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Cooperative relay tracking strategy for multi-agent systems with assistance of Voronoi diagrams ☆

Lijing Dong[a,b], Senchun Chai[b,*], Baihai Zhang[b], Sing Kiong Nguang[c], Xiangshun Li[d,c]

[a]*School of Mechanical, Electronic and Control Engineering, Beijing Jiaotong University, Beijing 100044, PR China*

[b]*School of Automation, Beijing Institute of Technology, Beijing 100081, PR China*

[c]*The Department of Electrical and Computer Engineering, The University of Auckland, Private Bag, 92019 Auckland, New Zealand*

[d]*School of Automation, Wuhan University of Technology, Wuhan, Hubei, 430070, PR China*

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Abstract

We study a class of multi-agent tracking system, in which targets are caught by cooperative agents in a monitoring area. A certain domain is monitored by a large number of agents, who are capable of sensing and mobility. In reality, agents are not able to measure absolute position of a target but able to measure the distances. However, when designing the tracking controller, the absolute position information is required. Therefore, we adopt trilateration algorithm to get the absolute position of a target through at least three distances. This monitored domain is overlapped by at least three monitoring agents and some redundant agents and with the assistance of knowledge of Voronoi diagrams it is divided into many Voronoi cells. Then, a cooperative relay tracking strategy is proposed such that during the tracking process, when a target enters a new Voronoi cell, the Voronoi site agent replaces one of the original tracking agents. This means that not only the topology but also the tracking agents switch, which is significantly different from the traditional switching topologies. With the introduction of redundant agents, this domain can deal with the situation that several targets share the same route. Finally, the proposed tracking strategy is verified by a set of simulations.

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*Corresponding author.

E-mail address: chaisc97@bit.edu.cn (S. Chai).

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1. Introduction

Multi-agent systems have become a hot research area in the recent two decades due to its wide applications to mobile robots, unmanned air vehicles (UAVs), autonomous underwater vehicles (AUVs), and satellites [1–11]. Tracking problem of a target is a typical issue of multi-agent systems [1,6–12]. Ref. [1] proposes a robust adaptive consensus tracking control scheme for a class of perturbed nonlinear multi-agent systems and [6] proposes a tracking approach based on potential function and behavior rules. In our previous works [9,10,13], we have investigated the tracking problem for one target under a time-varying topology.

In [11], the authors propose a relay pursuit scheme to capture a maneuvering target in a plane, on which a group of pursuers are distributed. At each instant of time, only one pursuer is assigned the task of capturing the moving target. The plane is divided into a number of regions by the pursuers with knowledge of Voronoi diagrams, a vigorous tool of dividing space [14–16]. In [11], the agents are governed by first-order integral kinematics and the target is tracked by a single pursuer without cooperation. There are some existing results on cooperative tracking [1,6–10,12] and single pursuer replay tracking [11,14]. However, there has not been any result on cooperative relay tracking. As discussed in the conclusion of [11], in some situations, capture will occur only if the pursuers cooperate. In that sense, relay pursuit strategies are viewed as an intermediate option offering a simpler alternative for a multi-agent tracking problem involving multiple pursuers, whose solution is known to be very hard [17]. Another possible extension is to consider the case that agents are described by more realistic kinematics not just integral dynamics.

In light of the above discussion, we first propose cooperative relay tracking in this work. We consider a more complicated scenario where a certain area is monitored by a large number of mobile agents, described by nonlinear dynamics, which are more realistic kinematics since almost all the physical plants contain nonlinearity [18–20]. In practice, our proposed scheme can be appropriately used for protecting sensitive areas against offensive intrusion [21] and occasionally tracking targets in a typical military, environmental, or habitat monitoring applications [22]. When targets move into this area, a preset number of mobile agents begin tracking the targets. There are three kinds of interconvertible agents: monitoring agents, redundant agents and tracking agents. One kind of agents can transfer to another kind during the pursuit process. This can be sort of regarded as a police–thief game, in which the monitoring agents are police stations, a Voronoi cell is the range of a police station, redundant agents are patrol police and tracking agents are the police with mission of capture the thief. In some situations, a thief may evade if he is only tracked by one police, multiple police are clearly required to keep the thief from escaping. When a target enters the monitoring area, a preset number of nearest agents (either monitoring or redundant agents) become tracking agents, if a monitoring agent (police in a station) begins tracking the target, a redundant agent (patrol police) will move to the monitoring position (police station) and become a monitoring agent. At the end of tracking, i.e., a target is captured, the corresponding tracking agents will be released to become redundant agents, i.e. once the thief is captured, the corresponding police's tracking mission is finished then they are able to carry out other missions.

It is worth to mention that during the course of tracking, the tracking agents (police) need to cooperate with each other, and the relay scheme involves switching of topologies. The process of switching can be solved by associating it with Markov chain [23–27]. Moreover, when the target moves into a new Voronoi cell (a new police station's range), not only the topology switches but also one of the pursuers is replaced by the corresponding Voronoi site (a thief moves from one area to another area, the local police join the capture task), which induces jump of tracking errors. This makes it quite different and more difficult compared with the most existed switching systems. In addition, there might be several targets moving in this monitoring area, which has four possible cases according to the trajectories and moving time in the area. When they enter one Voronoi cell simultaneously, a priority mechanism is involved to help the Voronoi site agent make decision.

The main contributions of this paper is summarized as follows. Firstly, we extend the work of [11] to a more realistic scenario and the agents are with more realistic dynamics. Then, a cooperative relay tracking strategy is proposed to deal with the raised problem. Finally, we solve the jumping problem in tracking errors when topologies and tracking agents switch. The rest of this paper is organized as follows. In Section 2, some related preliminaries are presented. The details of tracking algorithm are addressed in Section 3. In Section 4, the controller of tracking agents is designed and the stability of the tracking systems is analyzed. Section 5 shows numerical examples and conclusions are drawn in Section 6.

2. Related preliminaries

The monitored 2-dimensional space is divided into a number of regions (Voronoi cells) with the assistance of Voronoi diagrams. A mathematical representation for Voronoi diagrams is addressed below.

