

# Assignment 3: PathTracer

## Jacob Hsiung

This project is completely about pathtracing just like its name. We start off with something simple. To begin with, we wrote a simple ray-generating algorithm so we can do ray tracing from there. Secondly, we implemented a intersection test, including one for triangle and another for sphere. At this point, we are able to do the simplest ray tracing. However, the problem was that, it took way too long to render just a low resolution image. This took us to the next part of the project which is the BVH Acceleration algorithm. We partition the space into several bounding boxes, and each bounding boxes will contain several primitives. This structure speeds up the ray tracing algorithm significantly. With the speed-up version of ray tracing, we can start on direct illumination and then global illumination. The first part of direct illumination is hemisphere sampling and importance sampling. The former sample the entire hemisphere at uniform distribution, so we end up sampling lots of non-light source object, causing the image to be noisy. This problem can be solved by the latter algorithm which is importance sampling. This algorithm directly loops through all the light source in the scene and sample its radiance. This guarantees that all of our samples are on the "important" source. We can achieve higher quality image with the same computational cost. After completing direct illumination, we can further extend our path tracing algorithm by enabling global illumination. In direct illumination, we are only considering radiance directly coming from light source, what we ignored is the radiance being reflected off the object. To include those radiance, we have to take the BRDF of different object into consideration. By using a recursive algorithm and Russian Roulette we can recursively calculate the radiance being reflected around. The details of the implementation are further discussed later. Last but not least, to make our sampling more efficient, we applied the method called adaptive sampling, which essentially stops sampling a pixel when its pixel value has converged.

## Part 1: Ray Generation and Intersection

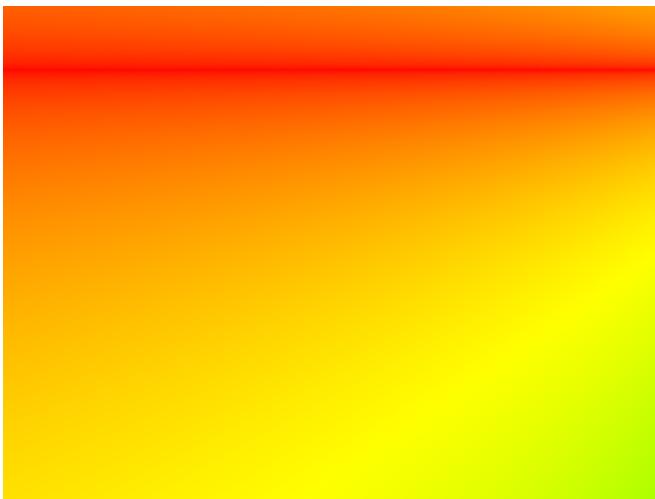


image with normal shading

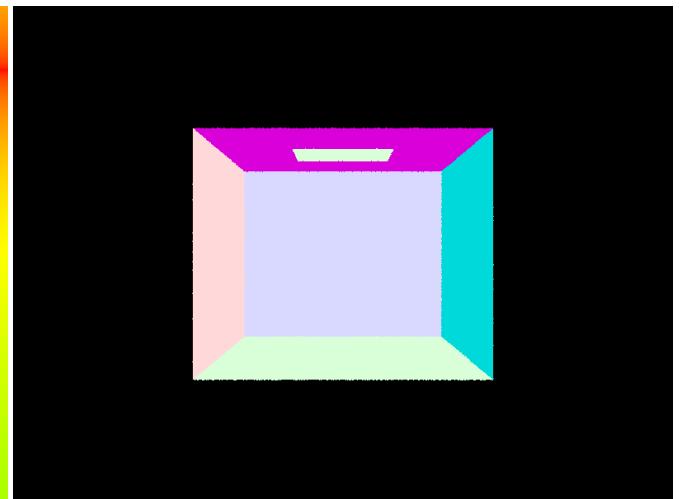
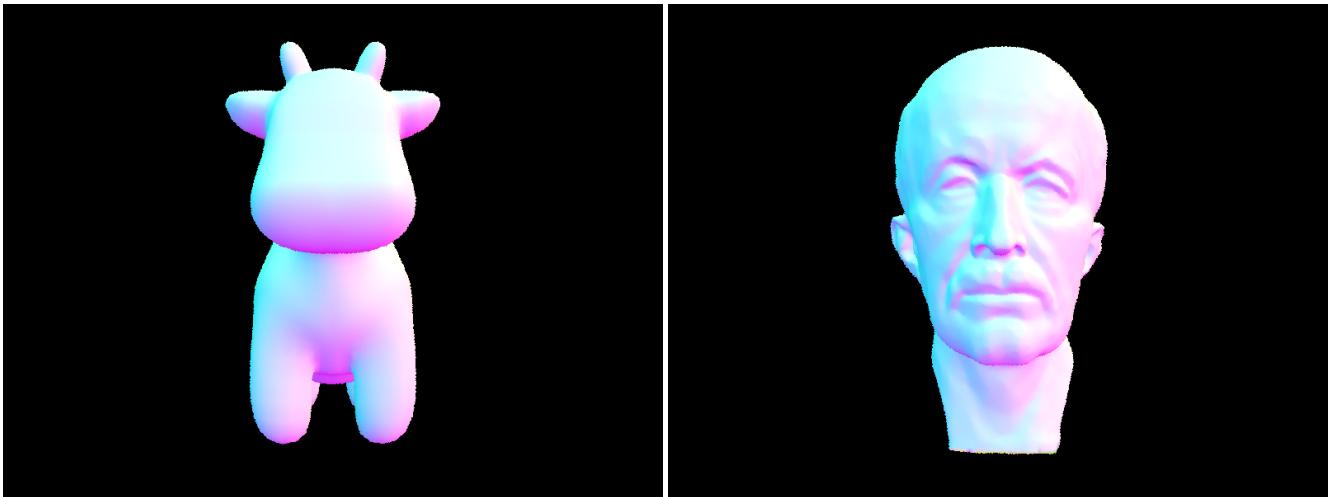


