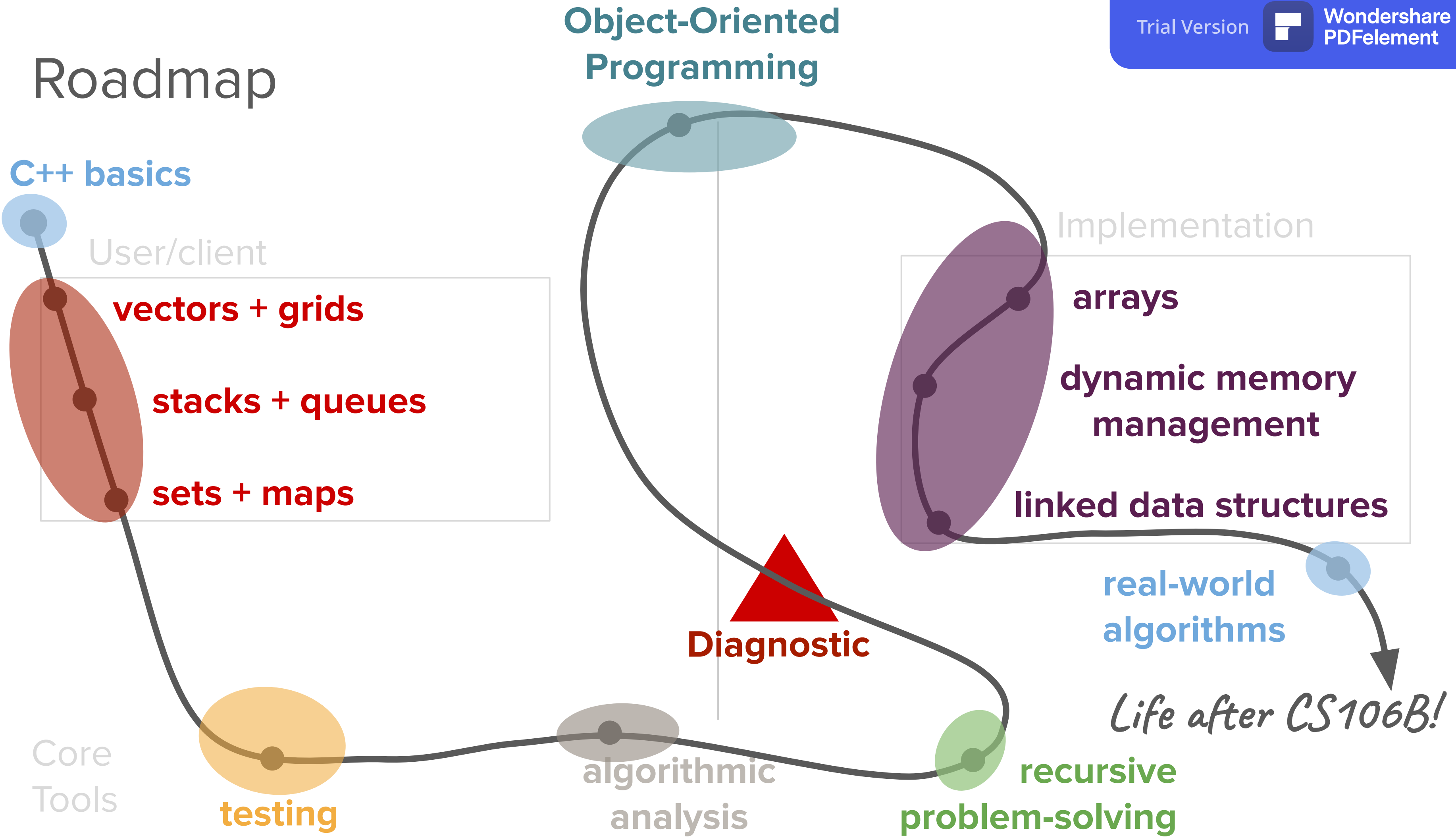
Recursive Fractals

What examples of recursion have you encountered in day-to-day life (not programming-related)?

(put your answers the chat)

Roadmap



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C++ basics
User/client
vectors + grids
stacks + queues
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Diagnostic
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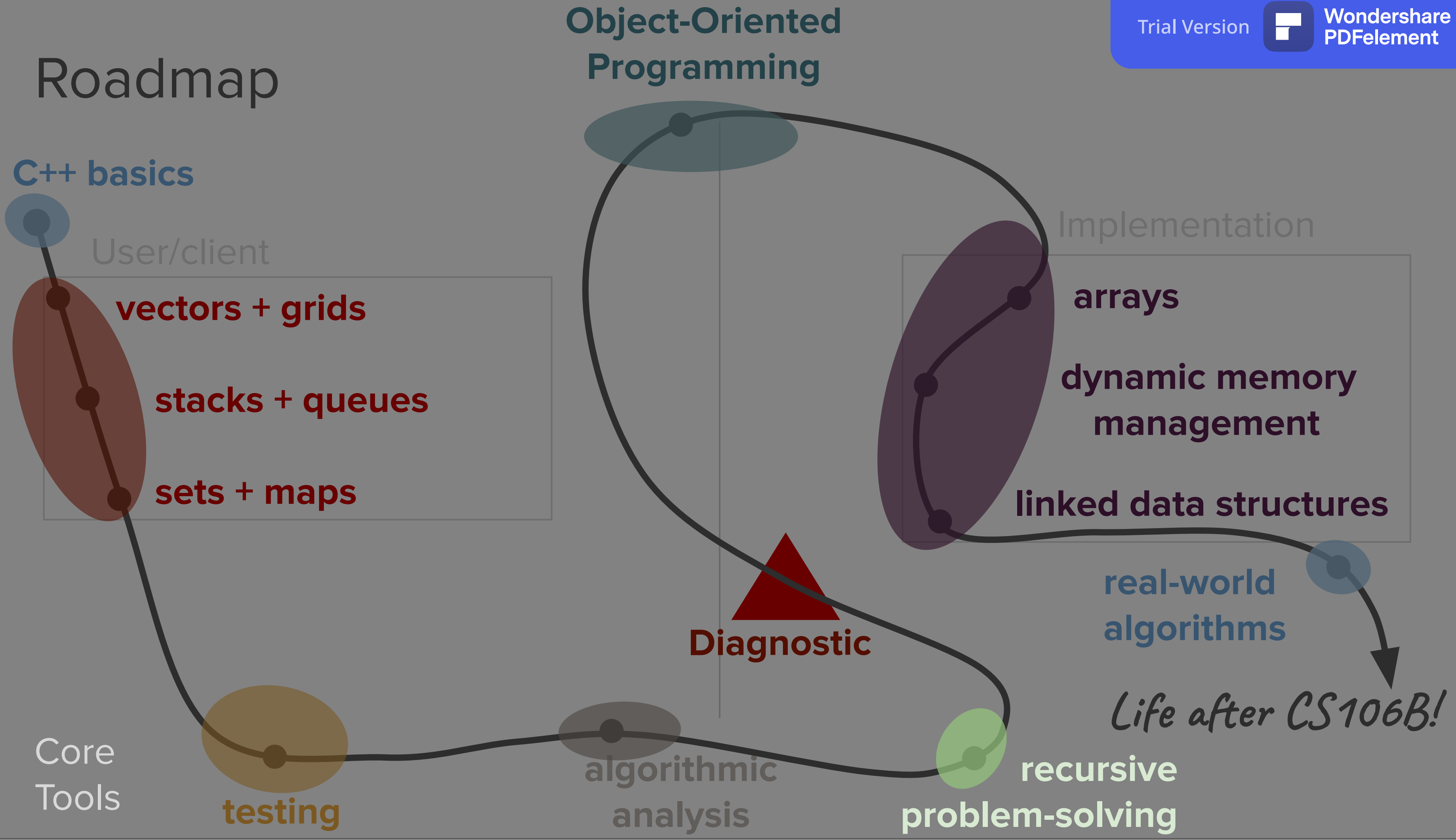
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Today's question

How can we use visual representations to understand recursion?

How can we use recursion to make art?

Today's topics

1. Review
2. Defining recursion in the context of fractals
3. The Cantor Set
4. The Sierpinski Carpet
5. Revisiting the Towers of Hanoi

Review

Definition

recursion

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
    - A recursive operation (function) is defined in terms of itself (i.e. it calls itself).

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.
    - Base case: Simplest form of the problem that has a direct answer.
    - Recursive case: The step where you break the problem into a smaller, self-similar task.

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
- Recursion has two main parts: the **base case** and the **recursive case**.
- The solution will get built up **as you come back up the call stack**.
    - The base case will define the “base” of the solution you’re building up.
    - Each previous recursive call contributes a little bit to the final solution.
    - The initial call to your recursive function is what will return the completely constructed answer.

Recursion Review

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- The solution will get built up **as you come back up the call stack**.

- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

Recursion Review

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

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- The solution will get built up **as you come back up the call stack**.

- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

Example:

`isPalindrome()`

Write a function that returns if a string is a palindrome

A string is a palindrome if it reads the same both forwards and backwards:

- isPalindrome(“level”) ➔ true
- isPalindrome(“racecar”) ➔ true
- isPalindrome(“step on no pets”) ➔ true
- isPalindrome(“high”) ➔ false
- isPalindrome(“hi”) ➔ false
- isPalindrome(“palindrome”) ➔ false
- isPalindrome(“X”) ➔ true
- isPalindrome(“”) ➔ true

Approaching recursive problems

- Look for self-similarity.

- Try out an example and look for patterns.
  - Work through a simple example and then increase the complexity.
  - Think about what information needs to be “stored” at each step in the recursive case (like the current value of **`n`** in each **`factorial`** stack frame).

- Ask yourself:
  - What is the base case? (What is the simplest case?)
  - What is the recursive case? (What pattern of self-similarity do you see?)

Discuss:

What are the base and recursive cases?

(breakout rooms)

isPalindrome()

- Look for self-similarity: **racecar**
    - Look at the first and last letters of “racecar” → both are ‘r’
    - Check if “aceca” is a palindrome:
        - Look at the first and last letters of “aceca” → both are ‘a’
        - Check if “cec” is a palindrome:
            - Look at the first and last letters of “cec” → both are ‘c’
            - Check if “e” is a palindrome:
                - **Base case**: “e” is a palindrome

What about the `false` *case?*

isPalindrome()

- Look for self-similarity: **high**
  - Look at the first and last letters of “high” ➜ both are ‘h’
  - Check if “ig” is a palindrome:
    - Look at the first and last letters of “ig” ➜ not equal
    - **Base case**: Return `false`

isPalindrome()

- **Base cases**:
  - isPalindrome(“”) ➜ `true`
  - isPalindrome(string of length 1) ➜ `true`
  - If the first and last letters are not equal ➜ `false`

- **Recursive case:** If the first and last letters are equal, isPalindrome(string) = isPalindrome(string minus first and last letters)

isPalindrome()

There can be multiple base (or recursive) cases!