Let X be a 2-dimensional space. We denote $\mathcal{N}=\{a_i, 1\le i\le N_a\}\subseteq X$, a set of N_a agents distributed in X. The Voronoi diagram is a way of dividing space into a number of regions, also called Voronoi cells. The agent in each Voronoi cell is a Voronoi site in the perspective of Voronoi diagram. The Voronoi cell, $V(a_i)$, associated with site a_i is the set of all points in X whose distances to a_i are not greater than their distances to the other sites $a_j, j\ne i$. In other words, if $d(x,a_i)$ denotes the distance between the point x and the site a_i, then

$$V(a_i)=\{x\in X|d(x,a_i)\le d(x,a_j)\ \text{for all}\ j\ne i\}. \tag{1}$$

Thereafter, the plane is divided by the Voronoi diagram, $V(A)$, into N_a independent cells for each

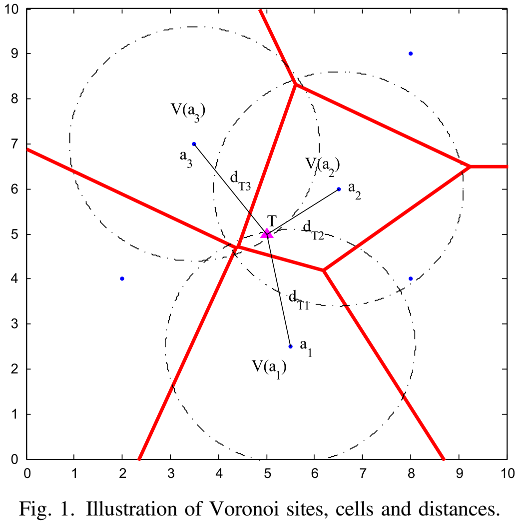

Fig. 1. Illustration of Voronoi sites, cells and distances.

site a_i.

$$V(A)=\{V(a_i), i=1,\dots,N_a\}. \tag{2}$$

Fig. 1 illustrates the concepts of Voronoi cells, sites and distances.

To maximize the control of a given region, distribution of agents should guarantee the coverage of the whole region. Ignoring boundary effects, and assuming each agent's sensing radius is R_s, then the corresponding sensing coverage area is πR_s^2.

In a 2-dimensional sensing field with an area S_c, the minimum number of agents N_m required to achieve 1-coverage is calculated by [15]

$$N_m=\frac{S_c}{\pi\left(\frac{\sqrt{3}R_s}{2}\right)^2}. \tag{3}$$

Regions overlapped by k agents are called k-coverage regions. Fig. 2 illustrates the k-coverage concept. k-Coverage can be regarded as k independent subsets of a 1-coverage network area. In other words, k-coverage of the network area can be obtained by deploying kN_m agents.

In this work, we adopt trilateration algorithm to determine the location of a target based on distance measurements by its neighbors. Trilateration algorithm is a typical localization algorithm, where the localization of a target is supported by at least three agents [28,29]. In a wide range of applications, such as habitat monitoring, smart environments and target tracking, accurate locations of agents are required in order to provide meaningful information and to efficiently route data through the network. Trilateration refers to the process of calculating a target's position based on measured distances between itself and at least three agents with known locations. Given the location of a target and an agent's Euclidean distance to the target, it is known that the agent must be positioned somewhere along the circumference of a circle centered at the anchor's position with a radius equal to the agent-target distance.

As shown in Fig. 1, the coordination of three Voronoi site agents are $(x_{a_1}, y_{a_1}), (x_{a_2}, y_{a_2})$ and (x_{a_3}, y_{a_3}), respectively. The measured distances between them with the target are $d_{T_1}, d_{T_2}, d_{T_3}$, respectively. Assume the coordination of the target is (x_T, y_T), then we have the following relationship of distances between agents and the target.

$$\begin{cases}\sqrt{(x_T-x_{a_1})^2+(y_T-y_{a_1})^2}=d_{T_1}\\ \sqrt{(x_T-x_{a_2})^2+(y_T-y_{a_2})^2}=d_{T_2}\\ \sqrt{(x_T-x_{a_3})^2+(y_T-y_{a_3})^2}=d_{T_3}\end{cases} \tag{4}$$

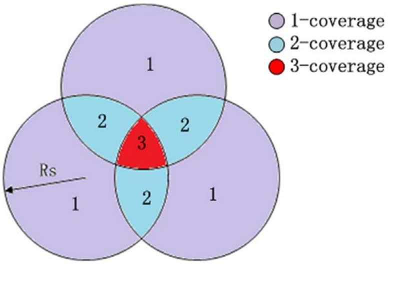

Fig. 2. k-Coverage.

After simple calculation, coordination of the target is obtained:

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = \begin{bmatrix} 2(x_{a1}-x_{a3}) & 2(y_{a1}-y_{a3}) \\ 2(x_{a2}-x_{a3}) & 2(y_{a2}-y_{a3}) \end{bmatrix}^{-1} \begin{bmatrix} x_{a1}^2-x_{a3}^2+y_{a1}^2-y_{a3}^2+d_{T_3}^2-d_{T_1}^2 \\ x_{a2}^2-x_{a3}^2+y_{a2}^2-y_{a3}^2+d_{T_3}^2-d_{T_2}^2 \end{bmatrix} \quad (5)$$

It can be seen that at least three agents are required for localization, which means this monitoring area should be 3-coverage. To achieve 3-coverage of a specific 2-dimensional area, with Eq. (3), the minimum number of agents can be calculated. Then, virtual force-based approach (VFA) is adopted in the distributed deployment process. Each agent behaves as a source giving a force to others. This force may be either attractive force or repulsive force. If two agents are too close, they exert repulsive forces to separate each other, otherwise, attractive forces are exerted. The total force F_i exerted on agent i is determined by the summation of all forces contributed by its neighbors. Under the effect of force F_i, agent i will move till the force is zero. In virtual force-based approach for deployment, every agent will eventually converge to a steady state. For the stability analysis of the virtual force-based approach, the readers are referred to [30].

For a target crossing a 3-coverage area, the tracking algorithm is triggered by the detection of a target. A target $\mathbf{t}$ is said to reside at agent a_i if target $\mathbf{t}$ is closest to agent a_i compared to all other agents in the monitoring area. For instance, in Fig. 1, the target resides at Voronoi site agent a_2. Every agent sends its range measurement information to agent a_2, and then agent a_2 estimates the location of $\mathbf{t}$ according to trilateration algorithm. The monitoring area is divided into Voronoi cells with each agent responsible for calculating the location of targets within its own cell with trilateration algorithm. The corresponding Voronoi diagram is shown in Fig. 3.

The agents in a 3-coverage area can monitor and estimate the locations of targets. However, if monitoring agents move to track a target, the 3-coverage will be destroyed. In this situation, when another target breaks into this area, we will fail to estimate its location. Therefore, redundant agents are introduced. Assume there are most N_t targets that may enter the monitoring area and each target is supposed to be tracked by N_f tracking agents. Then the number of redundant agents is $N_t \times N_f$ and the whole number of agents in this area is $N_a = 3N_m + N_t N_f$. The Voronoi diagram with redundant agents deployed is shown in Fig. 4.

