HW3

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$$s' \cdot a + t' \cdot b = gcd(a,b)$$

$$s' = s + \frac{k}{gcd(a,b)} \cdot b = s + k \cdot \frac{b}{d}$$

$$t' = t - \frac{k}{gcd(a,b)} \cdot a = t - k \cdot \frac{a}{d}$$

$$(s + \frac{kb}{d})a + (t - \frac{ka}{d})b = gcd(a,b)$$

$$= sa + \frac{kba}{d} + tb - \frac{kab}{d}$$

$\because$ The smallest next possible solution, we need to minimize $\frac{kb}{d}$ and $\frac{ka}{d}$, a and b are fixed, to minimum we need k become -1 or 1, (0 will become $s \cdot a + t b$) and we need the largest d to minimize $\frac{kb}{d}$ and $\frac{ka}{d}$, but $\frac{kb}{d}$ and $\frac{ka}{d}$ needs to be integer, so the largest d we can choose is $gcd(a,b)$. Because any value $> gcd(a,b)$ will cause to $d \nmid b$ or $d \nmid a$ 不能整除 in that situation $\frac{kb}{d}$ and $\frac{ka}{d}$ cannot become integer in the same time. And any larger solutions we can get by adjust k.

$\therefore$ $s' = s + \frac{k}{gcd(a,b)} \cdot b$, $t' = t - \frac{k}{gcd(a,b)} \cdot a$ in general solutions $s' \cdot a + t' \cdot b = gcd(a,b)$ include all solutions.