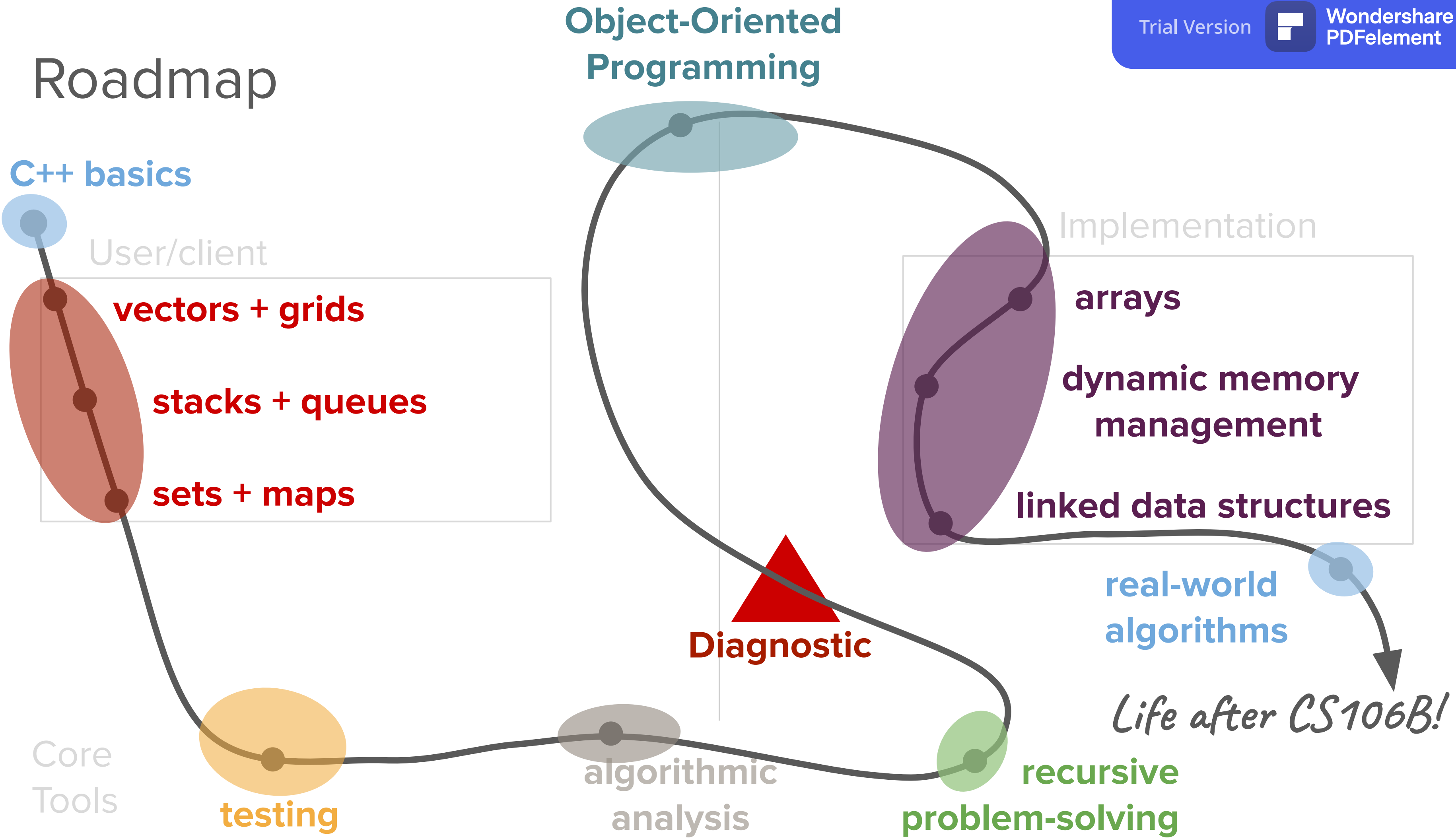
Binary Search Trees

What is your favorite type of tree?

(put your answers the chat - e.g. oak, redwood, maple, etc.)

Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
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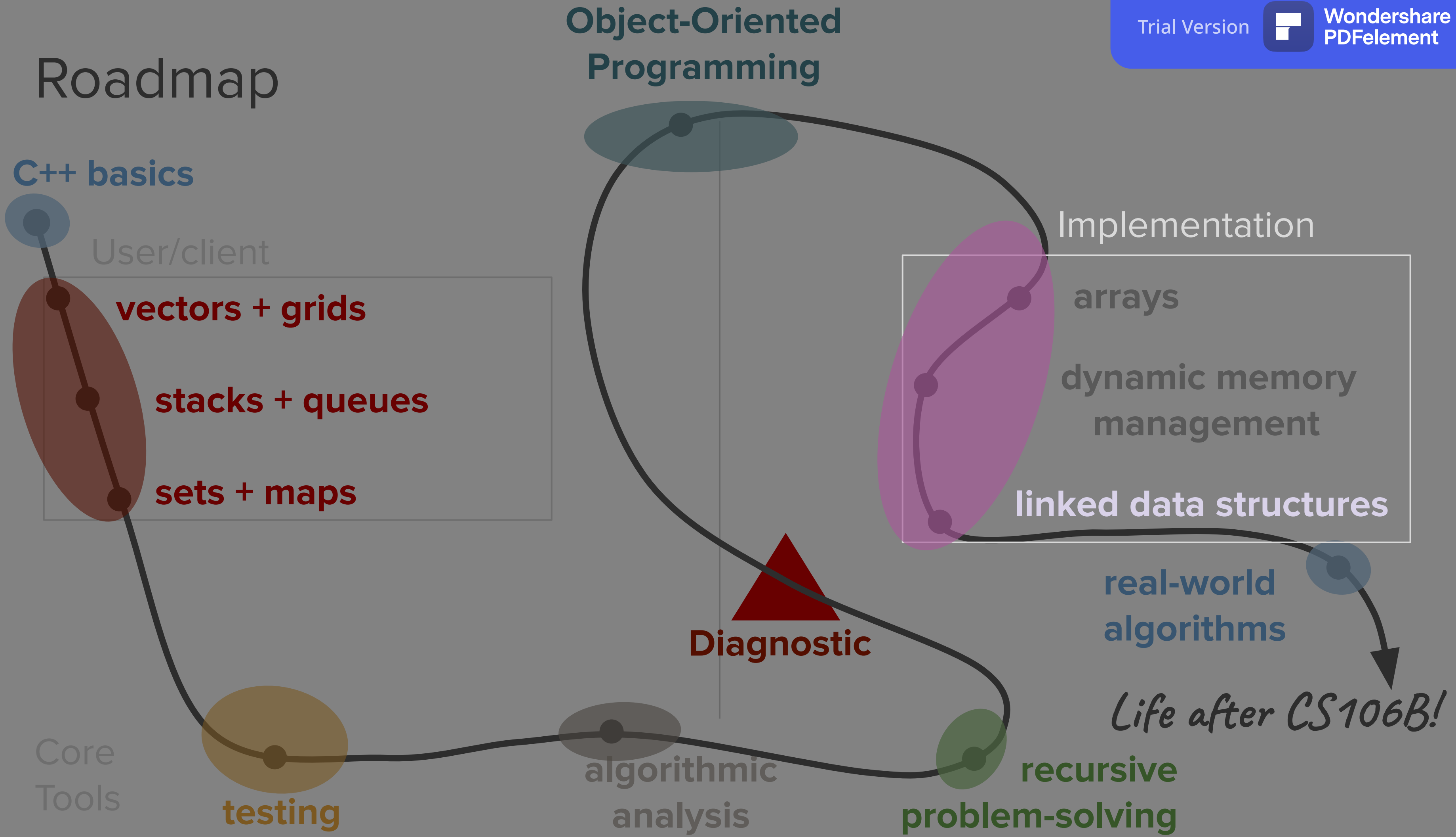
Roadmap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Today's questions

How can we take advantage of trees to structure and efficiently manipulate data?

Today's topics

1. What is a binary search tree (BST)?
2. Building efficient BSTs
3. Implementing Sets with BSTs

Review

[trees]

Definition

tree

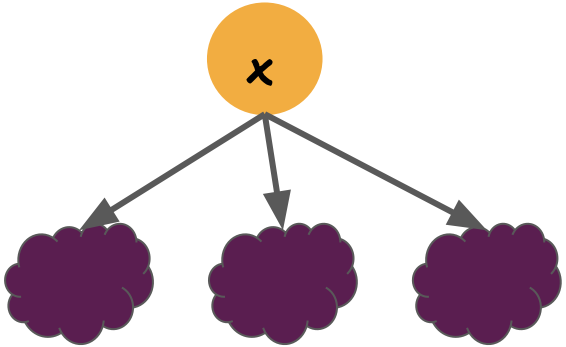
A tree is hierarchical data organization structure composed of a root value linked to zero or more non-empty subtrees.

What is a tree?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)



Tree terminology

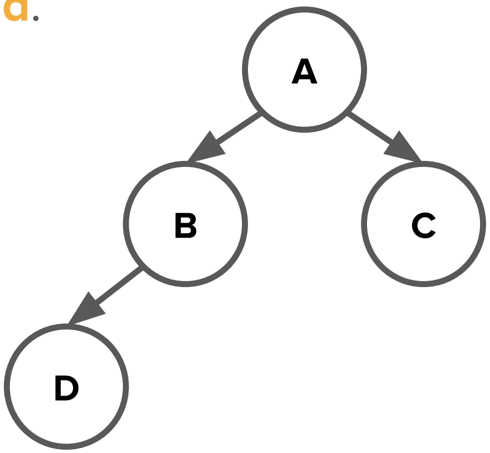
- Types of nodes
  - The **root** node defines the "top" of the tree.
  - Every node has 0 or more **children** nodes descended from it.
  - Nodes with no children are called **leaf nodes**.
  - Every node in a tree has exactly one **parent** node (except for the root node).

- Terminology for quantifying trees
  - A **path** *between two nodes* traverses edges between parents and their children, and **length** *of a path* is the number of edges between the two nodes.
  - The **depth** *of a node* is the length of the path (# of edges) between the root and that node.
  - The **height** *of a tree* is the number of nodes in the longest path through the tree (i.e. the number of **levels** in the tree).

Binary trees

- A **binary tree** is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.

- Typically, the two children of a node in a binary tree are referred to as the **left child** and the **right child**.



Binary trees

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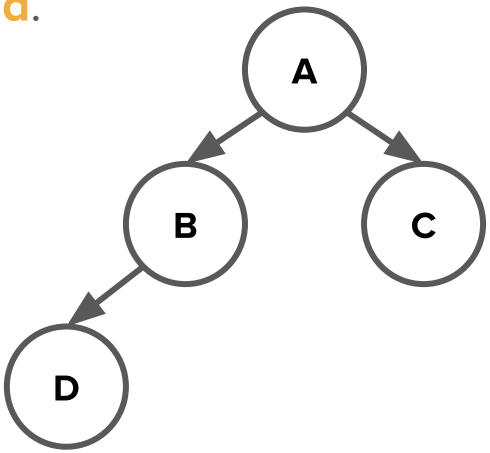