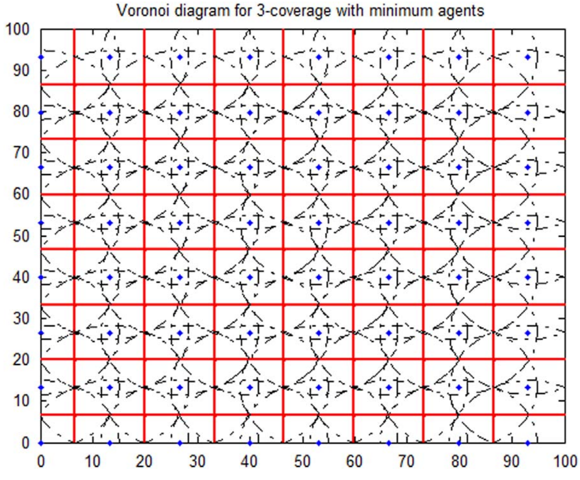

Fig. 3. Voronoi diagram for 3-coverage.

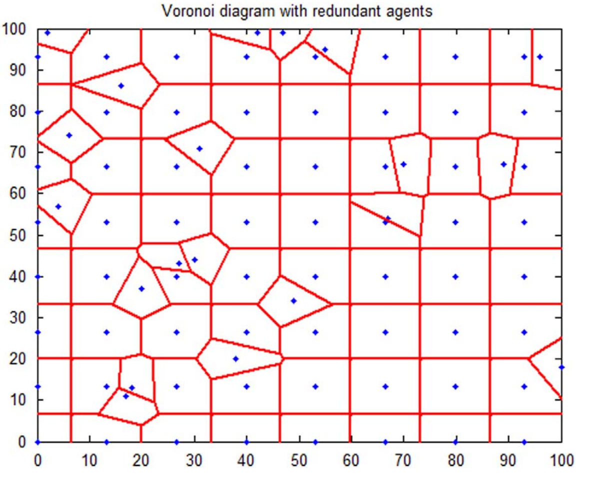

Fig. 4. Voronoi diagram with redundant agents.

Definition 1. An agent is called a monitoring agent if it is on the key position that assure the area is 3-overlapped.

Definition 2. An agent is called a tracking agent if it is tracking a target.

Definition 3. An agent is called a redundant agent if it is neither a monitoring nor tracking agent.

Definition 4. A Voronoi site agent can be either a monitoring or redundant agent.

Remark 1. Monitoring agents are deployed in specific locations to reach 3-coverage. However, redundant agents are initially randomly deployed in this area and do not participate in the virtual force deployment approach. An agent is able to determine its own role through comparing the distances to its neighboring agents. If the distances between an agent and more than three of its neighbors are equal, then it is a monitoring agent, otherwise, it is a redundant agent. In Fig. 3, all the agents are monitoring agents. In Fig. 4, except the monitoring agents, the others which are randomly deployed are called redundant agents. All the agents in Fig. 4 can be called Voronoi site agents.

3. Relay tracking algorithm

The structure of tracking algorithm is shown in Fig. 5. Initial deployment of agents based on virtual force approach has been addressed in Section 2. This section mainly focuses on relay tracking strategy.

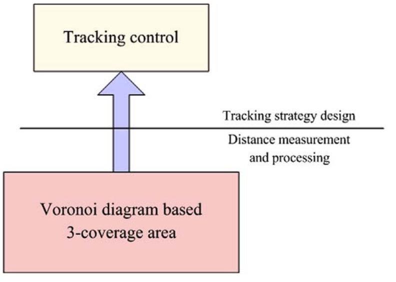

Fig. 5. Tracking structure.

Remark 2. A significant difference in our model for tracking targets is that we do not seek consensus of all the agents. In this paper, a large number (N_a) of agents are deployed on an interested area to carry out surveillance and tracking tasks. When a target enters this area, a distributed tracking application is activated. At each instant of time, N_f agents are assigned the task of capturing the maneuvering target, whereas all other agents in this region remain stationary. In other words, only N_f agents are supposed to be consensus with the target.

The cooperative relay tracking algorithm is addressed in detail as follows.

(1) Calculate the required number of agents and initialize the deployment guaranteeing the monitoring area is 3-overlapped.
(2) When a target goes into the monitoring area, the 3 neighboring agents $a_i, 1 \le i \le 3$, send their detected distances to the Voronoi site agent (local police station collects information about a thief from neighboring stations), where the target resides.
(3) Using trilateration algorithm, the Voronoi site agent estimates the target's location through three measurement distances (local station infers the thief's position with collected information). This means during the pursuit process only one agent is able to access the thief's position.
(4) Then the Voronoi site agent and $N_f - 1$ other nearest agents start tracking the target, becomes tracking agents.
(5) If tracking agent is a monitoring agent, in order to guarantee at least 3-coverage of this area, the nearest redundant agents move to the monitoring agent's previous location immediately. If the tracking agent is one of the redundant agents, the other agents stay at their original positions. Then recreate a new Voronoi diagram, in which the target and the tracking agents are exclusive.
(6) If the target is not captured in the first Voronoi cell, it will go to the next Voronoi cell. The new corresponding Voronoi site agent will become a tracking agent, meanwhile, one of the original tracking agents will quit tracking based on the distance discipline. The one which is furthest from the target quits. Then repeat from procedure 5 again.
(7) When a target is caught, it stops moving and releases the corresponding tracking agents to become redundant agents. This is called as release policy, with which there are more redundant agents in the domain. Repeat all the procedures until there is no more new and un-captured targets.

We note that the monitored area can tolerate moving of agents (addition or deletion of nodes for a local domain). Any changes in Voronoi diagram topology are solved in a local manner by means of well-known local algorithms [16], which means the moving of agents do not cause a global reassignment of Voronoi diagram.

4. Controller design and stability analysis

The agents in the monitoring area are identical and described as

$$\dot{x}_i(t) = f(t, x_i(t)) + u_i(t), \quad i \in \{1, 2, \ldots, N_a\} \tag{6}$$

where $x_i \in \mathbb{R}^2$ is position state of the i-th agent. $f(t, x_i(t))$ is a nonlinear vector-valued continuous function to describe the self-dynamics of i-th tracking agent. $u_i \in \mathbb{R}^2$ is the control input of the i-th agent.

The maneuvering targets are assumed to possess the following kinematics:

$$\dot{x}_{\mathbf{t}k}(t) = f(t, x_{\mathbf{t}k}(t)), \quad k \in \{1, 2, \ldots, N_{\mathbf{t}}\} \tag{7}$$

where $x_{\mathbf{t}k} \in \mathbb{R}^2$ is position state of the k-th target.

The objective of this paper is to design a tracking strategy which ensures that the tracking agents effectively catch the targets in a 2-dimensional space. In this work, we assume the nonlinear function $f(\cdot)$ in (Eqs. (6) and 7) satisfies the following constraint:

Assumption 1. There exists a nonnegative constant $l \geq 0$ such that

$$\| f(t, x_i) - f(t, x_{\mathbf{t}}) \| \leq l \| x_i - x_{\mathbf{t}} \|.$$

Assumption 1 is a Lipschitz-type condition, satisfied by many well-known systems including Lorenz system, Chen system, Lü system, Chua's circuit, and so on [31].