- **Base cases**:
  - isPalindrome(“”) → `true`
  - isPalindrome(string of length 1) → `true`
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isPalindrome()

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bool isPalindrome (string s) {
    if (s.length() < 2) {
        return true;
    } else {
      if (s[0] != s[s.length() - 1]) {
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      }
      return isPalindrome(s.substr(1, s.length() - 2));
    }
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isPalindrome() in action

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int main() {
    cout << boolalpha <<
         isPalindrome("racecar")
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    return 0;
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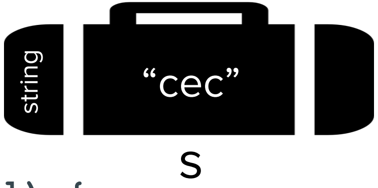

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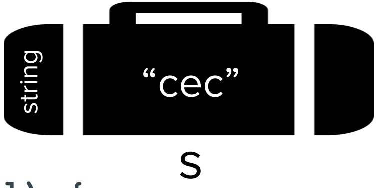

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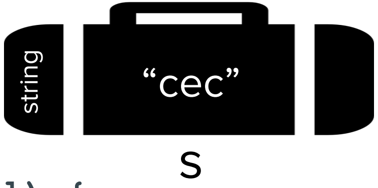

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    return 0;
}
```

Prints **true**!

How can we use visual representations to understand recursion?

Self-Similarity

Self-Similarity

- Solving problems recursively and analyzing recursive phenomena involves identifying **self-similarity**

- An object is **self-similar** if it contains a smaller copy of itself.

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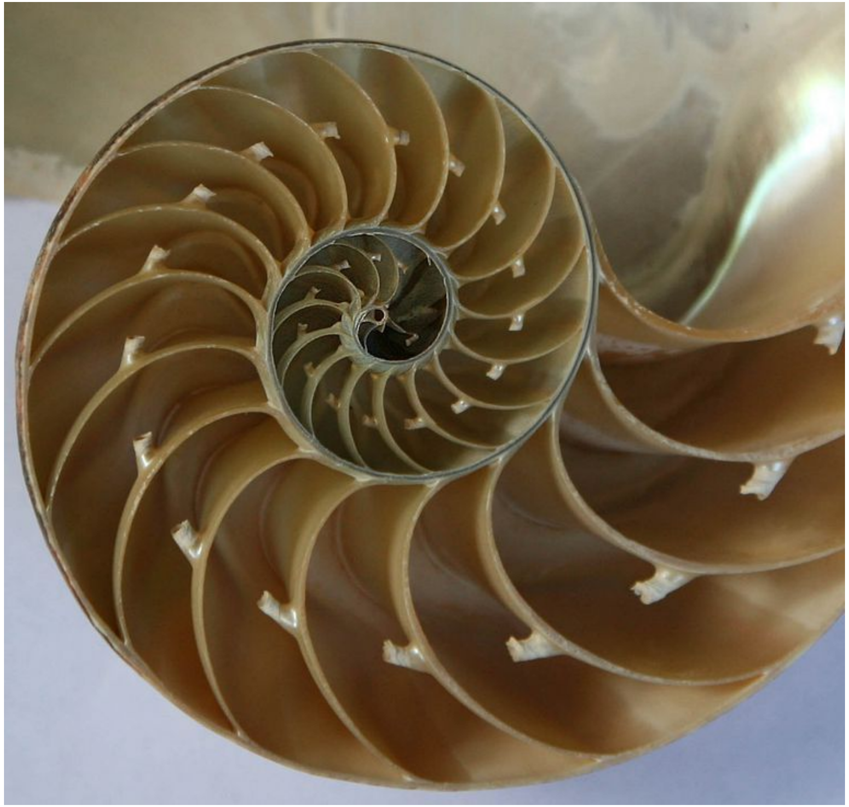

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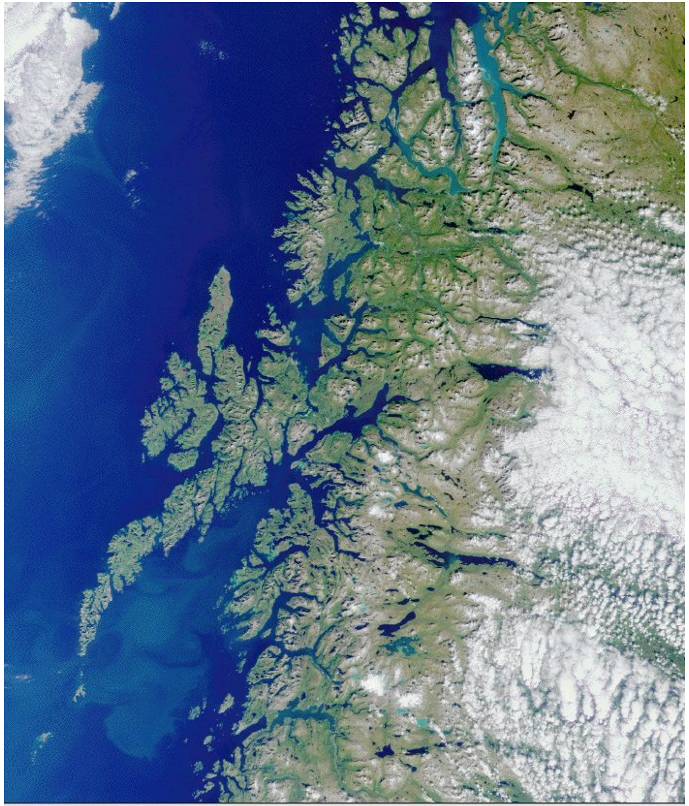


Self-Similarity

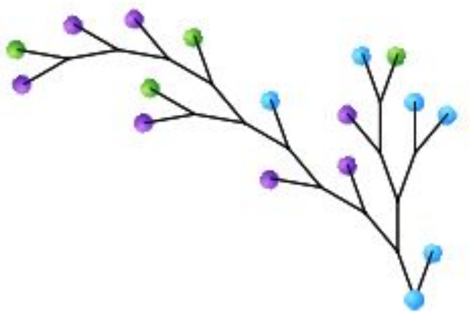
- Solving problems recursively and analyzing recursive phenomena involves identifying **self-similarity**