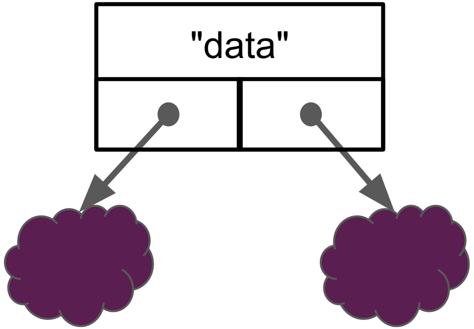

```
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

What is a tree in C++?

A tree is either...

An empty tree represented by **`nullptr`**, or...

A single **`TreeNode`,** with 0, 1, or 2 non-null pointers to other **`TreeNodes`**



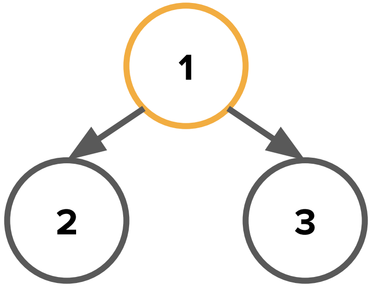
Building a tree

- Building a tree is very similar to the process of building a linked list.
- We create new nodes of the tree by dynamically allocating memory.
- We start by first creating the leaf nodes and then creating their parents.
- We integrate the parents into the tree by rewiring their `left` and `right` pointers to the already-created children.

Traversing a tree - recursively!

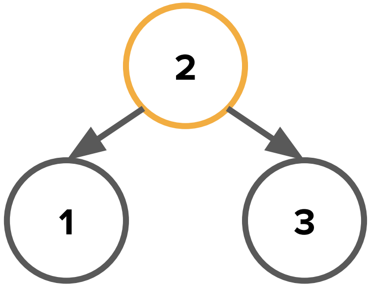
Pre-order

1. **"Do something" with the current node**
2. Traverse the left subtree
3. Traverse the right subtree



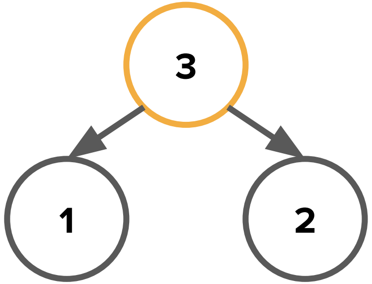
In-order

1. Traverse the left subtree
2. **"Do something" with the current node**
3. Traverse the right subtree



Post-order

1. Traverse the left subtree
2. Traverse the right subtree
3. **"Do something" with the current node**



Try it yourself:

Freeing a tree!

Key Idea: The distance from each element (node) in a tree to the top of the tree (the root) is small, even if there are many elements.

How can we take advantage of trees to structure and efficiently manipulate data?

Revisiting our levels of abstraction...

Levels of abstraction

What is the interface for the user?

How is our data organized?
(binary heaps, **BSTs**, Huffman trees)

What stores our data?
(arrays, linked lists, **trees**)

How is data represented electronically?
(RAM)

Abstr
Structures

Data Organization Strategies

Fundamental C++ Data Storage

Computer Hardware

ADT Big-O Matrix

- Vectors
  - .size() - O(1)
  - .add() - O(1)
  - v[i] - O(1)
  - .insert() - O(n)
  - .remove() - O(n)
  - .clear() - O(n)
  - traversal - O(n)
- Grids
  - .numRows()/.numCols() - O(1)
  - g[i][j] - O(1)
  - .inBounds() - O(1)
  - traversal - O(n^2)
- Queues
  - .size() - O(1)
  - .peek() - O(1)
  - .enqueue() - O(1)
  - .dequeue() - O(1)
  - .isEmpty() - O(1)
  - traversal - O(n)
- Stacks
  - .size() - O(1)
  - .peek() - O(1)
  - .push() - O(1)
  - .pop() - O(1)
  - .isEmpty() - O(1)
  - traversal - O(n)
- Sets
  - .size() - O(1)
  - .isEmpty() - O(1)
  - .add() - O(log(n))
  - .remove() - O(log(n))
  - .contains() - O(log(n))
  - traversal - O(n)
- Maps
  - .size() - O(1)
  - .isEmpty() - O(1)
  - m[key] - O(log(n))
  - .contains() - O(log(n))
  - traversal - O(n)

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  - .contains() – O(log(n))
  - traversal – O(n)

Levels of abstraction

What is the interface for the user?
(**Sets**, Maps, etc.)

How is our data organized?
(binary heaps, **BSTs**, Huffman trees)

What stores our data?
(arrays, linked lists, **trees**)

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(RAM)

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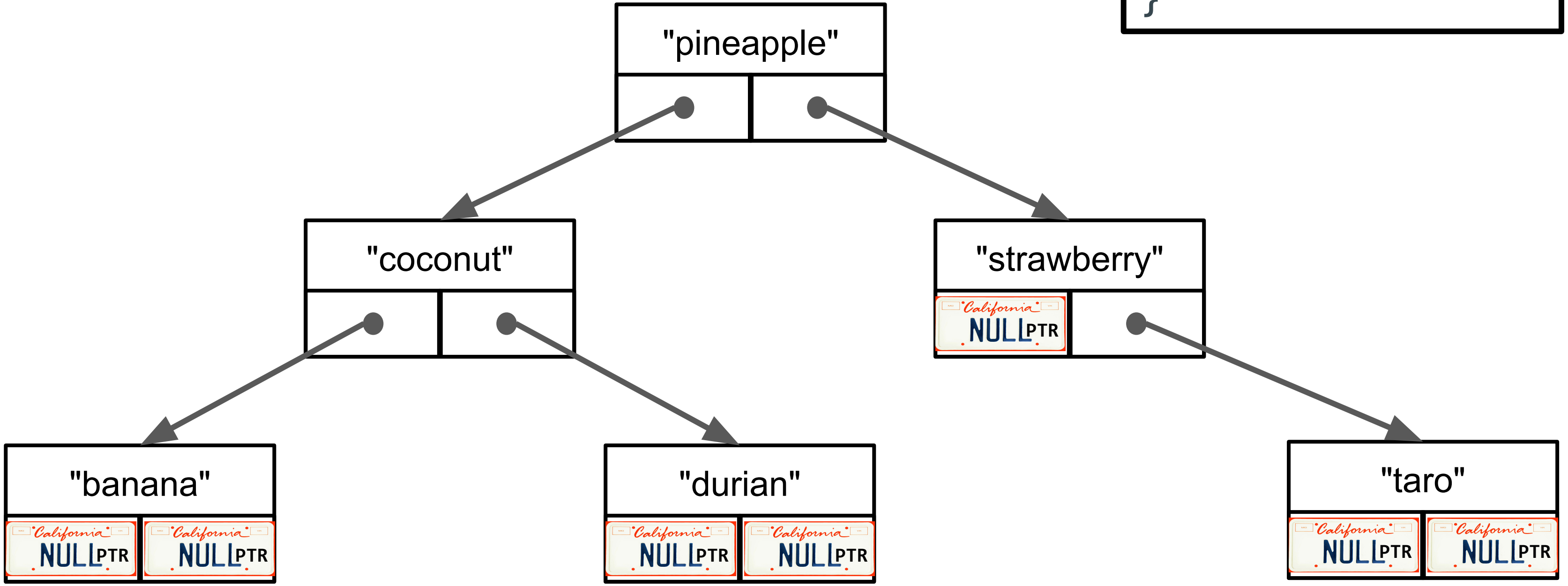
What is a binary search tree (BST)?

```
stru
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

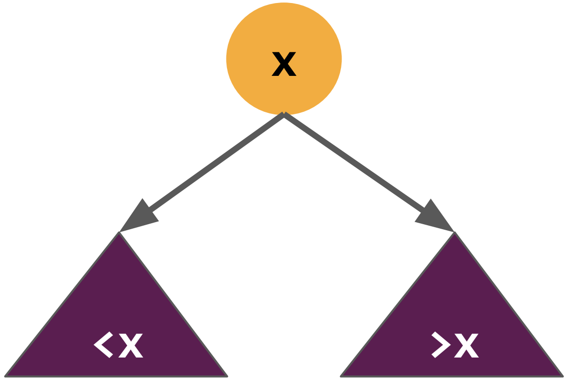
Building Trees Programmatically



A binary search tree is either...

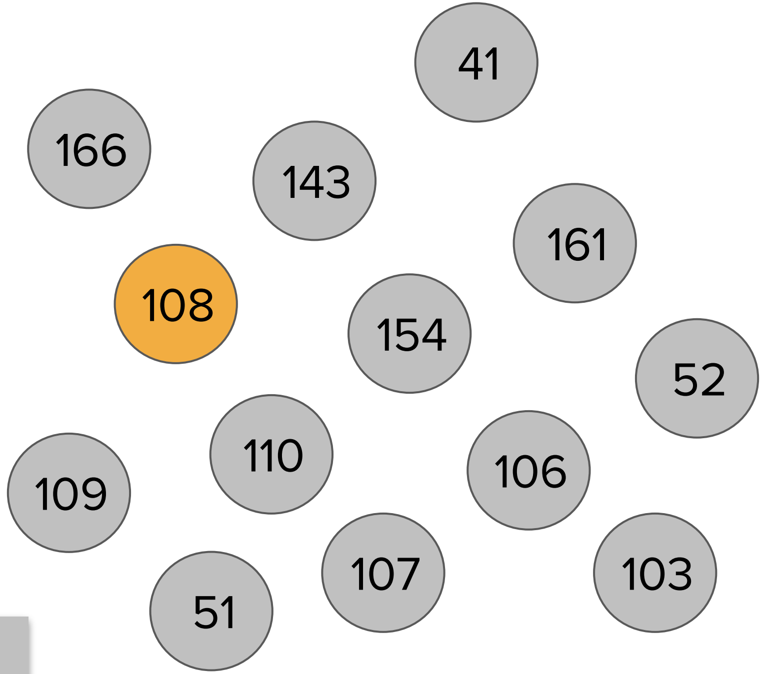
an empty data structure represented by nullptr or...

a single node, whose left subtree is a BST of smaller values than **x**...

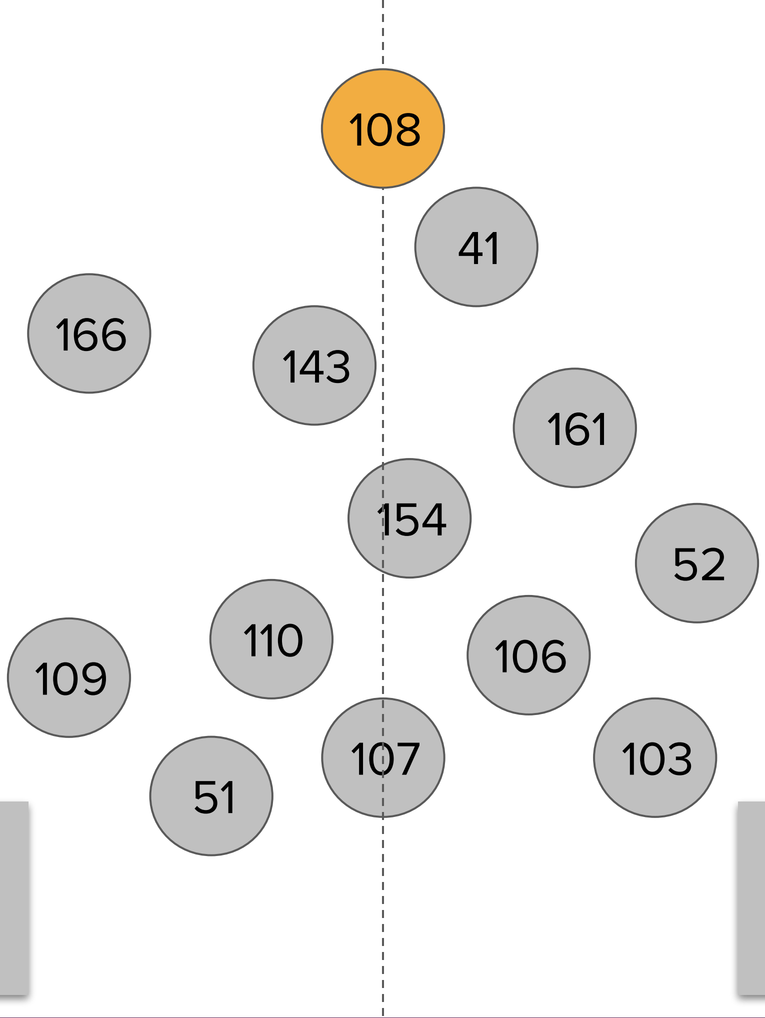


and whose right subtree is a BST of larger values than **x**.