In the view of graph theory, every agent can be treated as a node. Then the communication topology of tracking agents and the target can be treated as a dynamic graph. A weighted graph $G := \{\mathcal{N}, \mathcal{E}, \mathcal{A}\}$ is denoted by with a node set $\mathcal{N} = \{1, 2, \ldots, N_f\}$, an edge set $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N_f \times N_f}$ with nonnegative elements. The nodes within the communication range of node i are called the set of neighbors of node i, which is denoted by $\mathcal{N}_i = \{j | j \in \mathcal{N}, (j, i) \in \mathcal{E}\}$. When $j \notin \mathcal{N}_i$, which means node j is beyond the communication range of node i, $a_{ij} = 0$, otherwise $a_{ij} > 0$. If $a_{ii} \neq 0$, we say that node has self-loop. In this paper, it is assumed that no self-loop exists. b_i decides whether agent i is able to communicate the target or not. $b_i = 0$ means agent i cannot get the information of target, otherwise $b_i > 0$. Denote $\mathcal{B} = diag\{b_1, b_2, \ldots, b_{N_f}\}$.

The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N_f \times N_f}$ of graph G is defined as $\mathcal{L} := \mathcal{D} - \mathcal{A}$, where

$$\mathcal{D} := diag\left\{ \sum_{j \in \mathcal{N}_1} a_{1j}, \sum_{j \in \mathcal{N}_2} a_{2j}, \ldots, \sum_{j \in \mathcal{N}_{N_f}} a_{N_f j} \right\}.$$

The communication structure among agents may be static, but in many real-world applications, it may be time-varying. More recently, stochastic switching topologies have become a powerful tool to solve this problem. With the movement of the target, i.e., the target enters a new Voronoi cell, the agents which are tracking the target may change. This may result in change of topology, which can be treated as switching topologies driven by Markov chain. Then the Laplacian matrix turns into $\mathcal{L}_{\sigma(t)}$, where $\sigma(t), t \geq 0$ is a right-continuous Markov chain on the probability space taking values in a finite state set $S = \{1, 2, \ldots, N_s\}$, where N_s is the

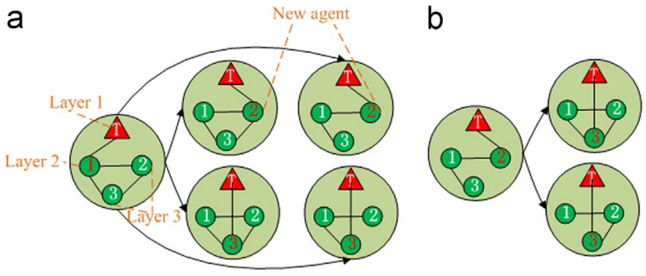

Fig. 6. Rules of switching of topology.

number of possible topologies. Corresponding generator $\Pi = (\pi_{ij})_{N_s \times N_s}$ is given by

$$Pr\{\sigma(t+h) = j | \sigma(t) = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j, \\ 1 + \pi_{ij}h + o(h), & i = j, \end{cases}$$

in which $h > 0$, $\pi_{ij} \geq 0$ is the transition rate from state i to j. When $i = j, \pi_{ii} = -\sum_{j \neq i} \pi_{ij}$.

Denote the communication graph by $\mathcal{G} = \{G(1), G(2), \ldots, G(N_s)\}$, where $G(k) = \{\mathcal{N}_{G(k)}, \mathcal{E}_{G(k)}, \mathcal{A}_{G(k)}\}$ is the communication graph. Denote the topology graph at time t as $\mathcal{G}_t$, then $\mathcal{G}_t = G(k)$ when $\sigma(t) = k$. Piecewise constant signal $\sigma(t)$ changes only when the target enters into a new Voronoi cell.

Remark 3. In this paper, the topology changes according to the movement of a target. When the target enters into a new Voronoi cell, the corresponding Voronoi site will become a tracking agent, meanwhile, one of the original tracking agents quits tracking. The one which is at the final layer of the topology quits. This means not only the topology changes but also the nodes, which is reflected by $\mathcal{N}_{G(k)}$. This is quite different from the traditional switching systems, in which only the topology switches.

Remark 4. It is possible that there exists a special case in which the target is captured in the first Voronoi cell. This means there is no longer switching in the capture process. In this paper, we consider the general case stated in Remark 3.

Fig. 6 illustrates the rules of the switching of topologies. We take the situation that $N_f = 3$ as an example. As analyzed in Section 3, only one agent can access the target's position, which means in the topology of the multi-agent system, only one of the $b_i, i = 1, \ldots, N_f$ is nonzero. When three tracking agents are supposed to track one target, as shown in Fig. 6, there are two cases during the evolution of topology. In case (a), there are three layers in the topology, the target is at layer 1, agent 1 is at layer 2, agent 2 and agent 3 are at layer 3. The final layer has two agents, therefore, when the target moves into a new Voronoi cell, the corresponding Voronoi site will either replace agent 2 or agent 3. Meanwhile, since agent 1 is in target's sense range and target is now at the new Voronoi site's sense range, then based on $R_c \geq 2R_s$, agent 1 can communicate with the new Voronoi site agent. That is why no matter which agent at layer 3 is replaced, the new agent, which is represented by noting the number in red, is always connected with agent 1. However, the new agent may connect or may not connect with the other(s) at layer 3. In case (b), there are 4 layers in the topology, and of course agent 3 is replaced. The switching process is stochastic and fits Markov property, i.e., the switching process is a Markov process.

Definition 5. Under stochastic switching topologies, the tracking agents (6) are said to track the target (7) successfully in the mean square sense if

$$\lim_{t\to\infty} \mathbb{E}(\|x_i(t)-x_{\mathbf{t}}(t)\|)\to 0, \quad i\in N_f. \tag{8}$$

in which $\mathbb{E}$ denotes for expectation.

Since we can only estimate the position of a target, we adopt the following format of control protocol for the i-th tracking agent. The tracking control protocol, consists of the position disagreement vectors between the i-th tracking agent and its neighboring agents, the position disagreement vector between the i-th tracking agent and the maneuvering target:

$$u_i(t) = -\alpha\left\{\sum_{j\in\mathcal{N}_i(t)} a_{ij}(\sigma(t))e_{ij}(t) + b_i(\sigma(t))e_i(t)\right\}, \tag{9}$$

where $\alpha>0$ is the control parameter to be designed. $e_{ij}(t)=x_i(t)-x_j(t)$ is the position disagreement vector between i-th tracking agent and j-th tracking agent. $e_i(t)=x_i(t)-x_{\mathbf{t}}(t)$ is the position disagreement vector between i-th tracking agent and the target. Then, from Definition 5, the tracking problem can be interpreted as stability problem of the following overall disagreement system:

