

Signature _____

Name Printed _____

Row _____ Seat _____

LEAVE THIS TEST CLOSED, FACE UP, UNTIL YOU ARE INSTRUCTED TO BEGIN

- This is a closed book test. Keep your desk clear (no handouts, notes, books, programs, or calculators).
- You will be provided with an extra sheet that shows all the constructors/methods in the templated classes.
- You should write all your answers clearly on these pages. Make sure that your final answers are easily recognizable. If you need extra space, use the **backs of these pages** for any scratch work. A short, direct answer using the right technical terms is likely to score a higher value than a long rambling answer.
- When you arrive, **pick up the exam and templated classes sheet at the front of the class**, and take your seat. Do not start until I announce when to start (at 2:00pm). When I call time (at 2:50pm), you are to stop writing, stand and **leave by any of the four exits (2 front, 2 back) dropping your exam in the boxes there** (there will also be a box on the stage). Please do this quickly and quietly.
- The number of points for each problem is clearly marked. In total, this test is worth 200 points, so during the 50 minute testing period, you should spend about 1/4 minute per point (4 points in 1 minute). That is about 12 minutes/problem. Not all problems are equally easy. If you get stuck on a problem, move on to another one; return if you have time.
- It is generally better to answer all questions briefly than to answer some completely and leave some blank.
- Hint: sometimes information useful for one problem on the test may be found in other problems on the test.
- When writing answers, it might be useful to **you** to draw pictures of small examples of lists, trees, etc.
- Solutions will be posted in the **Solutions** link on the course web-page soon after everyone takes the exam. I hope that graded exams will be returned next week.

Problem	Points	Score
1	41	41
2	48	89
3	59	148
4	52	200
	200	

[illegible]

1a. (20 pts). We can use an **ArrayMap** whose **keys** (**std::string**) are each associated with a **value** that is an **ArraySet** of **std::string**. For example, an **ArrayMap** named **test** with four associations might print as:

```
map[a->set[x,y,z], b->set[x,y], c->set[z], d->set[y,z]]
```

Write the **keys_with** function, whose two parameters are (a) first an **ArrayMap** as specified above and (b) and second a **std::string**. The **keys_with** function returns an **ArraySet** of all **keys** whose associated **set** contains the specified **std::string** parameter. Calling **keys_with (test,"y");** would return **set[a,b,d]**: only **c**'s associated **set** does not contain **y**, so it is not included in the returned result.

```
ics::ArraySet<std::string>
keys_with (const ics::ArrayMap<std::string,ics::ArraySet<std::string>>& map, std::string v) {
    ics::ArraySet<std::string> answer;
    for (auto kv : map)
        if (kv.second.contains(v))
            answer.insert(kv.first);
    return answer
}

//auto is the type: ics::pair<std::string,ics::ArraySet<std::string>>
```

1b. (12 pts) Assume that we have declared **ics::ArraySet<std::string> words;** write a code fragment that removes those values from **words**, whose **.size** method (callable for **std::string**) call returns a number strictly **>3**.

```
for (auto i = words.begin(); i != words.end(); ++i)
    if (i->size() > 3)
        i.erase(v);
```

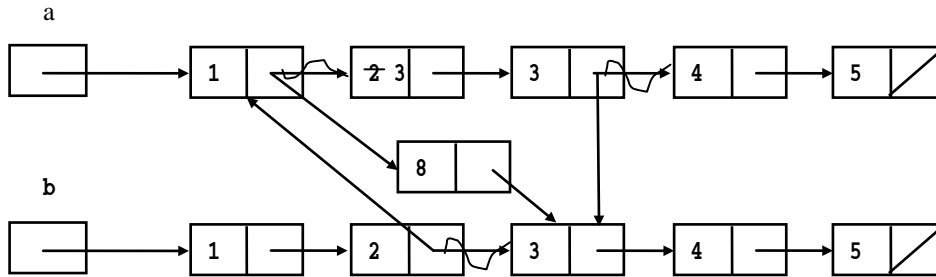
1c. (9 pts) For each problem below, fill in the blank with either **DT (Data Type)** or **DS (Data Structure)** depending on what you should primarily think about to solve the problem, and then briefly explain your answer.