Building a BST

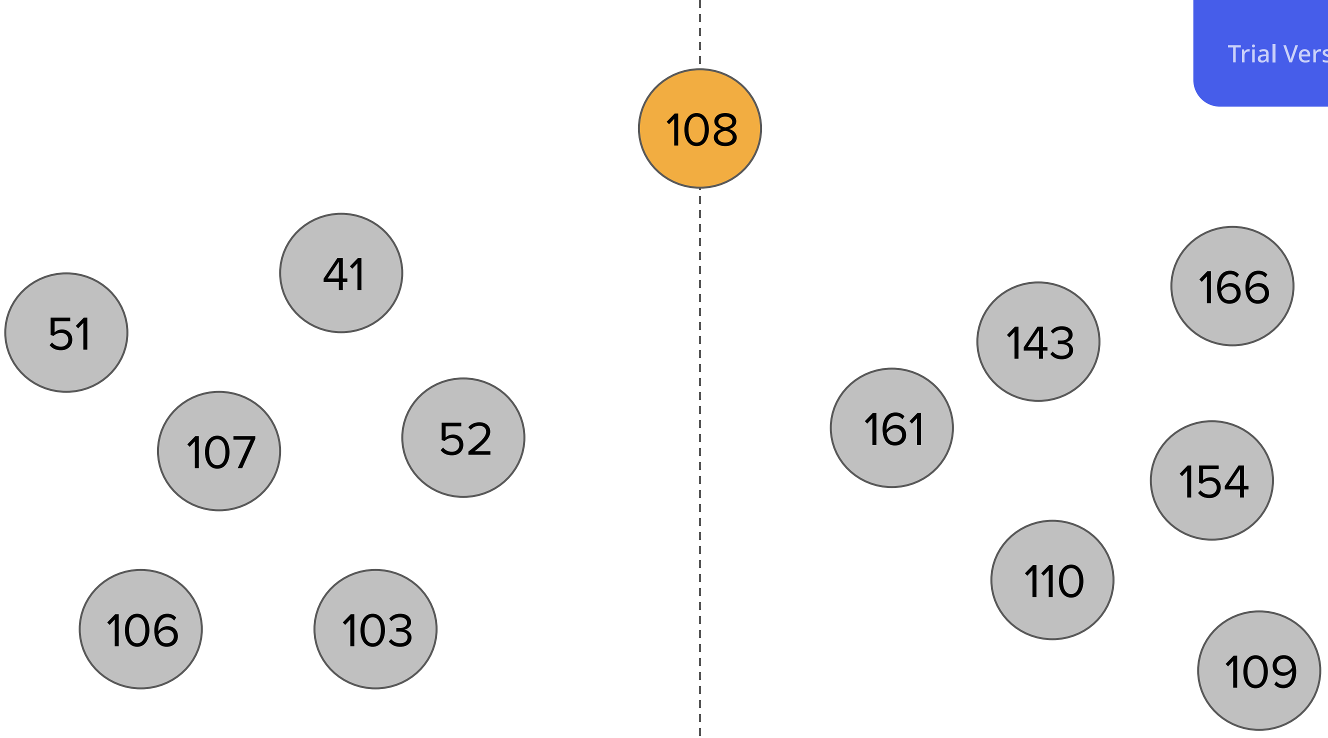


Pick the median element



Move elements less than 108 to this side

Move elements greater than 108 to this side



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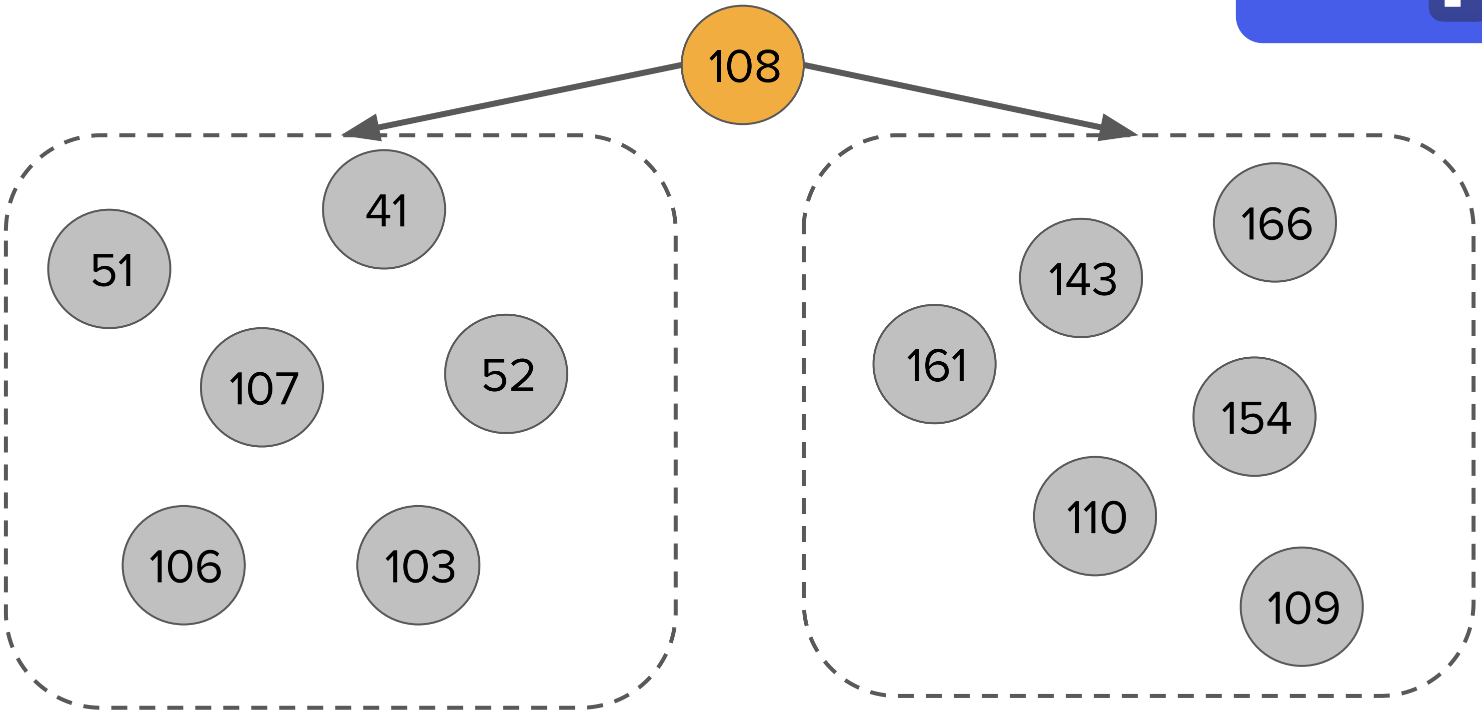
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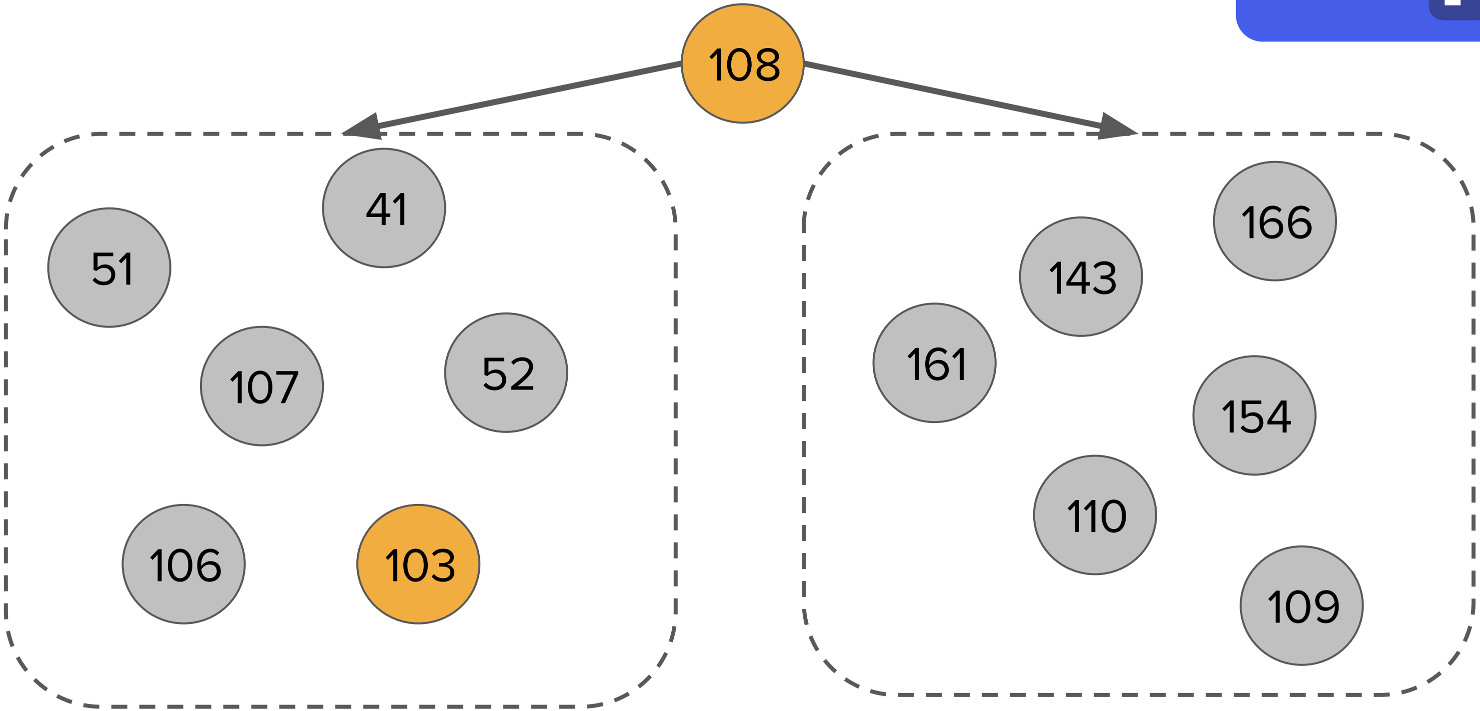
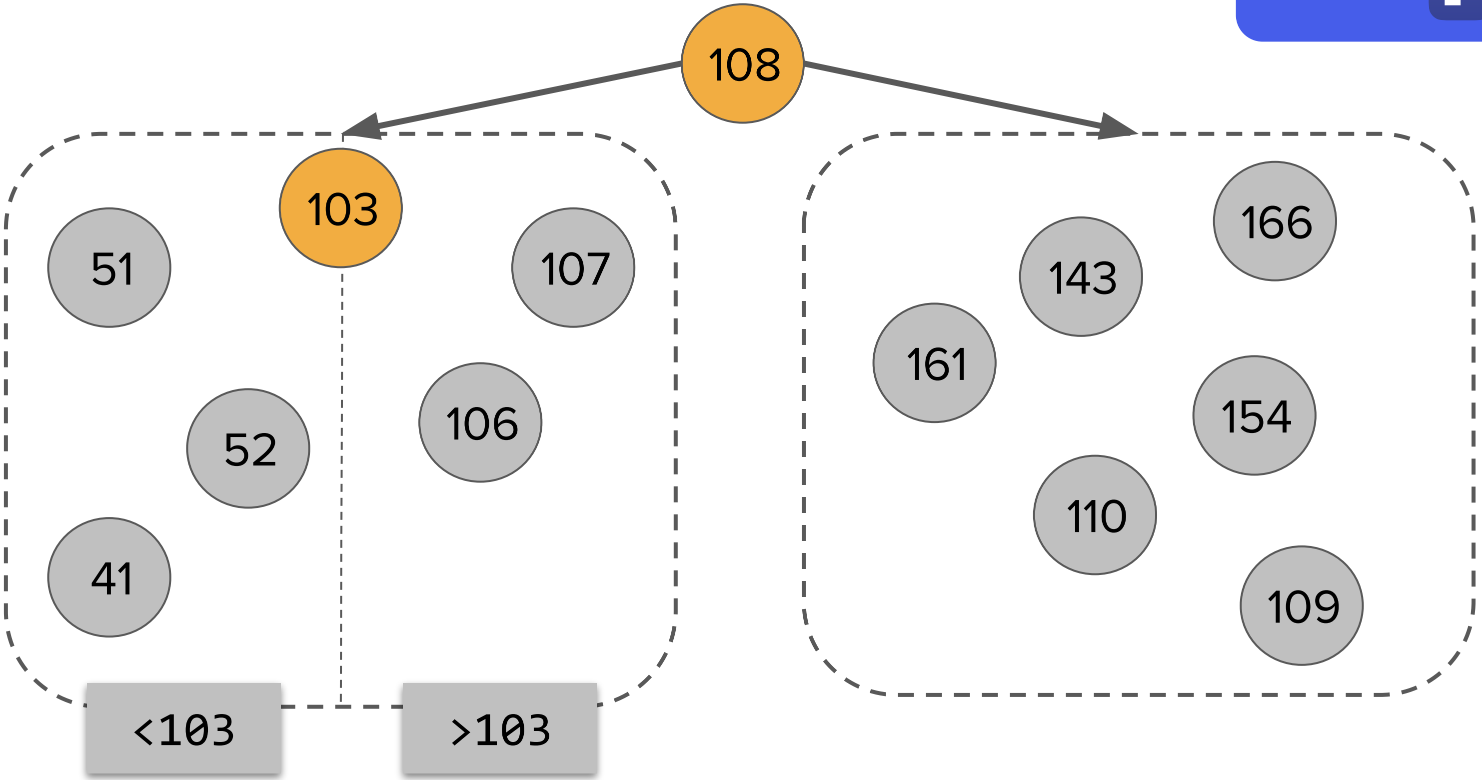


