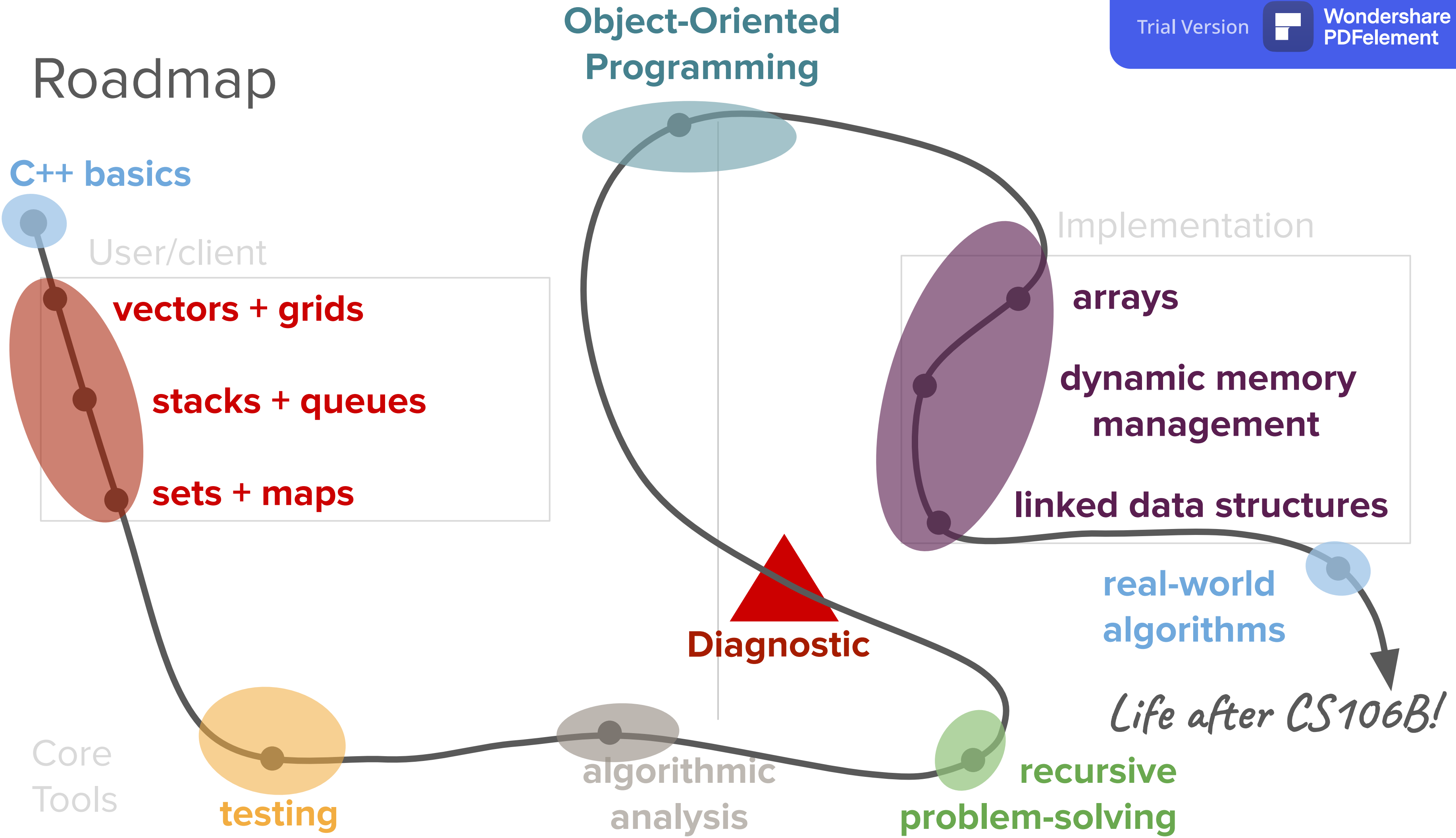
Trees

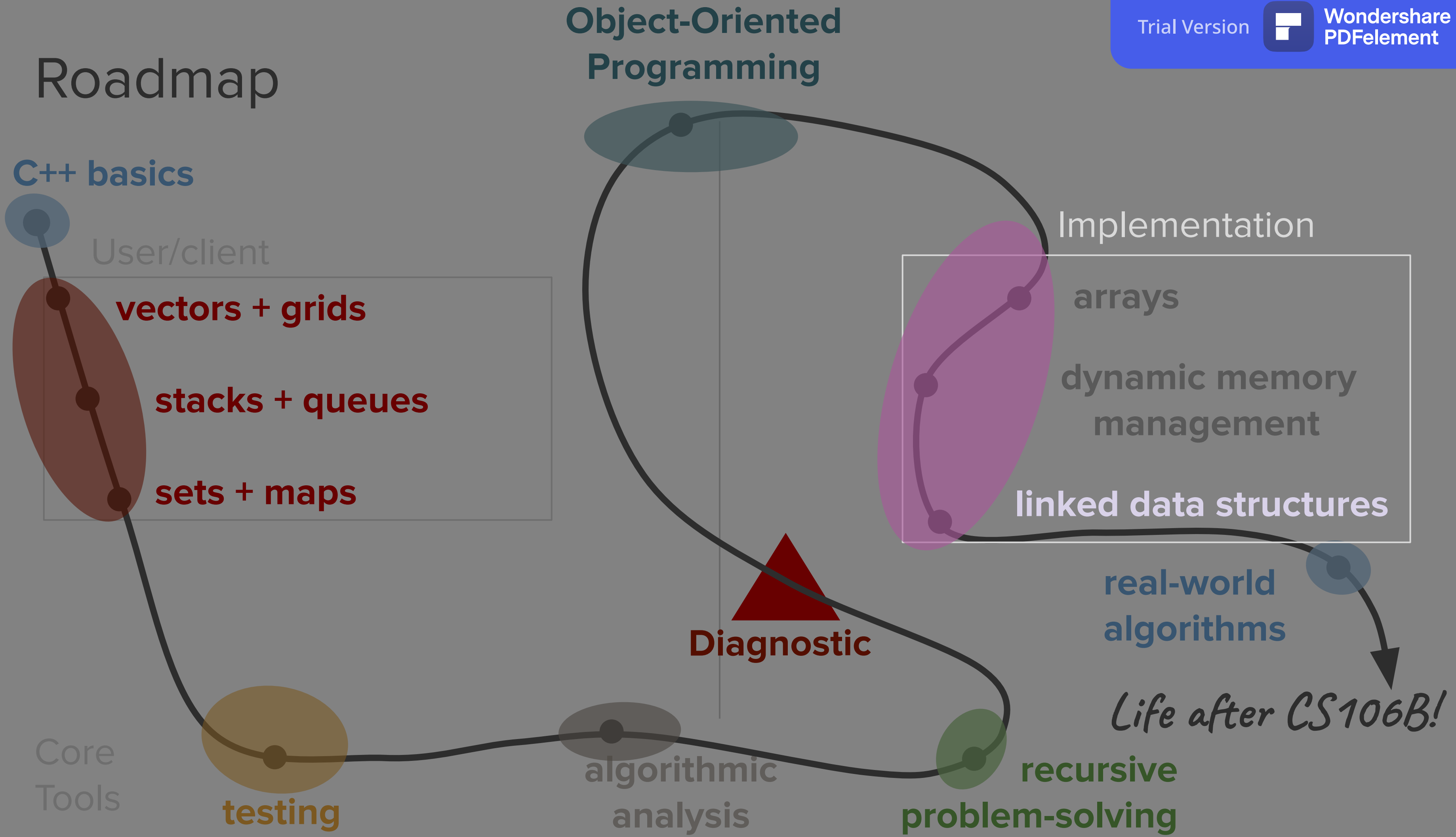
Is there any component of "Life after CS106B" that you would like us to focus on in our final lecture next week?

(put your answers the chat)

Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Roadmap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Today's questions

How can we better organize data stored in a linked data structure?

Today's topics

1. Linked Data Structure Overview

2. Introduction to Trees

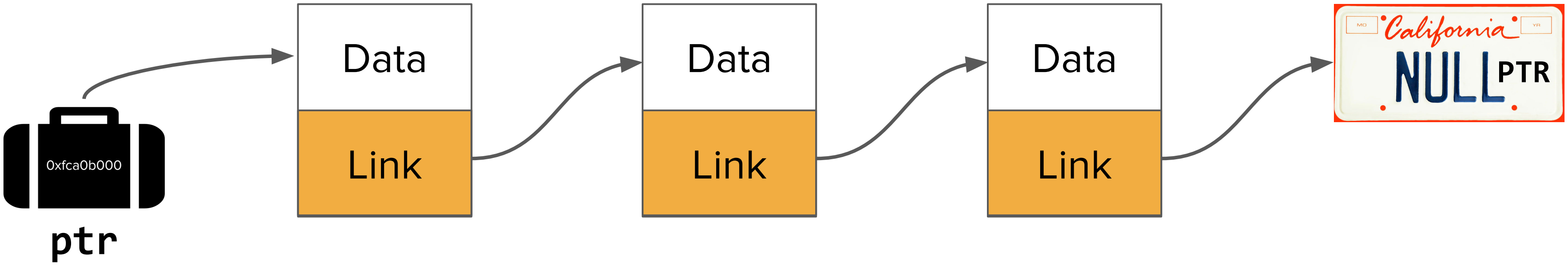
3. Trees in C++

Review

[linked data structures]

Linked Data Structures

- Last week, we explored linked lists, our first example of a **linked data structure**.



Linked Data Structures

- Last week, we explored linked lists, our first example of a **linked data structure**.

- Linked data structures are distinguished by the fact that they stored data in a **distributed** manner. This means that the data is stored across many different locations in computer memory.

- In order to organize this data, we had to **bundle data alongside pointers** in the concept of a "node."

- Using pointers allows us to **create links** to other nodes to impose structure.

Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
  - Insertion/removal of elements of a linked list was very quick because it only involved fast pointer rewiring operations. We never had to "shift" elements over to make room. 腾出地方
  - Because all the data was stored in dynamic memory, expanding the size of the linked list was very easy and never required an expensive "re-sizing" operation that had to copy all the data.

Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.

- However, we also ran into some limitations when it came to working with lists:
    - Data was organized in a linear structure, which meant the path to traverse between any two nodes (specifically between the front and a node later on in the list) could get very long.
    - Finding elements in a linked list is an `O(n)` operation, which can get slow when we want to store many elements.
    - We couldn't feasibly write recursive algorithms that traversed linked lists, due to stack frame limits that came into play since traversal algorithms required one stack frame per node.

Linked List Tradeoffs

- Storing data in a distributed (non-contiguous) manner had some distinct advantages over working with arrays.
- However, we also ran into some limitations when it came to working with lists.
- **Question:** Can we organize data in a linked data structure in such a way that the path between the "front" and any element in the structure is short (better than `O(n)`) even if there are many elements?

How can we better organize data stored in a linked data structure?

