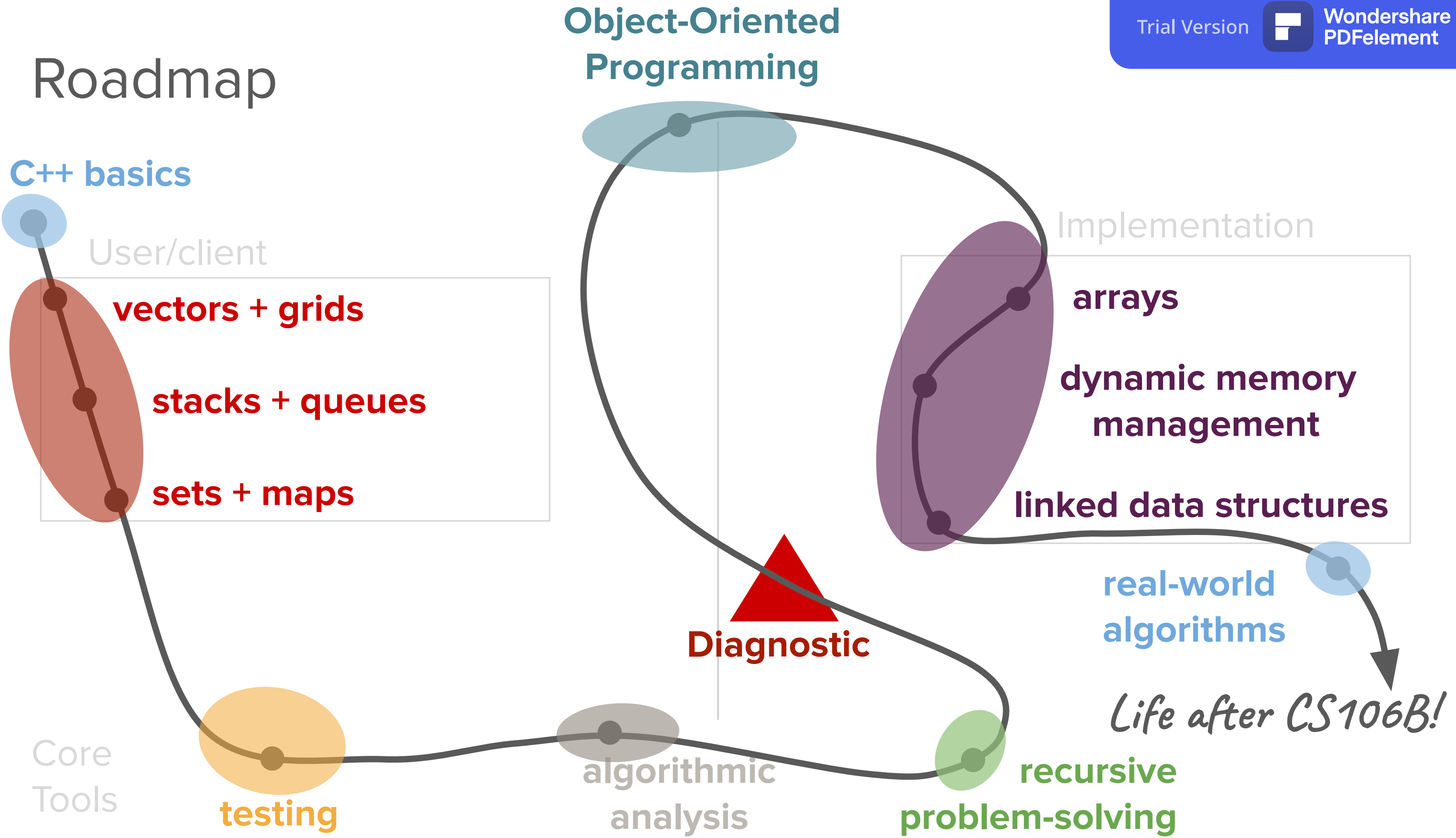
Introduction to Recursion

What's been the most challenging part of Assignment 2 for you so far?

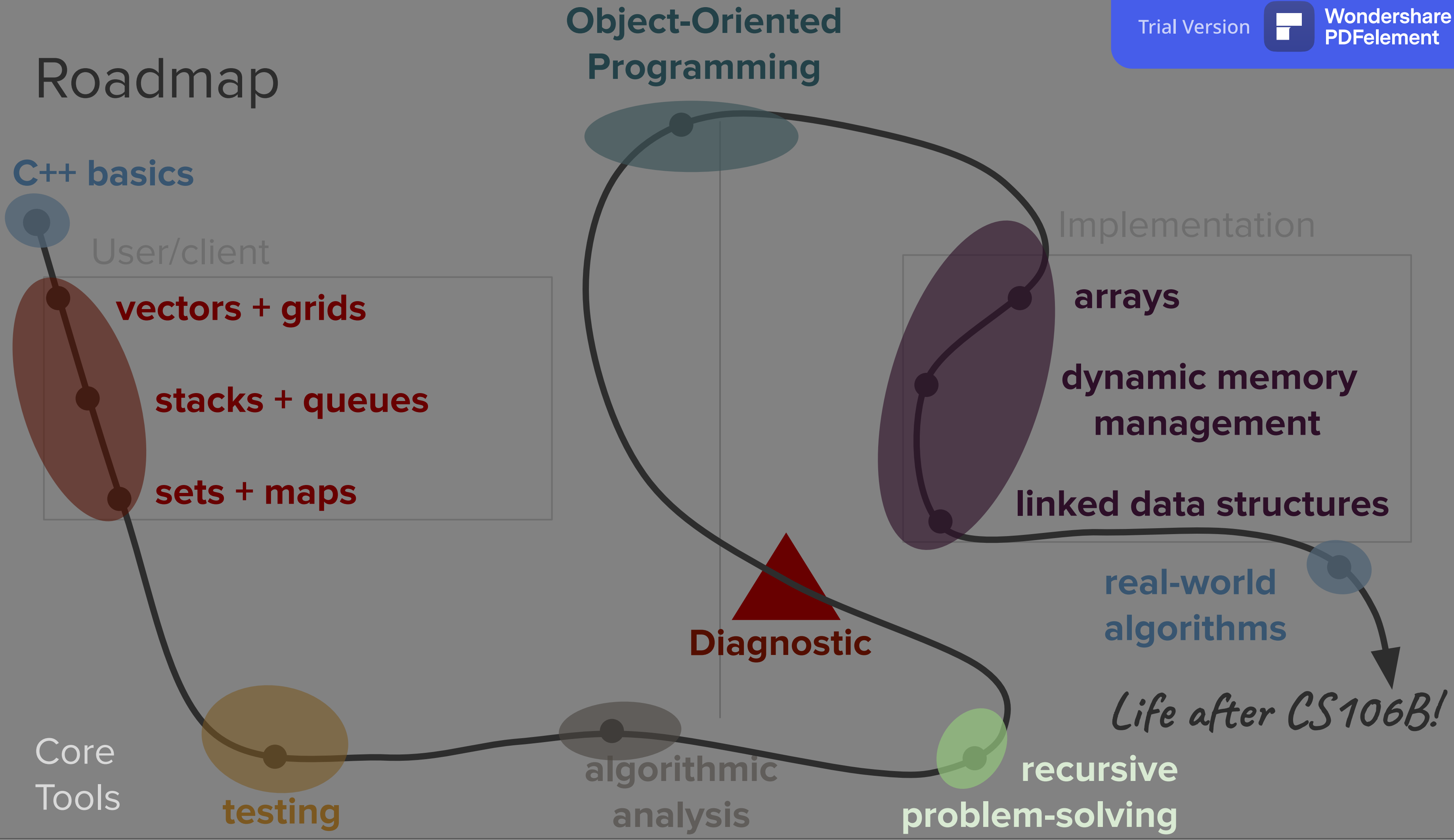
(put your answers the chat)

Roadmap



Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Roadmap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Today's question

How can we take advantage of self-similarity within a problem to solve it more elegantly?

Today's topics

1. Review
2. Defining recursion
3. Recursion + Stack Frames (e.g. factorials) 阶乘
4. Recursive Problem-Solving (e.g. string reversal)

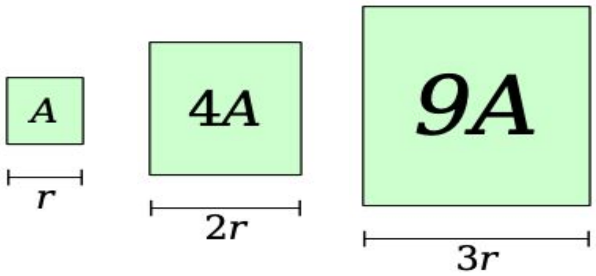
Review

(Big O)

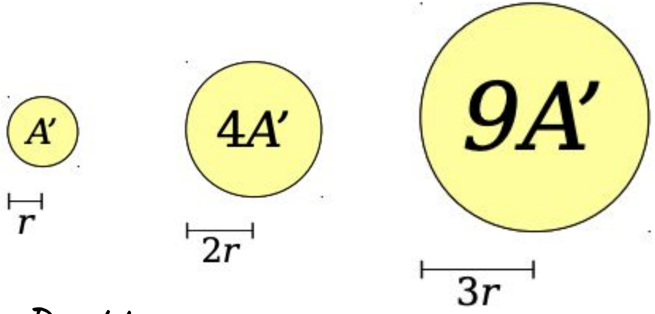
Big-O Notation

- **Big-O notation** is a way of quantifying the rate at which some quantity grows.
- Example:
  - A square of side length `r` has area $O(r^2)$.
  - A circle of radius `r` has area $O(r^2)$.

