APPENDIX of "Low-rank Tensor based One-step Multiple Kernel K-means Clustering"

1 Complete Experimental Results

Tab. 1,2 present the detailed information of datasets used in this paper, including the number of samples, kernels, and clusters.

Tab. 3,4,5,6 present the ACC, NMI, PUR, and RI (%) comparison of dozens of algorithms on ten benchmark datasets, and Tab. 7,8,9,10 present comparison results on four large-scale datasets, where the '-' indicates that the results are unavailable due to out-of-memory error or too long execution time. The best results are marked in bold. As observed, LTOMKKM demonstrates superior clustering performance compared to competitors on all datasets, verifying the effectiveness and efficiency of our proposed tensor manner. This simple idea with excellent performance will likely attract intensive research and application.

To show the learning process, we plot the loss and performance curves across iterations as shown in Fig. 1,3. As observed, the loss value converges quickly, and the performance usually increases and then keeps stable, sufficiently demonstrating the learning process's effectiveness and necessity.

To show the cluster partition learned by LTOMKKM, we plot visual results in Fig. 2,4. As seen, the learning process of LTOMKKM effectively learns clear and distinguishable cluster structures from fuzzy and indistinguishable initial partitions, further showing our proposed algorithm's effectiveness.

To evaluate two trade-off parameters introduced by LTOMKKM, ρ and λ, we study their sensitivity and effect and plot the results in Fig. 5,6. Note that these two parameters are only coarsely tuned in this algorithm instead of fine-tuning. As seen, the parameters apparently influence the performance, and meanwhile, LTOMKKM achieves stable clustering performance while ρ and λ are around 10^2.

Finally, to evaluate the efficiency of the algorithms, we report the time consumption and plot it in Fig. 7. Note that, for better clarity, we scaled the values based on the minimum value and adopted logarithmic values. As observed, our proposed LTOMKKM shows significant superiority over most usual MKC and tensor-based methods and considerably improves the clustering performance.

Dataset	#Samples	#Kernels	#Clusters
Caltech5	441	5	7
BBCSport	544	2	5
ProteinFold	694	12	27
Cora	2708	2	7
PsortPos	541	69	4
Plant	940	69	4
PsortNeg	1444	69	5
Nonpl	2732	69	3
LandUse	2100	3	21
4Area	4236	2	4

Table 1: Datasets information.

Dataset	#Samples	#Kernels	#Clusters
CCV	6773	3	20
SUNRGBD	10335	2	45
YoutubeFace	10153	5	31
Reuters	18758	5	6

Table 2: Large-scale datasets information.

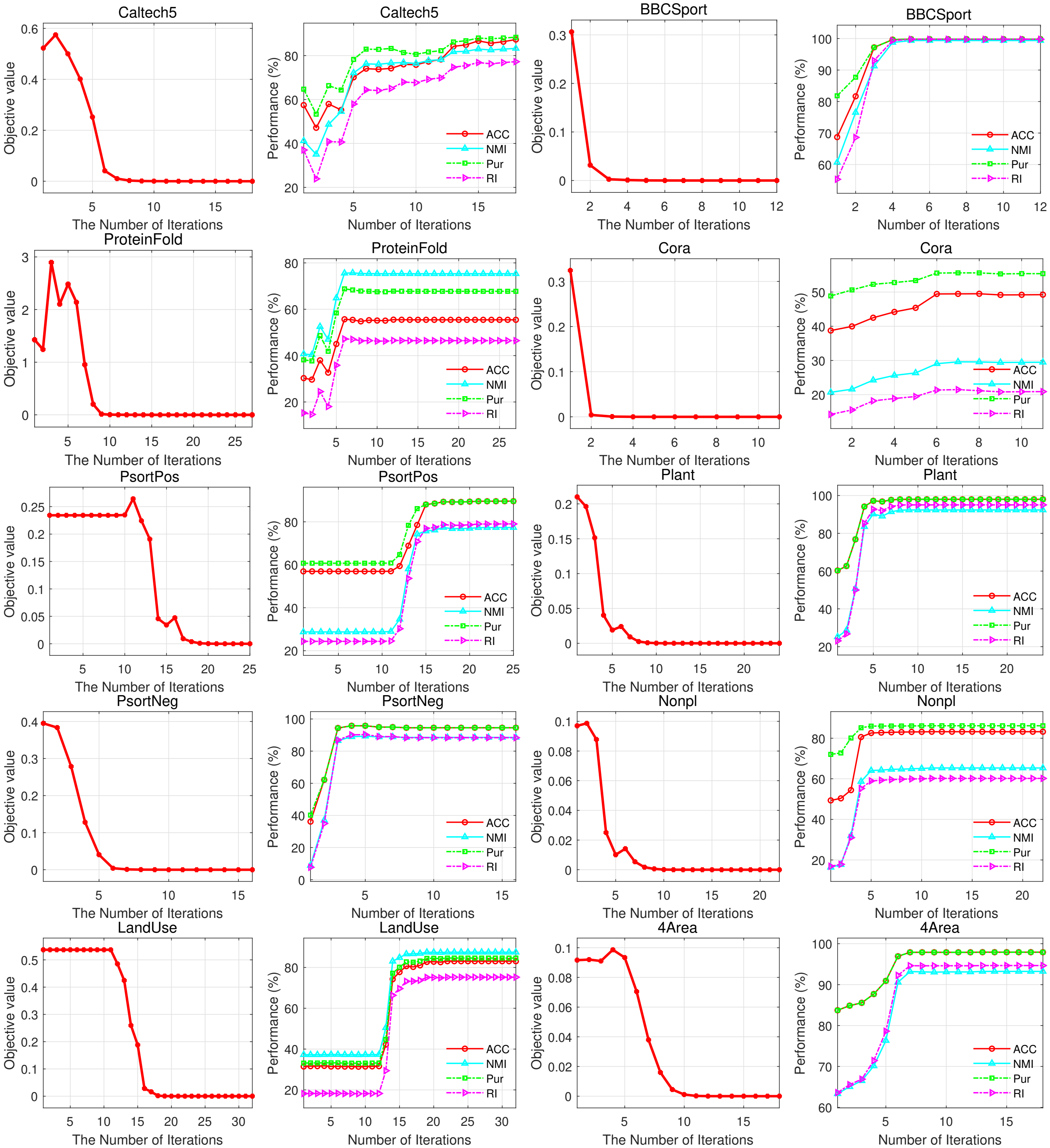

Figure 1: The loss curves across iterations and the evolution of the clustering performance of LTOMKKM.