- An object is **self-similar** if it contains a smaller copy of itself.

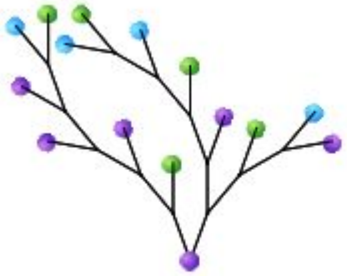
Self-similarity shows up in many real-world objects and phenomena, and is the key to truly understanding their formation and existence.

Graphical Representations of Recursion

Graphical Representations of Recursion

- Our first exposure to recursion yesterday was graphical in nature!
  - "Vee" is a recursive program that traces the path of a sprite in Scratch
  - The sprite draws out a funky tree-like structure as it goes along its merry way
- Graphical representations of recursion allow us to visualize the result of having **multiple recursive calls**
  - Understanding this "branching" of the tree is critical to solving challenging problems with recursion





Fractals

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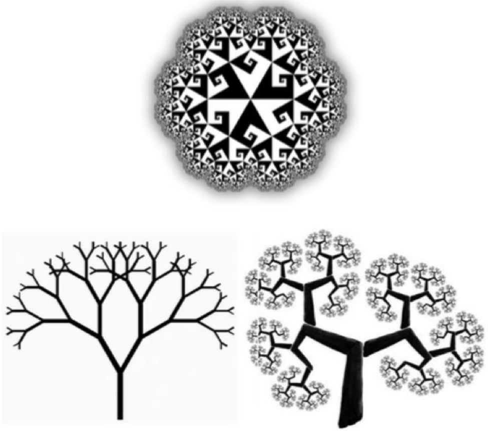
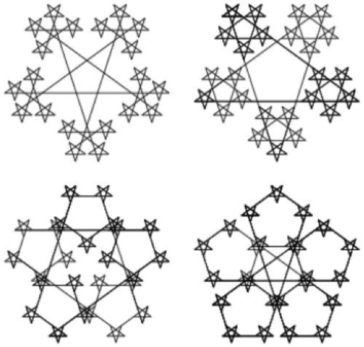
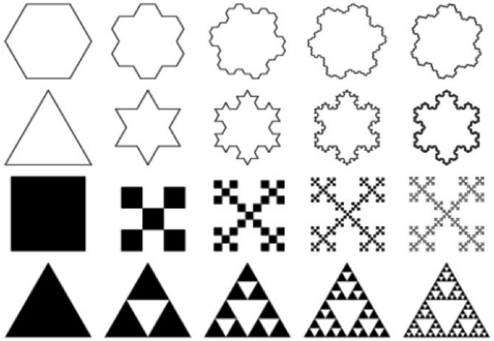
- A **fractal** is any repeated, graphical pattern.

- A fractal is composed of **repeated instances of the same shape or pattern**, arranged in a structured way.







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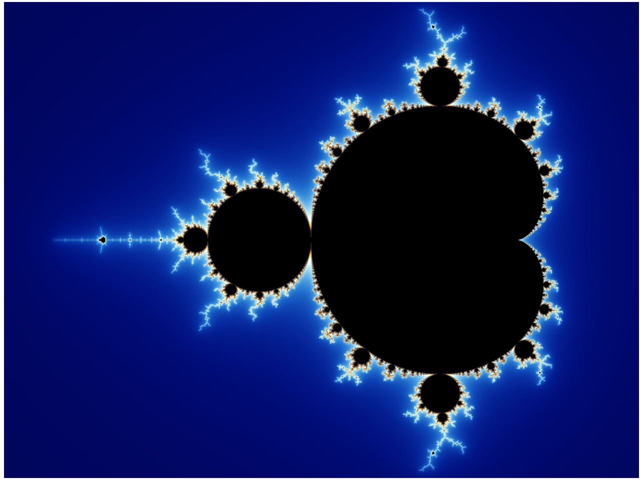
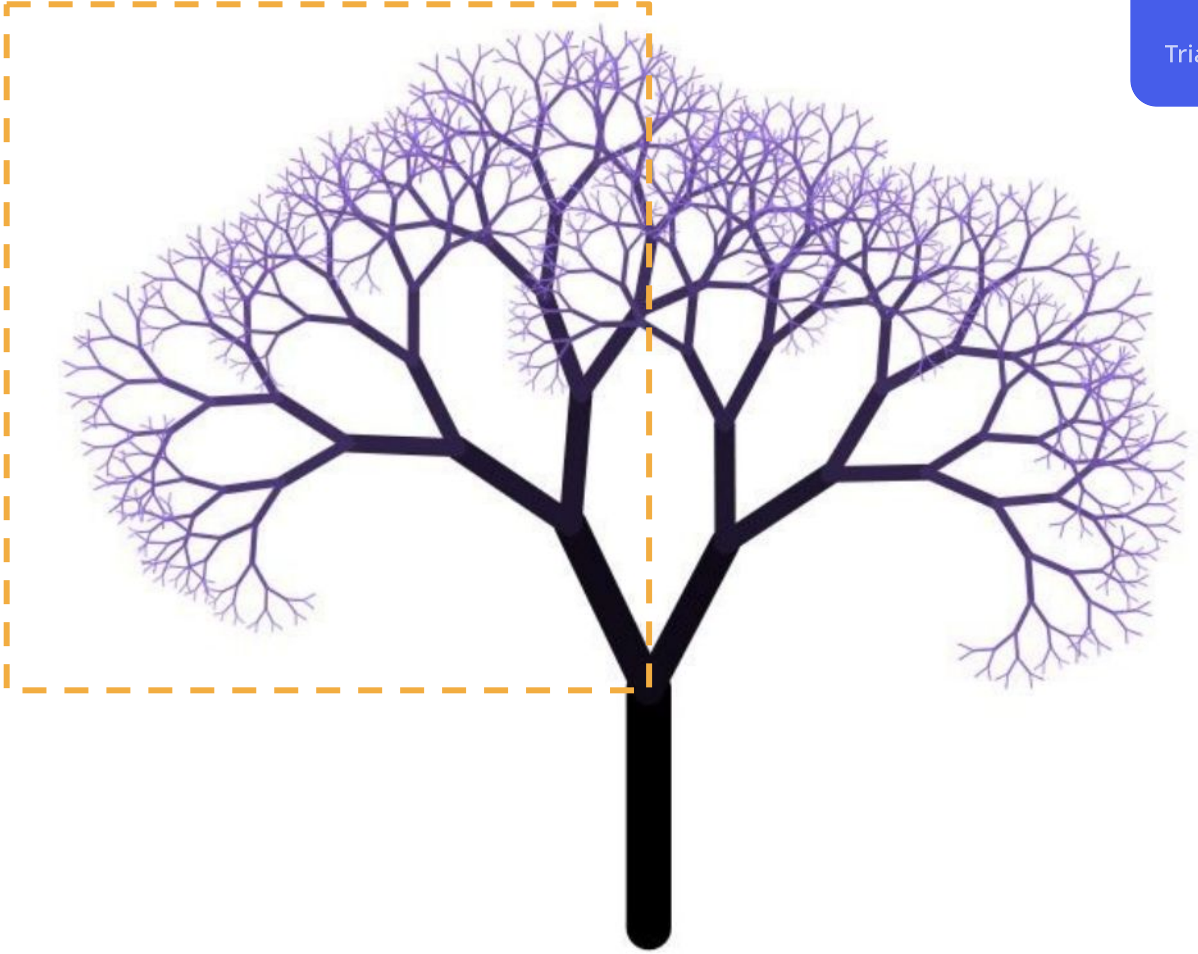
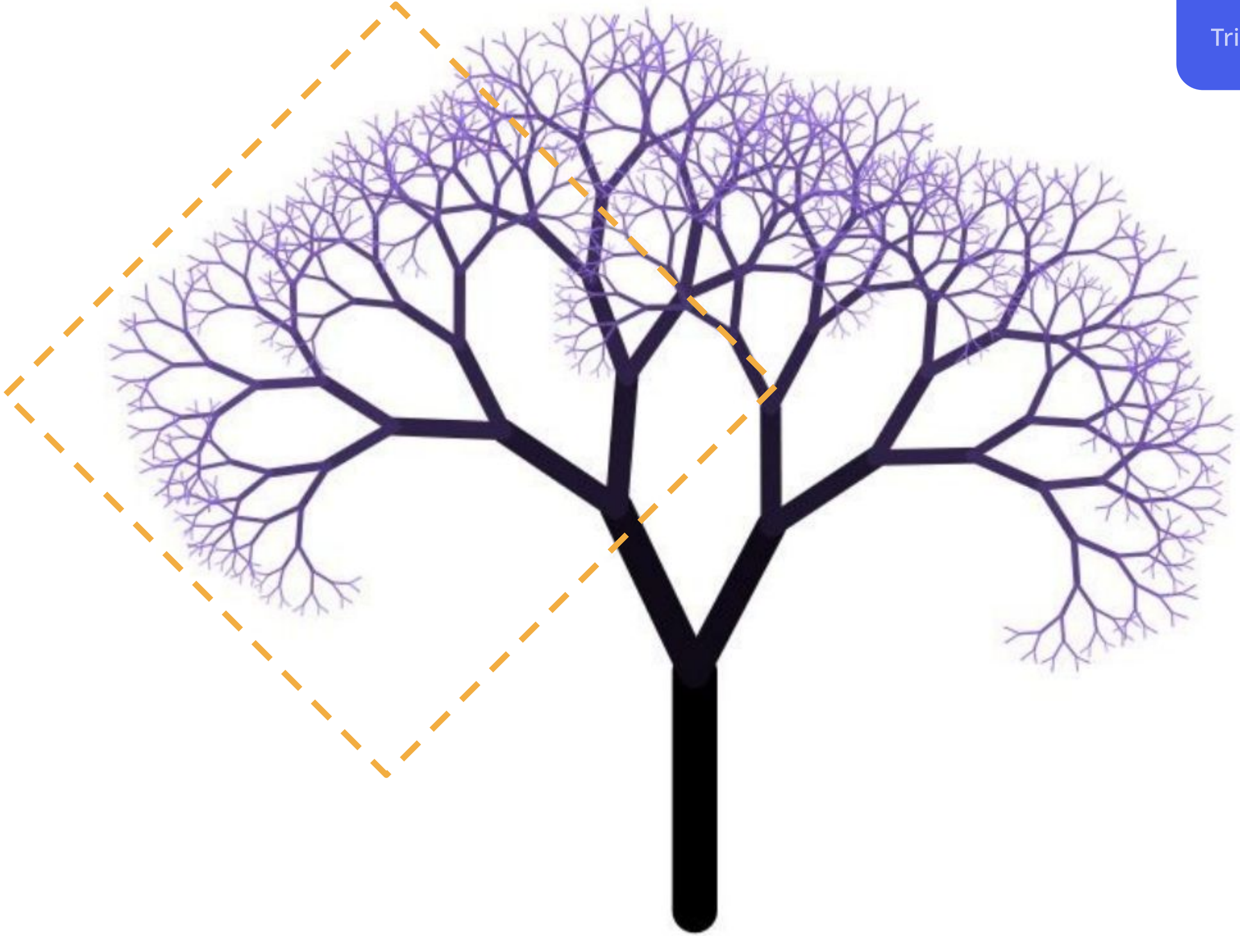
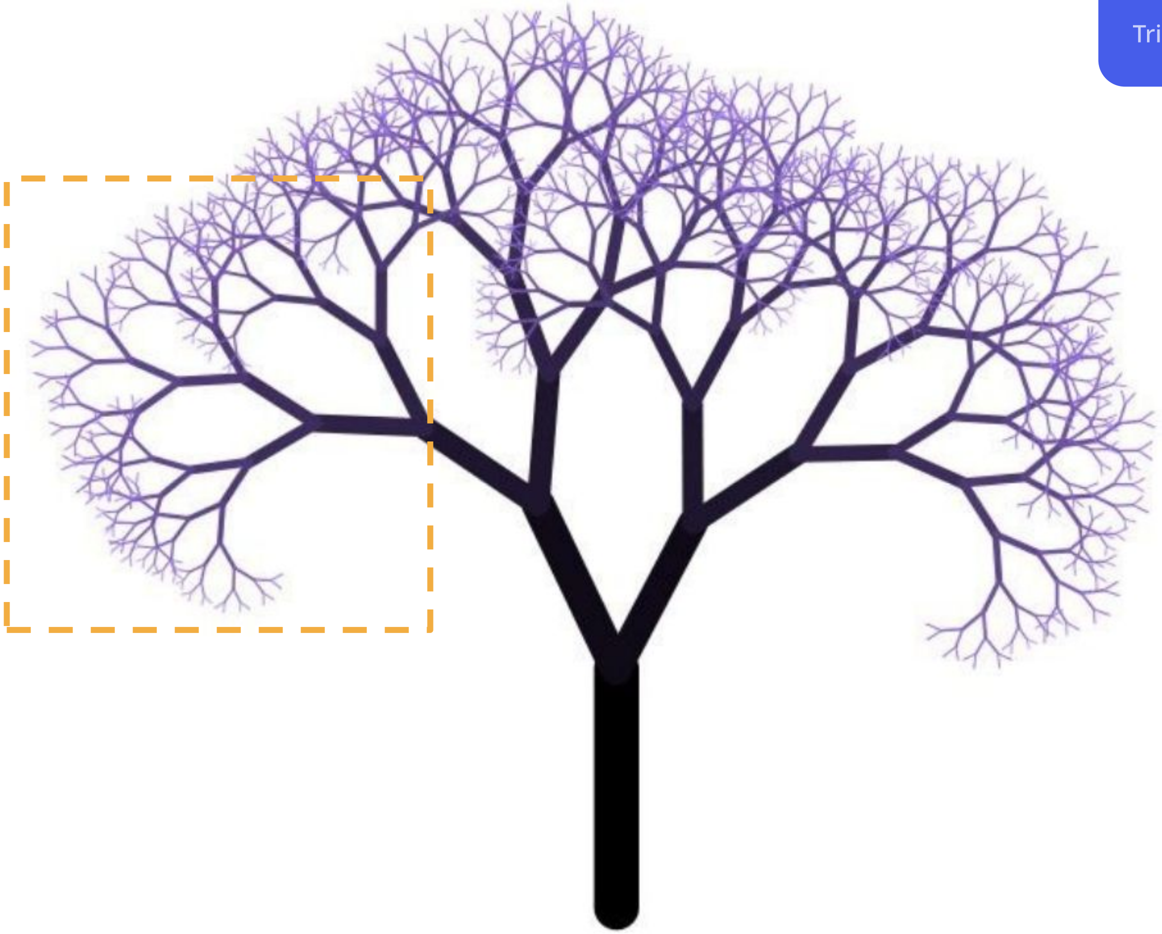
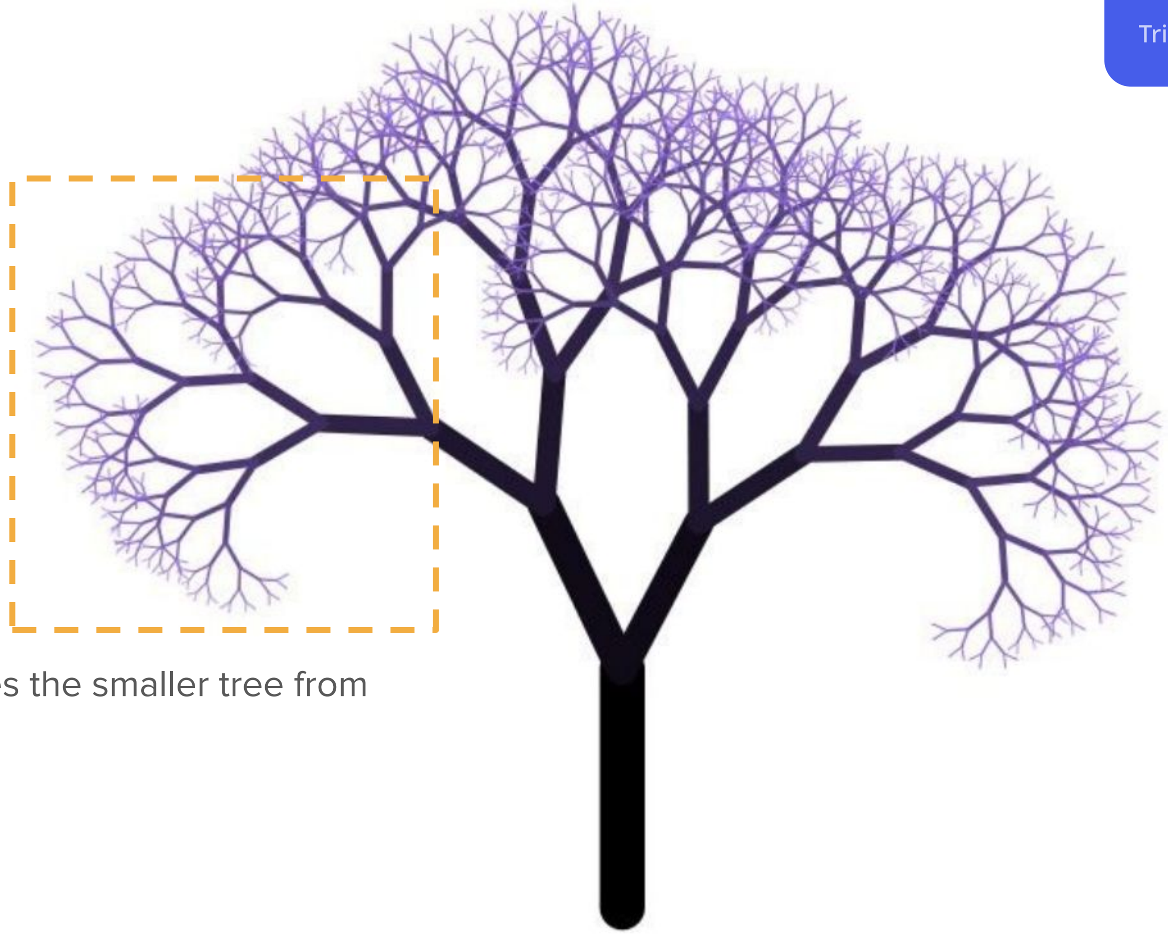


Understanding Fractal Structure

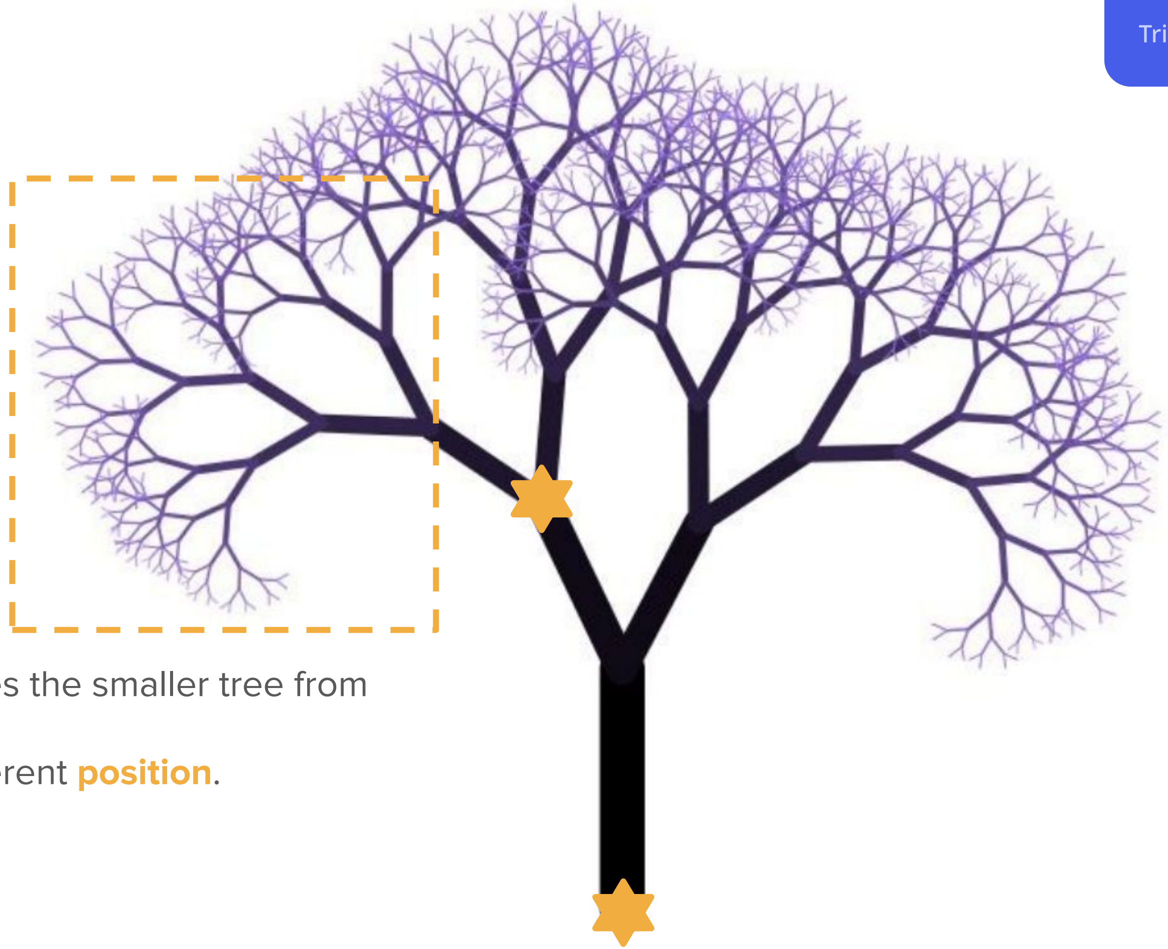






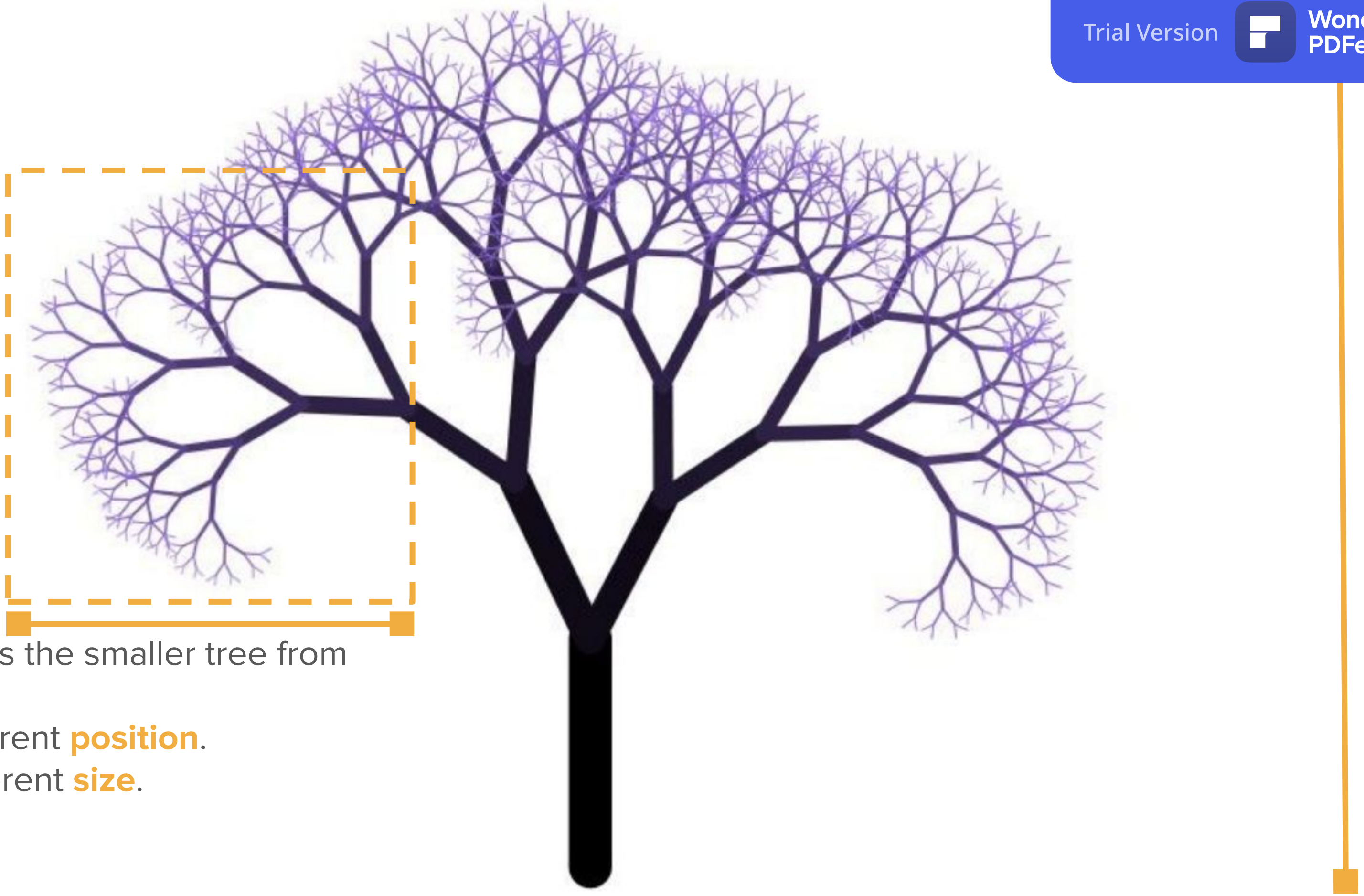


What differentiates the smaller tree from the bigger one?



What differentiates the smaller tree from the bigger one?

1. It's at a different **position**.



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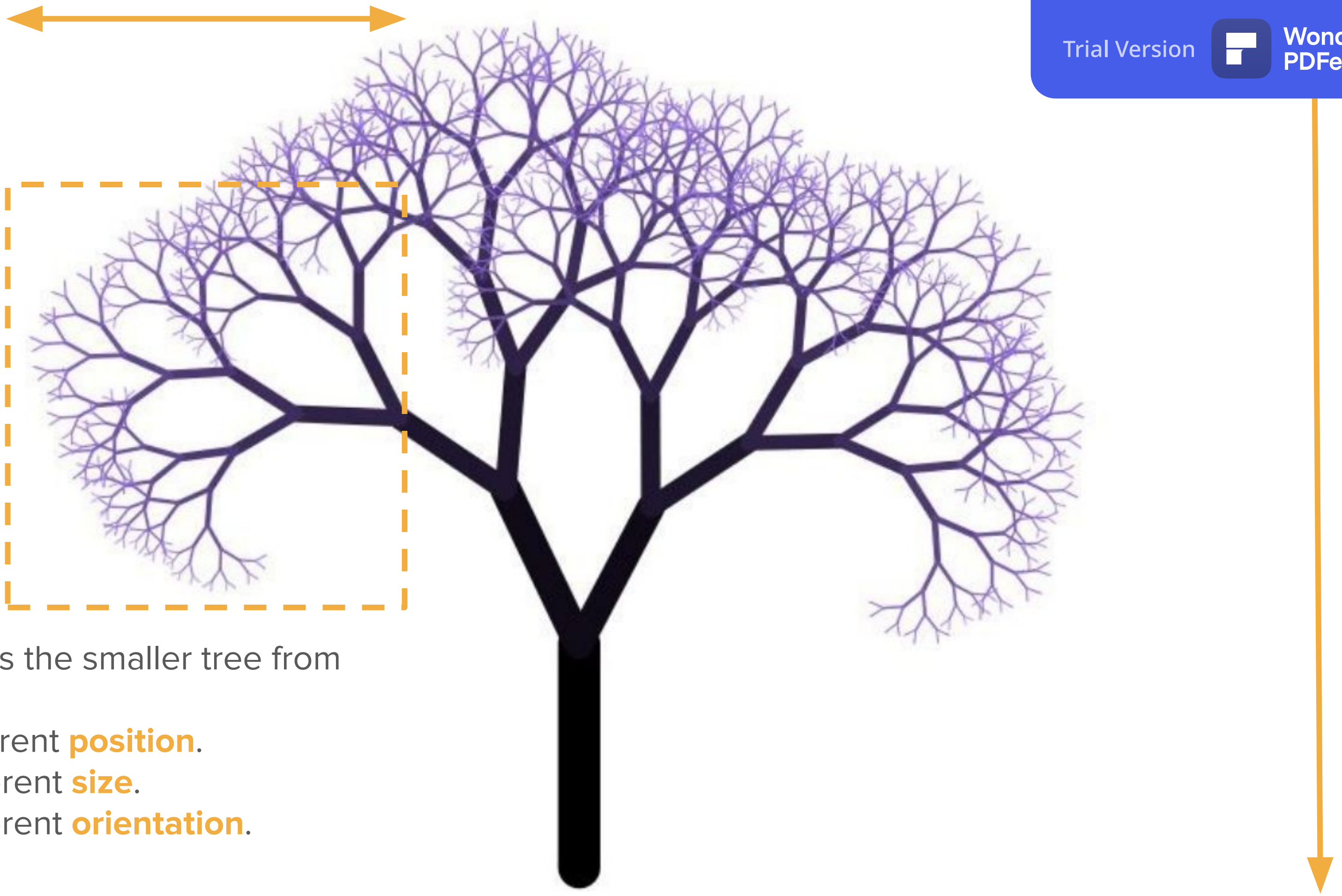