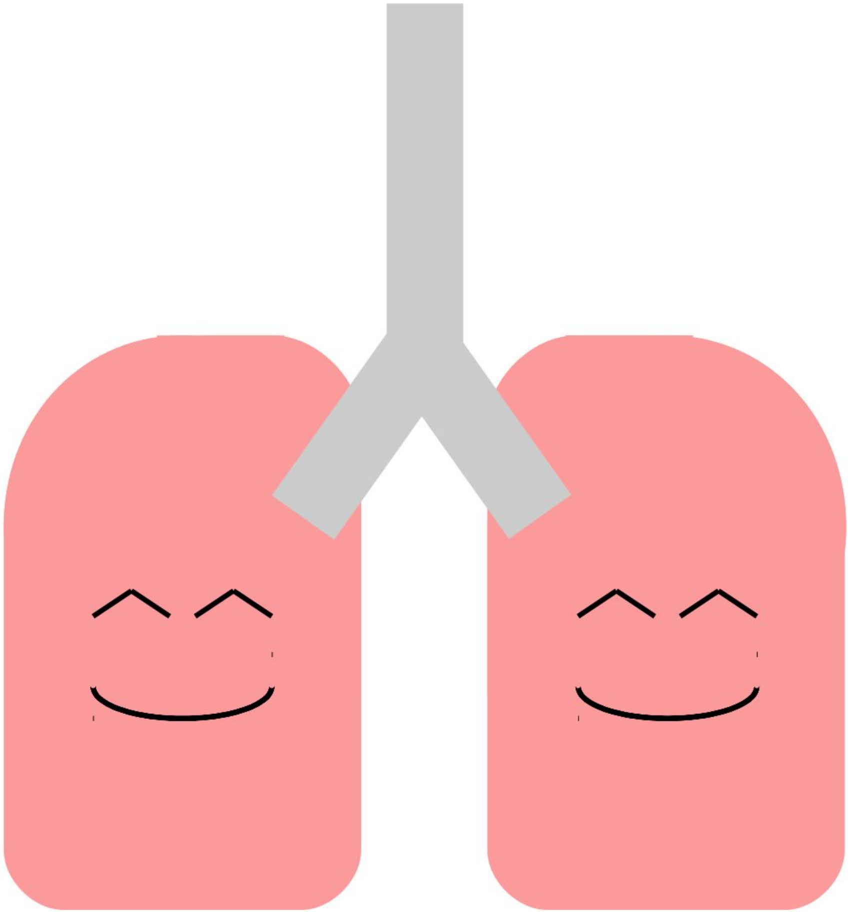
Interactive Exercise

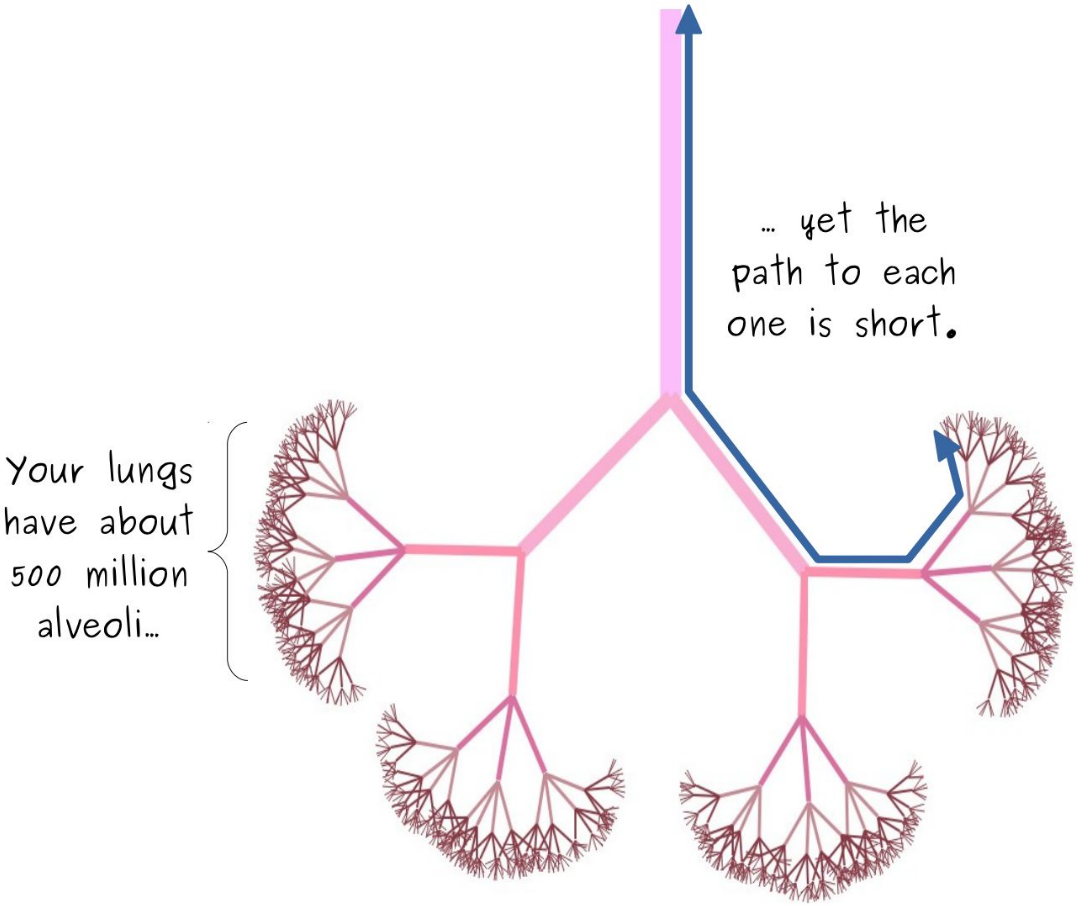
[borrowed from Keith Schwarz]

Take a deep breath.

And exhale…

Feel nicely oxygenated?



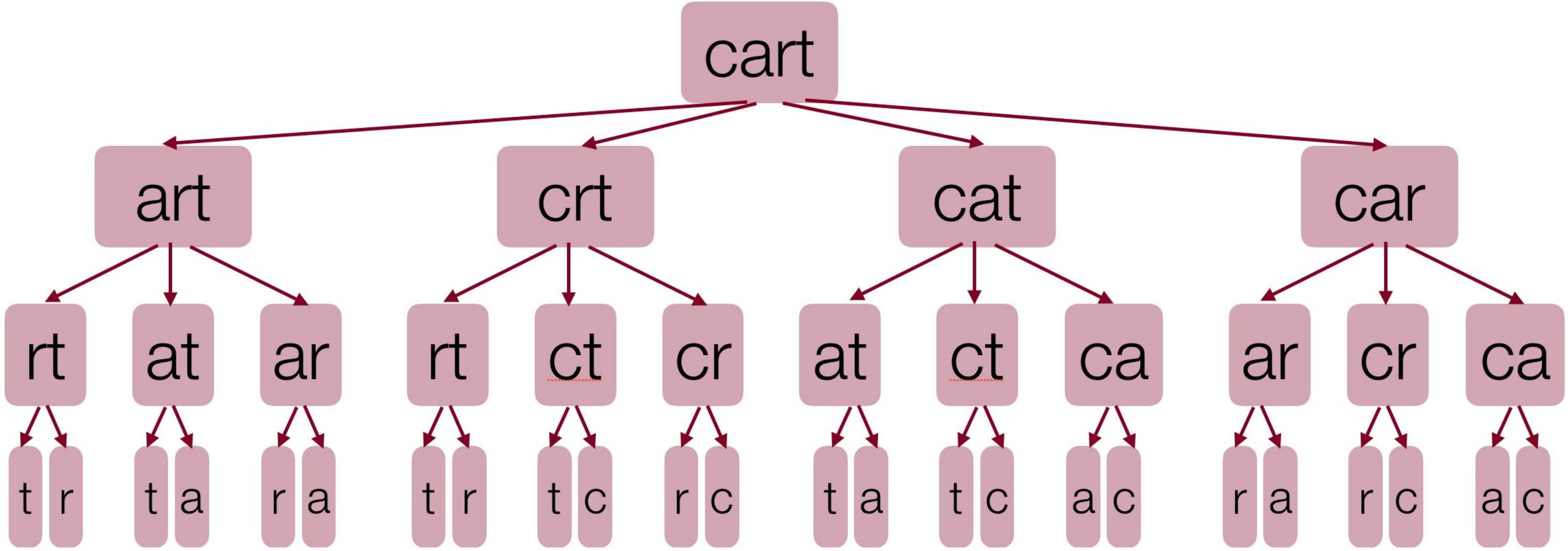

… yet the path to each one is short.
Your lungs have about 500 million alveoli…

Key Idea: The distance from each element in this structure to the top of the structure is small, even if there are many elements.

Trees

Throwback Thursday (on Monday)

- We've already seen trees before in this class... decision trees!



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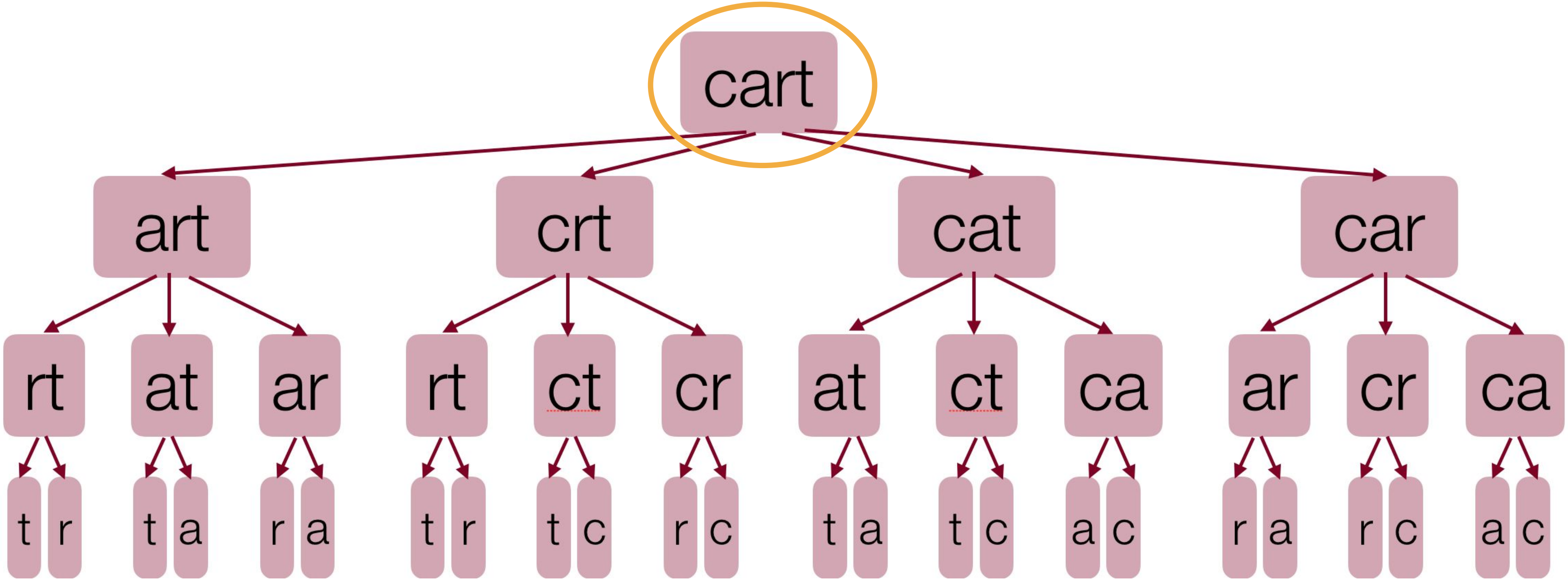

