P\left(Z \ge \frac{55-60}{2.236}\right) = P(Z \ge -2.236) = .9875$ and

$P(58 \le \bar{X} \le 62) = P\left(-.89 \le Z \le .89\right) = .6266$.

c. $E(X_1 - .5X_2 - .5X_3) = \mu - .5\,\mu - .5\,\mu = 0$, while

$V(X_1 - .5X_2 - .5X_3) = \sigma_1^2 + .25\sigma_2^2 + .25\sigma_3^2 = 22.5 \Rightarrow SD(X_1 - .5X_2 - .5X_3) = 4.7434$. Thus,

$P(-10 \le X_1 - .5X_2 - .5X_3 \le 5) = P\left(\frac{-10-0}{4.7434} \le Z \le \frac{5-0}{4.7434}\right) = P\left(-2.11 \le Z \le 1.05\right) = .8531 - .0174 =$ $.8357$.

d. $E(X_1 + X_2 + X_3) = 150$, $V(X_1 + X_2 + X_3) = 36 \Rightarrow SD(X_1 + X_2 + X_3) = 6$, so

$P(X_1 + X_2 + X_3 \le 200) = P\left(Z \le \frac{160-150}{6}\right) = P(Z \le 1.67) = .9525$.

Next, we want $P(X_1 + X_2 \ge 2X_3)$, or, written another way, $P(X_1 + X_2 - 2X_3 \ge 0)$.

$E(X_1 + X_2 - 2X_3) = 40 + 50 - 2(60) = -30$ and $V(X_1 + X_2 - 2X_3) = \sigma_1^2 + \sigma_2^2 + 4\sigma_3^2 = 78 \Rightarrow$

$SD(X_1 + X_2 - 2X_3) = 8.832$, so

$P(X_1 + X_2 - 2X_3 \ge 0) = P\left(Z \ge \frac{0-(-30)}{8.832}\right) = P(Z \ge 3.40) = .0003$.

6.4

a. $E\left(\bar{X} - \bar{Y}\right) = E\left(\bar{X}\right) - E\left(\bar{Y}\right) = \mu_1 - \mu_2$; $\bar{x} - \bar{y} = 8.141 - 8.575 = -.434$.

b. $V\left(\bar{X} - \bar{Y}\right) = V\left(\bar{X}\right) + V\left(\bar{Y}\right) = \sigma_{\bar{X}}^2 + \sigma_{\bar{Y}}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$ $\sigma_{\bar{X}-\bar{Y}} = \sqrt{V\left(\bar{X} - \bar{Y}\right)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$. The estimate would be $s_{\bar{X}-\bar{Y}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{1.66^2}{27} + \frac{2.104^2}{20}} = .5687$.

c. $\frac{s_1}{s_2} = \frac{1.660}{2.104} = .7890$.

d. $V\left(X - Y\right) = V\left(X\right) + V\left(Y\right) = \sigma_1^2 + \sigma_2^2 = 1.66^2 + 2.104^2 = 7.1824$.