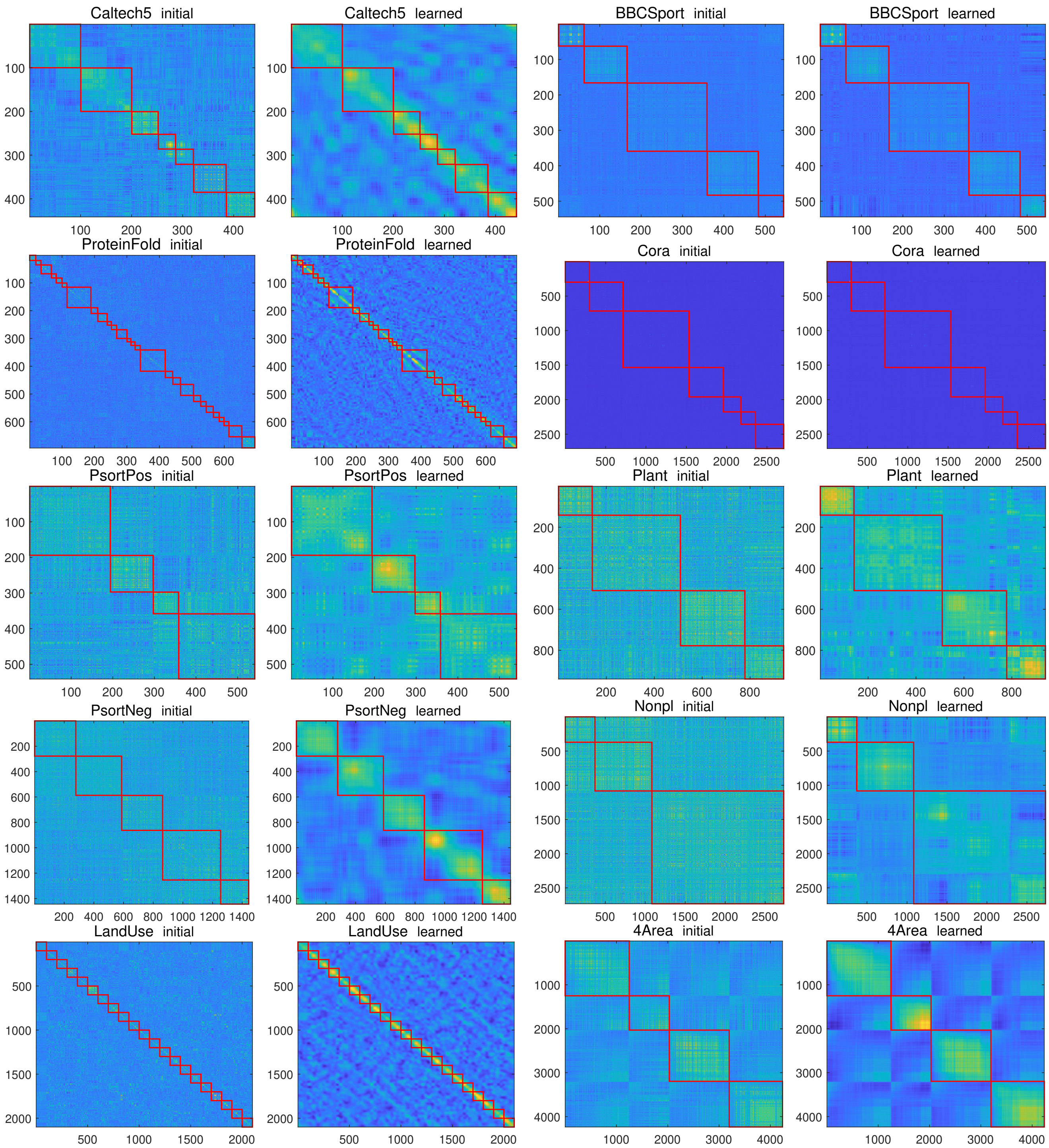

Figure 2: The initial (left) and learned (right) cluster partitions.

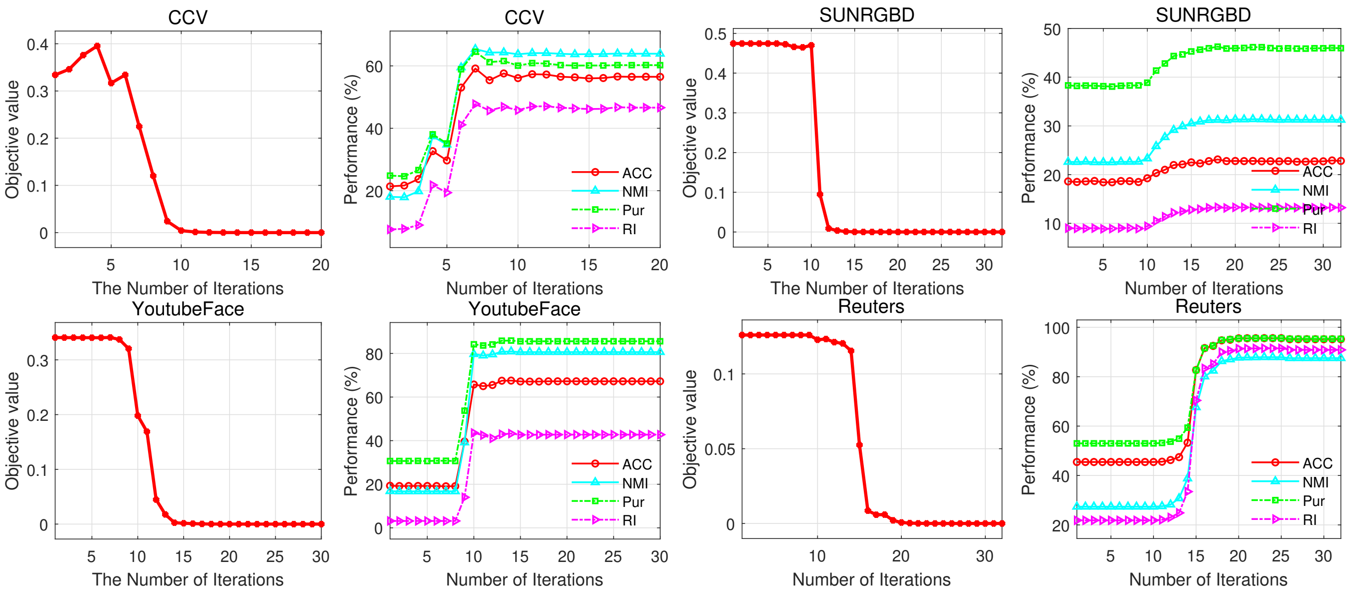

Figure 3: The loss curves across iterations and the evolution of the clustering performance for large-scale datasets of LTOMKKM.

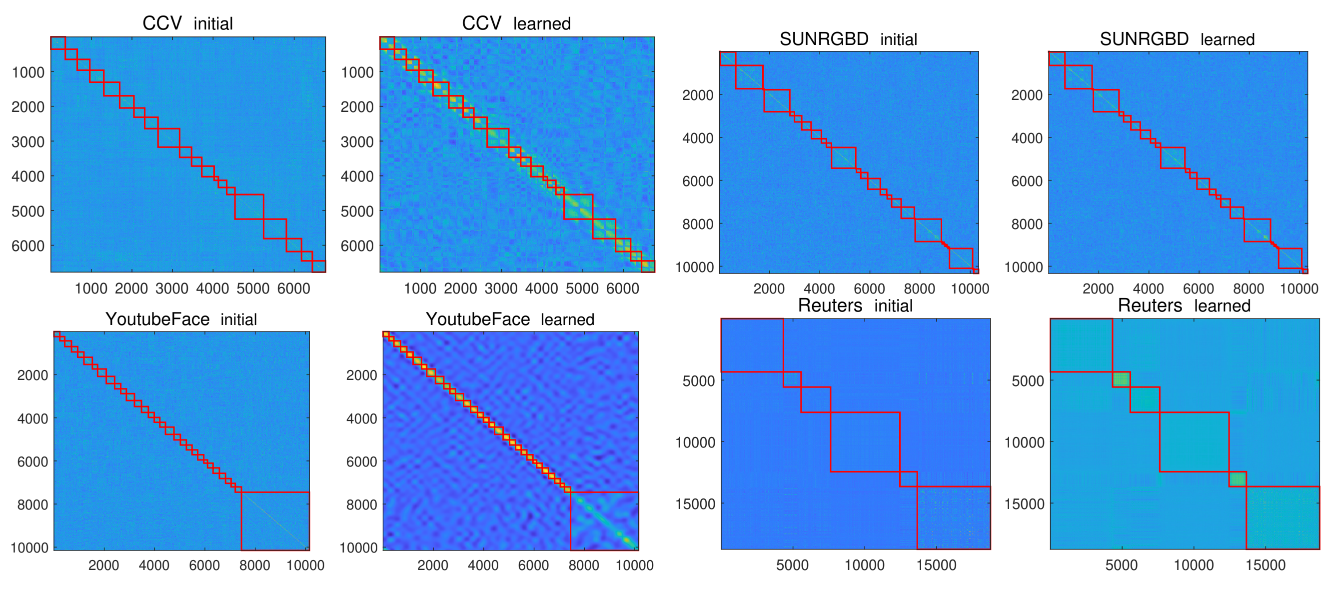

Figure 4: The initial (left) and learned (right) cluster partitions for large-scale datasets.