This just says that these quantities grow at the same relative rates. It does not say that they're equal!



Doubling r increases area 4x
Tripling r increases area 9x



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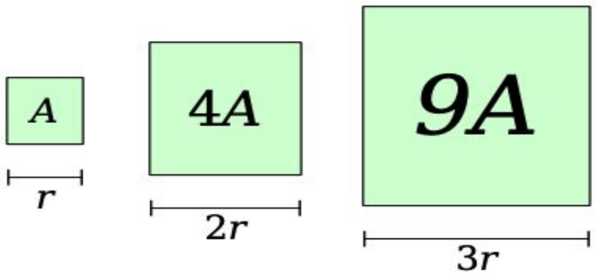
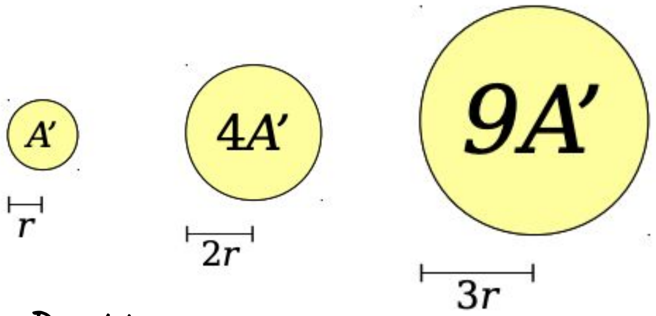
With respect to a given input variable.

Big-O Notation

- **Big-O notation** is a way of quantifying the **rate at which some quantity grows**.
- Example:
  - A square of side length `r` has area `O(r²)`.
  - A circle of radius `r` has area `O(r²)`.





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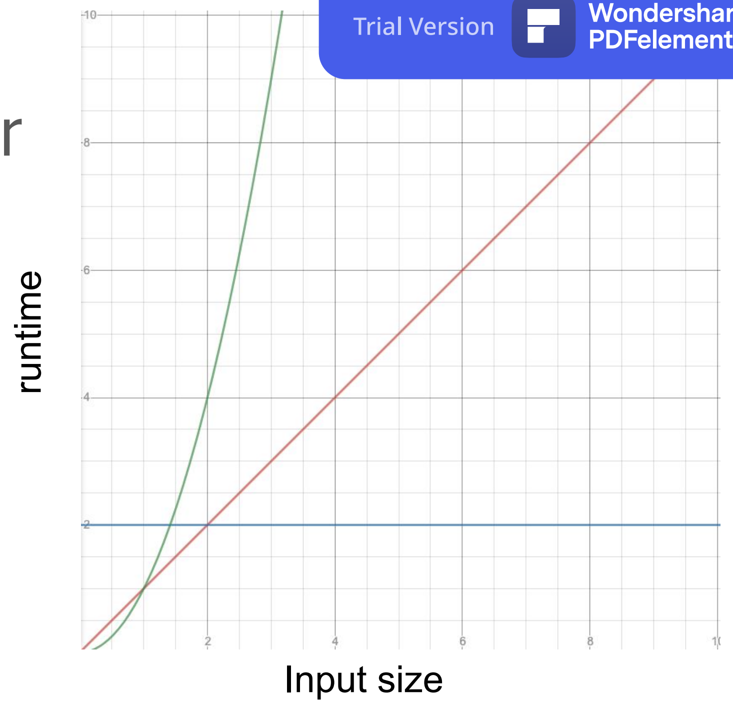
Efficiency Categorizations So Far

- Constant Time – O(1)
  - Super fast, this is the best we can hope for!
  - Euclid's Algorithm for Perfect Numbers
- Linear Time – O(n)
  - This is okay; we can live with this
- Quadratic Time – O(n^2)
  - This can start to slow down really quickly
  - Exhaustive Search for Perfect Numbers
- How do all the ADT operations we've seen so far fall into these categories?



ADT Big-O Matrix

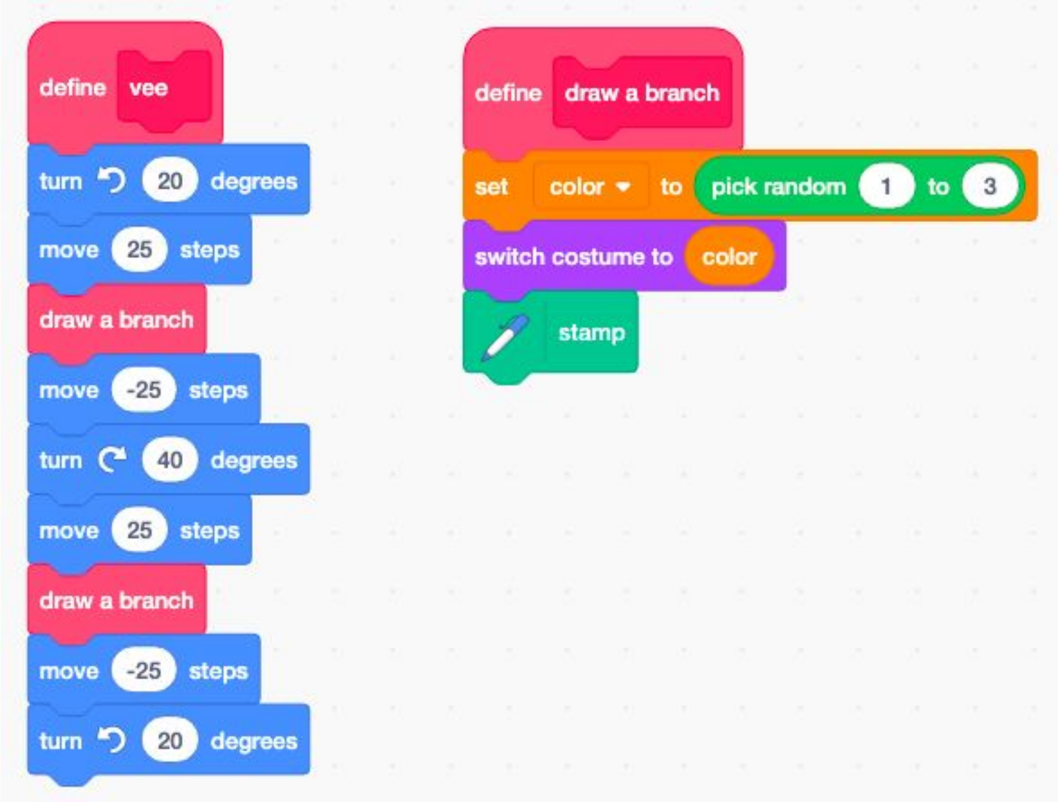
- Vectors
  - `.size() – O(1)`
  - `.add() – O(1)`
  - `v[i] – O(1)`
  - `.insert() – O(n)`
  - `.remove() – O(n)`
  - `.clear() – O(n)`
  - `traversal – O(n)}`
- Grids
  - `.numRows()/.numCols() – O(1)`
  - `g[i][j] – O(1)`
  - `.inBounds() – O(1)`
  - `traversal –` $O(n^2)$
- Queues
  - `.size() – O(1)`
  - `.peek() – O(1)`
  - `.enqueue() – O(1)`
  - `.dequeue() – O(1)`
  - `.isEmpty() – O(1)`
  - `traversal – O(n)`
- Stacks
  - `.size() – O(1)`
  - `.peek() – O(1)`
  - `.push() – O(1)`
  - `.pop() – O(1)`
  - `.isEmpty() – O(1)`
  - `traversal – O(n)`
- Sets
  - `.size() – O(1)`
  - `.isEmpty() – O(1)`
  - `.add() – ???`
  - `.remove() – ???`
  - `.contains() – ???`
  - `traversal – O(n)`
- Maps
  - `.size() – O(1)`
  - `.isEmpty() – O(1)`
  - `m[key] – ???`
  - `.contains() – ???`
  - `traversal – O(n)`

What is recursion?