image for intersection

The ray generation algorithm was rather straightforward. To generate a ray in the right direction, we first generate the ray using the fact that  $x_{max} = \tan(hFov)$  and  $y_{max} = \tan(vFov)$  in the image plane. Using these two values, we can interpolate the coordinates at the given  $x, y$  value, and the  $z$  coordinates are always set to -1. At this point, we can calculate the direction of this ray since the origin of the ray coincides with the origin of the coordinates. Furthermore, we have to normalize the direction of the ray and setting its  $min\_t$  to  $nClip$  and  $max\_t = fClip$ . The importance of setting these values will become obvious in the latter part of the project because it helps us determine whether or not the intersection between a ray and an object is valid. Lastly, the generated ray is in the camera coordinates, so we need to convert it into the world's coordinates, which can be done by multiplying with the  $c2w$  matrix. The triangle intersection algorithm I implemented was Moller Trumbore algorithm. According to the algorithm, the intersection point is given by  $O + tD = (1 - b1 \cdot b1) * P0 + b1 * p1 + b2 * p2$ .  $P0, P1, P2$  are the three vertices of the triangle and  $b1$  and  $b2$  are variables that we will calculate. We first define a few constants that are  $E1 = P1 - P0$ ,  $E2 = P2 - P0$ ,  $S = O - P0$ ,  $S1 = \text{cross}(D, E2)$ ,  $S2 = \text{cross}(S, E1)$ . With these constants,  $[t, b1, b2].T = 1 / (\text{cross}(S1, E1)) * [\text{dot}(S2, E2), \text{dot}(S1, S), \text{dot}(S2, D)].T$ . We then check if  $t$  is in the range of  $min\_t$  and  $max\_t$  of the ray,  $b1$  and  $b1$  are greater than 0,  $b1 + b2$  are smaller than 1, and  $t$  is greater than 0. If all of the above conditions are satisfied, then the intersection point is said to be within the triangle.

## Part 2: Bounding Volume Hierarchy

To construct the BVH Heirarchy. Firstly, I loop through all the primitives and add all of them to the root bounding box. I also added all of their centroid to a centroid bounding box, which is later used for splitting. If the total number of primitives in the bounding box is greater than the  $max\_leaf\_size$  allowed, then we have to split the bounding box into left and right node. I chose the axis with the greatest extent according to the centroid bounding box we just calculated. I split along the mid point of the axis, and separate all of the primitives into left and right vectors based on their centroid. After separation, I recursively called `construct_bvh` on the left and right node of the bounding box with the vector  $l$  and  $r$ . The recursion will end when the number of primitives in the bounding box is less than the  $max\_leaf\_size$ .



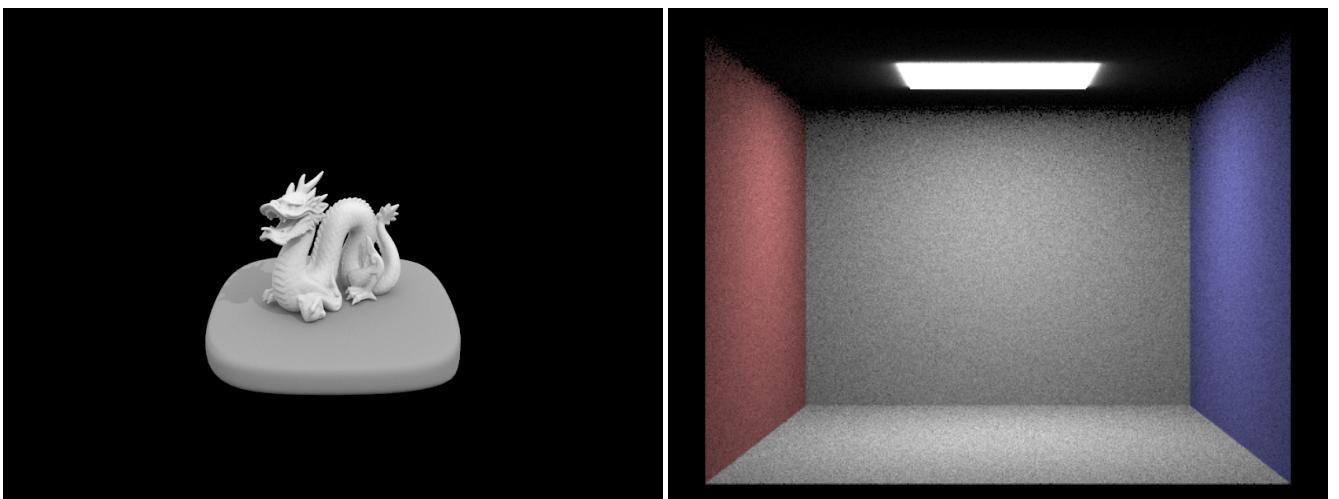
cow.png with BVH acceleration

MaxPlank.png with BVH acceleration

Before the BVH, cow.png takes about 62 seconds to render, but then is reduced to less than a second. The MW.png originally takes 239 seconds to render, but it was reduced to also less than a second. The reason behind the incredible speed up is because we decrease the number of intersection between the ray and objects. Instead of checking intersection primitive by primitive, we only check for intersection when the ray intersects the bounding box in the first place.

