HW5

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#1

1. MA(1) $X_t = u + \theta_t - 0.6\theta_{t-1}$

$$\gamma(h) = \begin{cases} 1.36 & h=0 \\ -0.6 & h=\pm 1 \\ 0 & \text{otherwise} \end{cases}$$

$$Var(\bar{X}) = \frac{1}{n}\sum_{h=-n}^{n}\left(1-\frac{|h|}{n}\right)\gamma(h) = \frac{1}{100}\sum_{h=-100}^{100}\left(1-\frac{|h|}{100}\right)\gamma(h)$$

$$= \frac{1}{100}\cdot 1.36 + \frac{1}{100}\cdot 2\cdot\frac{99}{100}\cdot(-0.6) = 0.00172$$

95% CI $= \bar{X} \pm 1.96\sqrt{Var(\bar{X})} = 0.157 \pm 0.081 = (0.076, 0.238)$

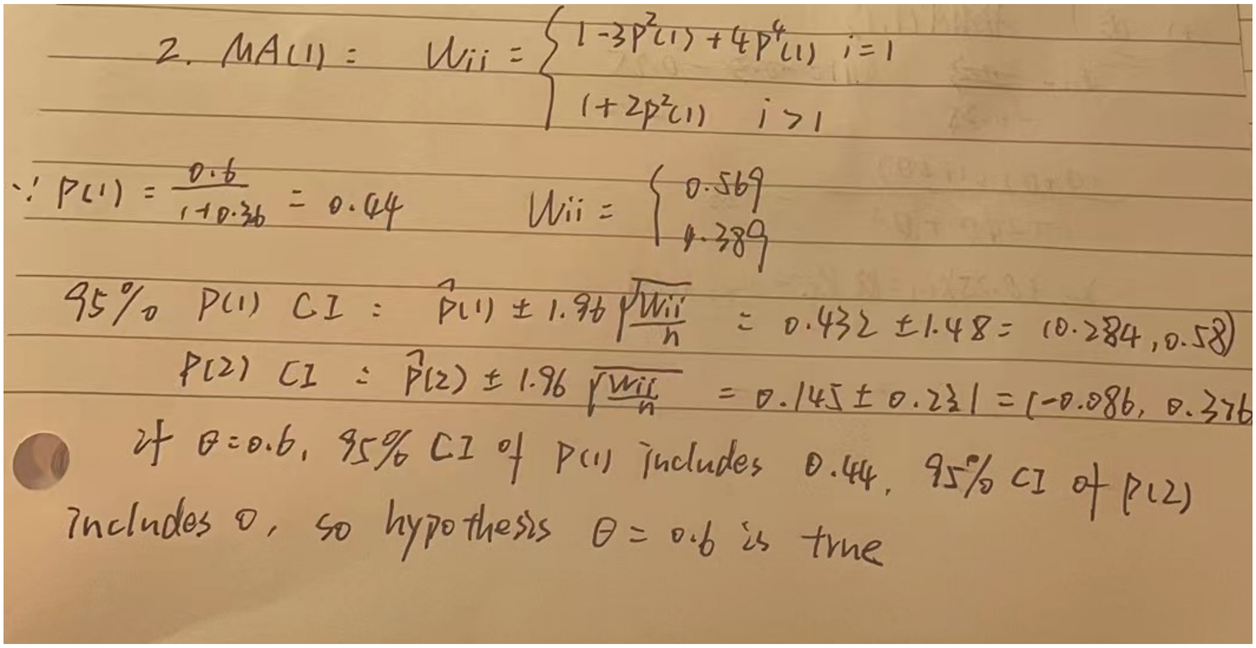
Since 0 is not included, so the data is not compatible with the hypothesis $u=0$

Figure 1: Problem 1

#2

#3 ##AR(2)

```
x=arima.sim(n=100, list(ar=c(1.5,-0.75)))
plot.ts(x)
title(main="Simulated Data from the AR(2) Process X(t)-1.5X(t-1)+0.75X(t-
2)=e(t)")
```



2. MA(1): $W_{ii} = \begin{cases} 1-3\rho^2(1)+4\rho^4(1) & i=1 \\ 1+2\rho^2(1) & i>1 \end{cases}$

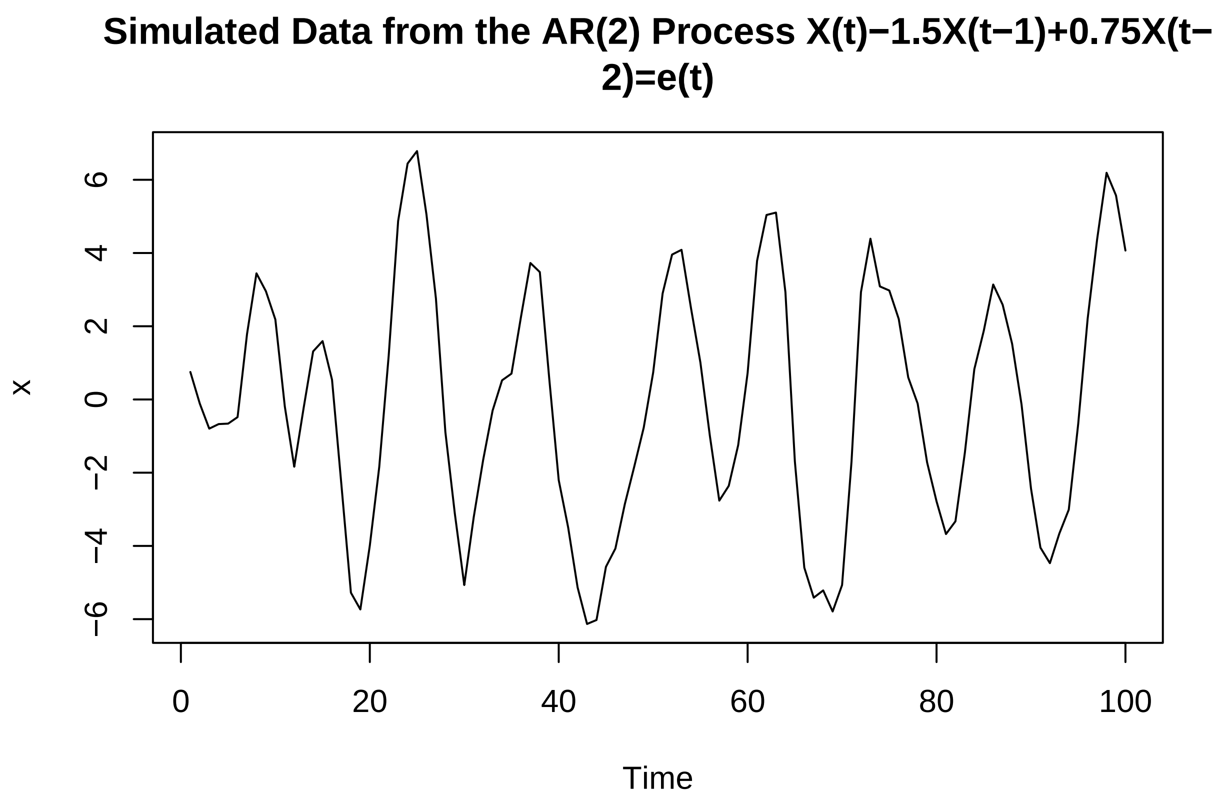
$\because \rho(1) = \frac{0.6}{1+0.36} = 0.44$ $\quad W_{ii} = \begin{cases} 0.569 \\ 1.389 \end{cases}$

95% $\rho(1)$ CI : $\hat{\rho}(1) \pm 1.96\sqrt{\frac{W_{ii}}{n}} = 0.432 \pm 1.48 = (0.284, 0.58)$

$\rho(2)$ CI : $\hat{\rho}(2) \pm 1.96\sqrt{\frac{W_{ii}}{n}} = 0.145 \pm 0.231 = [-0.086, 0.376$

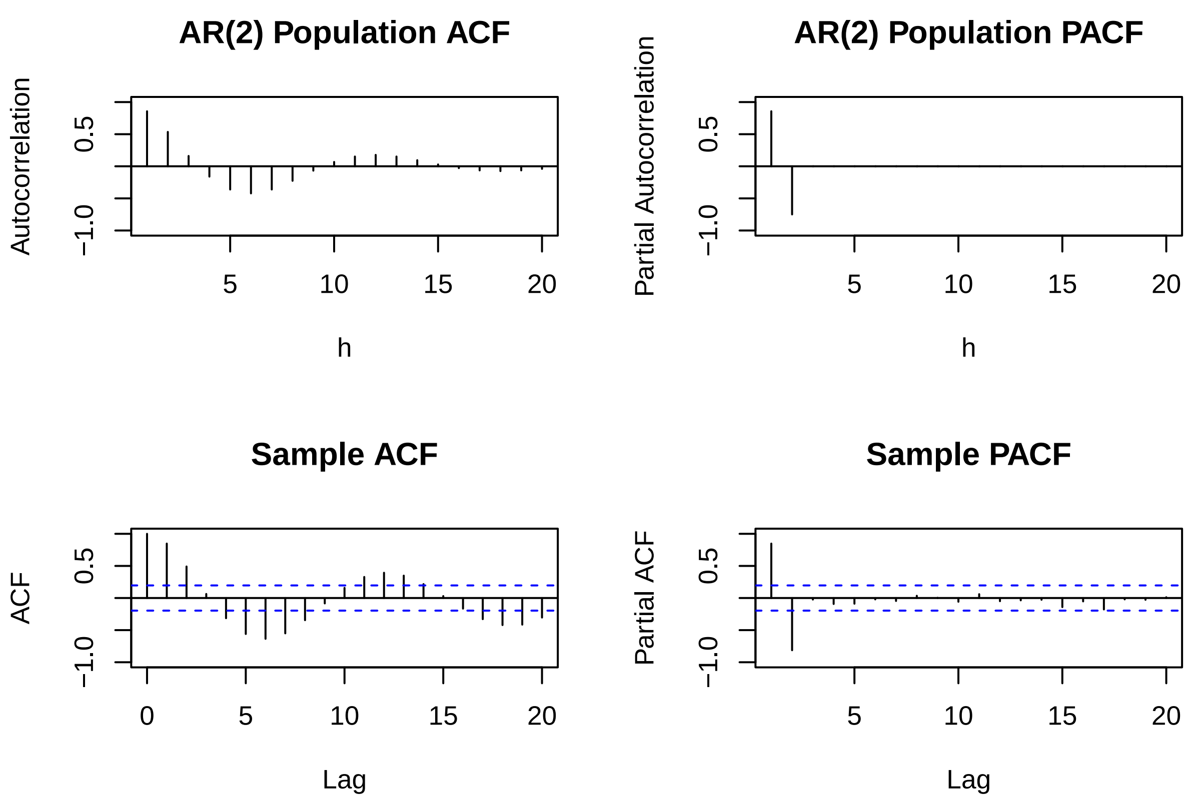
若 $\theta = 0.6$, 95% CI of $\rho(1)$ includes 0.44, 95% CI of $\rho(2)$ includes 0, so hypothesis $\theta = 0.6$ is true