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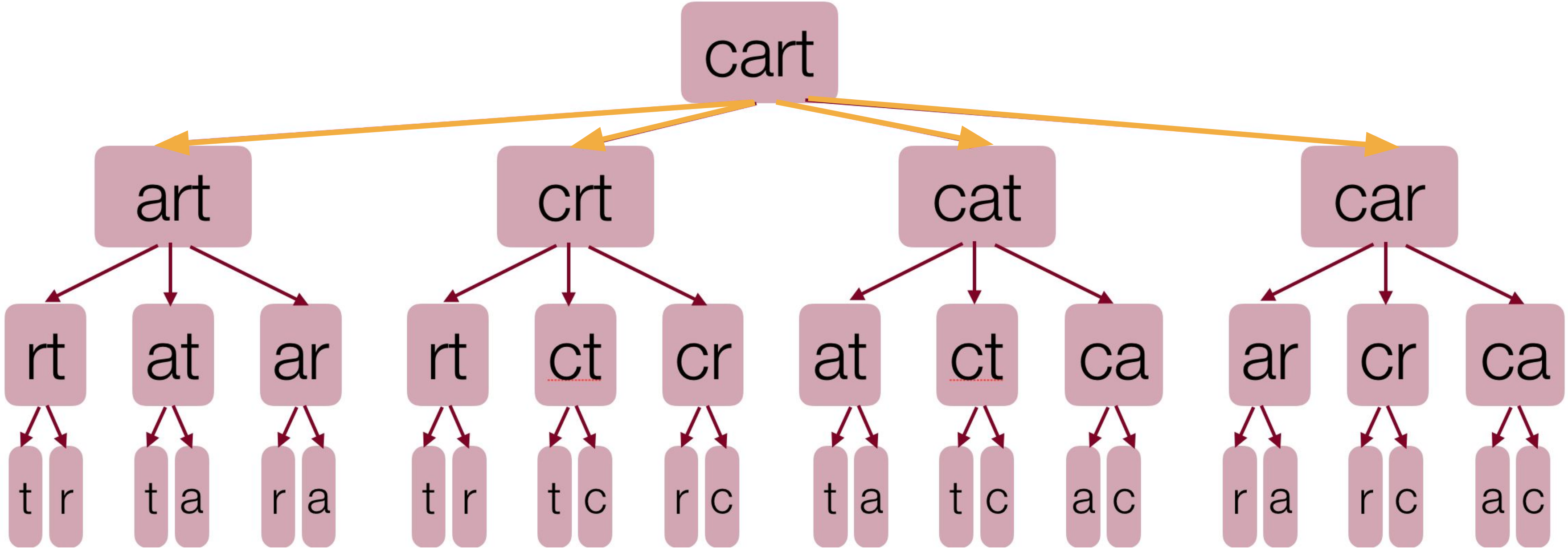

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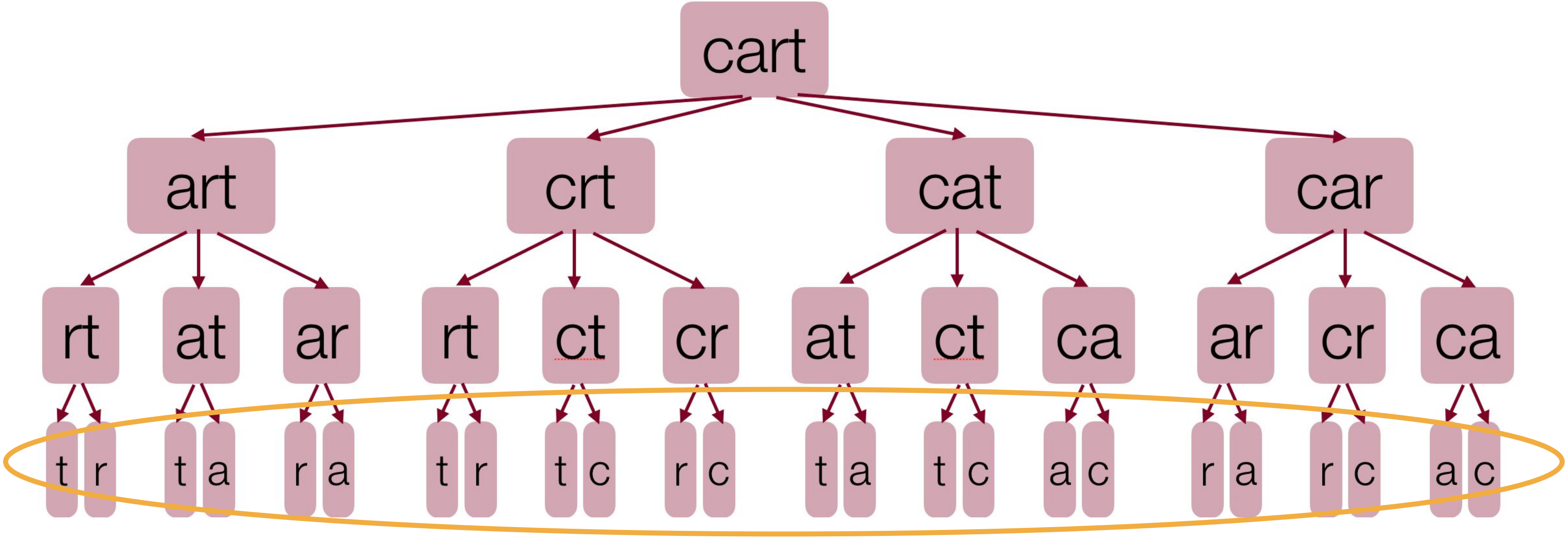
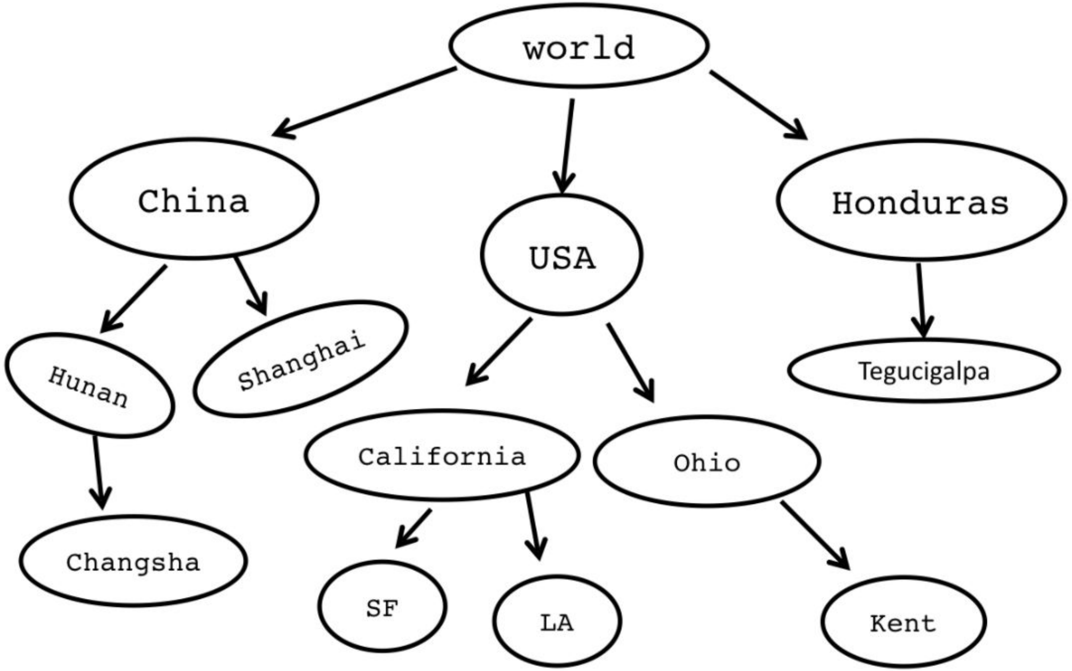


Trees in the Wild

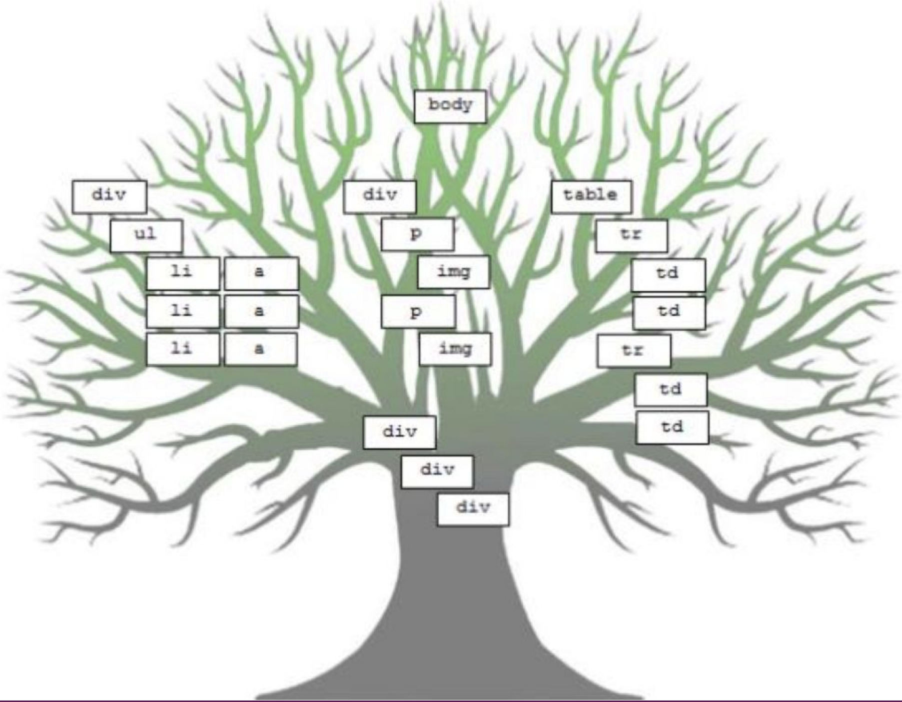
- Trees are useful in other ways besides just visualizing recursive backtracking.



Trees can be used to describe *hierarchies*.

Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

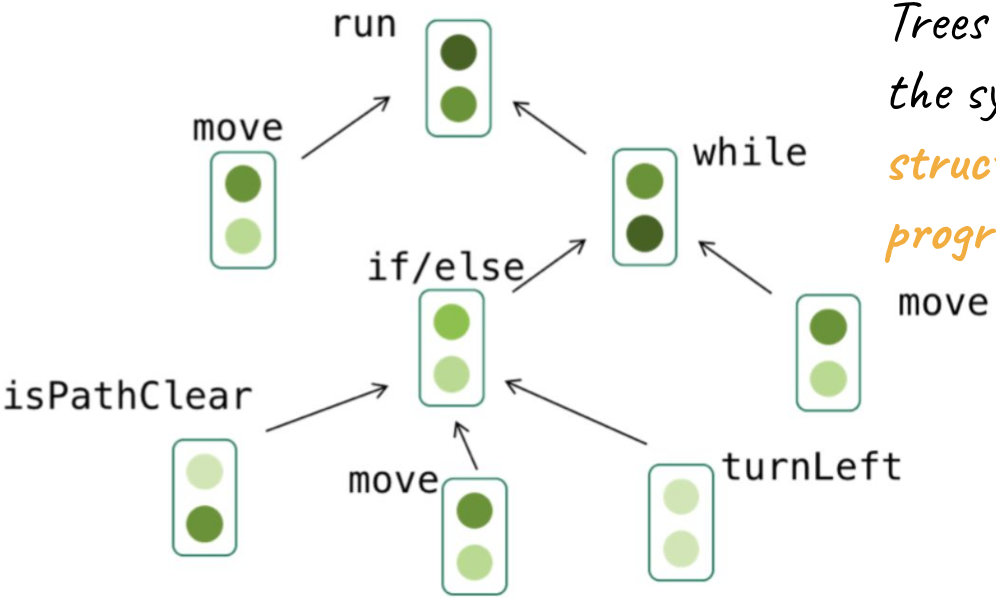


Trees are used to model the structure of websites.

Trees in the Wild

- Trees are useful in other ways besides just visualizing recursive backtracking.

```
def run() {
  move();
  while (notFinished()) {
    if (isPathClear()) {
      move();
    } else {
      turnLeft();
    }
    move();
  }
}
```



Trees describe the syntax structure of programs.

