1.54

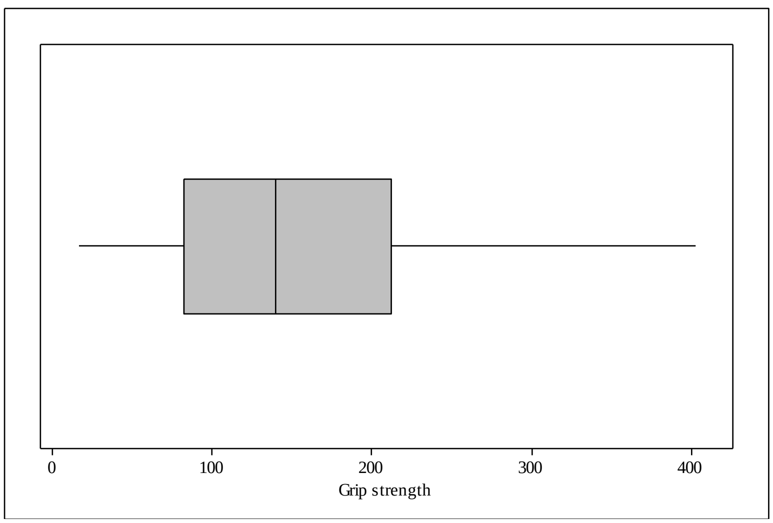
a. Minitab provides the stem-and-leaf display below. Grip strengths for this sample of 42 individuals are positively skewed, and there is one high outlier at 403 N.

```
 6     0  111234
 14    0  55668999
(10)   1  0011223444             Stem = 100s
 18    1  567889                 Leaf = 10s
 12    2  01223334
 4     2  59
 2     3  2
 1     3
 1     4  0
```

b. Each half has 21 observations. The lower fourth is the 11th observation, 87 N. The upper fourth is the 32nd observation (11th from the top), 210 N. The fourth spread is the difference: $f_s = 210 - 87 = 123$ N.

c. min = 16; lower fourth = 87; median = 140; upper fourth = 210; max = 403

The boxplot tells a similar story: grip strengths are slightly positively skewed, with a median of 140N and a fourth spread of 123 N.



d. inner fences: $87 - 1.5(123) = -97.5$, $210 + 1.5(123) = 394.5$
outer fences: $87 - 3(123) = -282$, $210 + 3(123) = 579$
Grip strength can't be negative, so low outliers are impossible here. A mild high outlier is above 394.5 N and an extreme high outlier is above 579 N. The value 403 N is a mild outlier by this criterion. (Note: some software uses slightly different rules to define outliers — using quartiles and interquartile range — which result in 403 N not being classified as an outlier.)

e. The fourth spread is unaffected unless that observation drops below the current upper fourth, 210. That's a decrease of $403 - 210 = 193$ N.

2.3

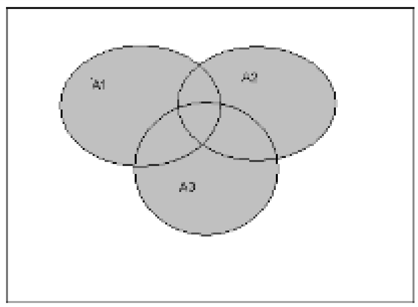
a. $A = \{SSF, SFS, FSS\}$.

b. $B = \{SSS, SSF, SFS, FSS\}$.

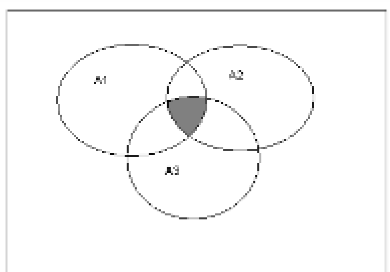
c. For event C to occur, the system must have component 1 working (S in the first position), then at least one of the other two components must work (at least one S in the second and third positions): $C = \{SSS, SSF, SFS\}$.

d. $C' = \{SFF, FSS, FSF, FFS, FFF\}$.
$A \cup C = \{SSS, SSF, SFS, FSS\}$.
$A \cap C = \{SSF, SFS\}$.
$B \cup C = \{SSS, SSF, SFS, FSS\}$. Notice that B contains C, so $B \cup C = B$.
$B \cap C = \{SSS\ SSF, SFS\}$. Since B contains C, $B \cap C = C$.

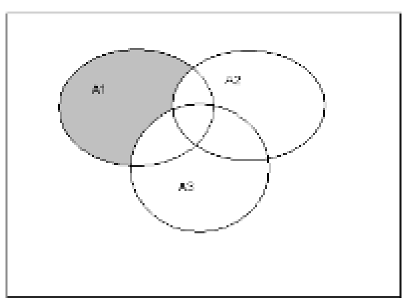
2.8



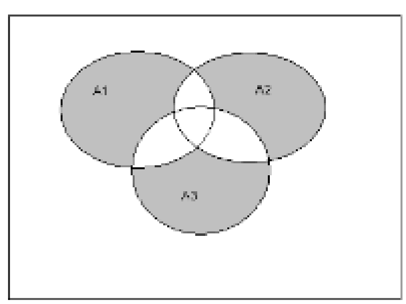
a. $A_1 \cup A_2 \cup A_3$



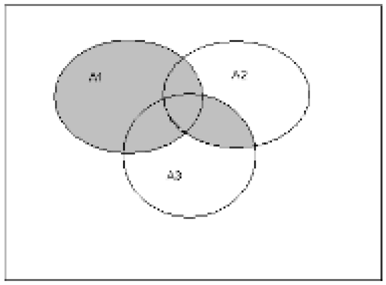
b. $A_1 \cap A_2 \cap A_3$



c. $A_1 \cap A_2' \cap A_3'$



d. $(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$



e. $A_1 \cup (A_2 \cap A_3)$

2.11

a. .07.