Figure 2: Problem 2



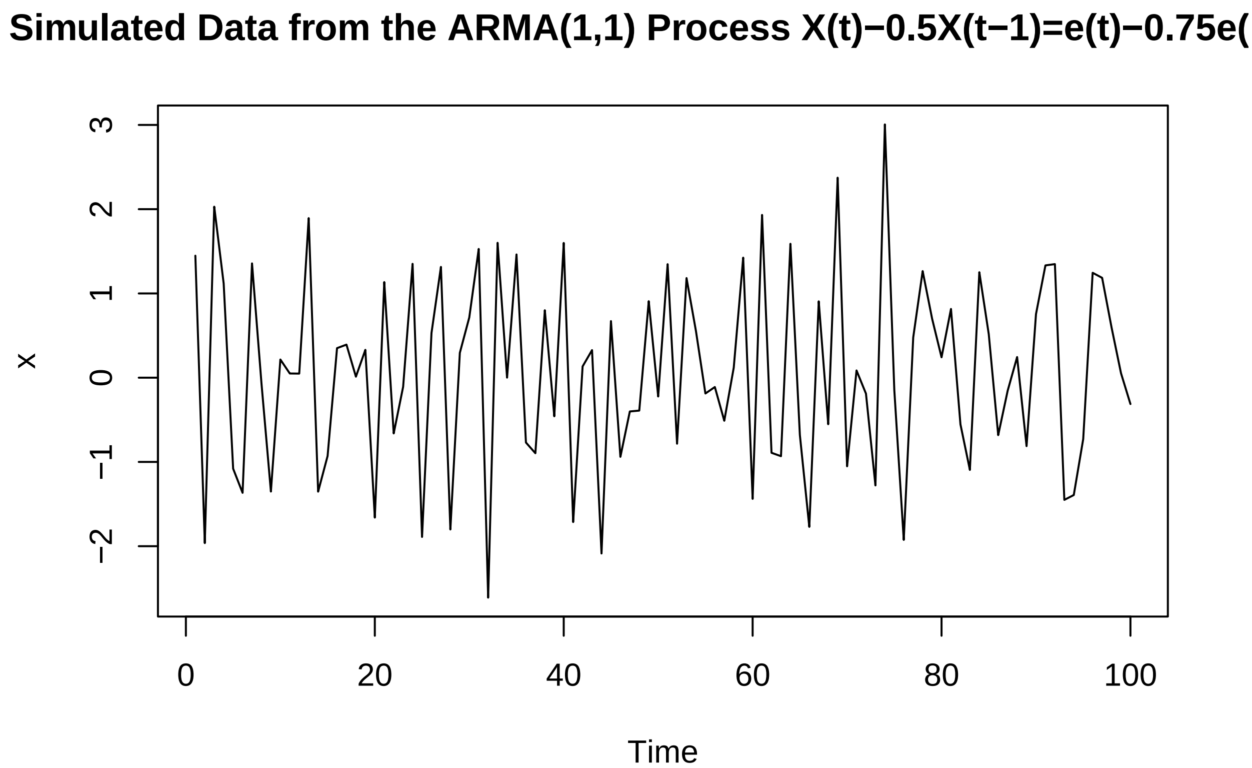

```
par(mfrow=c(2,2))
y = ARMAacf(ar=c(1.5,-0.75),lag.max = 20)
```

```
y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Autocorrelation", main = "AR(2) Population ACF")
abline(h = 0)
y = ARMAacf(ar=c(1.5,-0.75),lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial Autocorrelation", main = "AR(2) Population PACF")
abline(h = 0)
acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```

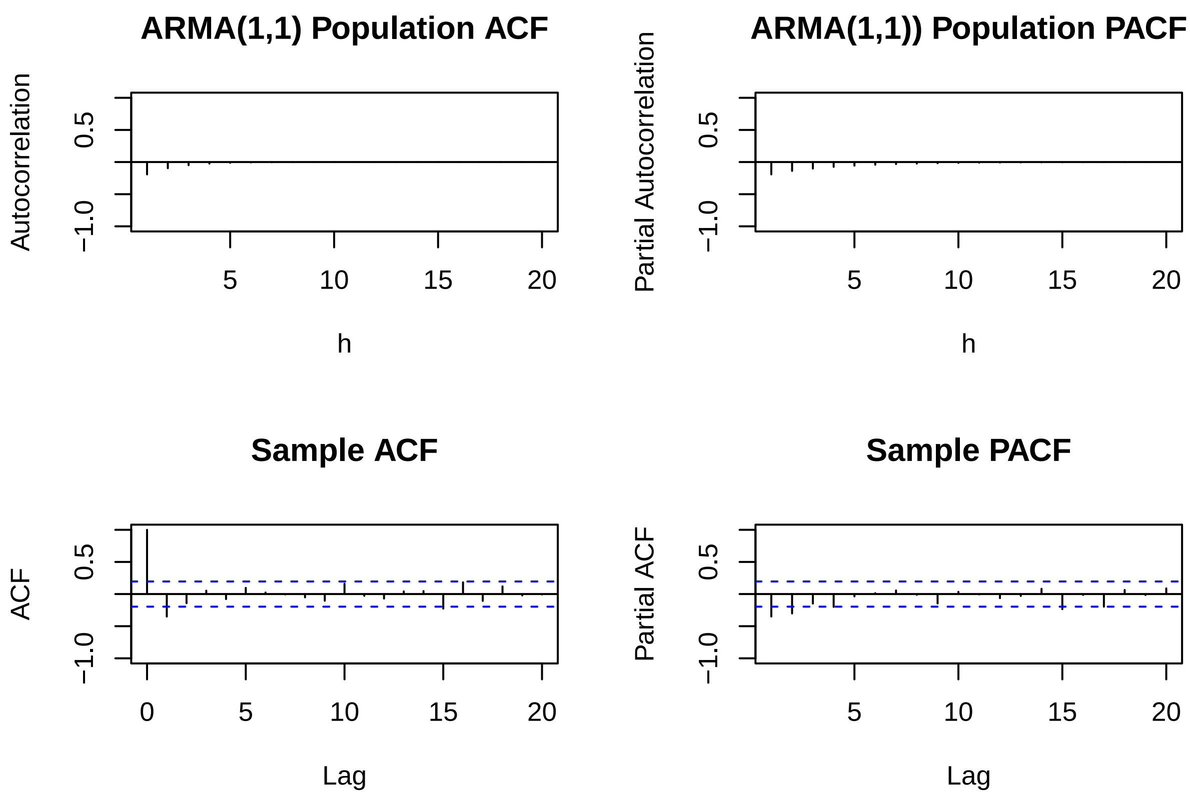


##ARMA(1,1)

```
x=arima.sim(n=100, list(ar=0.5,ma=-0.75))
plot.ts(x)
title(main="Simulated Data from the ARMA(1,1) Process X(t)-0.5X(t-1)=e(t)-0.75e(t-1)")
```

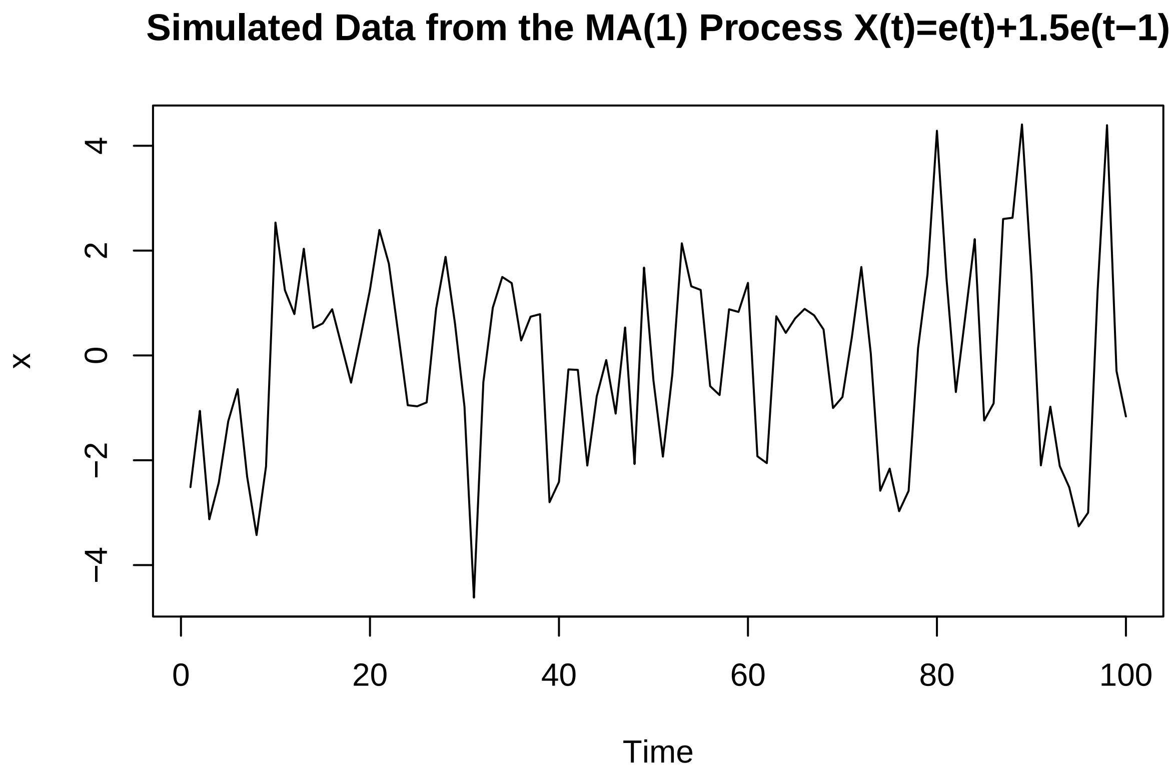