Activity: Vee

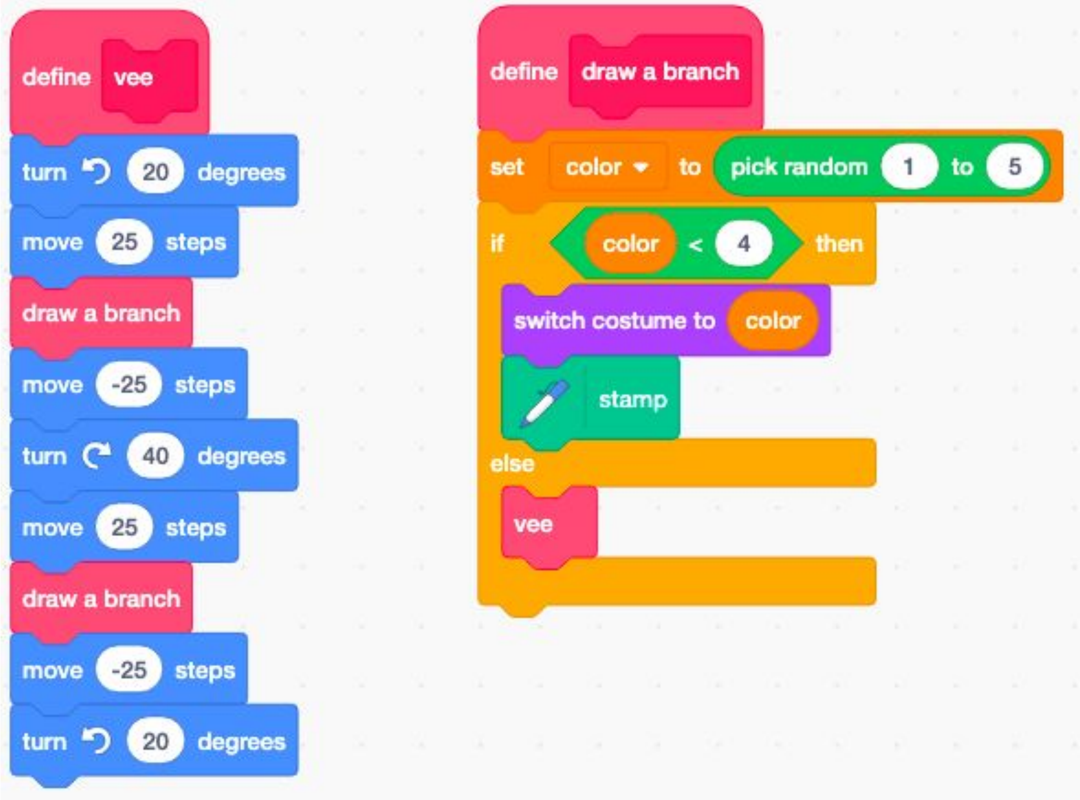
(https://scratch.mit.edu/projects/409796637/)

This code creates a “vee” shape with random colors.

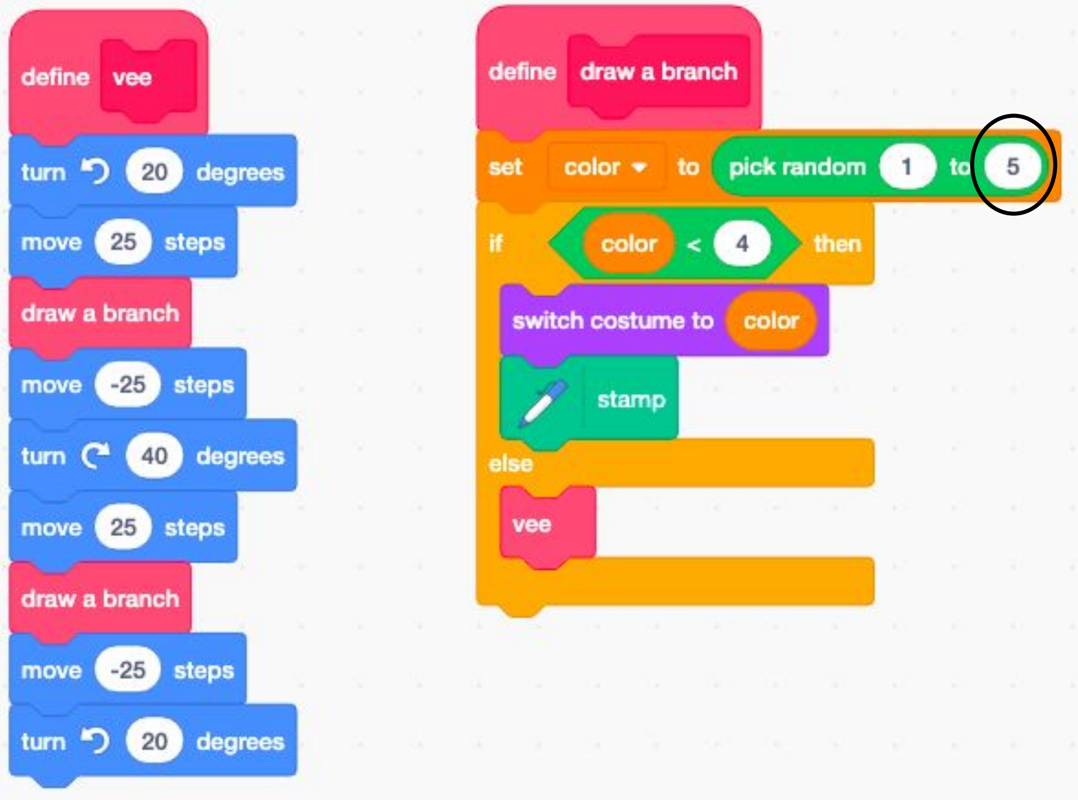


Based on Vee by Dan Garcia

Discuss in breakout rooms: What will this code do?



Discuss in breakout rooms: What will this code do?



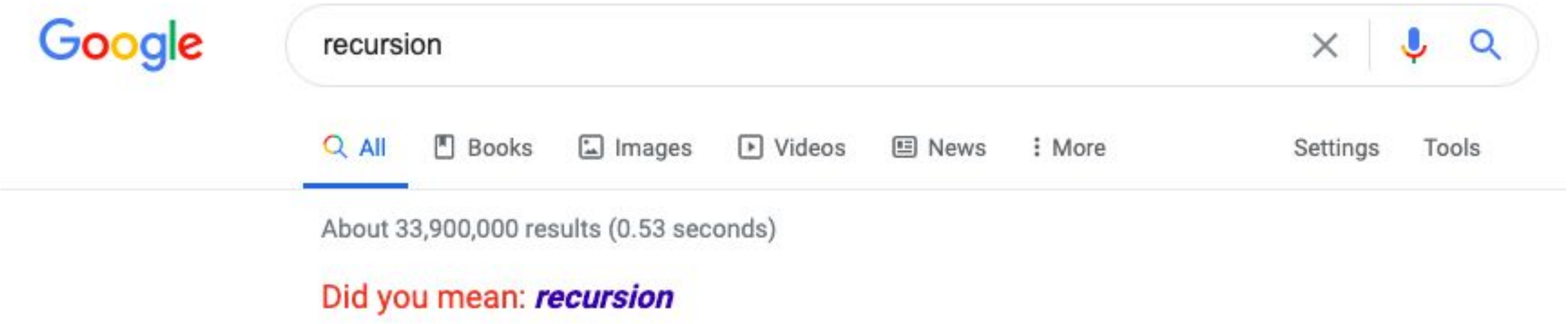
Notice the differences

Demo: Recursive Vee

(https://scratch.mit.edu/projects/409785610/)

What is recursion?

Wikipedia: “Recursion occurs when a thing is defined in terms of itself.”



Definition

recursion

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

What is recursion?

- A powerful substitute for iteration (loops)
  - We'll start off with seeing the difference between iterative vs. recursive solutions
  - Later in the week we'll see problems/tasks that can only be solved using recursion
- Results in elegant, often shorter code when used well
- Often applied to sorting and searching problems and can be used to express patterns seen in nature
- Will be part of many of our future assignments!

How many students are in a lecture hall?

A [non-COVID] analogy

How many students are in the lecture hall?

- Let's suppose I want to find out how many people are at lecture today, but I don't want to walk around and count each person.

招募

- I want to recruit your help, but I also want to minimize each individual's amount of work.

We can solve this problem recursively!

How many students are in the lecture hall?