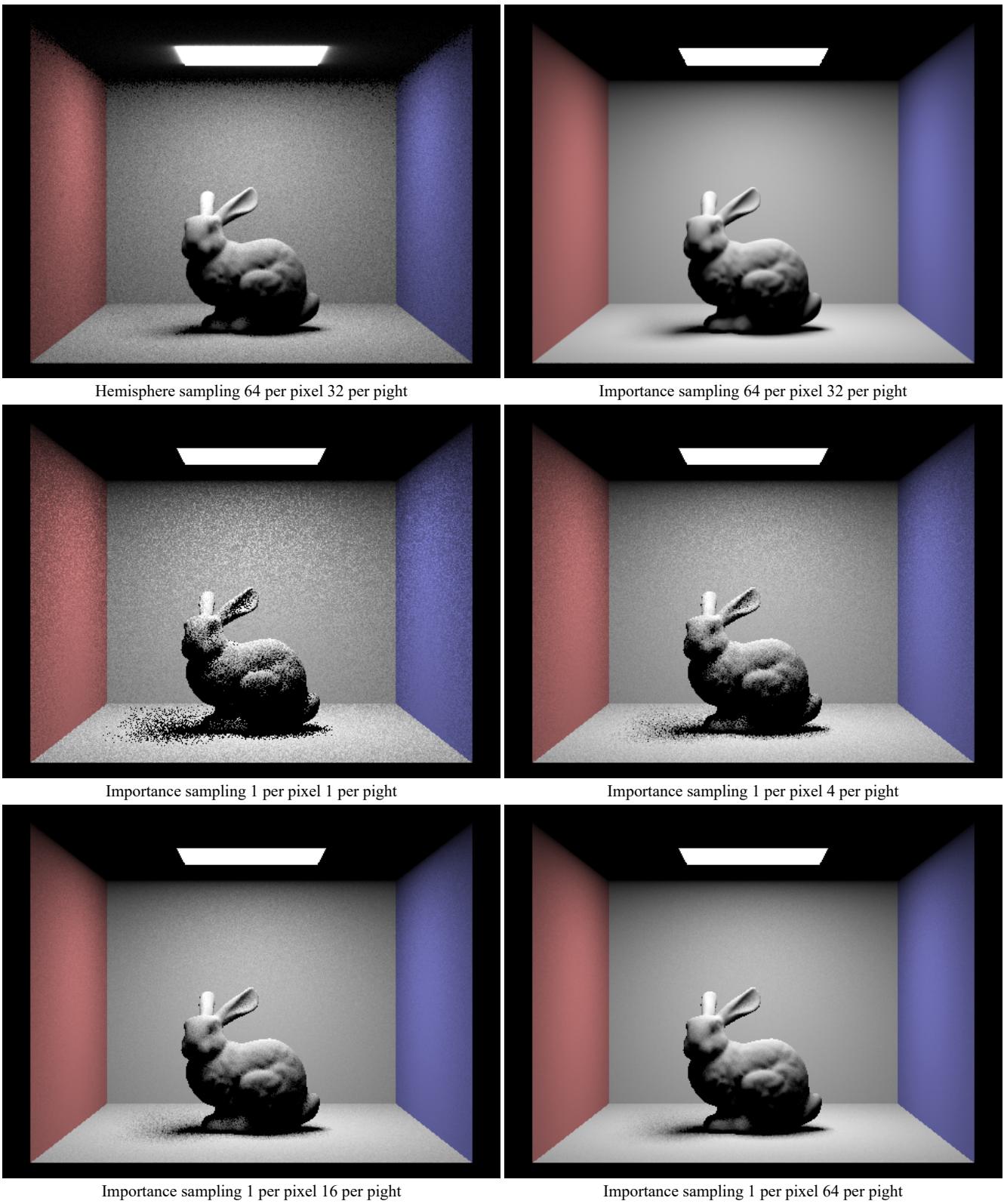
### Part 3: Direct illumination

In the first half of this part, we implemented the hemisphere sampling. In this method, we do not know where the light source is located in the hemisphere, so we randomly sample the ray in the hemisphere. It follows that the chance of hitting a light source is small, not to mention if it is a point light source. However, if our sample ray does intersect with a light source, we calculate how much radiance is emitted from the light source. If the sample ray does not intersect with a light source, we simply move on to the next sample. At the end, we have to normalize our sample by multiplying it with  $2\pi$  and dividing by the total number of samples taken. In the second half, we implemented the importance sampling. This method addresses the problem of hemisphere sampling, which is that lots of our sample rays don't intersect with the light source; hence, we cannot produce images with stable and high quality. To solve the problem, instead of randomly sampling the ray, we iterate over all the light sources, then sample a ray from the light source to the object. We then have to verify that this ray doesn't intersect anything when traveling from the light source to the object, so that the radiance emitted from the light source is directly onto the object. In the end, we get the BSDF of the object and multiply it with the incoming radiance from the light source and multiply it with the cosine of the angle and normalize it by dividing by the PDF sample distribution. There are two things to notice in importance sampling. First thing is that if the light source is a point light source, then we can reduce the number of samples to 1. Second thing is that when calculating the radiance, we have to take into consideration that this radiance is sampled with a probability PDF from the light source, so we have to normalize it by dividing PDF.