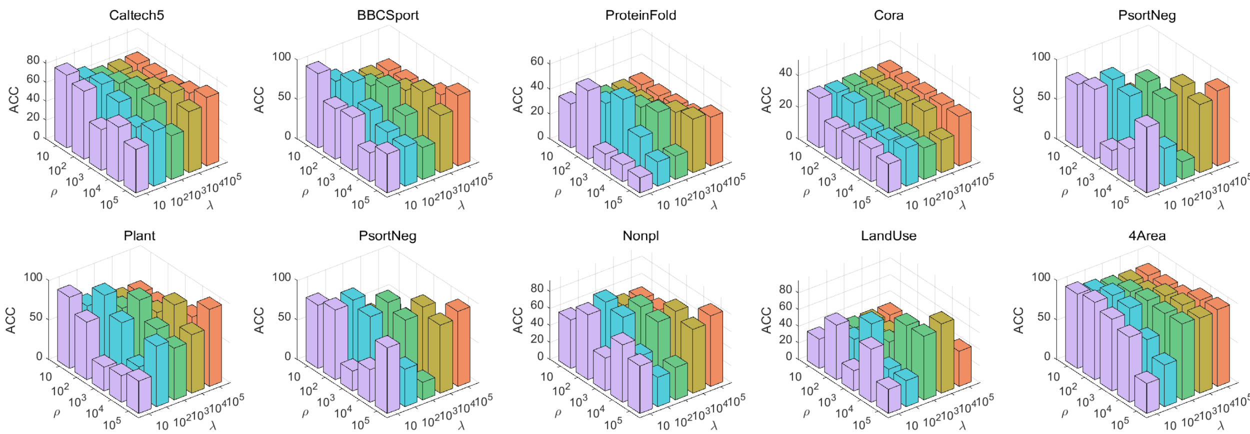

Figure 5: The parameter analysis of LTOMKKM with ρ and λ.

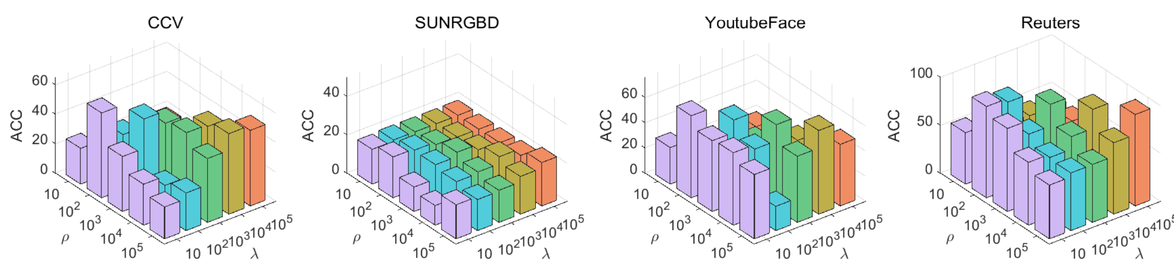

Figure 6: The parameter analysis for large-scale datasets of LTOMKKM with ρ and λ.

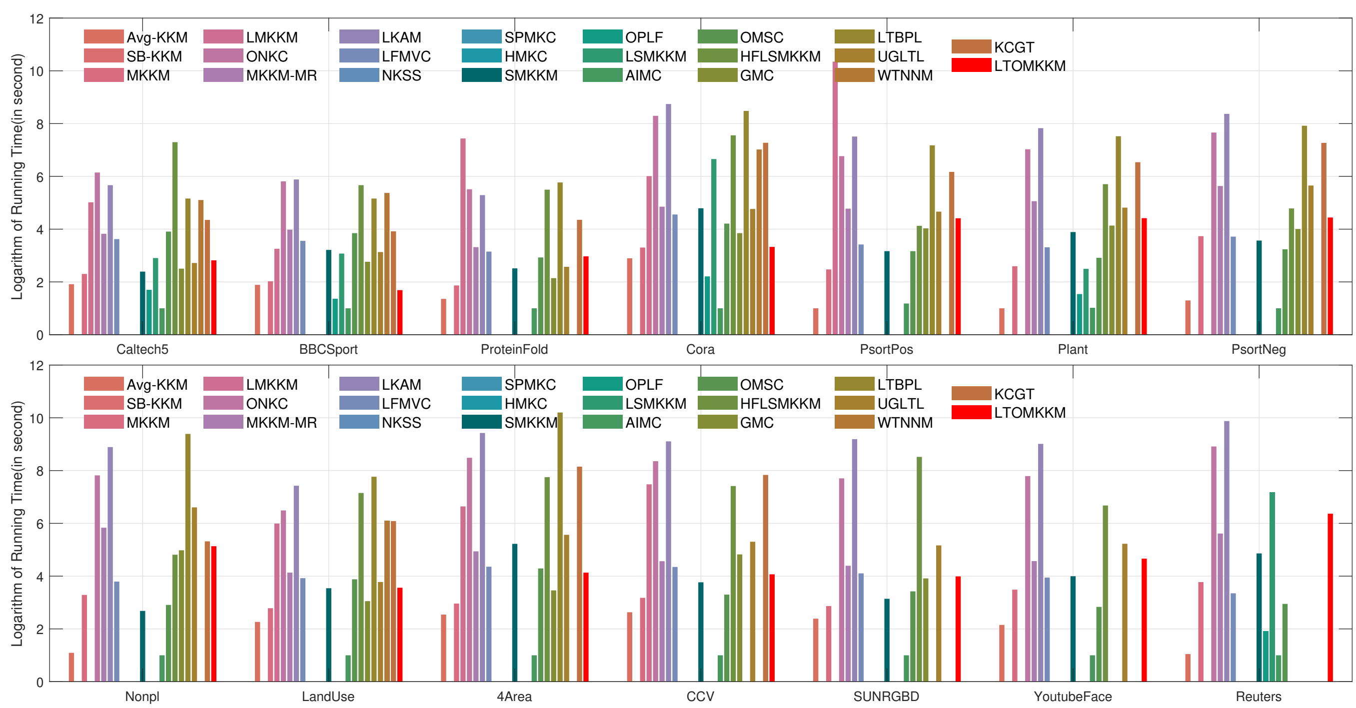

Figure 7: Time complexity comparison on benchmark and large-scale datasets.