- We'll focus on solving the problem for single "column" of students.
  - I go to the first person in the front row and ask: "How many people are sitting directly behind you in your 'column'?"
  - Student's algorithm:
    - If there is no one behind me, answer 0.
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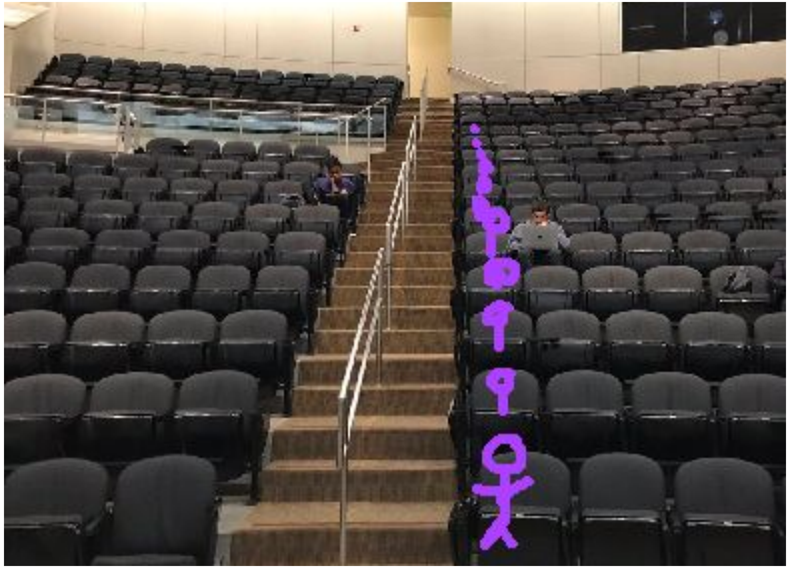

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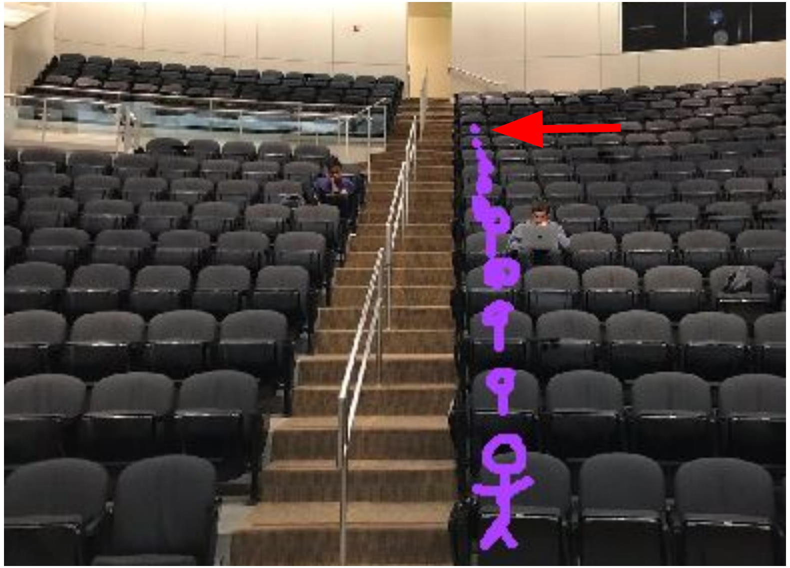

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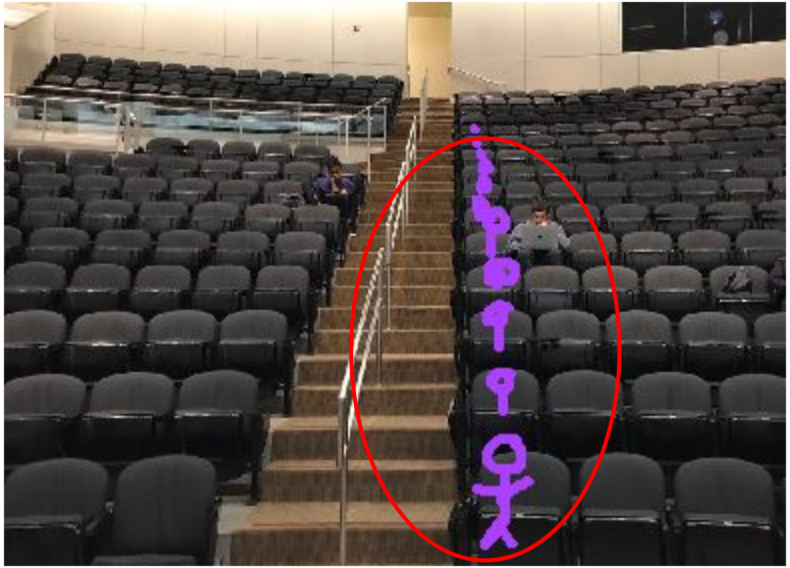

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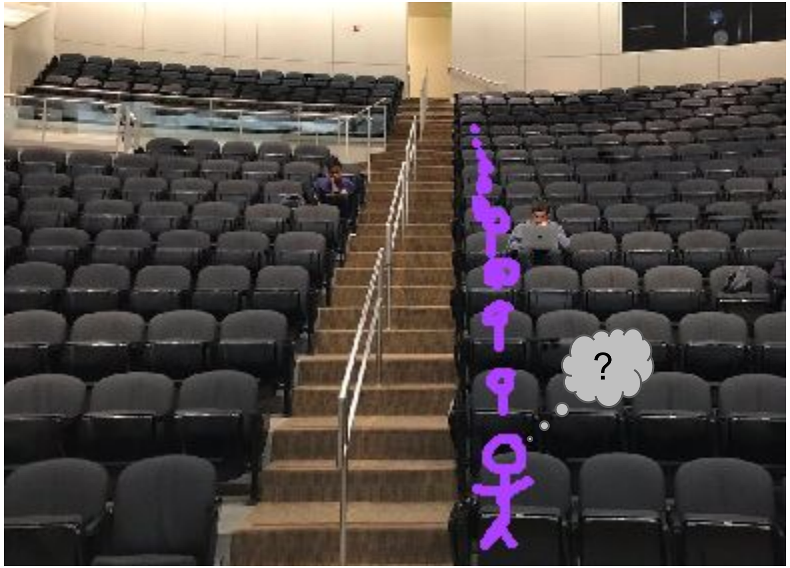

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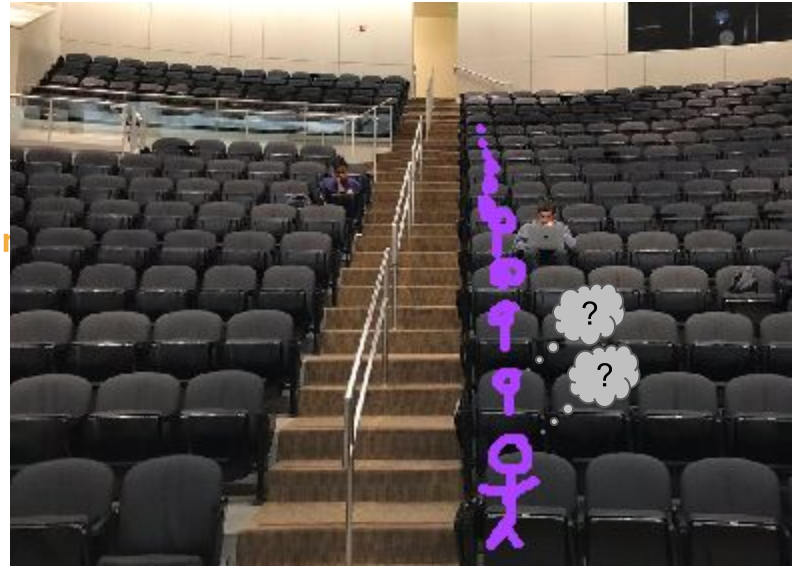

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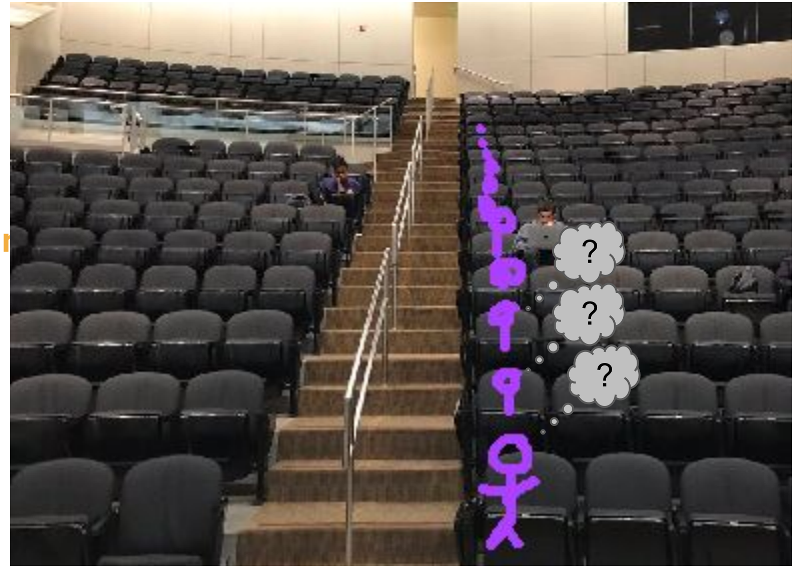

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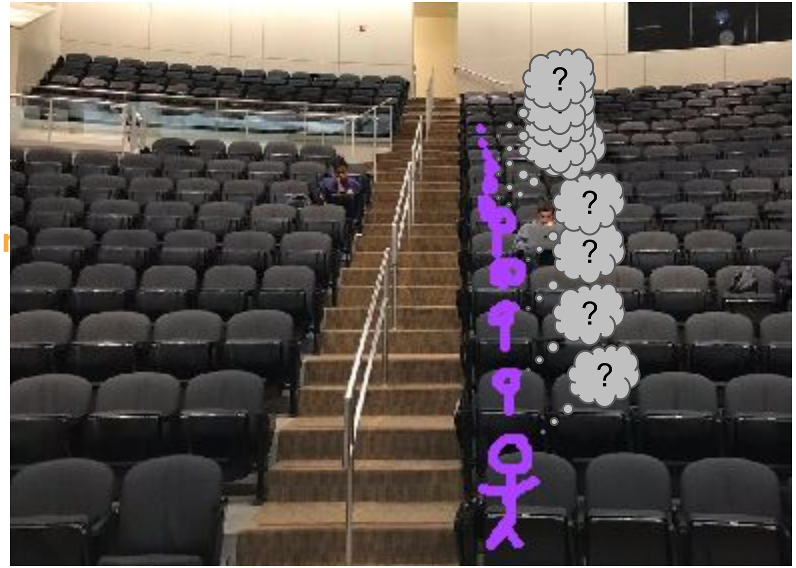