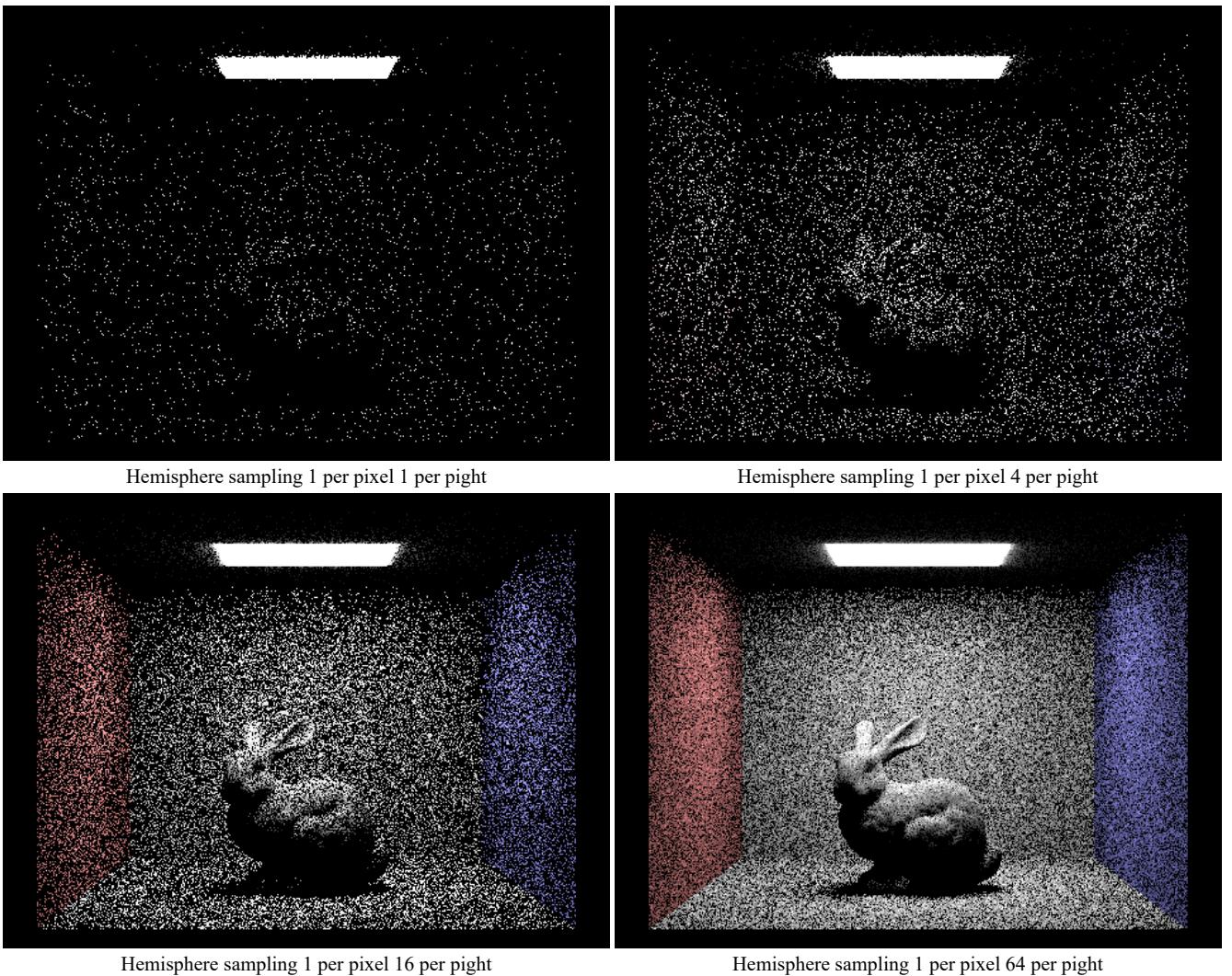


Importance sampling 64 per pixel 32 per light

Importance sampling 64 per pixel 32 per light



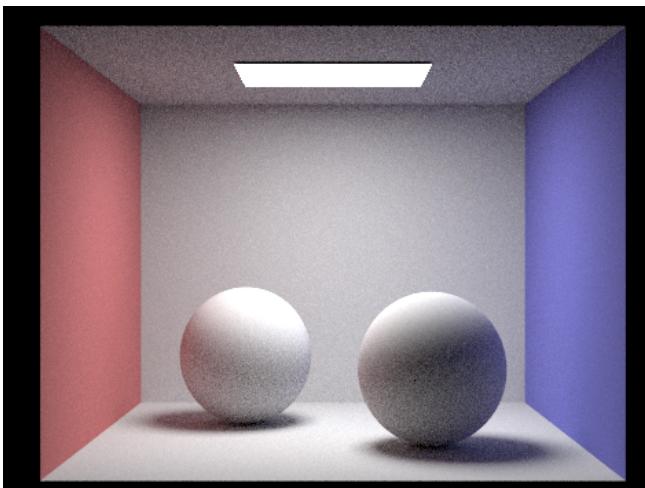
As we increase the number of samples for light, we can see that the noises decreases dramatically even though the sample per pixel is set to 1