- (a) Design your program: Data Types : use a data type that is correct for the code
- (b) Test your code by running it: Data Structures: run code with any implementation of the data type
- (c) Try to improve its efficiency: Data Structures: find a fast/compact implementation of the data type

2. See the class **LN** below. In your answers, you cannot use any other constructors or functions.

```
class LN {
public:
    LN (int v, LN* n = nullptr) : value(v), next(n) {}
    int value;
    LN* next;
};
```

2a. (16 pts) Given the following two linked lists, execute all the statements below, in the specified order, updating the linked lists. **Cross out** ALL values/pointers that are replaced and **Write in** new values/pointers. Use the list updated by one statement as the starting point for the next statement.



- 1) `a->next->value = b->next->next->value;`
- 2) `a->next->next->next = b->next->next;`
- 3) `b->next->next = a;`
- 4) `b->next->next->next = new LN(8, a->next->next->next);`

2b. (16 pts) Write an **iterative append_rear** function, which appends the specified **int** value (**v**) at the end of a linked list (**l**), which may be an empty list.

```
void append_rear(LN*& l, int v) {
    if (l == nullptr)
        l = new LN(v);
    else {
        for (LN* c = l; /* see body */; c = c->next)
            if (c->next == nullptr) {
                c->next = new LN(v);
                break;
            }
    }
}
```

```
LN* c = l;
while (c->next != nullptr)
    c = c->next;
c->next = new LN(v);
```

2c. (16 pts) Write a **recursive** `copy_deleting` function, which returns a **copy** of the **list** its argument points to, while deallocating all the **LN** objects in that **list**. So, if we executed `x = copy_deleting(x);` then all of the **LN** objects in the **list** `x` originally pointed to would be deallocated, and `x` would now point to a new **list** that contained all the same values from the original **list** `x` pointed to. **Hint:** My solution used one local variable, setting/using it once.

```
LN* copy_deleting(LN* l) {
    if (l == nullptr)
        return nullptr;
    else {
        LN* copy = new LN(l->value, copy_deleting(l->next));
        delete l;
        return copy;
    }
}
```

```
LN* copy_deleting(LN* l) {
    if (l == nullptr)
        return nullptr;
    else {
        LN* tail = copy_deleting(l->next);
        int v = l->value;
        delete l;
        return new LN(v, tail);
    }
}
```

3a. (26 pts) Write the **complexity class** for each of the following operations. Assume each data structure stores **N** values. **Caution:** some parts of this problem contain information that is **usesless/irrelevant** to the question asked.

- (a) $O(1)$ Remove value from position **i** of an **unsorted** array (no need to retain order of elements)
- (b) $O(N)$ Remove value from position **i** of a **sorted** array (must retain order of elements)
- (c) $O(N)$ Search of an **unsorted** array
- (d) $O(\log_2 N)$ Search of a **sorted** array
- (e) $O(\log_2 N)$ Search of a **well balanced** binary search tree
- (f) $O(N)$ Search of a **poorly balanced** binary search tree
- (g) $O(\log_2 N)$ Add a value to a Min-Heap
- (h) $O(1)$ Find (peek at) the minimum value from a Min-Heap
- (i) $O(\log_2 N)$ Remove the minimum value from a Min-Heap
- (j) $O(N)$ Build a heap from N values **offline**
- (k) $O(N \log_2 N)$ Build a heap from N values **online**
- (l) $O(N)$ Reverse a linked list (using the algorithm hand-simulated in class)
- (m) $O(N \log_2 N)$ Sort a list by copying its values to an array, sorting the array, copying its values back to the list

3b. (16 pts) Assume that function **f** is in the complexity class **$O(N \times (\log_2 N)^2)$** and that for **$N = 10^6$ (1,000,000)** the function runs in **8 seconds**.

Write a formula, **T(N)** that computes the approximate time that it takes to run (also in seconds) **f** for any input of size **N**. Show your work/calculations by hand, approximating logarithms, finish/simplify all the arithmetic.

From the formula, **$T(N) = c (O(N \times (\log_2 N)^2)$**) we have

$$8 = c(10^6 \times (\log_2 10^6)^2)$$

$$8 = c (10^6 \times 400)$$

$$8 = 4c10^8$$

therefore,

$$c = 2 \times 10^{-8}$$

$$\text{So, } T(N) = 2 \times 10^{-8} \times (N \times (\log_2 N)^2)$$

3c. (9 pts) Assume that we have recorded the following data when timing three functions (measured in milliseconds). Based on these, **fill in an estimate** for the complexity class for each method.