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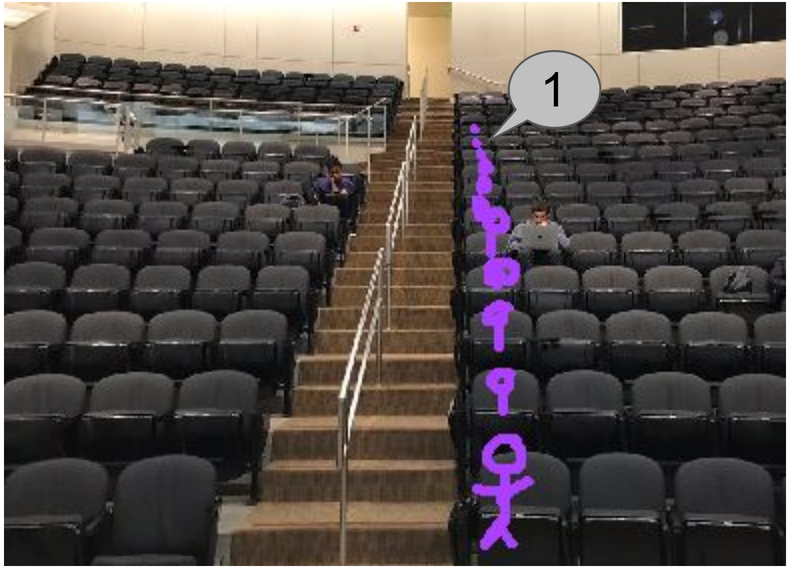

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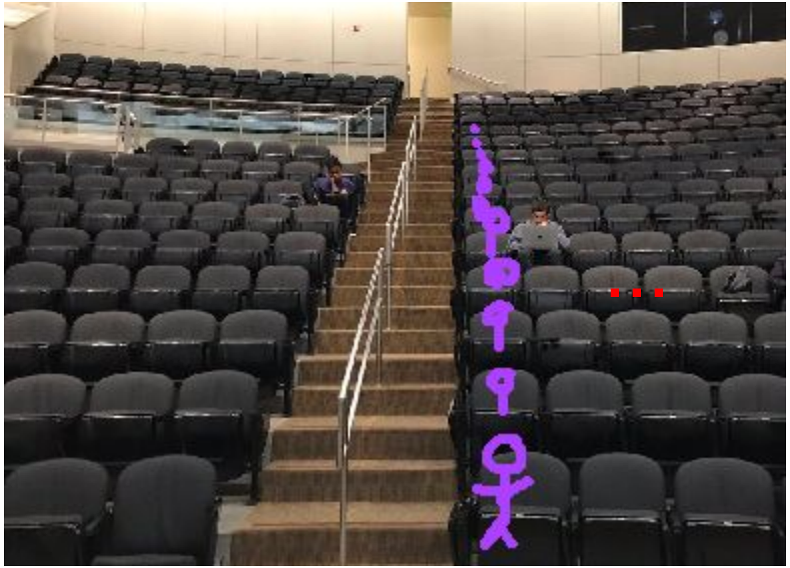

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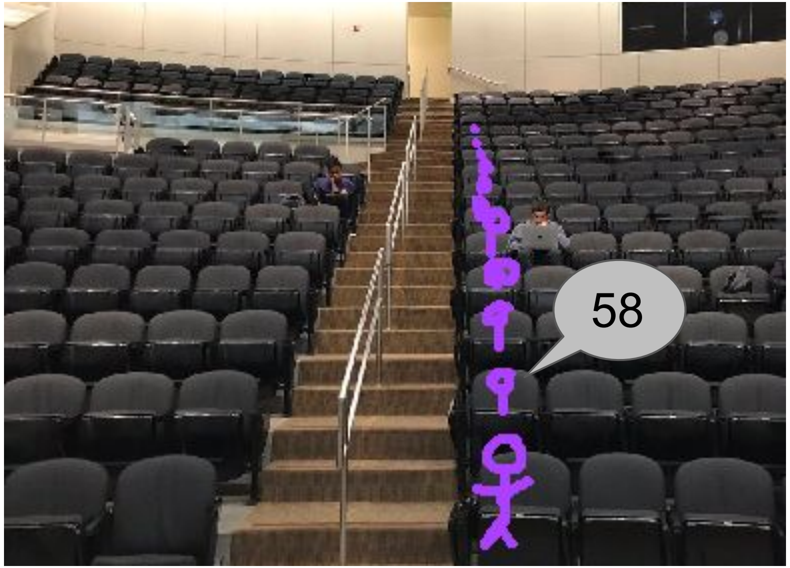

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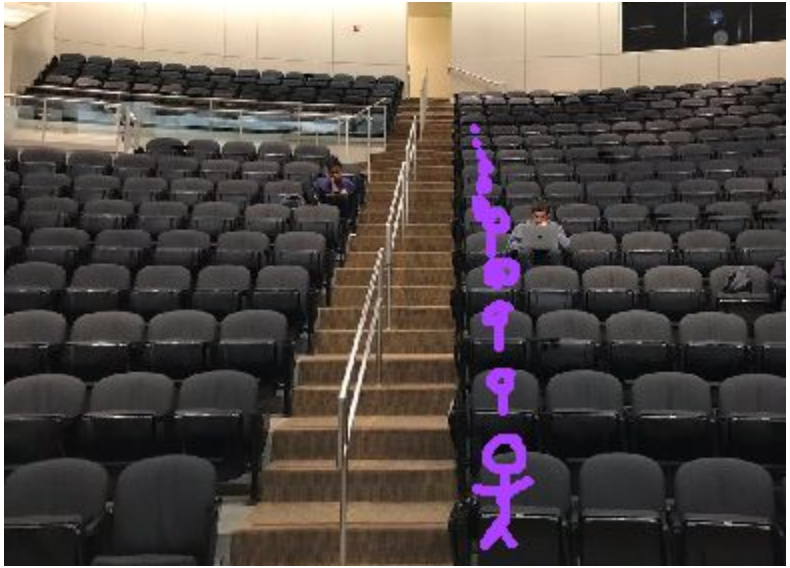

- Can generalize to the entire lecture hall!

Definition

recursion

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

Two main cases (components) of recursion

- Base case
  - The simplest version(s) of your problem that all other cases reduce to
  - An occurrence that can be answered directly

“If there is no one behind me, answer 0”

- Recursive case
  - The step at which you break down more complex versions of the task into smaller occurrences
  - Cannot be answered directly
  - Take the “recursive leap of faith” and trust the smaller tasks will solve the problem for you!

“If someone is sitting behind me … ”

Announcements

Announcements

- Assignment 2 is due Wednesday, 7/8.
- Assignment 3 will be released by the end of the day on Thursday.
- The mid-quarter diagnostic will cover through the end of this week (Thursday will be the last day of content covered).
- Please remember to only ask questions in the chat that are necessary for your immediate understanding!

Factorial example

Factorials

- The number **n factorial**, denoted **n!**, is

$$n \times (n - 1) \times \ldots \times 3 \times 2 \times 1$$

- For example,
  - `3! = 3 × 2 × 1 = 6.`
  - `4! = 4 × 3 × 2 × 1 = 24.`
  - `5! = 5 × 4 × 3 × 2 × 1 = 120.`
  - `0! = 1. (by definition)`
- Factorials show up in unexpected places. We'll see one later this quarter when we talk about sorting algorithms.
- Let's implement a function to compute factorials!

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
2! = 2 x 1!
1! = 1 x 0!
0! = 1
```

By definition!

Another view of factorials

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{otherwise} \end{cases}$$

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

堆栈结构

This is a "**stack frame**." One gets created each time a function is called.

- The "stack" is where in your computer's memory the information is stored.
- A "frame" stores all of the data (variables) for that particular function call.

Recursion in action

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    cout << "5! = " << n << endl;
    return 0;
}
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Recursion in action

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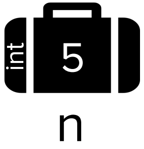

When a function gets called, a new stack frame gets created.

Recursion in action

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}
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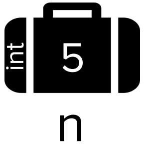

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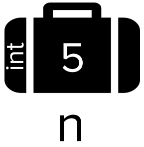

5

Recursion in action

```
int main() {

}
```

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int factorial (int n) {
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    }          5
}
```

n

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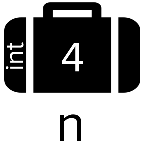

Every time we call **`factorial()`**, we get a new copy of the local variable **n** that's independent of all the previous copies because it exists inside the new frame.