```
par(mfrow=c(2,2))
y = ARMAacf(ar=0.5,ma=-0.75,lag.max = 20)
y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Autocorrelation", main = "ARMA(1,1) Population ACF")
abline(h = 0)
y = ARMAacf(ar=0.5,ma=-0.75,lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial Autocorrelation", main = "ARMA(1,1)) Population PACF")
abline(h = 0)
acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```

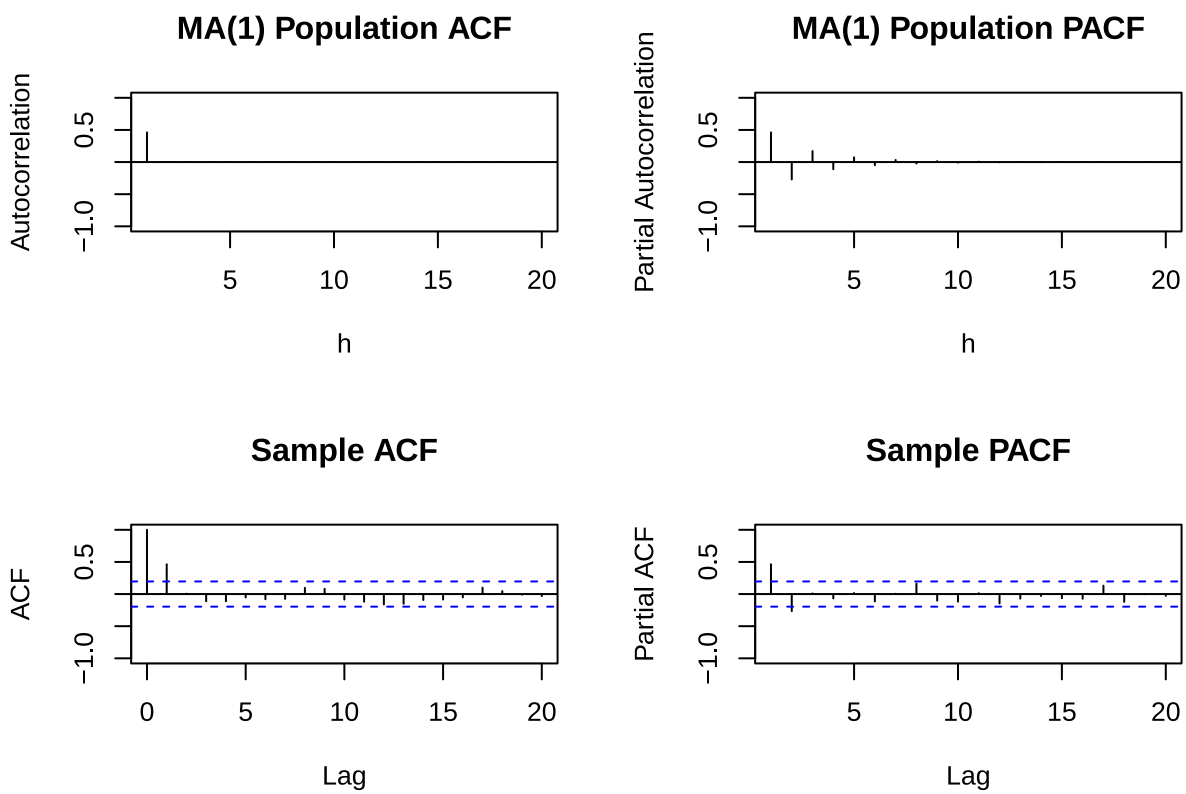


##MA(1)

```
x=arima.sim(n=100, list(ma=1.5))
plot.ts(x)
title(main="Simulated Data from the MA(1) Process X(t)=e(t)+1.5e(t-1)")
```

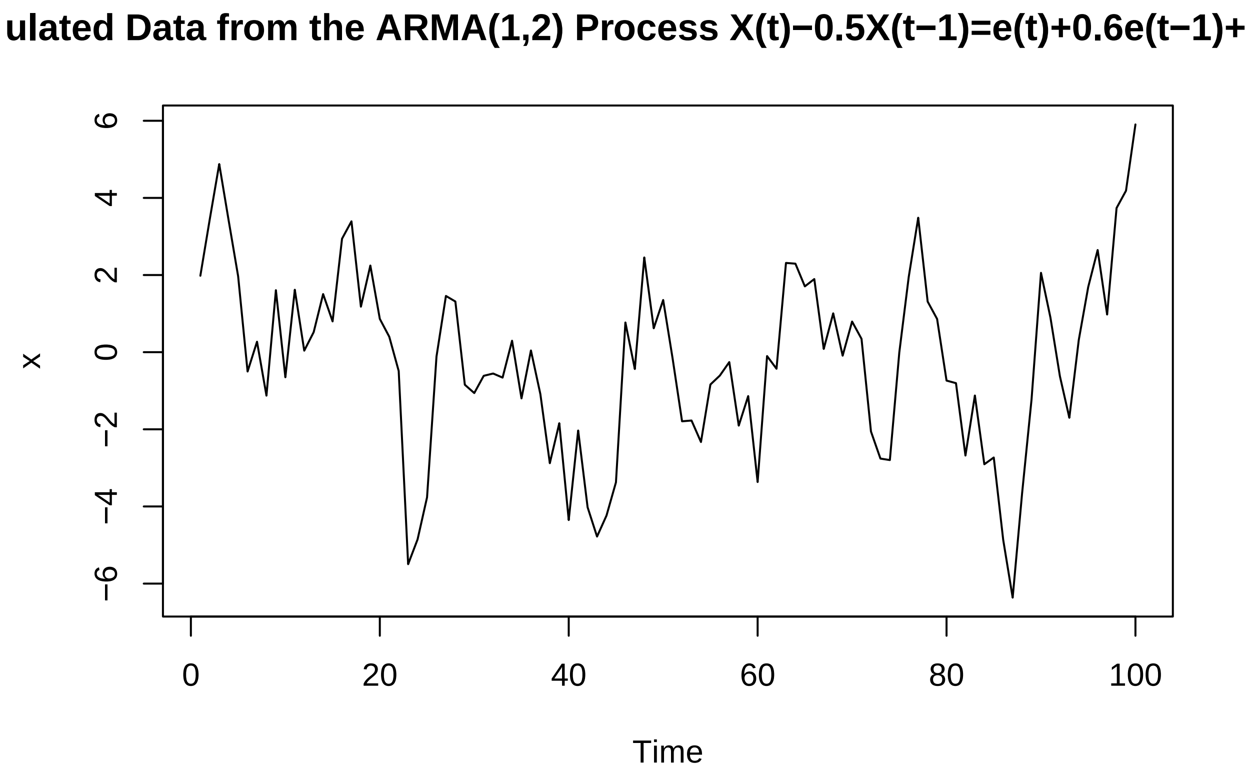



```
par(mfrow=c(2,2))
y = ARMAacf(ar=0,ma=1.5,lag.max = 20)
y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Autocorrelation", main = "MA(1) Population ACF")
abline(h = 0)
y = ARMAacf(ar=0,ma=1.5,lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial Autocorrelation", main = "MA(1) Population PACF")
abline(h = 0)
acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```

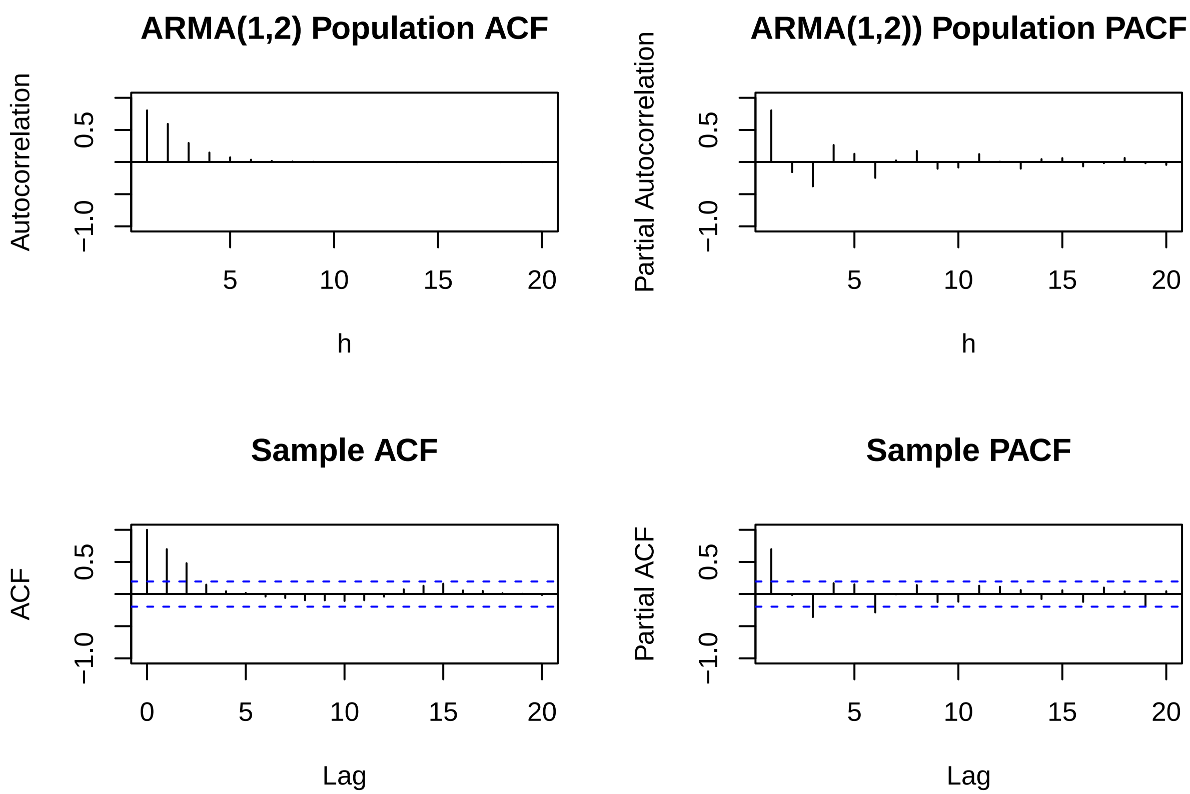


ARMA(1,2)