Trees in the Wild

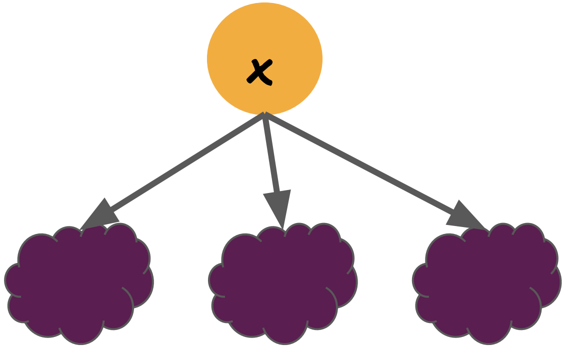
- Trees are useful in other ways besides just visualizing recursive backtracking.
- But, it is not a coincidence that we first saw them appear in conjunction with recursion.
- Trees are inherently defined recursively!

What is a tree?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)



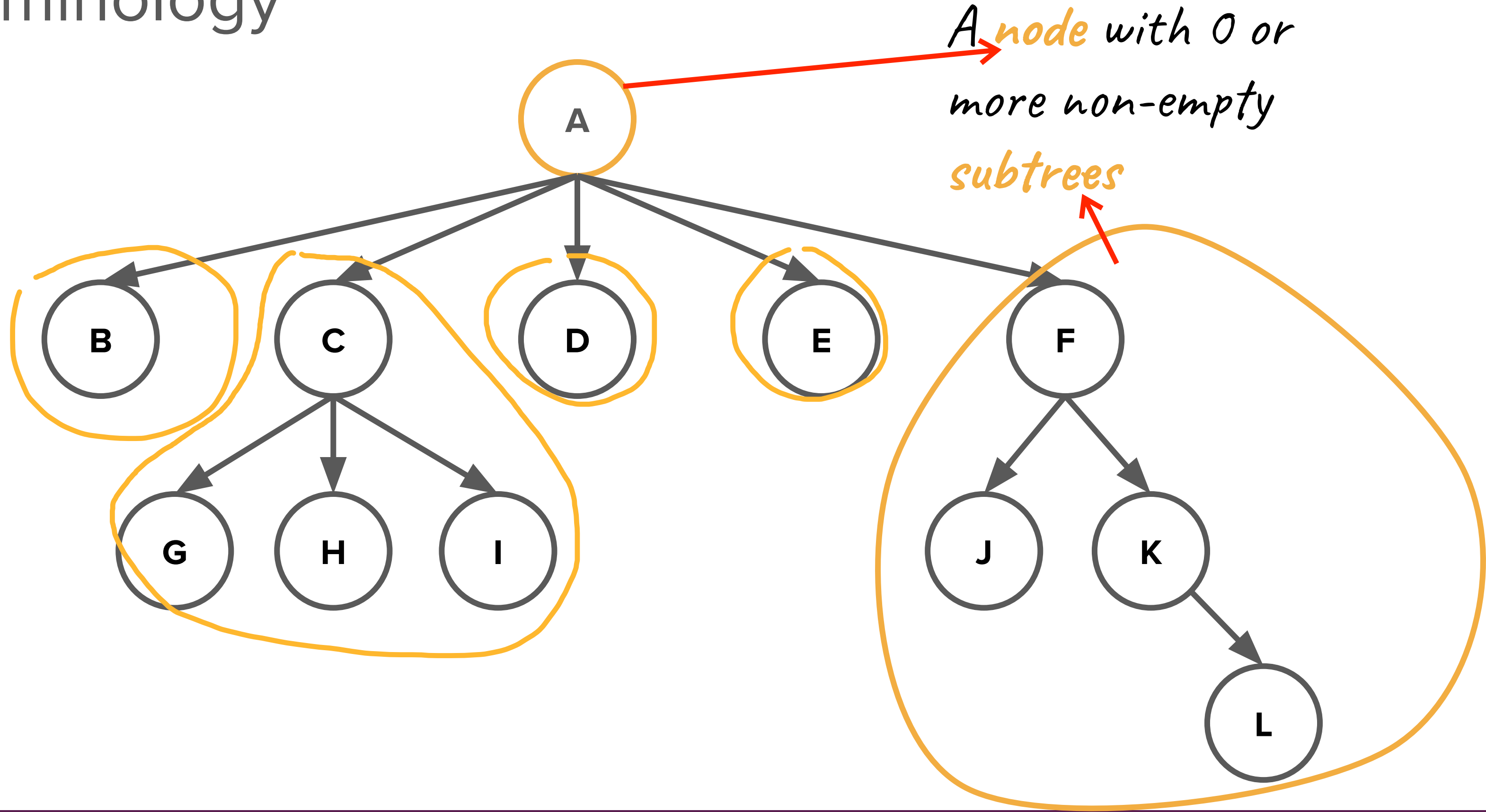
Definition

tree

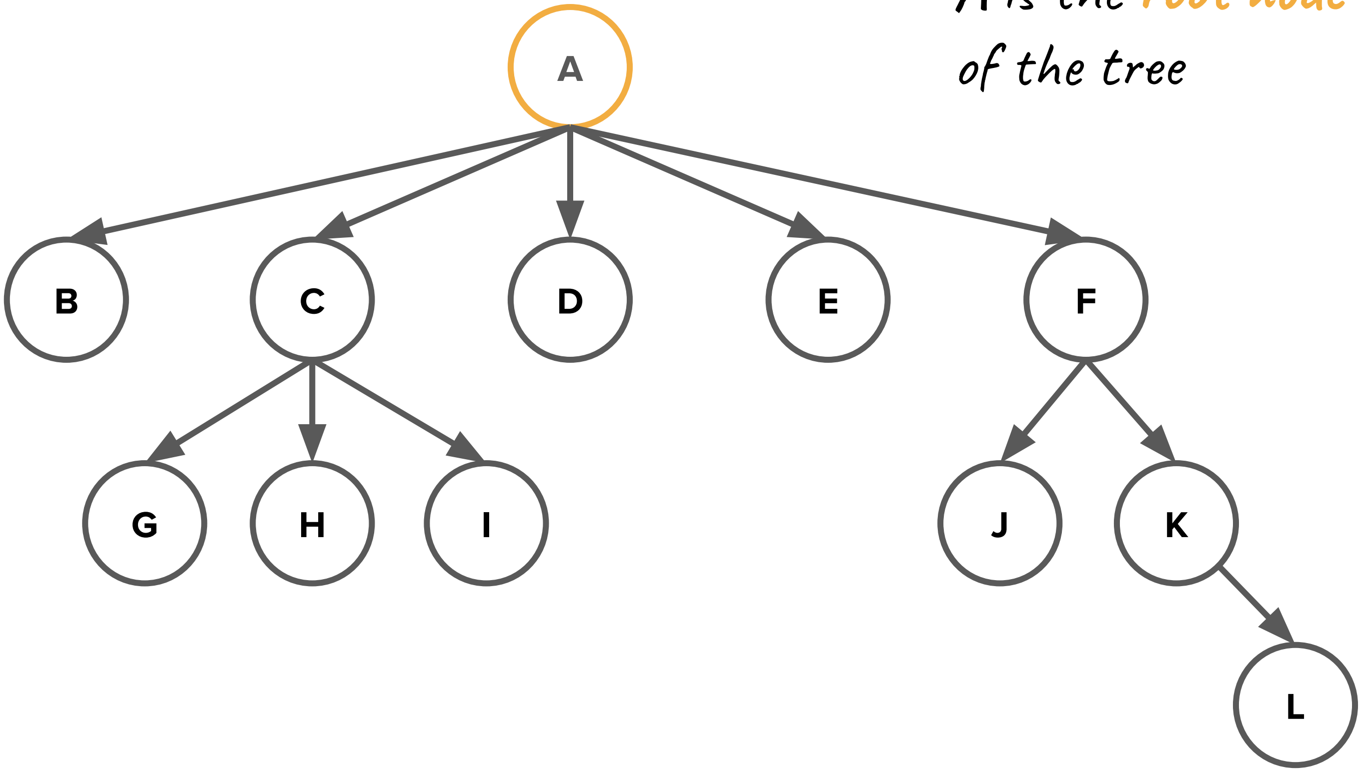
A tree is hierarchical data organization structure composed of a root value linked to zero or more non-empty subtrees.