$$\begin{aligned}&\dot{\mathcal{E}}(t) = F(t,\mathcal{E}(t)) - \alpha\tilde{\mathcal{L}}_{\sigma(t)}\mathcal{E}(t), t\neq\sigma^*(t),\\ &\mathcal{E}(\sigma^*(t^+)) = \mathcal{E}(\sigma^*(t^-)) - \Delta\mathcal{E}(\sigma^*(t)), t=\sigma^*(t),\end{aligned} \tag{10}$$

where $\mathcal{E}(t)=[e_1^T(t), e_2^T(t), \ldots, e_{N_f}^T(t)]^T$ is the collective position disagreement vector between tracking agents and the target, at time t. $\tilde{\mathcal{L}}_{\sigma(t)} = (\mathcal{L}_{\sigma(t)} + \mathcal{B}_{\sigma(t)}) \otimes I_2$ is the associated Laplacian matrix of the dynamic graph including the target. $F(t,\mathcal{E}(t)) = col\{f_1(t), f_2(t), \ldots, f_{N_f}(t)\}$ satisfies Assumption 1, where $f_i(t)=f(t,x_i(t))-f(t,x_{\mathbf{t}}(t)), i\in\mathcal{N}$ is the collective nonlinear self-dynamic disagreement to the target at time t. $\sigma^*(t)$ is the switching time. $\sigma^*(t^-)$ indicates the time before switching and $\sigma^*(t^+)$ represents the time after switching. $\Delta\mathcal{E}(\sigma^*(t))$ is the jump of tracking error.

Remark 5. As mentioned in Remark 3, when the topology switches, the tracking agent which is furthest to the target is replaced by the new Voronoi cell. This means the norm of tracking error $\mathcal{E}(t)$ decreases at every switching time $\sigma^*(t)$. The element in $\mathcal{E}(t)$ corresponding to the replaced tracking agent jumps, reflected by $\Delta\mathcal{E}(\sigma^*(t))$. For instance, if the ith tracking agent is replaced at time $\sigma^*(t)$, then $\|e_i(\sigma^*(t^+))\| \le \|e_i(\sigma^*(t^-))\|$ while $\|e_j(\sigma^*(t^+))\| = \|e_j(\sigma^*(t^-))\|, j\neq i, j\in\mathcal{N}$, which implies that $\|\mathcal{E}^T(\sigma^*(t^+))\| \le \|\mathcal{E}^T(\sigma^*(t^-))\|$, i.e., the jump of tracking error $\Delta\mathcal{E}(\sigma^*(t))$ is nonnegative.

Definition 6 (*[32]*). Assume that there exists a stochastic Lyapunov function $V(\mathcal{E}(t), i, t)$ for system (10) with Markov switching topologies such that $\boldsymbol{d}V(\mathcal{E})\le 0$, then the equilibrium solution of $\mathcal{E}=0$ of the stochastic differential equation (10) is stochastically stable. The operator $\boldsymbol{d}V$ is defined as [33]

$$\boldsymbol{d}V(\mathcal{E}(t), i, t) = \lim_{h\to 0}\frac{1}{h}\left\{\mathbb{E}V\left(\mathcal{E}(t+h), \sigma(t+h)\middle|\mathcal{E}(t), \sigma(t)=i\right) - V(\mathcal{E}(t), \sigma(t)=i)\right\}$$

$$\boldsymbol{d}V(\mathcal{E}(t),i,t) = V_t(\mathcal{E}(t),i,t) + V_{\mathcal{E}}(\mathcal{E}(t),i,t)\dot{\mathcal{E}}(t) + \sum_{j=1}^{N_s}\pi_{ij}V(\mathcal{E}(t),i,t), \tag{11}$$

where

$$V_t(\mathcal{E}(t),i,t) = \frac{\partial V(\mathcal{E}(t),i,t)}{\partial t}$$
$$V_{\mathcal{E}}(\mathcal{E}(t),i,t) = \left(\frac{\partial V(\mathcal{E}(t),i,t)}{\partial \mathcal{E}_1}, \frac{\partial V(\mathcal{E}(t),i,t)}{\partial \mathcal{E}_2}, \ldots, \frac{\partial V(\mathcal{E}(t),i,t)}{\partial \mathcal{E}_{N_f}}\right).$$

To analyze the tracking problem, the following lemmas are essential to introduce.

Lemma 1. *For any vectors x, y of appropriate dimensions, the following inequality holds*:

$$\pm 2x^T y \le x^T x + y^T y.$$

Lemma 2 (*[34]*). *For a given symmetric matrix $S = \begin{pmatrix} S_{11} & S_{12} \\ * & S_{22}\end{pmatrix}$, the following inequalities are equivalent.*

(1) $S<0$; (2) $S_{11}<0,\ \ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$; (3) $S_{22}<0,\ \ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

The following theorem provides a sufficient condition for successful tracking.

Theorem 1. *Consider the multi-agent system (10). Suppose that for given positive scalars α, l, there exist symmetric positive definite matrices P_i of appropriate dimensions such that*

$$\begin{pmatrix} -\alpha P_i\tilde{L}_i - \alpha\tilde{L}_i^T P_i + \sum_{j=1}^{N_s}\pi_{ij}P_j + I & lP_i \\ * & -I \end{pmatrix} < 0 \tag{12}$$

for all $i, j = 1, 2, \ldots, N_s$. Then the system (10) is stochastically stable, which means the tracking agents can track the target.

Proof. Let us select the Lyapunov function as

$$V(\mathcal{E}(t),i,t) = \mathcal{E}^T(t)P_i\mathcal{E}(t), \quad t \neq \sigma^*(t). \tag{13}$$

The derivative of Lyapunov function (13) along the trajectory of the system (10) is

$$\begin{aligned}
&\boldsymbol{d}V(\mathcal{E}(t),i,t) \\
&\quad = \dot{\mathcal{E}}^T(t)P_i\mathcal{E}(t) + \mathcal{E}^T(t)P_i\dot{\mathcal{E}}(t) + \sum_{j=1}^{N_s}\pi_{ij}\mathcal{E}^T(t)P_j\mathcal{E}(t) \\
&\quad = \mathcal{E}^T(t)\left(-\alpha P_i\tilde{L}_i - \alpha\tilde{L}_i^T P_i + \sum_{j=1}^{N_s}\pi_{ij}P_j\right)\mathcal{E}(t) + \mathcal{E}^T(t)P_iF(t,\mathcal{E}(t)) + F^T(t,\mathcal{E}(t))P_i\mathcal{E}(t).
\end{aligned}$$

Applying Lemma 1, we have

$$\boldsymbol{d}V(\mathcal{E}(t),i,t)$$

$$\leq \mathcal{E}^T(t)\left(-\alpha P_i\tilde{L}_i - \alpha\tilde{L}_i^T P_i + \sum_{j=1}^{N_s}\pi_{ij}P_j\right)\mathcal{E}(t) + \mathcal{E}^T(t)\mathcal{E}(t) + F^T(t,\mathcal{E}(t))P_iP_iF(t,\mathcal{E}(t)).$$