N	Time: Function 1	Time: Function 2	Time: Function 3
100	300	20	20
200	600	80	22
400	1,200	320	20
800	2,400	1280	22
1,600	4,800	5,120	22
Complexity Class Estimate	~doubles: $O(N)$	~ quadruples: $O(N^2)$	~ no real increase: $O(1)$

3d. (8 pts) Suppose functions **f** and **g** solve the same problem in running times $O(N)$ for **f** and $O(N \log_2 N)$ for **g**. Based on speed alone, when should we use **f** and when should we use **g**? Use one of the words **always**, **sometimes**, or **never** to start your answer, which should include **because** and a short justification.

For solving large problems...

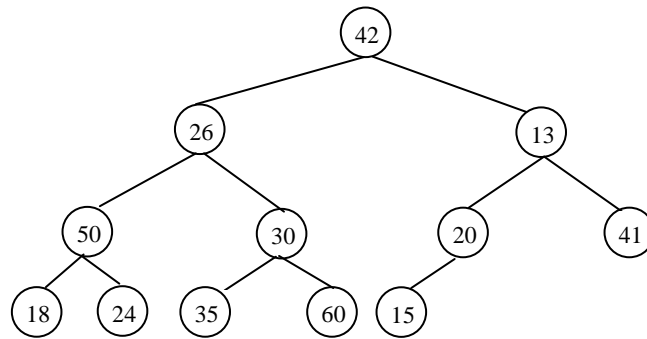
always use **f/never** use **g, because** the lower/higher complexity class is guaranteed to be faster/slower beyond some problem size.

For solving small problem...

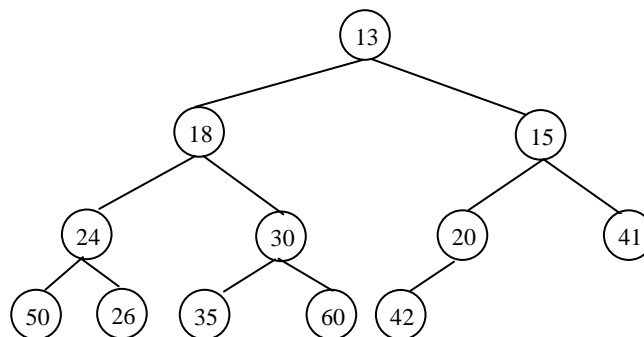
sometimes use **f/sometimes g, because** knowing their complexity class provides no guarantee which is faster for small problem sizes

4a. (8 pts) In the box below, draw a binary tree that **satisfies the structure property** of a **Min-Heap** using the values shown in array below: **values belong in a specific node based on their array index, using the standard encoding of a heap structure as an array**. This binary tree **does not satisfy the order property** for a **Min-Heap** (see part b).

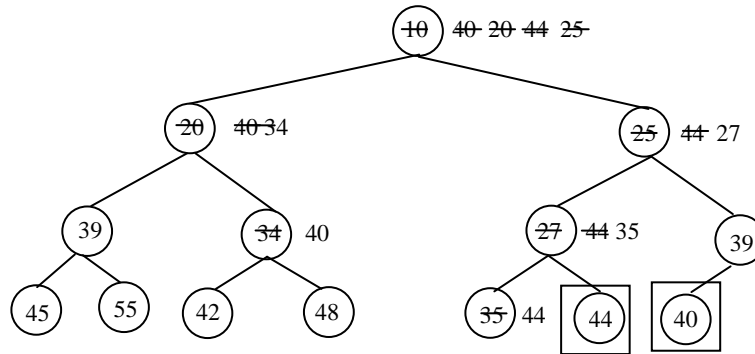
Index	0	1	2	3	4	5	6	7	8	9	10	11
Value	42	26	13	50	30	20	41	18	24	35	60	15



4b. (16 pts) In the box below, draw a binary tree representing a true **Min-Heap** (satisfying both the **order and structure properties**). To construct it, use the standard **offline algorithm** to process the structurally-correct **Min-Heap** from part 4a.



4c. (16 pts) Examine the **Min-Heap** below. Using the standard algorithm, remove the minimum value **twice**: in the tree, draw a box around node(s) that are no longer part of the tree; cross out changed values inside node(s) and write their new value(s) to their right.



4d. (12 pts) Examine the class **TN** below.

```

class TN {
public:
    //..constructors
    int value;
    TN* left;
    TN* right;
}

```

Write the **height** function, which computes the height of any node in any binary tree.

```

int height (TN* t) {
    if (t == nullptr)
        return -1;
    else
        return 1 + std::max( height(t->left), height(t->right) )
}

```