Algorithm	Caltech5	BBCSport	ProteinFold	Cora	PsortPos	Plant	PsortNeg	Nonpl	LandUse	4Area
Avg-KKM	59.8±3.4	63.2±1.4	29.0±1.5	30.7±0.8	56.9±0.5	61.3±0.9	41.0±1.4	49.7±0.2	31.6±0.8	83.0±0.0
SB-KKM	64.3±0.7	71.4±0.1	33.8±1.3	45.2±0.1	69.1±0.0	51.2±1.1	55.3±0.0	57.5±0.1	32.6±0.8	74.7±0.0
MKKM	52.8±4.0	63.0±1.5	27.0±1.1	25.3±0.4	60.7±0.5	56.1±0.6	51.9±0.3	49.3±0.2	28.6±0.6	74.5±0.0
LMKKM	53.5±1.1	63.9±1.4	22.4±0.7	22.5±0.2	61.8±0.5	-	-	-	24.3±0.5	73.7±0.0
ONKC	68.1±3.4	63.4±1.4	36.3±1.5	40.8±0.3	50.4±0.2	41.4±0.2	40.2±0.6	56.7±0.0	32.8±0.8	71.1±0.0
MKKM-MR	70.2±0.2	63.2±1.5	34.7±1.8	35.7±0.1	49.2±0.8	50.3±0.8	39.7±0.5	50.4±0.0	32.9±0.7	71.7±0.0
LKAM	68.7±3.6	73.9±0.5	37.7±1.2	34.5±0.1	53.1±0.9	47.6±0.0	40.5±0.4	55.0±0.0	32.4±0.5	49.9±3.8
LFMVC	71.3±2.7	76.4±2.9	33.0±1.4	41.0±1.1	53.2±0.3	59.5±0.6	45.5±0.3	48.7±0.2	32.6±0.6	83.3±0.3
NKSS	64.2±1.8	64.1±1.2	36.4±0.7	20.3±0.4	61.4±0.2	43.3±0.0	48.2±1.0	-	32.9±0.5	61.6±2.6
SPMKC	55.1±3.6	51.3±1.9	17.8±0.5	25.1±0.0	45.3±1.9	51.4±0.1	25.0±0.6	51.0±0.0	31.3±0.8	74.3±0.0
HMKC	74.4±1.4	91.1±3.7	35.3±1.5	47.4±0.2	59.0±0.1	64.2±0.1	49.1±0.0	54.3±0.1	26.5±0.6	89.2±0.0
SMKKM	69.5±2.6	64.2±1.6	34.7±1.9	35.7±0.1	43.7±0.2	49.5±0.5	41.5±0.0	52.0±0.0	32.4±0.9	70.8±0.0
OPLFMVC	70.0±3.5	89.2±3.2	31.1±2.6	43.2±2.7	55.6±1.9	47.3±3.1	46.1±2.3	45.4±0.6	31.2±1.1	65.3±1.5
LSMKKM	75.9±4.0	73.4±1.0	36.3±1.5	33.6±0.0	49.5±0.2	57.1±0.8	45.7±0.1	69.3±0.0	32.4±0.6	71.8±0.4
AIMC	67.8±0.0	70.4±0.0	33.6±0.0	41.8±0.0	53.4±0.0	47.9±0.0	45.4±0.0	45.9±0.0	32.0±0.0	63.1±0.0
OMSC	66.9±0.0	89.0±0.0	31.8±0.0	40.1±0.0	60.4±0.0	56.5±0.0	39.5±0.0	44.4±0.0	30.6±0.0	67.2±0.0
HFLSMKKM	45.2±0.1	51.6±1.3	33.8±1.1	18.6±0.0	44.3±0.1	43.6±0.1	31.3±0.6	76.7±0.0	31.7±0.6	59.0±2.0
GMC	70.1±0.0	88.2±0.0	29.3±0.0	37.6±0.2	37.7±0.0	39.4±0.0	25.2±0.0	48.3±0.0	25.4±0.9	29.5±0.0
LTBPL	80.0±2.0	96.5±0.0	32.1±1.1	47.6±0.0	46.4±0.0	48.2±0.0	29.1±0.0	63.9±0.0	31.5±0.8	42.7±1.2
UGLTL	78.0±0.0	99.1±0.2	51.1±1.7	16.7±0.3	74.5±0.2	68.6±1.2	92.2±0.0	77.2±0.0	25.3±0.9	71.5±0.0
WTNNM	83.0±0.0	95.2±0.0	43.2±1.7	39.1±0.0	71.5±0.0	68.0±0.1	64.8±0.0	61.7±0.0	40.1±0.7	65.2±0.0
KCGT	76.7±0.1	90.3±0.0	54.0±3.0	34.0±0.2	56.6±0.1	56.4±0.0	47.9±0.1	49.8±0.0	49.9±2.0	84.2±0.0
TKKM	**87.3**±4.9	**99.8**±0.0	**54.9**±2.7	**49.2**±0.1	**89.6**±0.4	**98.0**±0.0	**95.1**±3.0	**83.2**±0.0	**83.1**±4.4	**97.9**±0.0

Table 3: ACC (%) comparison (mean±std) on 10 benchmark datasets.

Algorithm	Caltech5	BBCSport	ProteinFold	Cora	PsortPos	Plant	PsortNeg	Nonpl	LandUse	4Area
Avg-KKM	59.7±2.9	43.5±1.1	40.3±1.3	15.7±1.4	28.7±0.2	26.5±0.9	17.4±0.7	17.2±0.5	37.3±0.4	62.2±0.0
SB-KKM	59.3±0.9	63.2±0.2	41.1±1.1	25.6±0.1	42.5±0.0	16.9±1.1	39.1±0.0	11.3±0.0	37.9±0.7	53.9±0.0
MKKM	53.1±3.4	43.6±1.2	38.0±0.6	9.5±0.2	35.5±0.8	19.5±0.5	32.2±0.2	15.0±0.5	32.9±0.5	53.8±0.0
LMKKM	51.0±1.3	44.0±0.8	34.7±0.6	6.7±0.3	37.2±0.3	-	-	-	29.0±0.4	52.7±0.0
ONKC	62.3±2.3	43.5±1.1	44.4±0.9	23.1±0.3	25.4±0.2	10.5±0.2	21.0±0.7	11.8±0.0	38.6±0.7	46.2±0.0
MKKM-MR	65.4±0.3	43.5±1.1	43.7±1.2	18.9±0.2	21.1±0.7	20.4±0.4	21.6±0.4	14.8±0.0	38.7±0.7	47.0±0.0
LKAM	64.0±2.5	65.4±1.0	46.2±0.6	16.1±0.1	24.5±0.5	13.9±0.0	21.8±0.8	16.0±0.0	37.9±0.4	21.6±2.0
LFMVC	65.4±2.1	58.9±3.0	41.7±1.1	22.4±0.5	24.9±0.3	23.4±0.8	18.8±0.3	13.0±0.1	37.7±0.5	62.2±1.5
NKSS	57.6±1.2	51.3±0.4	46.5±0.5	1.0±0.1	36.6±1.1	14.2±0.0	25.9±1.3	-	38.4±0.4	38.8±1.1
SPMKC	51.9±1.1	29.9±3.1	27.3±0.5	4.4±0.0	9.8±1.4	24.2±0.0	3.2±0.3	11.7±0.0	38.2±0.7	53.2±0.0
HMKC	69.9±1.4	78.2±4.4	45.3±1.1	**30.6**±0.1	28.8±0.1	32.9±0.4	24.9±0.0	10.7±0.0	31.7±0.5	69.7±0.1
SMKKM	64.2±1.3	44.4±1.0	44.4±1.1	18.8±0.2	23.8±0.3	16.9±0.9	19.1±0.1	11.2±0.0	37.9±0.5	45.8±0.0
OPLFMVC	66.2±2.7	78.7±3.0	40.0±2.0	24.8±1.9	27.7±1.0	13.3±1.0	21.3±1.9	8.4±0.4	35.6±0.8	50.6±3.4
LSMKKM	71.7±2.8	65.0±1.4	45.2±1.2	15.7±0.1	24.0±0.1	20.8±1.0	17.0±0.0	22.6±0.0	38.1±0.6	44.6±2.2
AIMC	64.0±0.0	69.6±0.0	42.9±0.0	22.9±0.0	24.2±0.0	13.9±0.0	17.9±0.0	7.9±0.0	36.4±0.0	36.1±0.0
OMSC	64.8±0.0	73.5±0.0	38.0±0.0	18.1±0.0	28.0±0.0	20.9±0.0	12.9±0.0	7.5±0.0	33.8±0.0	41.5±0.0
HFLSMKKM	42.7±0.4	35.4±1.2	44.7±0.6	2.0±0.0	22.6±0.2	18.6±0.1	8.3±0.3	32.0±0.0	38.7±0.5	37.3±0.8
GMC	66.6±0.0	78.3±0.0	25.9±0.0	16.3±0.1	2.6±0.0	0.8±0.0	1.6±0.0	9.8±0.0	31.1±0.7	0.9±0.0
LTBPL	72.4±1.2	88.6±0.0	43.4±0.7	26.5±0.0	17.9±0.0	10.7±0.0	6.0±0.0	8.3±0.0	37.8±0.8	9.8±0.8
UGLTL	81.8±0.0	96.7±0.6	73.2±1.2	0.4±0.1	65.9±0.3	43.3±0.7	83.4±0.1	48.9±0.0	33.9±1.2	56.3±0.1
WTNNM	**85.9**±0.1	85.1±0.0	51.0±0.8	22.0±0.0	44.5±0.0	35.1±0.1	42.6±0.0	21.5±0.0	45.7±0.5	37.7±0.0
KCGT	75.0±0.2	82.1±0.0	**75.7**±1.7	13.8±0.4	36.9±0.2	23.1±0.0	19.5±0.1	5.7±0.0	61.8±0.9	65.2±0.0
TKKM	83.3±2.2	**99.4**±0.0	75.2±1.0	29.5±0.1	**77.3**±0.4	**92.3**±0.0	**89.0**±2.9	**65.3**±0.0	**87.5**±1.6	**93.2**±0.0

Table 4: NMI (%) comparison (mean±std) on 10 benchmark datasets.