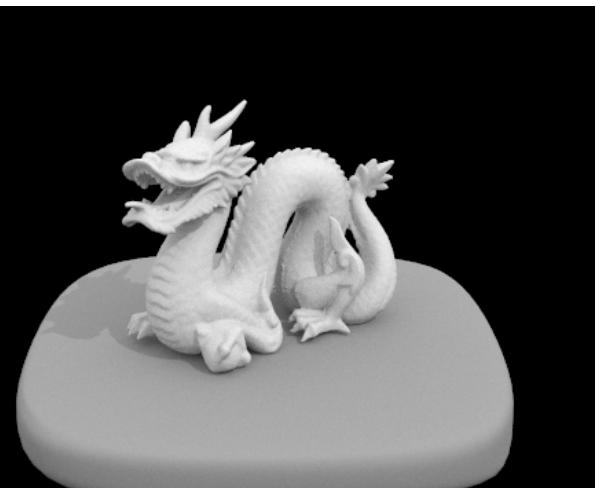
Comparing the result of Hemisphere sampling and Importance sampling, we can see that with the same parameters, importance sampling can achieve better quality. This is very obvious if we are sampling 1 ray per pixel. In Hemisphere sampling, we have a big chance of missing the light source resulting in a black pixel, causing lots of noise in our image. However, in Importance sampling, we will be sampling in the light source direction.

## Part 4: Global illumination

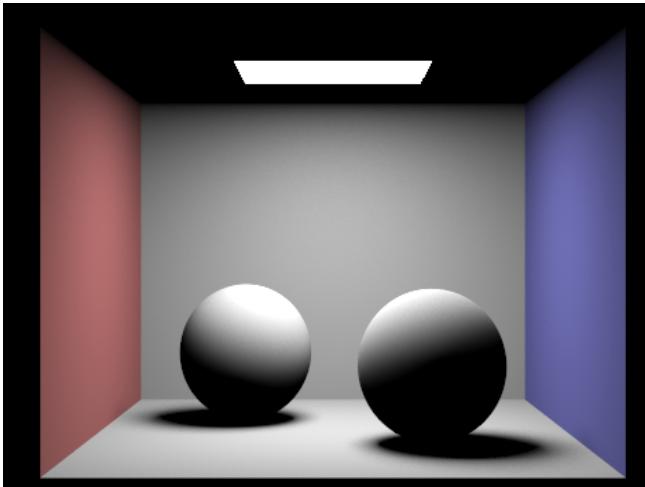
We already implemented zero bounce and one bounce radiance in part 3 of this project. However, there are still lots of noise in our image. To produce better quality image, we have to consider radiance that bounces more than 1 time. We implement the `at_least_one_bounce` function which calls `one_bounce` function in the beginning and recursively calls itself. In each recursive call to the function, it sample a new direction from the bsdf of the object and generate a ray in that direction. If the ray intersects with an object, it recursively call itself with the new intersect object and the ray that just intersect with it. To prevent infinite recursion, we use the Russian Roulette method, which terminates the recursion with a probability  $p$ . This  $p$  is chosen by the user. Besides Russian Roulette, every ray has a member variable `depth`. In each recursion, we decrease the value of `depth` by 1, when the `depth` is 0, we also terminate the recursion. In the end of the algorithm, we add together `zero_bounce` and `at_least_one_bounce` to get the estimate of global illumination.