Pick the median element of the left side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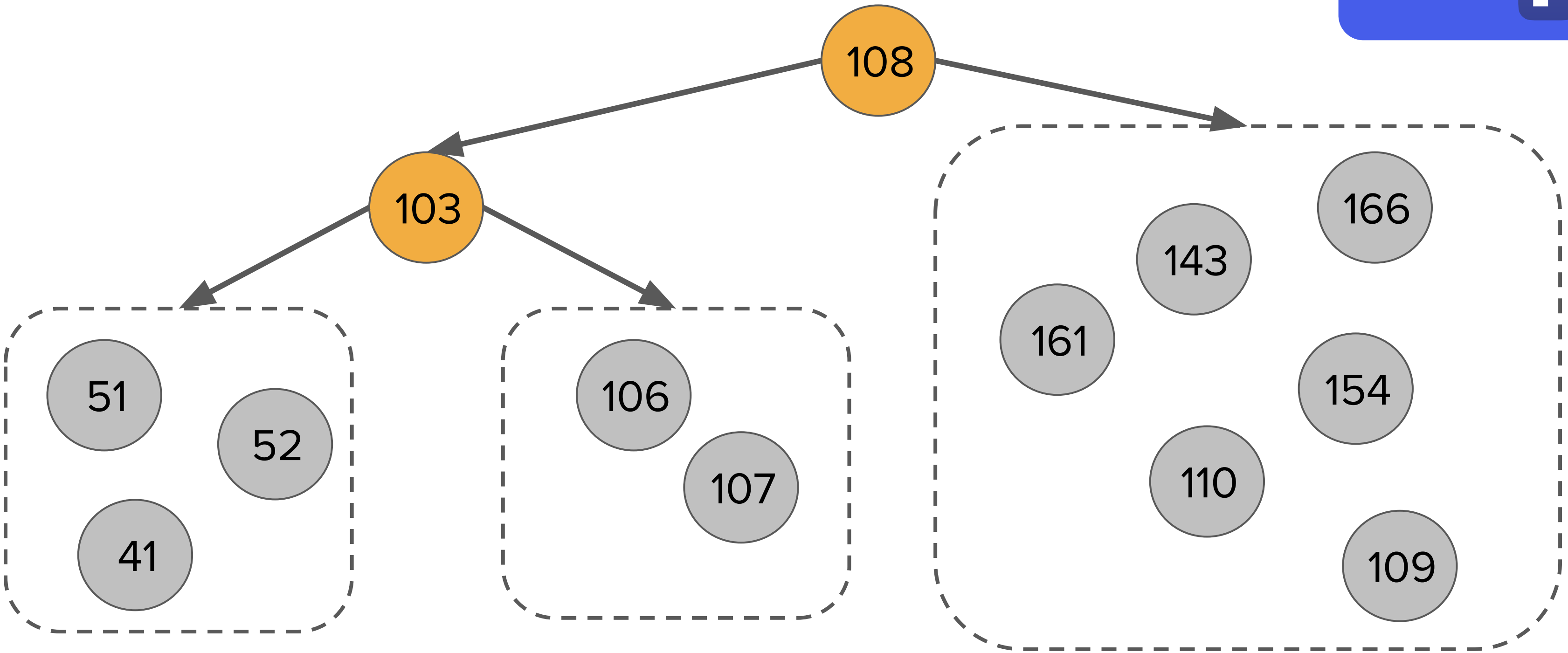
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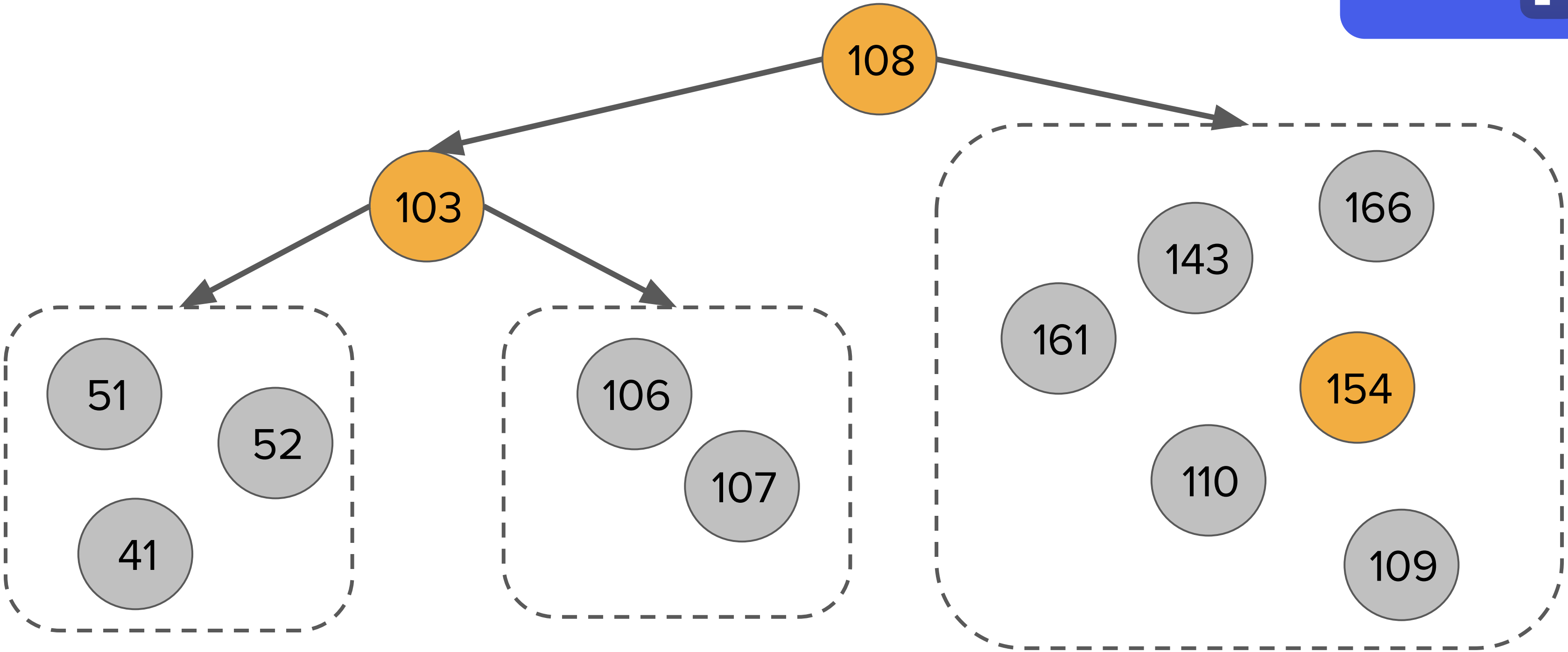
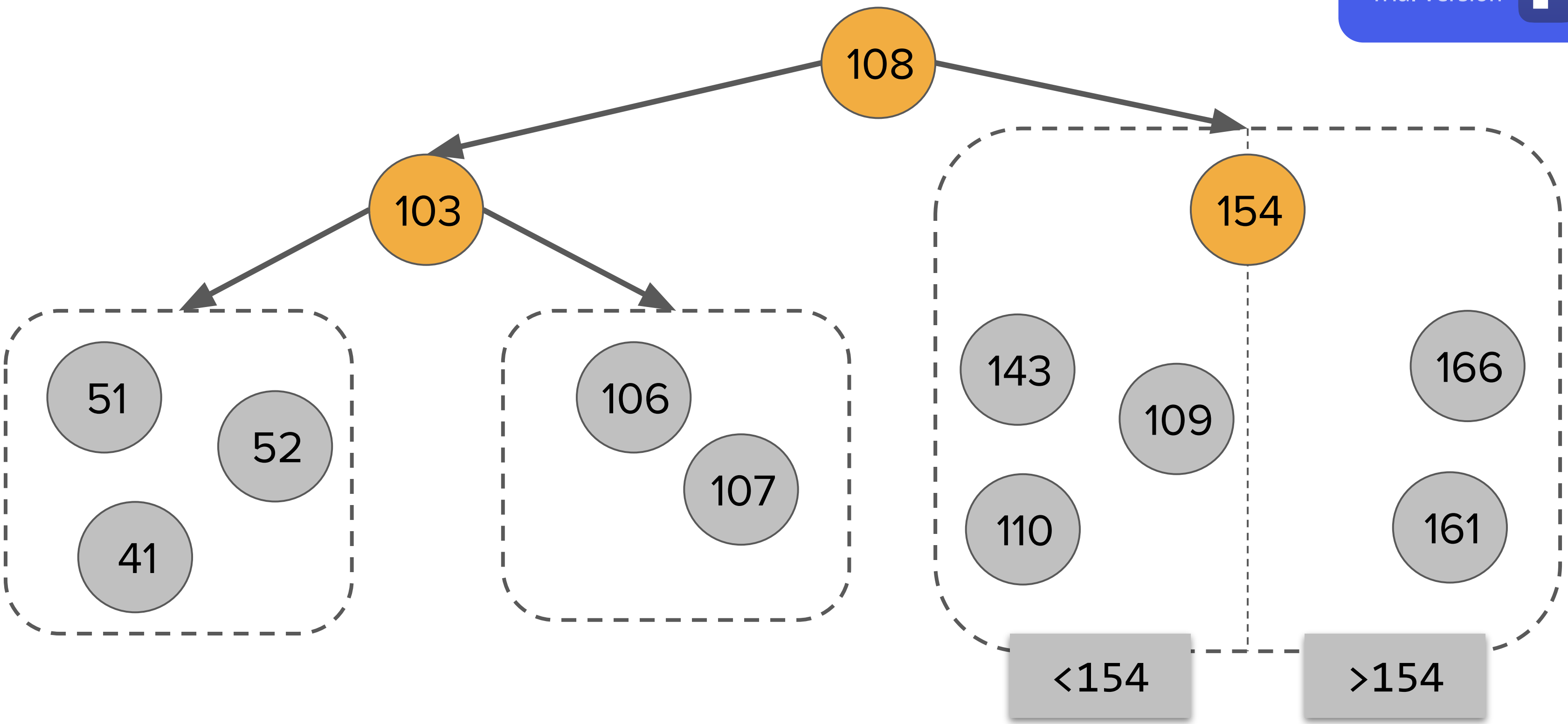


Pick the median element of the right side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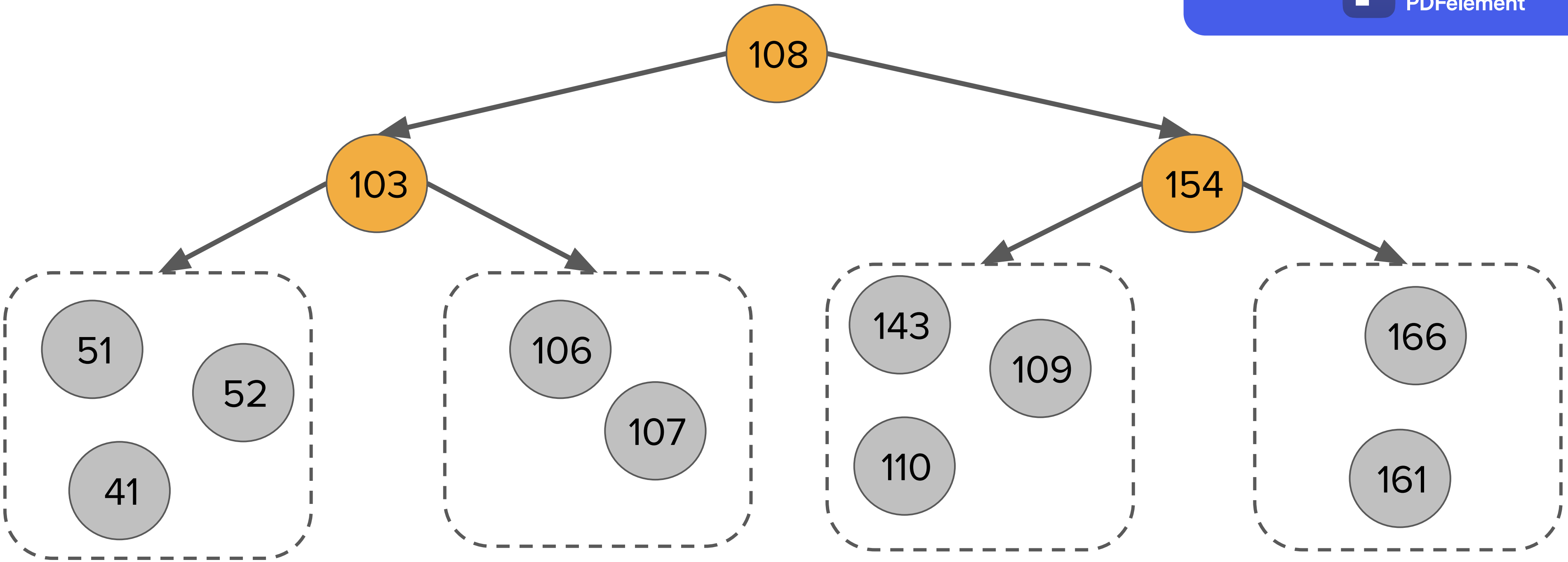
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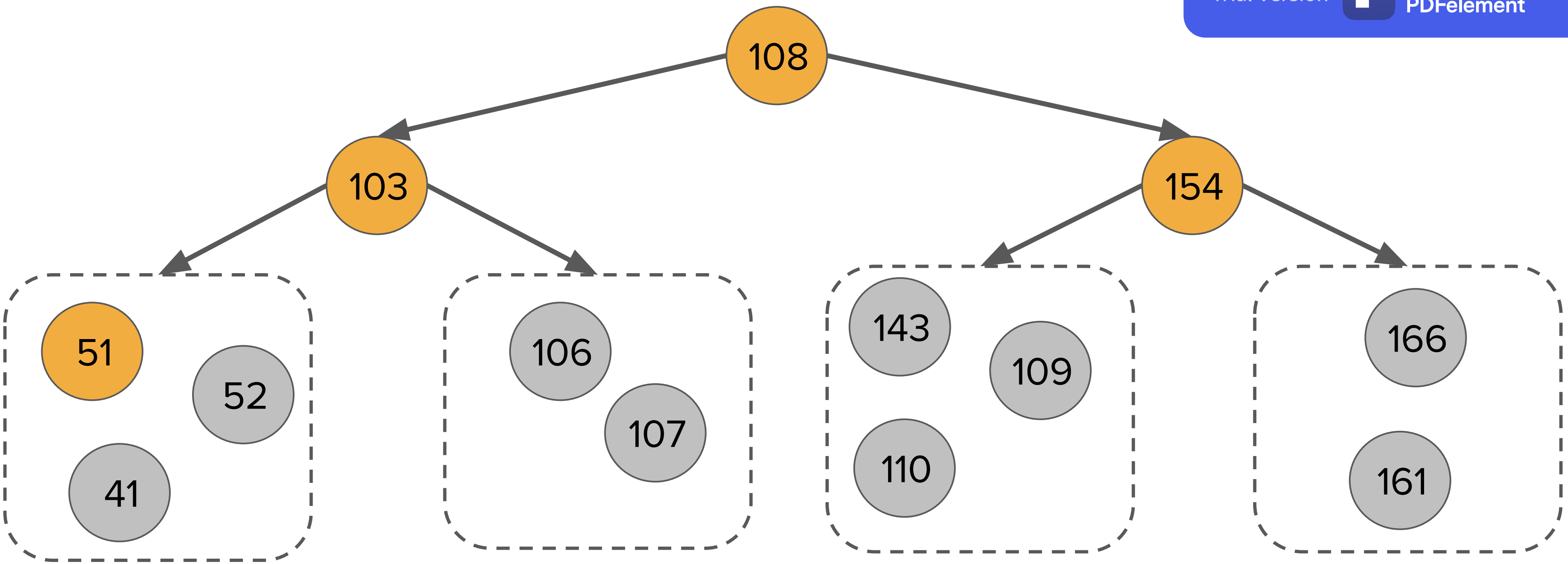
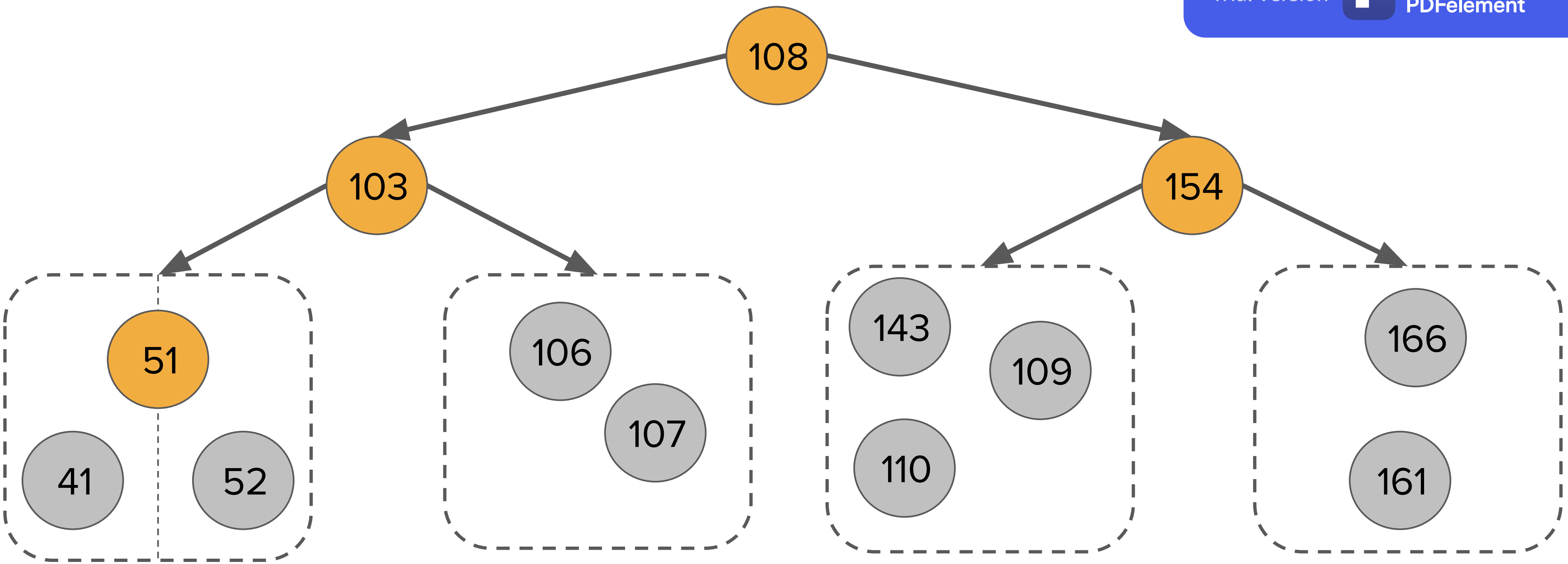