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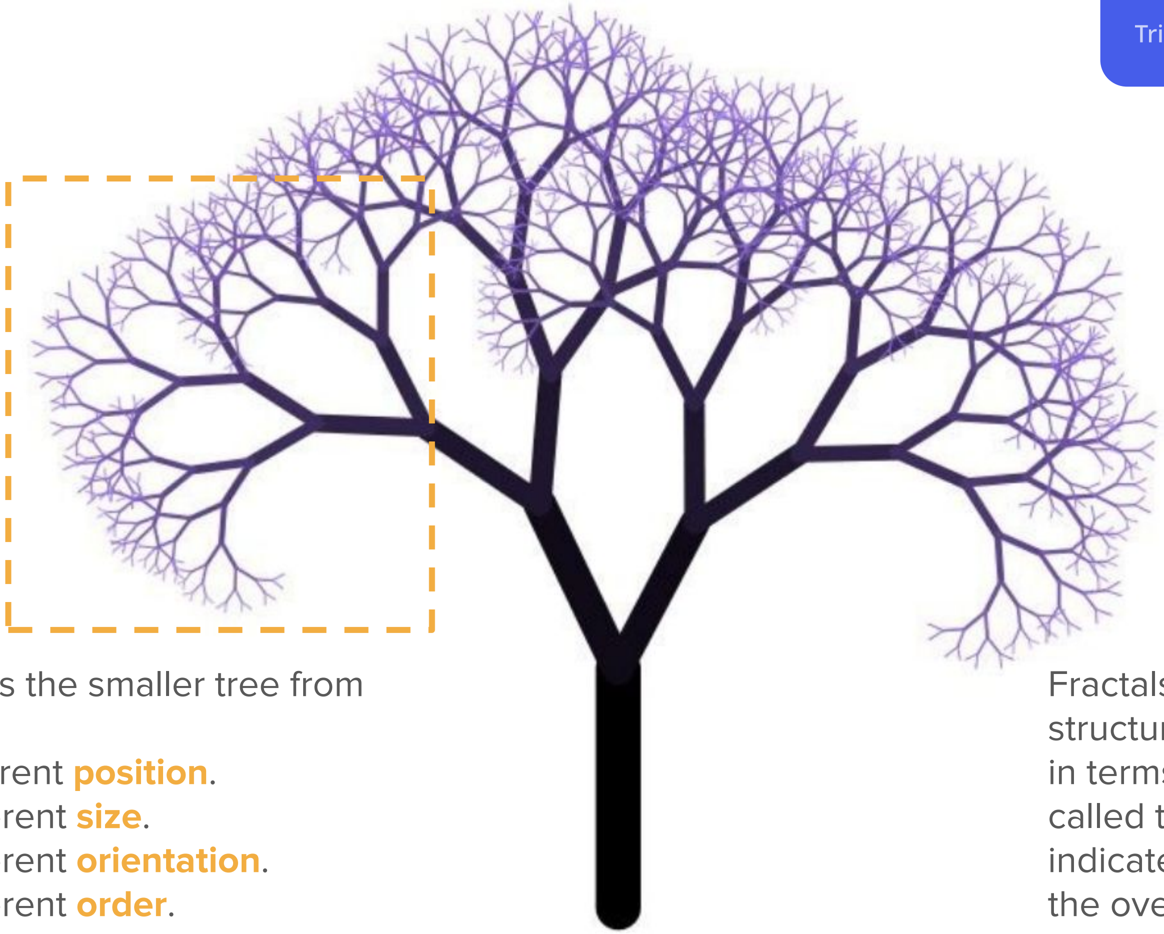

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Fractals and self-similar structures are often defined in terms of some parameter called the **order**, which indicates the complexity of the overall structure.

An order-0 tree

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An order-1 tree

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An order-2 tree

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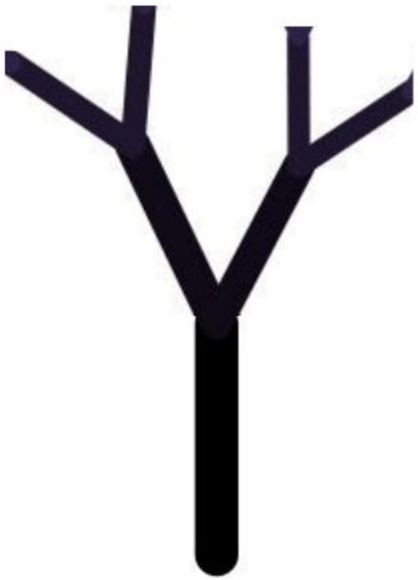

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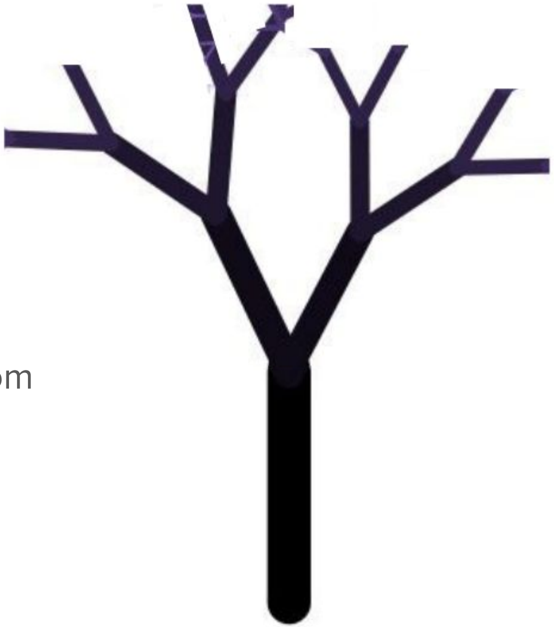
An order-4 tree



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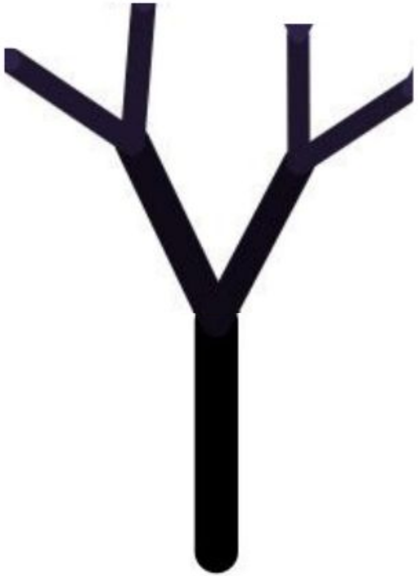
An order-3 tree

An order-0 tree is nothing at all.

An order-`n` tree is a line with two smaller order-`(n-1)` trees starting at the end of that line.



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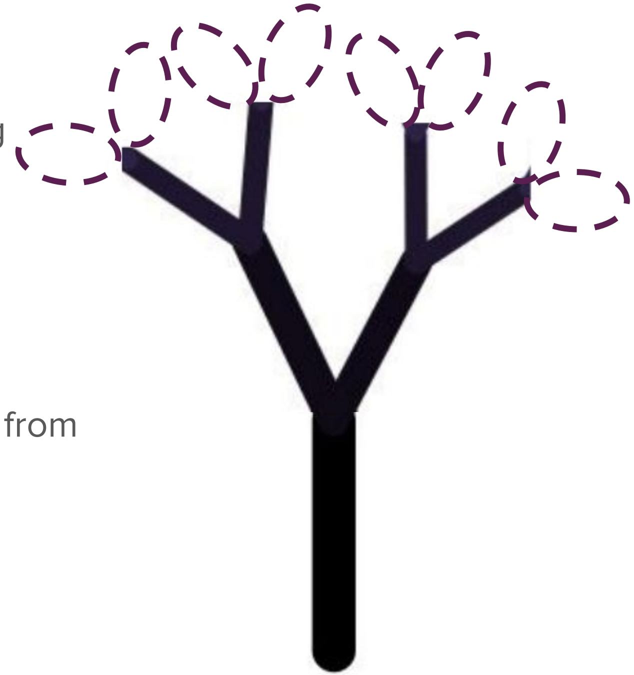

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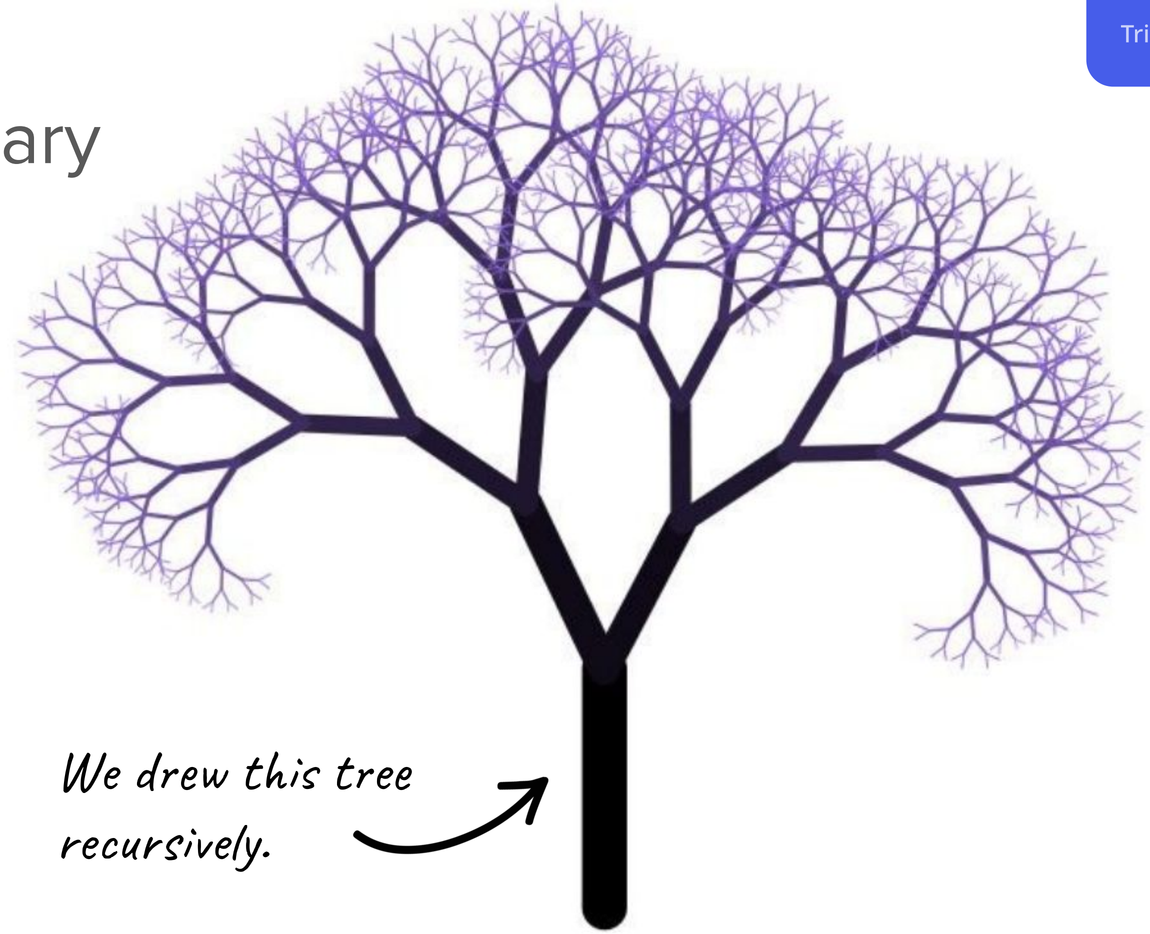
3. It has a different **orientation**.
4. It has a different **order**.

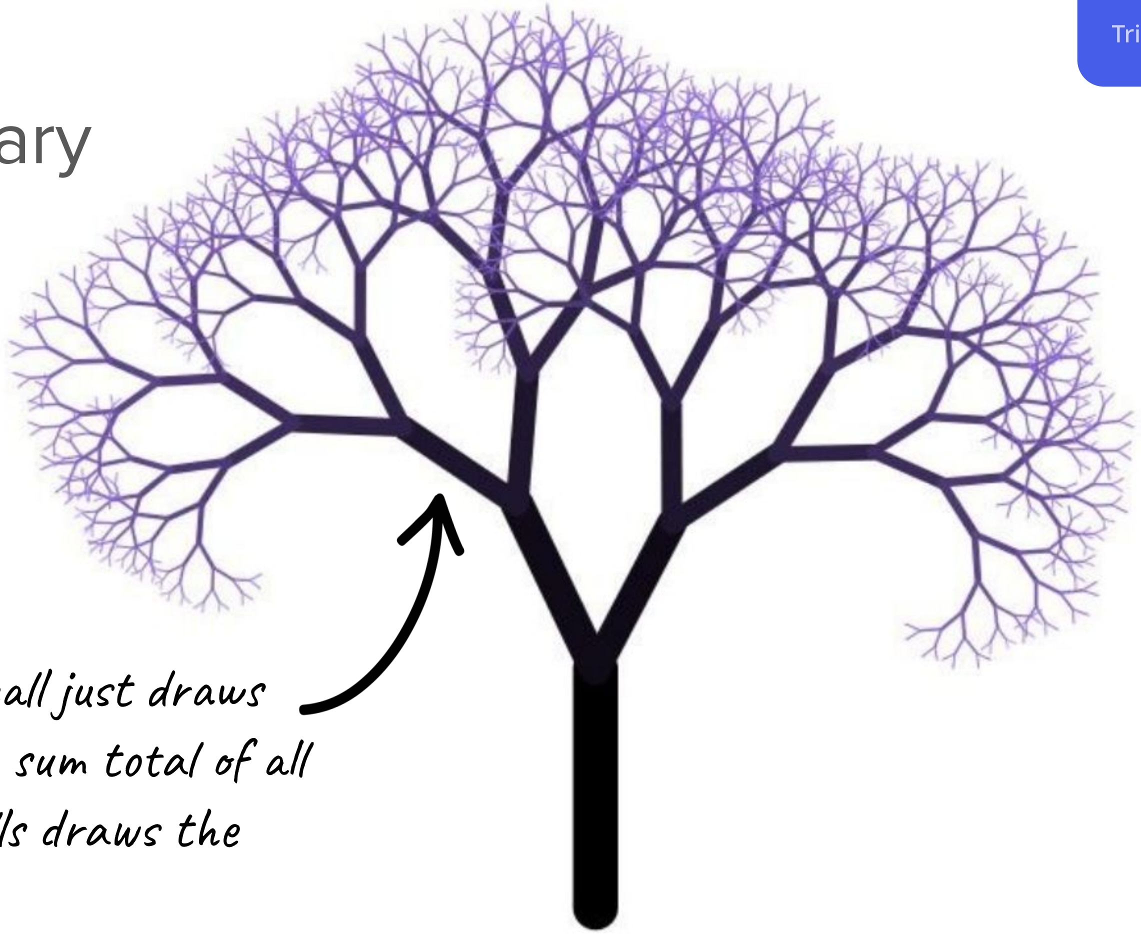
Fractals and self-similar structures are often defined in terms of some parameter called the **order**, which indicates the complexity of the overall structure.

In Summary



We drew this tree recursively.

In Summary



Each recursive call just draws one branch. The sum total of all the recursive calls draws the whole tree.

An Awesome Website!

http://recursivedrawing.com/

Announcements

Announcements

- Assignment 1 grades and commented feedback will be released on Paperless by the end of the day today.