```
x=arima.sim(n=100, list(ar=0.5,ma=c(0.6,1.2)))
plot.ts(x)
title(main="Simulated Data from the ARMA(1,2) Process X(t)-0.5X(t-1)=e(t)+0.6e(t-1)+1.2e(t-2)")
```

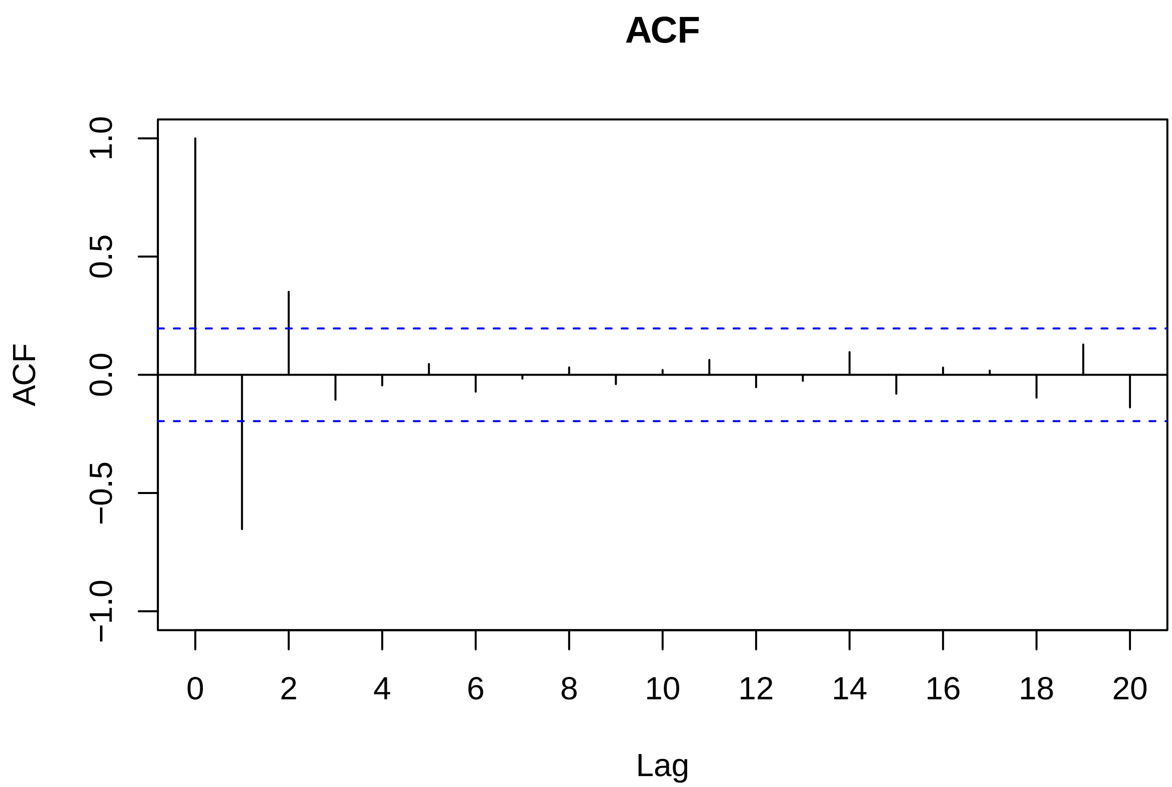



```r
par(mfrow=c(2,2))
y = ARMAacf(ar=0.5,ma=c(0.6,1.2),lag.max = 20)
y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Autocorrelation", main = "ARMA(1,2) Population ACF")
abline(h = 0)
y = ARMAacf(ar=0.5,ma=c(0.6,1.2),lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial Autocorrelation", main = "ARMA(1,2)) Population PACF")
abline(h = 0)
acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```

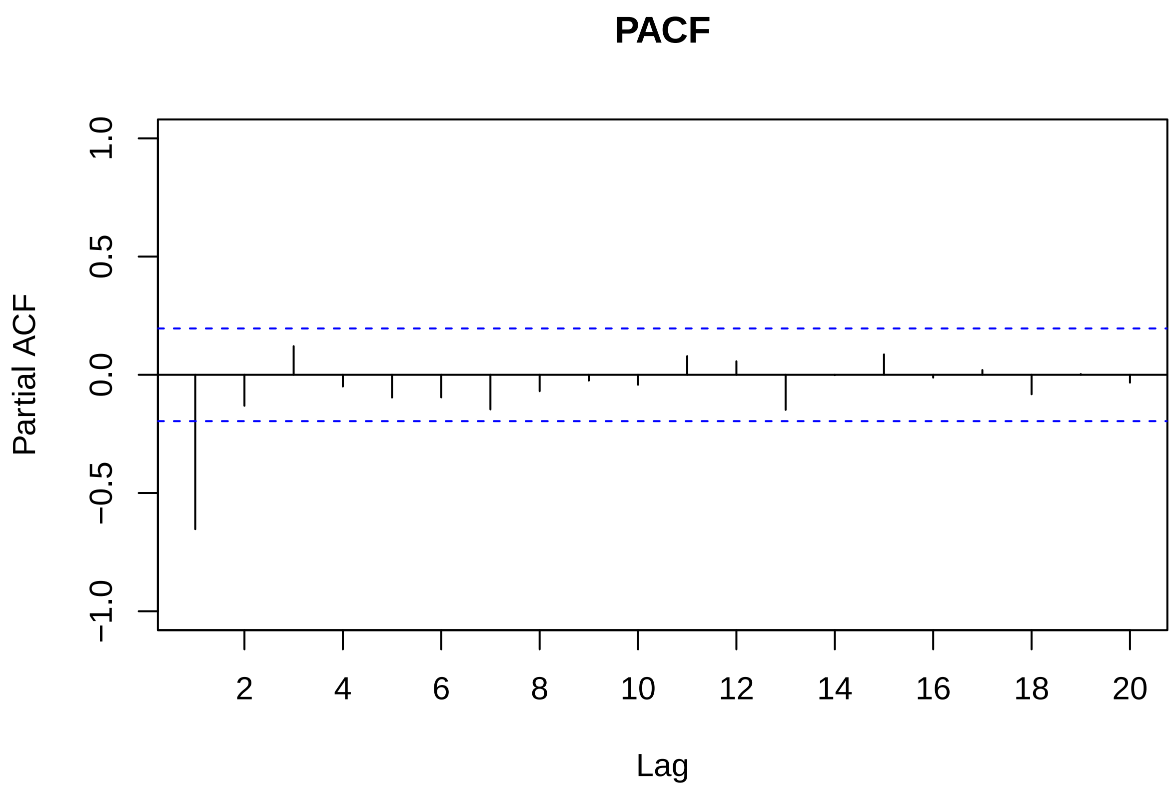


#4 ##(a) X(t)+0.7X(t-1)=et

```
a=arima.sim(n=100, list(ar=-0.7))
acf(a,main="ACF",ylim=c(-1,1),xaxp=c(0,20,10))
```

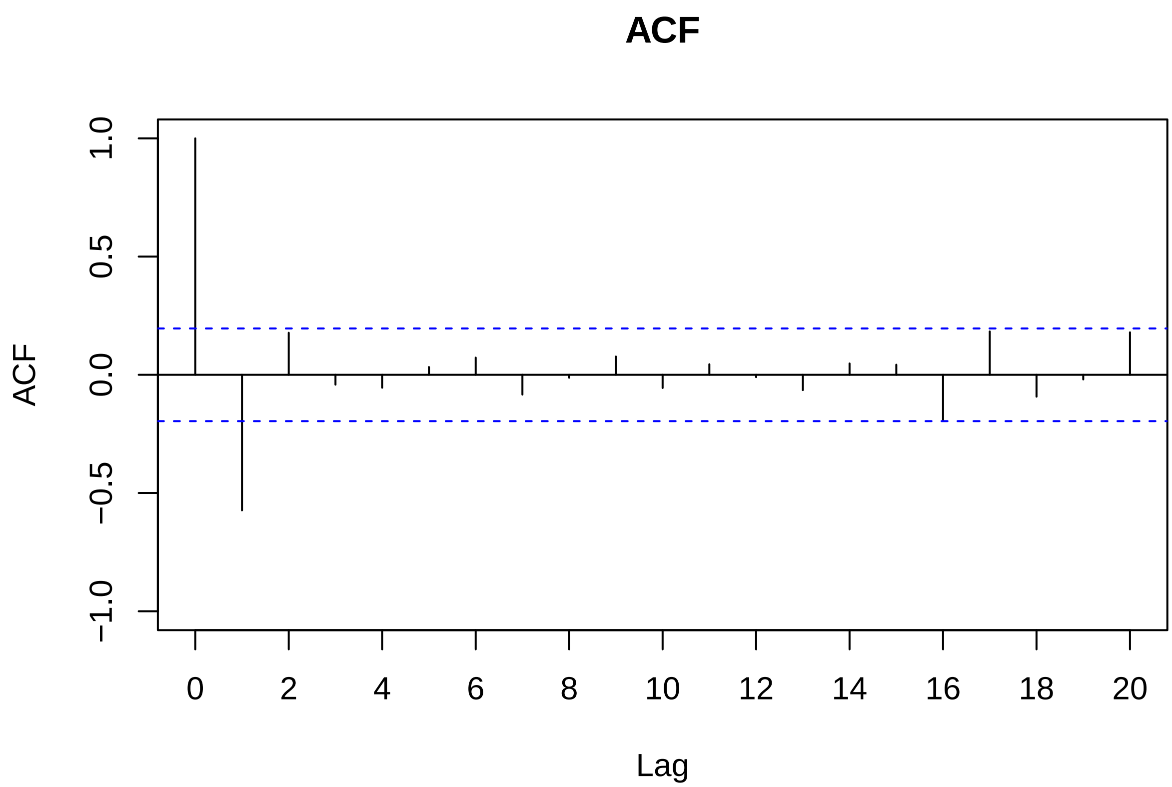



```
pacf(a,main="PACF",ylim=c(-1,1),xaxp=c(0,20,10))
```

