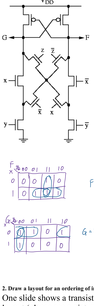
HW1

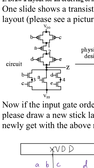
Wednesday, 5 March 2025 2:38 PM



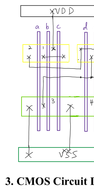
x	y	z	F	G
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

F = ?
G = ?

Please com
table and t



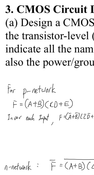
$F = xz + xy + yz$



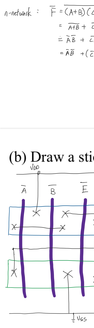
$G = \bar{x}\bar{z} + \bar{x}\bar{y} + \bar{y}\bar{z}$

2. Draw a layout for an ordering of input nets (15 points)

One slide shows a transistor design and a possible stick layout (please see a picture below)

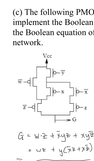


Now if the input gate ordering is changed to a, b, c, d, e, please draw a new stick layout. Then, compare what you newly get with the above result and comment.

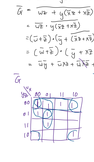


3. CMOS Circuit Design (20 points)

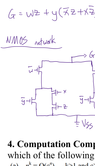
(a) Draw a CMOS logic gate F=(A+B)(CD+E). Draw the transistor-level (PMOS, NMOS) schematic. Please indicate all the names of signals (F, A, B, C, D, E) and also the power/ground.

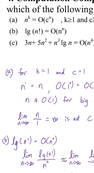


$G = \bar{F} = \overline{(A+B)(CD+E)}$

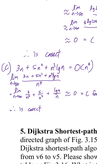


(b) Draw a stick diagram for the design in (a).





(c) The following PMOS network was designed to implement the Boolean function G(w,x,y,z). Please write the Boolean equation of G. Also complete the NMOS network.



$G = w\bar{z} + \bar{x}y + x\bar{y}\bar{z}$

$\bar{G} = w\bar{x} + y(\bar{x}z + x\bar{z})$

$= w\bar{x} + y(\bar{x}z + x\bar{z})$

$= \overline{w\bar{x} + y(\bar{x}z + x\bar{z})}$

$= \overline{w\bar{x}} \cdot \overline{y(\bar{x}z + x\bar{z})}$

$= (z + \bar{x}) \cdot (\bar{y} + \overline{(\bar{x}z + x\bar{z})})$

$= (z + \bar{x}) \cdot (\bar{y} + \overline{(\bar{x}z)} \cdot \overline{(x\bar{z})})$

$= (z + \bar{x}) \cdot (\bar{y} + (x + \bar{z}) \cdot (\bar{x} + z))$

$= \bar{y}z + \bar{y}\bar{x} + x\bar{x} + xz + \bar{x}\bar{z} + z\bar{z}$

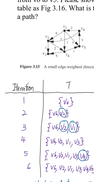
$= \bar{y}z + \bar{y}\bar{x} + xz + \bar{x}\bar{z}$

$\overline{\bar{x}\bar{z}} = \bar{x} + \bar{z}$

$\overline{(\bar{x}z)(x\bar{z})} = \bar{x}z + x\bar{z}$

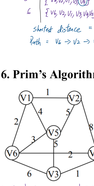
$\bar{G} = \bar{y}\bar{x} + \bar{y}z + xz + \bar{x}\bar{z}$

$= \bar{x}(\bar{y} + z) + z(\bar{y} + x)$



$G = w\bar{z} + y(\bar{x}z + x\bar{z})$

NMOS network



4. Computation Complexity (5 points) Determine which of the following is correct.

(a) $n^2 = O(n^3)$, $n \geq 1$ and $n \leq 1$.