Recursion in action

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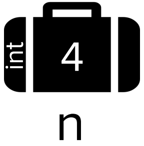

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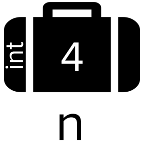


Recursion in action

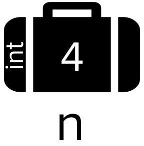
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

```
int main() {

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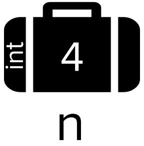
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

```
int main() {

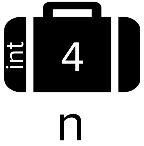
}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

$\boxed{n}$ = 4



Recursion in action

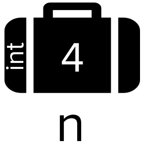
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }          4
}
```



Recursion in action

```
int main() {

}
```

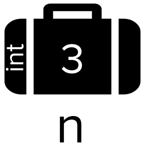
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

```
int main() {

}
```

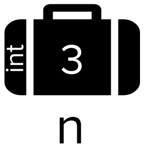
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



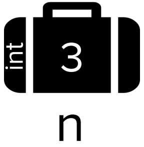
Recursion in action

```
int main() {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

```
int main() {

}
```

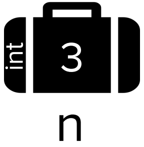
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

n

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

3

n

Recursion in action

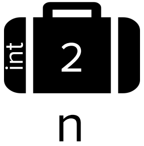
```
int main() {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

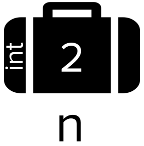
```
int factorial (int n) {
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



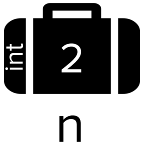
Recursion in action

```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    if (n == 0) {
                        return 1;
                    } else {
                        return n * factorial(n-1);
                    }
                }
            }
        }
    }
}
```



Recursion in action

```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    if (n == 0) {
                        return 1;
                    } else {
                        return n * factorial(n-1);
                    }
                }
            }
        }
    }
}
```



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
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```
int factorial (int n) {

}
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```
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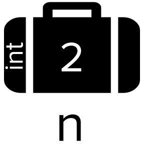
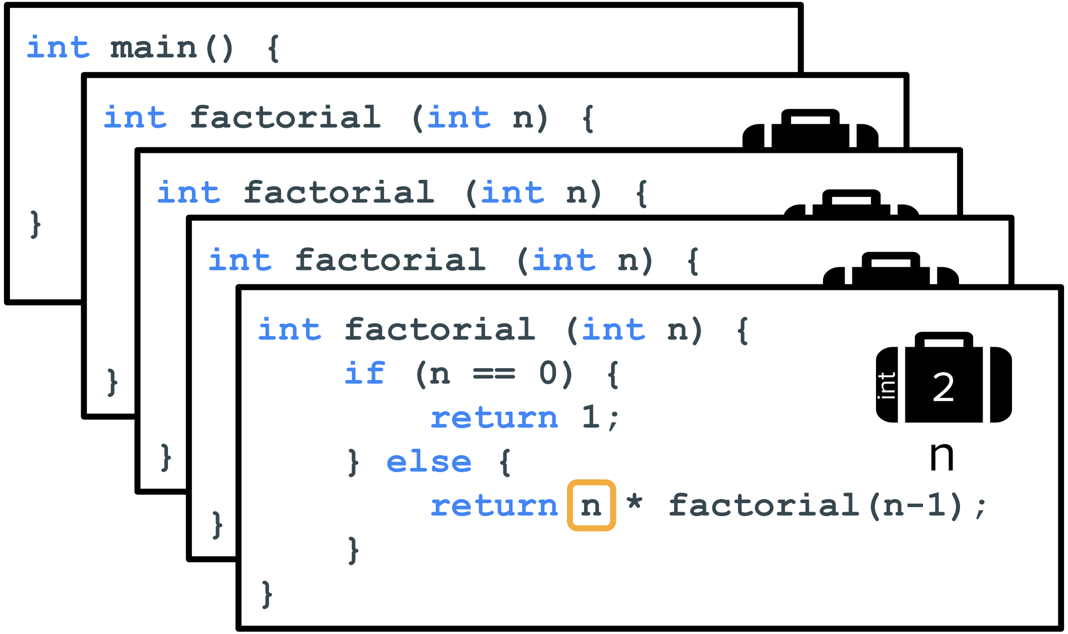
```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

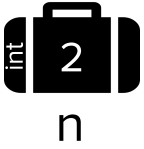
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }           2
}
```



Recursion in action

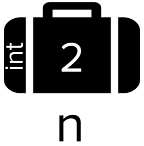
```
int main() {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }            2
}
```



Recursion in action

```
int main() {

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```

```
int factorial (int n) {

}
```

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}
```

```
int factorial (int n) {

}
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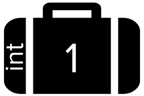
```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



n

Recursion in action

```
int main() {

}
```

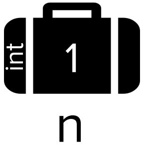
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

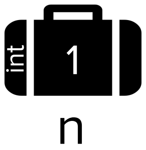
```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

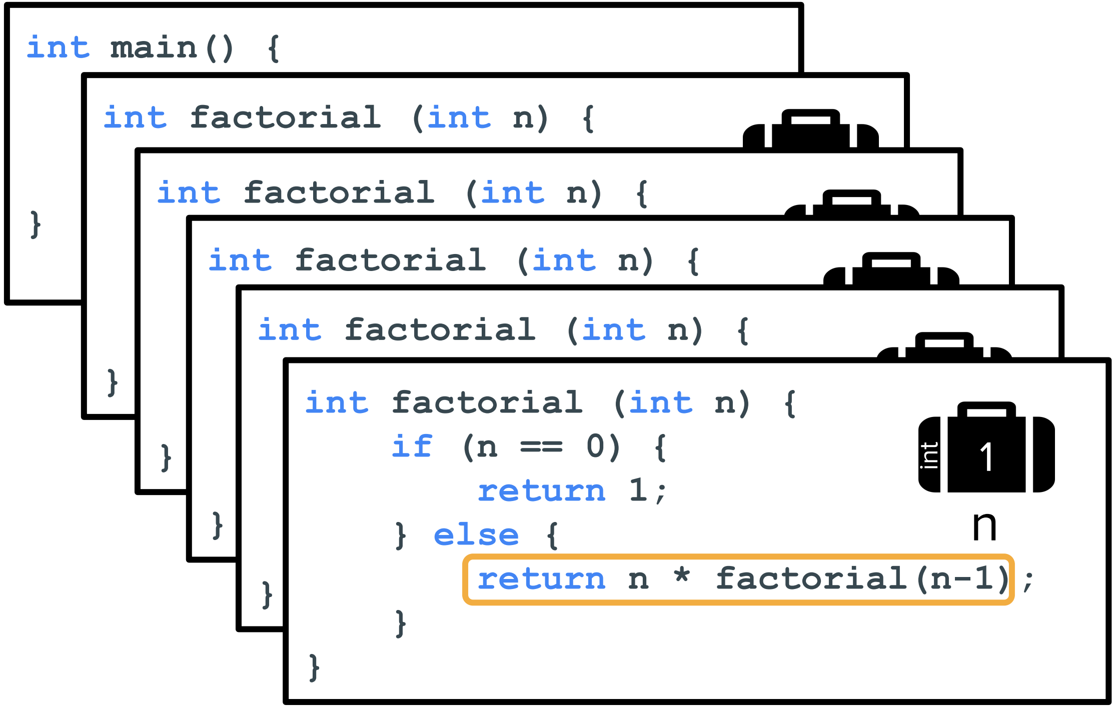


Recursion in action

```
int main() {
}
int factorial (int n) {
}
int factorial (int n) {
}
int factorial (int n) {
}
int factorial (int n) {
}
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action