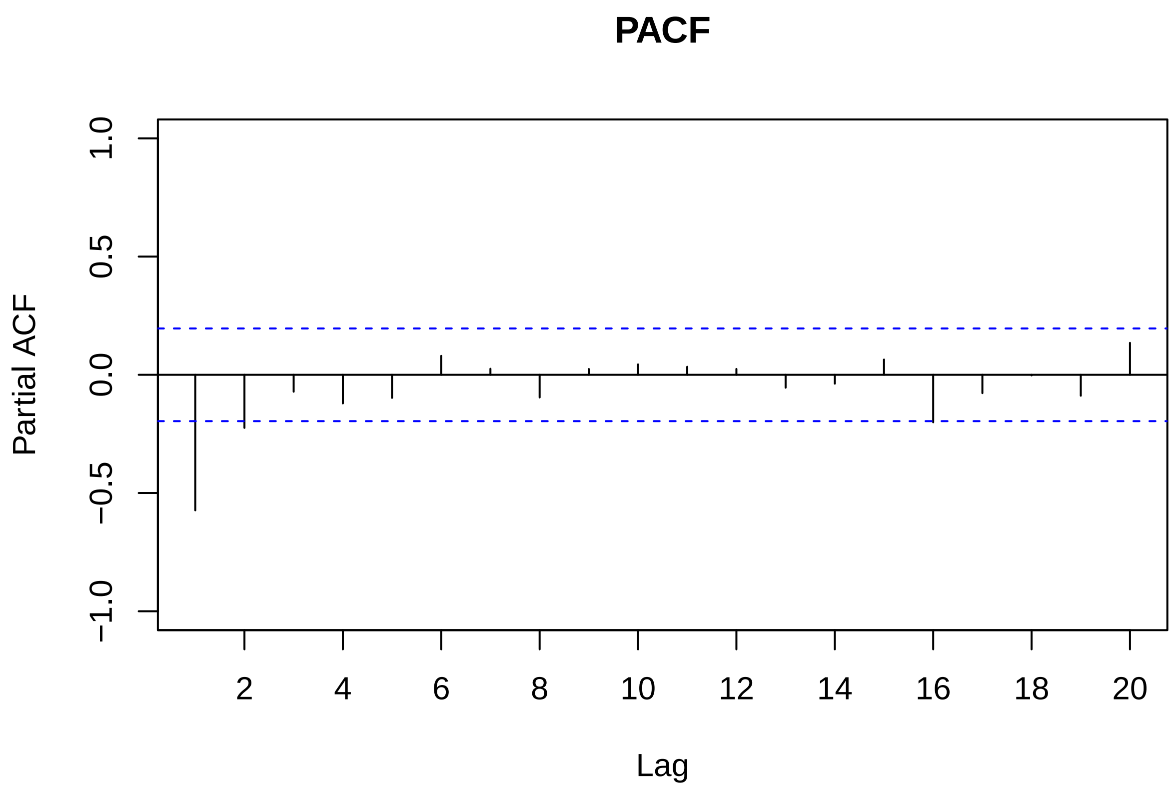


##(b) e(t)-0.5e(t-1)=X(t)

```
b=arima.sim(n=100, list(ma=-0.5))
acf(b,main="ACF",ylim=c(-1,1),xaxp=c(0,20,10))
```

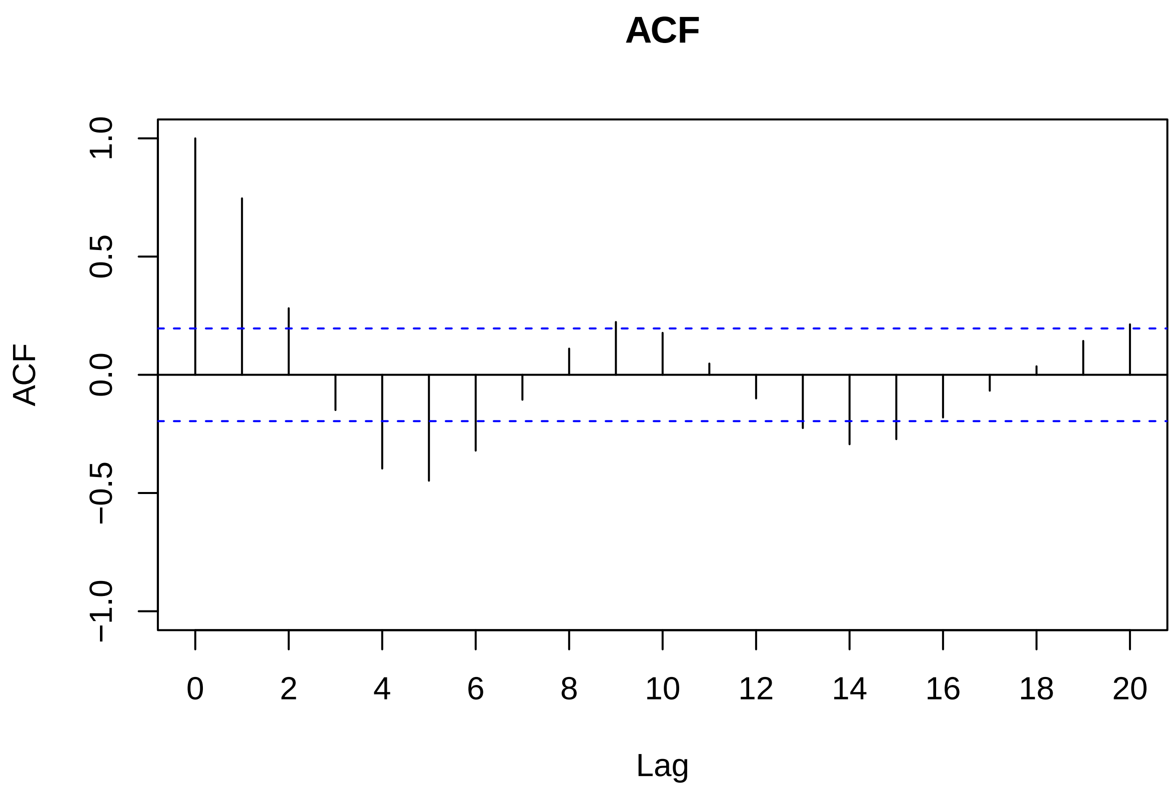



```
pacf(b,main="PACF",ylim=c(-1,1),xaxp=c(0,20,10))
```

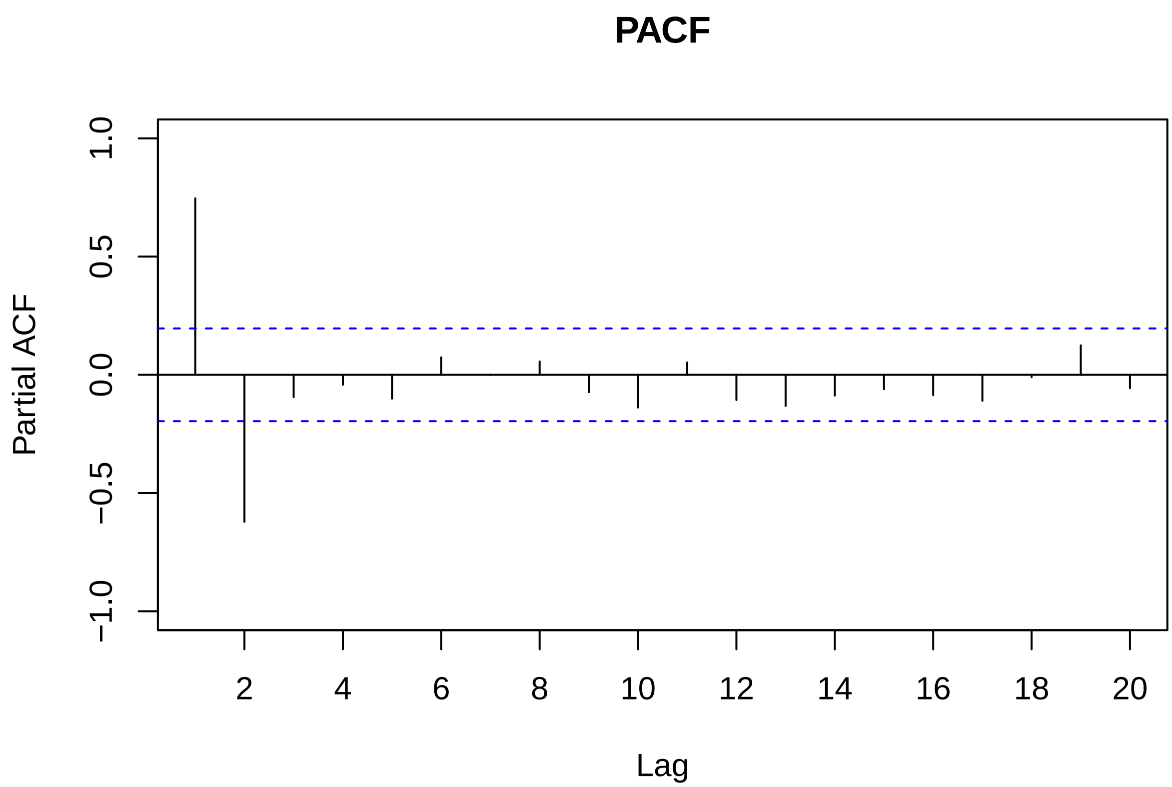


##(c) X(t)-1.3X(t-1)+0.75X(t-2)=e(t)

```
a=arima.sim(n=100, list(ar=c(1.3,-0.75)))
acf(a,main="ACF",ylim=c(-1,1),xaxp=c(0,20,10))
```

