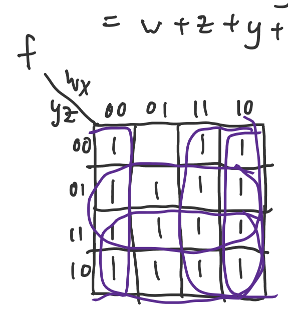
HW3

Tuesday, 13 May 2025 10:31 AM

1.) $f = wx\bar{y} + w\bar{y}z + w + xz + \bar{w}xy + \bar{w}\bar{x} + \bar{w}\bar{x}y\bar{z}$

$= w + xz + xy + \bar{x}$

$= w + z + y + \bar{x}$



$f = w + \bar{x} + z + y$

(a) $f = w + \bar{x} + z + y$

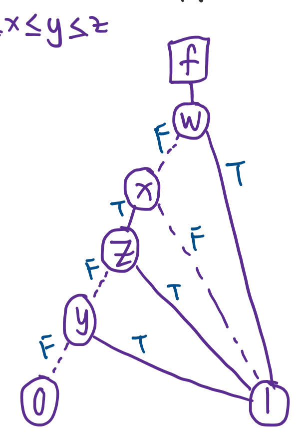
$f_w = 1$

$f_{\bar{w}} = \bar{x} + z + y$, $f_{\bar{w}x} = z + y$, $f_{\bar{w}xz} = 1$

$f_{\bar{w}\bar{x}} = 1$ $f_{\bar{w}x\bar{z}} = y$, $f_{\bar{w}x\bar{z}y} = 1$

$f_{\bar{w}x\bar{z}\bar{y}} = 0$

$w \le x \le y \le z$



(b) $f = w \vee \bar{x} \vee z \vee y$

$f = 0$, need $w = 0$, $\bar{x} = 0$, $z = 0$, $y = 0$ Maxterms (represent $f = 0$)

$\Rightarrow w = 0, x = 1, z = 0, y = 0$ $\Rightarrow f(w, x, y, z) = M_4$

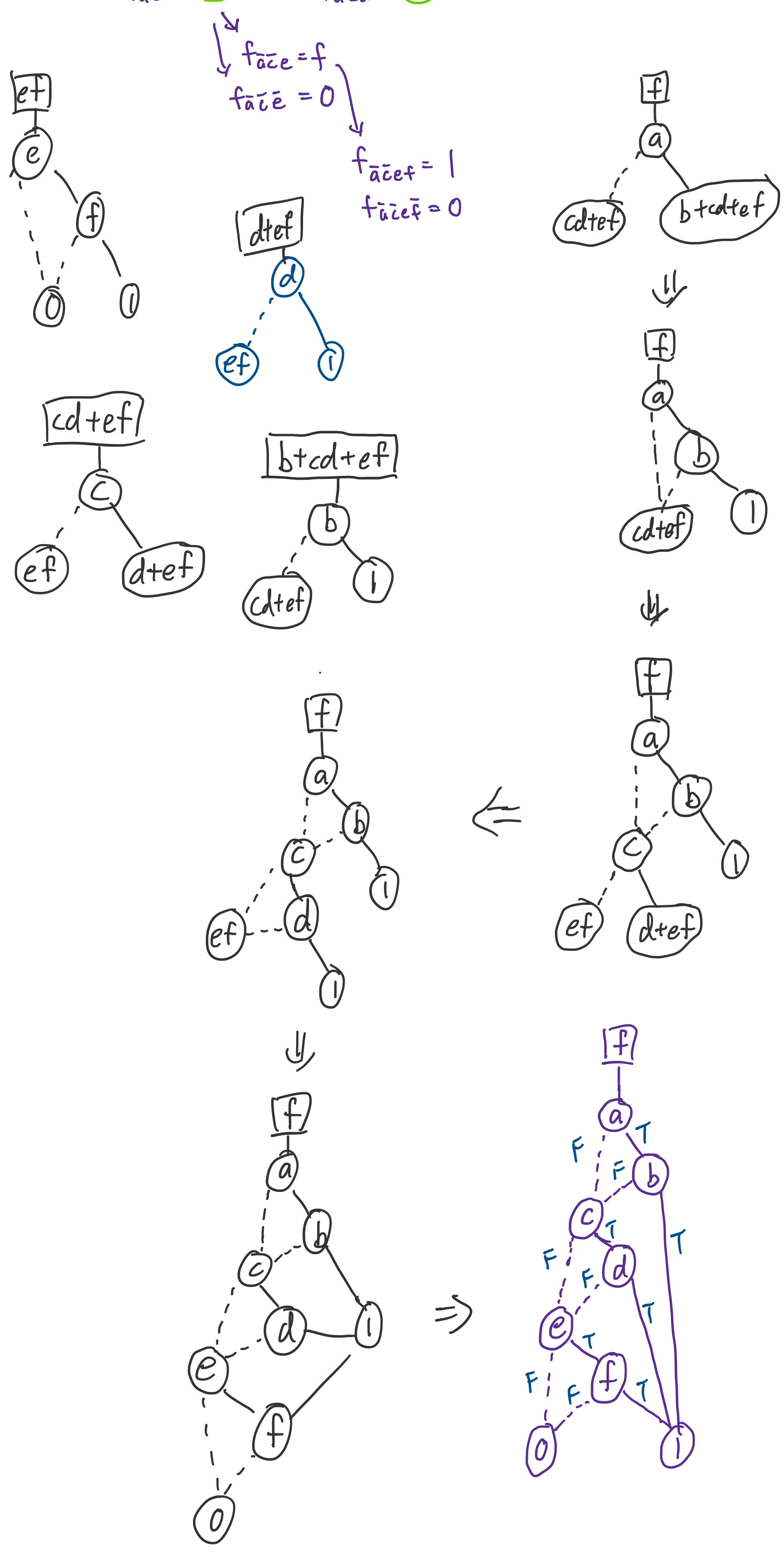
2.) $f = ab + cd + ef$

$f_a = b + cd + ef \rightarrow f_{ab} = 1$, $f_{a\bar{b}} = cd + ef$ $\rightarrow f_{a\bar{b}c} = d + ef \rightarrow f_{a\bar{b}cd} = 1$, $f_{a\bar{b}c\bar{d}} = ef$; $f_{a\bar{b}\bar{c}} = ef$

