MA585

Homework 5

1. Sample mean $\overline{X} = 0.157$ was computed from a sample of size 100 generated from a $MA(1)$ process with mean μ and $\theta = -0.6$, $\sigma^2 = 1$. Construct an approximate 95% CI for μ. Are the data compatible with the hypothesis that $\mu = 0$?

2. Suppose you have a sample of size 100 and obtained $\widehat{\rho}(1) = 0.432$ and $\widehat{\rho}(2) = 0.145$. Assuming that data was generated from a $MA(1)$ process, construct a 95% CI for $\rho(1)$ and $\rho(2)$. Based on these two confidence intervals, is the data consistent with a $MA(1)$ model with $\theta = 0.6$?

3. For each one of the following ARMA processes, choose parameters such that the process is causal and invertible. In each case, use the arima.sim function in R to generate a sample realization of size 100. Generate a time series plot of the simulated series, and in each case plot both population and sample ACF and PACF.

(i) AR(2)
(ii) ARMA(1,1)
(iii) MA(1)
(iv) ARMA(1,2)

R Note: acf and pacf functions in R can be used to obtain sample acf and pacf. The following code was used to generate the plots for AR(2) example in lecture note 4.

```
x=arima.sim(n=100, list(ar=c(1.5,-0.75)))
plot.ts(x)
title(main="Simulated Data from the AR(2) Process X(t)-1.5X(t-1)+0.75X(t-
2)=e(t)")
par(mfrow=c(2,2))
y = ARMAacf(ar=c(1.5,-0.75),lag.max = 20)
y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Autocorrelation", main = "AR(2) Population ACF")
abline(h = 0)
y = ARMAacf(ar=c(1.5,-0.75),lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial Autocorrelation", main = "AR(2) Population PACF")
abline(h = 0)
acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```

For general ARMA processes, you modify the list of parameters as ar=c(..,..), ma=c(..,...). For example, to simulate a sample realization from an ARMA(2,1) process, use

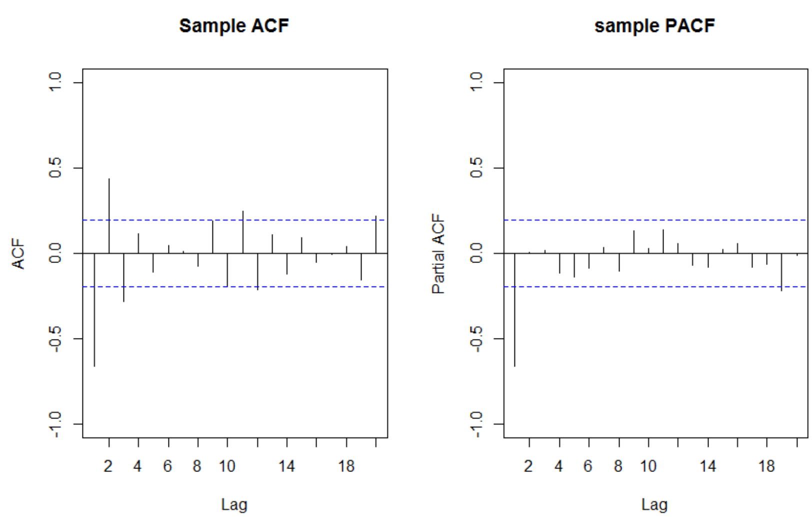
>arima.sim(n=100,list(ar=c(1.5,-0.75), ma=c(0.3))) # n=100 observations from X(t)-1.5X(t-1)+0.75X(t-2)=e(t)+0.3e(t-1).