- Assignment 2 is due tomorrow at 11:59pm PDT.
- Assignment 3 will be released by the end of the day on Thursday.
- Make sure to check out our weekly announcement posts on Ed – there's lot of important info contained there!
- Please send us your OAE accommodation letters if you haven't already!

How can we use recursion to make art?

C++ Stanford graphics library

Graphics in CS106B

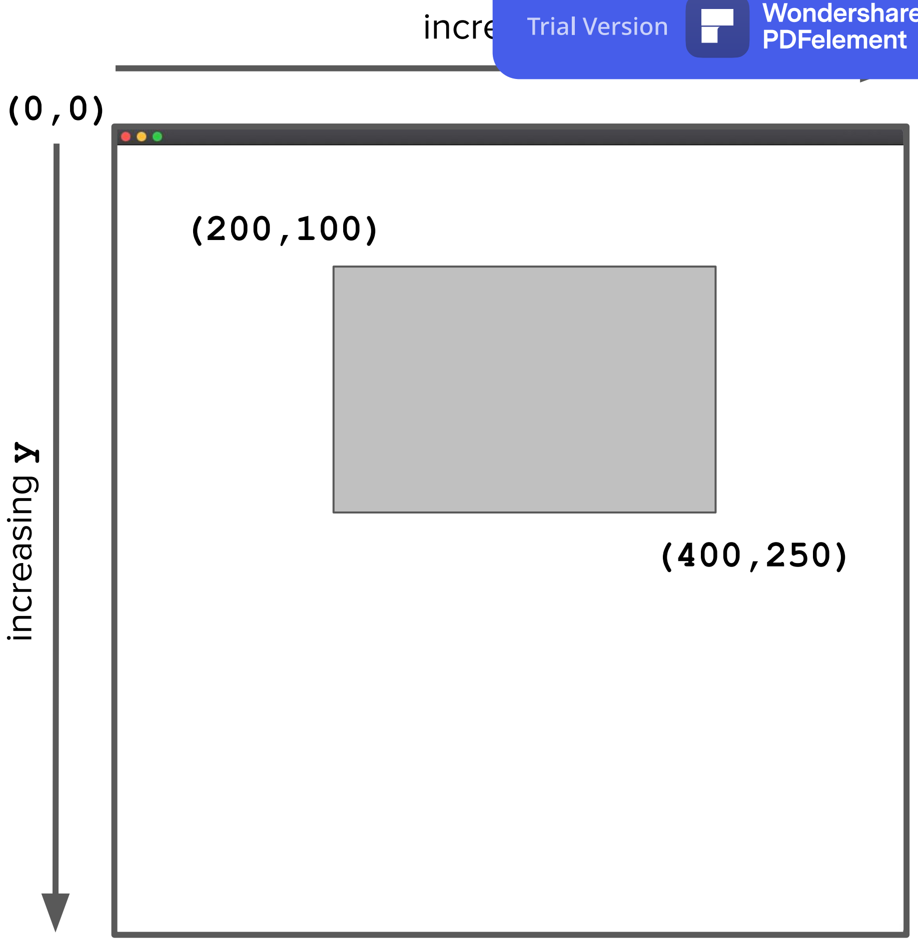
- Creating graphical programs is not one of our main focuses in this class, but a brief crash course in working with graphical programs is necessary to be able to code up some fractals of our own.

- The Stanford C++ libraries provide extensive capabilities to create custom graphical programs. The full documentation of these capabilities can be found in the official documentation.

- We will abstract away almost all of the complexity for you via provided helper functions.
  - There are two main classes/components of the library you need to know: **`GWindow`** and **`GPoint`**

GWindow

- A **GWindow** is an abstraction for the graphical window upon which we will do all of our drawing.
- The window defines a coordinate system of x-y values
  - The top left corner is `(0, 0)`
  - The bottom right corner is `(windowWidth, windowHeight)`
- All lines and shapes drawn on the window are defined by their `(x,y)` coordinates

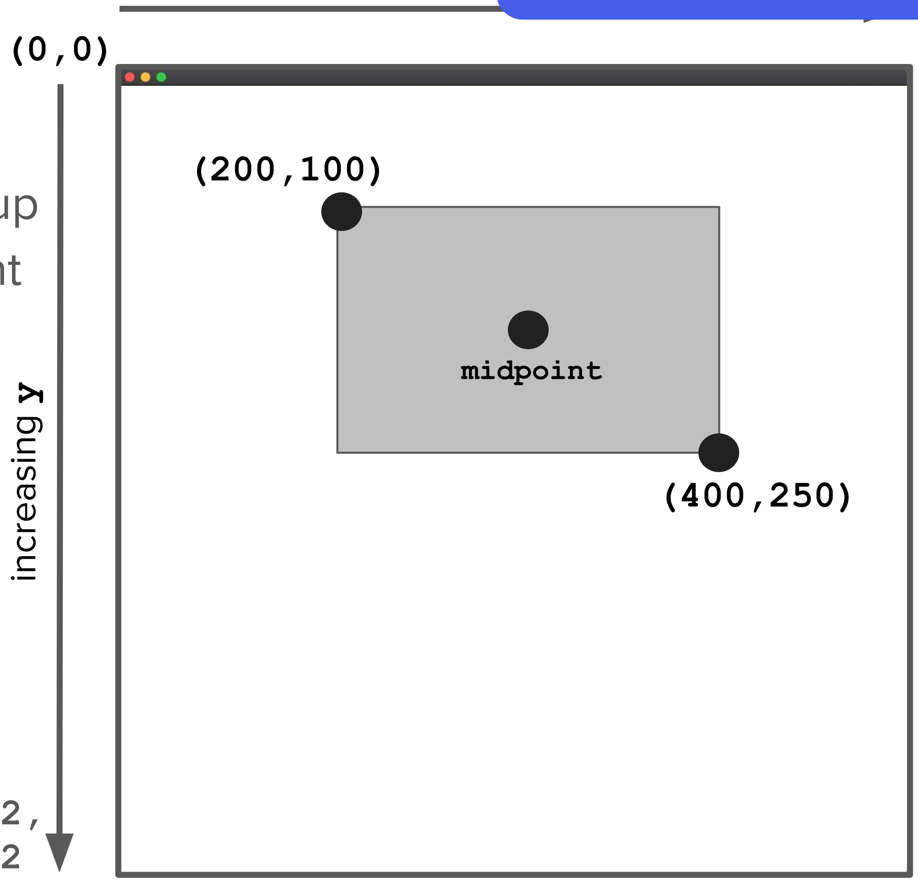


GPoint

- A **GPoint** is a handy way to bundle up the x-y coordinates for a specific point in the window.
    - Very similar in functionality to the **GridLocation** struct we learned about before!

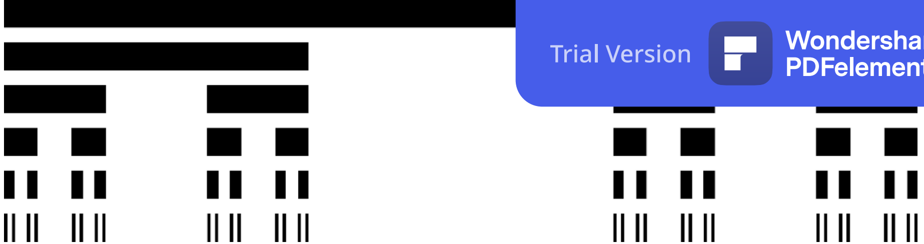
```
GPoint topLeft(200, 100);
GPoint bottomRight(400, 250);
drawFilledRect(topLeft, bottomRight);