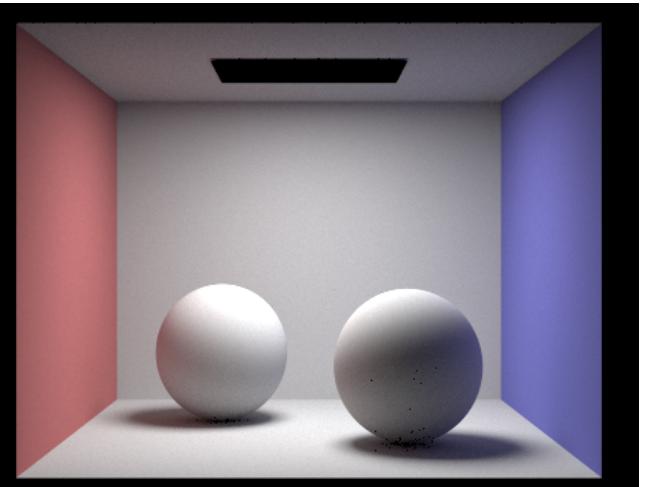
Global Illumination of Spheres



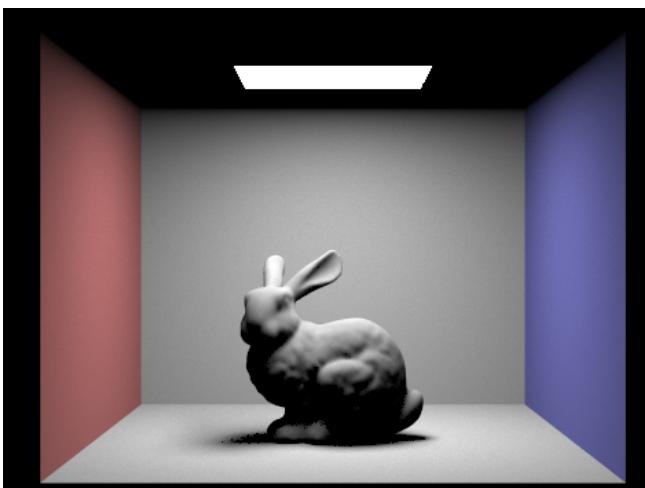
Global Illumination of Dragon



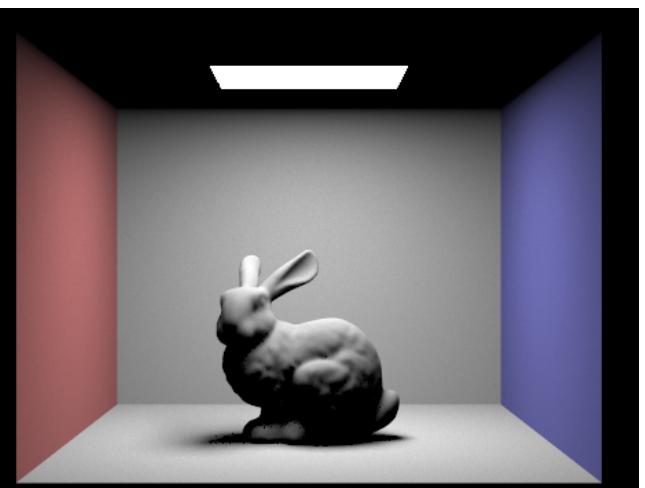
Direct illumination



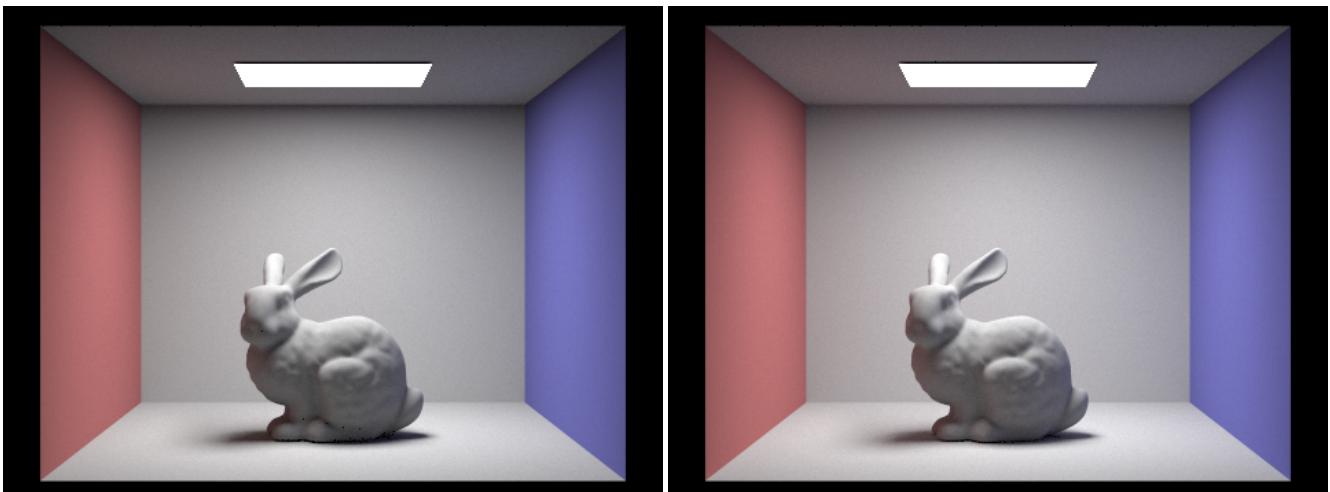
indirect illumination



1024 samples per pixel, 1 sample per light, Depth = 0