Also, the following code was used to generate the MA(2) example. Note how the option ci.type="ma" affects the confidence bounds for sample acf.

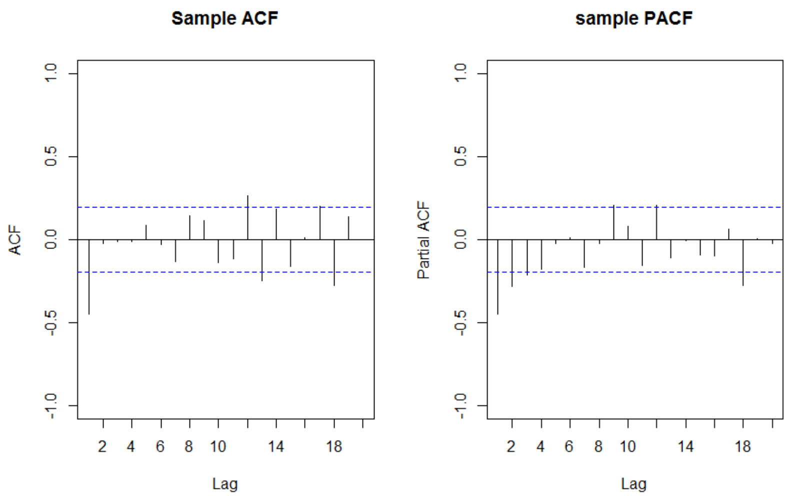
```
par(mfrow=c(3,1))
y=arima.sim(100, model=list(ma=c(-1.5,0.75)))
plot.ts(y,main="Sample Realization from a MA(2) Process")
acf(y,xlim=c(1,20), ylim=c(-0.6,0.6),xaxp=c(0,20,10), main="Sample ACF",ci.type="ma")
pacf(y,xaxp=c(0,20,10), ylim=c(-0.6,0.6), main="sample PACF")
```

4. The graphs below show the sample ACF and PACF of three time series of length 100 each. On the basis of the available information, choose an ARMA model for the data. You need to identify the order of the model and, if possible, provide approximate values of the ARMA parameters ϕ and θ. Justify your answer.

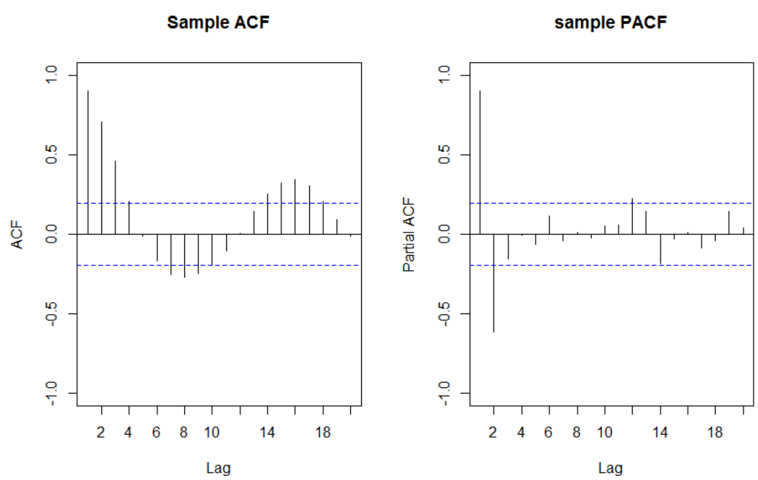
a.



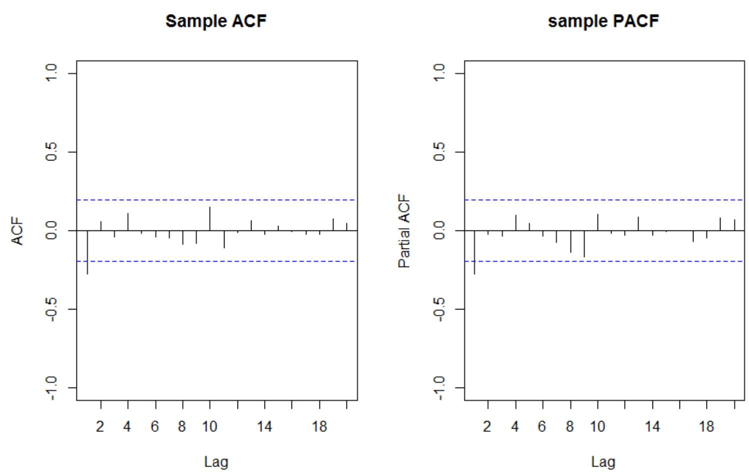
b.


Sample ACF
sample PACF
ACF
Partial ACF
1.0
0.5
0.0
-0.5
-1.0
2 4 6 8 10 14 18
Lag

c.


Sample ACF
sample PACF
ACF
Partial ACF
1.0
0.5
0.0
-0.5
-1.0
2 4 6 8 10 14 18
Lag

d.



5. For a series of length 169, we find that $\widehat{\rho}(1) = 0.41, \widehat{\rho}(2) = 0.32, \widehat{\rho}(3) = 0.26, \widehat{\rho}(4) = 0.21$,and $\widehat{\rho}(5) = 0.16$.What ARMA model fits this pattern of autocorrelations? Justify your answer.

6. A stationary time series of length 121 produced sample partial autocorrelation of $\phi_{11} = 0.8$, $\phi_{22} = -0.6$,$\phi_{33} = 0.08$, and $\phi_{44} = 0.00$. Based on this information alone, what model would we tentatively specify for the series?

7. Consider the annual sunspots data in R given in data file sunspot.year (yearly numbers of sunspots from 1700 to 1988). If you don't know what sunspots are, check on the internet.

(i) Plot the time series and describe the features of the data.

(ii) Generate a new time series by transforming the data as newsunspot=sqrt(sunspot.year). Plot the new time series. Why is the square-root transformation necessary?

(ii) Plot ACF and PACF of the transformed data. Based on these plots, propose a plausible model and justify your answer.