Tree Terminology

Tree Terminology



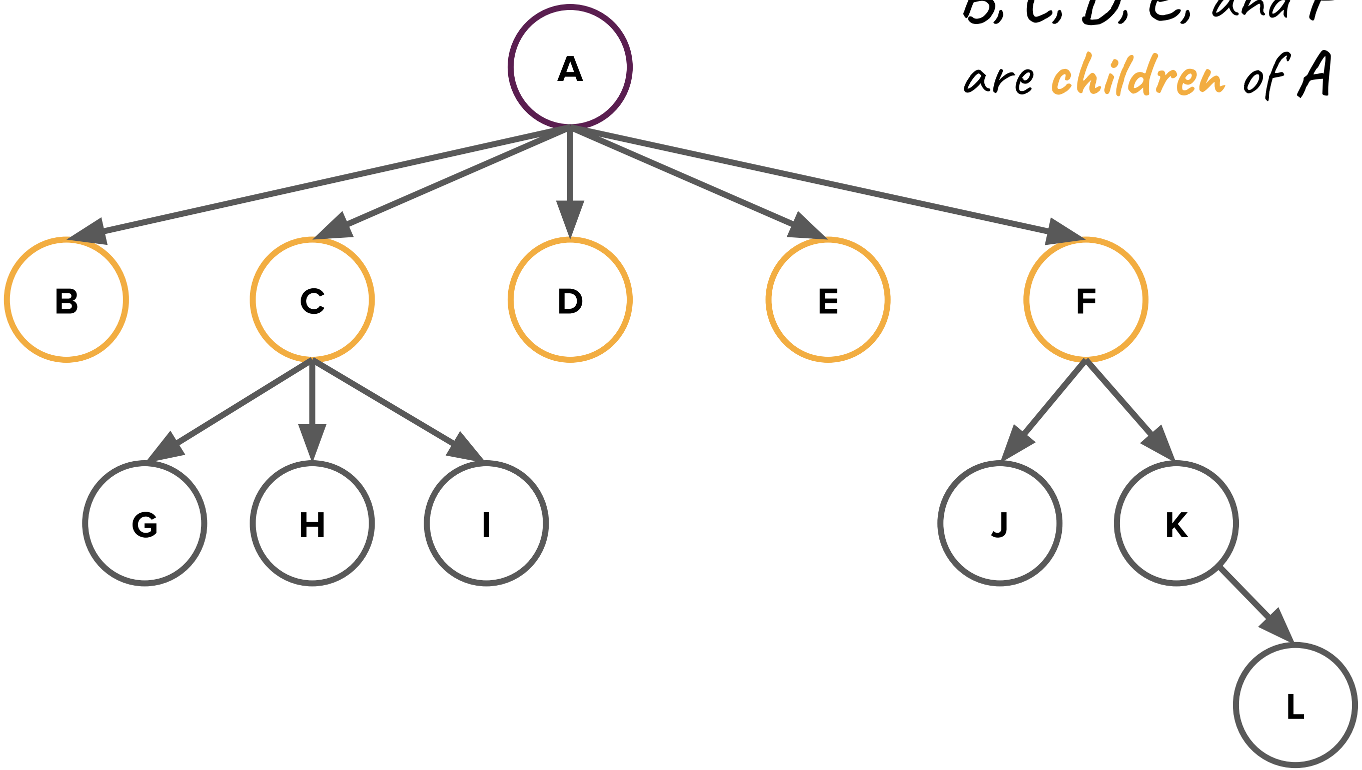
Tree Terminology



A is the root node
of the tree

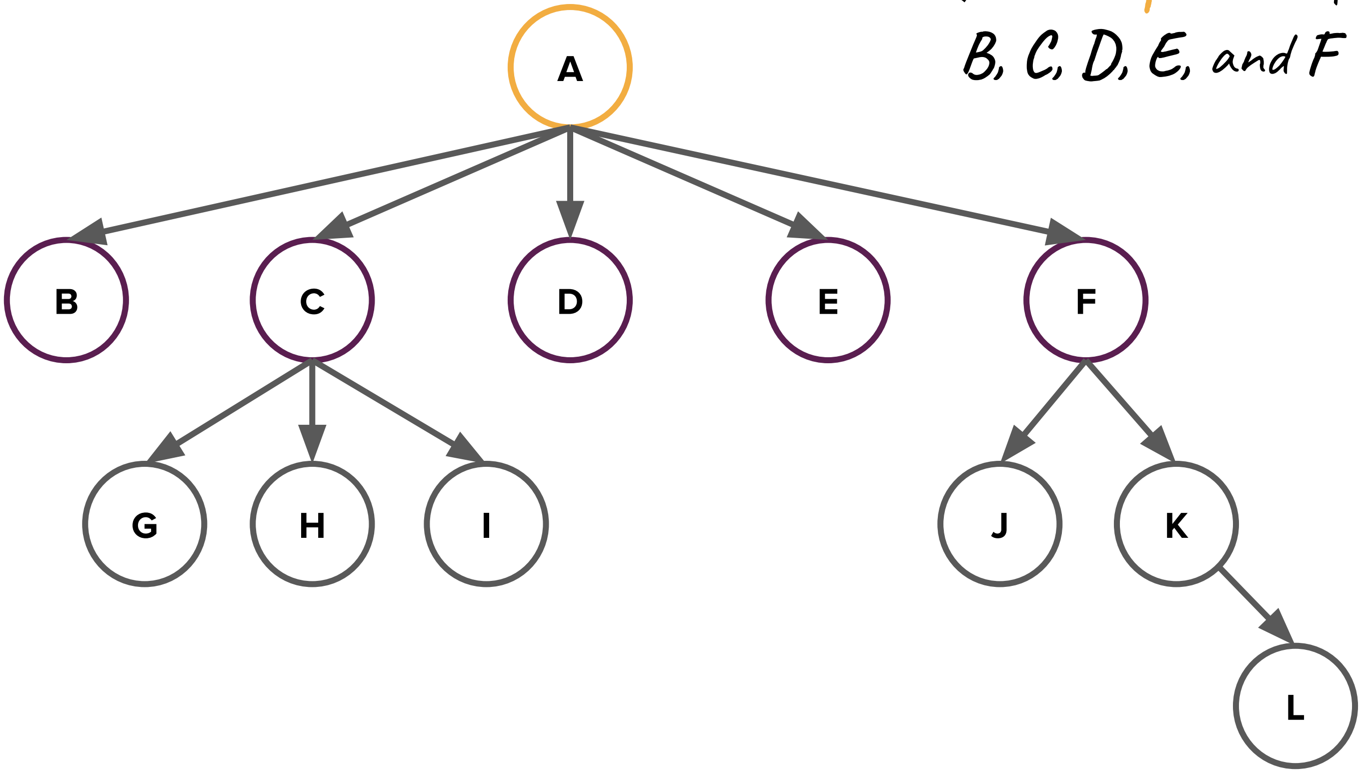
Tree Terminology

B, C, D, E, and F are children of A



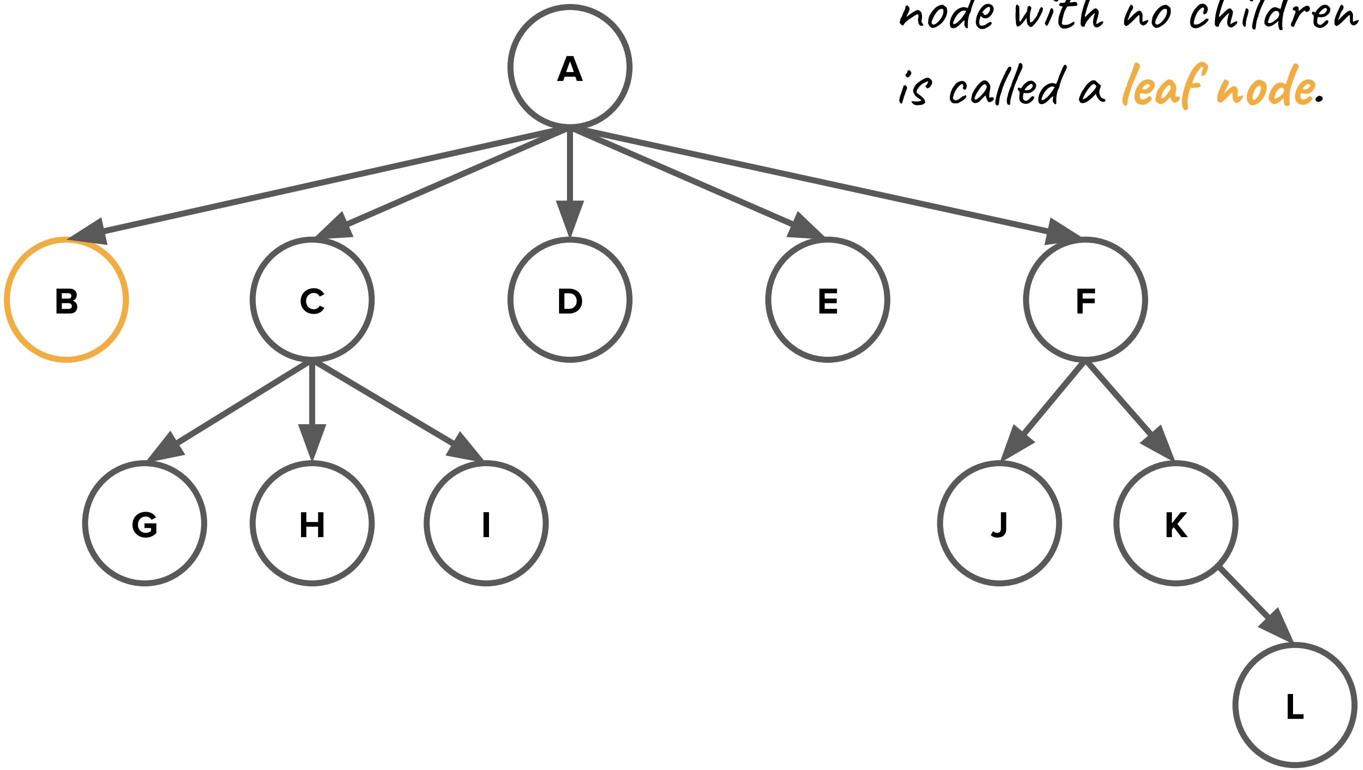
Tree Terminology

A is the parent of
B, C, D, E, and F



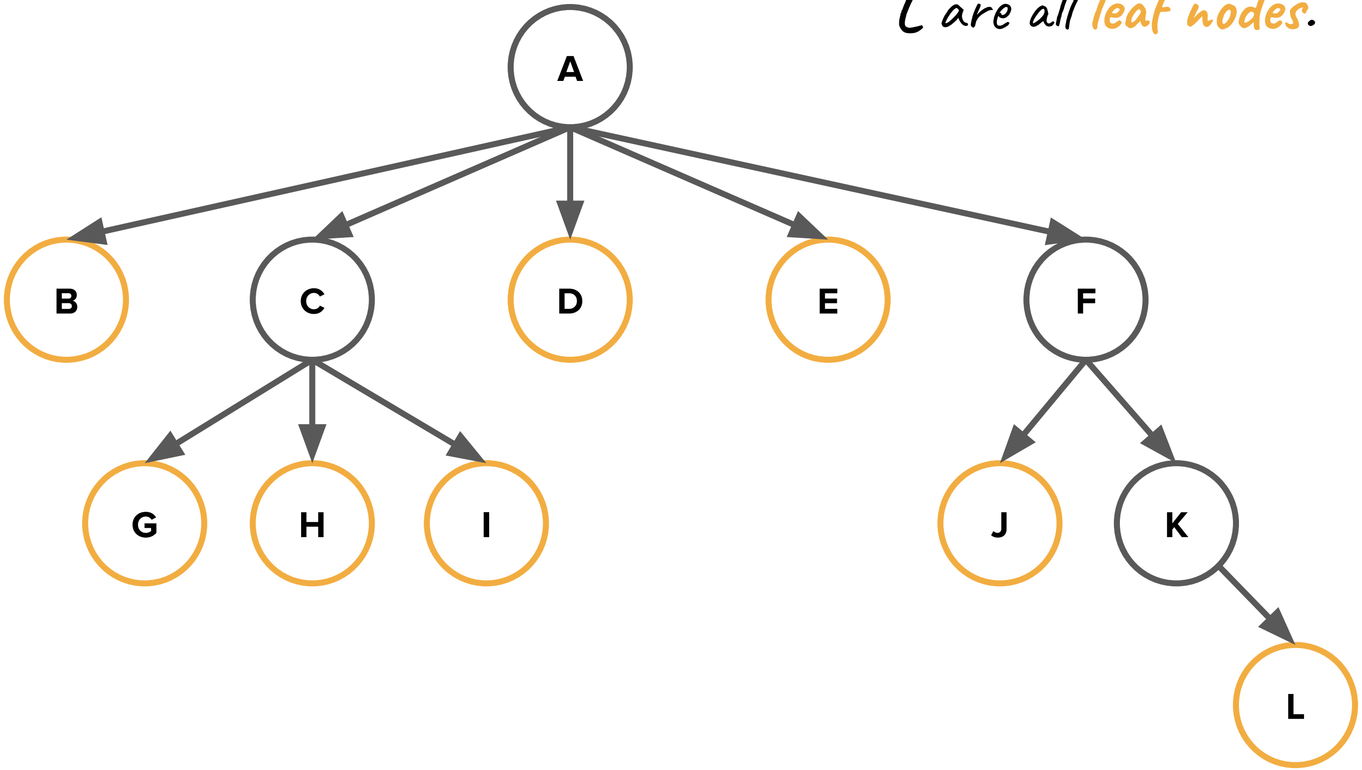
Tree Terminology

B has no children. A node with no children is called a leaf node.