(b) $\lg(n!) = O(n^n)$

(c) $3n + 5n^2 + n^3 \lg n = O(n^4)$

(a) for $k = 1$ and $c = 1$

$n^1 = n$, $O(1) = O(1)$

$n \neq O(1)$ for big n

$\lim_{n \to \infty} \frac{n}{1} = \infty$ is not C

(b) $\lg(x!) = O(x^x)$

$\lim_{n \to \infty} \frac{\lg(n!)}{n^n} = \lim_{n \to \infty} \frac{\lg(\sqrt{2\pi n}(\frac{n}{e})^n)}{n^n}$

$= \lim_{n \to \infty} \frac{\lg\sqrt{2\pi n} + n\lg n - n\lg e}{n^n}$

$= \lim_{n \to \infty} \frac{\frac{1}{2}\lg 2\pi + \frac{1}{2}\lg n + n\lg n - n\lg e}{n^n}$

$\geq 0 < C$ for $C \geq 0$

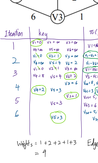
$\therefore$ is correct

(c) $3n + 5n^2 + n^3 \lg n = O(n^4)$

$\lim_{n \to \infty} \frac{3n + 5n^2 + n^3 \lg n}{n^4} = \lim_{n \to \infty} \frac{3}{n^3} + \frac{5}{n^2} + \frac{\lg n}{n} \geq 0 < C$ for $C > 0$

$\therefore$ is correct

5. Dijkstra Shortest-path Algorithm **(15 points)** For the directed graph of Fig. 3.15 in the textbook, use the Dijkstra shortest-path algorithm to find the shortest path from v6 to v5. Please show all the iterations in the same table as Fig 3.16. What is the shortest distance, and such a path?



$V_5 = V_6$

$V_6 = V_6$

Figure 3.15 A small edge-weighted directed graph.

Iteration	T	v_1	v_2	v_3	v_4	v_5	v_6
1	$\{v_6\}$	2	2	∞	∞	∞	0
2	$\{v_6, v_1\}$	2	2	3	∞	∞	0
3	$\{v_6, v_1, v_2\}$		2	3	5	∞	0
4	$\{v_6, v_1, v_2, v_3\}$			3	5	4	0
5	$\{v_6, v_1, v_2, v_3, v_5\}$				5	4	0
6	$\{v_6, v_1, v_2, v_3, v_5, v_4\}$						

(v_6, v_1, v_2)

(v_6, v_2, v_3)

(v_6, v_3, v_5)

Shortest distance = 4

Path : $v_6 \to v_3 \to v_5 \to v_6$

6. Prim's Algorithm for MST (15 points)

Iteration	key	$\pi(v)$	Update to MST
1	$v_1: 1$ $v_2: 4$ $v_3: 5$ $v_4: \infty$ $v_5: \infty$ $v_6: \infty$	$v_1 = nil$	v_1
2	$v_2: 4$ $v_3: 5$ $v_4: \infty$ $v_5: 2$ $v_6: 6$	$v_2 = v_1$ $v_3 = v_1$ $v_4 = nil$ $v_5 = v_1$ $v_6 = v_1$	v_1, v_5
3	$v_2: 4$ $v_3: 3$ $v_4: 8$ $v_6: 6$	$v_2 = v_1$ $v_3 = v_5$ $v_4 = v_5$ $v_6 = v_1$	v_1, v_5, v_3
4	$v_2: 4$ $v_4: 8$ $v_6: 6$	$v_2 = v_1$ $v_4 = v_5$ $v_6 = v_1$	v_1, v_5, v_3, v_2
5	$v_4: 8$ $v_6: 6$	$v_4 = v_5$ $v_6 = v_1$	v_1, v_5, v_3, v_2, v_6
6	$v_4: 8$	$v_4 = v_5$	$v_1, v_5, v_3, v_2, v_6, v_4$

Weight = 1 + 2 + 3 + 1 + 3 = 9

Edges: $\{(v_1, v_5), (v_5, v_3), (v_1, v_2), (v_5, v_4), (v_1, v_6)\}$

7. Branch and Bound (15 points)

Use the branch and bound algorithm to solve the TSP problem in above Figure HW-6. Start and end with V1. Show the search tree as Fig. 5.6 in the textbook. How many nodes do you have in your search tree?

43 nodes