Pick the median element of the left side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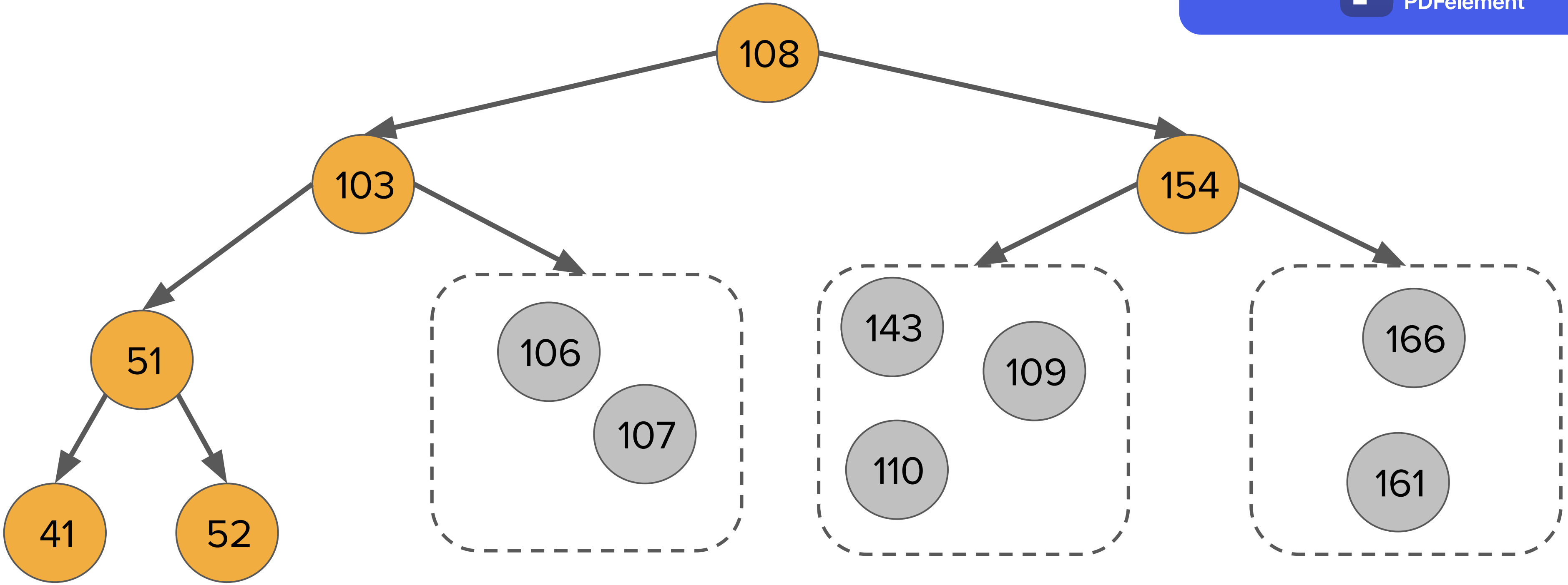
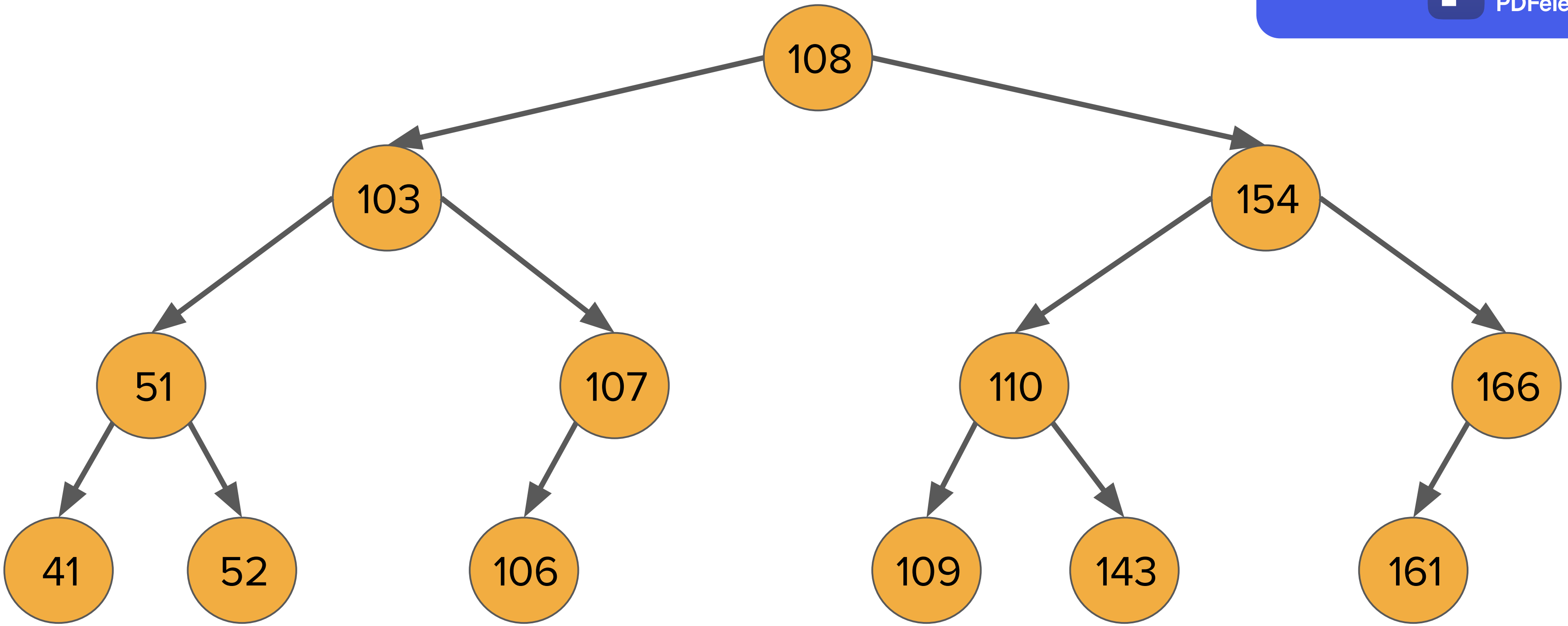


Keep repeating this process
for all the subtrees!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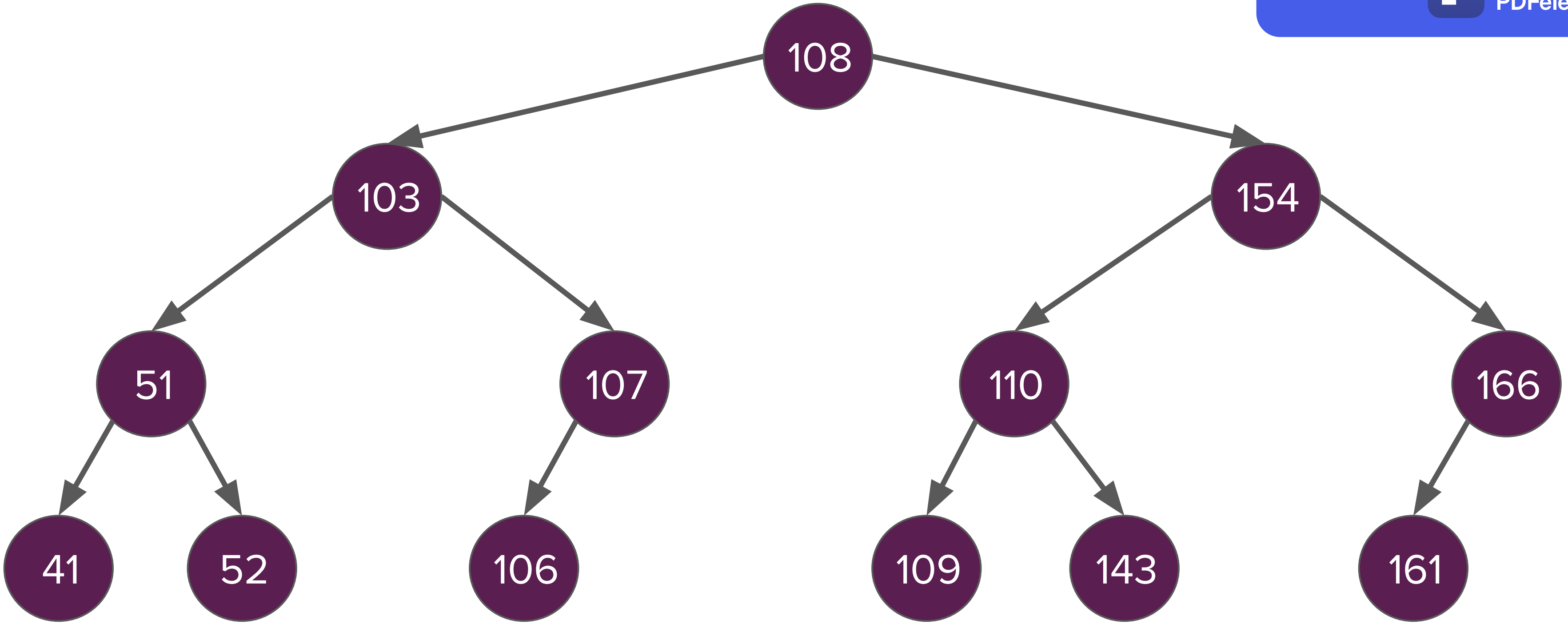
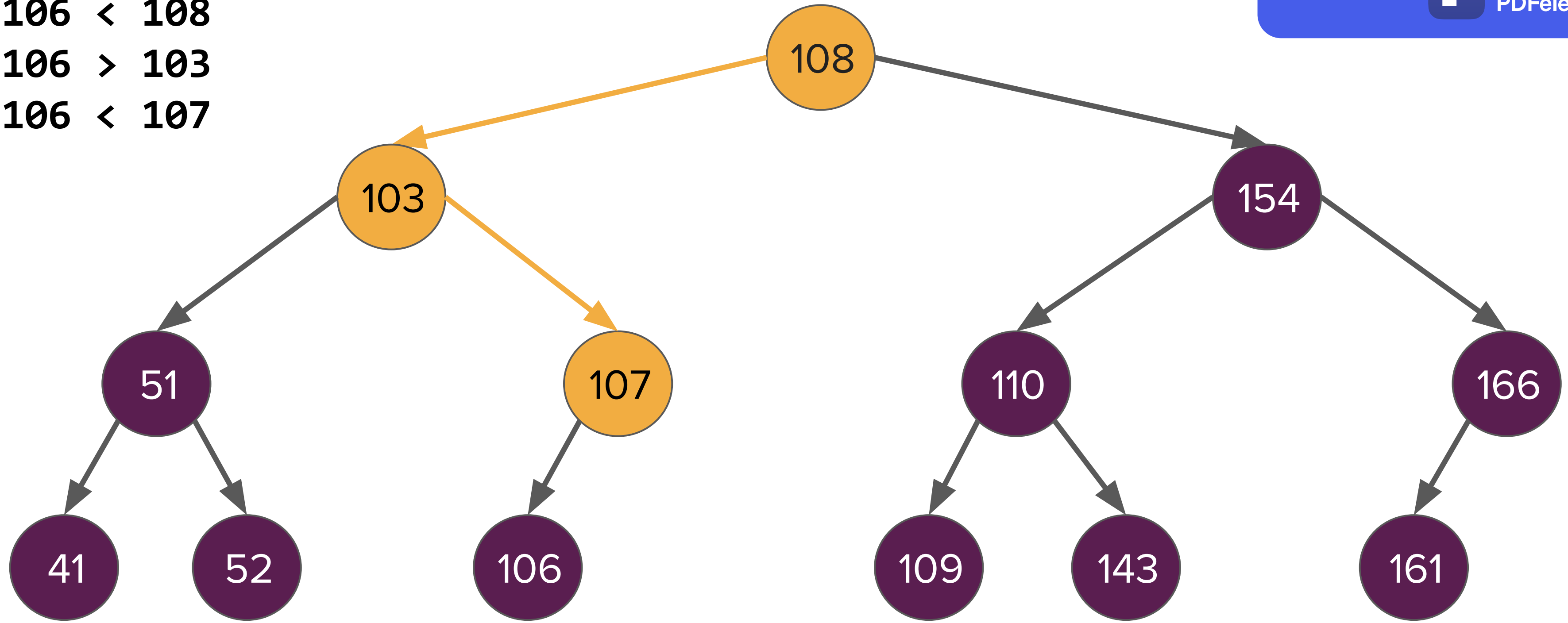