b. .15 + .10 + .05 = .30.

c. Let A = the selected individual owns shares in a stock fund. Then $P(A)$ = .18 + .25 = .43. The desired probability, that a selected customer does <u>not</u> shares in a stock fund, equals $P(A')$ = 1 – $P(A)$ = 1 – .43 = .57. This could also be calculated by adding the probabilities for all the funds that are not stocks.

2.26

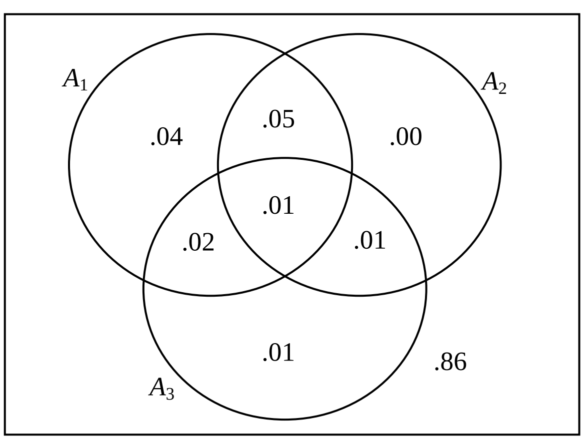
These questions can be solved algebraically, or with the Venn diagram below.

a. $P(A_1') = 1 - P(A_1)$ = 1 – .12 = .88.

b. The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Solving for the intersection ("and") probability, you get $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$ = .12 + .07 – .13 = .06.

c. A Venn diagram shows that $P(A \cap B') = P(A) - P(A \cap B)$. Applying that here with $A = A_1 \cap A_2$ and B = A_3, you get $P([A_1 \cap A_2] \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) =$ .06 – .01 = .05.

d. The event "at most two defects" is the complement of "all three defects," so the answer is just 1 – $P(A_1 \cap A_2 \cap A_3)$ = 1 – .01 = .99.



2.29

a. There are 26 letters, so allowing repeats there are $(26)(26) = (26)^2 = 676$ possible 2-letter domain names. Add in the 10 digits, and there are 36 characters available, so allowing repeats there are $(36)(36) = (36)^2 = 1296$ possible 2-character domain names.

b. By the same logic as part **a**, the answers are $(26)^3 = 17{,}576$ and $(36)^3 = 46{,}656$.

c. Continuing, $(26)^4 = 456{,}976$; $(36)^4 = 1{,}679{,}616$.

d. P(4-character sequence is already owned) = 1 – P(4-character sequence still available) = 1 – $97{,}786/(36)^4 = .942$.

2.35

a. Since there are 20 day-shift workers, the number of such samples is $\binom{20}{6} = 38{,}760$. With 45 workers total, there are $\binom{45}{6}$ total possible samples. So, the probability of randomly selecting all day-shift workers is $\dfrac{\binom{20}{6}}{\binom{45}{6}} = \dfrac{38{,}760}{8{,}145{,}060} = .0048$.

b. Following the analogy from **a**, P(all from the same shift) = P(all from day shift) + P(all from swing shift) + P(all from graveyard shift) = $\dfrac{\binom{20}{6}}{\binom{45}{6}} + \dfrac{\binom{15}{6}}{\binom{45}{6}} + \dfrac{\binom{10}{6}}{\binom{45}{6}} = .0048 + .0006 + .0000 = .0054$.

c. P(at least two shifts represented) = 1 – P(all from same shift) = 1 – .0054 = .9946.

d. There are several ways to approach this question. For example, let A_1 = "day shift is unrepresented," A_2 = "swing shift is unrepresented," and A_3 = "graveyard shift is unrepresented." Then we want $P(A_1 \cup A_2 \cup A_3)$.

$P(A_1)$ = P(day shift unrepresented) = P(all from swing/graveyard) = $\dfrac{\binom{25}{6}}{\binom{45}{6}}$,

since there are 15 + 10 = 25 total employees in the swing and graveyard shifts. Similarly, $P(A_2) = \dfrac{\binom{30}{6}}{\binom{45}{6}}$ and $P(A_3) = \dfrac{\binom{35}{6}}{\binom{45}{6}}$. Next, $P(A_1 \cap A_2)$ = P(all from graveyard) = $\dfrac{\binom{10}{6}}{\binom{45}{6}}$.