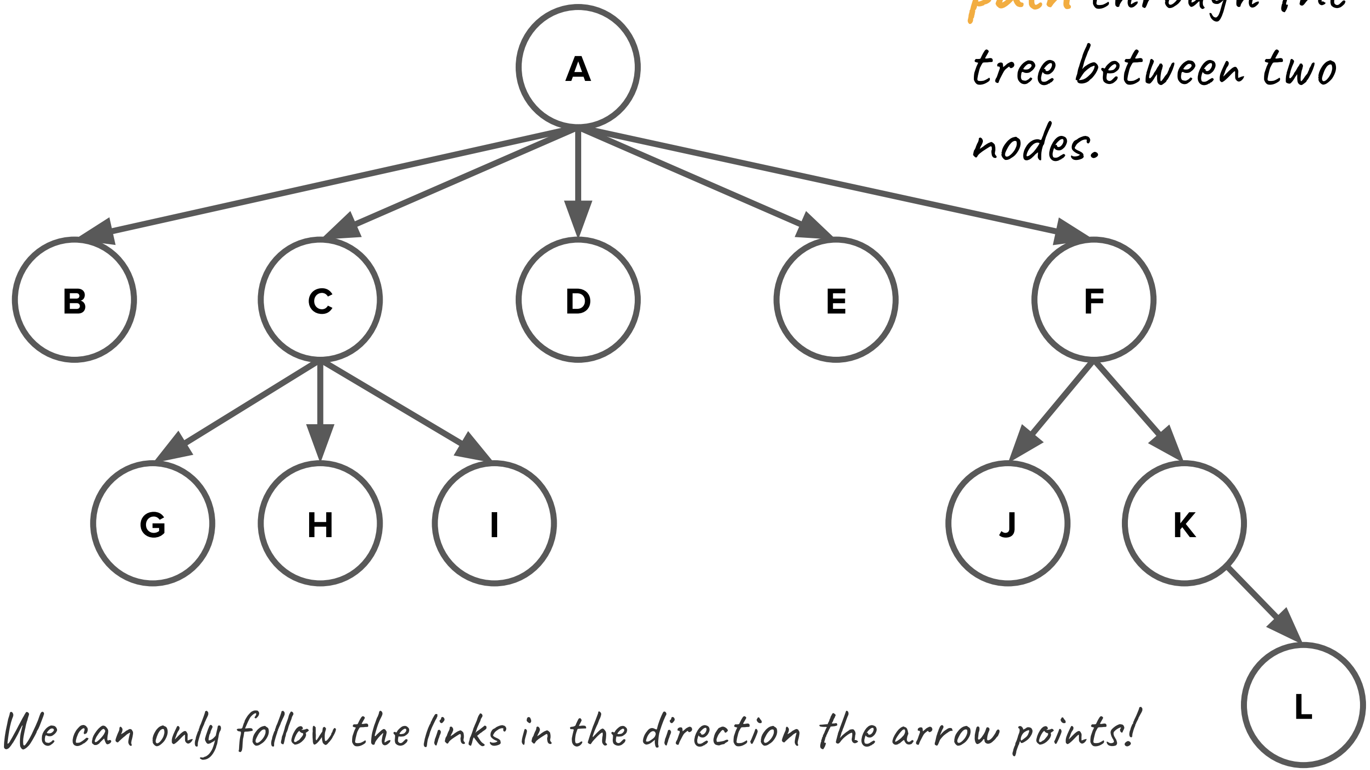


Tree Terminology

B, G, H, I, D, E, J, and L are all leaf nodes.



Tree Terminology

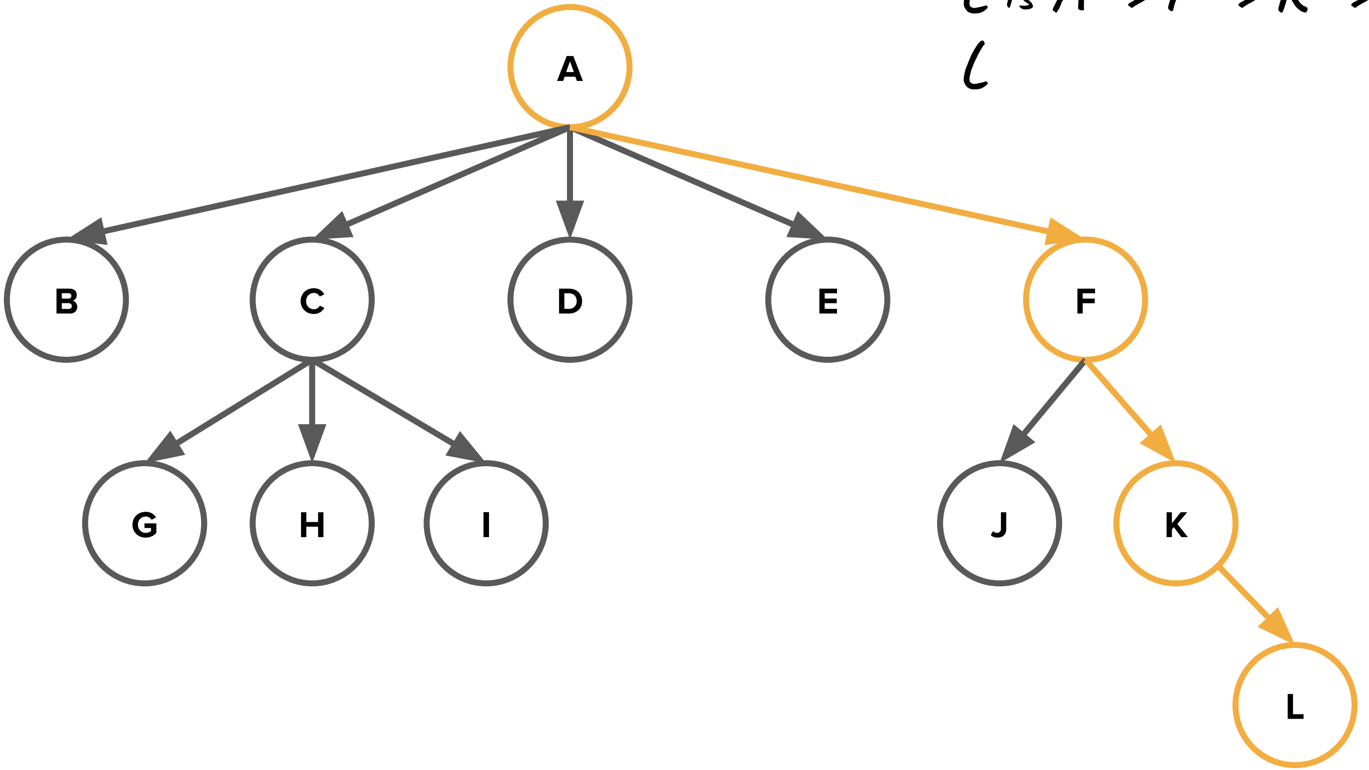


We can define a *path* through the tree between two nodes.

Note: We can only follow the links in the direction the arrow points!

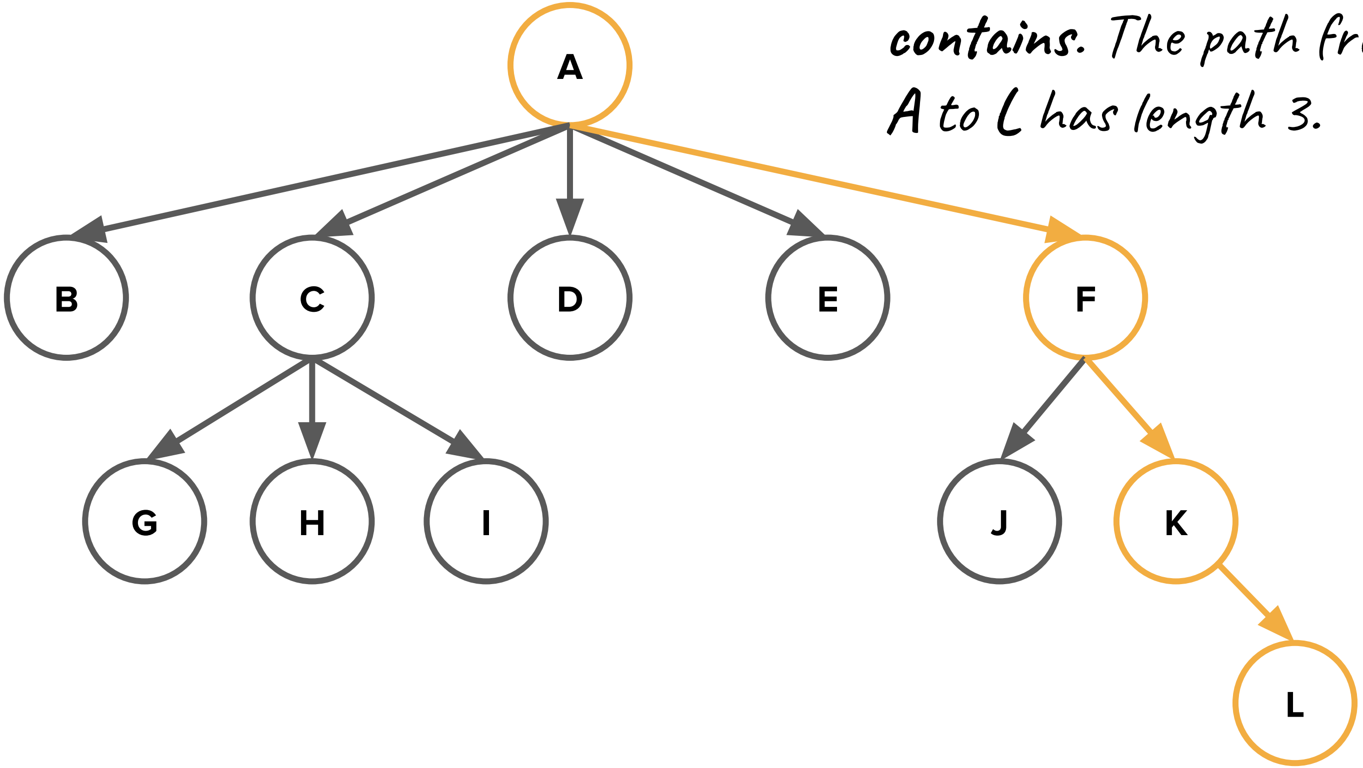
Tree Terminology

The path from A to L is A -> F -> K -> L



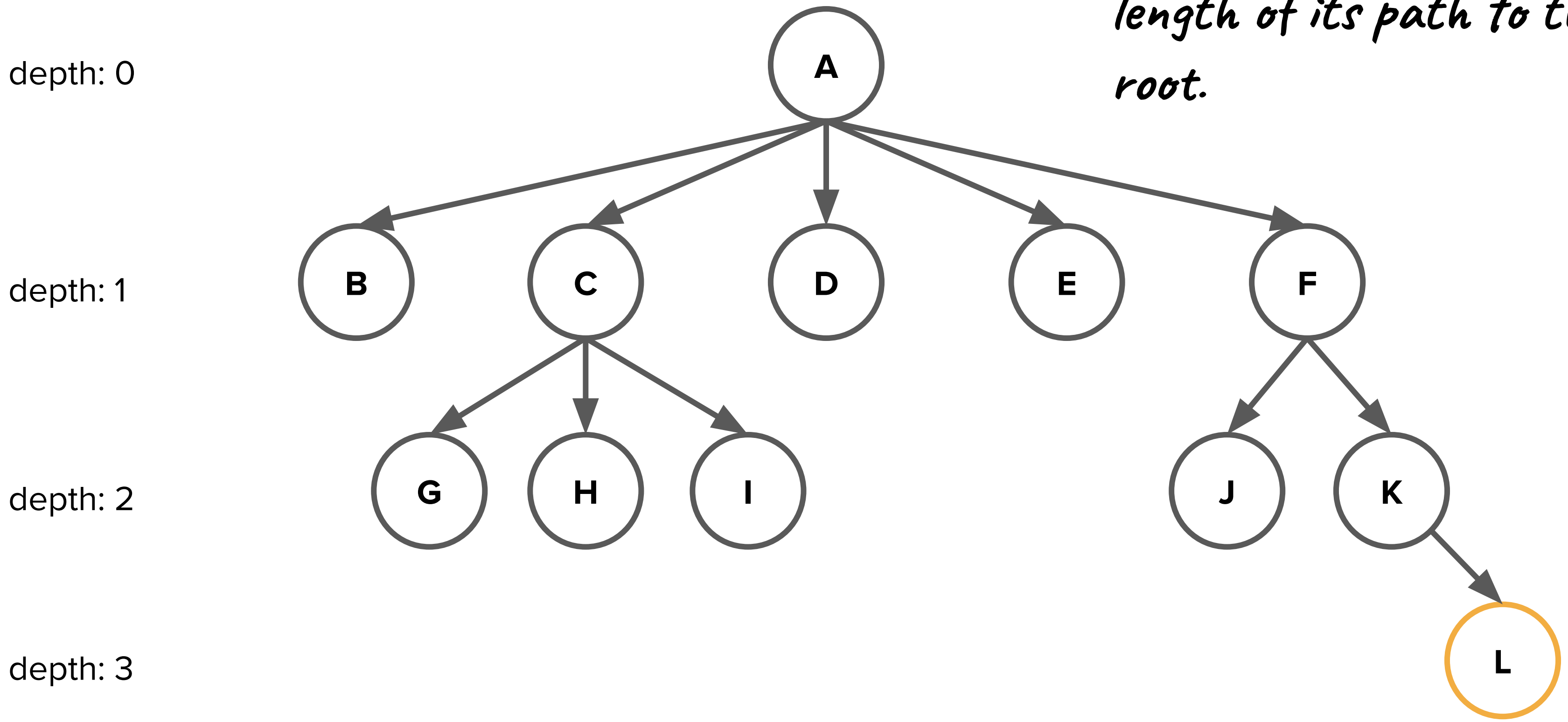
Tree Terminology

The length of the path is number of edges it contains. The path from A to L has length 3.



