

# ECE276A PR2 Report

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## I. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) is pivotal in robotics, enabling autonomous systems to understand their surroundings and their locations with minimal human intervention. In this project, I implemented SLAM on a differential-drive robot equipped with a wheel encoder, an Inertial Measurement Unit (IMU), 2-D LiDAR, and an RGBD camera, and reconstructed detailed 2D occupancy map, 2D floor texture map, and 3D texture map.

The process began with the construction of a motion model trajectory based on the differential-drive kinematics, utilizing data from the wheel encoder and IMU. Concurrently, an observation model trajectory was estimated using LiDAR scans and the Iterative Closest Points (ICP) method. Subsequently, I applied a Factor Graph and loop closure techniques to refine and optimize the trajectory. Lastly, I leveraged the optimized trajectory to generate detailed maps, showcasing the potential.

## II. PROBLEM FORMULATION

In this project, the robot trajectory, discretized as a set of pose and time stamp tuples, is the core of all computation. Denote the robot's pose at given time stamp  $t$  as  $P_t$ . The pose can be represented in both two-dimensional (2D) and three-dimensional (3D) spaces, with the z-axis value fixed at zero.

$$P_t = \begin{cases} \begin{matrix} \text{2D} \\ \left[ \begin{matrix} x \\ y \\ \theta \end{matrix} \right] \end{matrix} & \in \mathbb{R}^3 \\ \begin{matrix} \text{3D} \\ \left[ \begin{matrix} R_{yaw}(\theta) & \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \\ \mathbf{0} & 1 \end{matrix} \right] \end{matrix} & \in \mathbb{R}^{[4 \times 4]} \end{cases}$$

This dual representation enables seamless transition between 2D and 3D spatial analyses. The initial pose  $P_0$  at time 0 is defined as the origin of the world frame serving as the reference for all subsequent poses.

$$P_0 = \begin{Bmatrix} \vec{0}_3 \\ I_4 \end{Bmatrix}$$

### A. Motion Model: IMU & Differential-drive Kinematics

Consider a small time interval  $\tau$  between two time stamp  $t$  and  $t+1$ . Suppose wheel encoder reported  $\delta d$  and IMU's yaw

reading reported  $\delta\theta$  during this interval, the motion of robot can be approximate as a arc:

$$P_{t+1}^{(2M)} = P_t^{(2M)} + \begin{bmatrix} \delta d \cdot \text{sinc}\left(\frac{\delta\theta}{2}\right) \sin\left(\theta_t + \frac{\delta\theta}{2}\right) \\ \delta d \cdot \text{sinc}\left(\frac{\delta\theta}{2}\right) \sin\left(\theta_t + \frac{\delta\theta}{2}\right) \\ \delta\theta \end{bmatrix}$$

in which, the superscript  $^{(2M)}$  means this is a 2D motion model trajectory,

### B. Observation Model: LiDAR & ICP

The ICP algorithm can estimate relative transformation between two given 3D point cloud: source  $S$  and target  $T$ . It achieved this by iterates over these steps:

- 1) Find point association for all source points using closest distance:

$$\Delta = \left\{ \left( s, \arg \min_{t \in T} \|s - t\| \right) \mid \forall s \in S \right\}$$

- 2) Estimate relative transformation  ${}_tT_s$  using Kabsch algorithm:

$${}_tT_s = \arg \min_T \sum_{(s,t) \in \Delta} \|t - Ts\|$$

- 3) Update  $S \leftarrow {}_tT_s S$  and repeat step 1 until no improvement.

Meanwhile, 2D-LiDAR scan reports a set of range and angle tuple  $L = \{(r_i, \phi_i)\}$ , which can be converted to a set of 3D point cloud in body frame using following equations.

$$\text{PC}_{\text{LiDAR frame}} = \left\{ \begin{bmatrix} r_i \sin \phi_i \\ r_i \cos \phi_i \\ 0 \end{bmatrix} \mid \forall (r_i, \phi_i) \in L \right\}$$

$$\begin{bmatrix} \text{PC} \\ 1 \end{bmatrix} = {}_{\text{body}}T_{\text{LiDAR}} \begin{bmatrix} \text{PC}_{\text{LiDAR frame}} \\ 1 \end{bmatrix}$$

Leveraging ICP, I can estimate relative transformation between two LiDAR scan and compose observation model trajectory:

$${}_tT_{t+1}^{(O)} = \text{ICP}(S = \text{PC}_{t+1}, T = \text{PC}_t)$$

$$P_t^{(3O)} = P_0 \prod_{f=1}^t {}_{f-1}T_f^{(O)}$$

in which, the superscript  $^{(3O)}$  means this is a 3D observation trajectory.

*C. Factor Graph and Loop Closure*

*D. Mapping*

1) *Occupancy:*

2) *Texture:*

### III. TECHNICAL APPROACH

[25 pts] Technical Approach: Describe your technical approach to SLAM and texture mapping.

*A. Time Stamp*

interpolation linear cubic

*B. wheel encoder*

average of all 4 wheels

*C. 2D 3D pose conversion*

*D. ICP*

partial ICP

*E. RGBD*

distance hist and remove

### IV. RESULTS

[15 pts] Results: Present your results, and discuss them – what worked, what did not, and why. Analyze the impact of loop closure detection and pose graph optimization on the accuracy of the robot trajectory estimate and the resulting map quality. Make sure your results include (a) images of the trajectory and occupancy grid map over time constructed by your SLAM algorithm and (b) textured maps over time. If you have videos, include them in your submission zip file and refer to them in your report.