Algorithm	Caltech5	BBCSport	ProteinFold	Cora	PsortPos	Plant	PsortNeg	Nonpl	LandUse	4Area
Avg-KKM	69.8$_{\pm 3.3}$	68.1$_{\pm 0.7}$	37.4$_{\pm 1.7}$	41.5$_{\pm 1.3}$	60.7$_{\pm 0.2}$	61.3$_{\pm 0.9}$	43.3$_{\pm 1.0}$	72.5$_{\pm 0.2}$	33.2$_{\pm 0.5}$	83.0$_{\pm 0.0}$
SB-KKM	73.2$_{\pm 0.6}$	78.8$_{\pm 0.1}$	39.4$_{\pm 1.2}$	52.5$_{\pm 0.1}$	69.1$_{\pm 0.0}$	53.2$_{\pm 0.5}$	61.6$_{\pm 0.0}$	65.3$_{\pm 0.1}$	34.4$_{\pm 0.8}$	74.7$_{\pm 0.0}$
MKKM	66.3$_{\pm 3.3}$	68.2$_{\pm 0.8}$	33.7$_{\pm 1.1}$	36.1$_{\pm 1.0}$	66.7$_{\pm 0.7}$	56.1$_{\pm 0.6}$	56.6$_{\pm 0.2}$	71.2$_{\pm 0.2}$	30.0$_{\pm 0.5}$	74.5$_{\pm 0.0}$
LMKKM	65.9$_{\pm 0.4}$	68.4$_{\pm 0.7}$	31.2$_{\pm 1.0}$	35.0$_{\pm 0.2}$	68.0$_{\pm 0.5}$	-	-	-	26.2$_{\pm 0.5}$	73.7$_{\pm 0.0}$
ONKC	73.7$_{\pm 1.8}$	68.1$_{\pm 0.7}$	42.7$_{\pm 1.3}$	48.6$_{\pm 0.3}$	60.8$_{\pm 0.1}$	49.0$_{\pm 0.1}$	44.7$_{\pm 0.3}$	62.3$_{\pm 0.1}$	34.6$_{\pm 0.8}$	71.1$_{\pm 0.0}$
MKKM-MR	74.4$_{\pm 0.1}$	68.0$_{\pm 0.7}$	41.9$_{\pm 1.4}$	47.0$_{\pm 0.1}$	56.1$_{\pm 0.4}$	56.7$_{\pm 0.1}$	44.7$_{\pm 0.4}$	60.4$_{\pm 0.0}$	34.9$_{\pm 0.6}$	71.7$_{\pm 0.0}$
LKAM	74.2$_{\pm 2.8}$	79.4$_{\pm 0.5}$	43.7$_{\pm 0.8}$	43.3$_{\pm 0.1}$	61.0$_{\pm 0.1}$	54.5$_{\pm 0.0}$	45.3$_{\pm 0.5}$	61.6$_{\pm 0.1}$	34.5$_{\pm 0.6}$	50.6$_{\pm 2.7}$
LFMVC	76.4$_{\pm 1.9}$	76.7$_{\pm 2.7}$	39.3$_{\pm 1.5}$	47.2$_{\pm 0.5}$	57.1$_{\pm 0.2}$	59.5$_{\pm 0.6}$	48.2$_{\pm 0.3}$	69.7$_{\pm 0.1}$	34.5$_{\pm 0.7}$	83.3$_{\pm 0.3}$
NKSS	70.3$_{\pm 1.3}$	72.6$_{\pm 0.2}$	44.8$_{\pm 0.6}$	30.3$_{\pm 0.0}$	66.3$_{\pm 0.3}$	46.1$_{\pm 0.0}$	54.3$_{\pm 1.6}$	-	34.9$_{\pm 0.3}$	64.1$_{\pm 1.8}$
SPMKC	64.7$_{\pm 0.9}$	56.1$_{\pm 1.5}$	23.7$_{\pm 0.7}$	34.3$_{\pm 0.0}$	48.5$_{\pm 1.3}$	59.0$_{\pm 0.1}$	27.8$_{\pm 0.1}$	63.9$_{\pm 0.0}$	34.4$_{\pm 0.5}$	74.3$_{\pm 0.0}$
HMKC	78.0$_{\pm 1.3}$	91.1$_{\pm 3.7}$	42.9$_{\pm 1.9}$	**56.8**$_{\pm 0.3}$	61.2$_{\pm 0.0}$	64.2$_{\pm 0.1}$	53.0$_{\pm 0.0}$	64.1$_{\pm 0.0}$	28.8$_{\pm 0.6}$	89.2$_{\pm 0.0}$
SMKKM	73.7$_{\pm 1.1}$	68.7$_{\pm 0.9}$	41.8$_{\pm 1.5}$	47.0$_{\pm 0.1}$	57.6$_{\pm 0.1}$	54.3$_{\pm 0.3}$	42.2$_{\pm 0.1}$	60.4$_{\pm 0.0}$	34.1$_{\pm 0.7}$	70.8$_{\pm 0.0}$
OPLFMVC	76.8$_{\pm 1.6}$	89.6$_{\pm 2.1}$	36.4$_{\pm 2.6}$	50.7$_{\pm 2.3}$	59.8$_{\pm 1.0}$	50.5$_{\pm 2.4}$	49.5$_{\pm 2.5}$	64.3$_{\pm 0.9}$	33.1$_{\pm 1.1}$	73.7$_{\pm 3.5}$
LSMKKM	80.7$_{\pm 2.4}$	79.2$_{\pm 0.5}$	42.6$_{\pm 1.5}$	42.3$_{\pm 0.0}$	53.7$_{\pm 0.1}$	58.5$_{\pm 1.1}$	47.2$_{\pm 0.1}$	70.6$_{\pm 0.0}$	34.1$_{\pm 0.7}$	71.8$_{\pm 0.4}$
AIMC	76.6$_{\pm 0.0}$	80.5$_{\pm 0.0}$	38.9$_{\pm 0.0}$	48.4$_{\pm 0.0}$	56.4$_{\pm 0.0}$	55.5$_{\pm 0.0}$	47.6$_{\pm 0.0}$	61.9$_{\pm 0.0}$	33.8$_{\pm 0.0}$	63.1$_{\pm 0.0}$
OMSC	76.4$_{\pm 0.0}$	89.0$_{\pm 0.0}$	37.2$_{\pm 0.0}$	42.4$_{\pm 0.0}$	63.0$_{\pm 0.0}$	57.6$_{\pm 0.0}$	43.5$_{\pm 0.0}$	63.0$_{\pm 0.0}$	32.0$_{\pm 0.0}$	67.2$_{\pm 0.0}$
HFLSMKKM	56.3$_{\pm 0.1}$	61.4$_{\pm 0.1}$	41.8$_{\pm 0.9}$	30.3$_{\pm 0.0}$	49.9$_{\pm 0.1}$	54.7$_{\pm 0.0}$	32.8$_{\pm 0.6}$	76.7$_{\pm 0.0}$	35.0$_{\pm 0.5}$	61.5$_{\pm 1.4}$
GMC	73.5$_{\pm 0.0}$	88.2$_{\pm 0.0}$	32.6$_{\pm 0.0}$	43.0$_{\pm 0.0}$	38.4$_{\pm 0.0}$	39.7$_{\pm 0.0}$	27.1$_{\pm 0.0}$	66.8$_{\pm 0.0}$	29.6$_{\pm 0.8}$	31.6$_{\pm 0.0}$
LTBPL	82.3$_{\pm 0.7}$	96.5$_{\pm 0.0}$	38.7$_{\pm 0.8}$	48.7$_{\pm 0.0}$	48.2$_{\pm 0.0}$	49.7$_{\pm 0.0}$	31.2$_{\pm 0.1}$	63.9$_{\pm 0.0}$	34.3$_{\pm 1.0}$	45.2$_{\pm 0.8}$
UGLTL	85.3$_{\pm 0.0}$	99.1$_{\pm 0.2}$	64.8$_{\pm 2.0}$	30.2$_{\pm 0.0}$	86.4$_{\pm 0.1}$	70.2$_{\pm 0.3}$	92.2$_{\pm 0.0}$	81.8$_{\pm 0.0}$	26.7$_{\pm 0.9}$	75.3$_{\pm 0.0}$
WTNNM	**89.8**$_{\pm 0.0}$	95.2$_{\pm 0.0}$	49.3$_{\pm 1.1}$	46.9$_{\pm 0.1}$	71.5$_{\pm 0.0}$	68.0$_{\pm 0.1}$	68.1$_{\pm 0.1}$	68.0$_{\pm 0.0}$	42.2$_{\pm 0.5}$	65.2$_{\pm 0.0}$
KCGT	83.9$_{\pm 0.1}$	90.3$_{\pm 0.0}$	**68.5**$_{\pm 2.2}$	41.9$_{\pm 0.3}$	57.2$_{\pm 0.1}$	56.4$_{\pm 0.0}$	47.9$_{\pm 0.1}$	61.4$_{\pm 0.0}$	52.4$_{\pm 1.5}$	84.2$_{\pm 0.0}$
TKKM	88.4$_{\pm 2.8}$	**99.8**$_{\pm 0.0}$	67.9$_{\pm 2.0}$	55.4$_{\pm 0.1}$	**89.6**$_{\pm 0.4}$	**98.0**$_{\pm 0.0}$	**95.1**$_{\pm 3.0}$	**86.1**$_{\pm 0.0}$	**84.6**$_{\pm 3.6}$	**97.9**$_{\pm 0.0}$

Table 5: PUR (%) comparison (mean±std) on 10 benchmark datasets.