Similarly, $P(A_1 \cap A_3) = \dfrac{\binom{15}{6}}{\binom{45}{6}}$ and $P(A_2 \cap A_3) = \dfrac{\binom{20}{6}}{\binom{45}{6}}$. Finally, $P(A_1 \cap A_2 \cap A_3) = 0$, since at least one shift must be represented. Now, apply the addition rule for 3 events:

$$P(A_1 \cup A_2 \cup A_3) = \frac{\binom{25}{6}}{\binom{45}{6}} + \frac{\binom{30}{6}}{\binom{45}{6}} + \frac{\binom{35}{6}}{\binom{45}{6}} - \frac{\binom{10}{6}}{\binom{45}{6}} - \frac{\binom{15}{6}}{\binom{45}{6}} - \frac{\binom{20}{6}}{\binom{45}{6}} + 0 = .2885.$$

2.38

a. There are 6 75W bulbs and 9 other bulbs. So, P(select exactly 2 75W bulbs) = P(select exactly 2 75W bulbs and 1 other bulb) = $\dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \dfrac{(15)(9)}{455} = .2967$.

b. P(all three are the same rating) = P(all 3 are 40W or all 3 are 60W or all 3 are 75W) =

$$\frac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \frac{4+10+20}{455} = .0747 .$$

c. P(one of each type is selected) = $\dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = \dfrac{120}{455} = .2637$.

d. It is necessary to examine at least six bulbs if and only if the first five light bulbs were all of the 40W or 60W variety. Since there are 9 such bulbs, the chance of this event is

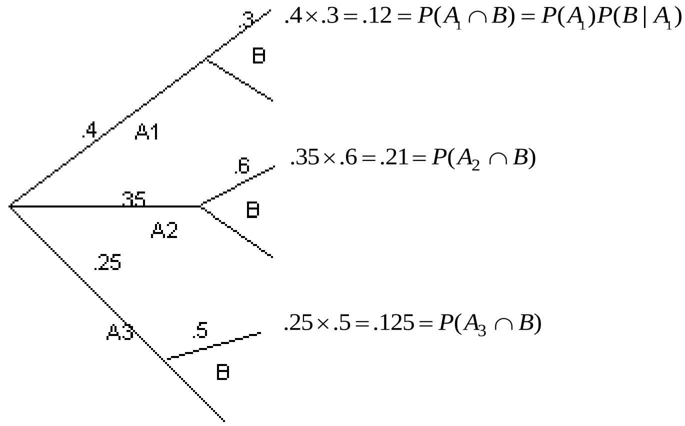
$$\frac{\binom{9}{5}}{\binom{15}{5}} = \frac{126}{3003} = .042 .$$

2.56

$$P(A \mid B) + P(A' \mid B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} = \frac{P(A \cap B) + P(A' \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

2.59

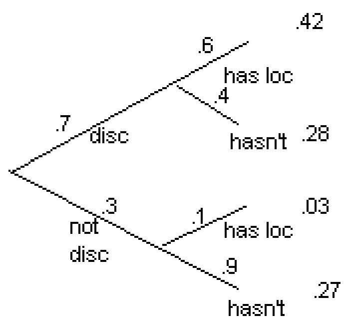
The required probabilities appear in the tree diagram below.



a. $P(A_2 \cap B) = .21$.

b. By the law of total probability, $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .455$.

c. Using Bayes' theorem, $P(A_1 \mid B) = \dfrac{P(A_1 \cap B)}{P(B)} = \dfrac{.12}{.455} = .264$; $P(A_2 \mid B) = \dfrac{.21}{.455} = .462$; $P(A_3 \mid B) = 1 - .264 - .462 = .274$. Notice the three probabilities sum to 1.



2.74

Using subscripts to differentiate between the selected individuals,
$P(O_1 \cap O_2) = P(O_1)P(O_2) = (.45)(.45) = .2025$.
$P(\text{two individuals match}) = P(A_1 \cap A_2) + P(B_1 \cap B_2) + P(AB_1 \cap AB_2) + P(O_1 \cap O_2) =$
$.40^2 + .11^2 + .04^2 + .45^2 = .3762$.

2.78

$P(\text{at least one opens}) = 1 - P(\text{none open}) = 1 - (.05)^5 = .99999969$.
$P(\text{at least one fails to open}) = 1 - P(\text{all open}) = 1 - (.95)^5 = .2262$.

2.80

Let A_i denote the event that component #i works (i = 1, 2, 3, 4). Based on the design of the system, the event "the system works" is $(A_1 \cup A_2) \cup (A_3 \cap A_4)$. We'll eventually need $P(A_1 \cup A_2)$, so work that out first: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = (.9) + (.9) - (.9)(.9) = .99$. The third term uses independence of events. Also, $P(A_3 \cap A_4)$ = (.9)(.9) = .81, again using independence.

Now use the addition rule and independence for the system:

$$\begin{aligned} P((A_1 \cup A_2) \cup (A_3 \cap A_4)) &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4)) \\ &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \times P(A_3 \cap A_4) \\ &= (.99) + (.81) - (.99)(.81) = .9981 \end{aligned}$$

(You could also use deMorgan's law in a couple of places.)