7.13

a. $\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} = 654.16 \pm 1.96\frac{164.43}{\sqrt{50}}$ = (608.58, 699.74). We are 95% confident that the true average CO_2 level in this population of homes with gas cooking appliances is between 608.58ppm and 699.74ppm

b. $w = 50 = \frac{2(1.96)(175)}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{2(1.96)(175)}{50} = 13.72 \Rightarrow$ $n = (13.72)^2$ = 188.24, which rounds up to 189.

7.18

90% lower bound: $\bar{x} - z_{.10}\frac{s}{\sqrt{n}} = 4.25 - 1.28\frac{1.30}{\sqrt{78}} = 4.06.$

7.32

We have n = 20, $\bar{x}$ = 1584, and s = 607; the critical value is $t_{.005,20-1} = t_{.005,19}$ = 2.861. The resulting 99% CI for μ is

$$1584 \pm 2.861\frac{607}{\sqrt{20}} = 1584 \pm 388.3 = (1195.7, 1972.3)$$

We are 99% confident that the true average number of cycles required to break this type of condom is between 1195.7 cycles and 1972.3 cycles.

7.34

n = 14, $\bar{x} = 8.48$, s = .79; $t_{.05,13} = 1.771$

c. A 95% lower confidence bound: $8.48 - 1.771\left(\frac{.79}{\sqrt{14}}\right) = 8.48 - .37 = 8.11$. With 95% confidence, the value of the true mean proportional limit stress of all such joints is greater than 8.11 MPa. We must assume that the sample observations were taken from a normally distributed population.

8.5

Let σ denote the population standard deviation. The appropriate hypotheses are H_0: σ = .05 v. H_a: σ < .05. With this formulation, the burden of proof is on the data to show that the requirement has been met (the sheaths will not be used unless H_0 can be rejected in favor of H_a. Type I error: Conclude that the standard deviation is < .05 mm when it is really equal to .05 mm. Type II error: Conclude that the standard deviation is .05 mm when it is really < .05.

8.16

a. $\alpha = P(T \geq 3.733$ when T has a t distribution with 15 df) =.001.

b. df = $n - 1 = 23 \Rightarrow \alpha = P(T_{23} \leq -2.500) = .01$.

c. df = 30 $\Rightarrow \alpha = P(T_{30} \geq 1.697$ or $T_{30} \leq -1.697) = .05 + .05 = .10$.

8.25

a. H_0: $\mu = 5.5$ v. H_a: $\mu \neq 5.5$; for a level .01 test, (not specified in the problem description), reject H_0 if either $z \geq 2.58$ or $z \leq -2.58$. Since $z = \dfrac{5.25 - 5.5}{.075} = -3.33 \leq -2.58$, reject H_0.

b. $1 - \beta(5.6) = 1 - \Phi\left(2.58 + \dfrac{(-.1)}{.075}\right) + \Phi\left(-2.58 + \dfrac{(-.1)}{.075}\right) = 1 - \Phi(1.25) + \Phi(-3.91) = .105$.

c. $n = \left[\dfrac{.3(2.58 + 2.33)}{-.1}\right]^2 = 216.97$, so use $n = 217$.

8.26

Reject H_0 if $z \geq 1.645$; $\dfrac{s}{\sqrt{n}} = .7155$, so $z = \dfrac{52.7 - 50}{.7155} = 3.77$. Since 3.77 is $\geq$ 1.645, reject H_0 at level .05 and conclude that true average penetration exceeds 50 mils.

8.29

a. The hypotheses are H_0: $\mu = 200$ versus H_a: $\mu > 200$. H_0 will be rejected at level $\alpha = .05$ if $t \geq t_{.05,12-1} = t_{.05,11} = 1.796$. With the data provided, $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{249.7 - 200}{145.1/\sqrt{12}} = 1.19$. Since 1.19 < 1.796, H_0 is not rejected at the $\alpha = .05$ level. We have insufficient evidence to conclude that the true average repair time exceeds 200 minutes.

8.37

a. The parameter of interest is p = the proportion of the population of female workers that have BMIs of at least 30 (and, hence, are obese). The hypotheses are H_0: $p = .20$ versus H_a: $p > .20$.
With $n = 541$, $np_0 = 541(.2) = 108.2 \geq 10$ and $n(1 - p_0) = 541(.8) = 432.8 \geq 10$, so the "large-sample" z procedure is applicable. Hence, we will reject H_0 if $z \geq z_{.05} = 1.645$.
From the data provided, $\hat{p} = \dfrac{120}{541} = .2218$, so $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \dfrac{.2218 - .20}{\sqrt{.20(.80)/541}} = 1.27$. Since 1.27 < 1.645, we fail to reject H_0 at the $\alpha = .05$ level. We do not have sufficient evidence to conclude that more than 20% of the population of female workers is obese.

b. A Type I error would be to incorrectly conclude that more than 20% of the population of female workers is obese, when the true percentage is 20%. A Type II error would be to fail to recognize that more than 20% of the population of female workers is obese when that's actually true.

c. The question is asking for the chance of committing a Type II error when the true value of p is .25, i.e. $\beta(.25)$. Using the textbook formula,

$$\beta(.25) = \Phi\left[\frac{.20 - .25 + 1.645\sqrt{.20(.80)/541}}{\sqrt{.25(.75)/541}}\right] = \Phi(-1.166) \approx .121.$$

8.51

Use Table A.8.

a. $P(t > 2.0)$ at 8df = .040.

b. $P(t < -2.4)$ at 11df = .018.

c. $2P(t < -1.6)$ at 15df = 2(.065) = .130.

8.55

Here we might be concerned with departures above as well as below the specified weight of 5.0, so the relevant hypotheses are H_0: $\mu = 5.0$ v. H_a: $\mu \neq 5.0$. Since $\frac{s}{\sqrt{n}} = .035$, $z = \frac{-.13}{.035} = -3.71$. Because 3.71 is "off" the z-table, P-value < 2(.0002) = .0004, so H_0 should be rejected.

8.58

μ = the true average percentage of organic matter in this type of soil, and the hypotheses are H_0: $\mu = 3$ v. H_a: $\mu \neq 3$. With $n = 30$, and assuming normality, we use the t test: $t = \frac{\bar{x} - 3}{s/\sqrt{n}} = \frac{2.481 - 3}{.295} = \frac{-.519}{.295} = -1.759$. The P-value = $2[P(\,t > 1.759\,)] = 2(\,.041\,) = .082$. At significance level .10, since $.082 \leq .10$, we would reject H_0 and conclude that the true average percentage of organic matter in this type of soil is something other than 3. At significance level .05, we would not have rejected H_0.