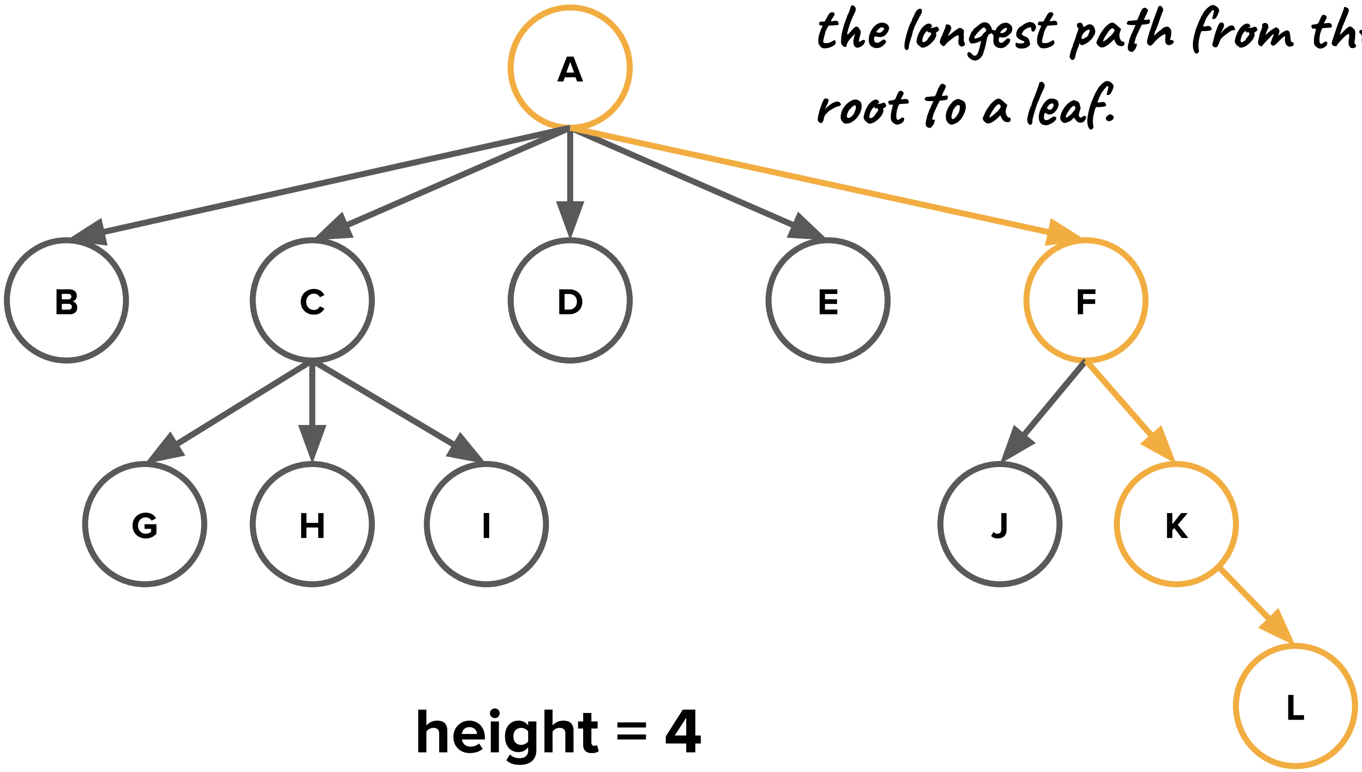
Tree Terminology

The depth of a node is the length of its path to the root.



Tree Terminology

The **height** can also be defined as the number of nodes along the longest path from the root to a leaf.



height = 4

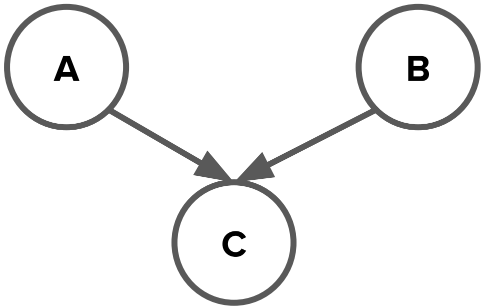
Tree Terminology Summary

- Every non-empty tree has a **root node** that defines the "top" of the tree.
- Every node has 0 or more **children** nodes descended from it. Nodes with no children are called **leaf nodes**.
- Every node in a tree has exactly one **parent** node (except for the root node).
- A **path** through the tree traverses edges between parents and their children.
- The **depth** of a node is the length of the path between the root and that node. A tree's **height** is the number of nodes in the longest path through the tree.

Tree Properties

Tree Properties

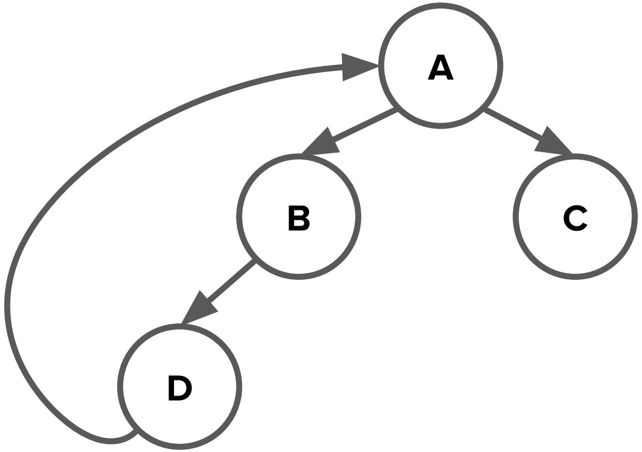
- Any node in a tree can only have one parent.



Not a tree!

Tree Properties

- Any node in a tree can only have one parent.
- The tree cannot have any cycles. That is, there should be no way to make a complete loop through the tree.



Not a tree!

Announcements

Announcements

- Assignment 5 is due on **Tuesday, August 4 at 11:59pm PDT**.

- Assignment 6 will be released by the end of the day on Wednesday and will be be due on **Wednesday, August 12 at 11:59pm PDT**. This is a hard deadline – there is **no grace period and no submissions will be accepted after this time**.

- Due to the end of quarter timeline, there will be **no revisions on Assignments 5 and 6.**

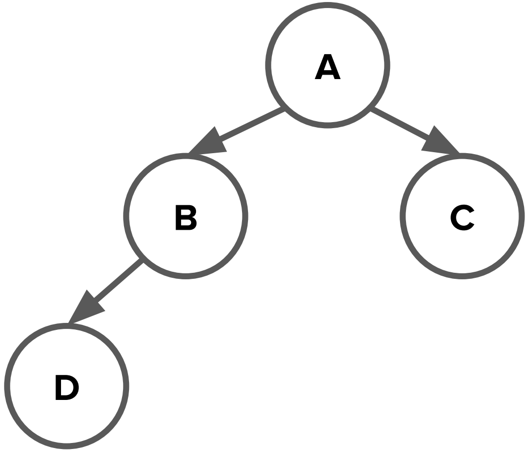
- Final project reports are due on Sunday, August 9 at 11:59pm PDT. You will have the opportunity to schedule your final presentation time after submitting.

Trees in C++

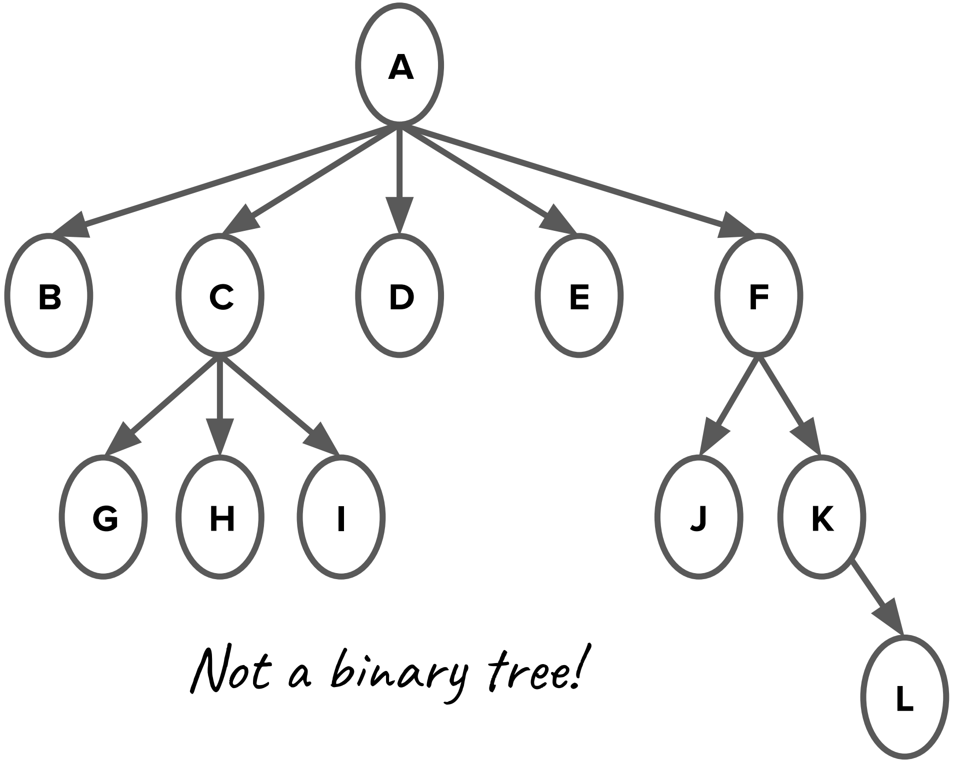
Binary Trees

- In general, we've seen that nodes in a tree can have variable numbers of children (subtrees) and sometimes very, very many.
- However, when working with trees in computer programs, it is common to work mostly with **binary trees**.
- A **binary tree** is a tree where every node has either 0, 1, or 2 children. No node in a binary tree can have more than 2 children.
- Typically, the two children of a node in a binary tree are referred to as the **left child** and the **right child**.

Binary Trees



Binary Tree!



Not a binary tree!

Building Trees Programmatically

- To build a tree in C++, we need a new version of the Node struct we've seen before.

- In this case, we want each Node to have a data value (like a linked list), but now we want two pointers, one to the left child, and one to the right child.