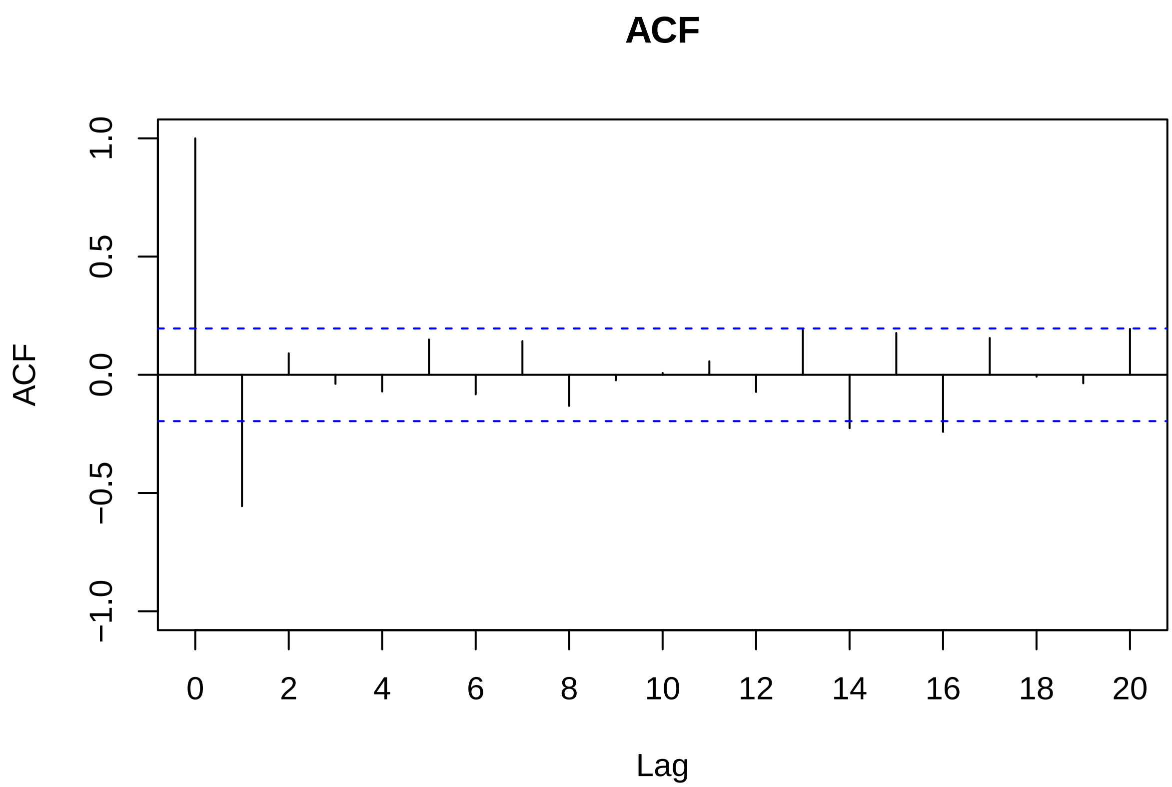



```
pacf(a,main="PACF",ylim=c(-1,1),xaxp=c(0,20,10))
```

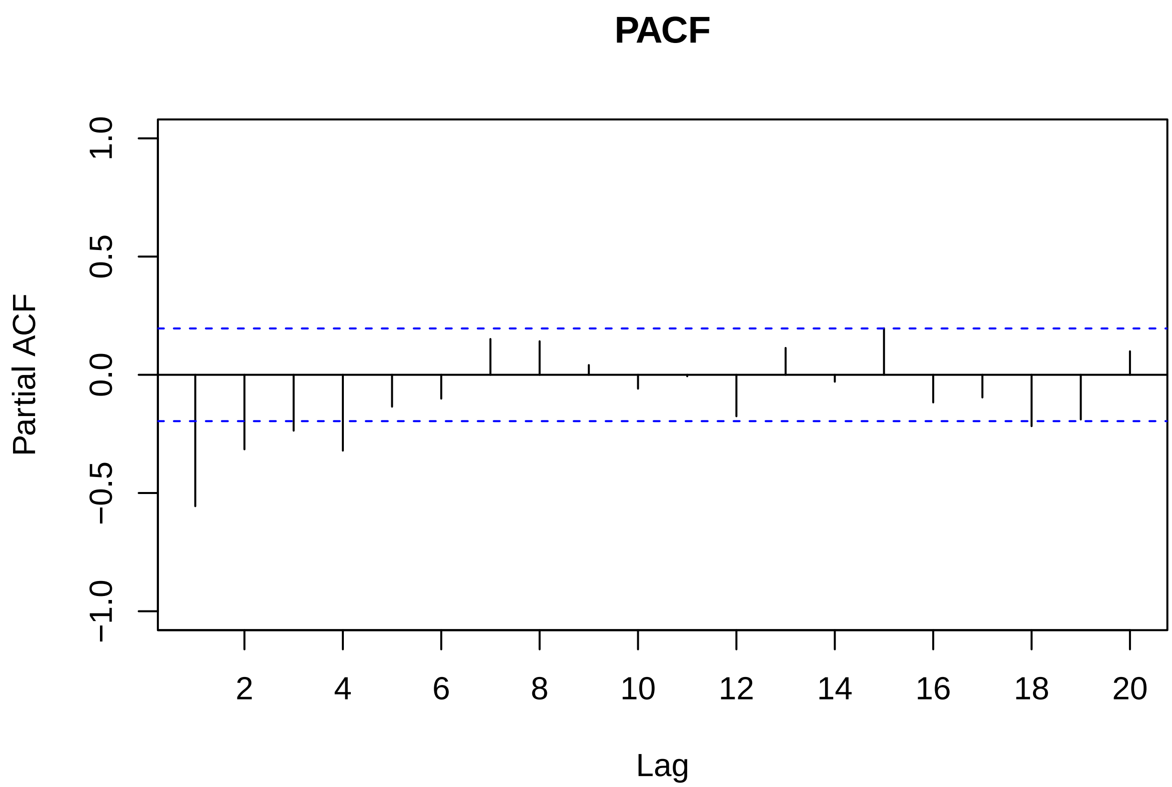


##(d) X(t)+0.25X(t-1)=e(t)-0.65e(t-1)

```
a=arima.sim(n=100, list(ar=-0.25,ma=-0.65))
acf(a,main="ACF",ylim=c(-1,1),xaxp=c(0,20,10))
```




```
pacf(a,main="PACF",ylim=c(-1,1),xaxp=c(0,20,10))
```



#5 X(t)-0.8X(t-1)=e(t)-0,5e(t-1)

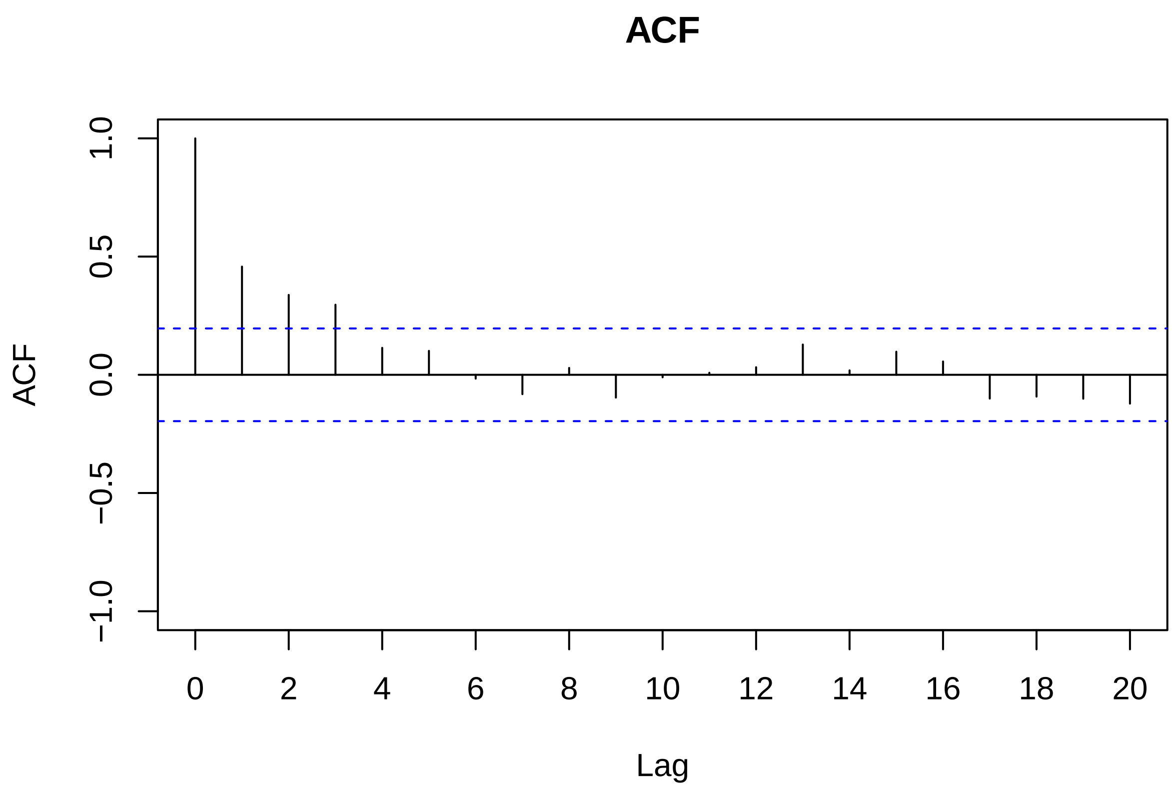
```
ARMAacf(ar =0.8,ma=-0.5,lag.max = 5)
```

```
##       0       1       2       3       4       5
## 1.00000 0.40000 0.32000 0.25600 0.20480 0.16384
```

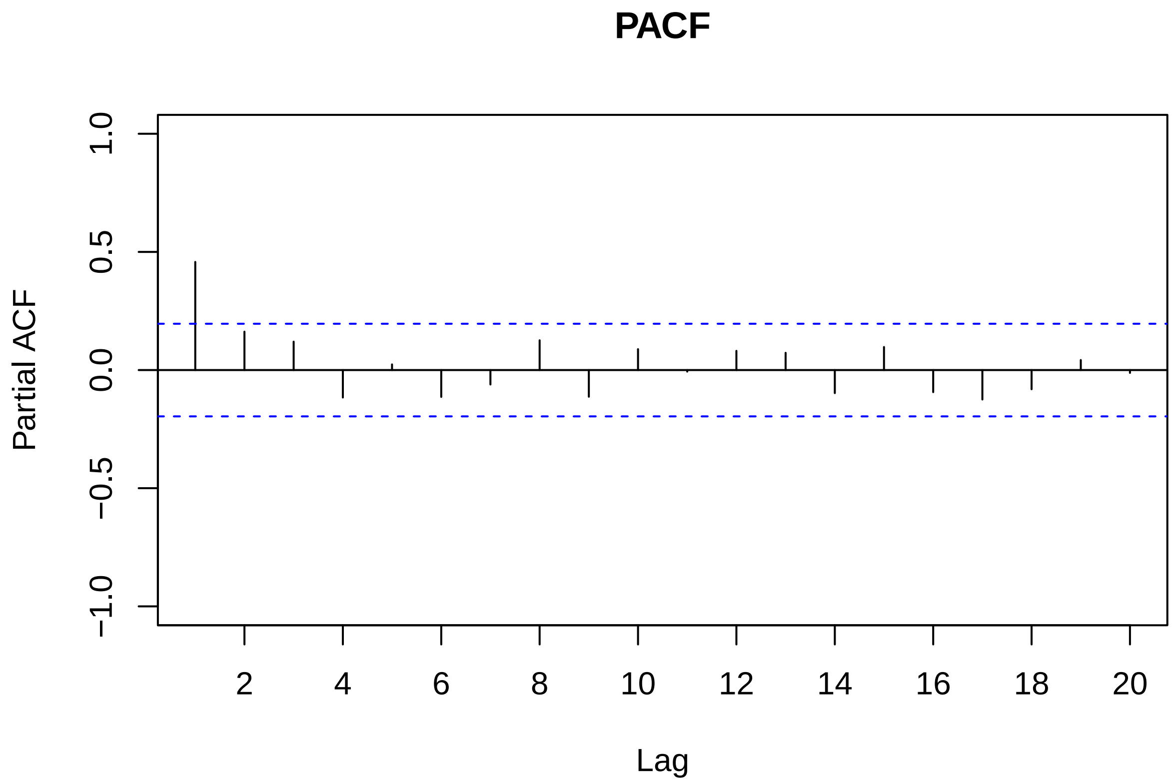
```
ARMAacf(ar =0.8,ma=-0.5,lag.max = 5,pacf=T)
```

```
## [1] 0.40000000 0.19047619 0.09411765 0.04692082 0.02344322
```

```
a=arima.sim(n=100, list(ar=0.8,ma=-0.5))
acf(a,main="ACF",ylim=c(-1,1),xaxp=c(0,20,10))
```




```
pacf(a,main="PACF",ylim=c(-1,1),xaxp=c(0,20,10))
```