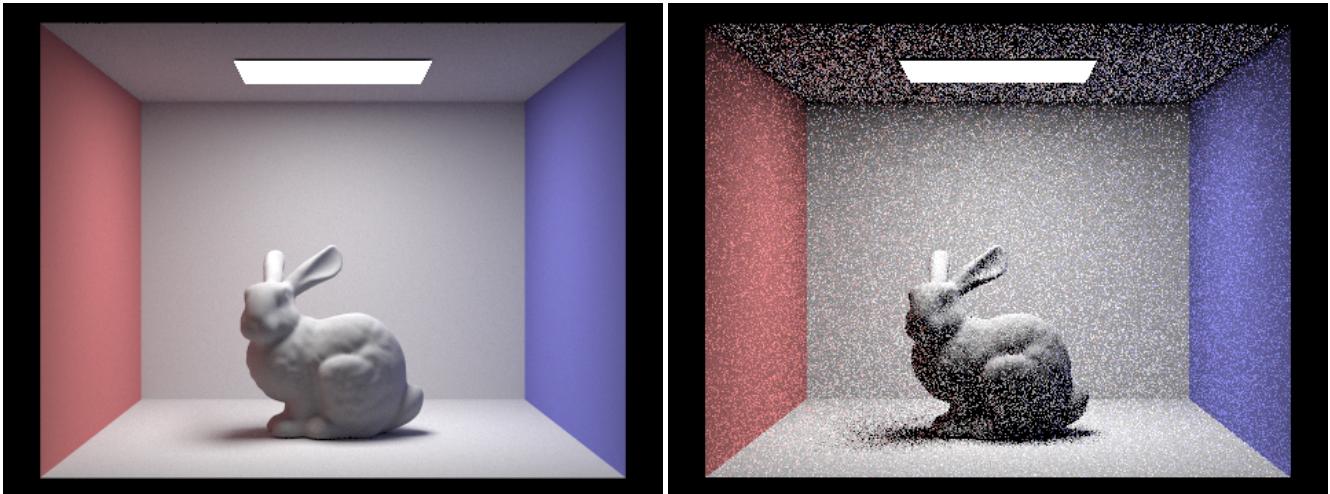


1024 samples per pixel, 1 sample per light, Depth = 1



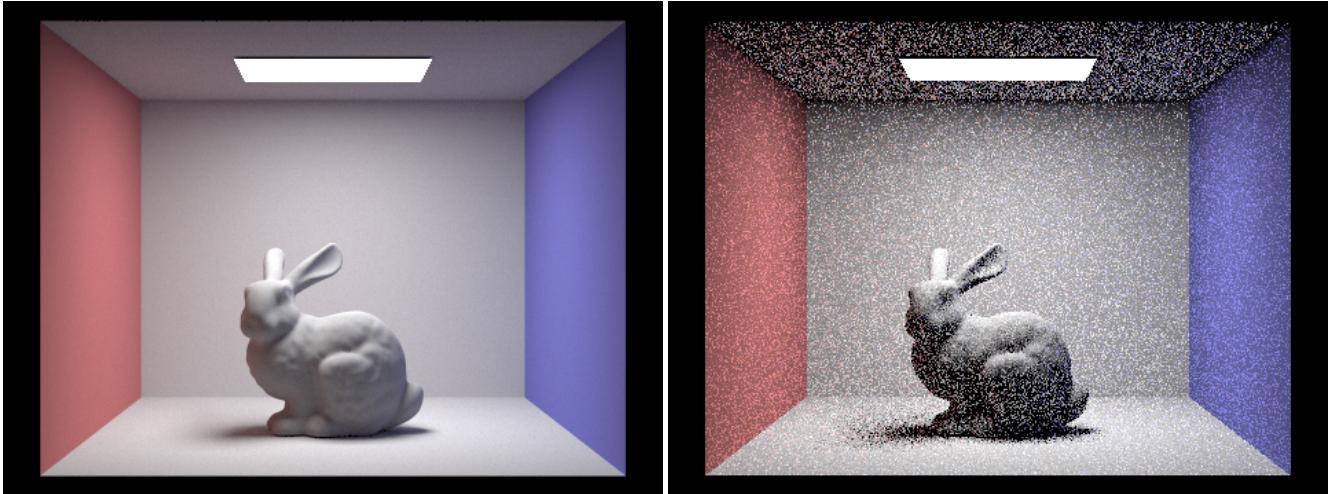
1024 samples per pixel, 1 sample per light, Depth = 2

1024 samples per pixel, 1 sample per light, Depth = 3



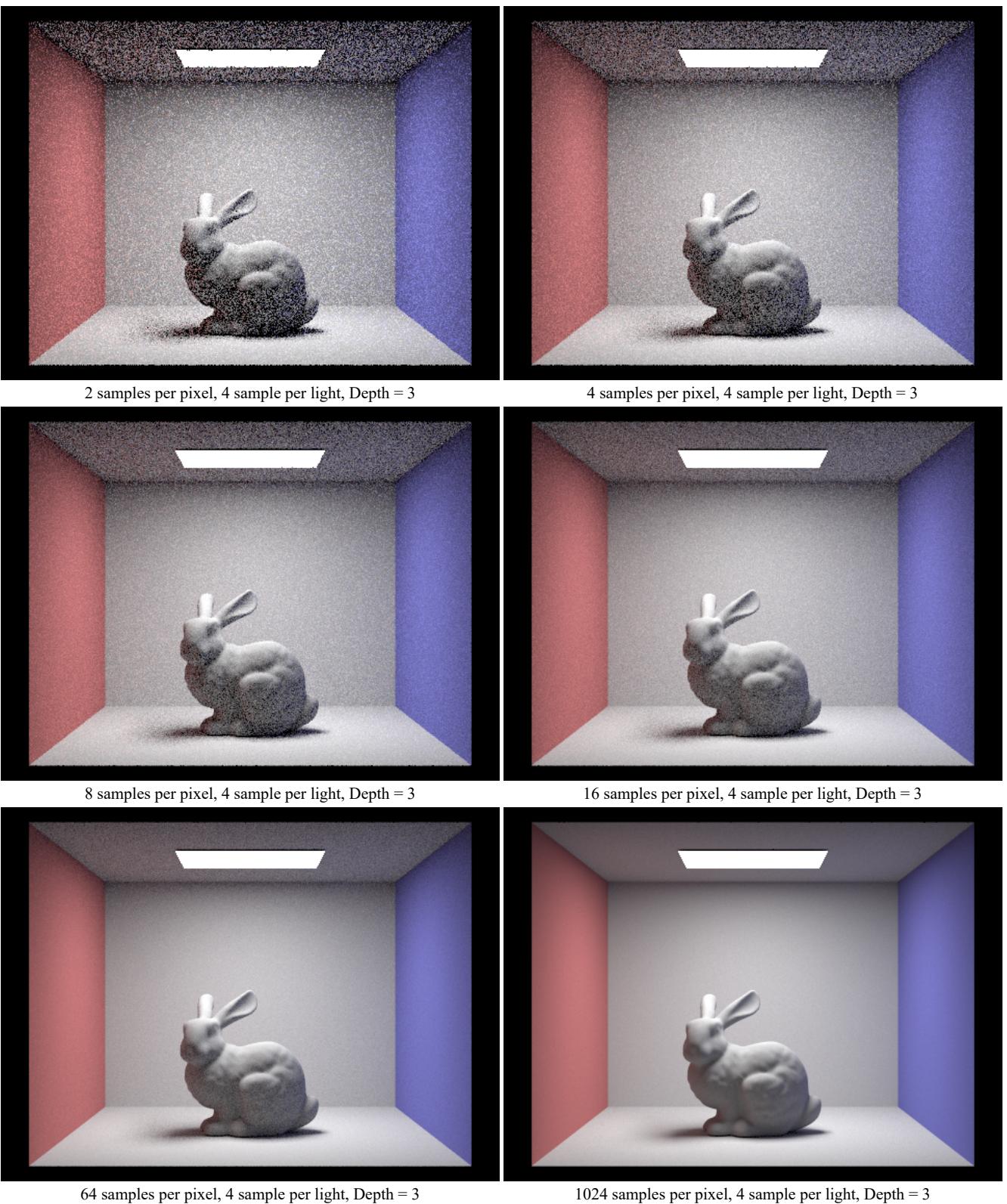
1024 samples per pixel, 1 sample per light, Depth = 100