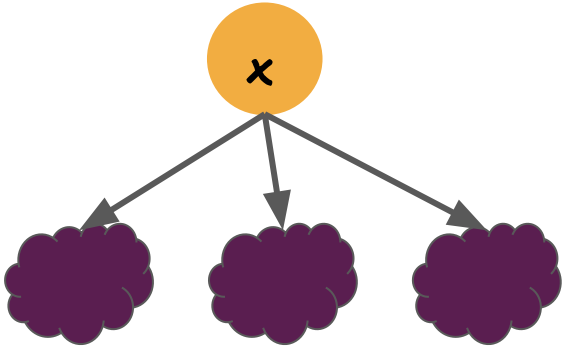
```
struct TreeNode {
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

What is a tree in C++?

A tree is either...

An empty data structure, or...

A single node (parent), with zero or more non-empty subtrees (children)

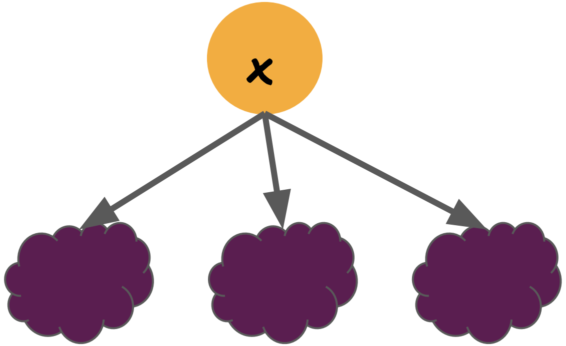


What is a tree in C++?

A tree is either...

An empty tree represented by **`nullptr`**, or...

A single node (parent), with zero or more non-empty subtrees (children)

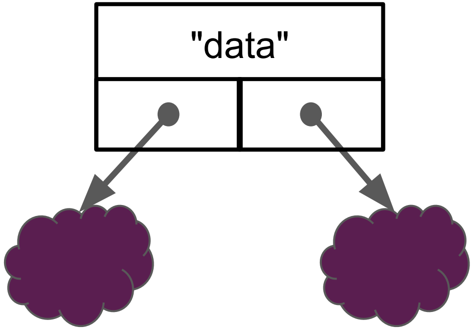


What is a tree in C++?

A tree is either...

An empty tree represented by **`nullptr`**, or...

A single **`TreeNode`**, with 0, 1, or 2 non-null pointers to other **`TreeNodes`**



Building Trees Programmatically

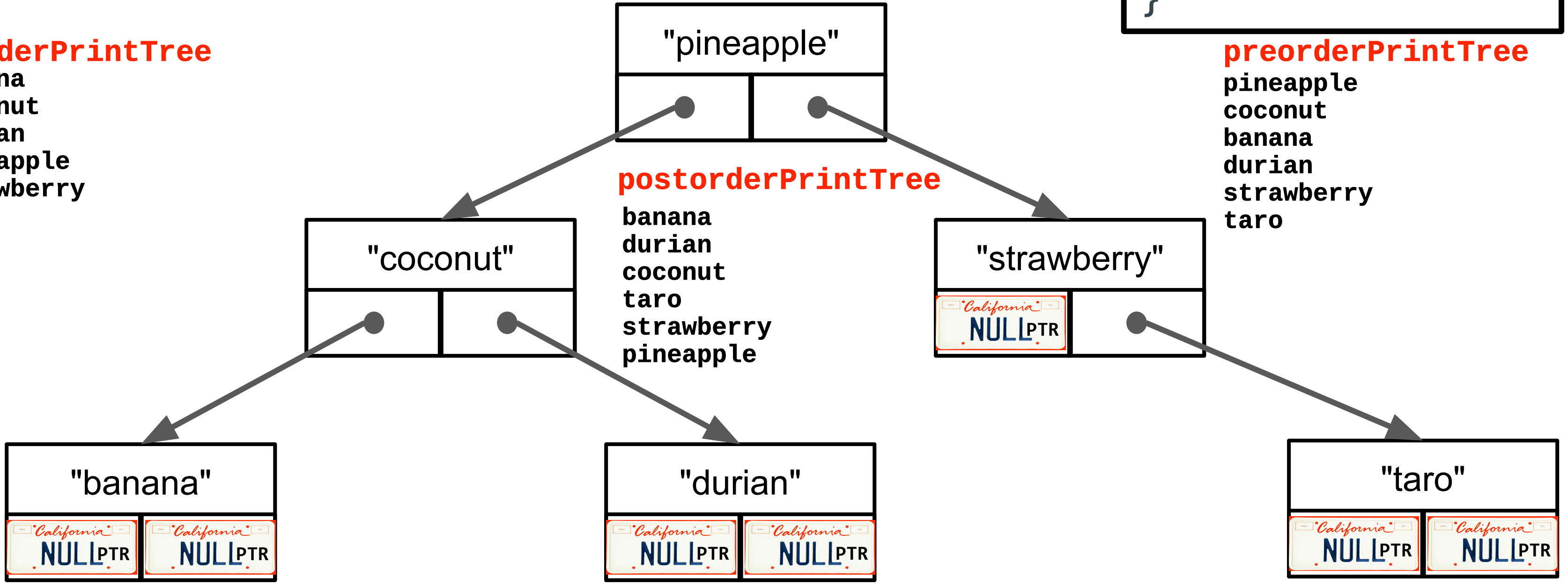
```
stru
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

```
stru
    string data;
    TreeNode* left;
    TreeNode* right;
}
```

Building Trees Programmatically

inorderPrintTree

```
banana
coconut
durian
pineapple
strawberry
taro
```



postorderPrintTree

```
banana
durian
coconut
taro
strawberry
pineapple
```

preorderPrintTree

```
pineapple
coconut
banana
durian
strawberry
taro
```

Note: Trees do not have to be complete, like heaps. Any node can have 0, 1, or 2 children.

Let's code it!

`buildExampleTree()`

Building a Tree Takeaways

- Building a tree is very similar to the process of building a linked list.
- We create new nodes of the tree by dynamically allocating memory.
- We integrate these new nodes into the tree by rewiring the **`left`** and **`right`** pointers of existing nodes in the tree.

Tree Traversals

Tree Traversals

- Often, we will want to "do something" with each node in a tree. Like linked lists, we can do so by **traversing the tree**. With the branching involved, this is a slightly more involved process than traversing a linked list!

- There are three main ways to traverse a binary tree:
  - Pre-order traversal
  - In-order traversal
  - Post-order traversal

- Due to the recursive nature of trees, all of these algorithms are most easily defined **recursively**.

Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
  - Traverse the left subtree
  - Traverse the right subtree

- For example purposes, let's have our "do something" to be printing the contents of the current node, which will allow us to print the overall tree.

Let's code it!

`preorderPrintTree()`

Pre-order Traversal

- The algorithm for a pre-order traversal is defined as follows:
  - "Do something" with the current node
  - Traverse the left subtree
  - Traverse the right subtree

- For example purposes, let's have our "do something" be printing the contents of the current node, which will allow us to print the overall tree.

- Output: **`pineapple coconut banana durian strawberry taro`**

In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree

Let's code it!

`inorderPrintTree()`

In-order Traversal

- The algorithm for an in-order traversal is defined as follows:
  - Traverse the left subtree
  - "Do something" with the current node
  - Traverse the right subtree

- Output: **`banana coconut durian pineapple strawberry taro`**

- Observation: The output of this traversal gives as all the values in alphabetical order. Is this a coincidence?
  - No! We'll see why tomorrow!

Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
  - Traverse the left subtree
  - Traverse the right subtree
  - "Do something" with the current node

Try it yourself!

`postorderPrintTree()`

Post-order Traversal

- The algorithm for a post-order traversal is defined as follows:
  - Traverse the left subtree
  - Traverse the right subtree
  - "Do something" with the current node

- Output: **`banana durian coconut taro strawberry pineapple`**

- Application: Freeing trees! (we'll see this in lecture tomorrow)

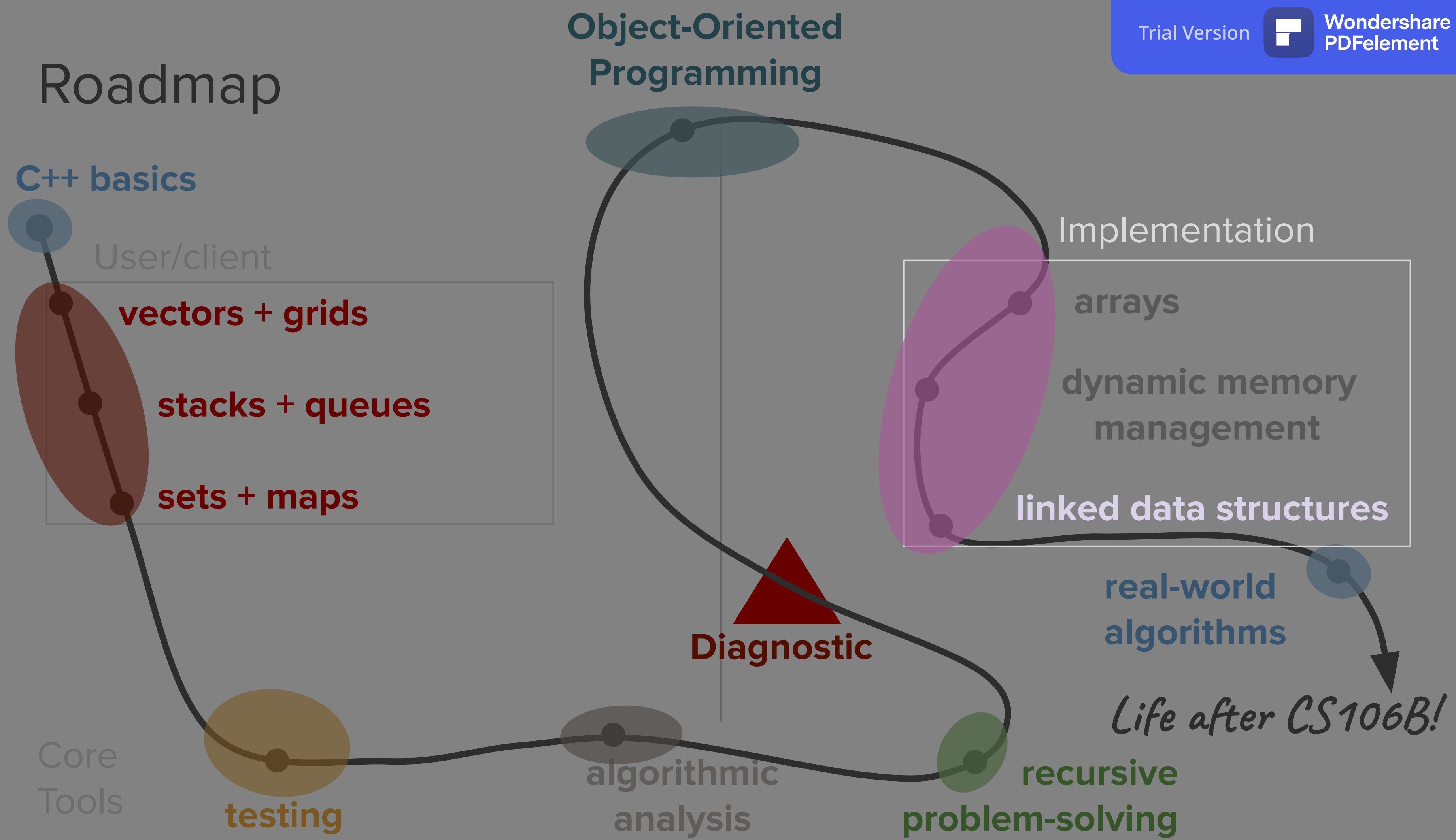
Summary

Trees Summary

- Trees allow us to organize information in a linked data structure such that the distance to any element is short, even if there are many elements.
- Trees organize nodes in a hierarchical manner, where each element contains connections to children nodes that exist "lower" in the tree.
- There are three main ways to traverse the nodes in a tree, and each type of traversal visits the nodes of the tree in a distinctly different order.

What’s next?

Roadmap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Binary Search Trees