Recursion in action

```
int main() {

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int factorial (int n) {

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```

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int factorial (int n) {

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        return 1;
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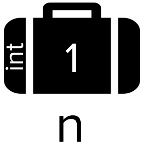
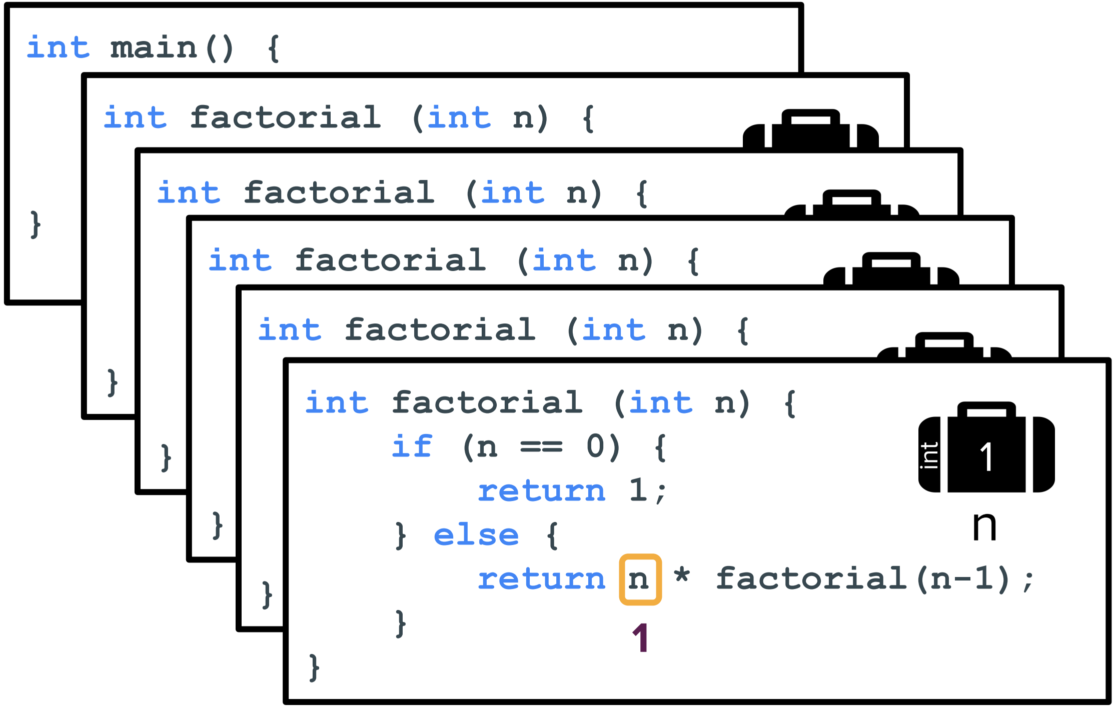
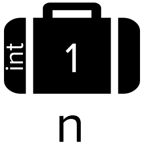


Recursion in action



Recursion in action

```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    int factorial (int n) {
                        if (n == 0) {
                            return 1;
                        } else {
                            return n * factorial(n-1);
                        }
                                   1
                    }
                }
            }
        }
    }
}
```



Recursion in action

```
int main() {
int factorial (int n) {
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int factorial (int n) {
int factorial (int n) {
int factorial (int n) {
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
}
}
}
}
}
}
```

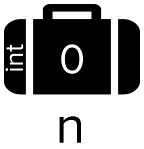

Recursion in action

```
int main() {

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```
int factorial (int n) {

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```

```
int factorial (int n) {

}
```

```
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```

```
int factorial (int n) {

}
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int factorial (int n) {

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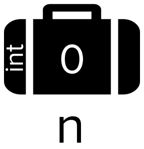
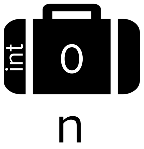
```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

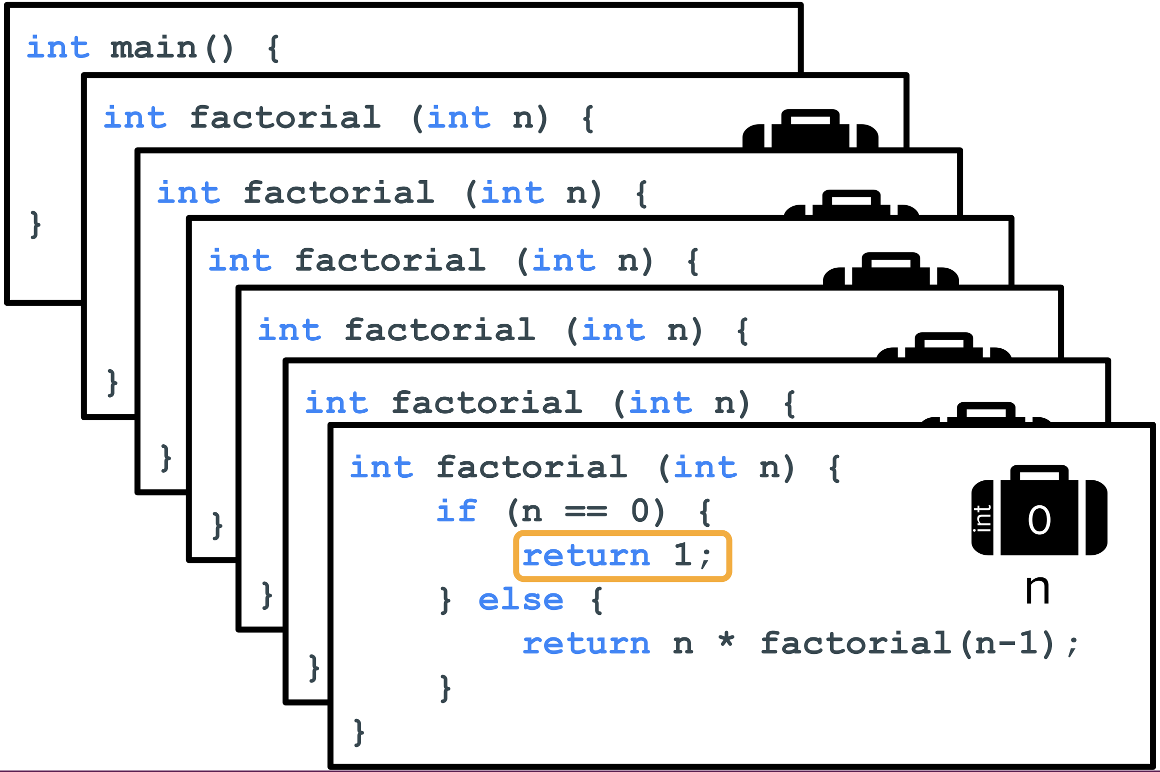


Recursion in action

```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    int factorial (int n) {
                        int factorial (int n) {
                            if (n == 0) {
                                return 1;
                            } else {
                                return n * factorial(n-1);
                            }
                        }
                    }
                }
            }
        }
    }
}
```



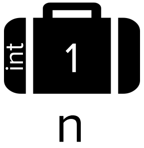
Recursion in action



Stack frames go away (get cleared from memory) once they return.

Recursion in action

```
int main() {
  int factorial (int n) {
    int factorial (int n) {
      int factorial (int n) {
        int factorial (int n) {
          int factorial (int n) {
              if (n == 0) {
                  return 1;
              } else {
                  return n * factorial(n-1);
              }
                        1
          }
        }
      }
    }
  }
}
```



Recursion in action

```
int main() {
}
```

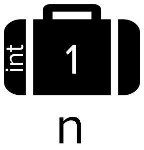
```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



1 1

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

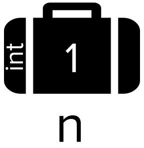
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }               1           1
}
```



Recursion in action

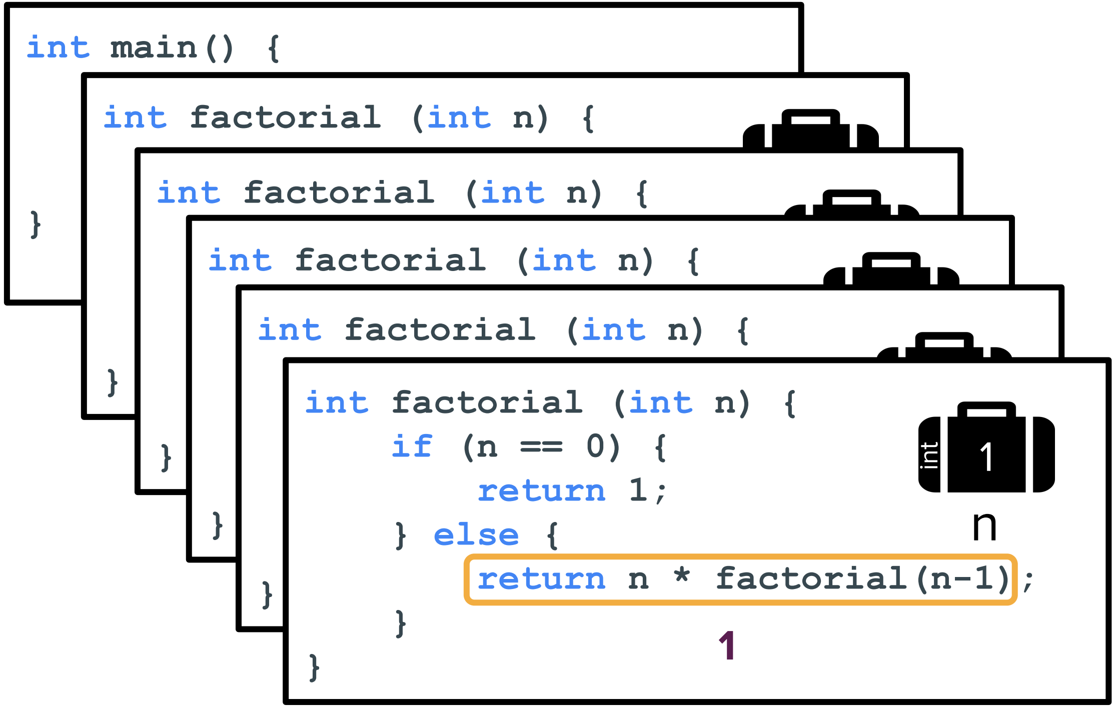
```
int main() {
  int factorial (int n) {
    int factorial (int n) {
      int factorial (int n) {
        int factorial (int n) {
          int factorial (int n) {
              if (n == 0) {
                  return 1;
              } else {
                  return n * factorial(n-1);
              }
          }
        }
      }
    }
  }
}
```

int 1

n

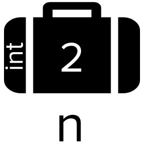
1 x 1

Recursion in action



Recursion in action