$f_{\bar{a}} = cd + ef \rightarrow f_{\bar{a}c} = d + ef \rightarrow f_{\bar{a}cd} = 1$, $f_{\bar{a}c\bar{d}} = ef$; $f_{\bar{a}\bar{c}} = ef$

$f_{\bar{a}\bar{c}e} = f \rightarrow f_{\bar{a}\bar{c}ef} = 1$, $f_{\bar{a}\bar{c}e\bar{f}} = 0$

$f_{\bar{a}\bar{c}\bar{e}} = 0$



3.) (a) $x - y == x\bar{y} + \bar{x}y$

$(f-g)\cdot h = (f\bar{g} + \bar{f}g)\cdot h$

$= f\bar{g}h + \bar{f}g\cdot h$

$f\cdot h - g\cdot h = fh\overline{gh} + \overline{fh}\cdot gh$

$= fh\cdot(\bar{g}+\bar{h}) + (\bar{f}+\bar{h})\cdot gh$

$= fh\cdot\bar{g} + 0 + \bar{f}gh + 0$

$= fh\bar{g} + \bar{f}gh = f\bar{g}\cdot h + \bar{f}g\cdot h$

Both are same $\Rightarrow (f-g)\cdot h = (f\cdot h - g\cdot h) \Rightarrow$ # proved

b) $f-(g-h) = (f-g)-h$

$f - (g\bar{h} + \bar{g}h) = f\,\overline{(g\bar{h}+\bar{g}h)} + \bar{f}(g\bar{h} + \bar{g}h)$

$\overline{g\bar{h}+\bar{g}h} = \overline{g\bar{h}}\cdot\overline{\bar{g}h} = (\bar{g}+\bar{\bar{h}})\cdot(\bar{\bar{g}}+\bar{h})$

$= (\bar{g}+h)\cdot(g+\bar{h})$

$= 0 + \bar{g}\bar{h} + gh + 0$

$= \bar{g}\bar{h} + gh$

$= f\cdot(\bar{g}\bar{h} + gh) + \bar{f}(g\bar{h} + \bar{g}h) = f\bar{g}\bar{h} + fgh + \bar{f}g\bar{h} + \bar{f}\bar{g}h$

$(f-g)-h = (f\bar{g} + \bar{f}g) - h$

$= (f\bar{g}+\bar{f}g)\bar{h} + \overline{(f\bar{g}+\bar{f}g)}\cdot h$

$= (f\bar{g} + \bar{f}g)\cdot\bar{h} + (\bar{f}\bar{g} + fg)\cdot h$

$= f\bar{g}\bar{h} + \bar{f}g\bar{h} + \bar{f}\bar{g}h + fgh$

Both can get same $\Rightarrow f-(g-h) = (f-g)-h \Rightarrow$ # proved

c) $\bar{f} = 1 - f$

$1 - f = 1\bar{f} + \bar{1}f$

$= \bar{f} + 0$

$= \bar{f}$

Both are same $\Rightarrow \bar{f} = 1 - f \Rightarrow$ # proved

d) f_x is positive cofactor , $fd = f_x - f_{x'}$

$f_{x'}$ is negative cofactor

$f = x\cdot f_x + \bar{x}\cdot f_{x'}$ (left)

$f_{x'} - x\cdot fd = f_{x'} \oplus x\cdot(f_x \oplus f_{x'})$

(right)

$= f_{x'} \oplus x\cdot(f_x\cdot\bar{f}_{x'} + \bar{f}_x\cdot f_{x'})$

$= f_{x'} \oplus (xf_x\cdot\bar{f}_{x'} + x\bar{f}_x\cdot f_{x'})$

$(\bar{x} + \bar{f}_x + \bar{\bar{f}}_{x'})\cdot(\bar{x} + f_x + \bar{f}_{x'})$

$= \bar{x} + \cancel{\bar{x}f_x} + \cancel{\bar{x}\bar{f}_{x'}} + \cancel{\bar{x}\bar{f}_x} + \bar{f}_x\bar{f}_{x'} + \cancel{\bar{x}f_{x'}} + f_xf_{x'}$

$= \bar{x} + \bar{f}_x\bar{f}_{x'} + f_xf_{x'}$

$= f_{x'}(x\cdot f_x\cdot\bar{f}_{x'} + x\bar{f}_x\cdot f_{x'}) + \bar{f}_{x'}\cdot(x\cdot f_x\cdot\bar{f}_{x'} + x\bar{f}_x\cdot f_{x'})$

$= \bar{x}f_{x'} + f_xf_{x'} + xf_x\bar{f}_{x'}$

$= \bar{x}f_{x'} + 1\cdot f_xf_{x'} + x\cdot f_x\cdot\bar{f}_{x'}$

$= \bar{x}f_{x'} + (x+\bar{x})\cdot f_xf_{x'} + xf_x\bar{f}_{x'}$

$= \bar{x}f_{x'} + xf_xf_{x'} + \cancel{\bar{x}f_xf_{x'}} + xf_x\bar{f}_{x'}$

$= \bar{x}f_{x'} + xf_x(f_{x'} + \bar{f}_{x'})$

$= \bar{x}f_{x'} + xf_x$

proved left == right