HOMEWORK 6

41. The tables below delineate all 16 possible (x_1, x_2) pairs, their probabilities, the value of $\bar{x}$ for that pair, and the value of r for that pair. Probabilities are calculated using the independence of X_1 and X_2.

(x_1, x_2)	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4
probability	.16	.12	.08	.04	.12	.09	.06	.03
$\bar{x}$	1	1.5	2	2.5	1.5	2	2.5	3
r	0	1	2	3	1	0	1	2

(x_1, x_2)	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
probability	.08	.06	.04	.02	.04	.03	.02	.01
$\bar{x}$	2	2.5	3	3.5	2.5	3	3.5	4
r	2	1	0	1	3	2	1	2

a. Collecting the $\bar{x}$ values from the table above yields the pmf table below.

$\bar{x}$	1	1.5	2	2.5	3	3.5	4
$p(\bar{x})$	.16	.24	.25	.20	.10	.04	.01

b. $P(\bar{X} \le 2.5) = .16 + .24 + .25 + .20 = .85$.

c. Collecting the r values from the table above yields the pmf table below.

r	0	1	2	3
$p(r)$	.30	.40	.22	.08

d. With $n = 4$, there are numerous ways to get a sample average of at most 1.5, since $\bar{X} \le 1.5$ iff the sum of the X_i is at most 6. Listing out all options, $P(\bar{X} \le 1.5) = P(1,1,1,1) + P(2,1,1,1) + \ldots + P(1,1,1,2) + P(1,1,2,2) + \ldots + P(2,2,1,1) + P(3,1,1,1) + \ldots + P(1,1,1,3)$
$= (.4)^4 + 4(.4)^3(.3) + 6(.4)^2(.3)^2 + 4(.4)^2(.2)^2 = .2400$.

46.

a. The sampling distribution of $\bar{X}$ is centered at $E(\bar{X}) = \mu = 12$ cm, and the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \dfrac{\sigma_{X}}{\sqrt{n}} = \dfrac{.04}{\sqrt{16}} = .01$ cm.

b. With $n = 64$, the sampling distribution of $\bar{X}$ is still centered at $E(\bar{X}) = \mu = 12$ cm, but the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \dfrac{\sigma_{X}}{\sqrt{n}} = \dfrac{.04}{\sqrt{64}} = .005$ cm.

c. $\bar{X}$ is more likely to be within .01 cm of the mean (12 cm) with the second, larger, sample. This is due to the decreased variability of $\bar{X}$ that comes with a larger sample size.

50.

a. $P(9{,}900 \le \bar{X} \le 10{,}200) \approx P\left(\frac{9{,}900-10{,}000}{500/\sqrt{40}} \le Z \le \frac{10{,}200-10{,}000}{500/\sqrt{40}}\right)$
$= P(-1.26 \le Z \le 2.53) = \Phi(2.53) - \Phi(-1.26) = .9943 - .1038 = .8905.$

b. According to the guideline given in Section 5.4, n should be greater than 30 in order to apply the CLT, thus using the same procedure for $n = 15$ as was used for $n = 40$ would not be appropriate.

53.

a. With the values provided,

$$P(\bar{X} \ge 51) = P\left(Z \ge \frac{51-50}{1.2/\sqrt{9}}\right) = P(Z \ge 2.5) = 1 - .9938 = .0062.$$

b. Replace $n = 9$ by $n = 40$, and

$$P(\bar{X} \ge 51) = P\left(Z \ge \frac{51-50}{1.2/\sqrt{40}}\right) = P(Z \ge 5.27) \approx 0.$$

60. Y is normally distributed with $\mu_Y = \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{3}(\mu_3 + \mu_4 + \mu_5) = -1$, and

$\sigma_Y^2 = \frac{1}{4}\sigma_1^2 + \frac{1}{4}\sigma_2^2 + \frac{1}{9}\sigma_3^2 + \frac{1}{9}\sigma_4^2 + \frac{1}{9}\sigma_5^2 = 3.167 \Rightarrow \sigma_Y = 1.7795.$

Thus, $P(0 \le Y) = P\left(\frac{0-(-1)}{1.7795} \le Z\right) = P(.56 \le Z) = .2877$ and

$$P(-1 \le Y \le 1) = P\left(0 \le Z \le \frac{2}{1.7795}\right) = P(0 \le Z \le 1.12) = .3686.$$