#6

#7 ##(i)

```
plot.ts(sunspot.year)
```

ARMA(1,1)

5. $\rho(1) = \dfrac{(\phi+\theta)(1+\phi\theta)}{1+2\phi\theta+\theta^2} = 0.41$

$\rho(2) = \phi\,\rho(1) = 0.32$

$\rho(3) = \phi^2\rho(1) = 0.26$

$\rho(4) = \phi^3\rho(1) = 0.21 \qquad \rho(5) = \phi^4\rho(1) = 0.16$

$\phi = 0.8125$

$$\frac{(0.8+\theta)(1+0.8\theta)}{1+1.6\theta+\theta^2} = 0.41$$

$$0.8+\theta+0.64\theta+0.8\theta^2 = 0.41+0.656\theta+0.41\theta^2.$$

$$0.39\theta^2 + \cancel{0.016}\ 0.984\theta + 0.39 = 0.$$

$$\theta^2+2.52\theta+1 = 1 \qquad \theta = \frac{-2.52 \pm \sqrt{2.25}}{2}.$$

$$\theta_1 = \cancel{3}\ -0.51 \qquad \theta_2 = -2.01$$

Figure 3: Problem 5

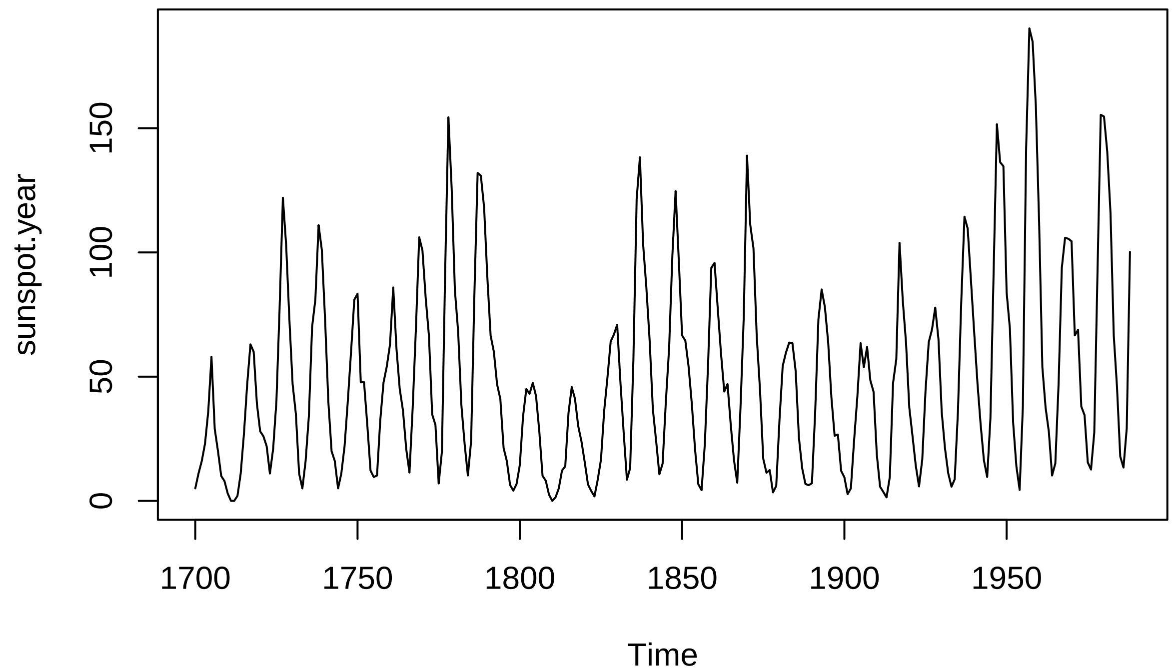
6. AR(2)

$\phi_{11} = \rho(1) = \dfrac{\phi_1}{1-\phi_2} = 0.8 \qquad \dfrac{\phi_1}{1+0.6} = 0.8 \qquad \phi_1 = 0.8\times 1.6 = 1.28$

$\phi_{22} = \phi_2 = -0.6$

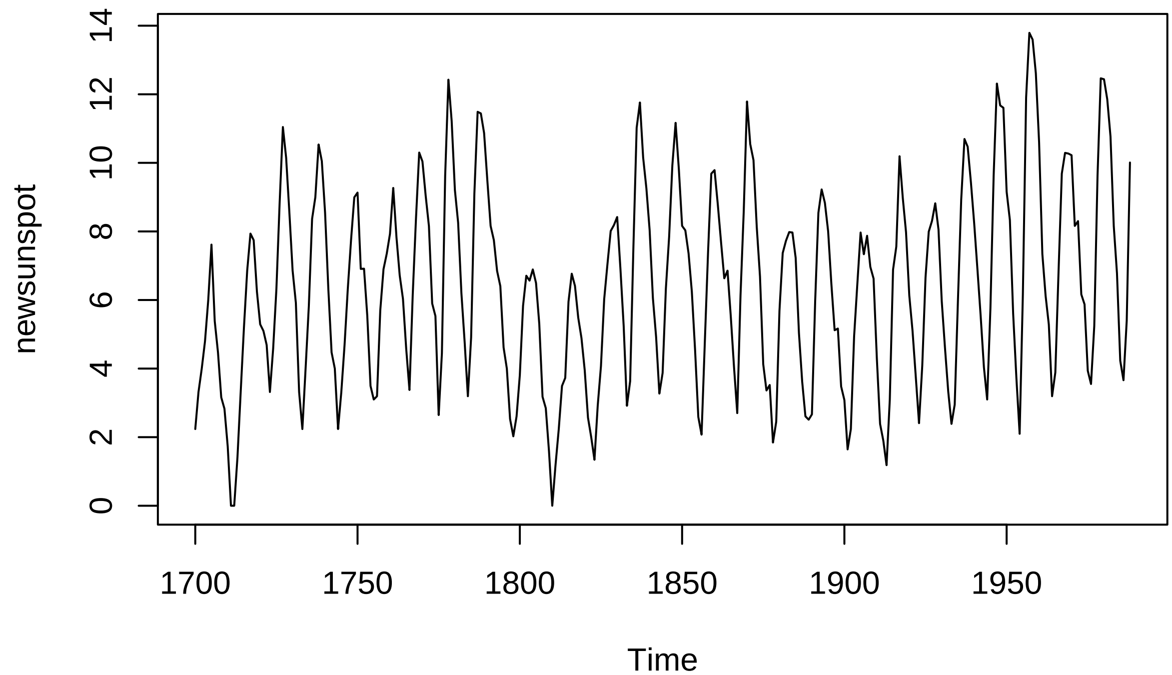
$$X_t - 1.28X_{t-1} + 0.6X_{t-2} = e_t$$

Figure 4: Problem 6



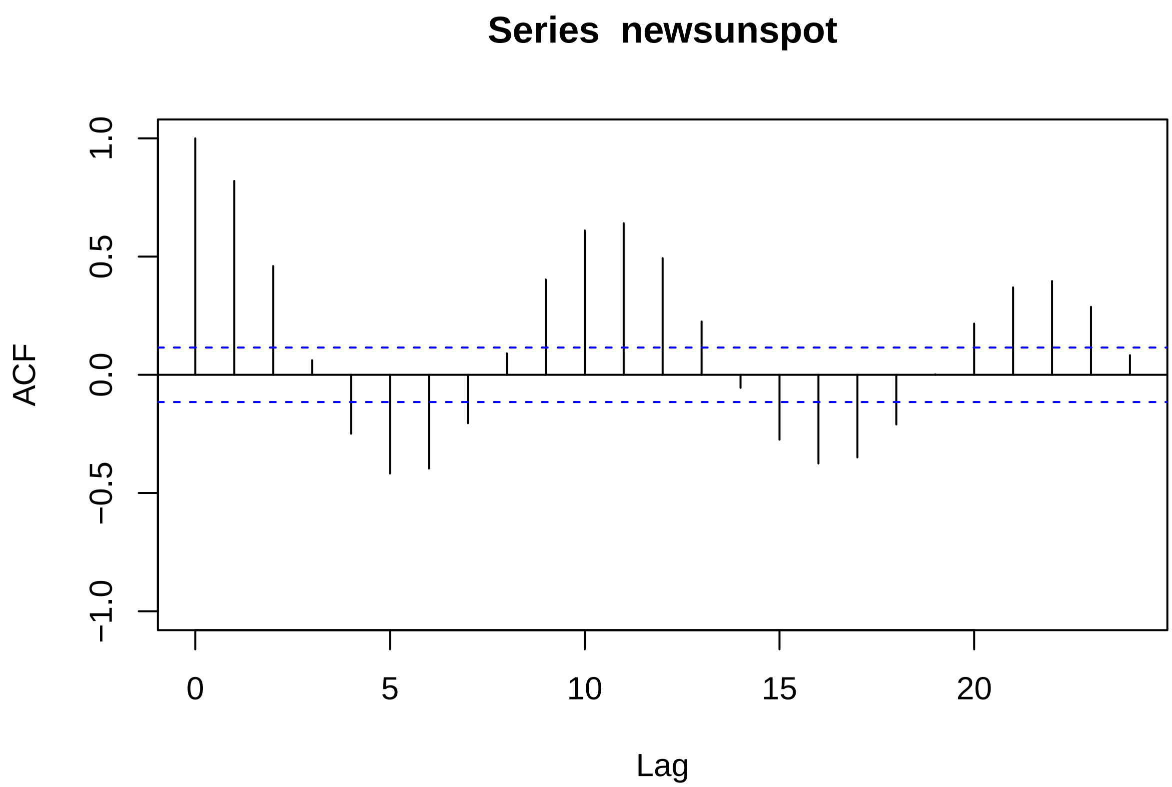
It's seems to be seasonal. ##(ii)

```
newsunspot <- sqrt(sunspot.year)
plot.ts(newsunspot)
```