Algorithm	Caltech5	BBCSport	ProteinFold	Cora	PsortPos	Plant	PsortNeg	Nonpl	LandUse	4Area
Avg-KKM	46.7$_{\pm 4.6}$	39.3$_{\pm 1.9}$	14.4$_{\pm 1.8}$	6.5$_{\pm 0.6}$	24.4$_{\pm 0.3}$	24.6$_{\pm 1.2}$	13.1$_{\pm 0.6}$	17.6$_{\pm 0.3}$	18.2$_{\pm 0.4}$	62.4$_{\pm 0.0}$
SB-KKM	49.5$_{\pm 0.9}$	60.4$_{\pm 0.2}$	15.1$_{\pm 1.2}$	19.9$_{\pm 0.1}$	35.0$_{\pm 0.0}$	13.9$_{\pm 0.9}$	31.6$_{\pm 0.0}$	16.8$_{\pm 0.1}$	19.5$_{\pm 0.4}$	55.5$_{\pm 0.0}$
MKKM	41.7$_{\pm 4.4}$	39.2$_{\pm 2.0}$	12.1$_{\pm 0.7}$	3.6$_{\pm 0.3}$	32.2$_{\pm 0.9}$	17.4$_{\pm 0.6}$	26.8$_{\pm 0.2}$	15.8$_{\pm 0.4}$	14.9$_{\pm 0.3}$	55.4$_{\pm 0.1}$
LMKKM	40.6$_{\pm 0.8}$	40.3$_{\pm 1.5}$	7.8$_{\pm 0.4}$	1.7$_{\pm 0.1}$	34.0$_{\pm 0.7}$	-	-	-	12.0$_{\pm 0.2}$	52.0$_{\pm 0.0}$
ONKC	55.3$_{\pm 3.8}$	39.5$_{\pm 1.9}$	18.0$_{\pm 1.1}$	15.6$_{\pm 0.4}$	21.4$_{\pm 0.2}$	9.8$_{\pm 0.1}$	16.9$_{\pm 0.3}$	14.2$_{\pm 0.0}$	19.5$_{\pm 0.5}$	47.1$_{\pm 0.0}$
MKKM-MR	58.0$_{\pm 0.3}$	39.3$_{\pm 1.9}$	17.2$_{\pm 1.5}$	11.4$_{\pm 0.1}$	18.9$_{\pm 0.3}$	19.0$_{\pm 0.2}$	16.9$_{\pm 0.3}$	8.5$_{\pm 0.0}$	19.8$_{\pm 0.4}$	48.0$_{\pm 0.0}$
LKAM	57.2$_{\pm 4.7}$	62.3$_{\pm 1.2}$	20.1$_{\pm 1.1}$	11.1$_{\pm 0.1}$	26.7$_{\pm 0.3}$	9.1$_{\pm 0.0}$	16.0$_{\pm 0.3}$	10.4$_{\pm 0.0}$	19.3$_{\pm 0.3}$	16.6$_{\pm 1.6}$
LFMVC	63.4$_{\pm 3.6}$	57.0$_{\pm 3.8}$	16.1$_{\pm 1.5}$	14.5$_{\pm 0.4}$	19.6$_{\pm 0.1}$	21.7$_{\pm 0.8}$	16.1$_{\pm 0.2}$	14.1$_{\pm 0.2}$	19.0$_{\pm 0.4}$	62.8$_{\pm 1.0}$
NKSS	46.2$_{\pm 2.5}$	44.3$_{\pm 0.6}$	18.5$_{\pm 0.6}$	0.4$_{\pm 0.1}$	30.0$_{\pm 0.2}$	9.2$_{\pm 0.0}$	19.9$_{\pm 0.5}$	-	19.4$_{\pm 0.5}$	35.9$_{\pm 1.7}$
SPMKC	39.4$_{\pm 4.0}$	21.8$_{\pm 3.5}$	4.4$_{\pm 0.3}$	1.5$_{\pm 0.0}$	6.8$_{\pm 1.3}$	19.1$_{\pm 0.0}$	0.1$_{\pm 0.2}$	11.2$_{\pm 0.0}$	16.4$_{\pm 0.8}$	43.4$_{\pm 0.0}$
HMKC	66.6$_{\pm 2.5}$	79.3$_{\pm 4.8}$	19.0$_{\pm 1.6}$	**23.4**$_{\pm 0.2}$	26.4$_{\pm 0.1}$	31.0$_{\pm 0.2}$	21.8$_{\pm 0.1}$	12.8$_{\pm 0.0}$	12.7$_{\pm 0.3}$	74.8$_{\pm 0.1}$
SMKKM	57.4$_{\pm 2.0}$	40.8$_{\pm 1.9}$	17.6$_{\pm 1.9}$	11.4$_{\pm 0.1}$	19.5$_{\pm 0.2}$	16.9$_{\pm 0.8}$	13.1$_{\pm 0.0}$	8.0$_{\pm 0.0}$	18.8$_{\pm 0.4}$	46.5$_{\pm 0.0}$
OPLFMVC	59.5$_{\pm 5.5}$	81.1$_{\pm 4.2}$	15.4$_{\pm 2.3}$	17.7$_{\pm 1.8}$	23.2$_{\pm 1.3}$	11.3$_{\pm 1.8}$	17.6$_{\pm 1.9}$	9.1$_{\pm 0.6}$	17.5$_{\pm 0.8}$	50.1$_{\pm 4.2}$
LSMKKM	72.0$_{\pm 6.3}$	61.6$_{\pm 1.8}$	19.9$_{\pm 1.2}$	9.9$_{\pm 0.0}$	18.5$_{\pm 0.1}$	19.7$_{\pm 1.4}$	13.8$_{\pm 0.0}$	35.0$_{\pm 0.0}$	19.1$_{\pm 0.4}$	43.1$_{\pm 0.4}$
AIMC	60.2$_{\pm 0.0}$	66.1$_{\pm 0.0}$	19.0$_{\pm 0.0}$	14.5$_{\pm 0.0}$	18.8$_{\pm 0.0}$	13.5$_{\pm 0.0}$	15.1$_{\pm 0.0}$	8.5$_{\pm 0.0}$	18.2$_{\pm 0.0}$	32.2$_{\pm 0.0}$
OMSC	55.7$_{\pm 0.0}$	74.7$_{\pm 0.0}$	15.9$_{\pm 0.0}$	11.3$_{\pm 0.0}$	26.7$_{\pm 0.0}$	20.8$_{\pm 0.0}$	9.2$_{\pm 0.0}$	7.8$_{\pm 0.0}$	16.5$_{\pm 0.0}$	39.7$_{\pm 0.0}$
HFLSMKKM	26.1$_{\pm 0.6}$	22.1$_{\pm 1.0}$	18.6$_{\pm 1.0}$	$-0.1_{\pm 0.0}$	12.4$_{\pm 0.1}$	12.0$_{\pm 0.0}$	4.0$_{\pm 0.2}$	48.8$_{\pm 0.0}$	18.3$_{\pm 0.3}$	30.4$_{\pm 0.3}$
GMC	56.7$_{\pm 0.0}$	82.6$_{\pm 0.0}$	2.9$_{\pm 0.0}$	9.8$_{\pm 0.0}$	0.2$_{\pm 0.0}$	0.0$_{\pm 0.0}$	$-0.5_{\pm 0.0}$	11.1$_{\pm 0.0}$	9.6$_{\pm 0.7}$	0.0$_{\pm 0.0}$
LTBPL	76.7$_{\pm 2.6}$	90.7$_{\pm 0.0}$	15.7$_{\pm 0.8}$	17.3$_{\pm 0.0}$	11.9$_{\pm 0.0}$	7.5$_{\pm 0.0}$	2.7$_{\pm 0.0}$	20.0$_{\pm 0.0}$	16.2$_{\pm 0.8}$	9.6$_{\pm 0.8}$
UGLTL	72.8$_{\pm 0.0}$	97.9$_{\pm 0.3}$	43.3$_{\pm 2.3}$	0.0$_{\pm 0.0}$	67.9$_{\pm 0.3}$	42.8$_{\pm 0.4}$	82.8$_{\pm 0.1}$	46.3$_{\pm 0.0}$	16.0$_{\pm 0.9}$	57.2$_{\pm 0.1}$
WTNNM	**81.9**$_{\pm 0.0}$	88.8$_{\pm 0.0}$	24.4$_{\pm 1.1}$	12.4$_{\pm 0.0}$	39.2$_{\pm 0.1}$	31.6$_{\pm 0.1}$	40.1$_{\pm 0.1}$	17.8$_{\pm 0.0}$	24.4$_{\pm 0.6}$	35.6$_{\pm 0.0}$
KCGT	71.0$_{\pm 0.1}$	77.5$_{\pm 0.0}$	**47.4**$_{\pm 3.4}$	5.5$_{\pm 0.1}$	25.3$_{\pm 0.1}$	18.3$_{\pm 0.0}$	17.2$_{\pm 0.1}$	6.1$_{\pm 0.0}$	39.4$_{\pm 1.1}$	63.1$_{\pm 0.0}$
TKKM	77.3$_{\pm 3.5}$	**99.8**$_{\pm 0.0}$	46.3$_{\pm 2.4}$	20.9$_{\pm 0.1}$	**79.0**$_{\pm 0.6}$	**95.0**$_{\pm 0.0}$	**89.2**$_{\pm 5.3}$	**60.2**$_{\pm 0.0}$	**75.2**$_{\pm 4.0}$	**94.7**$_{\pm 0.0}$