In light of Assumption 1, the above equation turns to be

$$\boldsymbol{d}V(\mathcal{E}(t),i,t) \leq \mathcal{E}^T(t)\left(-\alpha P_i\tilde{L}_i - \alpha\tilde{L}_i^T P_i + \sum_{j=1}^{N_s}\pi_{ij}P_j + I + l^2 P_iP_i\right)\mathcal{E}(t).$$

By inequality (12) and Lemma 2, it is easy to see that $\boldsymbol{d}V(\mathcal{E}(t),i,t) < 0$ holds. At the switching times $\sigma^*(t)$, since $\|\mathcal{E}^T(\sigma^*(t^+))\| \leq \|\mathcal{E}^T(\sigma^*(t^-))\|$, we have

$$\mathcal{E}^T((\sigma^*(t^+))P_i\mathcal{E}(\sigma^*(t^+)) - \mathcal{E}^T(\sigma^*(t^-))P_i\mathcal{E}(\sigma^*(t^-)) \leq 0.$$

Therefore, the switching accelerates the degradation of tracking errors, which will be verified in Example 3.

Then according to Definition 6, we can obtain that Theorem 1 ensures the stochastic stability of the disagreement system (10). The disagreement between the tracking agents and the target tends to 0 when $t \to \infty$, i.e., tracking agents finally track the target according to Definition 5. This completes the proof. □

The above analysis is definitely suitable for the situation in which only one target is considered. If we consider multi targets, there could be four possible cases based on when and where targets enter the monitoring area. Here are the detail descriptions and mathematical statements of the four cases.

Case 1. At all the time, there is only one target in the monitoring area, i.e., no new target enter into this area until the existed target got caught. The mathematical statement of this case is

$$T_{int}(\mathbf{t}_i) \cap T_{int}(\mathbf{t}_j) = \varnothing, \quad i \neq j \in \{1,2,\ldots,N_{\mathbf{t}}\}, \tag{14}$$

where $T_{int}(\mathbf{t}_i)$ means the time interval from target $\mathbf{t}_i$ enters into the monitoring area till it is captured. In this case, proof of the successful tracking is the same with that in Theorem 1 due to that we just need to deal with each target separately.

Case 2. There are several targets in the monitoring area at one time, but the targets are far-field distributed. They enter this area from various directions, they are far away from each other and there is no interaction between them. The mathematical statement of this case is

$$\begin{aligned} &T_{int}(\mathbf{t}_i) \cap T_{int}(\mathbf{t}_j) \neq \varnothing, \\ &R(\mathbf{t}_i) \cap R(\mathbf{t}_j) = \varnothing, \quad i \neq j \in \{1,2,\ldots,N_{\mathbf{t}}\}, \end{aligned} \tag{15}$$

where $R(\mathbf{t}_i)$ means the route of target $\mathbf{t}_i$. In this case, the analysis of capturing each target is independent, we can still prove the stability of tracking system using Theorem 1.

Case 3. Several targets might enter the monitoring target along a same route over a time interval. The time interval makes sure that no two targets in one Voronoi cell at the same time. The mathematical statement of this case is

$$T_{int}(\mathbf{t}_i) \cap T_{int}(\mathbf{t}_j) \neq \varnothing,$$

$$R(\mathbf{t}_i) \cap R(\mathbf{t}_j) \neq \emptyset, \quad i \neq j \in \{1,2,\ldots,N_{\mathbf{t}}\},$$
$$T_{V_{a_k}}(\mathbf{t}_i) \cap T_{V_{a_k}}(\mathbf{t}_j) = \emptyset, \quad k \in \{1,2,\ldots,N_a\}, \tag{16}$$

where $T_{V_{a_k}}(\mathbf{t}_i)$ is the time interval of target $\mathbf{t}_i$ that resides at Voronoi cell $V(a_k)$. In this case, even though the targets share a route, they are passing Voronoi cells at different time, this means that the approach in Theorem 1 for capturing each target is also applicable.

Case 4. Several targets are in the monitoring area at the same time. The targets are close to each other, a target could interact with another. The mathematical statement of this case is

$$T_{int}(\mathbf{t}_i) \cap T_{int}(\mathbf{t}_j) \neq \emptyset, \quad i \neq j \in \{1,2,\ldots,N_{\mathbf{t}}\},$$
$$T_{V_{a_k}}(\mathbf{t}_i) \cap T_{V_{a_k}}(\mathbf{t}_j) \neq \emptyset, \quad k \in \{1,2,\ldots,N_a\}, \tag{17}$$

where $T_{V_{a_k}}(\mathbf{t}_i)$ is the time interval of target $\mathbf{t}_i$ that resides at Voronoi cell $V(a_k)$. In this case, when several targets are in a same Voronoi cell, the Voronoi site agent tracks the nearest target, the topology of other target tracking system stay static.

5. Numerical examples

In this section we present numerical examples to show the effectiveness and correctness of the main results derived above. Consider a scenario where a group of three tracking agents tracking a maneuvering target in a 2-dimensional space. Since the monitoring area is 3-overlapped, when a target enters into this area, with the initialization principle addressed in Section 3, the three tracking agents can communicate with each other. Then the topology of the tracking system with three tracking agents is always as shown in the first graph in Fig. 7. According to the rules of

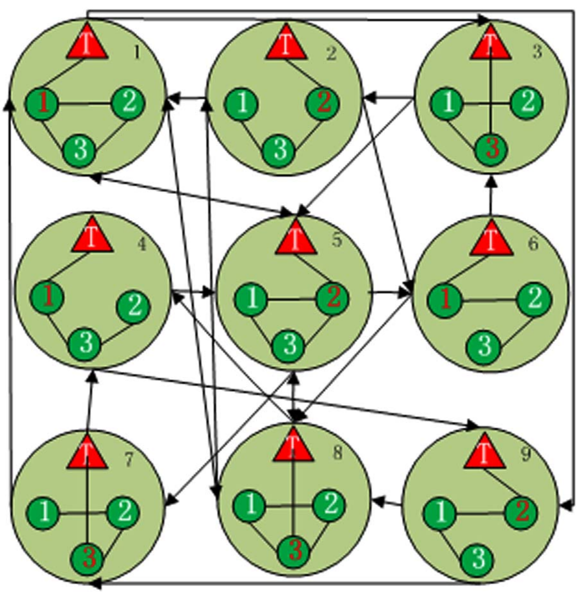

Fig. 7. Possible topologies when N_f=3.

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evolution of a topology, there are nine possible topologies when the tracking number is set as $N_f = 3$.