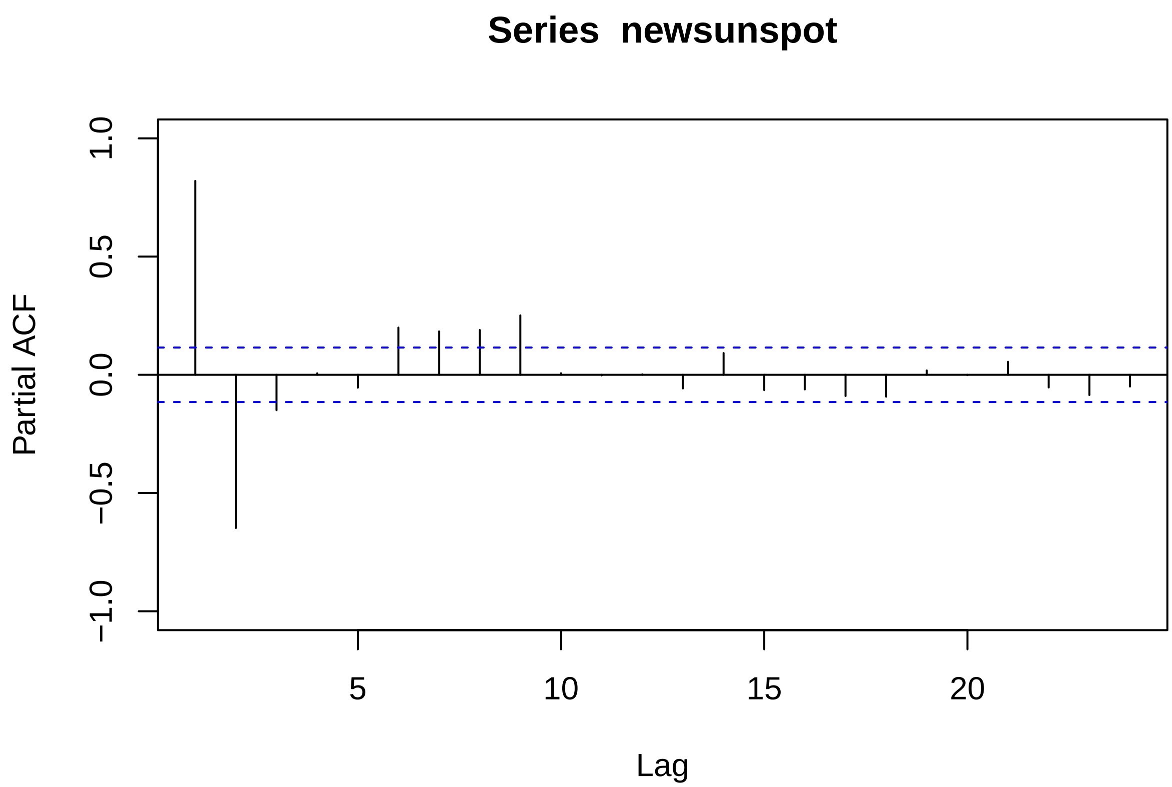


square-root transformation makes the variation more uniform. ##(iii)

```
acf(newsunspot,ylim=c(-1,1))
```

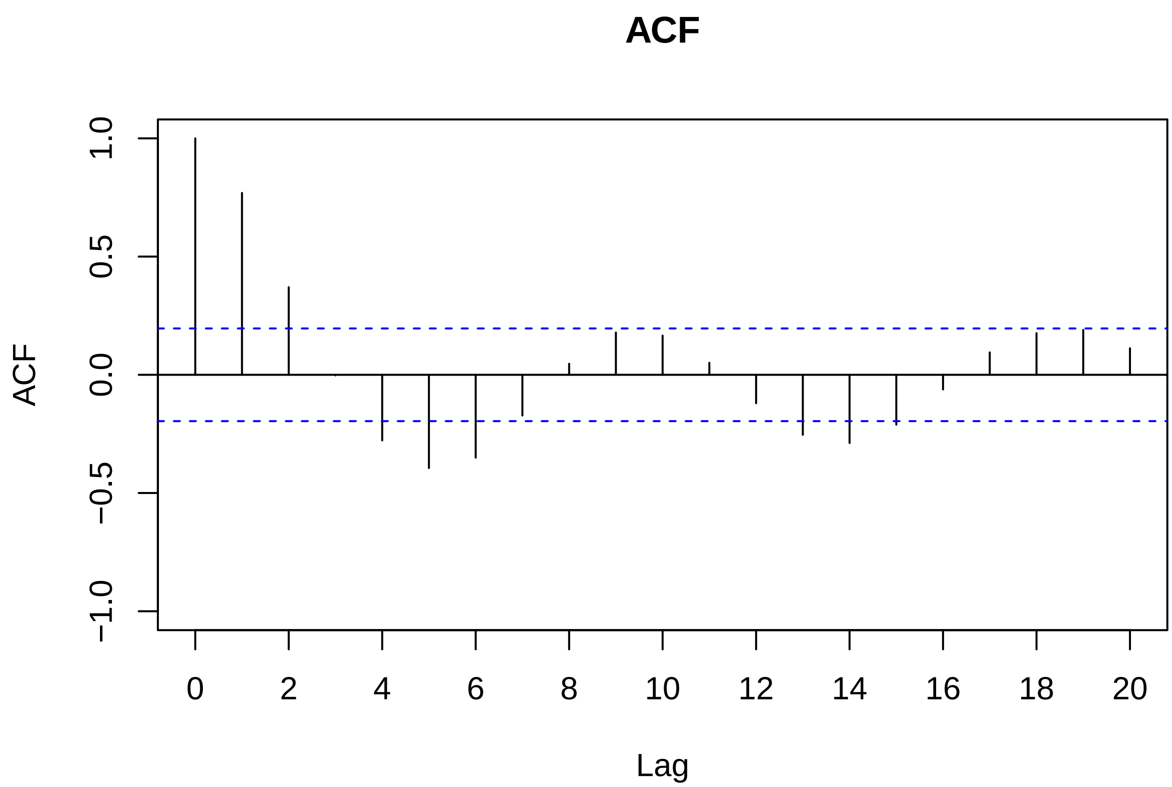



```
pacf(newsunspot,ylim=c(-1,1))
```

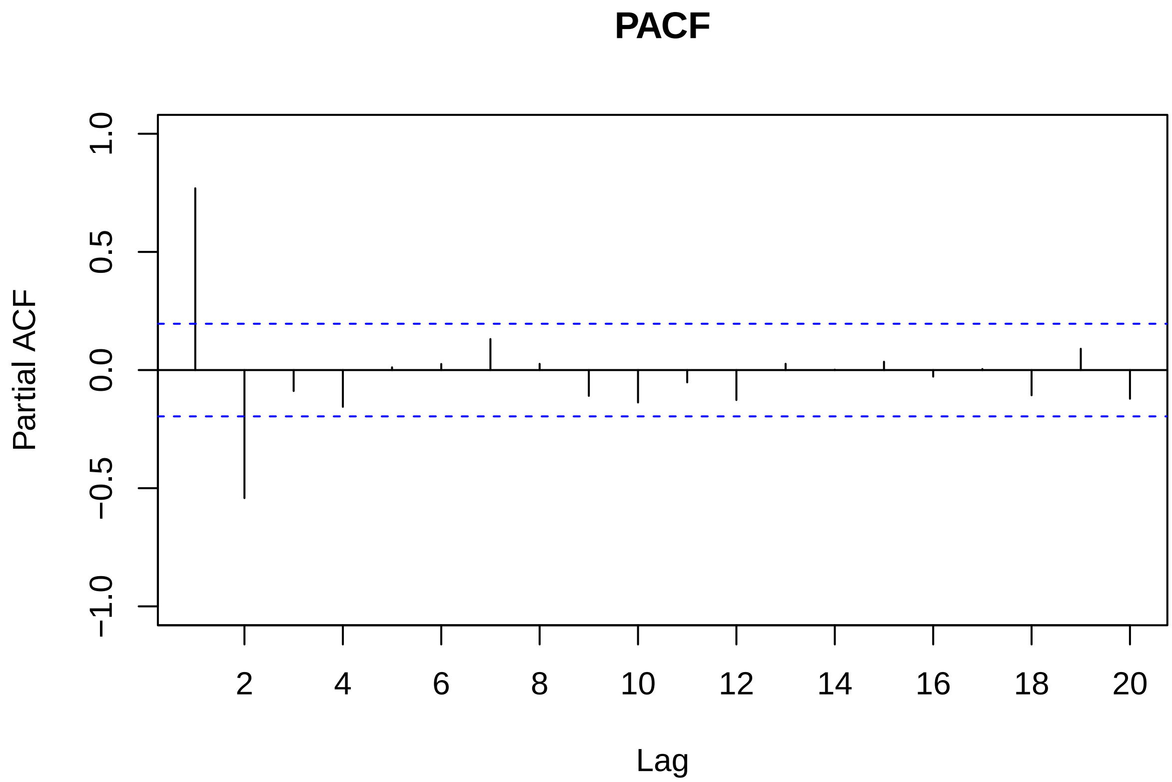


Xt-1.36X(t-1)+0.7X(t-2)=e(t)

```
a=arima.sim(n=100, list(ar=c(1.36,-0.7)))
acf(a,main="ACF",ylim=c(-1,1),xaxp=c(0,20,10))
```




```
pacf(a,main="PACF",ylim=c(-1,1),xaxp=c(0,20,10))
```


PACF
Partial ACF
1.0
0.5
0.0
−0.5
−1.0
2
4
6
8
10
12
14
16
18
20
Lag