GPoint midpoint = {
(topLeft.getX() + bottomRight.getX())/ 2,
(topLeft.getY() + bottomRight.getY())/ 2
};
```



Cantor Set example

Cantor Set



- The first fractal we will code is called the "Cantor" fractal, named after the late-19th century German mathematician Georg Cantor.

- The Cantor fractal is a set of lines where there is one main line, and below that there are two other lines: each ⅓ of the width of the original line, with one on the left and one on the right (with a ⅓ separation of whitespace between them)

- Below each of the other lines is an identical situation: two ⅓ lines.

- This repeats until the lines are no longer visible.

An order-0 Cantor Set

An order-1 Cantor Set

An order-2 Cantor Set

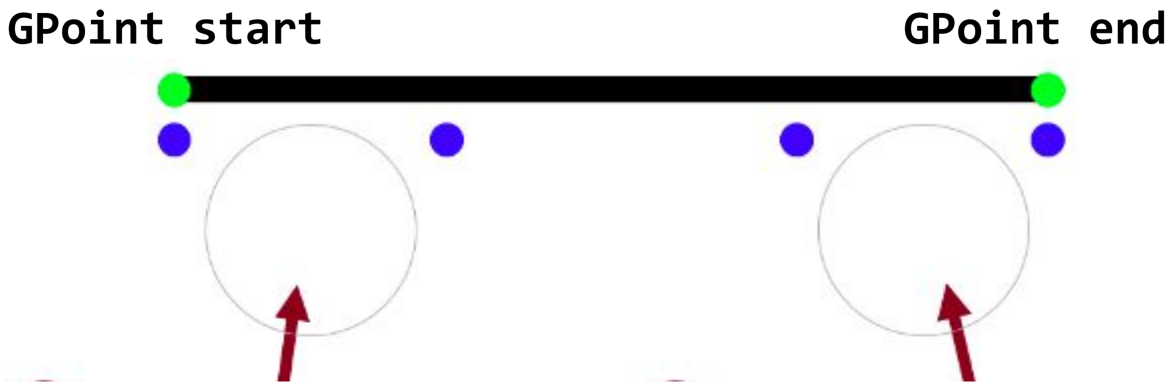
An order-6 Cantor Set

Another Cantor Set

Also a Cantor Set

How to draw an order-n Cantor Set

1. Draw a line from **`start`** to **`end`**.



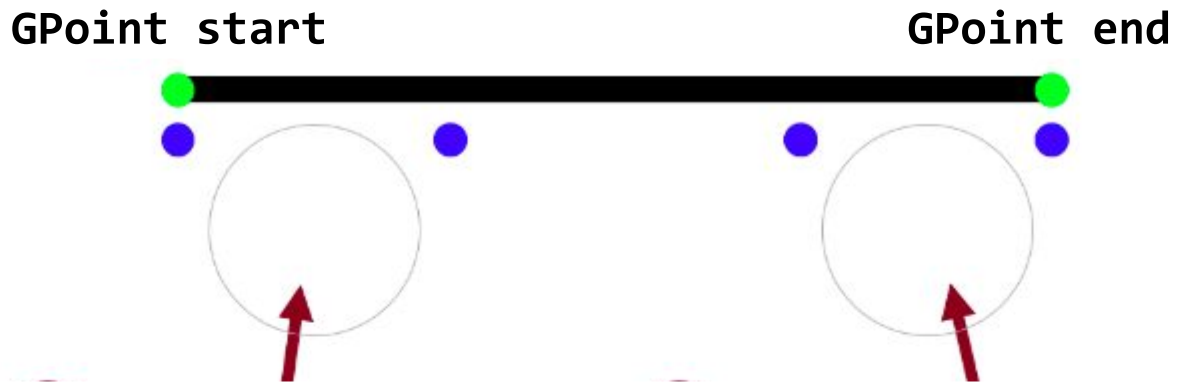
2. Underneath the left third, draw a Cantor Set of order-(**`n - 1`**).
3. Underneath the right third, draw a Cantor Set of order-(**`n - 1`**).

How to draw an order-n Cantor Set

Base case:

`order == 0`

1. Draw a line from **`start`** to **`end`**.

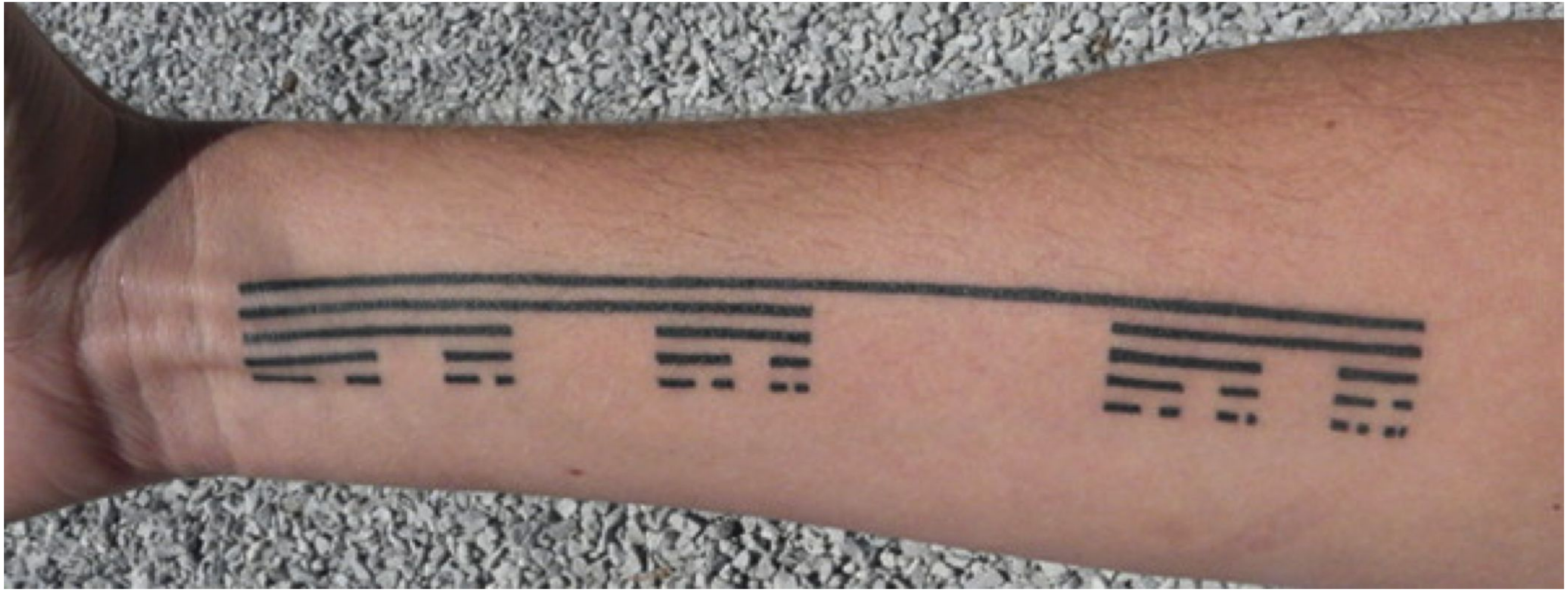


2. Underneath the left third, draw a Cantor Set of order-(**`n - 1`**).
3. Underneath the right third, draw a Cantor Set of order-(**`n - 1`**).

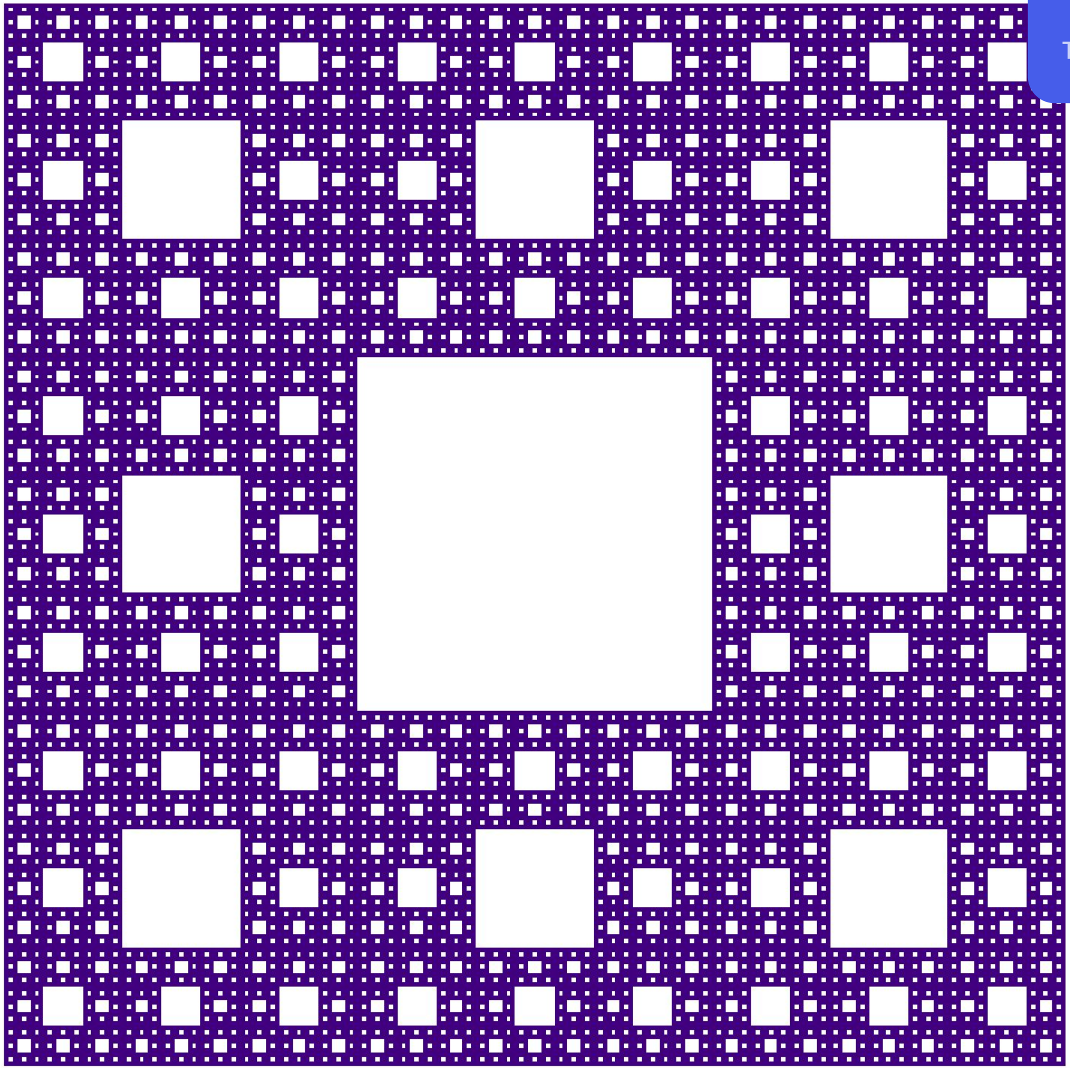
Cantor Set demo

[Qt Creator]

Real-world application of the Cantor Set

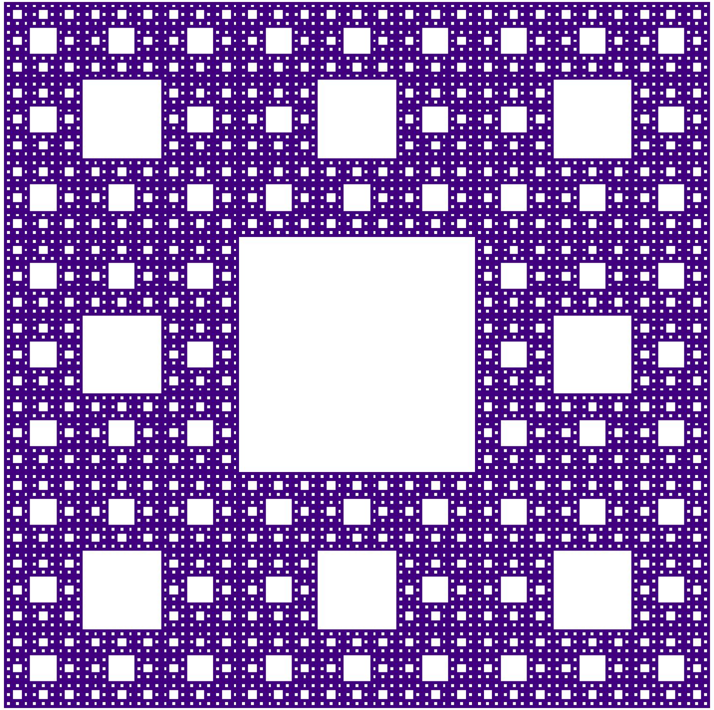


Sierpinski Carpet example

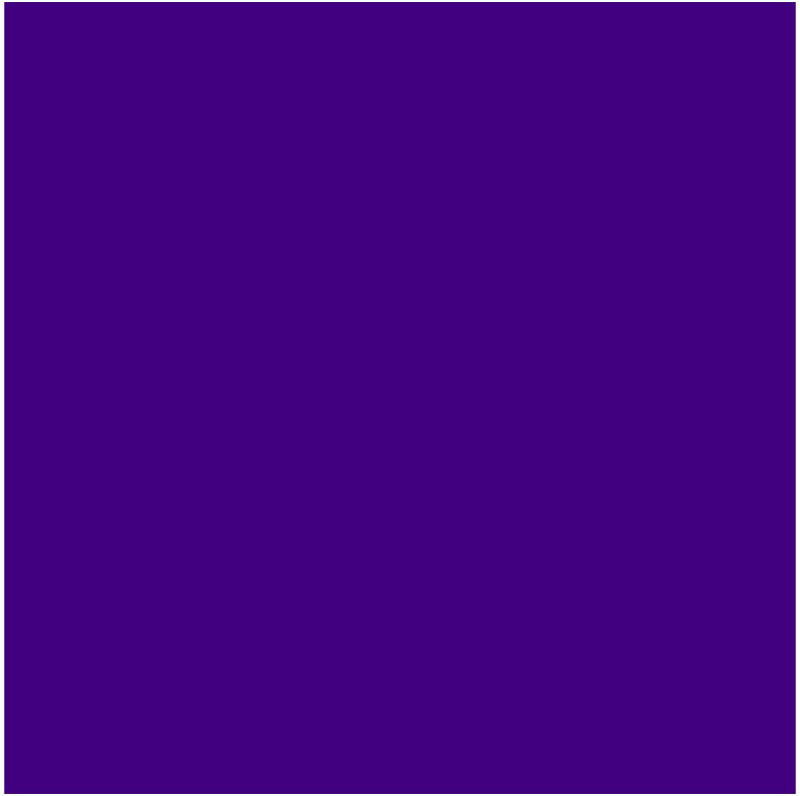


Sierpinski Carpet

- First described by Wacław Sierpiński in 1916
- A generalization of the Cantor Set to two dimensions!
- Defined by the subdivision of a shape (a square in this case) into smaller copies of itself.
  - The same pattern applied to a triangle yields a Sierpinski triangle, which you will code up on the next assignment.

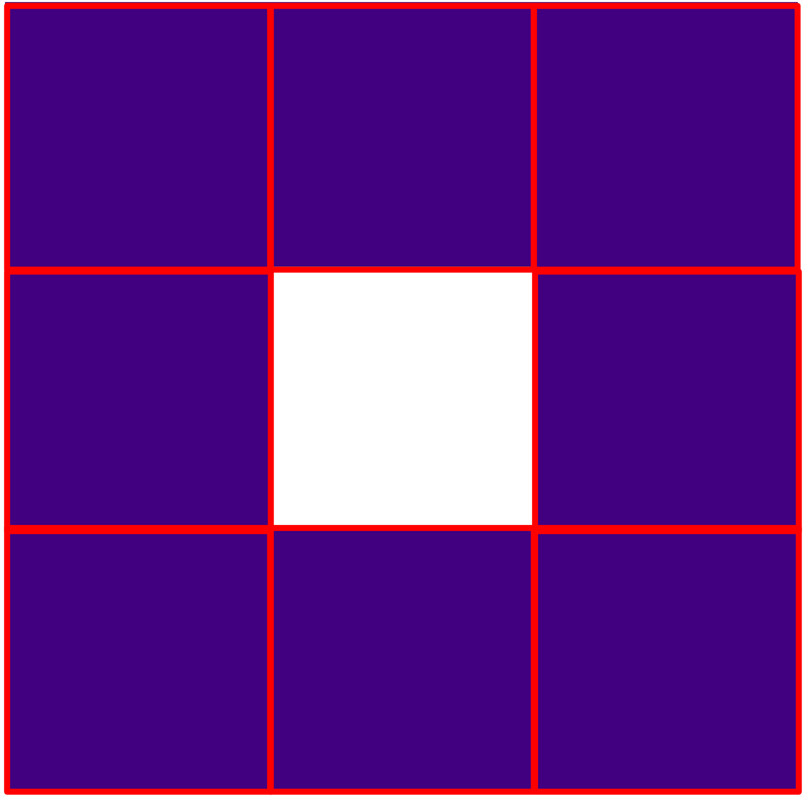


An order-0 Sierpinski Carpet



An order-1 Sierpinski Carpet

An order-1 carpet is subdivided into eight order-0 carpets arranged in this grid pattern

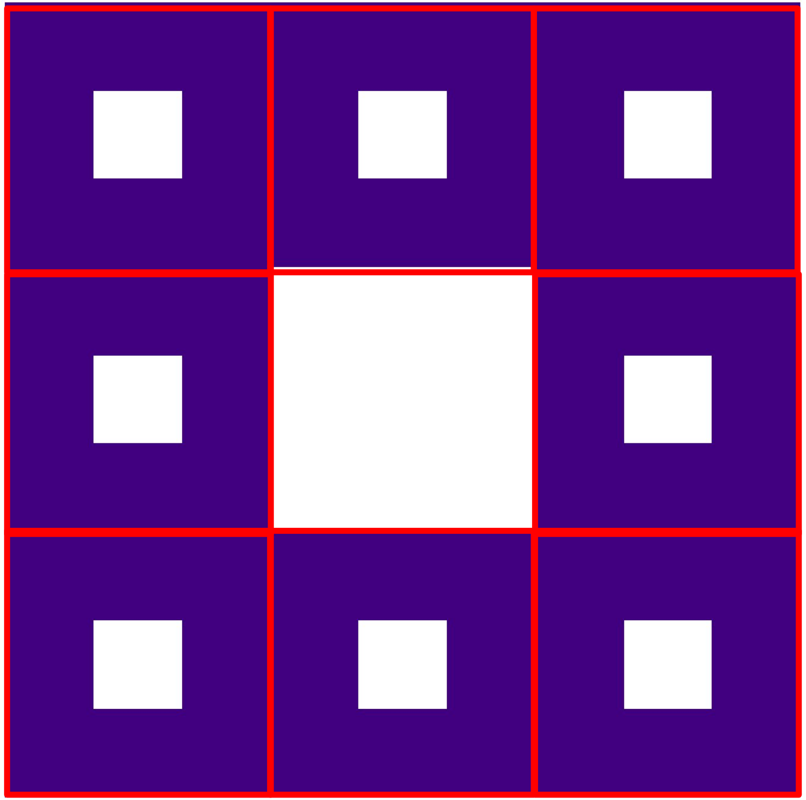


An order-2 Sierpinski Carpet

???

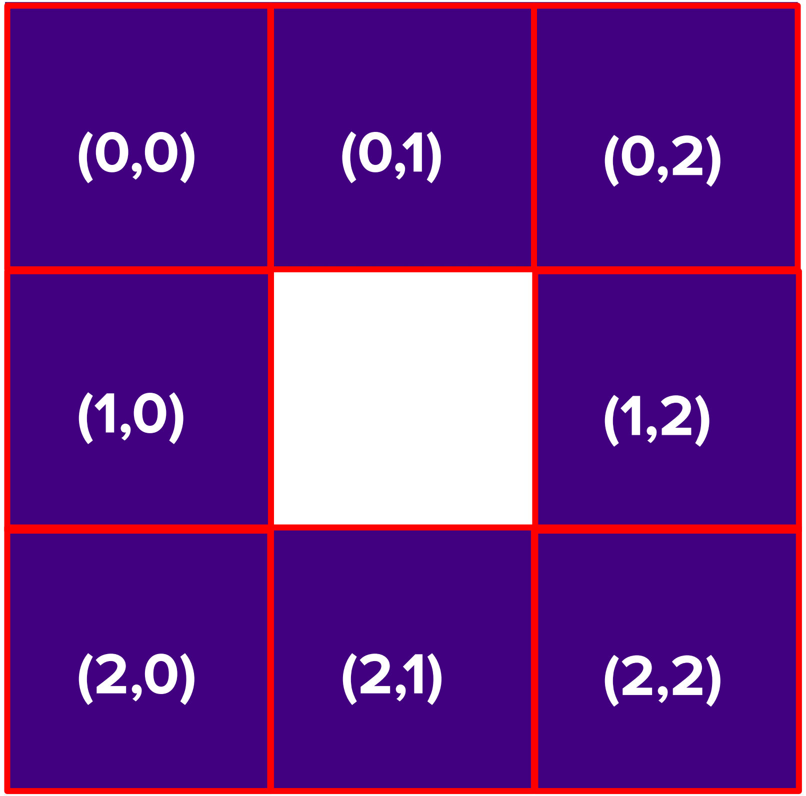
Practice and Discuss:
Draw the order-2 Sierpinski carpet. What are the base and recursive cases that define this fractal?
(breakout rooms)

An order-2 Sierpinski Carpet



Sierpinski Carpet Formalized

- Base Case (order-0)
  - Draw a filled square at the appropriate location
- Recursive Case (order-n, n ≠ 0)
  - Draw 8 order n-1 Sierpinski carpets, arranged in a 3x3 grid, omitting the center location
    - i.e. Draw an n-1 fractal at (0,0), draw an n-1 fractal at (0,1), draw an n-1 fractal at (0,2)…



Sierpinski Carpet Pseudocode (Take 1)