The nonlinear dynamics of the tracking agents and target are

$$f(t, x(t)) = 250\sin(0.0035x(t)) + 200\cos(2.5t) + 10\sin(2.5t). \tag{18}$$

The control parameter is chosen as $\alpha = 3.2$. Consider there are at most 5 targets that may enter a $1000 \times 1000\ \mathrm{m}^2$ monitoring area, the initial deployment of agents is shown in Fig. 8. The time of the first target that breaks into this domain is considered as initial time. Coordination of the location where the first target enters this area is (0,100).

It is easy to find that case 3 and case 4 are more complicated than case 1 and case 2. Actually, case 1 and case 2 can be reflected in case 3 and case 4. Therefore, we give two examples under case 3 and case 4 respectively.

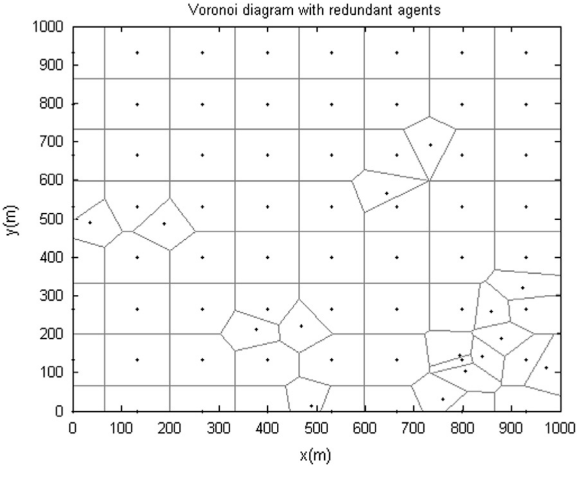

Fig. 8. Initial deployment of agents.

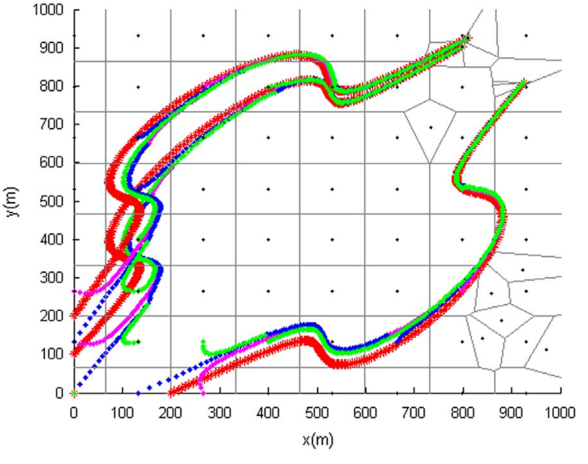

Fig. 9. Tracking trajectories for case 3.

Example 1. This is an example for case 3. At time steps of 50 and 150, two other new targets enter at locations (0, 200) and (200, 0) respectively. The third target is isolated to the other two targets, and the other two targets meet during the pursuit. The tracking process is illustrated in Fig. 9. As can be seen clearly in this figure, the discontinuous trajectories represent switching of tracking agents.

Comparing the two figures, we can find that in Fig. 9, at the end of the trajectories, there are more redundant agents monitoring this area. This is because we adopt the release policy, then when a target is caught, its tracking agents are released to become redundant agents.

Figs. 10, 11 and 12 show tracking errors on *x*-axis, *y*-axis and the norm of tracking errors respectively. Figs. 10 and 11 obviously reflect the switching signal. Every switching of topology

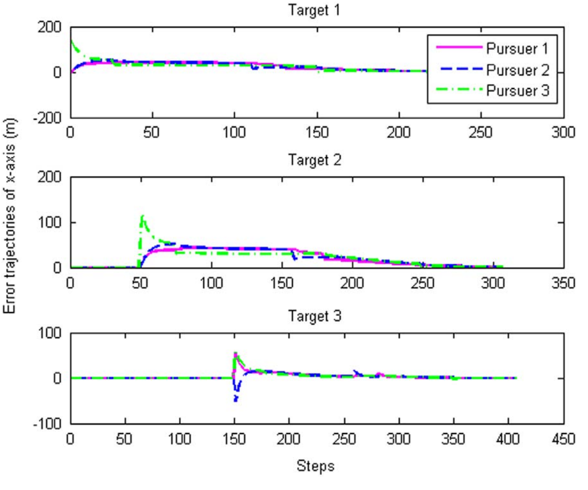

Fig. 10. Error trajectories of tracking on *x*-axis for case 3.

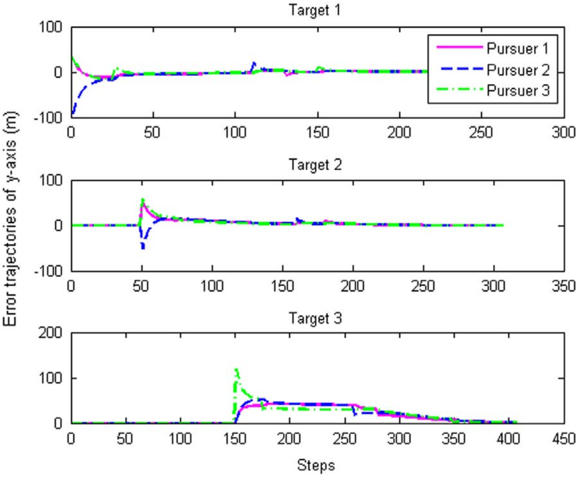

Fig. 11. Error trajectories of tracking on *y*-axis for case 3.

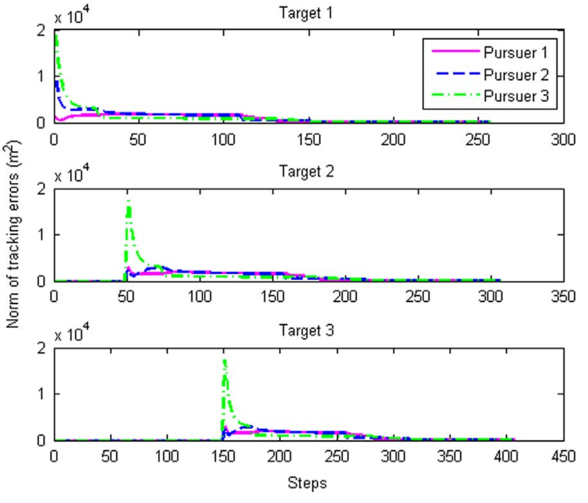

Fig. 12. Norm of tracking errors for case 3.

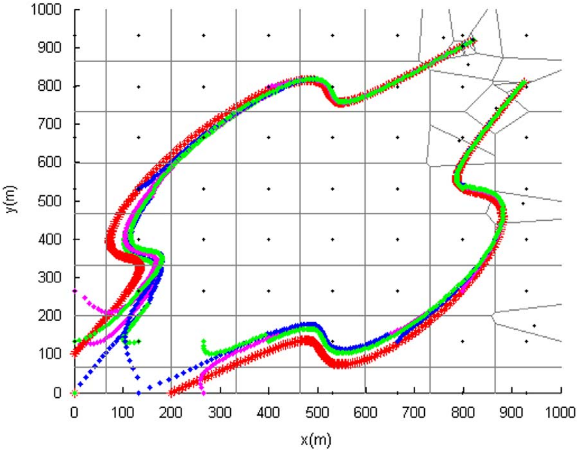

Fig. 13. Tracking trajectories for case 4.

and tracking agents brings a decrease jump in tracking error. However, the decrease may not in both x-axis and y-axis, then the norm of tracking errors is depicted in Fig. 12, in which the signal decreases every time a switching occurs.