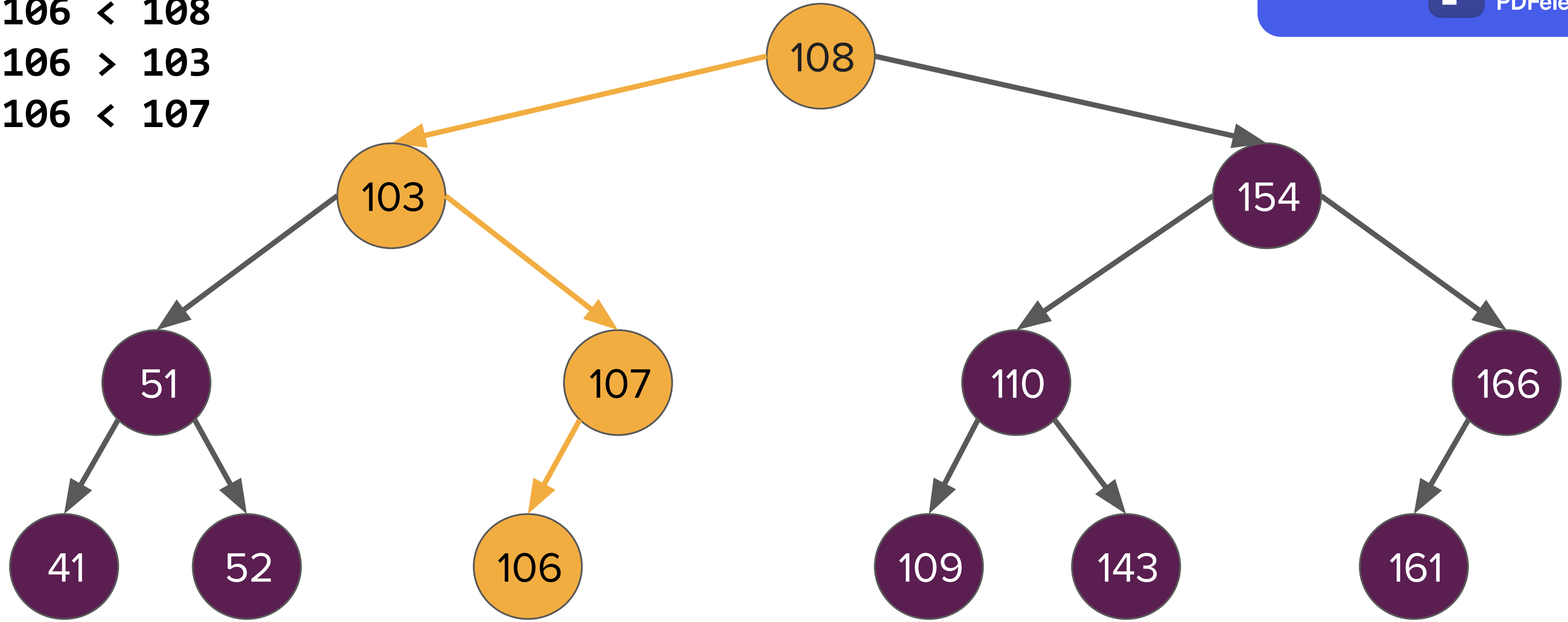
There are 13 nodes in the tree, but the path to each node is short (~`O(log 13)`)!

`106 < 108`
`106 > 103`
`106 < 107`

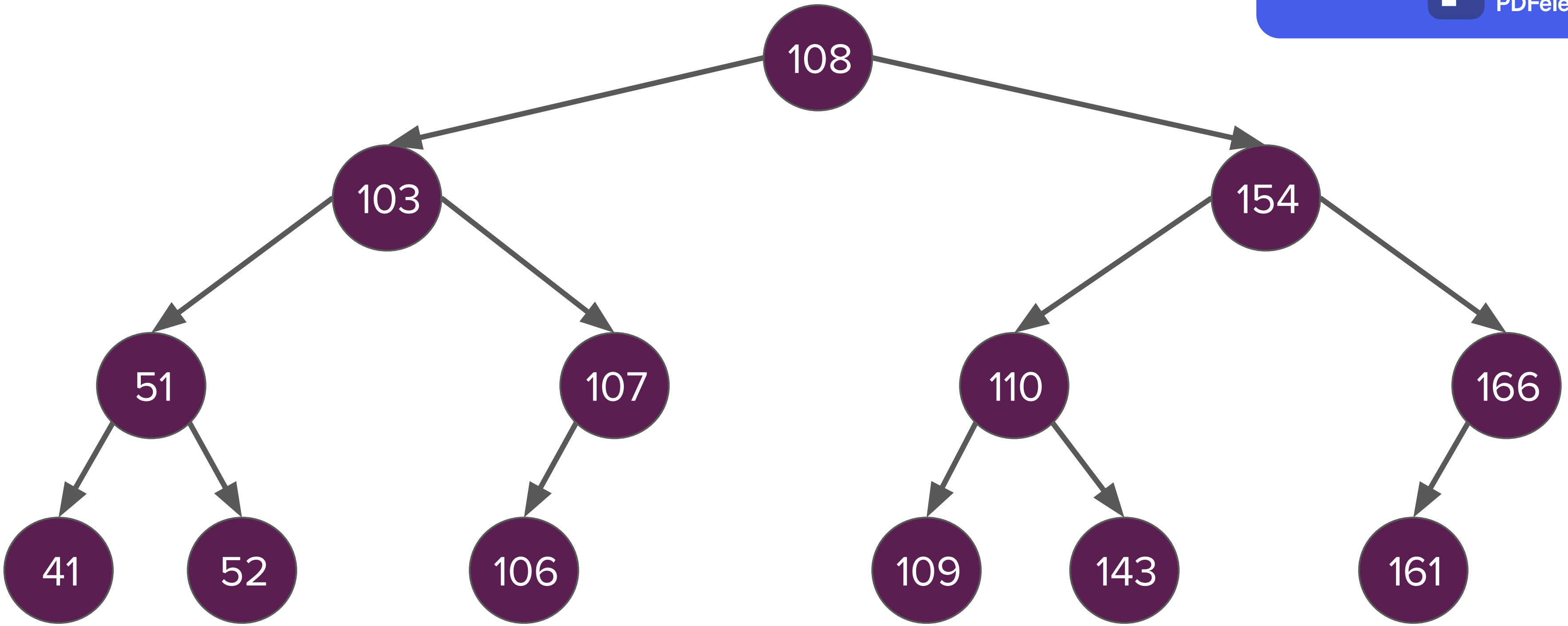


How could we check if **106** is in this tree?

106 < 108
106 > 103
106 < 107



We found **106** so we're done!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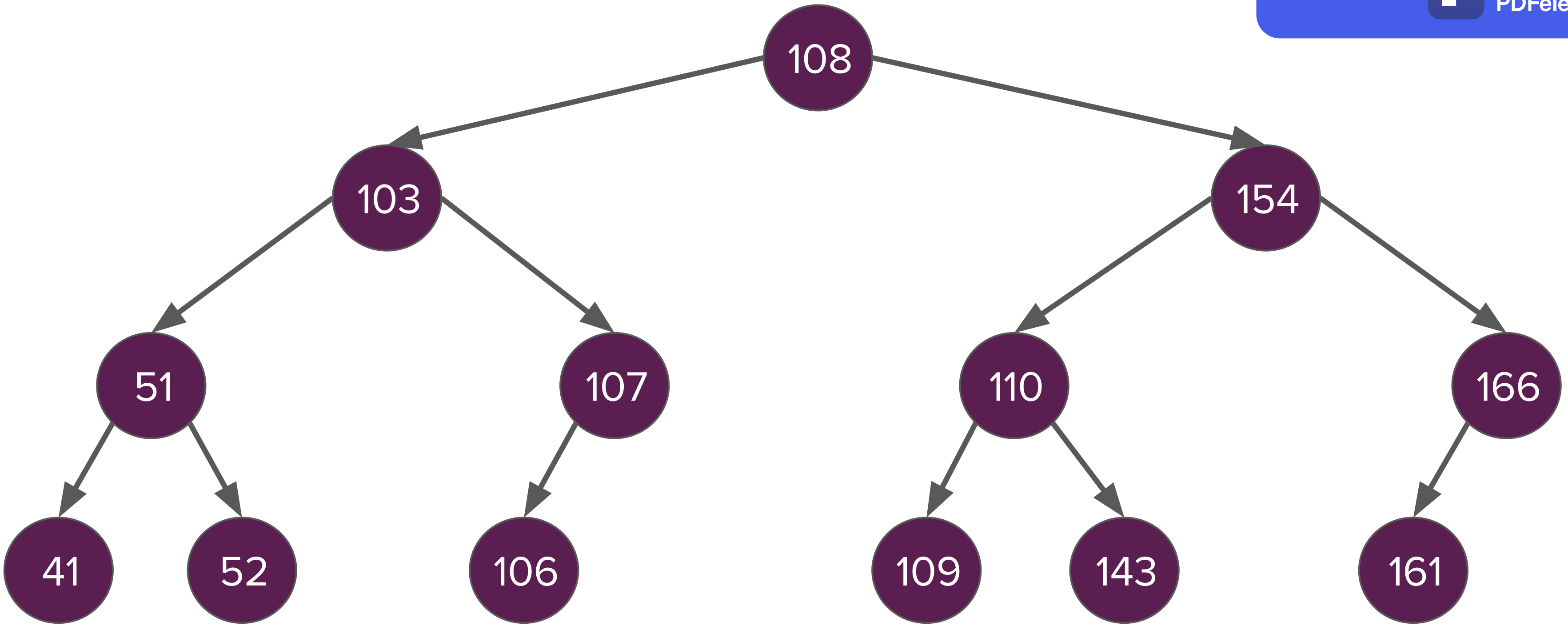
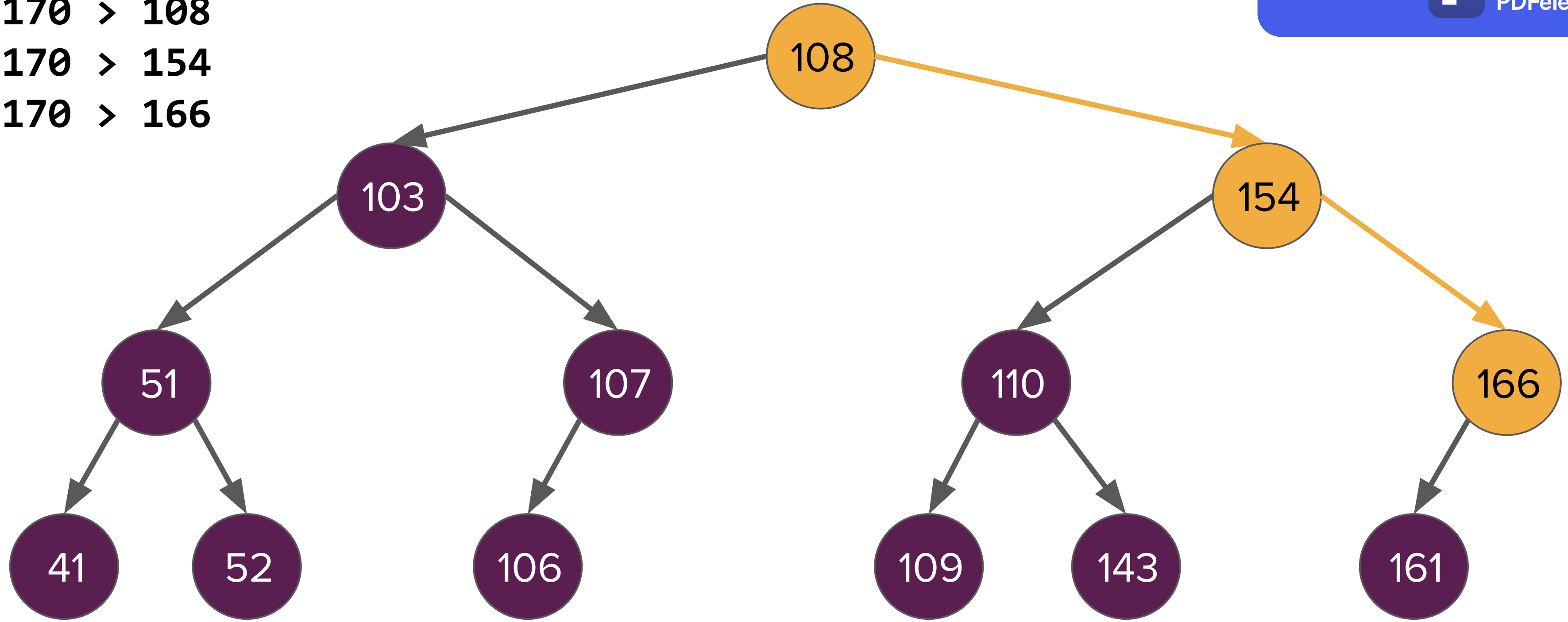