```
drawSierpinskiCarpet (x, y, order):
    if (order == 0)
        drawFilledSquare(x, y, BASE_SIZE)
    else
        drawSierpinskiCarpet(newX(x, y, 0, 0), newY(x, y, 0, 0), order -1)
        drawSierpinskiCarpet(newX(x, y, 0, 1), newY(x, y, 0, 1), order -1)
        drawSierpinskiCarpet(newX(x, y, 0, 2), newY(x, y, 0, 2), order -1)
        drawSierpinskiCarpet(newX(x, y, 1, 0), newY(x, y, 1, 0), order -1)
        drawSierpinskiCarpet(newX(x, y, 1, 2), newY(x, y, 1, 2), order -1)
        drawSierpinskiCarpet(newX(x, y, 2, 0), newY(x, y, 2, 0), order -1)
        drawSierpinskiCarpet(newX(x, y, 2, 1), newY(x, y, 2, 1), order -1)
        drawSierpinskiCarpet(newX(x, y, 2, 2), newY(x, y, 2, 2), order -1)
```

Sierpinski Carpet Pseudocode (Take 1)

```
drawSierpinskiCarpet (x, y, order):
    if (order == 0)
        drawFilledS
    else
        drawSierpin                                    , 0, 0), order -1)
        drawSierpin                                    , 0, 1), order -1)
        drawSierpin                                    , 0, 2), order -1)
        drawSierpin                                    , 1, 0), order -1)
        drawSierpin                                    , 1, 2), order -1)
        drawSierpin                                    , 2, 0), order -1)
        drawSierpin                                    , 2, 1), order -1)
        drawSierpin                                    , 2, 2), order -1)
```

This isn't very pretty, can we do better?

Sierpinski Carpet Pseudocode (Take 2)