Example 2. This is an example for case 4. At the initial time, target 1 and target 2 move into the monitoring area simultaneously, and share the same route. At time steps of 150, a new target enters this area from another direction. The third target is isolated to the other two targets. The tracking trajectories are illustrated in Fig. 13. When target 1 and target 2 move into this area at the same time and same location, three nearest agents join the tracking team of target 1, the initial tracking agents for target 2 is far-field distributed.

Figs. 14, 15 and 16 show tracking errors on x-axis, y-axis and the norm of tracking errors respectively. No matter the targets are isolated or not, they are captured eventually, which vividly shows the feasibility and effectiveness of our proposed tracking strategy.

Example 3. To verify the advantage of proposed tracking strategy, we compare the tracking performance between the relay pursuit strategy and the non-switching pursuit strategy. In the two tracking strategies, the initial tracking agents are the same, and it is assumed the target enters this monitored area from position (0,100). However, in the non-switching pursuit strategy, the tracking agents do not switch when the target enters a new Voronoi cell.

Fig. 17 shows norm of tracking errors with two tracking strategies. As can be seen from the figure, it is obvious that the target can be tracked successfully within a shorter time with

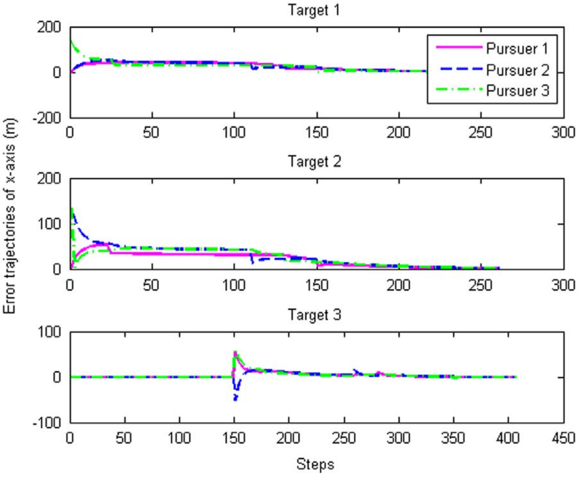

Fig. 14. Error trajectories of tracking on x-axis for case 4.

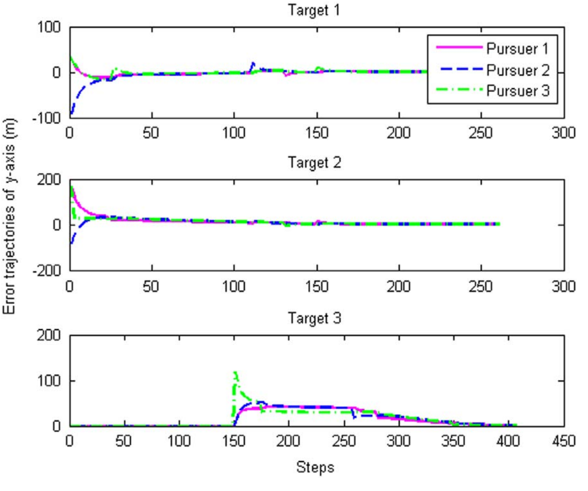

Fig. 15. Error trajectories of tracking on y-axis for case 4.

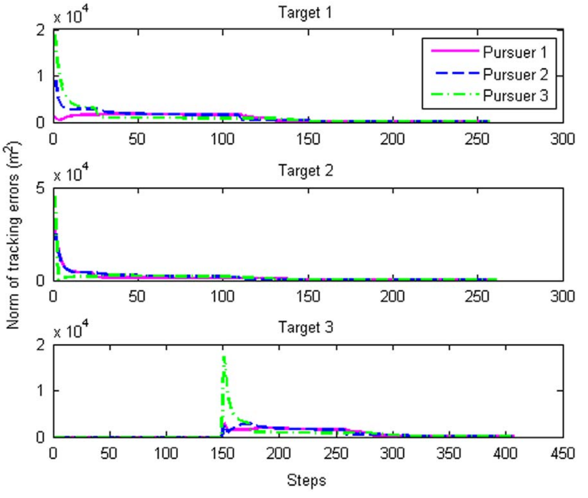

Fig. 16. Norm of tracking errors for case 4.

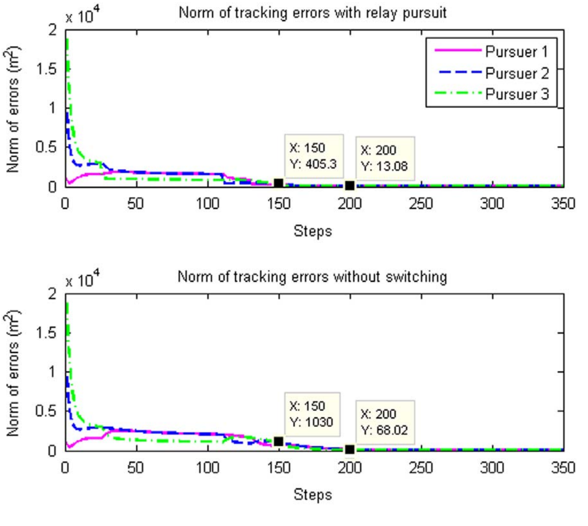

Fig. 17. Norm of tracking errors comparison.

proposed tracking strategy. The norms of tracking error of one tracking agent at steps 150 and 200 are marked. Here, only one tracking agent is marked since all the three are at approximate magnitude. Specifically, at step 150, the norm of tracking error with proposed strategy is 405.3, which is overwhelmingly less than its counterpart value 1030 of the non-switching strategy.

6. Conclusions

This paper has proposed a cooperative relay tracking strategy to capture targets intruding a domain monitored by a large number of smart agents. Trilateration algorithm is used to calculate the location of a target, which is key information for designing a tracking controller. Since a target is tracked by several tracking agents, the tracking agents need to cooperate and communicate with each other. The domain is divided into many Voronoi cells with assistance of Voronoi diagram. In the tracking process, the tracking agent which is able to access the location of a target is relayed by a series of Voronoi sites. When a target is moving into a new Voronoi cell, the Voronoi site replaces one of the tracking agents, bringing switching of topologies. The switching problem of topologies is solved by modelling it as Markov chain process and the switching of tracking agents is reflected by the jump of tracking errors. Simulation results prove the advantage of proposed tracking algorithm.

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