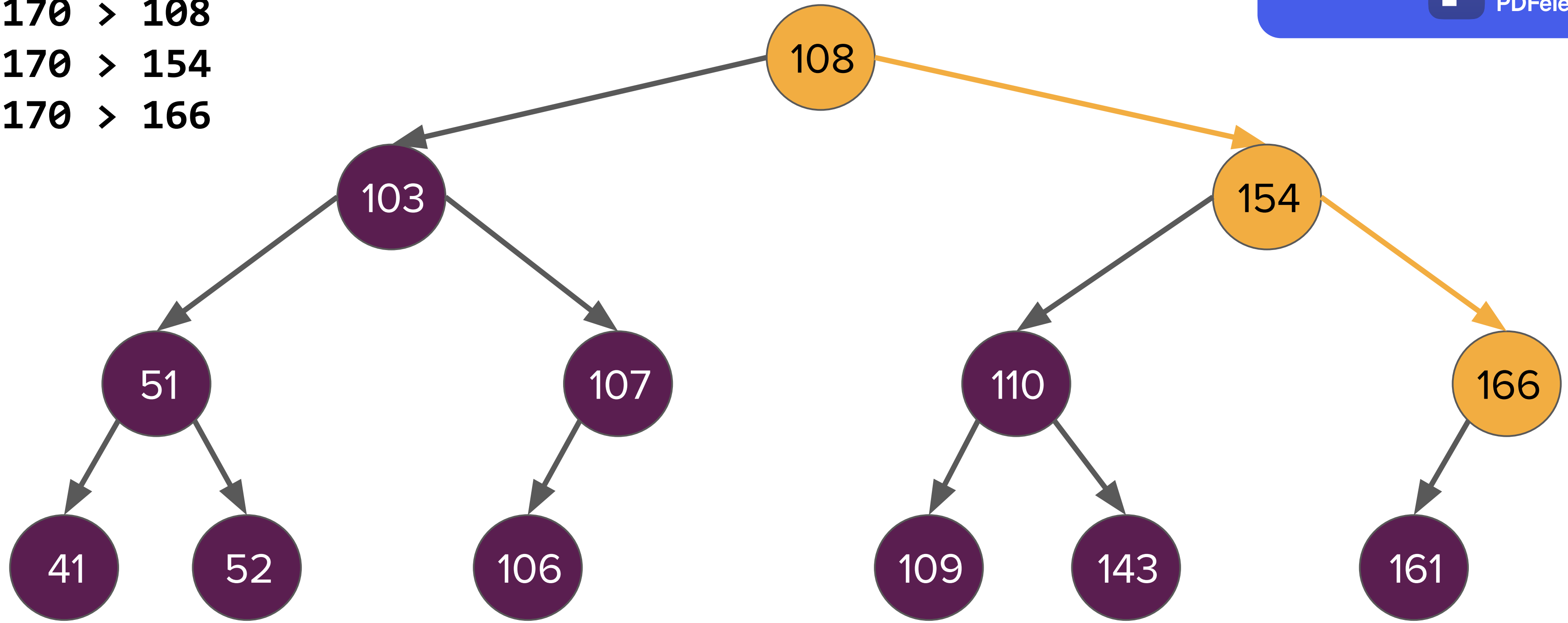
How could we check if **`170`** is in this tree?

170 > 108
170 > 154
170 > 166



How could we check if **170** is in this tree?

`170 > 108`
`170 > 154`
`170 > 166`



Right child is **`nullptr`** so we're done!

Building a BST

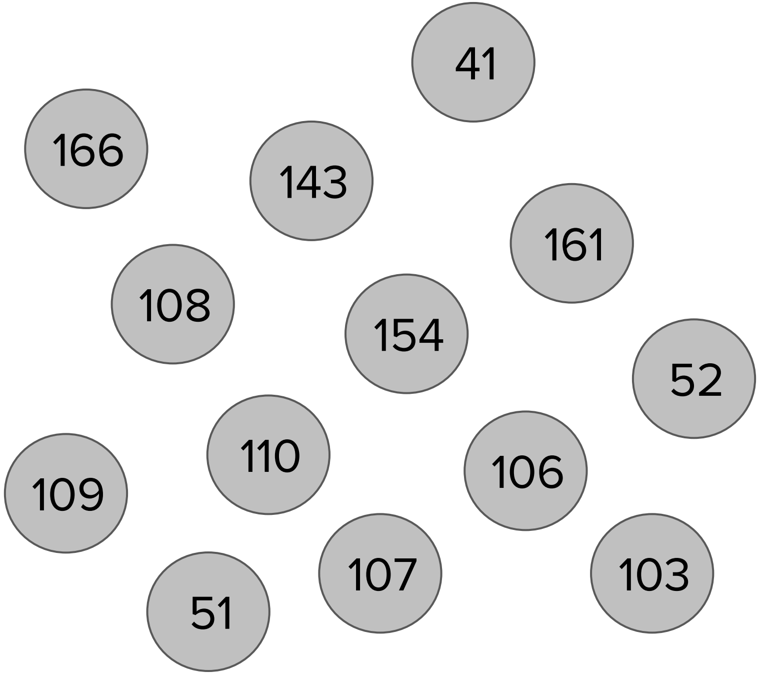
- An **optimal BST** is built by repeatedly choosing the median element as the root node of a given subtree and then separating elements into groups less than and greater than that median.

Building a BST

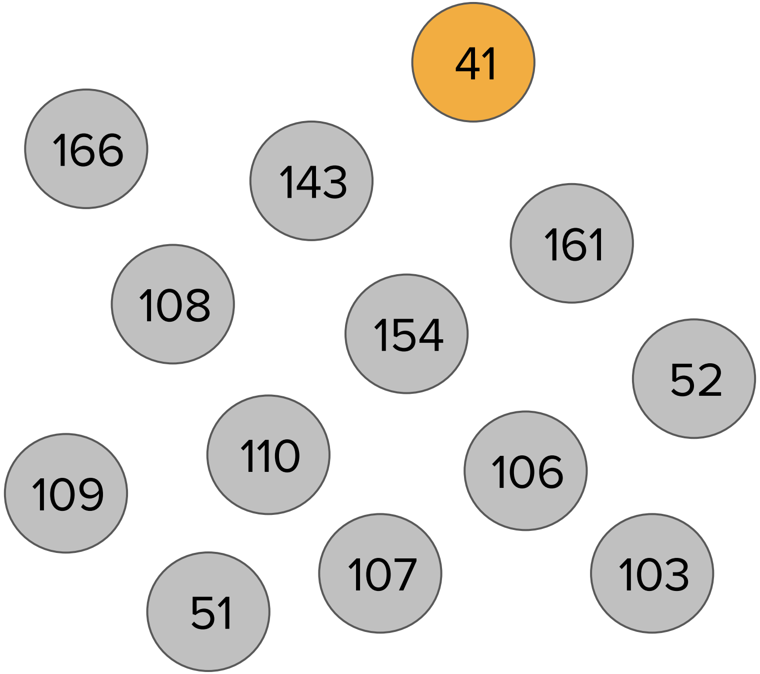
- An **optimal BST** is built by repeatedly choosing the median element as the root node of a given subtree and then separating elements into groups less than and greater than that median.

What does "optimal" mean?

What if we didn’t choose the median?



Let’s choose the smallest element instead…



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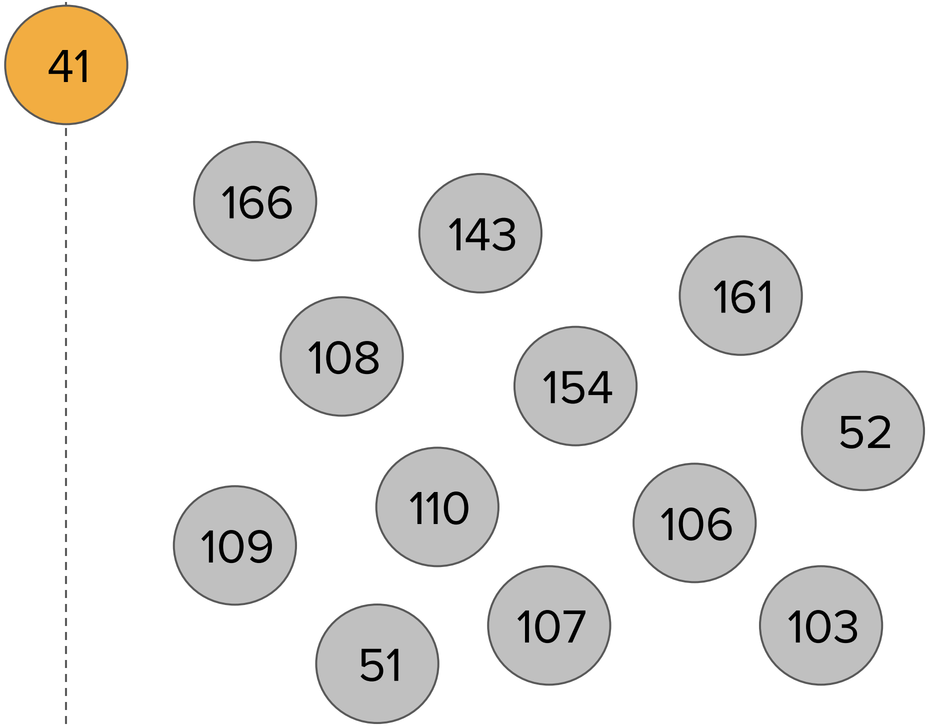

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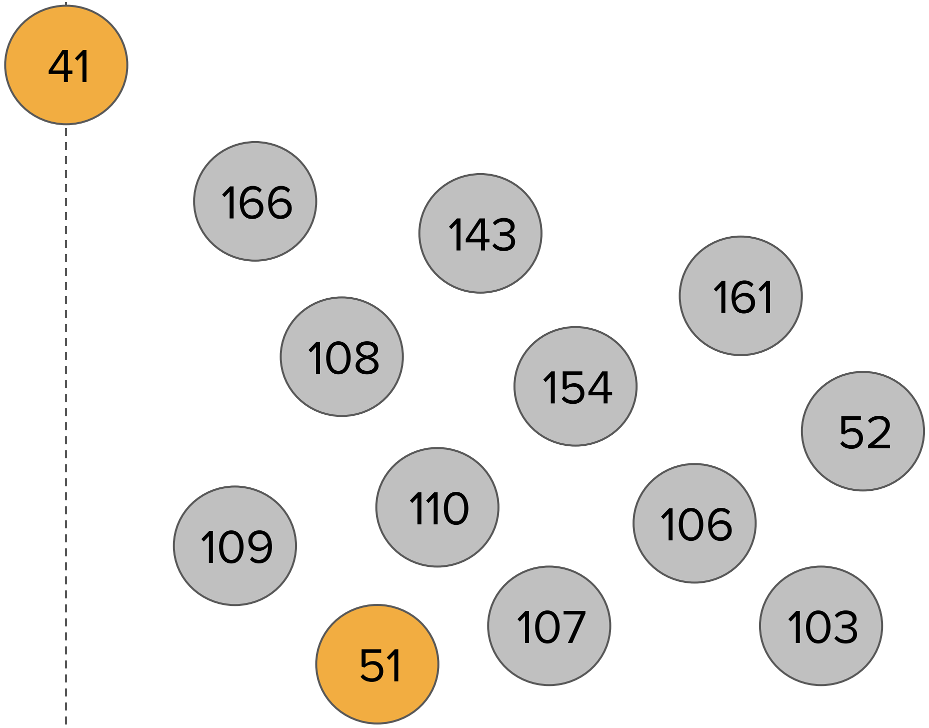
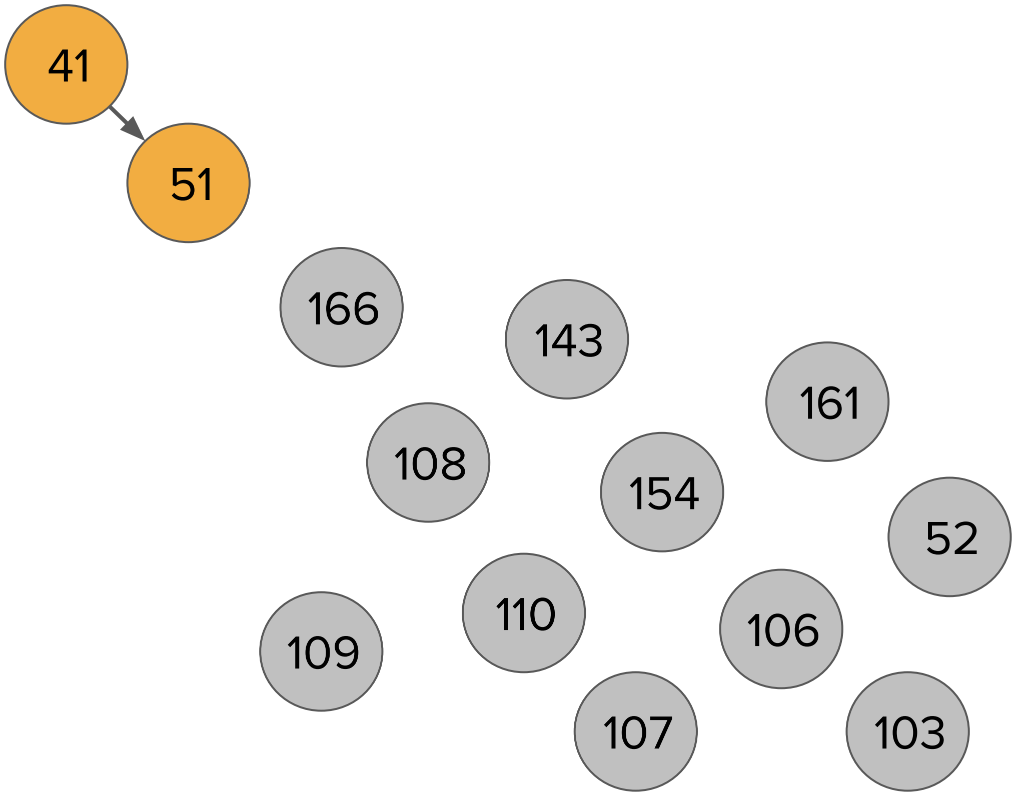