Table 6: RI (%) comparison (mean±std) on 10 benchmark datasets.

Algorithm	CCV	SUNRGBD	YoutubeFace	Reuters
Avg-KKM	$19.6_{\pm 0.6}$	$18.5_{\pm 0.5}$	$19.3_{\pm 0.7}$	$45.5_{\pm 1.5}$
SB-KKM	$20.1_{\pm 0.2}$	$17.6_{\pm 0.3}$	$16.8_{\pm 0.5}$	$47.2_{\pm 0.0}$
MKKM	$18.0_{\pm 0.5}$	$17.2_{\pm 0.6}$	$11.5_{\pm 0.2}$	$45.4_{\pm 1.5}$
LMKKM	$18.6_{\pm 0.1}$	-	-	-
ONKC	$22.4_{\pm 0.3}$	$19.8_{\pm 0.5}$	$19.4_{\pm 0.5}$	$41.8_{\pm 1.2}$
MKKM-MR	$21.2_{\pm 0.9}$	$19.5_{\pm 0.5}$	$18.9_{\pm 0.6}$	$46.2_{\pm 1.4}$
LKAM	$20.4_{\pm 0.3}$	$19.6_{\pm 0.5}$	$21.3_{\pm 1.0}$	$45.5_{\pm 0.0}$
LFMVC	$25.1_{\pm 0.5}$	$18.6_{\pm 0.6}$	$27.3_{\pm 0.4}$	$45.7_{\pm 1.6}$
NKSS	$20.0_{\pm 0.2}$	-	-	-
SPMKC	$16.2_{\pm 0.2}$	-	-	-
HMKC	$32.8_{\pm 0.5}$	$18.5_{\pm 0.3}$	$34.9_{\pm 0.6}$	$46.9_{\pm 0.4}$
SMKKM	$22.2_{\pm 0.7}$	$19.2_{\pm 0.5}$	$20.6_{\pm 0.6}$	$45.5_{\pm 0.7}$
OPLFMVC	$23.7_{\pm 0.9}$	$20.4_{\pm 0.6}$	$22.7_{\pm 1.3}$	$43.9_{\pm 1.0}$
LSMKKM	$21.5_{\pm 0.9}$	$19.4_{\pm 0.7}$	$23.1_{\pm 0.8}$	$47.1_{\pm 1.0}$
AIMC	$24.5_{\pm 0.0}$	$20.2_{\pm 0.0}$	$23.2_{\pm 0.0}$	$43.2_{\pm 0.0}$
OMSC	$25.1_{\pm 0.0}$	$20.1_{\pm 0.0}$	$22.2_{\pm 0.0}$	$42.4_{\pm 0.0}$
HFLSMKKM	$18.5_{\pm 0.3}$	$17.4_{\pm 0.2}$	$18.5_{\pm 0.3}$	$37.5_{\pm 0.8}$
GMC	$16.8_{\pm 0.4}$	$16.3_{\pm 0.1}$	-	-
LTBPL	-	-	-	-
UGLTL	$43.7_{\pm 1.3}$	$15.3_{\pm 0.8}$	$63.5_{\pm 1.3}$	-
WTNNM	$47.7_{\pm 0.0}$	-	-	-
KCGT	$39.7_{\pm 0.5}$	$22.7_{\pm 0.6}$	-	-
TKKM	$\mathbf{57.5}_{\pm 2.8}$	$\mathbf{22.8}_{\pm 1.0}$	$\mathbf{68.2}_{\pm 2.4}$	$\mathbf{95.2}_{\pm 1.9}$

Table 7: ACC (%) comparison (mean±std) on large-scale datasets.

Algorithm	CCV	SUNRGBD	YoutubeFace	Reuters
Avg-KKM	$16.8_{\pm 0.4}$	$22.6_{\pm 0.3}$	$16.8_{\pm 0.7}$	$27.4_{\pm 0.4}$
SB-KKM	$17.7_{\pm 0.1}$	$21.3_{\pm 0.2}$	$16.4_{\pm 0.3}$	$25.5_{\pm 0.0}$
MKKM	$15.0_{\pm 0.4}$	$21.2_{\pm 0.4}$	$12.1_{\pm 0.1}$	$27.3_{\pm 0.4}$
LMKKM	$14.4_{\pm 0.1}$	-	-	-
ONKC	$18.5_{\pm 0.2}$	$23.6_{\pm 0.2}$	$17.4_{\pm 0.3}$	$22.3_{\pm 0.4}$
MKKM-MR	$18.0_{\pm 0.4}$	$23.5_{\pm 0.3}$	$16.9_{\pm 0.4}$	$25.3_{\pm 0.7}$
LKAM	$17.6_{\pm 0.2}$	$23.9_{\pm 0.3}$	$19.4_{\pm 0.7}$	$29.9_{\pm 0.0}$
LFMVC	$20.1_{\pm 0.3}$	$22.6_{\pm 0.4}$	$24.5_{\pm 0.3}$	$27.4_{\pm 0.4}$
NKSS	$16.9_{\pm 0.2}$	-	-	-
SPMKC	$12.1_{\pm 0.1}$	-	-	-
HMKC	$27.6_{\pm 0.2}$	$23.2_{\pm 0.3}$	$32.6_{\pm 0.3}$	$30.4_{\pm 0.5}$
SMKKM	$18.2_{\pm 0.3}$	$23.1_{\pm 0.4}$	$18.4_{\pm 0.5}$	$27.7_{\pm 0.2}$
OPLFMVC	$18.1_{\pm 0.7}$	$21.2_{\pm 0.3}$	$20.6_{\pm 0.8}$	$24.8_{\pm 1.5}$
LSMKKM	$17.8_{\pm 0.4}$	$23.4_{\pm 0.3}$	$20.7_{\pm 0.4}$	$27.0_{\pm 0.6}$
AIMC	$19.0_{\pm 0.0}$	$20.5_{\pm 0.0}$	$20.9_{\pm 0.0}$	$24.3_{\pm 0.0}$
OMSC	$19.1_{\pm 0.0}$	$14.5_{\pm 0.0}$	$18.6_{\pm 0.0}$	$24.7_{\pm 0.0}$
HFLSMKKM	$15.1_{\pm 0.2}$	$22.9_{\pm 0.2}$	$17.6_{\pm 0.1}$	$18.6_{\pm 0.8}$
GMC	$15.6_{\pm 0.2}$	$13.2_{\pm 0.1}$	-	-
LTBPL	-	-	-	-
UGLTL	$58.9_{\pm 0.5}$	$22.6_{\pm 0.2}$	$80.9_{\pm 0.5}$	-
WTNNM	$38.9_{\pm 0.1}$	-	-	-
KCGT	$35.9_{\pm 0.3}$	$\mathbf{33.1}_{\pm 0.2}$	-	-
TKKM	$\mathbf{66.0}_{\pm 0.9}$	$31.3_{\pm 0.4}$	$\mathbf{81.1}_{\pm 0.9}$	$\mathbf{87.5}_{\pm 1.8}$