1024 samples per pixel, 1 sample per light, Depth = 100



1024 samples per pixel, 1 sample per light, Depth = 100

1 samples per pixel, 4 sample per light, Depth = 3

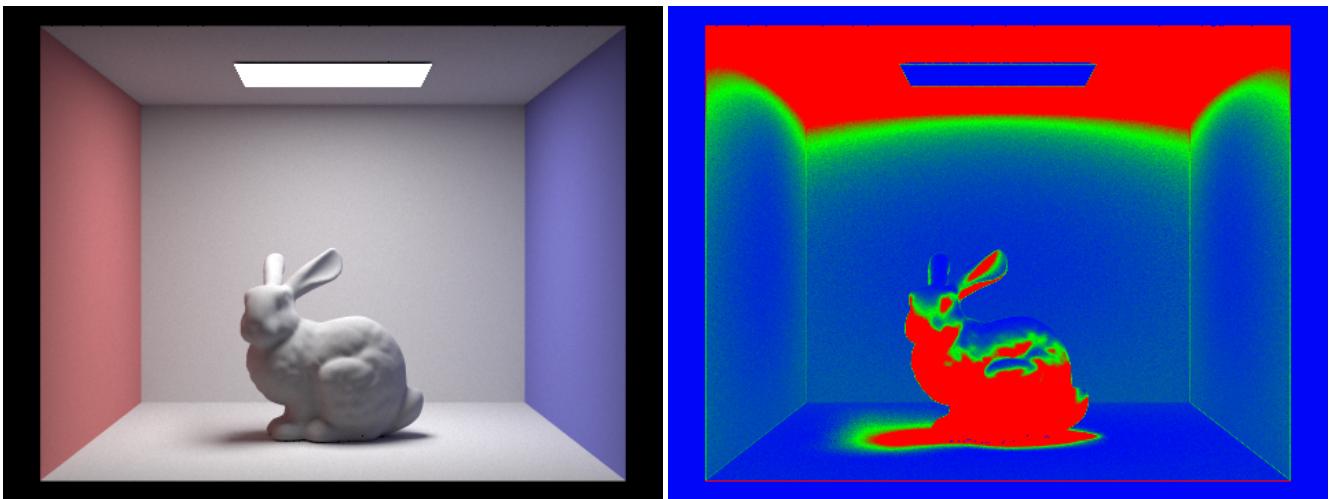


For CBunny.dae, when we set the number of samples to 1024 per pixels and render image with `max_ray_depth = 0,1,2,3,100`. As we increase the `max_ray_depth`, the image become brighter and brighter because they are brighten by the gloabel illumination.

On the other hand, when we set the samples per light to 4 and sample per pixel = 1,2,4,8,16,64,1024. The noise in our image become less and less. The image become clearer as we increase the sample per pixel.

## Part 5: Adaptive Sampling

Adaptive concept is a relatively straightforward concept. In the past, we sample every pixel a set amount of times no matter what the value is. In adaptive sampling, however, we stop sampling the pixel if the value of the pixel converge. We can check if the value of the pixel converges by calculating its  $I$ , which is equal to  $1.96 * \text{the standard deviation} / \sqrt{\text{number of samples}}$ . If  $I$  is less than the `maxTolerance * mean`; then we stop sampling the pixel.



2048 samples per pixel, 1 sample per light, Depth = 5

Image of sampling rate

From the image of sampling rate, we can see that, different from the uniform samling rate accross the image. We stop sampling the light source very quickly because the value of the pixel converges. However, notice that places with shadows often requires more samples per pixel before its value converges.