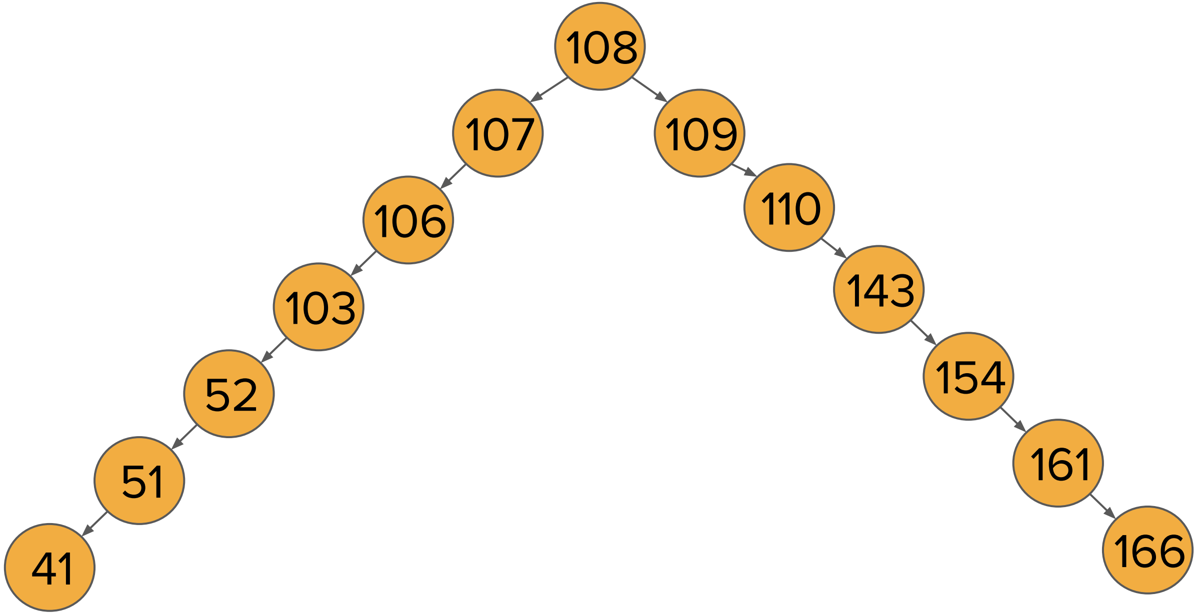
Let’s choose the smallest element instead...



Now our longest path is O(n)!

Takeaways

- There are multiple valid BSTs for the same set of data.
  - Another example with the previous dataset:



Takeaways

- There are multiple valid BSTs for the same set of data.
- How you construct the tree/the order in which you add the elements to the tree matters!
- A binary search tree is **balanced** if its height is `O(log n)`, where **n** is the number of nodes in the tree (i.e. left/right subtrees don't differ in height by more than 1).
  - Lookup, insertion, and deletion with balanced BSTs all operate in `O(log n)` runtime.
  - **Theorem**: If you start with an empty tree and add in random values, then with high probability the tree is balanced. → take CS161 to learn why!
  - A **self-balancing** BST reshapes itself on insertions and deletions to stay balanced (how to do this is beyond the scope of this class).

Announcements

Announcements

- Assignment 5 is due on **tonight at 11:59pm PDT**.

- Assignment 6 will be released by the end of the day tomorrow and will be due on **Wednesday, August 12 at 11:59pm PDT**. This is a hard deadline – there is **no grace period and no submissions will be accepted after this time**.

- Due to the end of quarter timeline, there will be **no revisions on Assignments 5 and 6.**

- Final project reports are due on **Sunday, August 9 at 11:59pm PDT**. You will have the opportunity to schedule your final presentation time after submitting.

Implementing Sets with BSTs

We’re going to implement a Set using a BST!

- Our Set will only store strings as its data type.

```
struct TreeNode {
   std::string data;
   TreeNode* left;
   TreeNode* right;

   // default constructor does not initialize
   TreeNode() {}
   // 3-arg constructor sets fields from arguments
   TreeNode(std::string d, TreeNode* l, TreeNode* r) {
       data = d;
       left = l;
       right = r;
   }
};
```

We’re going to implement a Set using a BST!

- Our Set will only store strings as its data type
- We have a header file that will include a public interface already defined.

OurSet Public Interface

```
class OurSet {
public:
     OurSet();    // constructor
    ~OurSet();    // destructor

    bool contains(string value);
    void add(string value);
    void remove(string value);
    void clear();
    int size();
    bool isEmpty();
    void printSetContents();

private:
     /* To be defined soon! */
};
```

We’re going to implement a Set using a BST!

- Our Set will only store strings as its data type
- We have a header file that will include a public interface already defined.
- As we write the Set methods, think about how their runtimes would change for a balanced vs. an unbalanced BST.
  - Note: Actual sets are self-balancing, but we won’t go into the details of how to implement that!

How do we design **`OurSet`**?

We must answer the following three questions:

1. Member functions: *What public interface should* **`OurSet`** *support? What functions might a client want to call?*

2. Member variables: *What private information will we need to store in order to keep track of the data stored in* **`OurSet`**?

3. Constructor: *How are the member variables initialized when a new instance of* **`OurSet`** *is created?*

`OurSet` Public Interface

```
class OurSet {
public:
     OurSet();     // constructor
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    bool contains(string value);
    void add(string value);
    void remove(string value);
    void clear();
    int size();
    bool isEmpty();
    void printSetContents();

private:
     /* To be defined soon! */
};
```

Let’s code it!

(constructor, destructor, `clear()`, etc.)

OurSet Public Interface

```
class OurSet {
public:
     OurSet();    // constructor
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    void clear();
    int size();
    bool isEmpty();
    void printSetContents();

private:
     /* To be defined soon! */
};
```
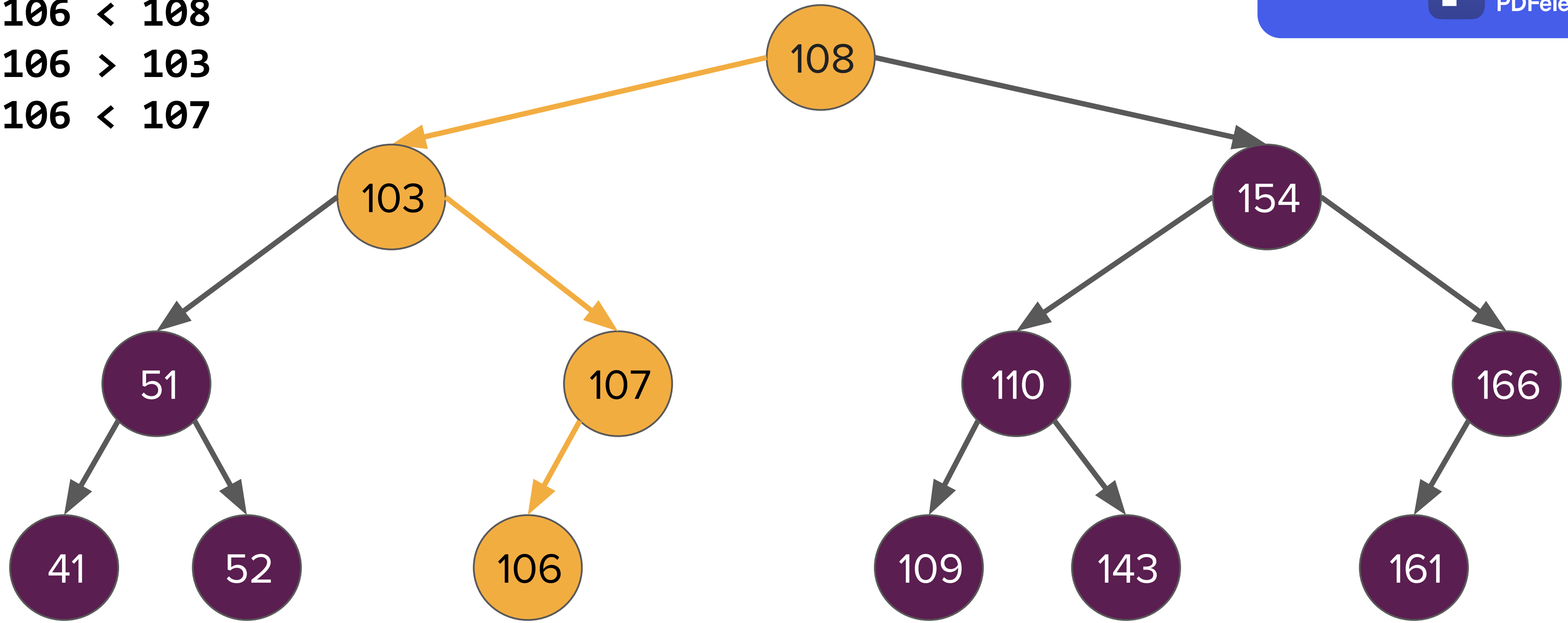

106 < 108
106 > 103
106 < 107



We found **106** so we're done!

Let's code it!

(`contains()`, `add()`)

`OurSet` summary

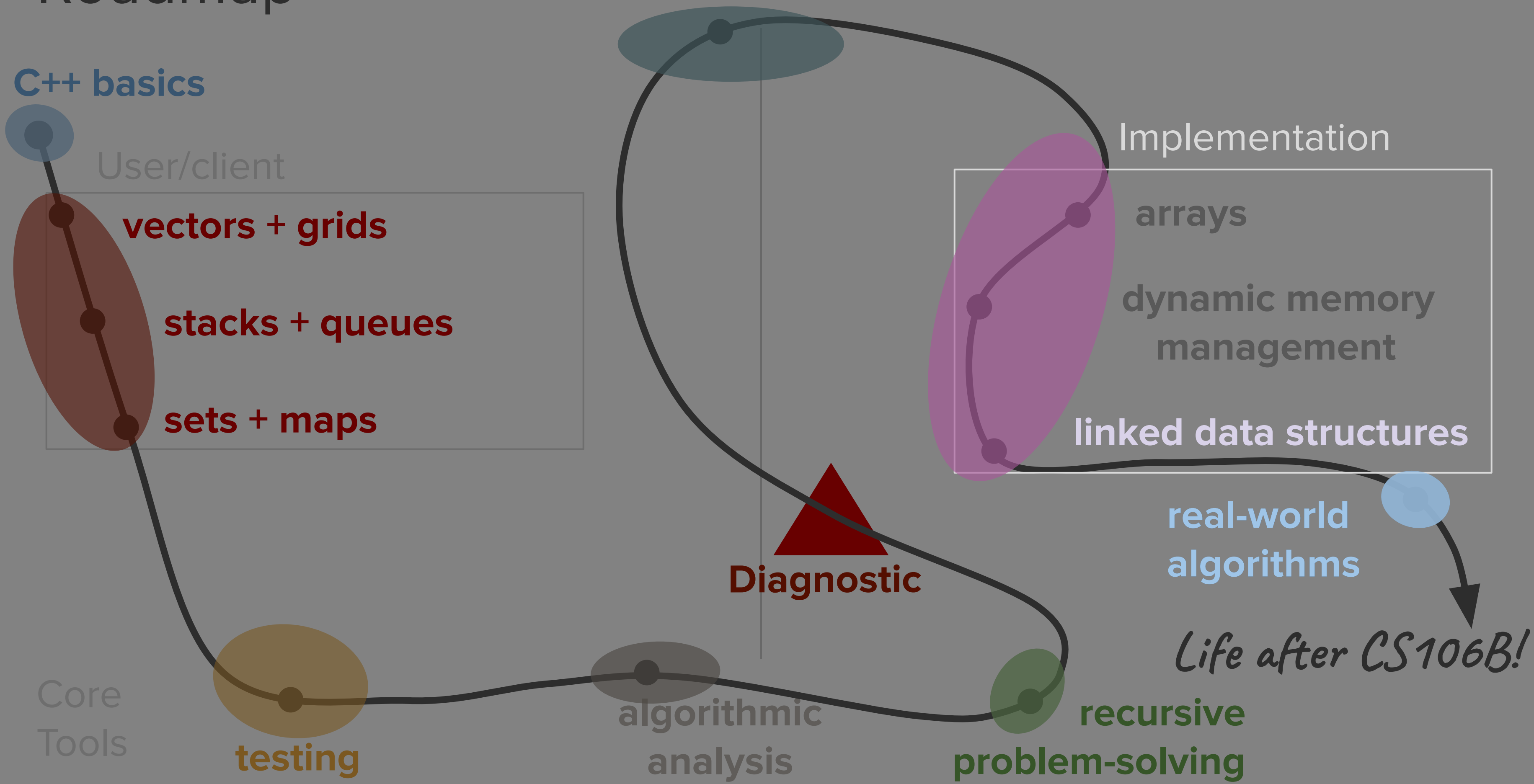
- Our tree utility functions (**`inorderPrint`**, **`freeTree`**) showed up as private member functions/helpers!
  - In-order traversal prints our elements in the correctly sorted order!

- Using a BST allowed us to take advantage of recursion to traverse our data and get an O(log n) runtime for our methods.

- Rewiring trees can be complicated!
  - Make sure to consider when nodes need to be passed by reference.
  - Check out the remove method after class if you're interested in seeing an example of tree rewiring (you won't be required to do anything this complex with tree rewiring).

What's next?

Roadmap



Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Levels of abstraction

What is the interface for the user?

How is our data organized?
(binary heaps, BSTs, **Huffman trees**)

What stores our data?
(arrays, linked lists, **trees**)

How is data represented electronically?
(RAM)

Abstr
Structures

Data Organization Strategies

Fundamental C++ Data Storage

Computer Hardware

Huffman coding