Table 8: NMI (%) comparison (mean±std) on large-scale datasets.

Algorithm	CCV	SUNRGBD	YoutubeFace	Reuters
Avg-KKM	$23.7_{\pm 0.5}$	$38.2_{\pm 0.7}$	$30.7_{\pm 0.8}$	$53.0_{\pm 0.4}$
SB-KKM	$23.3_{\pm 0.2}$	$36.5_{\pm 0.4}$	$28.6_{\pm 0.5}$	$53.9_{\pm 0.0}$
MKKM	$22.2_{\pm 0.5}$	$36.2_{\pm 0.7}$	$26.8_{\pm 0.1}$	$52.9_{\pm 0.5}$
LMKKM	$22.0_{\pm 0.1}$	-	-	-
ONKC	$24.6_{\pm 0.3}$	$39.6_{\pm 0.6}$	$30.7_{\pm 0.6}$	$52.6_{\pm 0.3}$
MKKM-MR	$23.7_{\pm 0.7}$	$39.4_{\pm 0.6}$	$30.2_{\pm 0.4}$	$52.2_{\pm 0.6}$
LKAM	$23.3_{\pm 0.2}$	$39.6_{\pm 0.4}$	$31.1_{\pm 0.8}$	$55.4_{\pm 0.0}$
LFMVC	$28.2_{\pm 0.4}$	$38.1_{\pm 0.6}$	$36.3_{\pm 0.6}$	$53.2_{\pm 0.4}$
NKSS	$23.6_{\pm 0.3}$	-	-	-
SPMKC	$20.8_{\pm 0.3}$	-	-	-
HMKC	$36.5_{\pm 0.4}$	$38.7_{\pm 0.5}$	$42.2_{\pm 0.9}$	$53.9_{\pm 0.1}$
SMKKM	$25.3_{\pm 0.5}$	$39.0_{\pm 0.6}$	$32.1_{\pm 0.5}$	$53.3_{\pm 0.0}$
OPLFMVC	$26.9_{\pm 0.8}$	$36.7_{\pm 0.6}$	$32.1_{\pm 1.0}$	$51.7_{\pm 1.2}$
LSMKKM	$24.7_{\pm 0.6}$	$38.8_{\pm 0.7}$	$32.2_{\pm 0.8}$	$52.9_{\pm 0.2}$
AIMC	$28.6_{\pm 0.0}$	$36.2_{\pm 0.0}$	$33.2_{\pm 0.0}$	$52.8_{\pm 0.0}$
OMSC	$27.9_{\pm 0.0}$	$24.5_{\pm 0.0}$	$30.4_{\pm 0.0}$	$49.8_{\pm 0.0}$
HFLSMKKM	$21.5_{\pm 0.3}$	$37.9_{\pm 0.2}$	$29.1_{\pm 0.1}$	$46.9_{\pm 0.8}$
GMC	$20.9_{\pm 0.4}$	$22.4_{\pm 0.2}$	-	-
LTBPL	-	-	-	-
UGLTL	$53.1_{\pm 0.9}$	$32.9_{\pm 0.5}$	$84.5_{\pm 0.9}$	-
WTNNM	$49.7_{\pm 0.0}$	-	-	-
KCGT	$43.4_{\pm 0.3}$	$\mathbf{46.3}_{\pm 0.5}$	-	-
TKKM	$\mathbf{62.1}_{\pm 1.7}$	$46.0_{\pm 0.7}$	$\mathbf{85.7}_{\pm 1.4}$	$\mathbf{95.3}_{\pm 1.3}$

Table 9: PUR (%) comparison (mean±std) on large-scale datasets.

Algorithm	CCV	SUNRGBD	YoutubeFace	Reuters
Avg-KKM	$6.6_{\pm 0.2}$	$8.9_{\pm 0.3}$	$3.2_{\pm 0.2}$	$21.8_{\pm 1.4}$
SB-KKM	$6.7_{\pm 0.1}$	$8.0_{\pm 0.1}$	$2.6_{\pm 0.1}$	$23.6_{\pm 0.0}$
MKKM	$5.7_{\pm 0.2}$	$8.1_{\pm 0.3}$	$1.4_{\pm 0.0}$	$21.8_{\pm 1.4}$
LMKKM	$5.6_{\pm 0.0}$	-	-	-
ONKC	$7.7_{\pm 0.1}$	$9.7_{\pm 0.2}$	$3.3_{\pm 0.1}$	$20.3_{\pm 0.3}$
MKKM-MR	$7.2_{\pm 0.3}$	$9.6_{\pm 0.3}$	$3.1_{\pm 0.2}$	$23.1_{\pm 0.6}$
LKAM	$6.9_{\pm 0.1}$	$9.9_{\pm 0.3}$	$3.8_{\pm 0.3}$	$24.1_{\pm 0.0}$
LFMVC	$9.4_{\pm 0.2}$	$9.0_{\pm 0.2}$	$6.2_{\pm 0.1}$	$22.1_{\pm 1.6}$
NKSS	$6.2_{\pm 0.2}$	-	-	-
SPMKC	$4.2_{\pm 0.1}$	-	-	-
HMKC	$14.0_{\pm 0.2}$	$8.8_{\pm 0.2}$	$9.2_{\pm 0.2}$	$22.6_{\pm 0.5}$
SMKKM	$7.5_{\pm 0.2}$	$9.4_{\pm 0.3}$	$3.7_{\pm 0.1}$	$22.1_{\pm 0.8}$
OPLFMVC	$7.9_{\pm 0.6}$	$9.9_{\pm 0.4}$	$4.8_{\pm 0.4}$	$20.6_{\pm 0.5}$
LSMKKM	$7.3_{\pm 0.3}$	$9.4_{\pm 0.3}$	$4.3_{\pm 0.3}$	$21.6_{\pm 0.2}$
AIMC	$9.0_{\pm 0.0}$	$9.6_{\pm 0.0}$	$5.0_{\pm 0.0}$	$20.0_{\pm 0.0}$
OMSC	$7.8_{\pm 0.0}$	$0.5_{\pm 0.0}$	$4.2_{\pm 0.0}$	$17.8_{\pm 0.0}$
HFLSMKKM	$6.3_{\pm 0.1}$	$8.3_{\pm 0.1}$	$3.1_{\pm 0.1}$	$13.6_{\pm 0.7}$
GMC	$5.6_{\pm 0.2}$	$0.3_{\pm 0.0}$	-	-
LTBPL	-	-	-	-
UGLTL	$35.8_{\pm 1.1}$	$7.2_{\pm 0.5}$	$39.8_{\pm 0.8}$	-
WTNNM	$26.1_{\pm 0.0}$	-	-	-
KCGT	$21.4_{\pm 0.3}$	$\mathbf{13.6}_{\pm 0.2}$	-	-
TKKM	$\mathbf{48.6}_{\pm 2.4}$	$13.2_{\pm 0.5}$	$\mathbf{44.2}_{\pm 2.2}$	$\mathbf{90.9}_{\pm 2.9}$

Table 10: RI (%) comparison (mean±std) on large-scale datasets.