```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    if (n == 0) {
                        return 1;
                    } else {
                        return n * factorial(n-1);
                    }              2
                }
            }
        }
    }
}
```



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

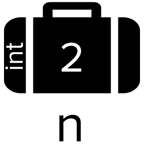
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }           2           1
}
```



Recursion in action

```
int main() {

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```

```
int factorial (int n) {

}
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int factorial (int n) {

}
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```
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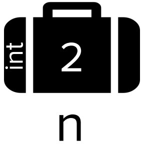
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }          2              1
}
```



Recursion in action

```
int main() {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
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```

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int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```
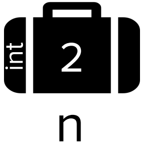



2 x 1

Recursion in action

```
int main() {

}
```

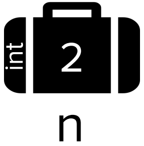
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

2



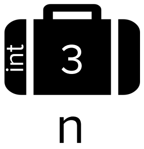
Recursion in action

```
int main() {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

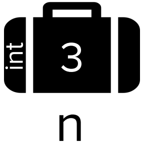
```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



3

Recursion in action

```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                if (n == 0) {
                    return 1;
                } else {
                    return n * factorial(n-1);
                }          3           2
            }
        }
    }
}
```



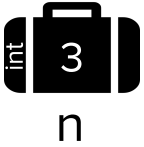
Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

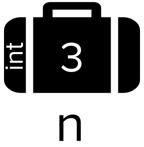
```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



3 2

Recursion in action

```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                if (n == 0) {
                    return 1;
                } else {
                    return n * factorial(n-1);
                }
            }
        }
    }
}
```



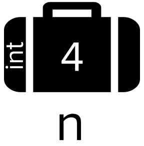
6

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }         4
}
```

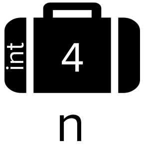


Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



4

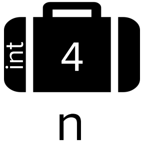
6

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



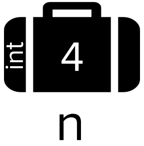
4 6

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



4 x 6

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

n

24

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

5

n

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

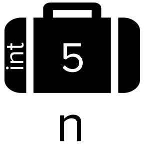
n

5 24

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }          5           24
}
```



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

5 x 24

n

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

n

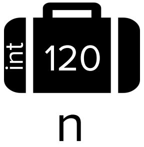
120

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```



Recursive vs. Iterative

[Qt Creator]

Reverse string example

How can we reverse a string?

Suppose we want to reverse strings like in the following examples:

"dog" → "god"

"stressed" → "desserts"

"recursion" → "noisrucer"

"level" → "level"

"a" → "a"

Approaching recursive problems

- Look for self-similarity.

- Try out an example.
  - Work through a simple example and then increase the complexity.
  - Think about what information needs to be “stored” at each step in the recursive case (like the current value of **`n`** in each **`factorial`** stack frame).

- Ask yourself:
  - What is the base case? (What is the simplest case?)
  - What is the recursive case? (What pattern of self-similarity do you see?)

Discuss:

What are the base and recursive cases?

(breakout rooms)

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed ➔ desserts**
    - Take the s and put it at the end of the string.
    - Then reverse “tressed”:
        - Take the t and put it at the end of the string.
        - Then reverse “ressed”:
            - Take the r and put it at the end of the string.
            - Then reverse “essed”:
                - …
                    - Take the d and put it at the end of the string.
                    - **Base case**: reverse “” ➔ get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed → desserts**
  - **Take the s and put it at the end of the string.**
  - **Then reverse “tressed”:**
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - …
    - Take the d and put it at the end of the string.
    - **Base case**: reverse “” → get “”

```
reverse(“stressed”)=
    reverse(“tressed”)+’s’
```

```
reverse(“tressed”) =
    reverse(“ressed”)+’t’
```

```
reverse(“ressed”)=
    reverse(“essed”)+’r’
```

```
reverse(“d”)=
    reverse(“”)+’d’
```

```
reverse(“”)=“”
```

How can we reverse a string?

- **Recursive case:** reverse(str) = reverse(str without first letter) + first letter of str
- **Base case**: reverse(“”) = “”

Depending on how you thought of the problem, you may have also come up with:

- **Recursive case:** reverse(str) = last letter of str + reverse(str without last letter)
- **Base case**: reverse(“”) = “”

Let’s code it!

(live coding)

Summary

Summary

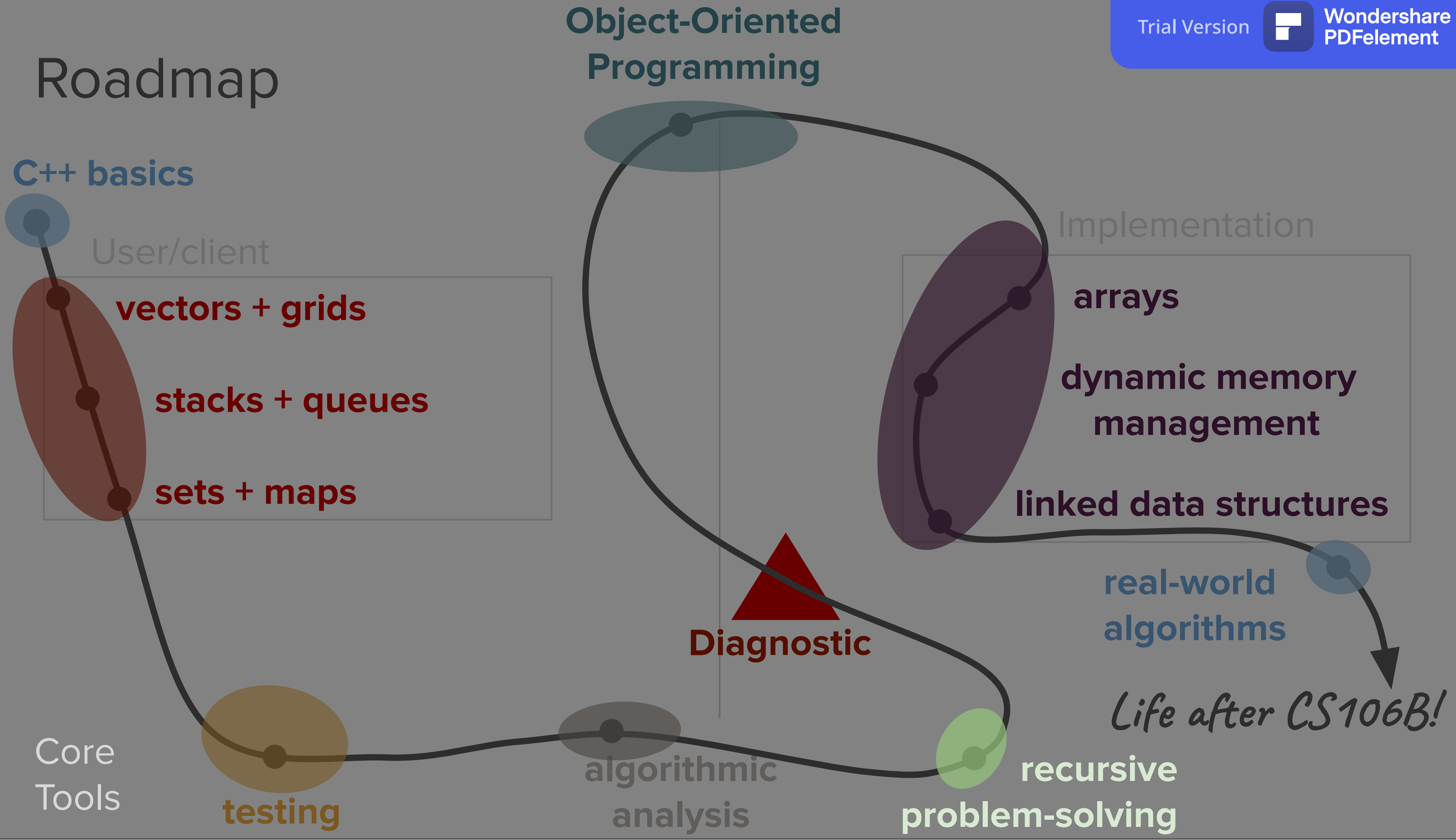
- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
  - A recursive operation(function) is defined in terms of itself (i.e.it calls itself) .
- Recursion has two main parts: the **base case** and the **recursive case**.
  - Base case: Simplest form of the problem that has a direct answer.
  - Recursive case: The step where you break the problem into a smaller, self-similar task.
- The solution will get built up **as you come back up the call stack**.

The base case will define the "base" of the solution you're building up.
Each previous recursive call contributes a little bit to the final solution.
The initial call to your recursive function is what will return the completely constructed answer

- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

What’s next?

Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Fractals