```
drawSierpinskiCarpet (x, y, order):
    if (order == 0)
        drawFilledSquare(x, y, BASE_SIZE)
    else
        for row = 0 to row = 2:
            for col = 0 to col = 2:
                if (col != 1 || row != 1):
                    x_i = newX(x, y, row, col)
                    y_i = newY(x ,y, row, col)
                    drawSierpinskiCarpet(x_i, y_i, order - 1)
```

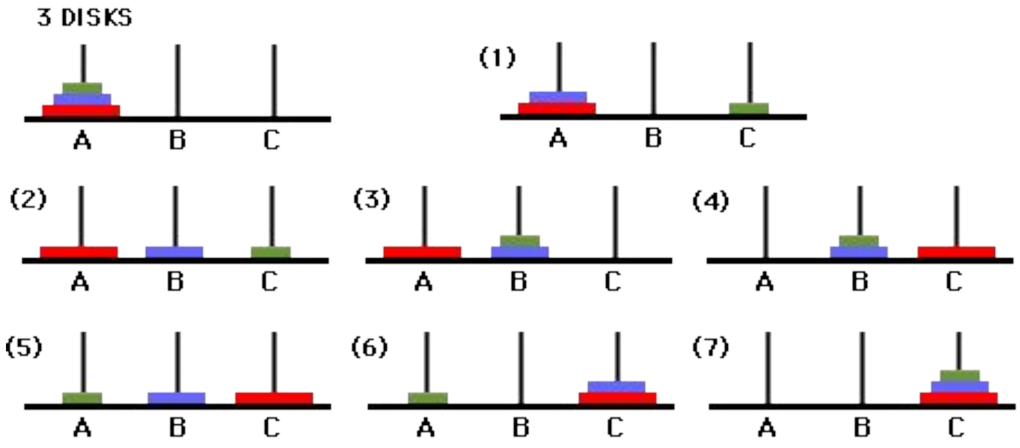
Iteration + Recursion

- It’s completely reasonable to mix iteration and recursion in the same function.
- Here, we’re firing off eight recursive calls, and the easiest way to do that is with a double for loop.
- Recursion doesn’t mean “the absence of iteration.” It just means “solving a problem by solving smaller copies of that same problem.”
- Iteration and recursion can be very powerful in combination!

Revisiting the Towers of Hanoi

[Recursive Part 2: Electric Boogaloo]

Pseudocode for 3 disks



(1) Move disk 1 to destination
(2) Move disk 2 to auxiliary
(3) Move disk 1 to auxiliary
(4) Move disk 3 to destination
(5) Move disk 1 to source
(6) Move disk 2 to destination
(7) Move disk 1 to destination

Homework before tomorrow's lecture

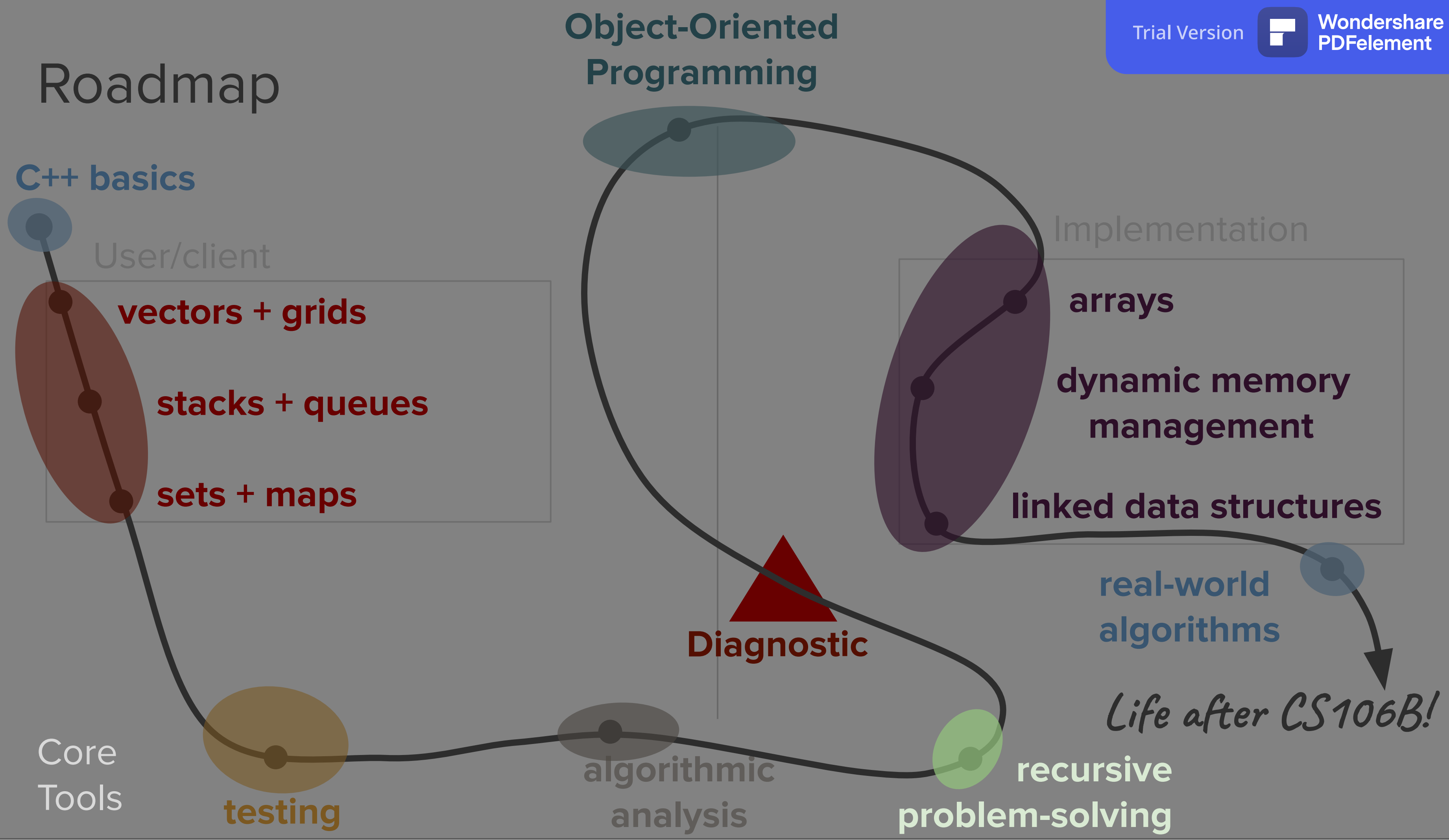
- Play Towers of Hanoi: https://www.mathsisfun.com/games/towerofhanoi.html

- Look for and write down patterns in how to solve the problem as you increase the number of disks. Try to get to at least 5 disks!

- **Extra challenge** (optional)**:** How would you define this problem recursively?
    - Don't worry about data structures here. Assume we have a function **`moveDisk(X, Y)`** that will handle moving a disk from the top of post **`X`** to the top of post **`Y`**.

What’s next?

Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Advanced Recursion Examples